Hyperbolic Discounting and Secondary Markets[1]

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Abstract

We study the effect of hyperbolic discounting on competitive equilibria in secondary markets for a durable good. Under exponential discounting, secondary markets are irrelevant in our model. They do not affect the price in the initial period and are neutral to the allocation. Under hyperbolic discounting, secondary markets are not neutral: they do affect price and allocation. The price in the unique competitive Markov equilibrium is lower than the price in the absence of secondary markets. This affects the equilibrium supply of the durable good in the initial period.

We characterise all stationary competitive equilibria in terms of prices. In particular, we obtain that there are stationary competitive equilibria in which trade occurs in each period and the allocation of the durable good is inefficient. Furthermore, we show that there exist competitive equilibria with increasing, decreasing, and cycling price paths, despite the stationarity of the market environment.

Keywords: hyperbolic discounting, secondary markets, durable good, time inconsistency

JEL classification: L11, D40, D91

1 Introduction

In this paper, we consider a sequence of perfectly competitive secondary markets for durable goods. These durable goods may be, for example, collector's items such as coins, stamps, Meissner porcelain figures, etc. We analyse a simple market environment which allows us to contrast the impact of hyperbolic discounting on market outcomes relative to exponential discounting. The environment is stationary; that is, there is no depreciation and no additional supply of the durable good over time. The only intertemporal aspect of our model is the durability of the traded goods.

In our model, perfectly competitive secondary markets are neutral to allocations if consumers are exponential discounters. Specifically, the initial price and the incentive to provide the initial supply are not affected by the absence or presence of secondary markets, which are simply additional opportunities to trade. Furthermore, the allocation of the durable good is efficient. That is, consumers above a marginal type buy the good in each period, whereas consumers below that marginal type never buy the durable good.

Hyperbolic discounting applies a higher discount rate to the near future than to the distant future. Such discounting implies a conflict between today's preferences and future preferences. Time inconsistency potentially matters in our durable goods environment, where consumers incur a cost at the date of purchase and receive a continuing stream of benefits from consumption over time.

The main question addressed in this paper is whether (and to what extent) the presence of secondary markets has an impact on market outcomes if consumers are hyperbolic discounters. As we will explain, under hyperbolic discounting, the equilibrium price, initial supply and the set of consumers served may change through the introduction of secondary markets.

With hyperbolic discounting, current and future incarnations play an intrapersonal game. A competitive equilibrium in our model satisfies two conditions: (i) for a given price path, the strategy profile of each consumer forms a subgame perfect equilibrium in the intrapersonal game, and (ii) the price path is such that the market clears in each period. In the absence of secondary markets, future incarnations are given no choice: the initial period incarnation decides whether to consume the durable good in each period or never. Certain consumer types have first period incarnations which ideally would like to commit to future consumption and prefer not to buy the good at present. These types have an incentive to delay consumption. This incentive remains for later incarnations, conflicting with the ideal consumption path of the first period incarnation. In the presence of secondary markets, certain types may therefore end up never consuming, although they would be better off if all incarnations decided to buy and consume in each period.

As far as we are aware, this is the first paper to consider hyperbolic discounting in a durable good environment. Motivated by an overwhelming evidence in the psychology literature on time-inconsistency, hyperbolic discounting has received a lot of attention in the economics literature recently.[1] The seminal paper in economics is Strotz (1956). Recent work on hyperbolic discounting has focused on task performance in decision problems (O'Donoghue and Rabin, 1999a,b,c, Carrillo and Mariotti, 2000, Brocas and Carrillo, 1999) and on intertemporal consumption and savings decisions (Laibson, 1997, Harris and Laibson, 1999, Luttmer and Mariotti, 2000).[2] In the light of the work on task performance, we want to emphasise some characteristics of our framework. We assume that consumers are *sophisticated* in that they are aware of the intrapersonal game they play.[3] Another feature of our model is that the durable good can be purchased in each period. Hence, the "tasks" to buy the durable good are not mutually exclusive.[4] In contrast to the papers on task performance, the decision problem is embedded into a market environment so that the cost of performing the task is endogenous.

The outline of the paper is as follows. In Section 2, we present the model. In Section 3, we first characterise competitive equilibrium in the benchmark case of exponential discounting. For the case of hyperbolic discounting, we then analyse the (unique) competitive equilibrium when secondary markets close in finite time, and the (unique) competitive Markov equilibrium when secondary markets never close. We obtain the following non-neutrality results: the initial price decreases in the number of periods in which the good may be traded in secondary markets. Hence, a monopolist (or Cournot oligopolists) that provides the initial supply has an incentive to close the secondary market. Moreover, a profit-maximising monopolist supplies less the larger is the number of periods in which secondary markets are open. In Sections 4 and 5, we consider non-Markovian strategies in secondary markets which never close. In Section 4, we introduce collusive strategy profiles in which a deviation triggers a change of the consumption decision in all future periods. Such a collusive strategy profile supports a high equilibrium price,

[1] For a discussion and references to the psychological literature, see for instance Ainslee (1992) or Loewenstein and Prelec (1992). See also the experiments documented in Thaler (1981).

[2] Other work includes Akerlof (1991), Caillaud, Cohen, and Jullien (1996), Benabou and Tirole (1999).

[3] Note, however, that the consumer's degree of sophistication matters only for non-Markovian intrapersonal strategies. An incarnation's Markov strategy is independent of the consumer's degree of sophistication.

[4] This also holds e.g. in Carrillo and Mariotti, 2000, but not in O'Donoghue and Rabin, 1999a,b,c.

and restores the equilibrium allocation in the absence of secondary markets. We then characterise the set of equilibrium outcomes in all stationary equilibria. In some of these equilibria, trade in the durable good occurs in each period and the allocation of the durable good is inefficient. This shows that additional markets can lead to allocative inefficiency with hyperbolic discounters. In Section 5, we show that non-stationary competitive equilibria may obtain in our stationary environment. In particular, we construct examples with increasing, decreasing, and cycling price paths. Section 6 concludes.

2 The model

We consider a discrete time, infinite horizon model of a durable good market. Time is labelled by $t = 0, 1, 2, ...$ In each period, consumers may spend their disposable income on the durable good and a Hicksian composite commodity.

There is a unit mass of heterogeneous consumers with unit demand for the durable good. Consumers differ only in their valuations for this good, which is parameterised by v. Consumer type v is time-independent and uniformly distributed on $[0, 1]$.[5]

Let us first consider a particular period t. Direct instantaneous utility is of the form $u(x_t, y_t; v) = vx_t + y_t$, where $x_t \in \{0, 1\}$ and $y_t \geq 0$ denote the period t consumption of the durable good and the Hicksian composite commodity, respectively. Type v can be interpreted as the utility derived from consuming the durable good in one period, which is measured in the units of the Hicksian composite commodity.

The Hicksian composite commodity is perishable, and its price normalised to one in each period. This normalisation is justified since we do not allow for income transfers over time. That is, we rule out saving and borrowing as well as forward markets for the durable good.[6] In each period, consumers have

[5]In particular, the durable good provides a constant utility stream. That is, there is no depreciation in the quality of the good and consumers' tastes do not change over time. – The uniformity assumption on the unit interval is made for convenience; we could work with any continuous distribution function.

[6]As long as the interest rate on savings r is less than $(1 - \beta\delta)/(\beta\delta)$, consumers would not like to save. If r is greater, consumers would like to postpone consumption of the nondurable good indefinitely (for prices of the nondurable that are fixed as above) because instantaneous utility functions are linear in the nondurable good. We could analyse a model with savings in which consumers' instantaneous utility functions are strictly concave in y_t to the effect that consumers maintain a positive consumption stream of the nondurable good. We conjecture that the nonneutrality of secondary markets for the durable good also holds in such a model. We do not pursue this avenue because (i) in a partial equilibrium environment our model appears to be the natural model to start with; in particular, (ii) introducing savings into a model with utility functions that are not linear

income m (which, for simplicity, is independent of v). Disposable income in period t, m'_t, is equal to $m + p_t$ if the consumer inherits an endowment of one unit of the durable good which can be sold at price p_t. Otherwise, disposable income m'_t is equal to m. To summarise, disposable income is $m'_t = m + p_t x_{t-1}$. Here, we have introduced the convention that a consumer with $x_{t-1} = 1$ always sells his endowment on the market. If he keeps the durable good he simply sells and buys it back at price p_t.

A consumer, who buys $x_t \in \{0, 1\}$ units of the durable good at price p_t and y_t units of the Hicksian composite commodity, faces the budget constraint $m'_t - p_t x_t - y_t \geq 0$. We assume that income m is sufficiently large such that a consumer can always afford to buy one unit of the durable good.

Following Strotz (1956) and Phelps and Pollak (1968), each consumer is composed of a sequence of incarnations indexed by their period of control over consumption. Consumer type v's period t incarnation chooses his consumption in period t so as to maximise the discounted sum of present and future instantaneous utilities. Discounting is of the exponential or hyperbolic form. In the latter case, the rate of substitution between periods t and $t+1$ is larger than the one between $t+s$ and $t+s+1$ for $s \geq 1$. Direct utility is of the form

$$U_t(\{x_s\}_{s=t}^{\infty}, \{y_s\}_{s=t}^{\infty}; v) = u(x_t, y_t; v) + \beta \sum_{s=1}^{\infty} \delta^s u(x_{t+s}, y_{t+s}; v).$$

where $\beta \in (0, 1]$ and $\delta \in (0, 1)$. If $\beta = 1$, then consumers are exponential discounters and hence "time consistent", otherwise they are hyperbolic discounters and "time inconsistent".

