When are Auctions Best?

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Abstract

We compare the two most common bidding processes for selling a company or other asset when participation is costly to buyers. In an *auction* all entry decisions are made prior to any bidding. In a *sequential bidding process* earlier entrants can make bids before later entrants choose whether to compete. The sequential process is more efficient because entrants base their decisions on superior information. But pre-emptive bids transfer surplus from the seller to buyers. Because the auction is more conducive to entry in several ways it usually generates higher expected revenue.

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1 Introduction

A large body of both academic and practitioner opinion argues that sellers benefit from the simultaneous competition of an auction. The United States legal system strongly encourages boards of directors that receive a serious takeover offer to hold auctions as the way to maximize shareholder value.[1] Still, many assets ranging from private companies to most real estate are sold through sequential processes in which sellers use the threat of refusing an offer and waiting for more competitors as their weapon for negotiating a price. Many attractive, sophisticated buyers ranging from Warren Buffet to private equity firms to large petrochemical companies say they avoid auctions because the risk of expending costly effort for no return is too large.[2]

Participating in a sales process is generally costly, and a sequential search process economizes on those costs. If there are enough potential buyers and the seller is able to firmly commit at any time to its optimal minimum price then the seller can capture the gains from sequential entry.[3] But in markets such as that for corporate takeovers, it is generally the buyers who name prices, and a buyer may make a pre-emptive offer which can deter potential rivals from entering. Where such public pre-emptive bids are credible, their existence or absence conveys information which makes subsequent entry decisions more efficient, attracting *additional* bidders when the early ones turn out to be weak.[4] The issue for sellers who

[1]See, for example, Cramton and Schwartz (1991).

[2]Buffet states each year in his annual report under "Acquisition Criteria": "[W]e don't want to waste our time ... *We don't participate in auctions*." The italics are in the original.

Similarly, a managing director of a private equity fund says: "If ...I know it's a broad auction, even if it's a very nice business, ...I still may not take it ...seriously because [the] chances of winning are so slim. I just don't put the effort into it." Mergers and Acquisitions, "Surviving M&A Auction Rigors", December 2006, p.32.

[3]For example, see Riley and Zeckhauser (1983), and Milgrom (2004) who show that given an infinite stream of potential buyers with independent and identically distributed values from a known distribution a seller can achieve full efficiency and extract all expected surplus if it can commit to a firm posted price. McAfee and McMillan (1987) considered the case where, as in our auction model, costly entry is fully determined before any credible bids are made. In their model a seller who could choose an optimal entry fee could also extract all expected surplus. However, this mechanism would generate less revenue than an optimal posted price if there are enough potential bidders. On optimal mechanisms see, for example, Cremer, Spiegel, and Zheng (2006), Kjerstad and Vagstad (2000), McAfee and McMillan (1988), Menezes and Monteiro (2000), and the references therein. See Krishna (2002), Milgrom (2004), Menezes and Monteiro (2005), and Klemperer (2004) for syntheses of a large fraction of the auction literature.

[4]Fishman (1988) considered a model with two potential bidders, both of whom would have participated in an auction. In this case, a pre-emptive offer could deter entry by the second bidder but a sequential mechanism would not increase the number of bidders.

In addition to Fishman's pioneering paper, see also Hirshleifer and Png (1989) and Daniel and Hirshleifer (1998). These papers began the literature on jump bidding - see Avery (1998), Rothkopf and Harstad (1994), and Fujishima, McAdams, and Shoham (1999) for alternative discussions. Among the large theoretical can choose between simultaneous and sequential entry is whether the efficiencies generated by a sequential process are enough to offset the expected gains to buyers from preemptive bidding. That is, should a seller structure the sale process (for example, by timing the release of critical information) so that bidders are unable or unwilling to make early bids?

Bulow and Klemperer (1996) show under mild conditions that even a simple ascending auction with no reserve price, is more profitable than the best possible process that can be conducted with one fewer bidder. But with entry costs even if, as we will show, sequential mechanisms attract fewer bidders in expectation, they may attract more bidders when those bidders are most valuable. So is a simple auction still desirable?

This paper compares the two most commonly used and studied bidding processes in which entry is costly and the seller has limited power or information. In both processes, an unknown number of potential bidders make entry decisions sequentially, before learning their values. In an *auction*, no credible bidding is possible until all entry decisions have been taken. In a *sequential mechanism*, potential bidders arrive in turn. Each one observes the current price and bidding history and decides whether to pay the entry cost to learn its value. If it does, and if it succeeds in outbidding any current incumbent (who can respond by raising its own bid), it can also make any additional jump bid it wishes to attempt to deter further entry. In both bidding processes, we assume the seller does not have the power or credibility to commit to a take-it-or-leave-it minimum (reservation) price above a buyer's minimum possible value.

Our central result is that the straightforward, level-playing-field competition that an auction creates is *usually* more profitable for a seller than a sequential process, even though the sequential mechanism is more efficient (as measured by the winner's expected value less expected aggregate entry costs) because it is more informative.

We identify four factors that may cause the expected revenue between the auction and the sequential mechanism to differ. The first three all unambiguously favor the auction. Two of these factors are fairly straightforward. First, even in the most favorable circumstances, a sequential process could only be superior if the queue of potential bidders is sufficiently longer than the number that would compete in an auction. Second, in a sequential mechanism bidders who deter entry choose a price where the expected distribution of winning values is such that an additional entrant would expect to earn zero. By contrast, except in a

literature on endogenous entry into auctions, see Burguet and Sakovics (1996), Harstad (1990, 2003), McAfee and McMillan (1987), Menezes and Monteiro (2000), and the references therein.

knife-edge break-even case, bidders deterred from an auction face a distribution of winning values that make entry strictly unprofitable.

These two factors would be nullified with an infinite stream of potential bidders, and when parameters are such that the expected profits of the marginal bidder who does not enter the auction is exactly zero. The third factor is therefore crucial: because the value of the winning bidder is generally less dispersed in the sequential process, which is more likely to attract one high-value bidder but will never attract more than one, the expected value of the top bidder in the auction must be higher to deter entry, because dispersion makes the entrant's option to buy more valuable – this is just the standard consumer-theory result that consumers prefer more random prices.[5]

These three factors all speak to the general superiority of the auction in generating entry, and each factor provides a separate reason why the expected *value* of the winner in the auction must exceed the expected value of the winner in the sequential mechanism. But the seller ultimately cares about the expected *price* paid, and the comparison of expected prices is a somewhat different question. The fourth factor, then, involves the gap between expected winning values and expected revenues, and this can work either for or against auctions.

However, we show that for typical demand specifications a higher expected winner's value directly implies higher expected revenue. Furthermore, this fourth factor, like the third, depends upon the dispersion of winners' values being different in the two mechanisms. For the fourth factor to be important, the dispersion must be much greater in the auction than in the sequential mechanism, which in turn makes the third factor more significant. So even when the remaining two factors are unimportant, it is hard for the simple sequential mechanism to defeat the auction in raising expected revenue.

Part of the appeal of the auction is that the competition it creates allows the seller to do well independent of any knowledge of bidder values or any ability to exploit that knowledge. Arming either buyers or sellers with greater commitment power and the knowledge necessary to use it will modify the equilibrium in the sequential mechanism. If the seller can opportunistically subsidize entry it can increase expected revenue – mainly through the

[5] Put differently, a potential bidder is just like an investor considering the purchase of a stock-option, with the bidder's entry cost corresponding to the price of the option, so *ceteris paribus* the mechanism with the more uncertain price is more attractive – just as a stock option is more valuable the more volatile is the stock.

threat to subsidize.[6] The seller's ability to subsidise future bidders both prompts stronger types of current bidders to make higher deterring offers, and prevents some weaker types from deterring – so in many cases subsidies never actually need to be paid. But the sequential mechanism with subsidies can only be more profitable than an auction when the seller *both* has sufficient information and power to set the subsidies optimally *and* has enough potential bidders. And if a sequential *buyer* can make plausible threats to withdraw if a seller seeks additional bids then the advantage of the auction becomes larger.

In sum, the auction looks better than practical versions of the sequential mechanism on the revenue criterion.

Moreover, while the model suggests the sequential mechanism is more efficient than the auction, this may reflect the model's narrow view of efficiency. First, because the sequential mechanism gives more surplus to the earliest bidders, potential entrants may dissipate their surplus in a race to the front of the queue. In this case, net social surplus will equal seller revenue. And second, even if bidders' rents are not dissipated in this way, mechanisms that give sellers a greater share of the value of the asset sold provide greater incentives to create valuable assets. So since the auction usually raises more revenue, it may well be more socially efficient overall.

Our results are consistent with the observation that sellers of companies and other assets often try to run auctions, while buyers commonly try to avoid them. Indeed, in one recent poll of private equity firms approximately 90 percent said that they did not like to participate in auctions or buy businesses in auctions. Almost the same majority said that when acting as sellers they prefer an auction process.[7] While it is possible to design examples where bidders will prefer auctions, it will typically be in the interest of an early bidder to pre-empt the process if possible.

We begin (in section 2) with a model of a random number of symmetric risk-neutral potential bidders with search costs of finding out their private values of, for example, a company for sale.[8] We show (section 3) that auctions are less efficient but likely to raise

[6] If entry can both be subsidized *and taxed* without limit it is possible to have a fully efficient mechanism that extracts all surplus, regardless of the number of potential bidders.

[7] "Auction Process Roundtable", *Mergers and Acquisitions,* December 2006, pp. 31-32. Our model ignores any technological costs of choosing one selling mechanism over another (see e.g., Arnold and Lippman (1995), and Wang (1993, 1995, 1998)) or legal issues (see e.g., Cramton and Schwartz (1991), Gilson and Black (1995)). Nor do we discuss seller credibility, see, e.g. Lopomo (2000), McAdams and Schwarz (2007).

[8] Our basic model assumes bidders enter any auction sequentially (so the maximum number that enters is deterministic). However, we will show our results are essentially unaffected if we instead assume simultaneous entry into the auction by many potential bidders following mixed entry strategies as, for example, in Levin

more revenue than sequential bidding, and explain why. In section 4 we consider how bidding subsidies and other simple tactics such as lock-ups, break-up fees, and matching rights may be used by sellers to improve their expected revenue from the sequential mechanism. We also illustrate how increased commitment power on the part of buyers can reduce revenue. Section 5 concludes.

2 The Model

We compare two methods of selling an asset for which there is a queue of risk-neutral potential buyers. The probability that at least j potential bidders exist, given the existence of at least $j-1$, is ρ_j, $0 \leq \rho_j \leq 1$. We write $\{\rho_j\}_{j=1}^{\infty}$ for $\rho_1, \rho_2, \rho_3, \ldots$. Each buyer decides in turn whether or not to pay a cost c to learn its own private value, which is drawn independently from the distribution $F(v)$ with a continuous density $f(v)$; $F(\underline{v}) = 0$ and $F(\overline{v}) = 1$; $\underline{v}$ equals the seller's value,[9] and $\overline{v} \leqslant \infty$. All this is common knowledge among the buyers (the seller needs no knowledge of $F(\cdot)$, c, or $\{\rho_j\}_{j=1}^{\infty}$).

