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A Reconsideration of the Stiglitz-Weiss Model with a Discrete Number of Borrower Types

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Abstract

In this paper we show that the equilibrium in the Stiglitz-Weiss model (Stiglitz and Weiss, 1981) is a two-interest rate equilibrium. For this we use the true return-function for banks shown by Arnold (2005), the assumption of Bertrand competition and make a consideration for a discrete number of borrowers. Rationing only affects one group of the borrowers, i.e. the borrowers with a safe project. The risky group always receives the funds it demands.

Key word: credit rationing, asymmetric information, adverse selection

JEL classification: D82, E51, G21

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1 Introduction

In their seminal paper, Stiglitz and Weiss (1981) posed the question: “Why is credit rationed?”. More generally, they initiated a wide discussion of information asymmetries, especially in credit markets. They describe a model of adverse selection and credit rationing with imperfect information. Their paper has had a wide influence on the development of models with credit rationing, the sorting mechanism and the role of collateral as an incentive device (Bester, 1985), and the macroeconomic implications of capital market imperfections (Greenwald and Stiglitz, 1987).

In a perfect capital market with all information available to everyone banks give risk adjusted credits to borrowers of different types. Banks choose in a perfect competition the interest rate such that they achieve zero profits in equilibrium.

Stiglitz and Weiss (1981) by contrast consider an imperfect credit market in which banks cannot observe the types of the borrowers. With two borrower types, that means that a bank does not know whether a safe or a risky borrower is applying for credit. Stiglitz and Weiss (1981) assume that all borrower types have the same expected return, but riskier projects offer a higher return in case of success, at the cost of a lower probability of success compared to safe projects. Banks lend using standard debt contracts. At each interest rate, the expected profit is higher for the risky borrowers than for the safe borrowers. Therefore, the risky borrowers are willing to pay a higher interest rate and still make non-negative profit. So it would be effective for a bank to charge lower interest rates from the safe borrowers and higher interest rates from the riskier investors. But because banks lack the knowledge of the risk types, they set a common interest rate for both borrower classes, which yields zero profit. In an equilibrium with credit rationing à la Stiglitz and Weiss (1981), banks give credit to both risk types at a common interest rate, demand exceeds supply, and the borrowers who get funds are selected randomly. Safe borrowers effectively cross-subsidize risky borrowers. No credit is given at any other interest rate: “Potential borrowers who are denied credits would not be able to borrow even if they indicate a willingness to pay more than the market interest rate...” (Stiglitz and Weiss, 1981, p. 408). That means both safe and risky borrowers are rationed.

The banks’ return function, which is defined as the rate of return on lending, has two local maxima, namely at the two interest rates at which the respective borrower types drop out of the market. In the construction of a rationing equilibrium, Stiglitz and Weiss (1981) assume that the global maximum occurs at the lower of these two interest rates, i.e. when the safe borrowers cease to demand capital. Because of perfect competition, at the return-maximizing interest rate banks make zero profit. Most of the literature takes this assumption as given like Blanchard and Fisher (1989), Hillier and Ibrahimo (1993) or De Meza and Webb (1987). Some economists (Hillier (1997) and Walsh (2001)) assume the correct return function (explicitly shown by Arnold (2005)) in the case of a discrete number of borrower types, but they do not consider eventual further interest rates at the

equilibrium. Hillier (1997) is focused on the adverse selection of the borrowers and on the credit rationing interest rate, known by Stiglitz and Weiss (1981). He mentions that banks only give credit at the first maximum interest rate, because they do not want to lose the borrowers with the safe projects. Hillier (1997) also considers the second interest rate and names the possibility of a second interest rate in the equilibrium with the result that only safe borrowers are rationed. But he denoted the second interest rate as “unusual” and mentioned furthermore that the second interest rate solely occurs if the market is inefficient. Walsh (2001) examines two borrowers and many banks with the true return function. However, he concentrates only on the lower equilibrium interest rate, but not on the possible and also shown, second higher equilibrium interest rate.

Arnold (2005) demonstrates, however, that actually the return function generally attains its global maximum at the higher of the two interest rates under the maintained assumptions. This has important consequences. Suppose there is double Bertrand competition in the markets for both credit and deposits (cf. Stahl (1988) and Yanelle (1989)). Then, if there is excess demand for credit at the lower rate, it is profitable to give a small amount of credit at an interest rate close to the return-maximizing interest rate. So there is a second, higher, interest rate at which the banks’ return is equal to that belonging to the standard equilibrium interest rate. Suppose that residual supply equals residual demand at the higher interest rate. Then this two-interest rate allocation is the equilibrium of the Stiglitz and Weiss (1981) model with two borrower types.¹ Since only risky borrowers apply for credit at the higher interest rate, the cross-subsidization of risky borrowers is not existing at this interest rate, but continues to be present at the lower rate.

Whether or not credit rationing prevails in such a two-interest rate equilibrium becomes a matter of definition. Risky borrowers get the funds they demand, though some of them have to pay a higher interest rate than others. Safe borrowers, which are rationed at the first interest rate, would also be served at the higher second interest rate, but choose not to apply for funds. Hence, we are talking of the rationing of the safe borrowers.

In the following section two we show our model following Stiglitz and Weiss (1981) and the modification of the return function according to Arnold (2005). In the third section we consider the equilibrium and we conclude in the fourth section.

2 The Model Setup

We consider an economy in two periods. In the first period the indivisible investment, B , is made. In the second period the investor² receives a payoff $R_i \geq 0$. We consider a model with two types of borrowers, the safe type denoted as $i = 1$ and the risky type characterized as $i = 2$. Furthermore we denote the number of safe investors as N_1 and the number

¹An analogous result for the Stiglitz-Weiss model with a continuum of borrower types is shown in Arnold (2005).