Utility maximization implies that the budget constraint in each period is satisfied with equality. Hence, y_t can be replaced by $m'_t - p_t x_t$, where disposable income is equal to $m'_t = m + p_t x_{t-1}$. It is more convenient to work with incarnation t's indirect utility function conditional on $\{x_s\}_{s \geq t}$. In case the consumer never buys the durable good from t onward, i.e. $x_s = 0$ for all $s \geq t$, we normalise incarnation t's conditional indirect utility to 0. Consequently, incarnation t's conditional indirect utility does not reflect utility gains due to initial endowment effects in period t. If type v consumes the durable good in all periods from t onward, his incarnation t's conditional indirect utility is

$$V_t(\{p_s\}_{s \geq t}; v | \{x_s = 1\}_{s \geq t}) \quad = \quad (v - p_t) + \beta \sum_{s=1}^{\infty} \delta^s v$$

in y_t destroys the stationarity of the environment for the durable goods markets; (iii) in our model we are able to perform a simple welfare analysis.

$$= \frac{1-(1-\beta)\delta}{1-\delta}v - p_t.$$

More generally, incarnation t's indirect utility conditional on $\{x_s\}_{s\geq t}$ is

$$V_t(\{p_s\}_{s\geq t}; v|\{x_s\}_{s\geq t}) = x_t\,(v - p_t + \beta\delta p_{t+1}) + \beta\sum_{\tau=1}^{\infty}\delta^s x_{t+\tau}(v - p_{t+\tau} + \delta p_{t+\tau+1}).$$

At $t = 0$, there is an initial supply of q units of the durable good. This can be thought of as the aggregate supply of an industry, which we do not model explicitly at this point. For simplicity, no consumer has an initial endowment at $t = 0$. Below, we analyse the case of a monopolist who chooses q so as to maximise his profits. In each period, consumers are price takers. Given initial supply q, the equilibrium price p_0 is such that the durable good market clears in $t = 0$. From $t = 1$ onward, there is a perfectly competitive secondary market for the durable good. The good remains in constant supply of q, i.e. there is no additional production. Again, equilibrium price p_t is such that the market clears in period t.

A consumer's sequence of incarnations are assumed to play an *intrapersonal game*: period t incarnation makes his consumption choice, taking as given the strategies of all other incarnations (of the same consumer) and aggregate market conditions, as summarised by the price sequence $\{p_t\}_{t=0}^{\infty}$ in the durable goods market. Given $\{p_t\}_{t=0}^{\infty}$, a *pure strategy* of incarnation t in the intrapersonal game is a mapping from the private history into the action space $\{0, 1\}$. A consumer's private history in period t is summarised by the sequence of own past consumption of the durable good $\{x_s\}_{s\leq t-1}$. Note that $y_s = (m + p_s x_{s-1}) - p_s x_s$. Hence, we can suppress the sequence $\{y_s\}_{s\leq t-1}$ as part of the private history. A *mixed strategy* of incarnation t is a probability distribution over pure strategies. To generate a probability distribution over actions after a particular history, an incarnation uses a randomisation device. Let z_t be the realisation of the random variable with support Z in period t and let μ be a probability measure on Z. (These random variables are assumed to be independent across individual consumers at any given time and across incarnations of the same consumer.) A consumer's private history in period t includes the sequence of realised outcomes of the randomisation devices used by his past incarnations.[7] That is, the consumer's private history in period t is now summarised by $\{x_s, z_s\}_{s\leq t-1}$. Since an incarnation's conditional indirect utility is independent of his private history and of past prices,

[7] That is, we consider a randomisation device which is public among incarnations of a consumer in the intrapersonal game. This is similar to the public randomisation device used in the repeated game literature; see Fudenberg and Maskin (1986).

a *pure Markov strategy* of incarnation t, given the price sequence $\{p_t\}_{t=0}^{\infty}$, is an element of $\{0, 1\}$.

Given $\{p_t\}_{t=0}^{\infty}$, a consumer's *intrapersonal equilibrium* is a subgame perfect equilibrium (SPE) in the intrapersonal game played by the consumer's different incarnations. (Sometimes, we will restrict attention to pure strategies or Markov strategies.) A *competitive equilibrium* in the durable goods market consists of the set of strategy profiles in the intrapersonal game and a sequence of prices $\{p_t\}_{t=0}^{\infty}$ in the durable goods market such that (i) each strategy profile forms an intrapersonal equilibrium at prices $\{p_t\}_{t=0}^{\infty}$, and (ii) the durable goods market clears in each period $t = 0, 1, 2, ...$ With the restriction to pure strategies in the intrapersonal game we refer to a *competitive equilibrium in pure strategies.* With the restriction to Markov strategies in the intrapersonal game we refer to a *competitive Markov equilibrium.*

3 (Non-)neutrality of secondary markets

In this section, we investigate whether the existence of secondary markets from period 1 onward has any impact on the equilibrium price in period 0. The related question of interest is whether the monopolist offering the durable good in period 0 has an incentive to close down the secondary markets. Throughout, we assume that all incarnations use Markov strategies in the intrapersonal game. Before analysing the case of hyperbolic discounting, we consider the benchmark case of exponential discounting.

3.1 Exponential discounting: the neutrality of secondary markets

Let us analyse the competitive equilibrium in the durable goods market when consumers are exponential discounters, i.e., $\beta = 1$. Under exponential discounting, the interests of the different incarnations of a given consumer coincide in the following sense. Suppose the consumer's period t incarnation could control consumption not only in period t, but also in all subsequent periods. Given prices $\{p_s\}_{s=0}^{\infty}$, period t incarnation's optimal sequence of consumption is denoted by $\{x_s^t\}_{s \geq t}$. This consumption sequence would be consistent with the optimal consumption sequence of future incarnations: for any period $\tau \geq t$ incarnation, $\{x_s^t\}_{s \geq \tau} = \{x_s^\tau\}_{s \geq \tau}$. This shows that the solution to the period 0 incarnation's intertemporal decision problem under commitment, $\{x_s^0\}_{s \geq 0}$, can be sustained in an intrapersonal equilibrium without commitment.

Lemma 1 *Under exponential discounting, the solution to the period 0 incarnation's decision problem with commitment is the unique equilibrium of the intrapersonal game without commitment.*

Proof. Denote $\varphi_t \equiv v - p_t + \delta p_{t+1}$ and $\varphi_t^+ \equiv \max\{0, \varphi_s\}$. Then, period 0 incarnation's maximal utility can be written as $V_0 = \Sigma_{s=0}^{\infty} \delta^s \max\{0, \varphi_s\} = \Sigma_{s=0}^{\infty} \delta^s \varphi_t^+$. Similarly, period t incarnation's utility is $V_t = \Sigma_{i=0}^{\infty} \delta^i \varphi_{t+i}^+$. Suppose there exists an equilibrium of the intrapersonal game in which a period t incarnation's utility is different from V_t. Then there must exist a period t in which either $\varphi_t > 0$ and $x_t^* = 0$ or $\varphi_t < 0$ and $x_t^* = 1$. Suppose $\varphi_t > 0$ and $x_t^* = 0$. Hence, the associated utility of period t incarnation $V_t^* < V_t$. We have $V_t^* = \delta V_{t+1}^*$. Consider a deviation in period t, $x_t^1 = 1$. This implies that the corresponding utility is $V_t^1 = \varphi_t + \delta V_{t+1}^1$. In order to support x_t^* along the equilibrium path, we must have $\varphi_t \leq \delta(V_{t+1}^* - V_{t+1}^1)$. Denote by V_{t+i}^i the utility of incarnation $t+i$ after the i-th deviation starting from period t. We have $V_{t+i}^i \geq \varphi_{t+i}^+ + \delta V_{t+1+i}^{i+1}$. Thus we obtain

$$V_t^1 \geq \Sigma_{i=0}^{n} \delta^i \varphi_{t+i}^+ + \delta^n V_{t+1+n}^{n+1}$$

for all n. Since $\lim_{n\to\infty}(\Sigma_{i=0}^{n} \delta^i \varphi_{t+i}^+ + \delta^n V_{t+1+n}^{n+1}) = V_t$, x_t^* cannot be supported along the equilibrium path. The same argument applies to the case $\varphi_t < 0$ and $x_t^* = 1$. ■

Hence, a consumer's intrapersonal game can be solved as if it was a decision problem of the consumer's period 0 incarnation.

We start by analysing consumer v's intrapersonal game, given an arbitrary sequence of prices $\{p_s\}_{s=0}^{\infty}$. The equilibrium consumption choice in period t is given by

$$x_t = \begin{cases} 1 & \text{if } v - p_t + \delta p_{t+1} \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Hence, the marginal consumer type in period t, who is just indifferent between buying and not, is $\widehat{v}_t = p_t - \delta p_{t+1}$. In the intrapersonal equilibrium, all consumer types below $\widehat{v}_t$ do not consume the durable good, while all other types do. In a competitive equilibrium, markets clear in each period. For a given aggregate supply q, the marginal type is given by $\widehat{v}_t = 1 - q$ for all t. Hence, the equilibrium price in period t can be written as

$$p_t = \frac{1-q}{1-\delta} + \lim_{s\to\infty} \delta^s p_{t+s}.$$

If the price path is not allowed to explode exponentially, we obtain

$$p_t = \frac{1-q}{1-\delta} \text{ for all } t \geq 0. \tag{1}$$

That is, the equilibrium price is constant over time. Note that this is the price which would obtain in period 0 if the secondary markets were closed down in all subsequent periods. We thus have the following result.

Proposition 1 *Under exponential discounting ($\beta = 1$), the existence of a secondary market for used goods does not affect consumer choice. Also, it does not affect the monopolist's optimal choice of supply q (or, alternatively, price p_0).*

The neutrality of secondary markets under exponential discounting serves as a useful benchmark. Of course, in a richer model, secondary markets may play a role if consumers' evaluations change over time or goods can become faulty.

3.2 Hyperbolic discounting: the non-neutrality of secondary markets

Let us now turn to the equilibrium analysis when consumers are hyperbolic discounters, i.e., $\beta < 1$. In the intrapersonal game, we confine attention to Markov strategies.