In the *Auction* mechanism, players cannot make any credible bid commitments until after entry stops. So when potential buyers make their entry decisions they observe only how many bidders have entered thus far.[10] When entry has stopped, there is a standard English auction in which the bidder with the highest value wins and pays the value of the second highest.[11]

Our *Sequential* mechanism is one that is commonly assumed for the sale of a company (see, e.g., Fishman (1988)). The first bidder simply chooses an initial bid. Any subsequent entrant may enter an English auction with the incumbent high bidder, and the winner of that competition may then, if it wishes, make a jump bid before finding out whether any subsequent potential entrant(s) actually exist. We will see later that neither the seller's, nor the bidders', expected surpluses – and therefore also none of the important results – would be affected if jump bidding were also permitted prior to, or during, the competition between

and Smith (1994).

[9] The assumption that $\underline{v}$ equals the seller's value is not required for any of our Propositions, but simplifies the discussion in section 3.5.

[10] They do not know whether any subsequent potential entrant(s) actually exist, but this makes no difference to the important results - see note 23. Our results are also essentially unaffected if we assume simultaneous entry into the auction by many potential bidders following mixed entry strategies - see section 3.5.

[11] The Revenue Equivalence Theorem implies our results hold for any standard auction. See Myerson (1981) and Riley and Samuelson (1981).

the current incumbent and current entrant. Nor would it make any difference if the actual number of potential entrants was revealed in advance of the game, or at any point during the game.[12]

Bidders cannot lower bids that they have already made. Potential entrants observe all previous bids at the time they make their entry decisions, and when entry stops the current high bidder wins the asset at its bid.

We consider the perfect Nash equilibrium of the auction, which is unique except for knife-edge cases with two equilibria when the last bidder is indifferent to entering; in this case we take the worse case (the last bidder does not enter) for the auction.[13]

We also consider all the perfect Nash equilibria of the sequential mechanism in which any buyer with a value greater than or equal to some constant cutoff value v^* makes a high enough preemptive bid that all subsequent entry is deterred.[14] However, we focus most attention on the unique perfect sequential equilibrium (as is standard in contexts like this one – see below).[15] We refer to this *equilibrium* deterrring value of the $\underline{S}$equential mechanism as V_S.[16] We will refer to the minimum value that a bidder would need to have to deter if its value were $\underline{K}$nown by potential entrants as V_K.

To avoid trivialities, we assume there is at least one potential bidder and maybe more ($\rho_1 = 1$ and $\rho_2 > 0$), and that at least two bidders would be attracted into an auction if they exist.[17] The seller can demand a minimum price of $\underline{v}$ in both mechanisms but no higher. There is no discounting.

[12] See note 23.

[13] This assumption, which is anyway irrelevant for almost all values of the parameters, affects none of the results, but saves tedious discussion.

[14] We will note later that there are other equilibria that are not (perfect) "symmetric cutoff" equilibria, but they do not seem very plausible.

[15] In a perfect sequential equilibrium, no bidder wishes to deviate from the equilibrium if a potential entrant who observes an out-of-equilibrium price would assume, if possible, that the bidder's type is among some set K (a subset of $[\underline{v}, \overline{v}]$) such that (i) all types in K would benefit from the deviation *if* it was then inferred that the bidder's type was in K, and (ii) all types not in K would prefer not to deviate given the aforementioned inference.

This refinement is also variously called "credible" or "neologism-proof" or "F-G-P" after its developers (Grossman and Perry (1986) and Farrell (1993)) – though Farrell's definition is very slightly different, this is unimportant here; see also Fudenberg and Tirole's (1991) standard text. Technically, we use the natural extension of the refinement to infinite strategy spaces; also a bit more than perfect sequential is needed to select a unique equilibrium when $\rho_j = 1\ \forall j$, and in that case we just use the limit of the (unique) perfect sequential equilibria as $\rho_j \rightarrow 1\ \forall j$ (this is also the unique equilibrium that Farrell's definition chooses $\forall \{\rho_j\}_{j=1}^{\infty}$, because he requires all types in K would *strictly* benefit in part (i) above).

[16] This will also turn out to be the revenue - or $\underline{V}$alue to the $\underline{S}$eller - from running the sequential mechanism in the important special case $\rho_j = 1\ \forall j$ in this equilibrium.

[17] If either no bidder, or only one bidder, would enter the auction, both mechanisms would yield the same revenue. If $\rho_j = 0$, we define $\rho_k \equiv 0\ \forall k > j$.

Notation

We define the "marginal revenue" (or "virtual utility") of a type, v, as $MR(v) \equiv [v - \frac{1-F(v)}{f(v)}]$.[18] We write $v_i(k)$ for the actual i^{th} highest value among k bidders, and n^* for the maximum number of entrants into the auction. That is, n^* is the largest integer satisfying $\frac{E\{v_1(n)\}-E\{v_2(n)\}}{n} > c$ or, equivalently, $\int_{x=\underline{v}}^{\overline{v}}[F^{n-1}(x) - F^n(x)]dx > c$. We write $S(v)$ for the expected surplus of a bidder whose value is drawn randomly from $F(\cdot)$ and who competes in an ascending auction against a single bidder whose value is drawn independently from the same distribution, conditional on the latter bidder's value exceeding v. It is easy to compute that $S(v) = \int_{x=v}^{\overline{v}}[1 - F(x)][F(x) - F(v)]/[1 - F(v)]dx$.

3 Revenue, Efficiency, and Bidders' Surplus

3.1 Equilibria

The expected revenue from the auction equals the sum of expected revenue conditional on there being precisely j bidders times the probability that precisely j bidders exist, for all $j \in \{1, 2, ...n^*-1\}$, *plus* the expected revenue when the maximum possible number of bidders, n^*, enters times the probability that at least n^* potential bidders exist. That is:

Lemma 1: *The Auction's expected revenue*

$$= \sum_{j=1}^{n^*-1}(1-\rho_{j+1})\prod_{k=1}^{j}\rho_k \int\limits_{x=\underline{v}}^{\overline{v}} jf(x)F^{j-1}(x)MR(x)dx \qquad (1)$$
$$+\prod_{k=1}^{n^*}\rho_k \int\limits_{x=\underline{v}}^{\overline{v}} n^*f(x)F^{n^*-1}(x)MR(x)dx.$$

In the equilibrium of the sequential mechanism, the first entrant makes a jump bid that is high enough to deter subsequent entry if its value exceeds some cutoff value, v^*, and otherwise bids the minimum price $\underline{v}$ – just as Fishman originally showed in his two-bidder version of this model. Any subsequent entrant whose value exceeds the current high bid competes with the current incumbent, and the price then rises until the lower-value of these two bidders quits at the point at which the price reaches its value. If it was the current incumbent who quit, the new entrant will then jump bid to the higher price that deters subsequent entry if its value exceeds the cutoff value, v^*, but will otherwise not raise the

[18]Section 3.3 (see especially notes 33 and 35) discusses why it is helpful to analyse auctions using marginal revenues. We make *no* assumptions on the $MR(v)$ function.

price further prior to any further entry.[19] (If it was the entrant that quit, there is also no jump bid prior to further entry, in equilibrium, because the current incumbent's value must be below v^*.) If there is no jump bid, another potential bidder then enters, if one exists.

The deterrence value, v^*, cannot be lower than the value V_S that satisfies $S(V_S) = c$, because if it were, then a prospective entrant would profit from entering against a deterring bid (because entering and winning against a deterring bid would deter all future entry, and the gross surplus from competing against a single bidder whose value is known to be v^* or higher is by definition $S(v^*)$, and clearly $S(v^*) > c$ if $v^* < V_S$). But an incumbent with a type as low as V_S can deter entry only if it is pooled with types with higher values – if an incumbent's value was known to be as low as V_S, entry against it would be known to be profitable. So while $v^* = V_S$ is a perfect Nash equilibrium, there are other equilibria in which higher-value incumbents are separated. The perfect constant-cutoff Nash equilibrium in which the fewest types deter entry is the one in which the only types who deter are those who would deter if their actual values were known (namely those whose values at least equal the value V_K which satisfies $\int_{x=V_K}^{\overline{v}} [x - V_K] f(x)dx - c = 0$). In these equilibria a bidder with value V_S finds it too costly to pool with higher-value incumbents, and instead accommodates entry – but, as in Fishman's model, these equilibria are not perfect sequential; see below.[20]

At any time, the (lowest) *bid* that deters further entry (i.e., the deterring price at that time) is such that a bidder with value v^* is just indifferent about deviating to accommodating entry. That is, the bidder with value v^* is indifferent between making a deterring bid and deviating to a strategy of never jump bidding but always bidding the lowest price possible to beat any challenger until entry closes, or until the price exceeds v^*.[21]

This indifference condition is exactly as in Fishman's model in which just one bidder

[19] The timing of the jump bid is unimportant. If the entrant were permitted to jump bid prior to competing with the current incumbent (or during that competition), it would be indifferent about when it made its jump. If, for example, it makes its jump before that competition, it bids the expected value of the jump bid that it would have made after competing with the incumbent (which depends upon the price at which the current incumbent quits). Seller's revenue, and all bidders' surpluses, are unaffected. See note 23.

[20] Recall that we refer to the (minimum) deterrring value in the <u>S</u>equential mechanism as V_S, and the minimum value that would deter if the actual value were <u>K</u>nown by potential entrants as V_K. Equilibria with v^* even higher than V_K are ruled out by our assumption of a constant deterring value, and also fail not only the intuitive criterion, but also the weaker "test of dominated messages", even in any single period - see proof of Lemma 2 in Appendix. (There is also a plethora of perfect *non-constant*-cutoff equilibria; and even *non*-cutoff equilibria, for example, entry could be deterred only by some particular set of prices exceeding the price that would correspond to value V_S, but these seem particularly implausible.)