²We will use the terms firm, borrower, and investor interchangeably in this paper.

of the risky borrowers as N_2 . Each of these borrowers has one project to undertake. If the projects succeed, the return for type 1 is R_1 and for type 2 it is R_2 . We assume the payoff is zero if the projects fail. In this situation the investor loses her collateral C with $0 < C < B$.³ The probabilities for success are p_1 and p_2 with $0 < p_2 < p_1 < 1$.⁴ With $1 - p_1$ and $1 - p_2$, respectively, the project fails. Both projects have the same expected return, i.e. $E(R_i) = p_i R_i = \bar{R}$. So from $p_1 > p_2$ it follows that $R_1 < R_2$. Investors have to pay B with $\bar{R} > B$ for the project and they can get the money solely from the banks.

We assume at least four⁵ banks in the model, $k \in \mathcal{K} = \{1, \dots, K\}$, $4 \leq K \leq \infty$. A bank may give credit and eventually collects deposits. The banks and the investors are risk neutral and have rational expectations. Furthermore we assume the banks are engaged in a Bertrand competition, i.e. they compete in prices.

In the following we consider the credit and the deposit subgames. In order to get a competitive solution the credit subgame precedes the deposit subgame (cf. Stahl (1988)).

We consider the credit subgame at first. The market is characterized by asymmetric information. This means the banks have no information about the risk type of the borrower and give credit using standard debt contracts. Interest rates cannot be conditioned on the firm's risk type (Stiglitz and Weiss, 1981). The borrowers start applying for credit at the lowest interest rate. If they do not receive a credit they try to get credit at the next possible, higher, interest rate in the market, given that their expected profit is non-negative. If the borrower gets credit, she has to give collateral C to the bank. The bank k acts as shown in Arnold (2005) with the tuple $(r_k, \lambda_k) (\geq (0, 0))$, where r_k denotes the interest rate and λ_k characterizes the credit ceiling up to which the bank is willing to lend at interest rate r_k . If there are more than one bank offering credit at the same interest rate, the one with the highest λ_k serves the whole demand. If several banks bid the same λ_k , the bank which serves the market is chosen by a random tie-breaking rule with the same probability for each bank. So at each rate only one bank supplies the borrowers with capital. If λ_k of the chosen bank is not high enough to satisfy the whole demand, credit is rationed at this interest rate. This rationing is random, because banks cannot observe the type of the borrower. Credit given at an interest rate r_n is denoted as L_{r_n} .

Subsequently the deposit subgame takes place. We denote the supply of deposits with $L^s(\rho)$. This supply is strictly increasing in deposit rates. The banks act with the tuple $(\rho_k, \delta_k) (\geq (0, 0))$ as shown in Arnold (2005). The variable ρ_k denotes the deposit rate the bank is willing to pay and δ_k is the quantity of deposits the bank is willing to collect.⁶ The demand for deposits is served in order of decreasing deposit rates. If the depositor cannot

³Clearly this implies also $0 < C < (1 + r)B$ for $r \geq 0$.

⁴Note that with the notation "safe" we do not mean that the project's probability of success has to be equal to 1. It only means that the first type's probability of success is higher than that of type 2.

⁵Two banks offering multiple credit contracts would also do, but to assume there are four banks makes the consideration easier and the issue clearer.

⁶Because of the timing of this game, i.e. the deposit subgame following the credit subgame, a bank would just collect as much as it is lending as credits.

place the desired amount at the bank, she has to decide either to save at the next lower rate or not at all. If supply equals or exceeds demand at a deposit rate ρ_k , every bank collects the demanded amount of deposits. In the case of excess supply the depositors are randomly rationed. If there is higher demand than supply of deposits, then each bank receives an amount proportional to the quantity δ_k it demands. Additionally we assume that there is no secondary market for deposits at which a bank has the possibility of refinancing. If a bank fails to refinance this leads to an infinitely negative profit, i.e. $\pi \rightarrow -\infty$. The bank promises an amount of credit to the firms and could not collect the capital it needs to distribute the promised amount of capital. The other way around, collecting an amount of deposits without giving credit, leads to a negative profit, which is not infinite. The loss equals the amount of the deposit rates the bank has to pay to the depositors. In our model, however, we assume that a bank never fails to refinance the credit, if it offers a sufficiently high deposit rate.

3 Equilibrium

3.1 The Supply Function of the Borrowers

For the specification of the residual supply we refer to Arnold (2005) as nothing changes in the deposit subgame if we examine two types of borrowers instead of a continuum.

3.2 The Demand Function of the Borrowers

Because of the fact that we just consider two types of borrowers we have a different demand function for capital compared to Arnold (2005).

The profit of an investor is denoted by $\pi(R_i, r) = \max \{R_i - (1 + r)B, -C\}$ with $i = 1, 2$. The borrowers apply for credit if their expected profit is non-negative, i.e. if

$$E[\pi(R_i, r)] = \bar{R} - p_i(1 + r)B - (1 - p_i)C \geq 0. \quad (3.1)$$

We see that $E[\pi(R_2, r)] > E[\pi(R_1, r)]$ if, and only if,

$$\begin{aligned} \bar{R} - p_2(1 + r)B - (1 - p_2)C &> \bar{R} - p_1(1 + r)B - (1 - p_1)C \\ (1 + r)B &> C. \end{aligned} \quad (3.2)$$

And this is true for all non-negative r by the assumption $0 < C < B$ made above. From equation (3.1) it follows that the borrower types demand capital if

$$r \leq \begin{cases} \frac{\bar{R} - (1 - p_1)C}{p_1 B} - 1 : & \text{safe borrowers demand capital,} \\ \frac{\bar{R} - (1 - p_2)C}{p_2 B} - 1 : & \text{risky borrowers demand capital.} \end{cases} \quad (3.3)$$