Consider consumer v's intrapersonal game for a given price path $\{p_s\}_{s=0}^{\infty}$. Since we assume that all incarnations use Markov strategies, future incarnations' consumption decisions will be independent of the action taken by the current incarnation. Hence, the consumer's period t incarnation optimally decides to consume in period t if and only if $v - p_t + \beta\delta p_{t+1} \geq 0$. Consequently, in period t, all consumer types above the marginal type $\widehat{v}_t \equiv p_t - \beta\delta p_{t+1}$ choose $x_t = 1$; all types below $\widehat{v}_t$ select $x_t = 0$. Market clearing implies that $\widehat{v}_t = 1 - q$. The equilibrium price in period t is then given by

$$p_t = \frac{1-q}{1-\beta\delta} + \lim_{s\to\infty} (\beta\delta)^s \, p_{t+s}.$$

We assume again that the price path does not explode exponentially. We thus have

$$p^M \equiv p_t = \frac{1-q}{1-\beta\delta} \text{ for all } t \geq 0. \tag{2}$$

In the limit as $\beta \to 1$, we are back in the case of exponential discounting, and the equilibrium price is again given by (1).

Let us now compare this equilibrium with the one that obtains when the secondary markets are closed down after period $T \geq 0$. In the special case when $T = 0$, trade is only possible in the initial period. Intrapersonal strategies are allowed to depend in an arbitrary way on the history of the game.

For a given price path $\{p_s\}_{s=0}^{T}$, we can solve for the intrapersonal equilibrium by backward induction. Consider consumer v's period T incarnation. His conditional indirect utility from buying the durable good in period T is equal to $v - p + \beta\delta \sum_{s=0}^{\infty} \delta^s v$. Hence, independently of the history of the game, he optimally chooses

$$x_T = \begin{cases} 1 & \text{if } \left(\frac{1-\delta+\beta\delta}{1-\delta}\right) v - p_T \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Using backward induction, we can now solve for period $T-1$ incarnation's equilibrium strategy. Since period T incarnation's strategy does not condition on the action taken in $T-1$, period $T-1$ incarnation optimally chooses to consume if and only if $v - p_{T-1} + \beta\delta p_T \geq 0$. Using the same argument for all previous incarnations, the intrapersonal equilibrium strategies in periods 0 to $T-1$ are as before. That is, in the (unique) intrapersonal equilibrium all incarnations use Markov strategies. Market clearing implies that $\widehat{v}_t = 1 - q$ for all $t = 0, ..., T$, where $\widehat{v}_t$ is again the marginal consumer type in period t. From market clearing in period T, we obtain

$$\begin{aligned} p_T &= \frac{1-\delta+\beta\delta}{1-\delta}(1-q) \\ &= p^C. \end{aligned} \tag{3}$$

Market clearing in all previous periods implies the following equilibrium price in period $t \in \{0, ..., T\}$:

$$\begin{aligned} p_t &= (1-q)\sum_{s=0}^{T-t-1}(\beta\delta)^s + (\beta\delta)^{T-t} p_T \\ &= \frac{1-(\beta\delta)^{T-t}}{1-\beta\delta}(1-q) + (\beta\delta)^{T-t} p_T. \end{aligned}$$

Abusing notation, the equilibrium price may be rewritten as

$$p_t(T,q) = \frac{1-\delta+\delta(\beta\delta)^{T-t+1}(1-\beta)}{(1-\beta\delta)(1-\delta)}(1-q). \tag{4}$$

For a given final trading period T and an initial supply q, the equilibrium price is increasing over time: $p_t(T,q) < p_{t+1}(T,q)$ for all $t = 0, .., T-1$. The equilibrium price in a given period is lower if secondary markets close later: $p_t(T+1,q) < p_t(T,q)$ for $t = 0, ..., T$. In particular, as $T \to \infty$, the equilibrium price in any given period converges to the equilibrium price when the secondary markets never close and consumers use Markov strategies; this price p^M is given by (2). For a given price path, there exists a unique subgame perfect equilibrium (SPE) in each consumer's intrapersonal game, provided secondary markets close in finite time ($T < \infty$). In the limit as the final trading period T goes to infinity, this equilibrium converges to the unique Markov perfect equilibrium (MPE) of the infinite intrapersonal game ($T = \infty$). The uniqueness of equilibrium in each consumer's intrapersonal game for a given price path translates into the uniqueness of the competitive equilibrium.

Lemma 2 *Under hyperbolic discounting ($\beta \in (0,1)$) and given initial supply $q \in (0,1)$, if secondary markets close in finite time $T < \infty$, there exists a unique competitive equilibrium with a price path $\{p_t\}_{t=0}^{T}$ characterised by equation (4). If secondary markets do not close in finite time there exists a unique competitive Markov equilibrium with a price path $\{p_t\}_{t=0}^{\infty}$ characterised by equation (2).*

Under hyperbolic discounting, i.e. $\beta \in (0,1)$, secondary markets are no longer neutral. For a given initial supply of q units, the equilibrium price in period 0 is the larger, the earlier the secondary markets close down. This non-neutrality obtains although the set of consumers who buy the good along the equilibrium path is independent of the final trading period T. That is, all trade in the secondary markets is "trivial" (or "degenerate") in that the same set of consumers re-sell and re-purchase the good in each period.

Proposition 2 *For a given initial supply q, the competitive equilibrium price in the initial period, p_0, is the larger, the earlier secondary markets close down. However, the equilibrium allocation is independent of the final trading period.*

In order to understand the features of the competitive equilibrium consider the case where secondary markets never close down. Suppose the equilibrium price is p in all periods. If the period 0-incarnation could commit on the whole future consumption path, which path would he optimally choose?

Type v period t incarnation's conditional indirect utility from starting to buy in period $t+s \geq t+1$ and always thereafter is

$$\begin{aligned} V_t\left(p; v|\{x_{t+\tau}=0\}_{\tau=0}^{s-1}, \{x_{t+\tau}=1\}_{\tau=s}^{\infty}\right) &= \beta\delta^s\left(\frac{v}{1-\delta}-p\right) \\ &\geq 0 \iff \frac{v}{1-\delta} \geq p. \end{aligned}$$

Note that the period t incarnation of type $v \geq (1-\delta)p$ derives a positive conditional utility, independently of s, $s \geq 1$. Thus, if it is optimal to buy at all, then any period t incarnation would like to commit to buy from either today (t) or tomorrow onward $(t+1)$.

Type v period t incarnation's utility from buying in period t and always thereafter is

$$\begin{aligned} V_t\left(p; v | \{x_{t+\tau} = 1\}_{\tau=0}^{\infty}\right) &= \frac{1-\delta+\beta\delta}{1-\delta} v - p \\ &\geq 0 \iff \frac{(1-\delta+\beta\delta)v}{1-\delta} \geq p. \end{aligned}$$

If type v's period t incarnation could make commitments on future consumption, it would choose to consume in all periods, starting in the current period, rather than to wait another period if

$$\frac{1-\delta+\beta\delta}{1-\delta} v - p \geq \beta\delta \left(\frac{v}{1-\delta} - p \right),$$

which is equivalent to

$$v \geq (1-\beta\delta)p.$$

Hence, we obtain that any incarnation of type $v \in [(1-\beta\delta)p, 1]$ would like to commit to consume starting in the current period, any incarnation of type $v \in [(1-\delta)p, (1-\beta\delta)p)$ would like to commit to postpone consumption for a single period, and any incarnation of type $v \in [0, (1-\delta)p)$ would prefer never to consume.

However, commitments are not possible. Since the decision problem is stationary, we obtain therefore the following result with Markov strategies: for a given constant price p, in the intrapersonal equilibrium, it will be exactly the types $((1-\delta)p, (1-\beta\delta)p)$ that will decide not to buy in the current period nor in any future period. All incarnations of types in $\left(\frac{1-\delta}{1-\delta+\beta\delta}p, (1-\beta\delta)p\right)$, however, would be strictly better off by consuming the good in all periods; see the welfare discussion below.

Relating our result to Gul and Pesendorfer (1999, 2000), the only temptation which matters in our model is the temptation of certain types and incarnations, who in the absence of secondary markets would buy the good, to procrastinate so that these types end up not consuming in any period. The opposite temptation to buy the good at some point, although the consumer would refrain from buying in the absence of secondary markets, does not arise.

Proposition 2 suggests that a monopolist, who chooses the initial supply q so as to maximise his profit, may have an incentive to prevent future trade in the used good. Suppose the monopolist has a constant marginal cost of production $c \geq 0$. His decision problem in period 0 is then to set q so as to maximise $[p_0(T, q) - c]\, q$. Since the price $p_0(T, q)$ is of the form $k(T)(1 - q)$, the solution to this problem is given by

$$q^*(T) = \frac{k(T) - c}{2k(T)},$$

where $k(T) \equiv \left[1 - \delta + \delta(\beta\delta)^{T+1}(1 - \beta)\right] / [(1 - \beta\delta)(1 - \delta)]$. If $c = 0$, the monopolist's equilibrium supply is 1/2, independently of the closure time of secondary markets. If $c > 0$, however, the monopolist's supply is decreasing with closure time T. The equilibrium price in period 0 is given by $p_0^*(T) = [k(T) + c]/2$, and the monopolist's equilibrium profit by

$$\pi^*(T) = \frac{[k(T) - c]^2}{2k(T)}.$$

Both $p_0^*(T)$ and $\pi^*(T)$ are decreasing in closure time T. The earlier the monopolist can close down further trade, the better he is off. We state the effect on the initial supply as the following additional non-neutrality result.

Proposition 3 *A profit maximising monopolist optimally chooses a smaller initial supply, the later future trade ceases.*

Qualitatively, the same results hold in a Cournot oligopoly with a fixed number of firms. The effect on quantity is strengthened (ignoring integer effects) in a free entry equilibrium where each firm in the market has incurred a sunk cost. Proposition 3 obtains since the introduction of secondary markets shifts the demand curve in period 0 towards the origin.

