Intertemporal Substitution, Risk Aversion, and Economic Performance in a Stochastically Growing Open Economy*

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Abstract

The constant elasticity utility function implies that the intertemporal elasticity of substitution is the inverse of the coefficient of relative risk aversion. With empirical evidence suggesting that this relationship may or may not hold, studies of risk and growth should decouple these two parameters. This paper provides an analytical characterization and numerical simulations of the equilibrium of a stochastically growing small open economy under general recursive preferences. We show that errors committed by using the constant elasticity utility function, even for small violations of the compatibility condition, can be substantial. Our results suggest that the constant elasticity utility function should be employed with caution.

JEL classification: D81, D91, F43
Key words: Intertemporal substitution; Risk aversion; Stochastic growth

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*This paper was begun when the second author was visiting the Department of Economics at the University of California, Berkeley. It has benefited from presentation to workshops at the University of Mannheim and the University of Washington and to the Seventh International Conference on Computing in Economics and Finance held at Yale University, June 2001. The constructive comments of an anonymous referee are also gratefully acknowledged.

Revised version
June 2002

1. Introduction

Recently, there has been a growing interest in analyzing the effects of policy shifts and other shocks on macroeconomic performance, growth, and welfare in the context of intertemporal stochastic growth models. These studies have been conducted for both closed and open economies, and a variety of shocks have been considered. Beginning with Eaton (1981), authors such as Gertler and Grinols (1982), Smith (1996), Corsetti (1997), Grinols and Turnovsky (1998) and Turnovsky (2000) have analyzed the effects of both monetary and fiscal shocks in stochastically growing closed economies. Parallel to this, Turnovsky (1993), Devereux and Smith (1994) Grinols and Turnovsky (1994), Obstfeld (1994a), Asea and Turnovsky (1998), and Turnovsky and Chattopadhyay (2001) have analyzed the effects of monetary shocks, terms of trade shocks, productivity shocks, and tax changes on economic growth and welfare in small open economies.

In order to obtain closed-from solutions, both the production characteristics and the preferences must necessarily be restricted, and with few exceptions the existing literature assumes that the preferences of the representative agent are represented by a constant elasticity utility function.[1] While this specification of preferences is convenient, it is also restrictive in that two key parameters critical to the determination of the equilibrium growth path -- the intertemporal elasticity of substitution and the coefficient of relative risk aversion -- become directly linked to one another and cannot vary independently. This is a significant limitation and one that can lead to seriously misleading impressions of the effects that each parameter plays in determining the impact of risk and return on the macroeconomic equilibrium and its welfare.

Conceptually, the coefficient of relative risk aversion, *R* say, introduced by Arrow (1965) and Pratt (1964) is a static concept, one that is well defined in the absence of any intertemporal dimension. Similarly, the intertemporal elasticity of substitution, emphasized by Hall (1978, 1988), Mankiw, Rotemberg, and Summers (1985) and others, focuses on intertemporal preferences and is well defined in the absence of risk. A natural definition of the intertemporal elasticity of substitution is in terms of the percentage change in intertemporal consumption in response to a given percentage

[1] Two exceptions to this include Obstfeld (1994a) and Smith (1996).

change in the intertemporal price. For any utility function separable both over time and states, this measure equals the elasticity of the marginal utility with respect to consumption, ε say; McLaughlin (1995). The standard constant elasticity utility function has the property that both parameters ε and R are constant, though it imposes the restriction $R = 1/\varepsilon$, with the widely employed logarithmic utility function corresponding to $R = \varepsilon = 1$. Thus it is important to realize that in imposing this constraint the constant elasticity utility function is also invoking these separability assumptions.

The empirical evidence for both these parameters is quite far-ranging. Estimates for ε based on macro data range from near zero (say 0.1) by Hall (1988), Campbell and Mankiw (1989) to near unity by Beaudry and van Wincoop (1995). Epstein and Zin (1991) provide estimates spanning the range 0.05 to 1, with clusters around 0.25 and 0.7. More recent estimates by Ogaki and Reinhart (1999) suggest values of around 0.4, somewhat higher than the early estimates of Hall (1988). Estimates of ε based on micro data introduce further sources of variation. Attanasio and Weber (1993, 1995) find that their estimate of ε increases from 0.3 using aggregate data, to 0.8 for cohort data, suggesting that the aggregation implicit in the macro data may cause a significant downward bias in the estimate of ε. Atkeson and Ogaki (1996) and Ogaki and Atkeson (1997) find evidence to suggest that the intertemporal elasticity of substitution increases with household wealth.[2]

Estimates of R show even more dispersion. Epstein and Zin find values of R clustering around unity, consistent with the logarithmic utility function, while at the other extreme, issues pertaining to the "equity premium puzzle" induce authors to take R as high as 18 (Obstfeld, 1994a) or even 30 (Kandel and Stambaugh 1991). However, Constantinides, Donaldson, and Mehra, (2002) present alternative empirical evidence to suggest that R lies most plausibly in the range 2 –5, a range that appears to be gaining increasing acceptance. Again, further insight into the empirical evidence is provided by micro data where, using Pakistani village data, Ogaki and Zhang (2001) find that R decreases with wealth.