To describe the form of the demand function we look under which parameter combinations the critical interest rate for the safe borrower r^* is smaller than the critical interest rate for the risky borrower r^{max} . This gives

$$r^* < r^{max} \quad (3.4)$$

$$\Leftrightarrow \frac{\bar{R} - (1 - p_1)C}{p_1 B} - 1 < \frac{\bar{R} - (1 - p_2)C}{p_2 B} - 1 \quad (3.5)$$

$$\Leftrightarrow (p_1 - p_2)C < (p_1 - p_2)\bar{R}. \quad (3.6)$$

With the assumption $p_1 > p_2$ it follows

$$C < \bar{R}. \quad (3.7)$$

This condition is fulfilled for all admissible C and \bar{R} , because by assumption $\bar{R} > B$ and $B > C$. Hence in the interval between $r = 0$ and $r = r^*$ all borrowers demand credit. At the point r^{max} the following equation for the expected profit of a risky investor holds

$$E[\pi(R_2, r^{max})] = p_2[R_2 - (1 + r^{max})B] - (1 - p_2)C = 0. \quad (3.8)$$

It becomes clear that r^{max} is the highest possible interest rate at which borrowers apply for credit. Safe borrowers would not apply at this interest rate anymore as can be seen from equation (3.3).

It follows that the demand of capital, denoted as $L^D(r)$, is constant in two intervals. In the interval $[0, r^*]$ all borrowers, safe and risky, apply for credit. In the interval $(r^*, r^{max}]$ only risky borrowers apply. At interest rates above r^{max} also risky borrowers do not apply anymore, because the expected profit of the riskier project is then less than zero. Thus the demand function is constant in two intervals and the demand is higher in the first interval than in the second.

If we consider the development of the residual demand, demand only changes at r^* and r^{max} if no credit is given. The residual demand is denoted by $l_{r_n}^D(r_n)$.

Lemma 1 *If no credit is given below r^* , the residual demand at r^* satisfies*

$$l_{r^*}^D = L^D(r^*). \quad (3.9)$$

*The residual demand at r^{**} with L_{r^*} denoting credit given at r^* is*

$$l_{r^{**}}^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)} \right] L^D(r^{**}). \quad (3.10)$$

Let $r^* < r^{**}$ and $r' < r^{**}$, then the residual demand at r^{**} is given by

$$l_{r^{**}}^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)} - \frac{L_{r'}}{L^D(r')} \right] L^D(r^{**}). \quad (3.11)$$

Equations (3.9) and (3.10) are obvious. To prove equation (3.11) we have to consider two cases, i.e. $r^* < r' < r^{**}$ and $r' < r^* < r^{**}$. First suppose $r^* < r' < r^{**}$. The residual demand is, using equation (3.10) from lemma 1, given by

$$l_{r'}^D(r') = \left[1 - \frac{L_{r^*}}{L^D(r^*)} \right] L^D(r'). \quad (3.12)$$

The residual demand at r^{**} is given by

$$l_{r^{**}}^D(r^{**}) = \left[1 - \frac{L_{r'}}{l_{r'}^D(r')} \right] l_{r'}^D(r^{**}). \quad (3.13)$$

with $l_{r'}^D(r^{**})$ denoting the residual demand at r^{**} when all applications for credit at r' are withdrawn such that

$$l_{r'}^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)} \right] L^D(r^{**}). \quad (3.14)$$

Now inserting equation (3.14) and equation (3.12) into equation (3.13) gives

$$l_{r^{**}}^D(r^{**}) = \left[1 - \frac{L_{r'}}{\left[1 - \frac{L_{r^*}}{L^D(r^*)} \right] L^D(r')} \right] \left[1 - \frac{L_{r^*}}{L^D(r^*)} \right] L^D(r^{**}). \quad (3.15)$$

Rearranging implies equation (3.11), such that

$$l_{r^{**}}^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)} - \frac{L_{r'}}{L^D(r')} \right] L^D(r^{**}).$$

This proof is graphically shown in figure 1. The proof of the second case with $r' < r^* < r^{**}$ is analogous to the first one. Q.E.D.

If a borrower doesn't receive credit at the interest rate she is applying for, she can apply at the next higher interest rate. But if the expected returns of her project are not high enough, i.e. the credit at the next higher interest rate would lead to a loss, she does deny applying. There are two of such critical interest rates in our model. The first is r^* , above which all the safe borrowers stop applying for a credit. The second is r^{max} , above which the risky borrowers also stop demanding credit.

For example 100 borrowers apply for credit at r^* . If at this rate credit demand is higher than supply, for instance only 20% could be given as credit. So only 20% of the borrowers

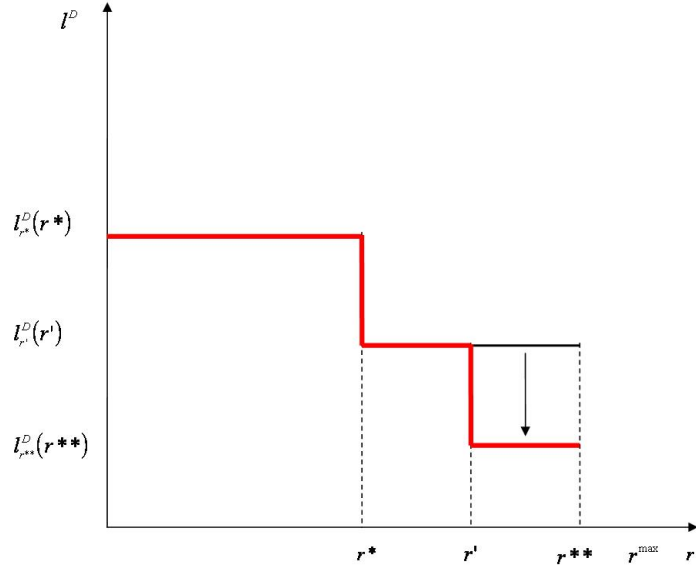


Figure 1: The Residual Demand at r^{**} in the Case $r^* < r' < r^{**}$

receive their full demand B .⁷ The borrowers who are not served at this interest rate, can apply at the next higher interest rate for the residual amount of capital or stop demanding credit. However, if 30 (safe) borrowers of the origin 100 borrowers are not able to apply at the next higher interest rate, only 70 borrowers apply at the next interest rate. And of these 70 (risky) borrowers 20% already had received the full credit at r^* . Thus residual demand in the interval $(r^*, r^{**}]$ is $l^D(r^{**}) = [1 - \frac{20}{100}] 70 = 56$. This Adverse Selection effect is shown in equation (3.10). Accordingly we can explain equation (3.11).