To conclude this section, we carry out a welfare analysis in the case of fixed supply. Observe that *a priori* it may not be possible to Pareto-rank the different allocations since some incarnations may be better off when others are worse off. An additional complication is the following: an incarnation's utility has been defined only over current and future consumption, but not over past consumption. The reason is that an incarnation cannot control past consumption; hence, the utility over past consumption does not matter for a positive analysis. However, an individual clearly cares about the past: memories of good events are preferred to memories of bad events. That is, when making utility comparisons also past consumption has to enter the utility. One way of formalising this idea is to view the past as a mirror of the future: consumption which is more distant from today is discounted more

(Caplin and Leahy, 1999).[8] Period t incarnation's direct utility is

$$U_t\left(\{x_s\}_{s=0}^{t}, \{y_s\}_{s=0}^{\infty}; v\right) = u(x_t, y_t; v) + \beta \sum_{s=0, s\neq t}^{\infty} \delta^{|t-s|} u(x_s, y_s; v)$$

His conditional indirect utility from buying in all periods $0, 1, 2, ...$ can then be expressed as

$$\begin{aligned} V_t(p; v|\{x_s\}_{s=0}^{\infty}) &= v + \beta\delta^t(v-p) + \beta\sum_{s=1}^{t-1}\delta^s v + \beta\sum_{s=1}^{\infty}\delta^s v \\ &= \frac{v}{1-\delta}\left(1-\delta+\beta\delta(2-\delta^t)\right) - \beta\delta^t p, \end{aligned}$$

which is strictly increasing in t.

If period 0 incarnation prefers this consumption stream to never buying, so will all other incarnations. This makes it possible to Pareto-rank the two consumption streams (never buying versus always buying): if $v \in \left(\frac{1-\delta}{1-\delta+\beta\delta}p, (1-\beta\delta)p\right)$ then all incarnations would prefer to consume in all periods rather than never to consume. Nevertheless, along the equilibrium path (competitive Markov equilibrium), they will never consume.

The result on Pareto-ranked consumption streams implies that the competitive Markov equilibrium is not stable with respect to mutual deviations by all incarnations of a consumer.

4 Stationary equilibria

In this section we fully characterise stationary equilibria in terms of their (constant) price paths. Stationary equilibria are equilibria in which the price

[8] There appear to be two natural alternative formulations. The first is that past consumption does not enter current utility. The second is to give earlier periods a greater weight than later periods. Specifically, period t incarnation's utility is given by

$$U_t\left(\{x_s\}_{s=0}^{t}, \{y_s\}_{s=0}^{\infty}; v\right) = \delta^t\left[u\left(x_t, y_t; v\right) + \beta\sum_{s=-t, s\neq 0}^{\infty}\delta^s u(x_s, y_s; v)\right].$$

In the case of exponential discounting ($\beta = 1$), this simplifies to

$$U_t\left(\{x_s\}_{s=0}^{t}, \{y_s\}_{s=0}^{\infty}; v\right) = \sum_{s=0}^{\infty}\delta^s u(x_s, y_s; v),$$

which is independent of t, i.e. the various incarnations of the same consumer agree on the evaluation of consumption streams. The conclusion of our welfare analysis carries over to both alternative formulations.

does not change over time. To the extent that the infinite time model is merely seen as an approximation of the finite time model with a long time horizon, the Markov perfect equilibrium is the appropriate equilibrium of the intrapersonal game. However, there is a qualitative difference between finite and infinite time. Non-Markovian strategies which live from future actions being based upon the past break down in finite time. If consumers are never sure that secondary markets will cease to exist, one should analyse also non-Markovian equilibria. Apart from the Markovian equilibrium we are particularly interested in an equilibrium in which each incarnation expects to end up in an eternal no-consumption situation if it does not buy the good himself because this situation corresponds to the equilibrium in the absence of secondary markets. Such strategies will allow incarnations to collude over time. A current deviation to no consumption has a long-run impact so that the trade-off between purchase and no purchase today looks different from the Markovian equilibrium.

4.1 Neutrality after all? Collusive strategies

We consider collusive strategies which enable a consumer to mimic the outcome in the absence of secondary markets. Remember that the absence of secondary markets implies an "all or nothing" decision in the initial period in which the market opens. To implement such a situation in the presence of secondary markets, a deviation from $x_t = 1$ to $x_t = 0$ in any period t must trigger a move from $x_s = 1$ to $x_s = 0$ for all future incarnations $s > t$.[9]

Implementing such strategies for constant price paths can be done merely by looking at past actions x_s, $s < t$, because the environment for the consumer remains stationary (for non-constant price paths see the next section). The basic idea of a collusive strategy is to keep consuming if and only if all previous incarnations decided to consume as well. If some previous incarnation decided not to consume, then the collusive strategy tells the present incarnation not to consume either. High types who consume even when using Markov strategies are exempted from this punishment.

Definition 1 *Given the stationary price p, the* collusive strategy profile *$\Sigma^C(p; v)$ in consumer v's intra-personal game is defined as follows:*

$$x_0 = \begin{cases} 1 & \textit{if } v \geq \frac{1-\delta}{1-\delta+\beta\delta} p, \\ 0 & \textit{otherwise,} \end{cases}$$

[9]Note the similarity to grim trigger strategies in the literature on repeated games (see e.g. Friedman, 1971).

$$x_t = \begin{cases} 1 & \textit{if } v \geq (1-\beta\delta)p, \\ 1 & \begin{array}{l} \textit{if } x_{t-s} = 1 \textit{ for all } s \in \{1, ..., t\} \\ \textit{and } v \in \left(\frac{1-\delta}{1-\delta+\beta\delta}p, (1-\beta\delta)p\right), \end{array} \\ 0 & \textit{otherwise,} \end{cases} \quad , t \geq 1.$$

Lemma 3 *For any stationary price p, the collusive strategy profile $\Sigma^C(p; v)$ forms an SPE in consumer v's intrapersonal game.*

Suppose all types use collusive strategies, i.e. they buy in period t if the consumption path starting at t, $\{x_s = 1\}_{s \geq t}$, gives a higher utility than $\{x_s = 0\}_{s \geq t}$. Otherwise, and if $v < (1 - \beta\delta)p$, they do not buy. Evaluating the indirect utility gives

$$V_t(p; v | \{x_s = 1\}_{s \geq t}) = v - p + \beta \sum_{s=1}^{\infty} \delta^s v$$

This expression is non-negative if $v(1 - \delta + \beta\delta)/(1 - \delta) \geq p$. In equilibrium, we must have $1 - v = q$ for market clearing. Consequently, the equilibrium price is

$$p^C = \frac{1 - \delta + \beta\delta}{1 - \delta}(1 - q),$$

provided all incarnations of all consumers use the collusive strategy.

Proposition 4 *Given initial supply q, the set of collusive strategy profiles $\Sigma^C(p^C; v)$, parameterised by v, and the stationary price path with*

$$p_t = p^C = \frac{1 - \delta + \beta\delta}{1 - \delta}(1 - q)$$

form a competitive equilibrium in the durable goods market.

Note that the collusive equilibrium restores the equilibrium outcome in the absence of secondary markets, both in terms of the equilibrium price path and the equilibrium allocation.

As pointed out in the previous section, there exist Pareto improvements over the equilibrium outcome reached by Markovian consumers. Furthermore, as has been shown with the previous result, a Pareto optimum can be implemented as a competitive equilibrium with non-Markovian consumers. Hence, the competitive Markov equilibrium may be viewed as an equilibrium in which incarnations fail to coordinate on more sophisticated strategies.[10]

[10] Suppose that a strategy is inherited by an incarnation from its previous incarnation.

4.2 Lower and upper bound on prices; allocative inefficiency

In this subsection, we provide tight upper and lower bounds on prices in a stationary equilibrium. We will show that the Markovian price $p^M = (1-q)/(1-\beta\delta)$ is the lower bound on prices and that the collusive price p^C is the upper bound on prices. Furthermore, we will construct a parameterised family of strategy profiles which "implements" any price in $[p^M, p^C]$ as a stationary equilibrium price; in equilibrium, the allocation of the final good is inefficient for any price in (p^M, p^C). That is, it is possible to construct *inefficient competitive equilibria.*

To sustain a low price, present consumption has to be discouraged. Suppose there exists an equilibrium with a price lower than p^M. In this case, some consumers with $v - p^M + \beta\delta p^M \geq 0$ must not buy with positive probability. If short-term utility is non-negative, i.e., $v - p + \beta\delta p \geq 0$, the worst punishment for buying today instead of not buying today is not to consume in all future periods instead of consuming, and the greatest reward for not consuming today is to consume in all future periods. Hence, we consider the deviation $\{x_s\}_{s\geq t} = \{1, 0, 0, 0, ...\}$ by period t incarnation from the sequence $\{0, 1, 1, 1, ...\}$. Subgame perfection requires that any deviation from $\{x_s\}_{s\geq t+1} = \{0, 0, 0, ...\}$, i.e., from the proposed deviation truncated at $t+1$, must not be profitable. Given the sequence $\{x_s\}_{s\geq t+1} = \{0, 0, 0, ...\}$, period $t+1$ incarnation has an incentive to deviate and consume independently of the future consumption path if $v - p + \beta\delta p \geq 0$. This shows that subgame perfection requires to consider deviation $\{x_s\}_{s\geq t} = \{1, 1, 0, 0, ...\}$ instead of $\{1, 0, 0, 0, ...\}$. The same argument can be applied to subsequent periods so that subgame perfections requires to consider deviation $\{1, 1, 1, 1, ...\}$. Such a deviation is profitable if $v - p + \beta\delta p \geq 0$. Hence, by contradiction, the worst punishment does not work.

Proposition 5 *The Markov price p^M is the lowest price that can be supported in any stationary competitive equilibrium.*

Proof. In Section 2, we have already shown that p^M can be supported in a stationary equilibrium by Markov strategies. Hence, it remains to be

Hence, a strategy is a genetic pattern and the composition of strategies possibly changes over time. One may then consider the evolution of a population of consumers who inherit the strategy of the previous consumers' incarnation (plus some mutation). Note that with sufficiently few mutations the collusive strategy has a higher "fitness" (for certain types) than the Markov strategy in that it gives a higher payoff to those inheriting this strategy. In such an evolutionary context, the collusive and not the Markovian outcome is predicted.

shown that a lower price cannot be supported in a stationary competitive equilibrium.

For a price $p < p^M$ to be supportable in a stationary equilibrium, there must be some type $v \geq 1 - q$ who does not buy the good in all periods at this price. We now want to show that there exists a profitable deviation for some incarnation of such a type.