Within this range of estimates one certainly cannot rule out the constraint $R = 1/\varepsilon$ being approximately satisfied. For example, $R = 2.5, \varepsilon = 0.4$ provides a plausible combination of

[2] Using a panel of Indian household data they find that the intertemporal elasticity of substitution increases from 0.50 for the poorest households to 0.8 for the richest households. Using aggregate data they find that since 1960 the mean for India is 0.27, while for the United States it is 0.40.

parameters for which the constant elasticity utility function is appropriate, and indeed, we shall consider this as representative of a realistic benchmark economy in simulations that we shall undertake. But, given the empirical evidence , $R\varepsilon$ may plausibly range from around 5 to 0.1, certainly well away from 1, as the constant elasticity utility function requires.

Several authors, including Epstein and Zin (1989) and Weil (1989) have represented preferences by a more general (non-separable) recursive function, which enables one to distinguish explicitly between ε and R. This is important for two reasons. First, conceptually, R and ε impinge on the economy in quite independent, and in often conflicting ways. They therefore need to be decoupled if the true effects of each are to be determined. Second, the biases introduced by imposing the compatibility condition $R = 1/\varepsilon$ for the constant elasticity utility function can be quite large, even for relatively weak violations of this relationship.

In this paper we apply the Epstein-Zin recursive preferences to a simple continuous time stochastic growth model of a small open economy. We shall focus on an agent having access to two assets yielding stochastic returns, and we shall identify the two assets as being domestic and foreign respectively. In this respect our analysis is related to Obstfeld (1994b) who introduces these more general preferences into a closed (one-asset) economy. But, by considering an open economy, we find that the differential effects of the two parameters, as well as their interaction, are much more complex, depending in part upon the respective risk characteristics of the two assets.

The paper begins by characterizing the stochastic equilibrium of the small open economy, identifying the closed economy as a useful benchmark. We first examine the impact of risk and return analytically and characterize the bias introduced by imposing the constant elasticity utility function. We supplement our analytical results with comprehensive numerical simulations. These have the advantage of illuminating the patterns of responses as the two key parameters, R and ε, vary, and emphasizing the role that portfolio substitution, a key element absent from the one asset economy, plays in the risk allocation process.

The structure of the equilibrium clarifies how the separation of R and ε is potentially important. Risk aversion impinges on the equilibrium through the portfolio allocation process and thus through the equilibrium risk that the economy is willing to sustain. It also determines the discounting for risk in determining the certainty equivalent level of income implied by the mean return on the assets. The intertemporal elasticity of substitution then determines the allocation of this certainty equivalent income between current consumption and capital accumulation (growth). The main conclusion of the numerical simulations is that the bias in using the constant elasticity utility function, even within the set of plausible parameters may be large, and further, qualitatively erroneous inferences may be drawn. The following are typical examples.

First, starting from the benchmark preference parameters $R = 2.5, \varepsilon = 0.4$, the constant elasticity utility function implies that doubling R to 5 (and thus simultaneously halving ε to 0.2) in a closed economy will reduce the equilibrium growth rate from 1.72% to 1.12%, whereas the unrestricted utility function implies that in fact the equilibrium growth rate will be raised to 1.84%. This represents an error of 0.72%, which in a long-run growth context compounds to a serious difference in economic performance. In the open economy, the errors are comparable, though they are sensitive to the relative riskiness of the domestic and foreign assets.

Second, in some cases the direction of the bias committed by the constant elasticity utility function in an open economy is reversed from what it would be in a closed economy. For example, suppose that the true preferences are $R = 5, \varepsilon = 0.4$. Our results show that increasing the risk on the domestic asset from $\sigma_y = 0.04$ to 0.05 will reduce welfare by 3.63%, whereas the constant elasticity utility function with preferences $R = 2.5, \varepsilon = 0.4$ will imply only a 1.79% reduction. In the corresponding open economy, the constant elasticity utility function will continue to understate the welfare losses as long as the foreign asset is at least as risky as the domestic. But it will mildly overstate the losses (-0.131% vs. –0.117%) if the domestic asset is the riskier one. A similar situation can emerge with the effects of the mean return.

Finally, in the case of an open economy, the constant elasticity utility function may wrongly predict the direction of effect, even for plausible parameters. For example, suppose the true preferences are represented by the parameters $R = 1, \varepsilon = 0.4$. Our results show that if the foreign asset is riskier, then an increase in the rate of return on the domestic asset from 8% to 8.5% will *reduce* the growth rate by 0.09 percentage points. In contrast, the corresponding constant elasticity utility functions, $R = \varepsilon = 1$ and $R = 2.5, \varepsilon = 0.4$, imply that the growth rate would *increase* by 0.06 and 0.07 percentage points, respectively.

The paper is structured as follows. Section 2 sets out the analytical framework. Sections 3 and 4 derive the formal implications for the closed economy and open economy respectively. Section 5 sets out the background to the calibrations, with the numerical results for the closed and open economy being discussed in Sections 6 and 7, respectively. Some final comments are provided in Section 8.