If $r' < r^*$ we can write the residual demand at r^* as

$$\begin{aligned} l_{r^*}^D(r^*) &= \left[1 - \frac{L_{r'}}{L^D(r')} \right] L^D(r^*) \\ &= L^D(r^*) - L_{r'}. \end{aligned} \tag{3.16}$$

This is valid because of $L^D(r') = L^D(r^*)$. If $r' \in (r^*, r^{**})$, i.e. credit given at r^* and r' , residual demand at r^{**} satisfies

⁷It is not possible that each borrower receives 20% of the demanded amount B . If that would be the case and in equilibrium each safe borrower would only receive 20% of B , she could not invest in any project, because she needs B completely. Hence, either a borrower receives all or nothing.

$$\begin{aligned}
l^D(r^{**}) &= \left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}) - L_{r'} \\
&= l^D(r') - L_{r'},
\end{aligned} \tag{3.17}$$

because demand at r' is equal to the demand at r^{**} , $L^D(r') = L^D(r^{**})$.

3.3 The Behavior of Banks

The repayment the bank receives per unit credit given is

$$\min\{(1+r)B, R_i + C\} = R_i - \pi(R_i, r). \tag{3.18}$$

Rearranging terms in equation (3.18) shows that the return of any project can be separated into two parts

$$R_i = \min\{(1+r)B, R_i + C\} + \pi(R_i, r). \tag{3.19}$$

The first part on the RHS of equation (3.19) is the repayment the bank receives. The second term is the profit of the investor. So the conclusion is clear, the more the bank gets the less the investor receives and vice versa.

Proposition 1 *There is a $r^{**} > r^*$ such that $\varrho(r^{**}) = \rho^*$ and $\varrho(r) > \rho^*$ for $r > r^{**}$.*

The banks cannot distinguish between the types of investors, but they know that only investors $E[\pi(R_i, r)] \geq 0$ with $i = 1, 2$ apply for credits. The aggregate expected repayment for all banks is given by

$$E(\pi_b) = \sum_{i=1}^2 N_i [p_i(1+r)B + (1-p_i)C] = \sum_{i=1}^2 N_i \{\bar{R} - E[\pi(R_i, r)]\}. \tag{3.20}$$

Considering only the interval $r \leq r^*$ gives

$$\begin{aligned}
E(\pi_b|r \leq r^*) &= (N_1 + N_2)\bar{R} - N_1[\bar{R} - p_1(1+r)B - (1-p_1)C] \\
&\quad - N_2[\bar{R} - p_2(1+r)B - (1-p_2)C] \\
&= N_1[p_1(1+r)B + (1-p_1)C] + N_2[p_2(1+r)B + (1-p_2)C].
\end{aligned} \tag{3.21}$$

However, the return function of a bank, denoted as $\varrho(r)$ is then given by

$$\begin{aligned}
\varrho(r|r \leq r^*) &= \frac{E(\pi_b|r \leq r^*)}{(N_1 + N_2)B} - 1 \\
&= \frac{\sum_{i=1}^2 N_i \{\bar{R} - E[\pi(R, i)]\}}{(N_1 + N_2)B} - 1 \\
&= \frac{N_1[p_1(1+r)B + (1-p_1)C] + N_2[p_2(1+r)B + (1-p_2)C]}{(N_1 + N_2)B} - 1. \tag{3.22}
\end{aligned}$$

Until $r = r^*$ all borrower types demand credit. The return function $\varrho(r)$ is increasing in the interest rate. Hence, the bank's return function in the interval $r \leq r^*$ attains its highest value at r^* . We denote this return $\varrho(r^*) = \rho^*$. Above r^* the safe borrowers' expected profit becomes negative and they leave the market. Only risky borrowers demand credit, because they can still achieve positive expected profits. So in the interval $r^* < r \leq r^{max}$ the expected repayment of the bank is given by

$$\begin{aligned}
E(\pi_b|r^* < r \leq r^{max}) &= N_2\bar{R} - N_2[\bar{R} - p_2(1+r)B - (1-p_2)C] \\
&= N_2[p_2(1+r)B + (1-p_2)C]. \tag{3.23}
\end{aligned}$$

At infinitesimal larger r than r^* the return function jumps down discontinuously to the value

$$\begin{aligned}
\varrho(r|r^* < r \leq r^{max}) &= \frac{E(\pi_b|r^* < r \leq r^{max})}{N_2B} - 1 \\
&= \frac{\bar{R} - E[\pi(R, i)]}{B} - 1 \\
&= \frac{p_2(1+r)B + (1-p_2)C}{B} - 1. \tag{3.24}
\end{aligned}$$

The return function is then again continuous and increasing in r until $r = r^{max}$. At $r = r^{max}$ the expected profit of the risky borrowers equals zero. At $r = r^{max}$ the return of the banks is $\varrho(r^{max}) = \bar{R}/B - 1$. This return for the bank is higher than the return shown in equation (3.22). The expected profit of the risky borrowers is zero, safe are not in the market anymore, and with equation (3.18) we see that the bank receives the whole return of the project.