Given the stationarity of both the game and the candidate equilibrium price, it is sufficient to consider (mixed) strategy profiles such that the expected utility (prior to randomisation) is the same for all incarnations of a given consumer. (Intuitively, if this were not the case, then some incarnations would have stronger incentives to deviate than others. By equalising the incentives to deviate for all incarnations, we make the "strongest case" in favour of the candidate equilibrium.)

The only way to equalise expected utility across incarnations is to have a constant probability α of consuming the good in each period. Since any punishment in the intrapersonal game must satisfy subgame perfection, we have to consider deviations from punishment paths as well. Hence, there must exist a sequence of probabilities $\{\alpha_i\}_{i=0}^{\infty}$ such that the following strategy profile forms an SPE in the intrapersonal game: each incarnation consumes the good with some probability, α_i say; if a past incarnation has deviated from this strategy (e.g. by consuming the good although, according to the used randomisation device, the incarnation should not have consumed), then all future incarnations consume the good with some (other) probability, α_{i+1} say, until another incarnation deviates...

Using the randomisation device, the "punishment phase $i+1$" is triggered in period $t+1$ if $x_t = 1$ and $z_t \notin Z_i \subseteq Z$ (or if $x_t = 0$ and $z_t \in Z_i$), where the probability of the realised random variable z_t being in Z_i is given by $\mu(Z_i) = \alpha_i$.

Let us now consider deviations of the following kind. After having observed the outcome of the randomisation device, the incarnation decides to deviate from its mixed strategy and consume the durable good. Suppose the different incarnations mix with probability α_0 and get "punished" with probability α_1. Then, an incarnation's conditional indirect utility from deviating (conditional on consuming today) is

$$v - (1 - \beta\delta)p + \beta \sum_{t=1}^{\infty} \delta^t \alpha_1 \left[v - (1 - \delta)p\right].$$

Let $a \equiv v - (1 - \beta\delta)p$ and $a' \equiv v - (1 - \delta)p$, and note that $a' > a > 0$ since $v \geq 1 - q$ and $p < p^M(q)$. Then, this deviations is not profitable if and only if

$$a-(\alpha_0-\alpha_1)\frac{\beta\delta}{1-\delta}a'\le 0.$$

More generally, we need

$$\alpha_i-\alpha_{i+1}\ge\left(\frac{1-\delta}{\beta\delta}\right)\frac{a}{a'}\text{ for all }i=0,1,2,...$$

Clearly, since $1\ge\alpha_i\ge 0$ for all i, this cannot hold. Hence, the proposed strategy profile does not form an SPE in the intrapersonal game. ■

The collusive price p^C is the lowest upper bound for the price in any stationary equilibrium. Before spelling this out in a proposition we provide the argument for pure strategies.

Let us first show that there does not exist an equilibrium with constant price $p\in(p^C,(1-q)/(1-\delta))$. Clearly, a necessary condition for a constant price in this interval to be sustainable, the marginal consumer of type $1-q$ has to be induced to buy the good at this price. Hence, we can confine attention to types v such that

$$v\in\left((1-\delta)p,\frac{1-\delta}{1-\delta+\beta\delta}p\right). \tag{5}$$

Note that an incarnation of such a type v is "happy" to consume in any future period in that the contribution to his utility from consumption in any period is positive. In contrast, the contribution to his utility from present consumption at price p is negative. That is, period t incarnation's most preferred consumption stream is $\{x_s\}_{s\ge t}=\{0,1,1,1,...\}$, and the worst is $\{1,0,0,0,...\}$. Hence, the best reward for consumption in the present period is consumption in all future periods: $\{1,1,1,....\}$, and the worst possible punishment for not consuming today is never to consume again: $\{0,0,0,....\}$. Hence, at date t, we compare consumption in all periods to no consumption. The first gives weakly higher utility than the latter if and only if

$$v-p+\beta\delta\sum_{s=0}^{\infty}\delta^s v\ge 0,$$

i.e.,

$$v\ge\frac{1-\delta}{1-\delta+\beta\delta}p,$$

which is in contradiction to (5). This completes the proof of the first claim.

We now claim that there does not exist an equilibrium with constant price p such that $p\ge(1-q)/(1-\delta)$. Since we are interested in the behavior of the marginal consumer, we can confine attention to types v such that

$$v\le(1-\delta)p.$$

Observe that any incarnation of such a type would never like to consume in that the contribution to his utility from consumption in any period is nonpositive.

Consider now the behavior of period T incarnation of type v. His (indirect) utility is bounded from below by

$$\beta\delta\left(\frac{v}{1-\delta}-p\right),$$

as period T incarnation may decide not to buy in T, and the worst possible punishment is consumption in all future periods. The contribution of no consumption in T and consumption thereafter to period $T-1$ incarnation's indirect utility is

$$\beta\delta^2\left(\frac{v}{1-\delta}-p\right).$$

Now, period T incarnation may decide to buy at T and at dates $\{T+t_k\}_k$, which gives him utility of

$$\begin{aligned} & v-(1-\beta\delta)p+\beta\sum_k \delta^{t_k}\left[v-(1-\delta)p\right] \\ \geq\ & \beta\delta\left(\frac{v}{1-\delta}-p\right), \end{aligned}$$

where the inequality follows from the fact that he may decide not to buy in T. The contribution of this consumption stream (from T onwards) to period $T-1$ incarnation's utility is

$$\begin{aligned} & \beta\delta\left[v-(1-\delta)p\right]+\beta\delta\sum_k \delta^{t_k}\left[v-(1-\delta)p\right] \\ \geq\ & \beta\delta^2\left(\frac{v}{1-\delta}-p\right)+\delta\left(1-\beta\right)\left(p-v\right) \\ >\ & \beta\delta^2\left(\frac{v}{1-\delta}-p\right) \text{ if } v<p \text{ (as assumed).} \end{aligned}$$

Repeating this exercise for all previous incarnations, we obtain that by not buying in $t=0$, period 0 incarnation can ensure himself a utility level of at least

$$\beta\delta^{T+1}\left(\frac{v}{1-\delta}-p\right),$$

which converges to zero as $T\to\infty$. In contrast, if period 0 incarnation decided to buy in $t=0$, his utility would be bounded from above by

$$v-(1-\beta\delta)p<0.$$

This concludes the argument.

Proposition 6 *The "collusive" price p^C is the highest price that can be supported in any stationary competitive equilibrium.*

Proof. We have already shown that p^C can be supported in a stationary competitive equilibrium by collusive strategies. Hence, it remains to be shown that a higher price cannot be supported in a stationary equilibrium.

For a price $p > p^C$ to be supportable in a stationary equilibrium, there must be some type $v \leq 1 - q$ who buys the good in some period at this price. We then have to show that there exists a profitable deviation for some incarnation of such a type.

The proof proceeds along the same lines as the proof of the previous proposition. ■

With Propositions 5 and 6, we have established that the set of prices of all stationary equilibria must be a subset of $[p^M, p^C]$.

Corollary 1 *Suppose p is the price in a stationary competitive equilibrium. Then, $p \in [p^M, p^C]$.*

Note that $\lim_{\beta \to 1} p^M = \lim_{\beta \to 1} p^C$, which means that the set of stationary equilibrium prices shrinks to a single price as consumers' discounting becomes exponential.

We can obtain any price $p \in [p^M, p^C]$ by assuming that a fraction $\lambda(p)$ of consumers play the collusive strategy in their intrapersonal game, while all others use Markov strategies. This requires that the population is heterogeneous with respect to their type v and with respect to their "personality", expressed by their intrapersonal strategies. For any $\lambda \in (0, 1)$, no trade between consumers occurs along the equilibrium path and the allocation of the durable good is inefficient: there is some low type v' with collusive intrapersonal strategies, who always buys the good along the equilibrium path, and a higher type $v'' > v'$ with a Markovian strategy profile, who never buys the good.

Consider instead the following population which is ex ante only heterogeneous with respect to their type v: at each point in time an incarnation chooses i.i.d. the collusive strategy with probability λ and the Markov strategy otherwise. Based on the randomisation device, a consumer with realisation $z_t \in Z^C \subseteq Z$ with $\mu(Z^C) = \lambda$ follows the collusive strategy, where a past deviation from collusion in period $s < t$ is only punished if $z_s \in Z^C$. We require that the realised population mean corresponds to this probability λ in each period. Being collusive means here to condition one's actions only on the actions of those past incarnations who also used collusive strategies.

Proposition 7 *The set of equilibrium prices which results from all equilibria with a probabilistic mix between collusive strategies and Markov strategies is the set* $[p^M, p^C]$.

Proof. First, find type $\widehat{v}$ which is indifferent between buying and not buying with the collusive strategy. Clearly, this type does not buy under the Markovian strategy. This marginal collusive type is given by

$$\widehat{v} - (1-\beta\delta)p + \frac{\beta\delta\lambda}{1-\delta}(\widehat{v} - (1-\delta)p) = 0,$$

i.e.,

$$\widehat{v} = \frac{(1-\delta)\left((1-\beta\delta)+\beta\delta\lambda\right)}{(1-\delta)+\beta\delta\lambda}p.$$

Hence, types $v \in [\widehat{v}, (1-\beta\delta)p)$ buy only in the periods in which the incarnation has drawn a collusive strategy, and types $v \in [(1-\beta\delta)p, 1]$ buy always. Market clearing implies

$$q = 1-(1-\beta\delta)p + \lambda p\left((1-\beta\delta) - \frac{(1-\delta)\left((1-\beta\delta)+\beta\delta\lambda\right)}{(1-\delta)+\beta\delta\lambda}\right) \equiv f(\lambda).$$

Since

$$\lim_{\lambda\to 1} f(\lambda) = 1 - \frac{1-\delta}{1-\delta+\beta\delta}p$$

and

$$\lim_{\lambda\to 0} f(\lambda) = 1-(1-\beta\delta)p,$$

for any price $p \in [p^M, p^C]$, we can find a $\lambda \in [0,1]$ such that this price is supported in equilibrium. Similarly, for any $\lambda \in [0,1]$, there exists a stationary equilibrium with price $p \in [p^M, p^C]$. ■

The example serves well to make another point. We observed in the previous example with a population of consumers which is heterogeneous with respect to their strategy profiles that the allocation of the durable good is inefficient. This result is confirmed in the present example. In addition, there is trade of the durable good because consumers may be Markovian in some period whereas they are collusive in others.