2. The Analytical Framework

The economy comprises a representative agent who faces the choice of consuming a single traded good, investing in that good, or investing in a foreign asset. The economic time unit is *h*, although following Svensson (1989) and Obstfeld (1994a) we shall let $h \to 0$. At time *t* the agent maximizes the intertemporal objective function $U(t)$ defined by the recursive relation

$$f([1-R]U(t)) = \left(\frac{1-R}{1-1/\varepsilon}\right)C(t)^{1-(1/\varepsilon)}h + e^{-\rho h} f\left([1-R]E_t U(t+h)\right) \tag{1}$$

where

$$f(x) = \left(\frac{1-R}{1-1/\varepsilon}\right)x^{(1-1/\varepsilon)/(1-R)}$$

and $C(t)$ denotes consumption, $\rho > 0$ represents the rate of time preference, $R > 0$ is the coefficient of relative risk aversion, and $\varepsilon > 0$ is the intertemporal elasticity of substitution.[3].

Domestic capital yields a stochastic rate of return over the period (t, t+dt)

$$dR_K = s dt + dy, \tag{2a}$$

where *dy* is a Brownian motion process having zero mean and variance $\sigma_y^2 dt$. Likewise, over the same period, the foreign asset yields the stochastic rate of return

[3] The conventional constant elasticity objective function obtains when $R = 1/\varepsilon$, when, as $h \to 0$ the objective function is of the conventional expected-utility form $U(t) = (1-R)^{-1} E_t \int_0^\infty C(s)^{1-R} e^{-\rho(s-t)} ds$.

$$dR_F = rdt + de \tag{2b}$$

where de is a Brownian motion process having zero mean and variance $\sigma_e^2 dt$. For simplicity, we shall assume that the two stochastic shocks are uncorrelated.

The agent's asset holdings are subject to the wealth constraint

$$W = K + F \tag{3}$$

where W denotes wealth, K is the agent's holding of the domestic asset (capital), and F is the foreign asset. The agent's optimization is subject to the stochastic wealth accumulation equation:[4]

$$dW = W\left[n_K dR_K + n_F dR_F\right] - Cdt \tag{4}$$

where n_K, n_F denote the portfolio shares of the two assets, with $n_K + n_F = 1$, and consumption over the time interval $(t, t+dt)$ proceeds at the non-stochastic rate Cdt.

Performing the optimization, leads to the equilibrium conditions:[5]

$$n_K = \frac{s-r}{R(\sigma_y^2 + \sigma_e^2)} + \frac{\sigma_e^2}{\sigma_y^2 + \sigma_e^2} \tag{5a}$$

$$n_F = 1 - n_K \tag{5b}$$

$$\psi = \varepsilon\left(n_K s + n_F r - \rho\right) + \frac{(1-\varepsilon)R}{2}\sigma_\psi^2 \tag{5c}$$

$$\frac{C}{W} = n_K s + n_F r - \psi \tag{5d}$$

$$\sigma_\psi^2 = n_K^2 \sigma_y^2 + n_F^2 \sigma_e^2 \tag{5e}$$

The macroeconomic equilibrium is a stochastic growth path along which all real quantities grow at the common stochastic growth rate

[4] For simplicity and without any essential loss of generality we abstract from labor income. The model can be extended to allow for endogenous labor income; see Turnovsky (2000) for an example in a closed economy, based on the constant elasticity utility function.
[5] The details of these calculations are available from the authors on request. The equilibrium has been derived using the procedure described by Swensson (1989) and employed by Obstfeld (1994a).

$$\frac{dW}{W} = \psi dt + du_{\psi} \tag{6}$$

where ψ denotes the mean growth rate and σ_{ψ} denotes the variability (standard deviation) of du_{ψ}. This equilibrium has a simple recursive structure. First, equations (5a) and (5b) jointly determine the portfolio shares, such that the risk-adjusted returns to the two assets are equalized. These expressions highlight the two determinants of the optimal portfolio shares. The first is the speculative component, which depends positively on the expected differential rate of return and inversely on the variances, while the second reflects the hedging behavior on the part of the investor and depends upon the relative variances associated with the returns on these two assets. Having obtained the portfolio shares, (5e) determines the equilibrium variance, σ_{ψ}^2, along the balanced growth path, with (5c) and (5d) then sequentially determining the equilibrium mean growth rate, ψ, and the consumption-wealth ratio C/W. Finally, the equilibrium must also satisfy the transversality condition, which in this case reduces to $C/W > 0$.[6]

Equations (5a) – (5e) highlight the distinct roles played by the two parameters R and ε in the determination of the equilibrium. It is seen that the portfolio shares, n_K, n_F, and therefore the equilibrium risk, σ_{ψ}, are both sensitive to R through the speculative component, but, being a purely static allocation decision, are independent of the intertemporal elasticity of substitution ε. The coefficient of risk aversion impinges on the C/W ratio, and thus on the mean growth rate, through two channels. The first is through the portfolio shares, which determine mean income, $sn_K + rn_F$, and its variance, σ_{ψ}^2. This is then converted to a "certainty-equivalent" quantity, $sn_K + rn_F - (R/2)\sigma_{\psi}^2$, with risk being "priced" at $R/2$. Any change in this risk-adjusted income measure leads to a positive income effect and a negative substitution effect on consumption, the net effect of which depends upon $(1-\varepsilon)$.