[21] If a bidder with value v^* is just indifferent about deviating, a bidder with a lower (higher) value would strictly gain (lose) by deviating, since higher-value types prefer strategies with higher probabilities of winning (and deviating strictly reduces the bidder's probability of winning). Not deviating therefore signals a value $\geq v^*$ and so successfully deters entry if $v^* \geq V_S$.

can deter. However, in our more general model in which many bidders have the possibility of deterring, the deterring *price* will generally change over the course of the game, since it depends on both the value of the previous high bidder, and the sequence of subsequent ρ_j's.[22] In our model, therefore, the expected revenue calculation would be extremely messy if done through computing the actual price contingent on any sequence of bidders, but the calculation is straightforward using the standard result that the expected revenue equals the expected marginal revenue, MR, of the winning bidder – since this only requires knowing the identity of the winner.[23] We illustrate the bidding process and revenue calculations in examples in the Appendix. Summarizing, we have (see Appendix for the details of the proof):

Lemma 2: *There exists a perfect Nash equilibrium of the Sequential mechanism in which any buyer with a value greater than or equal to v^* makes a bid that deters all subsequent entry if and only if $v^* \in [V_S, V_K]$*

$$\textit{where } V_S \textit{ satisfies} \qquad S(V_S) = c, \tag{2}$$

$$\textit{and } V_K \textit{ satisfies} \qquad \int_{x=V_K}^{\overline{v}} [x - V_K] f(x) dx - c = 0. \tag{3}$$

The Sequential mechanism's expected revenue

$$= v^*(1 - F(v^*)) \sum_{j=1}^{\infty} F^{j-1}(v^*) \prod_{k=1}^{j} \rho_k + \sum_{j=1}^{\infty} (1 - \rho_{j+1}) \prod_{k=1}^{j} \rho_k \int_{x=\underline{v}}^{v^*} j f(x) F^{j-1}(x) MR(x) dx. \tag{4}$$

Note that if $\rho_j = 1 \; \forall j$, the expected revenue $= v^*$.

In the unique perfect sequential equilibrium, $v^* = V_S$.

Example: If $F(v) = v$ for $v \in [0, 1]$ and ρ is constant ($\rho_j = \rho \leq 1 \; \forall j > 1$), any new entrant whose value is v^* or more deters entry by jump bidding to the price $v^* - \frac{1-\rho}{\rho} \log\left(\frac{1-\rho p}{1-\rho v^*}\right)$ ($> p$), after defeating the existing incumbent at price p.[24]

[22] For example, if the first bidder's *value* is between its deterring price and v^*, a second bidder who enters and defeats the first will still need to make at least a small jump to credibly signal that its value is at least v^*. (In the special case $\rho_j = 1 \; \forall j$ the deterring price always equals v^* - see next subsection.)

[23] The original result is due to Myerson (1981). Marginal-revenue analysis (following Bulow and Roberts (1989) and Bulow and Klemperer (1996)) makes it clear that expected revenues (and every bidder's expected surplus) would be unaffected by bidders knowing the number of potential entrants who actually exist in advance of bidding (assuming it is common knowledge that they know this), and/or by the timing of the entrant's jump-bidding (whether it is before, or during, or subsequent to competing with a current incumbent, or at all of these times) - because although the actual prices would of course depend upon these assumptions, the identity, and therefore the expected MR, of the winner would not.

[24] For the first entrant, $p = \underline{v}$ ($= 0$ in the example).

Expected revenue equals $\frac{v^*}{\rho}\left[1+\frac{(1-\rho)^2}{(1-\rho v^*)}\right]+\frac{2(1-\rho)}{\rho^2}\ln(1-\rho v^*)$.[25] In the unique perfect sequential equilibrium, $v^*= V_S = 1-\sqrt{6c}$. (Calculations are in the the Appendix.)

Not surprisingly, the comparison between the sequential mechanism and the auction depends crucially on which equilibrium is played in the sequential mechanism. However, only the $v^*= V_S$ equilibrium is perfect sequential. In other equilibria, bidders with values above V_S could gain from deviating to a bid that signals a value of (just) V_S or more if the potential entrant were then to infer the equilibrium deterring value is actually only V_S, *and* all other types of bidders would lose from making such a bid however the potential entrant were to respond;[26] so the potential entrant should make such an inference, and a bid that signals a value of at least V_S therefore does deter entry.[27]

A large literature argues that the perfect sequential equilibrium is the only reasonable one. Fishman (1988, 1989) restricts attention to this equilibrium in his analysis of the two-potential-bidders version of our model, arguing that only this one is "credible". Many other authors take the same view, and pay scant attention to alternative equilibria.[28] Shleifer and Vishny, for example, argue "The case for the minimum bid equilibrium is compelling.... There is no basis for [any alternative] belief, since it is common knowledge that all types would like to [pay] the lowest possible price."[29] Bagnoli, Gordon, and Lipman say "[We do not] clutter the text with details [of equilibria that are not perfect sequential]." Our own view is that even if this last position may be overly strong, we agree the unique perfect-sequential equilibrium – $v^*= V_S$ – deserves the most attention.[30]

[25] The deterring *price* is increasing in both p and ρ, and equals the deterring *value*, v^*, both in the limit as $p \to v^*$ and in the limit as $\rho \to 1$; as $\rho \to 0$, the deterring price equals price, p; as $\rho \to 0$ and 1, expected revenue equals 0 and v^*, respectively – see discussion at end of section 3.2.

[26] In fact, a perfect sequential equilibrium requires only that all other types would lose from making such a bid *if* the potential entrant makes the inference that they are types above V_S, so the refinement is (even) more compelling than usual in our context; see notes 15 and 29.

[27] As throughout the refinement literature, it can be debated whether the potential entrant should make such an inference but, for example, Riley's (2001) recent, authoritative Journal of Economic Literature survey of the signalling literature makes a strong argument for this (indeed he concludes that if a perfect sequential equilibrium fails to exist, then there is no credible equilibrium at all).

[28] For example, Bagnoli and Lipman (1996), Bagnoli, Gordon and Lipman (1989), Shleifer and Vishny (1986), Bernhardt and Scoones (1993), McCardle and Viswanathan (1994), and Che and Lewis (2006), among others.

[29] The Shleifer-Vishny model has some different features than ours, but in our model, just as in theirs, "in order to eliminate implausible equilibria, we do not rely on the Grossman-Perry method of treating types in the complement of K....In the cases we consider, types in the complement of K would not want to deviate regardless of what [those observing the deviation] believe." (Shleifer and Vishny, 1986, note 14). See our note 15 above. See also Gertner, Gibbons, and Scharfstein (1988).

[30] Other arguments – for example, one in the spirit of McLennan's (1985) "justifiable" equilibrium (if a

Perhaps the best argument against the v*$= V_S$ equilibrium is that the seller prefers the v*$= V_K$ equilibrium, and could perhaps in effect achieve it by changing the game slightly to a game that is similar to our game but in which the only reasonable equilibrium is v*$= V_K$. We therefore discuss this possibility in section 4, but otherwise focus primarily on the v*$= V_S$ equilibrium.

3.2 Efficiency

It is a standard result that the amount of entry into a private-value ascending auction is efficient (see Engelbrecht-Wiggans (1993)). So, since in both our mechanisms the last potential entrant makes its entry decision based on the possibility of having to compete in an ascending action with the strongest actual entrant, it is straightforward that all the entry decisions are socially efficient *given* the available information (details are in the Appendix):

Lemma 3: *Conditional on the information available to a potential entrant, it makes the socially correct decision about whether to enter in either of the sales mechanisms.*

Note that in the sequential mechanism the higher is v^*, the more efficient is the equilibrium. A social planner who could observe each bidder's value upon entry would choose entry until the highest value was such that the expected contribution of a new bidder was less than or equal to zero, that is, until a bidder was found with a value greater than or equal to V_K, since$\int_{x=V_K}^{\overline{v}}[x - V_K]f(x)dx - c = 0$. So the V_K equilibrium is socially optimal.

In other equilibria, entrants who find out their values are in the range $[v^*, V_K)$ pool with those above V_K and inefficiently prevent the further entry that would occur if their values were publicly observed. However, all these equilibria are more efficient than the auction, because they all provide prospective bidders with better information than the auction does. (A sequential bidder who learned whether there were zero or one incumbent bidders with values in excess of v^* would never revise its entry decision based upon the number of previous entrants; a bidder who knew the number of other entrants might revise its decision upon learning whether or not there was a bidder with a value of more than v^*.) Therefore, since all the entry decisions are socially efficient *given the available information*, the entry decision in the sequential mechanism is more efficient (in expectation) whenever the entry decisions

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player observes a price that he thought he would never see, he should revise his views about how the game is being played so that what he has observed is consistent with equilibrium behavior) – also suggest this is the only natural equilibrium.

in the two mechanisms differ.[31] Both mechanisms select efficiently among all bidders who actually enter, so:

Proposition 1: *The Sequential mechanism is more efficient than the Auction.*

Note that when $\rho_j = 1\ \forall j$ – the "infinite potential bidder case"– the lowest deterring type must bid its full value, v^*, to deter, since it will otherwise surely face entry by someone with at least as high a value and will therefore earn zero surplus. So the deterring *price* always equals v^* in this case.[32] Since the expected profit of any potential entrant, conditional on there being no previous bidder above the cutoff, is $\int_{x=v^*}^{\overline{v}}[x-v^*]f(x)dx-c$, (3) implies that in this case bidders all earn zero expected profits in the V_K equilibrium. So the V_K equilibrium is not only efficient but fully extractive if there is an infinite number of potential bidders.

3.3 Revenue

The auction usually beats the sequential mechanism on revenue.

We begin by showing that if the auction yields more revenue for some $\{\rho_j\}_{j=1}^{\infty}$, then it is also superior for any weakly lower sequence of $\{\rho_j\}_{j=1}^{\infty}$. The intuition is simply that the sequential mechanism's advantage is that it may attract more bidders than the auction. The proof is not immediate, though, because the sequential mechanism encourages bidders to signal with high bids in early stages so, for example, conditional on only one entrant actually existing ex-post, the sequential mechanism earns more revenue on average, ex-post, than the auction (which then earns the minimum possible price). Because the payments of a winning bidder are a function of other players' bids and possibly the sequence of subsequent ρ_j's, a direct computation and comparison of profits is both intricate and tedious.

However, because the expected revenue from any mechanism is the expectation of the MR of the winning bidder, MR analysis (following Bulow and Roberts (1989) and Bulow and Klemperer (1996)) allows us to calculate each winner's contribution to expected revenue strictly as a function of its expected MR, without reference to other bidders' values or

[31] Note when $v^* = V_S$ a potential entrant is just indifferent to entering, and so entry is a matter of indifference for efficiency as well. However, the entry decision is then strictly more efficient than in the auction when bidders cease entering the auction even though none have yet bid V_S.

(The extra entry that takes place in the sequential mechanism is thus "business-stealing" but not welfare-reducing, see Mankiw and Whinston (1986).)

[32] So also the seller's revenue equals v^* in this case – see Lemma 2, where this result can also be confirmed directly using (4).