At interest rates $r > r^{max}$ no borrower demands credit anymore. Since the banks' return function achieves its highest level at the maximum interest rate, there is an interest rate, denoted as r^{**} , in the interval $r^* < r \leq r^{max}$, at which the return equals ρ^* . Hence, $\varrho(r^*) = \varrho(r^{**}) = \rho^*$. At interest rates $r^{**} < r \leq r^{max}$ banks get higher returns than in the interval $r \leq r^*$. The return function of banks is drawn in figure 2.

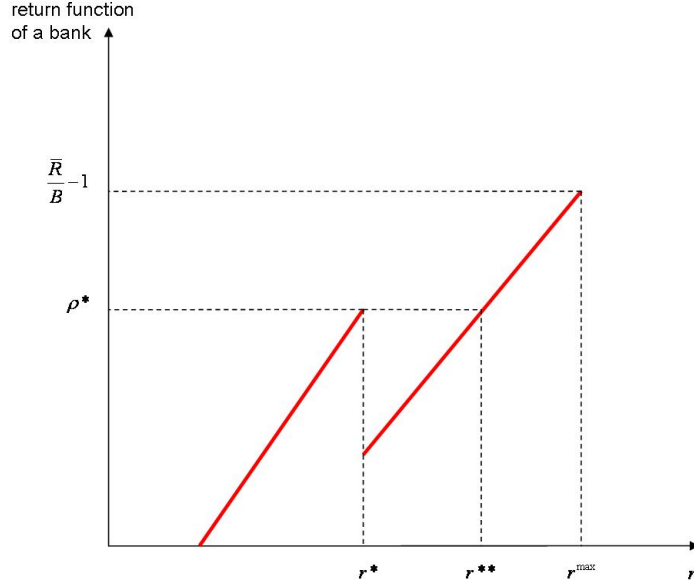


Figure 2: Return Function of a Bank

3.4 Subgame Perfect Equilibrium

Depending on the levels of credit supply at r^* and demand until and above r^* we observe different equilibria. If $L^S(\rho^*) > L^D(r^*)$ or $0 < L^S(\rho^*) < L^D(r^{**})$ we result in a market clearing equilibrium at interest rates $r < r^*$ with $\rho < \rho^*$ or at $r > r^*$ with $\rho > \rho^*$, respectively. As we concentrate our analysis of the equilibrium in this model on a equilibrium with credit rationing, we therefore assume like Stiglitz and Weiss (1981), that $L^D(r^{**}) < L^S(\rho^*) < L^D(r^*)$.

As a bank receives at interest rates $r \geq r^{**}$ at least the same return as at r^* , it follows that it is not possible that borrowers are only served at r^* , where they are rationed. The reason is that a bank could, due to positive residual demand at interest rates above r^* , achieve a positive expected profit by offering a small amount of credit close to the return maximizing interest rate, r^{max} , for example.

Instead, we consider the allocation at which banks raise deposits $L^S(\rho^*)$ by paying the deposit rate ρ^* to depositors. Suppose credit is only given at r^* and r^{**} and demand exceeds supply at the lower interest rate, but not at the higher rate, $L^D(r^{**}) < L^S(\rho^*) < L^D(r^*)$. Suppose further that credit given at r^* , L^* , is such that credit given at r^* and r^{**} together equals $L^S(\rho^*)$, where at r^{**} residual demand equals residual supply

$$L^* + \left[1 - \frac{L^*}{L^D(r^*)}\right] L^D(r^{**}) = L^S(\rho^*). \quad (3.25)$$

Thus banks give credit L^* at r^* and $L^S(\rho^*) - L^*$ at r^{**} and receive the return $\varrho(r^*) = \varrho(r^{**}) = \rho^*$ from borrowers at both interest rates. Recall that we assumed that the credit subgame precedes the deposit subgame and that at least two banks per interest rate r^* and r^{**} offer credit. The following theorem states that the two-interest rate allocation is an equilibrium of the model.

Theorem 1 *In the credit subgame, at least two banks set $(r_k, \lambda_k) = (r^*, L^*)$, while at least two other banks set $(r_k, \lambda_k) = (r^{**}, L^S(\rho^*) - L^*)$. In the deposit subgame the bank that serves the credit demand at r^* sets $(\rho_k, \delta_k) = (\rho^*, L^*)$ and the bank that gives credit at r^{**} sets $(\rho_k, \delta_k) = (\rho^*, L^S(\rho^*) - L^*)$. For all other banks, which are not giving credit, choosing $(\rho_k, \delta_k) = (0, 0)$ is optimal.*

We prove the theorem by showing that it is not profitable for any bank to deviate from these strategies in the deposit or in the credit subgame. We solve the model backwards and thus start with the deposit subgame.

In the deposit subgame we have to take into account the fact, that the amount of deposits a bank k_i has to raise to refinance its credit given l_{k_i} is already determined in the credit subgame. Suppose that a deposit rate, denoted as $\tilde{\rho}$, exists such that the amount of deposits raised by paying $\tilde{\rho}$ exactly suffices to refinance aggregate credit given.⁸ It is an optimal strategy for all banks k_i to set $(\rho_{k_i}, \delta_{k_i}) = (\tilde{\rho}, l_{k_i})$ in the deposit subgame. This is because $\rho_{k_i} < \tilde{\rho}$ or $\delta_{k_i} < l_{k_i}$ leads default and thus banks would have to face an infinite high loss, $\pi_{k_i} \rightarrow -\infty$. On the other hand, by bidding $\rho_{k_i} > \tilde{\rho}$ or $\delta_{k_i} > l_{k_i}$ banks would indeed be able to refinance. However, this would raise the cost of refinancing to $(1 + \rho_{k_i})d_{k_i}$. Since revenue is determined in the credit subgame, this is not profitable. It follows that the deposit subgame strategies described in the theorem represent a Nash equilibrium.