Of particular significance is the welfare of the representative agent, as the economy evolves along its stochastic equilibrium growth path. This can be shown to be given by

$$\Omega = \left(\frac{C}{W}\right)^{((1-R)/(1-\varepsilon))} \frac{W_0^{1-R}}{1-R} \tag{5f}$$

[6] This was first established by Merton (1969) and the argument applies to the present model.

Given the transversality condition (5f) implies $\Omega(1-R)>0$ and taking the differential of (5f) yields:

$$\frac{d\Omega}{(1-R)\Omega}=\frac{1}{(1-\varepsilon)}\cdot\frac{d(C/W)}{C/W} \tag{7}$$

which forms the basis for the analysis of the welfare effects undertaken below.

We shall compare the economy using this more general representation of preferences with those obtained from the conventional constant elasticity utility function, $\left(1/(1-\theta)\right)C^{1-\theta}$, according to which the implied coefficient of relative risk aversion and intertemporal elasticity of substitution are $\varepsilon_C = 1/\theta,\ R_C=\theta$, respectively. In interpreting the constant elasticity function, we shall assume

$$\theta=\lambda R+(1-\lambda)(1/\varepsilon) \qquad 0\le\lambda\le 1 \tag{8}$$

where λ reflects the relative weight assigned to R and $1/\varepsilon$ in the constant elasticity θ. Thus the biases of θ as measures of R and $1/\varepsilon$ are respectively,

$$\theta-R=\frac{(1-\lambda)}{\varepsilon}(1-R\varepsilon) \tag{9a}$$

$$\theta-\frac{1}{\varepsilon}=\frac{\lambda}{\varepsilon}(R\varepsilon-1) \tag{9b}$$

The elasticity θ is an unbiased measure of both parameters simultaneously if and only if $R\varepsilon=1$. Otherwise it understates (overstates) R and overstates (understates) $1/\varepsilon$ if $R\varepsilon>(<)1$, and in general, $R\varepsilon-1$ serves as a measure of the incompatibility of the constant elasticity utility assumption.

3. Closed Economy

It is convenient to begin with a closed economy. This is obtained by setting $n_K=1, n_F=0$ in the equilibrium (5a) – (5e), which reduces drastically to[7]

$$\psi=\varepsilon(s-\rho)+\frac{(1-\varepsilon)R}{2}\sigma_y^2 \tag{5c'}$$

$$\frac{C}{W}=s-\psi \tag{5d'}$$

[7] The corresponding equilibrium volatility is $\sigma_\psi=\sigma_y$

From these relationships we immediately infer the following:

3.1 Higher Mean Return

$$\frac{\partial\psi}{\partial s}=\varepsilon;\quad \frac{\partial(\mathrm{C/W})}{\partial s}=1-\varepsilon;\quad \frac{\partial\Omega/\partial s}{\Omega(1-R)}=\frac{1}{1-\varepsilon}\frac{\partial(C/W)/\partial s}{C/W}=\frac{1}{C/W} \tag{10}$$

A higher mean return to (domestic) capital has both a positive income effect and a negative substitution effect on consumption, the net effect of which depends upon $1-\varepsilon$, thus leading to more consumption if and only if the intertemporal elasticity of substitution is less than unity, as the empirical evidence suggests. But even in the unlikely event that consumption declines, the growth rate increases, as does intertemporal welfare.

3.2 Higher Risk

$$\frac{\partial\psi}{\partial\sigma_y^2}=(1-\varepsilon)\frac{R}{2};\quad \frac{\partial(\mathrm{C/W})}{\partial\sigma_y^2}=-(1-\varepsilon)\frac{R}{2};\quad \frac{\partial\Omega/\partial\sigma_y^2}{\Omega(1-R)}=\frac{1}{1-\varepsilon}\frac{\partial(C/W)/\partial\sigma_y^2}{C/W}=-\frac{R/2}{C/W} \tag{11}$$

Higher risk reduces the certainty equivalent return to capital, by an amount that is proportional to the degree of risk aversion, R. This has both a negative income effect and positive substitution effect on consumption, lowering consumption, and raising the equilibrium growth rate as long as $\varepsilon<1$. Welfare always declines, as long as agents are risk averse.

The impact of volatility on growth has been extensively analyzed empirically, with mixed findings. Kormendi and Meguire (1985) obtain a positive relationship, while a more recent study by Ramey and Ramey (1995) finds a negative relationship. Given the empirical evidence favoring $\varepsilon<1$, the response in (11) is consistent with the earlier evidence, although its magnitude is small. As we shall see in Section 4 below, the introduction of a second asset, with the portfolio adjustment, introduces much more flexibility into the risk-growth relationship.