$\{\rho_j\}_{j=1}^{\infty}$.[33]

The key is that neither the number of bidders who would enter an auction if they exist, n^*, nor the entry-deterring value is affected by any of the ρ_j (of course, the entry-deterring prices would be affected), so the effect of reducing any ρ_j is simply to shift probability from the events that the jth and subsequent bidders exist, to the event that the $j-1$st bidder is the last, without changing the expected marginal revenues conditional on these different events. Since the auction is always more profitable if there are n^* or fewer bidders,[34] while adding additional bidders beyond n^* makes a contribution to profits in the sequential mechanism but cannot affect the auction, the result follows straightforwardly (see Appendix for details):[35]

Lemma 4: *For any given value of v^*, if the Auction is more profitable in expectation than the Sequential mechanism for some sequence $\{\rho_j\}_{j=1}^{\infty}$ then the Auction is more profitable in expectation for any weakly lower sequence of $\{\rho_j\}_{j=1}^{\infty}$.*

We can now show that the auction is superior in expected revenue to the sequential mechanism under likely conditions:

Proposition 2: *The Auction is more profitable in expectation than the Sequential mechanism*

if *(i) the equilibrium of the latter is its unique perfect sequential equilibrium and $S(\cdot)$ is convex (as it is for economists' most commonly used distributions of demand)*

and/or if *(ii)* $\prod_{j=3}^{j=n^*+1} \rho_j$ *is sufficiently small (i.e., the probability of at least n^*+1 po-*

[33] For example, as noted above, conditional on only one entrant actually existing, *ex post* the auction's expected profits are strictly less than the sequential mechanism's but, of course, the two mechanisms' expected *marginal revenues* are equal, because the real contribution of the first entrant is the same in both mechanisms. (More generally, marginal-revenue analysis focuses on the real contribution of each potential bidder to profits - whether by bidding high, or whether by encouraging others to bid higher earlier because of the threat the bidder poses in which cases some of the profits are realised in states in which the bidder does not actually exist.)

[34] This is essentially Myerson (1981)'s standard result that the simple auction maximises revenue among mechanisms that always result in a sale. But unlike Myerson we do *not* need to assume "downward sloping" marginal revenue or any other regularity assumption.

[35] For the reason given above, this argument would fail if expressed in terms of state-by-state revenues rather than state-by-state *marginal* revenues. This is exactly as in ordinary monopoly theory: by setting a higher or lower price than the optimum a monopolist can raise the contribution to profits (price *less* cost) of some units, but it will weakly lower the contribution to marginal revenue *less* marginal cost of *every* unit sold (since the optimum is set at *MR*=*MC*); that is, there is state-by-state dominance in marginal revenues, but not in prices. (See Bulow and Klemperer (1996) for another application of state-by-state dominance in *MR*s where there is no dominance in prices or revenues.)

tential entrants existing, conditional on at least two existing, is sufficiently small).

Proof: Part (i): by Lemma 4, if the result holds for $\rho_j = 1\ \forall j$, it holds for all $\{\rho_j\}_{j=1}^{\infty}$. Also, $S(V_S) = c \geq E\{S(v_2(n^*))\} \geq S\{E(v_2(n^*))\}$, in which the equality is the entry condition for the sequential mechanism (equation (2)); the first inequality is the entry condition for the auction (an $(n^*{+}1)^{st}$ potential entrant would not want to enter the auction given that it would compete against a bidder whose value exceeds the second-highest of the n^* previous entrants' values, $v_2(n^*)$); and the second inequality is Jensen's inequality (if $S(\cdot)$ is convex). So, since $S'(\cdot) < 0$, we have $V_S \leq E(v_2(n^*))$. Since if $\rho_j = 1\ \forall j$, the expected revenue from the auction $= E(v_2(n^*))$, and the revenue from the sequential mechanism $= v^*$ (see Lemma 2), $v^* = V_S$ suffices for the result.

Part (ii): under this condition, the probability of $n^*{+}1$ or more potential bidders is sufficiently small that the greater expected marginal revenue (EMR) of the winning bidder in the auction, when there are at least 2 and no more than n^* potential bidders (see the discussion leading to, and proof of, Lemma 4), dominates the possibly-greater EMR of the sequential mechanism when more than n^* potential bidders exist.□

There are three reasons why the auction is usually *strictly* more profitable than the sequential mechanism. These correspond to the facts that each of the first three inequalities in the proof of part (i) (namely $\{\rho_j\}_{j=1}^{\infty} \leq 1$, $c \geq E\{S(v_2(n^*))\}$, and $E\{S(v_2(n^*))\} \geq S\{E(v_2(n^*))\}$) are usually strict; they also correspond to the factors listed in the introduction.

First, there is never an infinite stream of potential bidders in practice (i.e., $\rho_j < 1$, for some[36] $j > 2$). Since the advantage of the sequential mechanism is only when a long tail of bidders actually enters, reducing the likely number of potential bidders reduces its chance of beating the auction, as discussed in Lemma 4.

Second, the jump-bids in the sequential mechanism are "fine-tuned" to make entry just barely unattractive to an additional bidder, while the expected profit of the marginal bidder who does not enter the auction is generally strictly negative (i.e., $c > E\{S(v_2(n^*))\}$).

Third, surplus from entry conditional on the value of the second highest bidder is likely to be convex (i.e., $E\{S(v_2(n^*))\} > S\{E(v_2(n^*))\}$). If $S(v)$ were the surplus obtained by a new entrant who faces a bidder with a value *equal to* v (and so faces a price equal to v), the inequality would be immediate from the standard consumer-theory result that a buyers'

[36] To see the need for $j > 2$, see the proof of Lemma 4.

surplus is convex in the price it faces.[37] However, because $S(v)$ is the surplus obtained by a new entrant who faces a bidder with a value *above* v, the inequality combines two separate factors (a) the consumer theory result applied to the *values* of the winners of the mechanisms, and (b) the relationship between the *values* of the winners and the *prices* they pay.

Both issues depend upon the fact that even if there were an infinite number of potential bidders, and the *expected* values of the winners of the two mechanisms were the same, the *dispersion* of values faced by a new entrant into the auction would be greater than the dispersion of values faced by a new entrant into the sequential mechanism. More precisely, the distribution of the winner's value in the sequential mechanism would second-order stochastically dominate the distribution of the winner's value in the auction in the sense of Rothschild and Stiglitz (1970), because the former value is distributed according to $F(\cdot)$ truncated below at V_S, while the latter value is distributed according to a convex function of $F(\cdot)$, namely $F^{n^*}(\cdot)$.

To understand this, see Figure 1 which pictures the case of uniform $F(\cdot)$, so that the truncated density is a constant above V_S, while the density of $F^{n^*}(\cdot)$ is increasing. [INSERT FIGURE 1 HERE]. So the truncated density must be the higher at V_S, since both densities integrate to 1. It must therefore also be the lower at $\overline{v}$ if both densities have the same mean. So the densities cross just twice, and the truncated distribution is therefore less risky. Considering a general $F(\cdot)$ simply distorts this picture without affecting the "two-crossings" property.[38]

Factor (a), then, is that, conditional on the two processes having the same expected winning value, a new entrant would find the auction more attractive because its distribution of winning values is more dispersed.

Factor (b) – namely the relationship between the *value* of the winner and the *price* it pays – depends on the relationship of MR to v. Since the expected revenue of any mechanism is the expected MR of the winning bidder, the same second-order stochastic dominance relationship would make the auction more (less) profitable if MR is convex (concave) in v,

[37] The entrant can be thought of as having an option to buy at the value of the highest competitor and, for any given value of the entrant, that option becomes more valuable as the distribution of the prospective purchase price becomes more dispersed.

[38] For $v > \widetilde{v}$, the ratio of the densities of $\phi(F(\cdot)\ )$ and $(v \mid v \geq \widetilde{v})$ is $[\phi'(F(\cdot))\ f(\cdot)]/[\frac{f(\cdot)}{1-F(\widetilde{v})}]$ which is increasing if $\phi''(\cdot) > 0$. In this case the former density is first higher (for $v < \widetilde{v}$), then lower, and finally higher again, and so is second-order stochastically dominated by, the latter, if the expectations are equal.

if the means of the expected high values were the same.

So all four factors go in the same direction if $MR(v)$ is convex $(\frac{d^2}{dv^2}MR(v) \geq 0)$[39], and this is therefore a *very* easily sufficient condition for expected revenue to be greater in the auction than in the sequential mechanism.

For the demand functions most commonly used by economists — linear, exponential, and constant elasticity — $MR(v)$ is affine (i.e., $\frac{d^2}{dv^2}MR(v) = 0$ everywhere),[40] so the auction clearly raises more revenue in all of these cases.

As an example, consider bidders with valuations uniformly distributed on $[0, 1]$, that is $F(v) = v$ for $v \in [0, 1]$. This corresponds to bidders forming a linear demand curve $p = 1 - q$, in the limiting case where there are m such bidders each demanding $1/m$ units at their values, as $m \to \infty$. If $c = .06$, the auction would attract up to $n^*= 3$ bidders and yield expected revenue of .50 if there are at least three potential bidders. A fourth bidder would have an expected loss of exactly .01 and so would not enter.

To have the same expected winner's value, even with an infinite supply of potential bidders, the sequential mechanism's cutoff, V_S, would need to be .50 - see Figure 1. But to reduce a new entrant's expected profits to $-.01$, a bidder in the sequential mechanism would have to credibly signal only that its value exceeded $1 - \sqrt{.3} \approx .452$.[41] So roughly $.50 - .45 = .05$ of the extra revenue of the auction over that from the sequential mechanism is attributable purely to "convexity of surplus": although the auction yields an expected winner's value of .75, the sequential mechanism only needs an expected winner's value of .726 to reduce an entrant's net profits to -.01, because its winner's value is less dispersed. Because $MR(v) = 2v - 1$, with MR neither concave nor convex, the difference in the expected value of the winner (here, between .75 and .726) translates linearly into twice that difference in expected revenues.

If bidders pool on jump bids that make a new entrant's expected profits 0 instead of $-.01$,

[39] MR convexity is equivalent to concavity in the inverse hazard rate (or $2(h'(v))^2 \leq h(v)h''(v)$, in which $h(v) \equiv \frac{f(v)}{1-F(v)} =$ the hazard rate).

It is not hard to show that if expected demand is convex (i.e., $f'(\cdot) \leq 0$), a sufficient condition for $S(\cdot)$ to be convex is $h'(\cdot) \geq 0$.

[40] For all these cases, the ratio of the slope of MR to the slope of (inverse) demand (v) is constant: for uniformly-distributed signals $(F(v) = \frac{v-\underline{v}}{\overline{v}-\underline{v}})$, which generate linear demand, the ratio of the slope of MR to the slope of demand $= \frac{1}{2}$; for constant-elasticity distributed signals $(F(v) = 1 - (\frac{v}{\underline{v}})^{\eta},\ \eta < -1)$, the ratio $= \frac{\eta}{\eta+1}$; and for exponentially-distributed signals $(F(v) = 1 - e^{-\lambda(v-\underline{v})})$, which generate log-linear demand, the ratio $= 1$.