Now consider the credit subgame. To analyze the equilibrium of this game, we take the optimal strategies in the deposit subgame as a given. We show that it does not yield positive expected profits if banks deviate from the given strategies.

The highest credit ceiling initially set by at least one bank at interest rate r_n is denoted as $\bar{\lambda}_{r_n}$. Because of the random tie-breaking rule one bank gives credit L^* at r^* and another bank lends $L^S(\rho^*) - L^*$ at r^{**} . Suppose that the corresponding highest credit ceilings ($\bar{\lambda}_{r^*} = L^*$ and $\bar{\lambda}_{r^{**}} = L^S(\rho^*) - L^*$, respectively) are initially offered by at least two banks at each interest rate. Clearly, at all other interest rates no bank offers a credit ceiling as credit is only given at r^* and r^{**} . In the following we use the symbol Δ to denote a change in the variable.

Now we consider what happens to aggregate credit given if a bank chooses one of the following strategies:

1. A bank makes an additional credit supply at an interest rate r' , at which no credit is given before. The previous given credit ceilings persist.

⁸Note that $\tilde{\rho}$ does not necessarily have to equal ρ^* . E.g., if credit is given at $r' < r^*$ additionally to the strategies described in the theorem, then $\tilde{\rho} > \rho^*$.

2. A bank offers a higher credit ceiling $\Delta\bar{\lambda} > 0$ than each existing credit ceiling at interest rate r^* or r^{**} .

The question is how the aggregate credit given $\Delta(\sum_n L_{r_n})$ is effected by these strategies.

Lemma 2 *At new interest rates below r^* or in the situation at which more credit is supplied at r^* , additional credit given is created. At all other interest rates above r^* no additional aggregate credit given is created.*

The following figure 3 and table 1 show the five different cases that occur using the two strategies mentioned above. Residual demand implies if $\Delta\bar{\lambda}_{r'} > 0$ then $\Delta L_{r'} > 0$. This

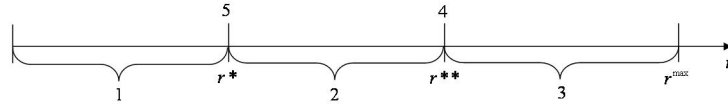


Figure 3: Illustration of the Possible Cases

$\bar{\lambda}_{r_n} \setminus r_n$	$r' < r^*$	r^*	$r^* < r' < r^{**}$	r^{**}	$> r^{**}$
initial case	0	L^*	0	$L^S(\rho^*) - L^*$	0
1	> 0	L^*	0	$L^S(\rho^*) - L^*$	0
2	0	L^*	> 0	$L^S(\rho^*) - L^*$	0
3	0	L^*	0	$L^S(\rho^*) - L^*$	> 0
4	0	L^*	0	$> L^S(\rho^*) - L^*$	0
5	0	$> L^*$	0	$L^S(\rho^*) - L^*$	0

Table 1: The Five Cases

leads either to $\Delta(\sum_n L_{r_n}) > 0$ or to $\Delta(\sum_n L_{r_n}) = 0$, as we will explain below. If there is no residual demand $\Delta\bar{\lambda}_{r_n} > 0$ implies $\Delta L_{r'} = 0$ and thus $\Delta(\sum_n L_{r_n}) = 0$. Now we look at the first strategy. A bank offers credit at an interest rate, at which no credit is given before.

If we consider interest rates r' that are lower than r^* , case 1, we can assume $L^D(r') = L^D(r^*)$. The demand of credit is constant in this interval. Here we split our consideration into three subcases depending on the amount of the new credit offered.

The additional supply at r' is offered in such an amount that there is still residual demand at r^{**} , i.e. $\lambda_{r'} < L^D(r^*) - L^*$. Hence credit is given at three interest rates, r' , r^* , and r^{**} . Credit given at the three interest rates is then $L_{r'} = \lambda_{r'}$, L^* at r^* , and at r^{**} credit given equals residual demand, as the credit ceiling offered at that interest rate is still $\bar{\lambda}_{r^{**}} = L^S(\rho^*) - L^*$ and because of the additional credit supply at r' it exceeds residual

demand

$$\begin{aligned}
L_{r^{**}}^{new} = l_{new}^D(r^{**}) &= \left[1 - \frac{L_{r'} + L^*}{L^D(r^*)}\right] L^D(r^{**}) \\
&< \\
\bar{\lambda}_{r^{**}} = L^S(\rho^*) - L^* &= \left[1 - \frac{L^*}{L^D(r^*)}\right] L^D(r^{**}) = L_{r^{**}}^{old}.
\end{aligned} \tag{3.26}$$

Comparing new and old credit given at all interest rates yields an increase in aggregate credit given

$$\Delta \sum_n L_{r_n} = L_{r'} \left[1 - \frac{L^D(r^{**})}{L^D(r^*)}\right] > 0. \tag{3.27}$$

If there is no excess demand at r^* , the credit ceiling offered at r' , $\lambda_{r'}$, has to satisfy the following condition

$$L^D(r^*) - L^* \leq \lambda_{r'} < L^D(r^*). \tag{3.28}$$

Hence, all the residual demand is satisfied at r^* . At interest rates above r^* no residual demand is existing anymore and so no credit is given there. That can be seen in equation (3.26) in which the fraction in brackets is equal to one and so the bracket equals zero. In this case credit is given at two interest rates, $L_{r'}$ at r' and $L_{r^*} \leq L^*$ at r^* . Aggregate credit given equals $L^D(r^*)$ which is higher than aggregate credit given before, $L^S(\rho^*)$. Adverse selection does not take place. All borrowers receive the amount of capital they demand. Hence, credit given rises, i.e. $\Delta L_{r'} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0$, compared to the initial situation with credit rationing.