3.3 Constant Elasticity Utility and Bias

Setting $R=1/\varepsilon=\theta$ in (5c'), the equilibrium growth rate implied by the constant elasticity utility function is (with the corresponding growth rate subscripted by C):

$$\psi_C = \frac{1}{\theta}(s-\rho) + \frac{(\theta-1)}{2}\sigma_y^2 \tag{5c”}$$

The effects of productivity and risk on the mean growth rate with the constant elasticity utility function are thus

$$\left(\frac{\partial\psi}{\partial s}\right)_{\mathrm{C}} = \frac{1}{\theta};\ \left(\frac{\partial\psi}{\partial\sigma_y^2}\right)_{\mathrm{C}} = \frac{\theta-1}{2} \tag{12}$$

Subtracting these quantities from the corresponding expressions in (10) and (11), and noting the definition of θ, we see that the effects implied by the constant elasticity utility function deviate from the “true effects” (those stemming from the more general utility function) by the amounts:

$$\left(\frac{\partial\psi}{\partial s}\right)_C - \frac{\partial\psi}{\partial s} = \frac{\lambda\varepsilon}{\theta}\left(\frac{1}{\varepsilon} - R\right) \tag{13a}$$

$$\left(\frac{\partial\psi}{\partial\sigma_y^2}\right)_C - \frac{\partial\psi}{\partial\sigma_y^2} = \frac{(1-\lambda-\varepsilon)}{2}\left(\frac{1}{\varepsilon} - R\right) \tag{13b}$$

In the event that the compatibility condition $R\varepsilon = 1$ is met, then the constant elasticity responses give correct estimates of the true responses. However, if this condition is not met, the responses are biased, and the biases can be large, for even a small degree of incompatability.

As a plausible example, assume $R = 2.5, \varepsilon = 0.2$. In this case the true effects of mean return and variance on the growth rate are $\partial\psi/\partial s = 0.2$, $\partial\psi/\partial\sigma_y^2 = 1$. In the case that $\lambda = 0$, so that the constant elasticity utility function places all the weight on ε (i.e. $\theta = 1/\varepsilon$), $(\partial\psi/\partial s)_C = 0.2$, $(\partial\psi/\partial\sigma_y^2)_C = 2$. In this case, the constant utility function correctly predicts the effect of higher productivity on the mean growth rate. However, it overstates the effect of the variance on the growth rate by 100%! In the other polar case where $\lambda = 1$, so that the constant elasticity utility function implies $\theta = R$, $(\partial\psi/\partial s)_C = 0.4$, $(\partial\psi/\partial\sigma_y^2)_C = 0.75$. Now the constant elasticity utility function overstates the effect of return on growth by 100% and now understates the true effect of risk by 25%. Since the difference between $\varepsilon = 0.4$ and $\varepsilon = 0.2$ lies well within typical statistical margins of error, these prediction errors are certainly of practical significance.

One point worth observing is that as long as one imposes the plausible restrictions $0<\varepsilon<1, R\geq 1$, so that $\theta\geq 1$, the predictions of the effects of a higher mean return and higher risk on the mean growth rate obtained from the constant utility function are qualitatively correct, although the biases may be large in magnitude.[8] As we will see, this does not hold when we move to the open economy.

4. The Open Economy

We turn now to the open economy. Substituting for the equilibrium portfolio shares n_K, n_F from (5a) and (5b) into (5c) and (5e), the equilibrium solutions for the mean and variance of the growth rate, ψ, and σ_ψ^2 are given by:

$$\psi=\left[\left(\frac{1+\varepsilon}{2R}\right)(s-r)^2+\varepsilon\left(s\sigma_e^2+r\sigma_y^2\right)+(1-\varepsilon)\frac{R}{2}\sigma_e^2\sigma_y^2\right]\frac{1}{\left(s\sigma_e^2+r\sigma_y^2\right)}-\varepsilon\rho \tag{14a}$$

$$\sigma_\psi^2=\left(\frac{(s-r)^2}{R^2}+\sigma_e^2\sigma_y^2\right)\frac{1}{\left(\sigma_e^2+\sigma_y^2\right)} \tag{14b}$$

with the implied value for the consumption-wealth ratio being

$$\frac{C}{W}=\frac{(s-r)^2}{R\left(\sigma_y^2+\sigma_e^2\right)}+\frac{s\sigma_e^2+r\sigma_y^2}{\sigma_y^2+\sigma_e^2}-\psi \tag{14c}$$

As we shall see, a crucial determinant of the effects of structural changes on the equilibrium is the size of the equilibrium portfolio shares, n_K, n_F, relative to the variance-minimizing portfolio shares, $\tilde{n}_K, \tilde{n}_F$. The latter are defined to be the portfolio shares chosen by an agent wishing to minimize the volatility σ_ψ^2 and are obtained by minimizing (5e) with respect to n_K, n_F to yield

$$\tilde{n}_K=\frac{\sigma_e^2}{\sigma_y^2+\sigma_e^2};\quad \tilde{n}_F=\frac{\sigma_y^2}{\sigma_y^2+\sigma_e^2}$$

With this notation, the equilibrium portfolio shares are

[8] In the unlikely case that $\varepsilon>1$ the effects of an increase in σ_y^2 implied by the constant elasticity utility function may be qualitatively in the wrong direction.

$$n_K = \frac{s-r}{R\left(\sigma_y^2+\sigma_e^2\right)} + \tilde{n}_K; \qquad n_F = \frac{r-s}{R\left(\sigma_y^2+\sigma_e^2\right)} + \tilde{n}_F \tag{15}$$