Proposition 8 *In any competitive competitive equilibrium which is induced by intrapersonal equilibria with a probabilistic mix between collusive strategies and Markov strategies,* $\lambda \in (0,1)$, *some units of the durable good change hands and the allocation of the durable good is inefficient.*

Recall that in the absence of secondary markets the resulting allocation is efficient. Hence, in our model with time-inconsistent consumers additional trading opportunities may generate allocative inefficiency.

Remark 1 *In the literature on repeated games, the question has been answered which restrictions on the discount parameter apply to sustain collusion if the number of punishment periods is finite (see Abreu, 1986, 1988). We can also address this question in our model. For this, the strategy profile has to be rewritten. Informally, a period t incarnation must be able to tell whether an action by some incarnation $s < t$ which is different from the action along the equilibrium path is a deviation which has to be punished or whether it is a punishment to some earlier deviation which itself is not to be punished. For a stationary price p the marginal consumer type with a punishment of k periods, v^k, solves*

$$v^k - p + \beta \sum_{t=1}^{k} \delta^t v + \beta \delta^k p = 0.$$

If a deviation which occurred within the last k periods triggers a punishment of k periods, the collusive price p^k is

$$p^k = \frac{1 + \beta\delta \frac{1-\delta^k}{1-\delta}}{1 + \beta\delta^{k+1}} (1 - q).$$

Clearly, $p^0 = p^M$ and $p^\infty = p^C$. Note that $p^k < p^{k+1}$, i.e., a longer punishment can support a higher price.

5 Non-stationary equilibria

5.1 Collusive strategy when prices are non-stationary

In this section, we show that non-stationary equilibria may obtain in our stationary durable goods environment. Analysing non-stationary equilibria makes it necessary to allow consumers to adopt different types of strategies over time. The strategy to be adopted may be interpreted as the "personality" of a particular incarnation so that we will say that a consumer's incarnation has a "collusive personality" or a "Markovian personality".

We introduce some notation. Denote by $v_t^M = p_t - \beta\delta p_{t+1}$ the consumer type with a Markovian personality who is indifferent between buying and not buying the durable good. Denote by v_t^C the consumer type with a collusive personality who would be indifferent between buying and not buying, given a constant price $p = p_t$, and presuming that all future incarnations who have a collusive personality do not deviate from the associated strategy. For the moment, consider the case in which all future incarnations of a collusive incarnation also have a collusive personality, then the marginal consumer type would be $v_t^C = \frac{1-\delta}{1-\delta+\beta\delta} p_t$. If, in some period, there is a positive probability $\gamma_{t,t+\tau} \leq 1$ of transition from the collusive personality in period t to the Markovian in period $t+\tau$, the marginal consumer v_t^C solves

$$v - p_t + \beta\delta p_{t+1} + \beta \sum_{\tau=1}^{\infty} \gamma_{t,t+\tau} \delta^{\tau} (v - p_{t+\tau} + \delta p_{t+\tau+1}) = 0.$$

The idea for a collusive strategy was that a current deviation from $x_t = 1$ to $x_t = 0$ triggers no purchase in all future periods instead of purchase. For collusion along a sequence of incarnations to work, there must neither be a date T after which it is optimal not to consume in any case nor a date after which it is always optimal to consumer using the Markov strategy. In the former case, the collusive effect is limited to a finite number of periods and, using backward induction, collusion cannot be sustained. Similarly, if a Markovian incarnation consumes in some distant future t sticking to the simple collusive strategy is not rational in $t-1$. Again, by backward induction collusion cannot be sustained. These two results are stated in the following lemmas.

Lemma 4 *Given the sequence of prices $\{p_t\}$, the period t incarnation of any consumer type $v < v_t^M$ such that there exists a period T with $v > v_s^M$, for all $s \geq T$, chooses no consumption $x_t = 0$, $t \geq 0$, along the equilibrium path in any subgame perfect equilibrium of the intrapersonal game.*

Lemma 5 *Given the sequence of prices $\{p_t\}$, any consumer with type v such that there exists a period T with $v < v_s^C$, for all $s \geq T$, chooses no consumption $x_t = 0$, $t \geq 0$, along the equilibrium path in any subgame perfect equilibrium of the intrapersonal game.*

With a nonconstant sequence of prices it may also become optimal to consume after some period $t \geq 1$ if all future incarnations follow the collusive strategy defined in the previous section, given the truncated history following t. We postulate that an earlier choice of no consumption is punished by future incarnations if consumption was rational and if the respective incarnation had a collusive and not a Markovian personality.

We will now formally define a collusive strategy profile given any sequence of prices $\{p_t\}$. Consumption of the durable good along the equilibrium path is denoted by $\widetilde{x}_t$. The collusive strategy is constructed such that a deviation from $x_{t-\tau} = 1$ to $x_{t-\tau} = 0$ in period $t-\tau$ is punished in period t whenever the consumer, being collusive, should have consumed in this period ($\widetilde{x}_{t-\tau} = 1$).

Definition 2 *Given the sequence of prices* $\{p_t\}$*, the* extended collusive strategy profile $\widetilde{\Sigma}^C(\{p_t\}; v)$ *in consumer* v*'s intra-personal game is defined as follows:*

$$
\begin{aligned}
x_0 &= \widetilde{x}_0 \\
x_t &= \begin{cases} 0 & \text{if } \exists \tau > 0 : \widetilde{x}_{t-\tau} = 1,\ x_{t-\tau} = 0, \\ & \text{and } v < v_t^M \\ \widetilde{x}_t & \text{otherwise,} \end{cases} \qquad t \geq 1 \\
\widetilde{x}_t &= \begin{cases} 0 & \text{if } v < \min\{v_t^M, \max_{s \geq t} v_s^C\} \\ 0 & \text{if } v \in (\min_{s \geq t} v_s^M, v_t^M) \\ 1 & \text{otherwise,} \end{cases}
\end{aligned}
$$

Lemma 6 *For any sequence of prices* $\{p_t\}$*, the extended collusive strategy profile* $\widetilde{\Sigma}^C(\{p_t\}; v)$ *forms an SPE in consumer* v*'s intrapersonal game.*

Since a consumer with a collusive personality in period t may have had past Markovian incarnations and he has to interpret each past consumption decision also with respect to whether the corresponding incarnation had a collusive or a Markovian personality, the definition of a strategy profile incorporating this shift of personality along incarnations has to include the history of personalities of a consumer. We do this by assuming that past realisations of the randomisation device are common knowledge in the intrapersonal game. Each consumer has a private history $\{x_\tau, z_\tau\}_{\tau < t}$. If the period t incarnation of a particular consumer has a collusive personality, we write $z_t \in Z^C$. A Markovian incarnation is denoted by $z_t \in Z^M$.

Definition 3 *Given the sequence of prices* $\{p_t\}$ *and personalities* $\{z_t\}$*, the* strategy profile $\Sigma^*(\{p_t\}, \{z_t\}; v)$ *in consumer* v*'s intra-personal game is defined as follows:*

$$
\begin{aligned}
x_0 &= \begin{cases} \widetilde{x}_0 & \text{if } z_0 \in Z^C \text{ and } v < v_0^M \\ x_0^M & \text{if } z_0 \in Z^M \text{ or } v \geq v_0^M \end{cases} \\
x_t &= \begin{cases} x_t^C & \text{if } z_t \in Z^C \text{ and } v < v_t^M \\ x_t^M & \text{if } z_t \in Z^M \text{ or } v \geq v_t^M \end{cases} \quad t \geq 1 \\
x_t^M &= \begin{cases} 0 & \text{if } v < v_t^M \\ 1 & \text{otherwise} \end{cases} \\
x_t^C &= \begin{cases} 0 & \text{if } \exists \tau > 0 : \widetilde{x}_{t-\tau} = 1,\ x_{t-\tau} = 0, \text{ and } z_{t-\tau} \in Z^C \\ \widetilde{x}_t & \text{otherwise,} \end{cases} \\
\widetilde{x}_t &= \begin{cases} 0 & \text{if } v < v_s^C, \text{ for some } s \geq t, \\ 0 & \text{if } v \in (\min_{s \geq t} v_s^M, v_t^M), \\ 1 & \text{otherwise,} \end{cases}
\end{aligned}
$$

Lemma 7 *For any sequence of prices $\{p_t\}$ and personalities $\{s_t\}$, the strategy profile $\Sigma^*(\{p_t\}, \{z_t\}; v)$ forms an SPE in consumer v's intrapersonal game.*

Note that different versions of a collusive strategy can be formulated which also give rise to an SPE of the intrapersonal game. In such an equilibrium, outcome $\{x_t\}$ may also differ from the outcome in the SPE from above for nonmonotone sequences $\{v_t^M\}$ or $\{v_t^C\}$. Since we will not consider such sequences in the following subsections on increasing and decreasing price paths, we avoid such arbitrariness in the corresponding result.

5.2 Increasing price paths

In this subsection we consider increasing price paths. Suppose that in the initial period there is a positive mass of consumers using Markov strategies. In each period, a share of the Markovians becomes collusive. Once an incarnation has drawn the collusive strategy, all future incarnations of this consumer will do so as well. Since there are only transitions from the Markovian to the collusive strategy, we only have to look at population weights for each type v and do not need to specify how such transitions are made for each consumer who "inherits" a Markovian strategy in a particular period. One possible specification is that each consumer knows that with some probability he will become collusive and that this probability may be time dependent but identical across consumers in each period. The realisation of the corresponding random variable, which determines the strategy to be adopted, is assumed to be common knowledge among current and future incarnations of a consumer.

Note that when prices are increasing permanent consumption seems less and less attractive. Recall that v_t^C denotes the marginal consumer type when using collusive strategies for constant prices $p = p_t$. The sequence v_t^C is increasing and only consumer types above $v_\infty^C = \lim_{t\to\infty} v_t^C$ possibly buy the good (see Lemma 5).

We first consider an example in which some consumers change strategy only from period 0 to period 1.