[41] This is because $S(1 - \sqrt{.3}) = .05$, and so if $c = .06$ a new bidder would have an expected loss of .01.

then the cutoff value is lowered to .40,[42] extending the auction's advantage by another .05. Finally, if there were only three potential bidders (so $\rho_j = 1$ for $j \leq 3$ and zero otherwise), the "finite supply of bidders" reduces the sequential mechanism's expected revenue by .05 more, to about .35 while leaving the auction's revenue unchanged at .50. This example is described in detail in the Appendix.

Of course, by choosing c so that an additional entrant is barely deterred in an auction (so $c = .05$ here) and $\rho_j = 1\ \forall j$ the last two effects are nullified. But even then, for the seller to prefer the sequential mechanism, the lower dispersion of the sequential mechanism's high value (worth the difference between .50 and .45 in the above example) must be compensated by concavity in the MR function. However, for MR concavity to matter much would require a relatively large dispersion in the high value in the auction relative to the sequential mechanism. And the larger that dispersion, the larger the expected value of the high value must be in the auction (relative to the sequential mechanism) to deter entry, making it that much more difficult for the sequential mechanism to catch up — even ignoring the other two factors. So, for the seller to prefer the sequential mechanism requires several stars to be aligned — and for the seller to realize that they are so.

Nevertheless, it is possible to find counterexamples – in particular by choosing a distribution with a marginal revenue function that is essentially flat at higher values (so that the higher expected v_1 does little for the auction) and that conveniently becomes sharply lower just below V_S (so that the auction is disproportionately punished for low outcomes) — and then combining this with a large stream of potential bidders and a carefully chosen c. For such an example, assume bidders are equally likely to have the values 0, $\frac{8}{11}$, and 1 (or a continuous approximation to this three point distribution) and $\rho_j = 1\ \forall j$. The expected revenue from the sequential mechanism then exceeds the expected revenue from the auction if (but only if) $c \in (\frac{80}{1782}, \frac{81}{1782})$, in which case c is low enough that $V_S = \frac{8}{11}$ and yet high enough that only 3 bidders enter the auction, so the greater chance that revenue will be 0 in the auction matters more than the possibility that the auction's revenue will be 1.[43]

If $v^*= V_K$ the sequential mechanism is both socially optimal and fully extractive, as we have seen in section 3.2, and therefore it *must* beat the auction in expectation if (but only

[42] This is because $S(.4) = .06 = c$, so $V_S = .4$.

[43] To calculate the continuous distribution analogue, note that a deterred entrant expects zero profits from entry if $v^*= V_S$, so a buyer with value $\frac{8}{11}$ enters with probability $\frac{11c}{1-11c}$. The expected value of the winner in the auction, .892, comfortably exceeds the expected value of the winner in the sequential mechanism (between .864 and .865 depending on c).

if) the number of potential bidders is sufficiently greater than n^*.

In short, there is a reasonable presumption that the expected price is higher in the auction than in the sequential mechanism.

3.4 Bidders' Preferences

Since the seller's profits are usually higher in the auction, but social welfare is lower, bidders' surplus must usually be lower also. We now show that not only does this result usually hold for total bidders' surplus, but that it is even more likely to hold for the surplus of the first bidder to enter. So the casual-empirical observation that the first bidder to enter will generally seek to pre-empt an auction is borne out. We begin by showing (see Appendix for details):

Lemma 5: *In the unique perfect sequential equilibrium of the Sequential mechanism, fewer bidders enter, in expectation, than in the Auction.*

The intuition is that the sequential mechanism sorts bidders better, in that entry stops as soon as one bidder with a high enough value to deter future entrants is found, while the auction attracts a random number of high-value bidders and therefore on average needs more bidders to achieve the same degree of entry deterrence. We now have (see Appendix):

Lemma 6: *The first bidder obtains a larger fraction of the total bidder surplus, in expectation, in the unique perfect sequential equilibrium of the Sequential mechanism than in the Auction.*

The intuition is that not only are there fewer bidders on average among whom to split the surplus (see the previous Lemma), but also the surplus is tilted towards the first bidder in the sequential mechanism, because that bidder sometimes deters others in that mechanism, while all of the first n^* bidders always enter the auction if they exist.

It follows straightforwardly (see Appendix) that:

Proposition 3: *The Auction yields lower total bidder surplus, surplus per participating bidder, and surplus of the first entrant, in expectation, than the Sequential mechanism* if *(i) the equilibrium of the latter is its unique perfect sequential equilibrium, and $S(\cdot)$ is convex* and/or if *(ii)* $\prod_{j=3}^{j=n^*+1} \rho_j$ *is sufficiently small.*

If both conditions (i) and (ii) fail, and the sequential mechanism has $v^*= V_K$ when $\rho_j = 1\ \forall j$, the auction is more attractive to all bidders. It is also possible to find examples in which all bidders prefer the auction with $v^*= V_S$ when $\rho_j = 1\ \forall j$ (and $S(\cdot)$ is not convex), but parameters must be chosen very carefully – such examples are clearly even more restricted than those for which the seller prefers the sequential mechanism.[44]

In short, bidders, especially those "at the front of the queue" (who are the most influential), mostly prefer sequential mechanisms; our model supports the advice that bidders should avoid auctions if possible.

3.5 Simultaneous (Random) Entry into the Auction

No important result is affected if potential bidders make simultaneous, instead of sequential, entry decisions into the auction.[45] The logic of Proposition 2 holds exactly as before: because the second-highest value in the auction is random the auction price must be higher than the sequential mechanism's price to deter entry, if $S(\cdot)$ is convex.[46] And since the random-entry auction is less efficient than the auction of our basic model (bidders have less information when they make their entry decisions), the auction remains less efficient than the sequential mechanism (Proposition 1). Obviously, our other Propositions are also unaffected.

[44] The example we gave in the previous subsection, for which the seller preferred the sequential mechanism, is also one in which the auction is the most attractive mechanism for bidders in total surplus, surplus per participating bidder, and surplus for the first entrant – of course, the set of $\{\rho_j\}_{j=1}^{\infty}$ for which these statements hold is even smaller than the set for which seller prefers the sequential mechanism. (If $\rho_j = 1\ \forall j$, the first bidder's expected surplus is $\frac{28}{297} - c$ and $\frac{27}{297} - c$ in the auction and the sequential mechanism, respectively.)

[45] For example, Reiley (2005) shows that random entry models the entry of bidders in internet auctions better than deterministic entry does.

[46] We can assume either that potential bidders only know the distribution of the number of other potential entrants when they make their entry decisions or as in, e.g., Levin and Smith (1994), that they know the actual number of potential entrants. In either case, the proof is essentially unchanged if we simply replace $v_2(n^*)$ by the actual second-highest value of whatever (random) number of bidders actually enter. (Although when no bidders actually enter a random-entry auction, an additional bidder would only have to pay the price the seller would receive absent his bid (instead of a higher price) this just makes entry more attractive, so reinforces the result that the auction must offer greater expected revenue to deter further entry.)

The random-entry auction may be either more or less profitable than the auction of our basic model. If the basic auction yields zero expected profits for bidders then it must be superior because it is more efficient. If, on the other hand, an additional bidder would only just be deterred from entering the basic auction, then random entry may entail n^*+1 bidders who almost always enter, which is superior to having just n^* enter for certain.

3.6 The Crucial Role of Jump Bidding

Prospective entrants in the sequential mechanism learn information about earlier bidders' values both by observing those bidders competing with one another, and by observing any pre-emptive jump bids. But the greater efficiency of the sequential mechanism relative to the auction is entirely because entrants compete until there is a single survivor prior to any further entry; given that competition, the survivor's ability to jump bid to deter further entry makes no difference to expected efficiency. The reason is that bidders fine-tune their jump bids to be the lowest possible to deter entry, that is, to make any prospective new entrant just indifferent about entering. So since prospective new entrants make socially efficient decisions given their information (Lemma 3), outcomes would be equally efficient in expectation if the jumps that are made did not deter entry, and so also equally efficient if jump bids were impossible (details are in the Appendix):

Proposition 4: The expected efficiency of the unique perfect sequential equilibrium of the Sequential mechanism would be unaffected if jump bids were not *possible.*

So on average jump bids have no effect on expected social surplus. But they play a key role in transferring profits from sellers to buyers. When there are no rents to be earned from bidder scarcity ($\rho_j = 1\ \forall j$) bidder profits are *entirely* due to the ability to make jump bids: absent jump bidding, a potential entrant knows only that the current incumbent's value exceeds the actual value of the second-highest-value entrant to date. So entry will continue until the value of the second-highest-value entrant is at least V_S, at which point the competition between the two highest-value entrants will demonstrate that at least one has a value above V_S, and there will be no subsequent entry. So, with an infinite supply of potential entrants, each new entrant knows it will win only if and when it beats one bidder whose value exceeds V_S. So it expects to earn zero profits (by equation (2)), that is:

Proposition 5: If jump bids were not *possible in the Sequential mechanism, and there is an infinite supply of potential entrants, all bidders would earn zero surplus.*

Because the "no jump bidding" mechanism would be both fully extractive (when $\rho_j = 1\ \forall j$) and as efficient as the perfect sequential equilibrium of our basic sequential mechanism, which is in turn more efficient than the auction, it must yield higher expected revenue than the auction in this case. (Of course, the auction remains more profitable if sufficiently few potential entrants are expected – Part (ii) of Proposition 2 holds exactly as before.) But, while these results help explain the role of jump bidding, it is unclear as a practical matter how a seller would be able to simultaneously enforce the requirement on sequential English auction bidding and the prohibition on jump bidding which are both necessary to implement this procedure. So we turn in the next section to more plausible modifications of the sequential mechanism.

4 Modifications of the Sequential Mechanism

4.1 Entry Subsidies

Since our analysis so far suggests that the auction is typically much more profitable than the sequential mechanism, the obvious question is whether the sequential mechanism can be improved for the seller. We now extend our basic model by assuming the seller also knows $F(\cdot), c$, and $\{\rho_j\}_{j=1}^{\infty}$.

Of course, if the seller can pre-commit to a set of rules and the full common knowledge assumptions obtain, it can extract all surplus by pre-committing to a sequence of entry fees and subsidies that assure efficient entry and no expected surplus for buyers. This seems unrealistic.[47] But it is natural to ask whether there is any simple modification of the basic sequential mechanism that yields an outcome corresponding to the seller's "favorite" equilibrium ($v^*= V_K$), while satisfying the perfect sequential equilibrium refinement that is usually imposed?

The answer is "yes", *provided* the seller has all this information. If the seller has the ability to partially subsidize new entry, this raises the signal (and therefore the price) needed to deter entry. In particular, if the size of the possible subsidy is chosen correctly, incumbent bidders with values between V_S and V_K are prevented from being able to pool with those above V_K. Even better, when the weaker types are separated from the stronger ones an unsubsidized entrant will compete against them. So while the *threat* of the subsidy is necessary, the subsidy never actually needs to be paid! The outcome is therefore exactly the same as the V_K equilibrium we discussed earlier, and is therefore more profitable than the auction for $\rho_j = 1\ \forall j$. So we have (for details of the proof, see the Appendix):

[47] For $\{\rho_j\}_{j=1}^{\infty}$ "sufficiently large" (i.e., "sufficiently close" to $\rho_j = 1\ \forall j$), another strategy that – if credible – can increase entry and so seller profitability is to commit to not allow the incumbent bidder to raise its bid in response to new competition. Edlin (2002) discusses regulatory regimes based on this principle.