If the new credit ceiling equals or exceeds demand at r' which occurs if $\lambda_{r'} \geq L^D(r^*)$, there is only one interest rate at which credit is given, i.e. r' . Clearly, this has exactly the same consequences for aggregate credit given as the last subcase.

Summarizing the first case yields the result that a bank increases aggregate credit given if it offers an additional supply at interest rates below r^* , i.e. $\Delta L_{r'} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0$. The reason for that is the positive residual demand that initially existed in this interval and the persisting credit offers at r^* and r^{**} . With the new lower interest rate the bank attracts all borrowers and credit rationing is reduced.

In case 2 we now look at an additional rate with $r^* < r' < r^{**}$. We again distinguish if supply is smaller, equal or bigger than residual demand. Additionally we have to take care like in the former case that the demand in this interval is constant.

So if the additional supply at r' falls short of residual demand at this interest rate, i.e. if $\lambda_{r'} < L^S(\rho^*) - L^*$, credit is given at three interest rates. The residual demand at higher rates decreases using equation (3.17) at exactly the same amount as additional credit is supplied, i.e. $\Delta l^D(r) = -L_{r'}$ for $r' < r \leq r^{**}$. With the additional credit supply a bank can only

attract borrowers of the risky type, because safe borrowers do not demand capital in this interest rate interval. At r^{**} no excess demand exists and no selection process is working. So no additional aggregate credit is created. Credit given is redistributed. Some of the risky borrowers just do have to pay a lower interest rate than initially, while nothing changes for the safe borrowers. Hence, $\lambda_{r'} > 0$ implies $\Delta L_{r'} > 0$ and it follows $\Delta(\sum_n L_{r_n}) = 0$.

In the other two cases, i.e. supply is equal to or higher than the residual demand at r' , the whole residual demand is satisfied at the new interest rate and no residual demand is existing at higher interest rates. Credit is then given at two interest rates, at r^* and r' . Also here no additional credit given is created.

If we look at interest rates with $r^{**} < r' < r^{max}$, case 3, the same result prevails, no additional credit is created. But here the reason is different. Above r^{**} no residual demand exists. Credit ceilings offered there cannot cause additional credit given, i.e. $\lambda_{r'} > 0 \Rightarrow \Delta L_{r'} = 0 \Rightarrow \Delta(\sum_n L_{r_n}) = 0$.

Now consider the second strategy where we assume that additional credit is supplied at the interest rate r^* or r^{**} .

In case 4 a bank offers a higher credit ceiling at r^{**} . This bank then becomes the bank with the highest credit ceiling at this interest rate. At r^{**} by assumption, no excess demand exists. So no additional credit given can be created. The bank simply adopts the role of the bank which was giving credit at r^{**} .

$$\Delta \bar{\lambda}_{r^{**}} > 0 \Rightarrow \Delta L_{r^{**}} = 0 \Rightarrow \Delta \left(\sum_n L_{r_n} \right) = 0. \quad (3.29)$$

Now let us consider the case 5 in that a bank offers an additional credit at r^* . Firstly, suppose the increased credit ceiling the bank offers is still smaller than the residual demand at r^* , $(1 + \Delta) \bar{\lambda}_{r^*} < l^D(r^*) = L^D(r^*)$. Clearly at r^* more credit is given because of excess demand, but to see what happens with aggregate credit given, we have, as in case one, to consider if credit at r^{**} changes and by how much. Credit is still given at two interest rates, $L_{r^*} = (1 + \Delta)L^*$ at r^* , and at r^{**} credit given is

$$L_{r^{**}} = l^D(r^{**}) = \left[1 - \frac{(1 + \Delta)L^*}{L^D(r^*)} \right] L^D(r^{**}). \quad (3.30)$$

The value in brackets is greater than zero and smaller than one. So if the bank increases the credit ceiling at r^* such that credit given at r^* rises by ΔL^* , then the residual demand decreases but by less than ΔL^* . That means the decrease in the residual demand does not compensate the whole increase of credit given at r^* . This is again a hint on the fact that only safe borrowers are rationed. So there is an increase in total credit given on the market, i.e. $\Delta L_{r^*} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0$.

Second, if the new highest credit ceiling at r^* equals or exceeds demand, credit is only given at that interest rate and no borrower is rationed, i.e. $L_{r^*} = L^D(r^*)$ and thus

$$\Delta L_{r^*} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0.$$

Hence, a bank increases aggregate credit given if it offers a higher credit ceiling at the first equilibrium interest rate, r^* . Q.E.D.

To find out, if any of the cases above appears in equilibrium, we have to consider the expected profit of the bank. Therefore we have to look at the deposit rate that the bank has to pay to its depositors. We have to keep in mind that the bank k_i is chosen by a random tie-breaking rule if more than one bank offers the same credit ceiling at one interest rate. So we have to form a function that shows the probability for a bank k_i to be chosen.

$$\psi_k = \begin{cases} 1, & \text{if } \lambda_{k_i} > \lambda_{k-1}, \text{ i.e. bank } k_i \text{ offers more credit than each other bank at } r_n \\ \frac{1}{k}, & \text{if } \lambda_{k_i} = \lambda_{k-1}, \text{ i.e. bank } k_i \text{ offers beside } k-1 \text{ other banks the highest } \lambda_{k_i} \\ 0, & \text{if } \lambda_{k_i} < \lambda_{k-1}, \text{ i.e. bank } k_i \text{ offers less than at least one other bank.} \end{cases}$$

By using this function we can write down the expected profit of bank k_i as

$$E(\pi_{k_i}) = \psi_{k_i} \varrho(r_{k_i}) l_{k_i} - \rho_{k_i} d_{k_i} \quad (3.31)$$

with l_{k_i} as credit given by bank k_i and d_{k_i} as the deposits collected by bank k_i . To maximize their expected profits the banks would just collect that amount of deposits they need to refinance its given credits. Hence, we can see that the aggregate deposits collected from banks can maximally reach the amount of aggregate credit given in the market. If there is excess demand of deposits, banks would receive a fraction of their demand. But we assume, that there is enough supply to satisfy the complete demand of banks. However, that does not mean that all borrowers receive credit. With this assumption we can conclude that a bank is always able to refinance its credit given completely by paying a sufficiently high deposit rate.