The intuition of our results is enhanced by focusing on the assets' beta coefficients. To do this we consider the equilibrium (market) portfolio, (n_K, n_F), the stochastic return on which is

$$dR_Q = r_Q dt + du_Q;$$

$$r_Q \equiv n_K s + n_F r; \qquad du_Q \equiv n_K dy + n_K de \equiv du_\psi$$

The beta coefficient of asset i (i = K, F) are defined by

$$\beta_K = \frac{\operatorname{cov}(du_Q, dy)}{\operatorname{var}(du_Q)}, \qquad \beta_F = \frac{\operatorname{cov}(du_Q, de)}{\operatorname{var}(du_Q)} \tag{16}$$

and thus

$$s-\eta = \beta_K(r_Q-\eta); \;\; r-\eta = \beta_F(r_Q-\eta)$$

where η is the expected return on an asset uncorrelated with du_Q. The coefficient of proportionality, β_i, reflects the degree of risk associated with the asset, the asset being more or less risky than the overall market according to whether its beta coefficient is greater or less than unity.

Evaluating (16),[9]

$$\beta_K = \frac{n_K\sigma_y^2}{n_K^2\sigma_y^2+n_F^2\sigma_e^2}; \qquad \beta_F = \frac{n_F\sigma_e^2}{n_K^2\sigma_y^2+n_F^2\sigma_e^2}$$

and using (15) we obtain

$$\beta_K = \frac{\sigma_y^2\left[(s-r)/R+\sigma_e^2\right]}{(s-r)^2/R^2+\sigma_e^2\sigma_y^2} = \frac{n_K\tilde{n}_F}{n_K^2\tilde{n}_F+n_F^2\tilde{n}_K} \tag{17a}$$

$$\beta_F = \frac{\sigma_e^2\left[(r-s)/R+\sigma_y^2\right]}{(s-r)^2/R^2+\sigma_e^2\sigma_y^2} = \frac{n_F\tilde{n}_K}{n_K^2\tilde{n}_F+n_F^2\tilde{n}_K} \tag{17b}$$

[9] If $\beta_i = 0$ the equilibrium return on the asset equals the riskless rate. It is also possible for $\beta_i < 0$, in which case the return is less than the riskless rate. This usually occurs if the return on the asset is negatively correlated with the market return. Since we are ruling this out, in our analysis $\beta_i < 0$ would correspond to $n_i < 0$, so that the agent is taking a short position in that asset.

These relationships highlight the intimate relationship between the equilibrium portfolio shares and the associated risk. The following special cases merit comment.

(i) Domestic and Foreign Assets Equally Risky $\beta_K = \beta_F$: Equating (17a) and (17b) this implies $s = r$ and equivalently, $n_K = \tilde{n}_K$, $n_F = \tilde{n}_F$ so that $\beta_K = \beta_F = 1$. With just two equally risky assets, their respective risks must be equal to the overall market risk.

(ii) Domestic Asset Riskier than Foreign Asset $\beta_K > \beta_F$: This implies $s > r$, so that $n_K > \tilde{n}_K$, $n_F > \tilde{n}_F$. Using the relationships

$$\beta_K - 1 = \frac{(n_K - \tilde{n}_K)n_F}{n_K^2\tilde{n}_F + n_F^2\tilde{n}_K}; \quad \beta_F - 1 = \frac{(\tilde{n}_K - n_K)n_K}{n_K^2\tilde{n}_F + n_F^2\tilde{n}_K}$$

we see that if $n_K > 0$, $n_F > 0$, then $\beta_K > 1, \beta_F < 1$.

(iii) Foreign Asset Riskier than Domestic Asset $\beta_F > \beta_K$: This implies $r > s$ and $\beta_F > 1 > \beta_K$.

4.1 Higher Mean Return on Domestic Asset

An increase in the mean return on the domestic asset has the following effects:

$$\frac{\partial\psi}{\partial s} = \varepsilon n_K + (n_K - \tilde{n}_K) \tag{18a}$$

$$\frac{\partial\sigma_\psi^2}{\partial s} = 2\left(\sigma_y^2 + \sigma_e^2\right)(n_K - \tilde{n}_K)R \tag{18b}$$

$$\frac{\partial(C/W)}{\partial s} = (1-\varepsilon)n_K \tag{18c}$$

$$\frac{\partial\Omega/\partial s}{(1-R)\Omega} = \frac{n_K}{C/W} > 0 \tag{18d}$$

The coefficient of relative risk aversion, R, affects these quantities only indirectly, through its impact on the equilibrium portfolio share, n_K. The intertemporal elasticity of substitution, ε, has a direct effect on the mean growth rate and an offsetting effect on the consumption-wealth ratio. It has no effect on volatility.

An increase in s has a positive direct effect on the growth rate, reflecting the extent, n_K, to which the agent holds the domestic capital stock. But at the same time, the higher return causes a portfolio shift toward the domestic asset. If $s > r$ so that $n_K > \tilde{n}_K$, this raises the total return further. It also increases domestic risk, which raises the mean growth rate as long as $\varepsilon < 1$. Taken together, these two effects are positive if and only if $n_K > \tilde{n}_K$. In the case that the foreign asset is sufficiently riskier than the domestic to yield $n_K < \tilde{n}_K/(1+\varepsilon)$ a higher return on domestic capital is growth-reducing. Finally, the effects on consumption and welfare are analogous to the corresponding effects in the closed economy, as in (10), but are scaled by the portfolio share n_K.