Proposition 6: *Consider the Sequential mechanism with the additional feature that after any buyer enters and bids, the seller has the ability but not the obligation to offer to subsidise up to* $c - S(V_K)$ *of the next potential entrant's cost of finding out its value. The unique perfect sequential equilibrium of this mechanism is more profitable in expectation than the Auction for a set of sequences of* $\{\rho_j\}_{j=1}^{\infty}$ *that includes* $\rho_j = 1\ \forall j$.[48] *Buyers with values below* V_K *cannot deter entry, but no subsidies are paid in equilibrium.*

Clearly the modified sequential mechanism will be more profitable than the auction for sequences of $\{\rho_j\}_{j=1}^{\infty}$ that are "close enough" to $\rho_j = 1\ \forall j$, but the argument of Proposition 2 part (ii) shows that for "sufficiently small" $\{\rho_j\}_{j=1}^{\infty}$ the auction remains more profitable.[49]

4.2 Lock-ups, Break-up Fees, and Matching Rights

The previous subsection shows that entry subsidies can make the sequential mechanism both more efficient and more profitable than the auction. By making entry subsidies larger still, the sequential mechanism can often be made even more profitable, by further raising the signal required to deter entry.[50]

However, if the seller has the ability to offer larger subsidies than $c - S(V_K)$, then subsidies must sometimes actually be paid – even in circumstances in which a subsidy would not have had to be paid if large subsidies were not possible – because bidders who fear *future* high subsidies to attract entry against them may need to be paid to be persuaded to enter today.[51] Furthermore, the gains to the seller from reducing buyer surplus are reduced by the inefficiency of the incremental entry induced by these larger subsidies.

The seller could do better than simply offering large subsidies if it could pre-commit to

[48] In fact, *any* perfect Nash equilibrium of the sequential mechanism with this feature is more profitable in expectation than the auction for a set of $\{\rho_j\}_{j=1}^{\infty}$ that includes $\rho_j = 1\ \forall j$.

[49] If, for example, the probability of an additional bidder is a constant, ρ (i.e., $\rho_j = \rho \leq 1\ \forall j > 1$) then, by the argument of Lemma 4, the modified sequential mechanism is more profitable than the auction if and only if $\rho \geq \widetilde{\rho}$ for some $\widetilde{\rho}$ ($\widetilde{\rho}$ depends on c and $F(\cdot)$).

[50] If, by contrast, the subsidy that can be paid to a new entrant is smaller than $c - S(V_K)$, some types of incumbent bidder with values below V_K can deter further entry. For uniform $F(\cdot)$, for example, the auction (without subsidies) is more profitable than the sequential mechanism ($\forall \{\rho_j\}_{j=1}^{\infty}$) if the allowable subsidy is less than about half of $c - S(V_K)$. (The critical fraction below which the auction is always superior is always in the range $\frac{1}{2} \pm \frac{1}{n^*+1}$.)

[51] In particular, when large subsidies are feasible a first bidder with a value above V_K may prefer to make the minimum allowable bid and risk entry if ρ_2 is small, rather than bid V_K to deter entry (because a lower bid that simply signals a *value* of V_K may not – by contrast with the low-subsidy case – be enough to deter entry); the second bidder may then require a subsidy since it may have to face future highly-subsidized entry (it will certainly require a subsidy if $\rho_j = 1\ \forall\ j > 2$).

avoiding future inefficient subsidies. In particular, it could benefit from a provision that prohibited it from paying any further subsidies to additional bidders if the current entrant signalled a value of at least V_K.

Such a combination of a subsidy to a current entrant and a promise to limit future subsidies parallels the common practice of sellers of public companies of offering "lock-ups" that include a break-up fee (equivalent to cash paid out of corporate assets) and "matching rights" that prohibit subsidies to future bidders in return for a sufficiently high bid.[52]

So our analysis suggests a role for lock-up-like combinations of current subsidies and promises about future subsidies, but the details will depend not only on the bargaining power of the seller against any new entrant, but also on the specification of the bargaining protocol between the seller and the current incumbent (since bargaining between these actors can lead to even more efficient outcomes). This is beyond the scope of the current paper.

However, we are sceptical of the broad use of subsidies in practice. Subsidies can easily be abused by a seller who wishes to bias the process. It is not uncommon to see the board of a target company granting subsidies (with matching rights) to an initial bidder that has made a deal with management, effectively deterring other bidders.[53] And potential first bidders may be deterred by the possibility of the board paying subsidies to a management-led counter-bid or other favored alternative bidder.[54]

Even putting aside both this moral hazard problem, and the problem of attracting non-

[52]A break-up fee guarantees the bidder a fixed sum in return for making its offer, payable if the deal fails to be completed – typically because of a topping offer by another bidder – and are typically accompanied by "matching rights". Since the break-up fee is a sunk cost once agreed to, it only distorts an auction if the recipient makes an initial offer above its value in order to receive it. See Ayres (1990). Che and Lewis (forthcoming) show the merits of breakup fees in Fishman (1988)'s model. (Sometimes buyers will offer sellers break-up fees, for example, as protection against a deal collapsing because of antitrust problems. Break-up fees are limited to 1% of the value of the bid in the UK, but can be much higher in the US.) Related issues are discussed by Rothkopf, Harstad and Fu (2003).

[53]Paying break-up fees to initial bidders is especially suspect. Consider, for example, the recent $33 billion buyout of hospital owner HCA, which gave the group that included management a $300 million break-up fee and matching rights in return for an offer that exceeded the pre-takeover valuation by about $3 billion. Since the management group was probably also better informed than any potential rival, it is no surprise that investment bankers were then unable to find a competing offer.

[54]For example, if $\rho_j = 1 \; \forall j$ and the first potential bidder perceives any risk that future subsidies will be larger than optimal, it will never enter, since it would be forced to either bid above V_K or to suffer subsidised competition.

A further concern arises if management can pay subsidies in shares. Giving shares is cheaper than paying cash, because the recipient values them more than the seller – either the recipient sells out at the sale price that would have resulted anyway, or the recipient is the final winner and so values them more than this, and because it gives the recipient a toe-hold advantage (see Bulow, Huang and Klemperer (1999)). So management has a temptation to offer a subsidy in shares even against a bid of V_K – but no bidder will then be prepared to make an original bid of V_K.

genuine bidders who pocket subsidies without investigating serious bids,[55] the tactics considered here depend sensitively on the seller's information. The basic sequential mechanism and the auction can be implemented by a seller who knows nothing about the distribution of bidders' values, or bidders' perception of that distribution, or their cost of entry. But the tactics here require knowledge of all these things. And with endogenous entry the implementation of an "optimal mechanism" is fraught with danger: there is a small difference between a tactic that extracts all surplus and one that discourages all entry. So while the ability to offer a limited special deal to a "white knight" involving paying its fees, or giving it cheap options or a lock-up, can increase expected revenue, this depends upon the seller being a revenue-maximiser with sufficient informational resources[56] and commitment powers to fine–tune its use of these tactics.

In addition, of course, even with modifications, the sequential mechanism beats the auction only if $\{\rho_j\}_{j=1}^{\infty}$ is "close enough" to $\rho_j = 1\ \forall j$, (perhaps unlikely in the context of company sales). So while a subsidy might be a useful device in some circumstances, at the very least one must handle with care.

4.3 Buyers' Bargaining Power

While sellers may be able to increase their leverage by threatening to subsidize entry, buyers may be able to improve their position, and reduce expected revenues in the sequential mechanism, if they can build a reputation for exiting if an offer they make is not accepted. In the extreme case where sequential buyers can all fully commit to making take-it-or-leave-it offers, and $\rho_j = 0$ for at least one (perhaps large) j, the unique subgame-perfect equilibrium is for the first bidder to make an offer of $\underline{v}$ with the seller accepting. (The game is trivial to solve recursively.)

The more general principle is that if there is a high enough chance that a buyer will leave then the seller may be forced to accept a price that would not deter future bidders, because of the risk of losing the current bidder. The increase in buyer's surplus from the

[55] Giving break-up fees and matching rights only in return for a "good" offer mitigate the second problem but without eliminating it (it is usually not hard for a winner to find an excuse to withdraw) and at the cost of possibly introducing inefficiency (see note 52).

[56] Not only must the seller have enough information about the distribution of the bidders' values, and the probability of further potential entrants being available, but the seller must also be known not to have too much information - if the seller is thought to have more information than the bidders about the probability of a further potential entrant, for example, then the current potential bidder may be reluctant to deal with the seller.

lower price plus the efficiency loss from reduced entry causes a double hit to the seller.

While it may be difficult for buyers to credibly commit to withdraw from a simple sequential process (and such a threat may be even less effective in an auction), it is perhaps more likely that prospective buyers will be able to threaten to withdraw if subsidies are offered to competitors – we often observe bidders negotiating for matching rights that effectively eliminate subsidies, in return for making an initial offer.

5 Conclusion

It is an old saw that "*An economist is someone who sees something working in practice, and asks whether it can work in theory.*"[57] Our bottom line is that auctions usually are best, in theory as well as in practice.

We have shown that buyers generally prefer to make pre-emptive bids that avoid auctions and force a sequential process. But if the seller initiates the sale it chooses an auction which is not as profitable for buyers.

The auction is better for the seller, even though it reveals information less efficiently, because it has three advantages related to entry. First, it requires only a limited number of potential bidders to achieve its maximum expected revenue; the sequential mechanism's advantage in being able to consider more buyers if earlier offers are too low is only relevant if the number of potential bidders significantly exceeds the number that would participate in an auction. Second, buyers' ability to make jump bids that just deter entry increase their expected profits at the expense of expected seller revenue. Third, the greater dispersion in the winner's value in the auction means its expected value must also be higher – and although it does not follow that the price it pays is necessarily higher, the greater entry means it usually will be.

When are auctions best? Unless the seller has the information and ability to credibly subsidize just the right amount of entry, the auction is better except under delicate conditions under which demand curvature, bidding costs, and a large supply of potential bidders align to overcome both the higher expected top value in the auction and the profits from pre-emptive bidding in the sequential mechanism. If a seller has only limited information about valuations, entry costs, or the number of bidders, or needs to standardize procedures

[57] We are not sure where this aphorism originates. The first recorded use we are aware of is by Ronald Reagan in a 1987 speech.

across many sales, an auction is its best bet for expected revenue. If the likely number of bidders is small, the advantage of an auction is particularly large.