In the following we have to compare for each case the values of the bank's return function with the deposit rate it would have to pay to refinance its deviating strategy. This is illustrated in table 2. The credit market equilibrium is shown in figure 4.

	case 1	case 2	case 3	case 4	case 5
banks receive from credit given	$< \rho^*$	$< \rho^*$	not possible	$= \rho^*$	$= \rho^*$
aggregate credit given and deposit demand	rises	unchanged	unchanged	unchanged	rises
banks have to pay to refinance	$> \rho^*$	$= \rho^*$	$= 0$	$= \rho^*$	$> \rho^*$
profit	< 0	< 0	$= 0$	$= 0$	< 0

Table 2: Profit in the Five Cases

At new interest rates below r^* a bank can attract firms by offering credit. This leads,

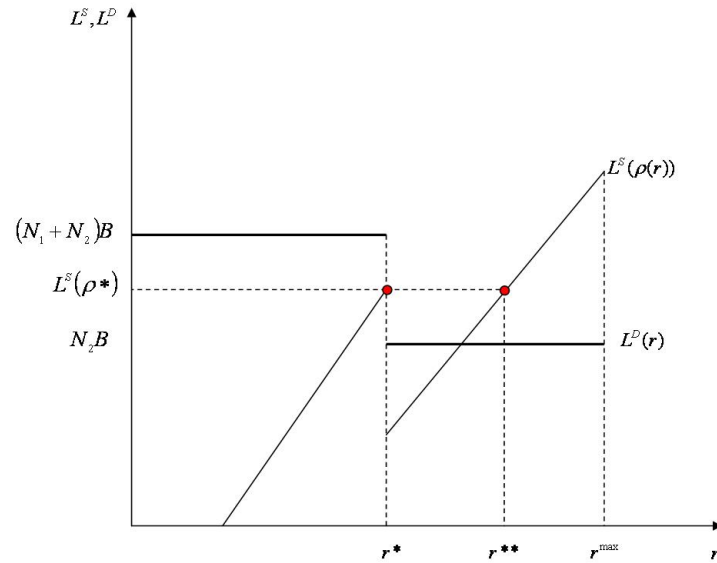


Figure 4: The Two-Interest Rate Equilibrium

according to case 1 in lemma 2, to a higher aggregate credit given.

Now we have to examine how the increase in the aggregate credit given affects the expected profit of the deviating bank. In the deposit subgame the aggregate demand for funds rises and thus the banks would have to pay a higher deposit rate than ρ^* to prevent default. This means that the banks would have to attain a higher return than $\varrho(r_k) = \rho^*$ in the credit subgame to achieve zero profits. However, this is evidently not possible in the interval $r < r^*$ if one considers the assumed shape of the banks' return function.

In case 2 credit is offered at an interest rate r' between r^* and r^{**} . We proved that aggregate credit given would then be unchanged as the additional offer only affects borrowers of the risky type that are not rationed in equilibrium anyway. However, this strategy does not yield positive expected profits for a bank, because its return in this interval $\varrho(r')$ is strictly smaller than ρ^* , while it would have to raise funds in the deposit subgame by paying exactly ρ^* .

Now consider case 3 where credit is offered at interest rates $r' > r^{**}$. From the definition of L^* in (3.25) follows that residual demand equals supply at the interest rate r^{**} . Thus above r^{**} there is no residual demand, consequently there is no possibility of creating positive returns.

Another possibility to deviate is shown in case 4. Here the bank deviates from the strategies shown in the theorem by setting a higher credit ceiling than the bank that served credit demand at r^{**} and thereby capturing all the credit demand at this interest rate and achieving the return rate $\varrho(r^{**}) = \rho^*$. Also this does not yield positive profits, but zero

profits, as there was no excess demand at r^{**} and thus credit given does not increase, and the bank would have to pay ρ^* in the deposit subgame. Hence, by deviating as in case 4 is the only possibility to serve some credit demand and not achieve negative expected profits. However, even this does not yield positive expected profits and thus there is no incentive to deviate from the proposed equilibrium strategy.

Setting a higher credit ceiling at r^* (case 5) leads to an increase in aggregate credit given and hence it is possible to create positive returns, $\varrho(r^*) = \rho^*$ in the credit subgame. However, expected profit in contrast decreases because the bank would again have to pay a higher deposit rate ($> \rho^*$) to refinance credit given.

This explains why in the credit subgame, setting $(r_k, \lambda_k) = (r^*, L^*)$ or $(r_k, \lambda_k) = (r^{**}, L^S(\rho^*) - L^*)$ is a Nash equilibrium.

It is necessary that at each interest rate at least two banks set the credit ceiling as described in the equilibrium tuple, because if one bank deviates from r^* or r^{**} , there has to be another bank that continues to offer credit at that interest rate. Otherwise deviating would possibly be profitable. This completes the proof of the theorem. Q.E.D.

4 Conclusion

In this paper is shown that the equilibrium in the Stiglitz-Weiss model (Stiglitz and Weiss, 1981) is a two-interest rate equilibrium. Arnold (2005) proofs this for a continuum of borrowers under Bertrand competition. We extended this consideration for a discrete number of borrowers. We found out that in the two interest-rate equilibrium some of the safe borrowers are rationed at the lower interest rate. Risky borrowers are also rationed at that interest rate, but all of them receive credit at the higher equilibrium interest rate. Hence, the cross-subsidization of risky borrowers is reduced in comparison to the pure credit rationing equilibrium in the model of Stiglitz and Weiss (1981).

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