4.2 Increase in Risk of Domestic Asset

An increase in the risk of the domestic asset has the following effects:

$$\frac{\partial \psi}{\partial \sigma_y^2} = -\frac{Rn_K}{2}\left[\varepsilon n_K + (n_K - 2\tilde{n}_K)\right] \tag{19a}$$

$$\frac{\partial \sigma_\psi^2}{\partial \sigma_y^2} = -n_K(n_K - 2\tilde{n}_K) = -2n_K(n_K - \tilde{n}_K) + n_K^2 \tag{19b}$$

$$\frac{\partial (C/W)}{\partial \sigma_y^2} = -Rn_K^2(1-\varepsilon) \tag{19c}$$

$$\frac{\partial \Omega / \partial \sigma_y^2}{(1-R)\Omega} = -\frac{R}{2}\frac{n_K^2}{C/W} < 0 \tag{19d}$$

An increase in σ_y^2 has two effects on the mean growth rate. On the one hand it induces a portfolio shift from the domestic to the foreign asset and whether this raises or lowers the growth rate depends upon whether $s \lessgtr r$. At the same time it has two effects on σ_ψ^2. There is a positive direct effect, n_K^2, and a portfolio adjustment effect that depends the portfolio composition relative to its variance minimizing structure. The net effect on the growth rate depends critically upon the relative sizes of n_K and $\tilde{n}_K$.

In the case that the two assets are equally risky, an increase in volatility of the domestic asset raises the overall volatility, increases the mean growth rate, reduces consumption (if $\varepsilon < 1$) and reduces welfare. If the domestic asset is sufficiently riskier than the foreign asset to yield

$$\frac{2\tilde{n}_K}{1+\varepsilon} < n_K < 2\tilde{n}_K$$

then (19a) and (19b) imply that higher volatility in the domestic asset leads to a reduction in the mean growth rate, consistent with the Ramey-Ramey (1995) results, accompanied by an increase in overall volatility. The likelihood of this increases with ε.

Finally, the symmetry of the model has enabled us to focus on the impact of the mean and variance of return of the domestic asset. The effects of corresponding changes in the foreign asset are analogous, leading to similar conclusions.

4.3 Constant Elasticity Utility and Bias

The effects of a higher return to the domestic asset and its risk on the mean growth rate and its volatility with the constant elasticity utility function are respectively

$$\left(\frac{\partial\psi}{\partial s}\right)_C = \left[\left(\frac{1+\theta}{\theta}\right)(s-r) + \sigma_e^2\right]\frac{1}{\theta\left(\sigma_y^2+\sigma_e^2\right)}$$

$$\left(\frac{\partial\psi}{\sigma_y^2}\right)_C = \left[-\frac{(1+\theta)}{2\theta^2}(s-r)^2 - \frac{\sigma_e^2}{\theta}(s-r) + \frac{(\theta-1)\sigma_e^4}{2}\right]\frac{1}{\left(\sigma_y^2+\sigma_e^2\right)^2}; \tag{20a}$$

$$\left(\frac{\partial\sigma_\psi^2}{\partial s}\right)_C = \left[\frac{2(s-r)}{\theta^2} + \sigma_y^2\sigma_e^2\right]\frac{1}{\left(\sigma_y^2+\sigma_e^2\right)}; \qquad \left(\frac{\partial\sigma_\psi^2}{\partial\sigma_y^2}\right)_C = -\frac{(s-r)^2}{\theta^2\left(\sigma_y^2+\sigma_e^2\right)^2} + \left(\frac{\sigma_e^2}{\sigma_y^2+\sigma_e^2}\right)^2 \tag{20b}$$

With the more general portfolio adjustment the expressions for the bias are more complex and further insight is obtained by focusing on the extreme cases $\lambda = 1, \lambda = 0$ and the corresponding values of $\theta = R, \theta = 1/\varepsilon$. The resulting expressions are presented in the Appendix.

In all cases the biases from using the constant elasticity index are proportional to the incompatibility factor $(R - 1/\varepsilon)$. Suppose $R > 1/\varepsilon$. Then, if $\lambda = 1$ the constant elasticity utility index will understate the effect of a higher return, s, on the growth rate, just as it does in the closed economy. But in contrast to the latter, when for $\lambda = 0$ there is no bias, the effect will be overstated or understated, depending upon whether the domestic asset is riskier or less risky than the foreign asset. The effects of risk on the growth path are essentially as in the closed economy. They are overstated by the constant elasticity utility function if $\lambda = 1$, and understated if $\lambda = 0$. If $\lambda = 1$, the constant elasticity utility function will correctly state the effects of both s and σ_y^2 on the volatility of the growth rate. When $\lambda = 0$ it will overstate or understate the effect of s on the volatility, depending upon the relative riskiness of the domestic asset, and it will always understate the effect of domestic volatility on overall volatility.