Example 1 The share of consumers who follow a collusive strategy in each period is λ_t with $\lambda_0 < \lambda_1 = ... = \lambda_\infty$. As stated above, if an incarnation in period t follows the collusive strategy we assume that all future incarnations will also be collusive. Hence, $(\lambda_1 - \lambda_0)/(1 - \lambda_0) = \mu(Z^C)$ where μ is a probability measure. Here, $\mu(Z^C)$ is equal to the share of consumers who become collusive in period 1 among those consumers who were Markovian in period 0.

The truncated model starting in $t = 1$ is stationary so that $p_t = p_{t+1}$, $t \geq 1$. We have

$$v_t^C = \frac{1-\delta}{1-\delta+\beta\delta} p_t$$

and $v_t^M = (1 - \beta\delta)p_t$. The market clearing condition for $t \geq 1$ is

$$q = \lambda_t(1 - v_t^C) + (1 - \lambda_t)(1 - v_t^M),$$

which yields the equilibrium price

$$p_t = \frac{1-\delta+\beta\delta}{\lambda_t(1-\delta) + (1-\lambda_t)(1-\beta\delta)(1-\delta+\beta\delta)}(1-q). \tag{6}$$

We postulate that $p_0 < p_1$. Hence, $v_0^C < v_1^C$ so that in period 0 only incarnations of type $v \geq v_1^C$ buy the durable good if they follow the collusive strategy. Clearly, $v_0^M = p_0 - \beta\delta p_1$. Under the hypothesis that $v_1^C \leq v_0^M \leq v_1^M$, from the market clearing condition for period 0,

$$q = \lambda_0(1 - v_1^C) + (1 - \lambda_0)(1 - v_0^M),$$

we obtain

$$p_0 = \frac{1}{1-\lambda_0}\left((1-q) + \frac{(1-\lambda_0)\beta\delta(1-\delta+\beta\delta) - \lambda_0(1-\delta)}{1-\delta+\beta\delta} p_1\right).$$

Substituting p_1, the price in period 0 is

$$p_0 = \frac{1-q}{1-\lambda_0} \frac{(\lambda_1 - \lambda_0)\left((1-\delta) + \beta\delta(1-\delta+\beta\delta)\right) + (1-\lambda_1)(1-\delta+\beta\delta)}{\lambda_1(1-\delta) + (1-\lambda_1)(1-\beta\delta)(1-\delta+\beta\delta)}. \tag{7}$$

If $\beta, \delta, q \in (0,1)$, and $\lambda_0 < \lambda_1$, it is easily checked that $p_1 > p_0 \geq 0$ and $v_1^C \leq v_0^M \leq v_1^M$, as assumed. Note that $v_1^C < v_0^M$ if and only if $\lambda_1 < 1$, i.e., there remains a positive share of Markovian consumers.

Above we have constructed an equilibrium in which $p_0 < p_1$. Our result can be summarised as follows.

Proposition 9 *There exist competitive equilibria with a nondecreasing price path such that $p_t < p_{t+1}$ for at least one period t.*

Consider the special case $\lambda_1 = ... = \lambda_\infty = 1$ in example 1. In this case, market clearing in period $t \geq 1$ implies $v_t^C = 1 - q$. Consequently, market clearing in period 0 implies $v_0^M = v_1^C$. We obtain a simple relationship between p_1 and p_0, namely

$$p_0 = (1-q) + \beta\delta p_1.$$

Example 2 Suppose now $\lambda_1 < ... < \lambda_{T-1} < \lambda_T = ... = \lambda_\infty = 1$. In period T and beyond our previous results hold. For market clearing we must have $v_t^M = v_T^C$ for any $t < T$. For any price p_t, $t < T$, we have

$$p_t = (1 - q) + \beta\delta p_{t+1}$$

so that

$$p_t = (1 - q)\frac{1 - (\beta\delta)^{T-t}}{1 - \beta\delta} + (\beta\delta)^{T-t} p_T.$$

Price in period T and all later periods is

$$p_T = \frac{1 - \delta + \beta\delta}{1 - \delta}(1 - q).$$

This determines the whole equilibrium price path provided that $p_1 < ... < p_T$. The condition $p_t < p_{t+1}$, $t < T$, is satisfied for all $\beta, \delta, q \in (0, 1)$. Hence, we have provided an example for a price path which is strictly increasing for the initial T periods and constant afterwards. Furthermore, in the example $p_T = p^C$. Note also that $\lim_{T\to\infty} p_0(T) = p^M$.

We now show that convergence to the stationary collusive price p^C is due to the fact that consumers with the collusive strategy become a set of full measure in finite time. If $\lambda_t < 1$ in finite time, then an equilibrium price path cannot converge to the collusive price p^C even if $\lim_{t\to\infty} \lambda_t = 1$.

Proposition 10 *Suppose that $\lambda_t < 1$ for all finite t. Then, in the limit as $t \to \infty$, any converging equilibrium price path $\{p_t\}$ can only converge to a price $p_\infty < p^C$.*

Proof. For an equilibrium price path to converge to the collusive price, $\{\lambda_t\}$ has to converge to 1 as time tends to infinity. If v_t^M is nondecreasing, which is equivalent to $\Delta p_t \equiv p_{t+1} - p_t \leq (\beta\delta)^{-1}\Delta p_{t-1}$, then demand from consumers with collusive strategies is $\lambda_t(1 - v_\infty^C)$ (see Lemma 5). If v_t^M is decreasing, then demand from consumers with collusive strategies is $\lambda_t(1 - v_t^M + v_\infty^M - v_\infty^C)$ (see Lemmas 4 and 5). In general, demand from consumers with collusive strategies is $\lambda_t(1 - v_\infty^C - \kappa_t)$, with $\kappa_t \geq 0$. Demand stemming from Markovian consumers is $(1 - \lambda_t)(1 - v_t^M)$. Market clearing in period t can be written as

$$q = \lambda_t\left(1 - v_\infty^C - \kappa_t\right) + (1 - \lambda_t)(1 - v_t^M).$$

Suppose that the equilibrium price path converges, i.e. for any $\varepsilon > 0$ one can find a t sufficiently large such that all prices starting with p_t are in the ε-neighborhood of some price $p_\infty(q)$. Since prices converge, $\lim_{t\to\infty} \kappa_t = 0$. This implies that $v_\infty^C = 1 - q$. For t sufficiently large, $v_\infty^C < v_s^M$ for all $s \geq t$. Since also $\kappa_t \geq 0$, the market clearing condition is violated for periods t sufficiently far in the future. ■

5.3 Decreasing price paths

In this subsection, we consider nonincreasing price paths and allow for changes from collusive strategies to Markov strategies. Suppose that a share λ_t of consumers follows a collusive strategy in period t and a share $1 - \lambda_t$ a Markov strategy. The sequence $\{\lambda_t\}$ is assumed to be nonincreasing over time.

Suppose there exists a nonincreasing path of consumer types which are indifferent between buying and not buying being Markovian. This sequence $\{v_t^M\}$ converges to some v_∞^M. Consequently for consumer types above v_∞^M an incarnation which has drawn the collusive strategy consumes if and only if it would also do so with a Markovian strategy. Since the price path is nonincreasing the sequence $\{v_t^C\}$ is also nonincreasing. This implies that all consumer types between v_t^C and v_∞^M who follow the collusive strategy buy the durable good in period t, provided $v_t^C < v_\infty^M$. Then the market clearing condition in period t can be written as

$$\begin{aligned} 1 - q &= \lambda_t(\max\{0, 1 - v_t^M\} + \max\{0, v_\infty^M - v_t^C\}) + (1 - \lambda_t) \max\{0, 1 - v_t^M\} \\ &= \max\{0, 1 - v_t^M\} + \lambda_t \max\{0, v_\infty^M - v_t^C\}. \end{aligned}$$

This market clearing condition has to hold in period t if the sequence $\{v_s^M\}$ is nonincreasing which is equivalent to $\Delta p_s \leq \beta\delta\Delta p_{s+1}$ for all s.

In this case, more consumers using the Markovian buy the durable good over time and more consumer using the collusive strategy buy it as well. Also, the share of Markovian increases. In order to possibly have an equilibrium, the average (over v) loss of consumption from reducing the size of collusive incarnations must exactly offset the average gain in consumption from increasing the size of consumers following the Markovian strategy. A simple example serves well to illustrate that a time path can be decreasing for some periods.

Example 3 Consider the following example in which, in period 0, all consumers follow the collusive strategy. In period 1, the share ρ follow a Markov strategy, whereas the share $(1 - \rho)$ keep playing according to the collusive strategy. The likelihood of experiencing such a strategy change over time is unrelated to the type v. The consumers who stick to a collusive strategy as period 1 incarnation follow this strategy in all future periods and also those consumers whose period 1 incarnation adopted the Markovian strategy follow the Markovian strategy in all future periods. Hence, from period 1 onward, the environment is stationary; i.e. the price path is constant: $p_1 = p_2 = ... = p_\infty$. For $t \geq 1$, the marginal consumer type who follows the Markov strategy is $v_t^M = (1 - \beta\delta)p_t$, and the marginal consumer type who follows the collusive strategy is

$$v_t^C = \frac{1-\delta}{1-\delta+\beta\delta} p_t.$$

Assume for the moment that $v_t^M \leq 1$, i.e., there exist consumer types who buy the durable good with a Markovian strategy in periods $t \geq 1$. Note that, for $t \geq 1$, $v_t^C = v_\infty^C$ so that the market clearing condition reads

$$q = \rho(1-(1-\beta\delta)p_1) + (1-\rho)\left(1 - \frac{1-\delta}{1-\delta+\beta\delta} p_t\right).$$

This yields the period t, $t \geq 1$, equilibrium price:

$$p_t = \frac{1-\delta+\beta\delta}{\rho(1-\beta\delta)(1-\delta+\beta\delta)+(1-\rho)(1-\delta)}(1-q).$$

In $t=0$, all consumers follow the collusive strategy. Note that $v_1^M < v_0^M$ because $p_2 = p_1 < p_0$ so that the sequence $\{v_t^M\}$ is indeed nonincreasing over time. The market clearing condition for period 0 is

$$\max\{0, 1-v_0^M\} + \max\{0, v_1^M - v_0^C\} = q$$

(see Lemma 4). Although no consumers follows the Markov strategy initially, we have to calculate the marginal consumer v_0^M because it partly determines which of those consumers who use the collusive strategy do buy the durable good in period 0.