Taking a broader perspective, this paper, and Bulow and Klemperer (1996), both show the power of competition. Bulow and Klemperer (1996) showed that the extra competition provided by a single extra bidder dominates the extra value of any clever negotiating strategies or regulatory schemes that might be designed to extract more rents from competing firms. The main result of this paper is that the straightforward, level-playing-field competition that a simple auction creates is likely to be more profitable than a sequential procedure that will sometimes attract more bidders and often ensure a higher minimum price, but prevents direct, simultaneous competition among all participants on equal terms.

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6 Appendix – Examples and some Proofs

Examples of the Equilibrium of the Sequential Mechanism:

Let bidders' values be drawn from the uniform distribution $F(v) = v$ for $v \in [0, 1]$. Then (2) $\implies$ $V_S = 1 - \sqrt{6c}$, that is, a potential entrant would be just indifferent to entering against an incumbent with a value in the range $[1 - \sqrt{6c}, 1]$ (the probability of defeating such an incumbent will be $\frac{1}{2}\sqrt{6c}$, and the expected profit conditional on doing so will be $\frac{1}{3}\sqrt{6c}$, so the net benefit from entering would be $\frac{1}{2}\sqrt{6c} \cdot \frac{1}{3}\sqrt{6c} - c = 0$).

Let $c = .06$, so $V_S = .4$. So – starting from any point of the game, and independent of past history or future $\{\rho_j\}$ – a bidder must signal that it has a value of at least .4 to deter further entry. It can do this by making a bid that, if its value were .4, would give it the same surplus if it successfully deterred entry as if it did not jump bid and accommodated further entry.

3-bidder Example

If there are exactly 3 bidders ($\rho_j = 1$ for $j \leq 3$, and zero otherwise), a first bidder with the deterring value of .4 who chose *not* to deter would win with probability $(.4)^2$, and earn $\frac{1}{3}(.4)$ conditional on winning, yielding an expected profit of .0213. So if the first bidder's value is .4 or more it will jump bid to $.4 - .0213 = .3787$ and there will be no further entry.

If the first bidder's value is less than .4, it will bid $\underline{v} = 0$, the second bidder will then enter, and the two bidders will raise the price continuously until the lower of their two actual values is reached. Call this price p. If it was the second entrant that quit, the third bidder then enters (it remains unprofitable for the first bidder to deter) and if the third bidder's value exceeds p, the price continues to rise until the lower of the two remaining bidders' values is reached.

If, instead, it was the incumbent bidder (that is, the first entrant) that quit at p, the new incumbent has the opportunity to jump bid to deter the final potential entrant. If the new incumbent had the minimum deterring value, .4, but did not jump bid, its expected profit from competing with the third bidder would be $p(.4 - p)$ (from the case in which the third value is below p) *plus* $\frac{1}{2}(.4 - p)^2$ (from the case in which the third value is between p and .4) equals $.08 - p^2/2$. So if the second bidder's value exceeds .4, it makes a jump bid from p to $.32 + p^2/2$ ($> p$) immediately after defeating the first bidder at the price p, and there is then no further entry. If its value is below .4 it does not jump bid, the third bidder then enters and, as before, if the third bidder's value exceeds p the price then rises again until the lower of the two remaining bidders' values is reached.

To compute expected revenue directly, note that with probability .6 the first bidder deters at price .3787. With probability $.6(.4) = .24$ the second bidder will deter the third bidder after beating the first bidder at price p uniformly distributed between 0 and .4 and the deterring price is $.32+p^2/2$, so the average price in this case equals $\int_{p=0}^{.4}(.32+p^2/2)dp = .3467$. With probability $.6(.4)^2 = .096$ neither of the first two bidders will deter and the third bidder's value is at least .4, so it will win at a price equal to the higher of the first two bidders' values, which is on average $(2/3).4 = .2667$. Finally, with probability $(.4)^3 = .064$ all three bidders have values below .4 and the expected price will be the expected second-highest of these three values, that is, $(1/2).4 = .2$. So total expected revenue equals $.6(.3787) + .24(.3467) + .096(.2667) + .064(.2) = .3488$.

This calculation can be made much more easily using MR analysis: with probability $.4^3$ all three bidders have values between 0 and .4 and so MRs uniformly distributed between -1 and -.2 (since "demand" is linear when values are uniformly distributed). In this case, the winner will be the highest-value bidder, and have the highest of these three MRs which is, on average, $-.4$. With the remaining probability at least one bidder will have a value above .4, and the first of these bidders in the queue will win yielding an expected MR of .4.[58] So total expected revenue equals $(.4)^3(-.4) + (1 - .4^3)(.4) = .3488$.

It is easy to check that the auction's expected revenue equals .5.

Constant-ρ Example

If instead $\rho_j = \rho \leq 1 \ \forall j \geq 2$, a new entrant with the deterring value of $v*$ who defeated an existing incumbent at price p but chose *not* to deter would win if no higher-value bidder entered, and would then pay the maximum of p and the highest among subsequent entrants' values. The probability that no subsequent entrant's value would exceed any amount $x \leq v*$ is $(1-\rho)[1 + \rho x + \rho^2 x^2 ...] = \left(\frac{1-\rho}{1-\rho x}\right)$, so the probability density of the highest among subsequent entrants' values equalling x would be $\frac{\partial}{\partial x}\left(\frac{1-\rho}{1-\rho x}\right) = \frac{\rho(1-\rho)}{(1-\rho x)^2}$ for $p \leq x \leq v*$, and this entrant's expected surplus would therefore be $\left[(v* - p)\left(\frac{1-\rho}{1-\rho p}\right) + \int_{x=p}^{v*}(v* - x)\frac{\rho(1-\rho)}{(1-\rho x)^2}dx\right] = \frac{1-\rho}{\rho}\ln\left(\frac{1-\rho p}{1-\rho v*}\right)$. So the deterring bid of any entrant whose value is $v*$ or more, and who defeats the existing incumbent at price p ($p = \underline{v} = 0$ for the first entrant) is $v* - \frac{1-\rho}{\rho}\ln\left(\frac{1-\rho p}{1-\rho v*}\right)$.

To calculate expected revenue, we find the expected MR of the winning bidder. The probability that there exists no bidder with a value above any $x \leq v*$ is $1 - \frac{1-x}{1-\rho x}$. So with probability $\frac{1-v*}{1-\rho v*}$ there will be a bidder with a value of at least $v*$, and so an expected MR of $v*$.[59] With probability density $\frac{\partial}{\partial x}\left(1 - \frac{1-x}{1-\rho x}\right) = \frac{1-\rho}{(1-\rho x)^2}$ the winner's value is $x < v*$, and $MR(x) = 2x - 1$, so the expected revenue from the mechanism is $v*\frac{1-v*}{1-\rho v*} + \int_{x=0}^{v*}(2x - 1)\frac{1-\rho}{(1-\rho x)^2}dx = \frac{v*}{\rho}\left[1 + \frac{(1-\rho)^2}{(1-\rho v*)}\right] + \frac{2(1-\rho)}{\rho^2}\ln(1-\rho v*)$.

In the perfect sequential equilibrium, $v* = V_S = 1 - \sqrt{6c}$ and, for $c = .06$, $v* = V_S = .4$

[58] The average MR of all bidders above any value $\widehat{v}$ equals $\widehat{v}$, since $\frac{1}{1-F(\widehat{v})}\int_{x=\widehat{v}}^{\overline{v}}(x - \frac{1-F(x)}{f(x)})f(x)dx = \widehat{v}$.

[59] The logic for these calculations is exactly as in the 3-bidder example above. As there, it is possible, but cumbersome, to compute expected revenue directly.

(as in the 3-bidder example above). If, for example, $\rho = .8$ the expected revenue $\approx$.29. (It is not hard to check the auction's expected revenue $\approx$.37.)

Proof of Proposition 3: Note that Lemmas 5 and 6 apply even for $v^* > V_S$ if $\prod_{j=3}^{j=n^*+1} \rho_j$ is sufficiently small (using the same argument as for Lemma 6, since the probability of $k > n^*$ entrants is sufficiently small), and the results are then immediate from these lemmas and Propositions 1 and 2.$\square$

Proof of Proposition 4: Absent jump bidding, potential entrants' information about the incumbent's value is only that it exceeds the value of the second-highest-value entrant to date (which the competition between the bidders allows it to observe). So entry will continue after the entry of the first bidder with a value above V_S. However, entry that takes place while there is just one bidder whose value exceeds V_S is on average neutral for efficiency, since revealing only that the incumbent's value exceeds V_S would leave potential entrants indifferent about entering. Conditional on there having been two entrants whose values exceed V_S (so that the higher of these two bidders' values must be drawn from a distribution of $F(\cdot)$ truncated somewhere above V_S) further entry is socially undesirable, but as soon as two such bidders have entered, the price will anyway be driven up to V_S or higher, thus deterring all further entry even without jump bidding.$\square$

Proof of Proposition 6: If, when the current high bid is p, a single final buyer enters, the joint benefit to the new bidder and the seller is $v_j - p$ if $v_j > p$, and 0 otherwise, where v_j is the new bidder's actual value, regardless of whether the incumbent outbids the entrant. So from (3) the seller would like to induce (at least) one additional entrant if $p < V_K$. Therefore, since the seller can if it wishes subsidise any potential bidder so that that bidder's total entry costs are no more than $S(V_K)$, a *price* below V_K would only deter entry if it signalled that the deterring incumbent's value was at least V_K. Obviously, no bidder with a value below V_K will be willing to set a price above V_K, so it follows that no bidder with a value below V_K will deter entry.

As in Lemma 2, in the unique perfect sequential equilibrium, the pool of deterring bidders is as large as possible, so all those with values above V_K *will* deter. Clearly no subsidy is then required to induce entry if the potential entrant faces only bidders who have not deterred (since an unsubsidized entrant earns at least zero in expectation if it will be able to buy at a price no more than V_K), and no feasible subsidy will induce entry once a deterring bid has been made. Therefore, no subsidies are actually paid in this equilibrium.

Furthermore, (3) and the discussion in section 3.2 then imply the equilibrium is both efficient and fully extractive when $\rho_j = 1 \; \forall j$, and so more profitable than the auction in this case.$\square$

Proof of Lemma 2: It is straightforward from the discussion in the text that it is a perfect Nash equilibrium for any entrant, immediately after entry, to jump bid to the price that deters any future entry, if and only if it has a value $\geq v*$, for some $v* \geq V_S$. Furthermore, there cannot be a constant deterring "cut-off" value $> V_K$, because if two bidders had values $> V_K$ but $< v*$ then a further new entrant would not be deterred and would face a minimum price $> V_K$. But its expected profits would be less than the profits from facing a fixed price of V_K, which from (3) are zero, which is a contradiction.[60] So $v* \in [V_S, V_K]$.

The expected revenue equals the expected marginal revenue, MR, of the winning bidder: the first term in (4) is $v*$ times the probability that the winner's value exceeds this cutoff value (see note 58); the second term in (4) (which may be negative) sums the probability that the game will end with exactly j bidders, all of whom have values less than $v*$, times the expected MR of the high bidder contingent on j bidders with values all below $v*$.

Finally, consider any sequential equilibrium in which, at some stage, some bidders with values $\geq V_S$ do not deter; and at that stage let $\widetilde{p}$ be the bid which type V_S would be just indifferent about jumping to *if* such a jump bid did deter future entry. Then at this stage of such an equilibrium (i) all types with values $\geq V_S$ would deviate from their equilibrium strategy to bidding $\widetilde{p}$ *if* doing so did deter (since types that were anyway deterring would thereby deter at a lower price, and other types above V_S gain more than type V_S gains by deterring and therefore strictly gain) *and* (ii) no other type would deviate from its equilibrium strategy to bidding $\widetilde{p}$ even if doing so did successfully deter all further entry. So a type of bidder with value $\geq V_S$ failing to deter cannot, at any stage, be part of a perfect sequential equilibrium. The unique perfect sequential equilibrium therefore has $v* = V_S$.[61]$\square$

Proof of Lemma 3: In both mechanisms, the direct private and social cost of entering is the same, that is, c.

In the auction the private and social contribution of each bidder is the same, namely zero for everyone other than the winner and the difference between the top two values for the winner. Since the expected contribution is therefore clearly also decreasing in the number

[60] Technically, this argument requires a positive probability of at least three potential entrants existing. However, any equilibrium in which the "cut-off value" is above V_K in *any* period not only fails the "intuitive criterion", but is not even robust to the "test of dominated messages" (see Cho and Kreps (1987), and Kreps (1990, p.436)): making the deterring bid corresponding to a "cut-off value" of V_K is strictly dominated for all types with values below V_K (however any potential entrant would respond, all these types would do better not to jump bid), so making this bid must be interpreted as signalling a value $\geq V_K$, and will therefore prevent further entry.

[61] Strictly, if $\rho_j = 1\ \forall j$ this argument proves only that this is the unique neologism-proof equilibrium. It does not quite prove it is the unique perfect-sequential equilibrium, because a bidder with value below V_S is *weakly* willing to bid V_S if $\rho_j = 1\ \forall j$, which spoils the exact Grossman-Perry argument (but not Farrell's). However, if bidders prefer no bid to a surely-losing one (e.g., there are small bidding costs) or of course if ρ_j < 1 for *any* j, *all* equilibria other than the constant-cutoff equilibrium with $v* = V_S$ fail Grossman-Perry's exact refinement, as well as Farrell's.

of entrants, the correct number enter.

In the sequential mechanism, if no bidder with a value of v^* or more has yet entered, the social contribution from one more entry is at least $\int_{x=v^*}^{\overline{v}}(x - v^*)f(x)dx \geq c$ so entry is socially efficient. Also, since the entrant will be able to deter entry through a bid no greater than v^*, it is also profitable to enter, even ignoring potential profits from winning with a bid, or even a value, below v^*. If a bidder with a value of v^* or more has already entered, it is inefficient and unprofitable for a new bidder to enter even ignoring the possibility of others entering later: the expected private and social contributions from entry would be $S(v^*) \leq c$ (from Lemma 2, $S(V_S) = c$ and $v^* \geq V_S$), so the private and social choices are again identical.□

Proof of Lemma 4: The expected revenue from either mechanism equals the expected marginal revenue (EMR) of the winner. Let $EMR_A(j)$ and $EMR_S(j)$ be the expected marginal revenue of the winner of the auction and of the sequential mechanism, respectively, conditional on exactly j potential entrants existing. $EMR_A(1) = EMR_S(1)$ (since the winner is the same in both mechanisms), $EMR_A(j) > EMR_S(j), \forall j \in [2, n^*]$ (since in these cases all the bidders enter the auction[62]), and $EMR_A(j) - EMR_S(j)$ is decreasing in j for $j \geq n^*$ (since for $j \geq n^*$ $EMR_A(j)$ is obviously constant, while $EMR_S(j)$ is increasing in j[63]).

Assume for some $\{\rho_j\}_{j=1}^{\infty}$ the auction yields higher expected revenue, hence higher EMR, than the sequential mechanism. Reducing ρ_k for any k increases the probability of the number of potential entrants equalling $k-1$, while correspondingly reducing the probability of the number exceeding $k-1$. So if $k-1 \geq n^*$, this leaves the auction with the higher EMR, since $EMR_A(j) - EMR_S(j)$ is decreasing.

If $k-1 < n^*$, the effect of a reduction from ρ_k to ρ_k' can be divided into three steps; first multiplying the probability of every possible number of potential entrants by the *same* fraction, ρ_k'/ρ_k, second *increasing* the probability of each number of potential entrants less than or equal to $k-1$ back to its original level, and third *increasing* the probability of $k-1$ potential entrants by multiplying it by $(1-\rho_k')/(1-\rho_k)$. Clearly each step leaves the auction

[62] This comparison of EMRs does *not* depend on MRs increasing in values (or any other regularity condition) since, if 0 or 1 bidder is above v^* the mechanisms sell to the same person, and if 2 or more are above v^* the sequential mechanism sells to an average bidder above v^* (who therefore on average has $MR = v^*$ – see note 58) whilst the auction sells to the highest one above v^*, who is therefore distributed above another one who has a value above v^* (and so on average has MR equal to that value above v^* – see note 58).

[63] This result, also, does *not* depend on MRs increasing in values or any other regularity condition. Conditional on the $(r+1)st$ value exceeding all previous values, the EMR of the $(r+1)st$ value equals the highest previous value (see note 58), which in turn exceeds the MR of that value, which is the MR of the previous winner. Moreover, conditional on the $(r+1)st$ value not exceeding all previous values, the EMR of the sequential mechanism is unchanged by the $(r+1)st$ entrant.

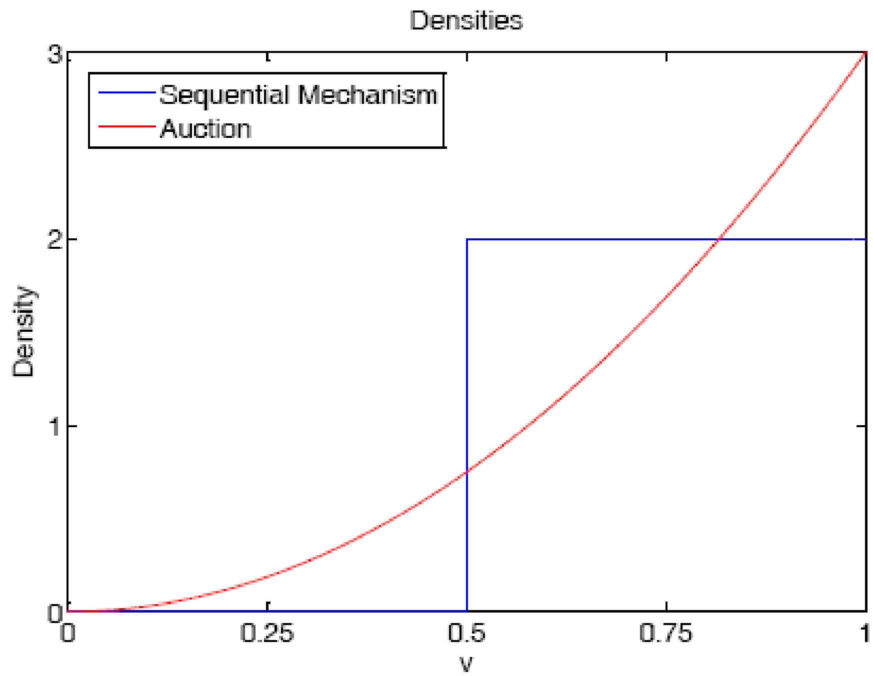
with the higher EMR. So for all k, reducing ρ_k leaves the auction with the higher EMR, and hence higher expected revenue.□

Proof of Lemma 5: For $\rho_j = 1\ \forall j$, assume for contradiction that the sequential mechanism has as many bidders as the auction in expectation. Then the distribution of highest values in the sequential mechanism must stochastically dominate the distribution in the auction, since entrants into the sequential mechanism win if and only if their values are v^* or more, while auction participants sometimes lose with values above v^* and sometimes win with values below v^*. So since an additional bidder's expected profits from entering the sequential mechanism are zero when $v^*= V_S$, its expected profits from entering the auction are positive, which is a contradiction. So the auction attracts more bidders in expectation.

Now, $\forall \{\rho_j\}_{j=1}^{\infty}$, the auction has more expected bidders attributable to potential bidder k, for $2 \leq k \leq n^*$ (since all these would participate in the auction and not all would participate in the sequential mechanism) and the sequential mechanism has more expected bidders attributable to potential bidder k, for any $k > n^*$. So since the auction has more expected bidders for $\rho_j = 1\ \forall j$, it also has more expected bidders if we reduce (by lowering ρ_k) the number of expected bidders attributable to all potential bidders from the kth on.□

Proof of Lemma 6:[64] Recall that expected surplus for all buyers and the seller would be unaffected if the number of potential entrants, k, were revealed in advance of the first bidder's entrance (see note 23). So consider the mechanisms with k revealed in advance. Observe that *conditional on actually entering* a potential entrant's expected surplus from the sequential mechanism is independent of its position in the queue (if its value is less than Vs it wins whenever its value exceeds all other $k-1$ signals; otherwise it wins with probability 1, and must pay the same that it would expect to pay if its value were Vs and it did not deter). So its expected share of surplus from the sequential mechanism equals the fraction of expected actual entrants that it represents, just as in the auction. Clearly there are fewer expected entrants in the sequential mechanism than the auction if $k \leq n^*$ (because all potential entrants then enter the auction) and, since the expected number of entrants in the sequential mechanism is less than n^* (since it is less than n^* even if $\rho_j = 1\ \forall j$, by Lemma 5), the same is true if $k > n^*$. So because the first bidder always enters, it expects a larger share of the buyers' surplus in the sequential mechanism than in the auction for any k, and therefore also on average.□

[64] If $\rho_j = 1\ \forall j$, the result is trivially true using the argument in the text, but in general, surplus is skewed towards the first bidder in the auction, as well as in the sequential mechanism.



Densities of winner's value for an Auction and Sequential Mechanism with identical expected winning bids and identical expected winner's values.

[Drawn for bidders' values uniformly distributed on $[0, 1]$;
for the Auction, $n^*= 3$, so expected winning bid $= \cdot 5$;
for the Sequential Mechanism we assume the same cutoff (winning) bid, $\cdot 5$,
so with an infinite number of potential entrants,
the expected winner's value $= \cdot 75$, in both cases.]