(i) Suppose $p_0 > p_1$, $v_0^C \leq v_1^M$, and $v_0^M \leq 1$. Marginal consumer type v_0^M is $v_0^M = p_0 - \beta\delta p_1$. A collusive type prefers not to buy irrespective of backward induction considerations if

$$v_0 - p_0 + \beta\delta p_1 + (1-\rho)\left(\frac{\beta\delta}{1-\delta} v_0 - \beta\delta p_1\right) < 0.$$

Note that the conditional indirect utility only includes the future discounted utility from consuming the durable good in case future incarnations of the consumers follow the collusive strategy because future incarnations following the Markovian strategy do not buy the durable good. In case period 0 incarnation deviates by not buying the durable good in period 0, also future incarnations following the collusive strategy would not buy it, which explains the right-hand side of the inequality. The marginal type v_0^C who is indifferent between buying and not buying is

$$v_0^C = \frac{1-\delta}{1-\delta+\beta\delta(1-\rho)}\left(p_0 - \beta\delta\rho p_1\right).$$

Inserting the expressions for v_0^M, v_1^M, and v_0^C into the market clearing equation gives

$$q = 1 - (p_0 - \beta\delta p_1) + (1-\beta\delta)p_1 - \frac{1-\delta}{1-\delta+\beta\delta(1-\rho)}\left(p_0 - \beta\delta\rho p_1\right),$$

and

$$p_0 = \frac{(1-\delta+\beta\delta(1-\rho))(1-q) + (1-\delta+\beta\delta(1-\rho)+(1-\delta)\beta\delta\rho)p_1}{2(1-\delta)+\beta\delta(1-\rho)}.$$

Inserting the expression for p_1 gives the candidate equilibrium price for period 0 which depends on the parameters β, δ, and ρ and on initial supply q.

$$\begin{aligned} p_0 \quad = \quad & \frac{1}{2(1-\delta)+\beta\delta(1-\rho)}\Bigg((1-\delta+\beta\delta(1-\rho)) + \\ & \frac{(1-\delta+\beta\delta(1-\rho)+(1-\delta)\beta\rho))(1-\delta+\beta\delta)}{\rho(1-\beta\delta)(1-\delta+\beta\delta)+(1-\rho)(1-\delta)}\Bigg)(1-q) \end{aligned}$$

(ii) It remains to be shown that $p_0 > p_1$ and $v_0^C \leq v_1^M$ and $v_0^M \leq 1$. For $\beta, \delta, q \in (0,1)$, the inequality $p_0 > p_1$ holds for all $\rho \in (0,1)$. The inequality $v_0^C \leq v_0^M$ holds if $\beta, \delta \in (0,1)$ and $\lambda \in [0,1]$.

In order to have $v_0^M \leq 1$, one must show $p_0 \leq 1 + \beta\delta p_1$. The condition is a polynomial in q, β, δ, and ρ which can be checked for a given choice of q and parameters β, δ, and ρ. For instance, the following restrictions imply that $v_0^M \leq 1$:

- $q \geq \frac{1}{2}$, $\beta > \delta$, and $\beta > \rho$, or
- $q \geq \frac{1}{2}$, $\beta \geq \frac{1}{2}$, and $\rho \leq \frac{1}{2}$.

Clearly, if $v_0^M \leq 1$ and $p_0 > p_1$, one has $v_t^M < 1$, which has been assumed above.[11]

With the example we have demonstrated that the equilibrium price can decrease from one period to another. This is stated in the following result.

[11] Obviously, for certain q and certain parameter values, there exist equilibria in which no consumer with a Markov strategy would buy the durable good in period $t = 0$. In order to use the results for periods $t \geq 1$ one has to check that $v_t^M \leq 1$, which, for instance, is implied by $q \geq (1-\beta)\delta$. To determine the equilibrium price p_0 in period 0, the market clearing equation for period 0 reads $\max\{0, v_1^M - v_0^C\} = q$. Equilibrium price p_0 (presuming the equilibrium exists) has to be even higher than the price which results from the above calculations.

If $v_t^M \geq 1$ for $t \geq 1$, then consumers with a Markov strategy never consume the durable good. This implies that the increase of the share of consumers with Markov strategies after period 0 is equivalent to a reduction in the number of consumers in the market, holding the distribution of consumer types fixed. Since the market has to clear, $v_0^C > v_1^C$ and $p_0 > p_1$.

Proposition 11 *There exist competitive equilibria with a nonincreasing price path such that $p_t > p_{t+1}$ for at least one period t.*

The allocative inefficiency takes a strong form in period 0: in this period all consumers are identical apart from differences in type ν and consumer types with $v \in [v_0^C, v_1^M] \cup [v_0^M, 1]$ buy the durable good in the initial period, which implies that, in equilibrium, there exist types v', v'' with $v' < v''$ with $x_0(v') = 1$, whereas $x_0(v'') = 0$. In words, some consumers with a relatively low willingness to pay buy the good whereas others with a higher willingness to pay do not buy the good.

5.4 Price cycles

This last subsection contains a very simple example of price cycles.

Example 4 Suppose that all consumers follow a revised collusive strategy, as defined below, in period 0 and all even periods and the Markovian strategy in all odd periods. Hence, we consider for almost all consumers a sequence $\{z_t\}$, $z_t \in Z^M$ if $t = 1, 3, ...$ and $z_t \in Z^C$ otherwise. This is to say that $\mu_t(Z^M) = 1$ if $t = 1, 3, ...$ and $\mu_t(Z^M) = 0$ otherwise. We consider the strategy profile Σ^* defined in the previous section according to which collusive incarnations only consider the actions of past collusive incarnations and ignore the actions taken by Markovian incarnations. Presuming that v_t^C and v_t^M are constant for $t = 0, 2, 4, ...$, this reads

$$x_0 = \begin{cases} 1 & \text{if } v \geq v_t^C, \\ 0 & \text{otherwise,} \end{cases}$$

$$x_t = \begin{cases} 1 & \text{if } v \geq v_t^M, \\ 1 & \text{if } x_{t-s} = 1 \text{ for all } s = 2, 4, ... \text{ where } s \leq t \\ & \text{and } v \in (v_t^C, v_t^M), \\ 0 & \text{otherwise,} \end{cases} \qquad t \geq 1$$

if $z_t \in C$ and

$$x_t = \begin{cases} 0 & \text{if } v < v_t^M \\ 1 & \text{otherwise} \end{cases} \quad t \geq 0$$

if $z_t \in M$.

We have $v_t^C = p_t(1 - \delta^2)/(1 - \delta^2 + \beta\delta^2)$, $t = 0, 2, 4, ...$ and market clearing for these periods implies that the price is

$$p_t = (1 - q)\frac{1 - \delta^2 + \beta\delta^2}{1 - \delta^2}.$$

Since in odd periods all consumers are Markovian we have $1 - q = v_t^M = p_t - \beta\delta p_{t+1}$, $t = 1, 3, 5, \ldots$. Substituting price in a period with collusive personalities for p_{t+1} we obtain

$$p_t = (1-q)\frac{(1+\beta\delta)(1-\delta^2)+\beta^2\delta^3}{1-\delta^2},\ t = 1, 3, 5, \ldots$$

Note that

$$p_t - p_{t+1} = (1-q)\frac{\beta\delta}{1-\delta^2}\left(\beta\delta(1-\delta^2) - \beta\delta^2(1-\beta\delta)\right).$$

Along the equilibrium path, the price cycles if this difference is positive, which is ensured by assumption $\beta > \delta$.

Let us summarise our result as follows.

Proposition 12 *There exist competitive equilibria with price cycles. In particular, $p_t > p_{t+1} < p_{t+2} = p_t$ for periods $t = 0, 2, 4, \ldots$*

6 Conclusion

In this paper, we have analysed perfectly competitive secondary markets for a durable good. In our market environment, secondary markets are neutral when consumers are exponential discounters: (i) the price in the initial period does not depend on whether secondary markets are opened for some periods or not; (ii) the incentives for the provision of initial supply by a monopolist or oligopolist are not affected by the existence of the secondary markets; and (iii) no trade occurs in the secondary markets. With exponential discounting the allocation is efficient; that is, consumers with higher evaluations buy the good in equilibrium, whereas those with lower valuations do not. When consumers are hyperbolic discounters all these results no longer hold. In the absence of secondary markets, a purchase of the durable good in the initial period implies a commitment to consume the good in all future periods. When secondary markets open, such a commitment is no longer possible and a consumer may procrastinate. We have obtained the following non-neutrality results:

1. The price in the initial period is decreasing with the number of periods in which secondary markets are open (Section 3, Proposition 2).

2. The initial supply by a monopolist (with positive marginal costs) is the smaller, the later secondary markets close (Section 3, Proposition 3). This result carries over to the case where the initial supply is provided by a group of oligopolistic producers.

3. When secondary markets never close, there are equilibria in which trade in the durable good occurs in each period (Section 4, Proposition 8).

4. When secondary market never close, there are inefficient competitive equilibria: consumers with a relatively low willingness to pay buy the durable good whereas others with a higher willingness to pay do not buy the durable good (Section 4, Proposition 8). This inefficiency may obtain even if consumers use strategies from the same "family" of strategy profiles (Subsection 5.3).

We have characterised the set of stationary equilibria in the case that secondary markets never close. Equilibrium prices are bounded from below by the Markovian price, and bounded from above by the collusive price. The latter coincides with the unique equilibrium price when secondary markets never open. Apart from stationary equilibria, there also exist equilibria with increasing, decreasing, and cycling price paths, despite the stationarity of the market environment.

While we consider the present setup as particularly useful for studying the effects of hyperbolic discounting, there may be other interesting durable good environments. For instance, it may be worthwhile to study the case where consumers' willingness to pay changes over time. One may also want to analyse the case where one or several firms provide additional supply of the durable good over time. This introduces a Coasian commitment problem on the side of the supplier(s).

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