The magnitudes of these biases can be substantial and will be illustrated in the numerical analysis carried out in Section 5. In contrast to the closed economy it is now possible through the portfolio adjustment for the responses implied by the constant elasticity function to be qualitatively incorrect. For example, we can show that if $R > 1/\varepsilon$ and

$$\frac{\theta}{1+\theta} < \frac{r-s}{\sigma_e^2} < \frac{\varepsilon R}{1+\varepsilon}$$

then $\left(\partial\psi/\partial s\right)_C < 0$, while in fact $\partial\psi/\partial s > 0$.

5. Calibration of Model

To obtain further insights we shall conduct a numerical simulation of the equilibrium. This will be based on the following parameters, representative of a small open economy:[10]

Rate of time preference	$\rho = 0.04$
Mean rates of return	$s = 0.08,\ 0.085$ $r = 0.08,\ 0.079,\ 0.088$
Stochastic shocks	$\sigma_y = 0.04,\ 0.05$ $\sigma_e = 0.04,\ 0.01414,\ 0.1131$
Coefficient of relative risk aversion	$R = 2/3,\ 1,\ 2.5,\ 5,\ 10$
Intertemporal elasticity of substitution	$\varepsilon = 1.5,\ 1,\ 0.4,\ 0.2,\ 0.1$

The starting point is a closed economy in which the mean rate of return on capital is 8% with a standard deviation of 4%, so that it is relatively volatile. The rate of time preference in this economy is 4% and remains so throughout the analysis. Our coefficients of relative risk aversion

[10] The returns and standard deviations are in percentages at annual rates. Thus $s = 0.08$ or 8% and $\sigma_y = 0.04$ or 4%.

span their respective plausible empirical ranges of 1 (logarithmic) – 10 and 1 to 0.1, respectively. In addition, because the qualitative behavior of the economy changes when $\varepsilon > 1$, we also consider $\varepsilon = 1.5$, and the corresponding value $R = 2/3$, as implied by the compatibility condition.

Table 1 yields a grid of equilibrium growth rates as ε and R vary across these specified values. The expressions in bold letters, along the diagonal, correspond to the combinations of ε and R that satisfy the compatibility condition $R\varepsilon = 1$, and thus correspond to the constant elasticity utility function. The shaded cell corresponds to the combination $R = 2.5,\ \varepsilon = 0.4$, which serves as a plausible benchmark, one that also satisfies the compatibility condition. It implies an equilibrium mean growth rate of 1.72%. Panels A and B of Table 2 then consider the equilibria for alternative combinations of ε and R that obtain when the standard deviation of the return is increased to 0.05 and the mean return is increased to 0.085, respectively.

When extended to an open economy, the base parameters for the domestic economy, $s = 0.08,\ \sigma_y = 0.04$, remain unchanged. Table 3 has three panels describing the equilibrium growth rate, corresponding to the relative riskiness of the two economies. Panel B describes the symmetric case where the domestic and foreign assets are equally risky. Note that the assumption of equal riskiness imposes the constraint $n_K / n_F = \tilde{n}_K / \tilde{n}_F = \sigma_e^2 / \sigma_y^2$. We impose the additional condition $\sigma_y = \sigma_e = 0.04$, which implies the symmetric equilibrium portfolio allocation $n_K = n_F = 0.5$. This implies an equilibrium growth rate of 1.6% with the corresponding volatility of 2.828%. Panel A is the case where the domestic asset is riskier than the foreign asset. It is parameterized by assuming for the benchmark preference parameters $R = 2.5,\ \varepsilon = 0.4$, that $\beta_K = 2, \beta_F = 0.5$, so that the domestic asset is twice as risky and the foreign asset half as risky as the overall market. This risk structure can be shown to imply $r = 0.079, \sigma_e = 0.01414$, a lower return and standard deviation than the domestic asset. At the benchmark it implies portfolio shares $n_K = 1/3, n_F = 2/3$ and an equilibrium growth rate and standard deviation of 1.59% and 1.63%, respectively. Panel C deals with the converse case where the foreign asset is riskier than the domestic. It is parameterized by assuming for the benchmark preference parameters $R = 2.5,\ \varepsilon = 0.4$, that $\beta_K = 0.5, \beta_F = 2$. This risk structure can be shown to imply $r = 0.088, \sigma_e = 0.1131$, a lower return and standard deviation than the domestic asset. At the benchmark it implies portfolio shares $n_K = 2/3, n_F = 1/3$ and an equilibrium growth rate and standard deviation of 1.87% and 4.63%, respectively.

The three cases are thus symmetric with respect to the risk characteristics of the two assets. Note that since the beta coefficient is a function of *R*, it changes as we move across the columns. Table 4 analyzes the effects of an increase in σ_y from 0.04 to 0.05 for the corresponding parameterizations of Table 3, while Table 5 analyzes similarly the effects of an increase in *s* from 0.08 to 0.085.

Tables 2, 4, and 5 report the changes in welfare resulting from changes in the mean and standard deviation of the return on the domestic asset. These quantities, reported as $d\Omega$, are equivalent variation measures, calculated as the percentage change in the initial stock of wealth necessary to maintain the level of welfare unchanged following the structural change in the asset return structure.

6. Numerical Results for Closed Economy

We now turn to the numerical results, beginning first with the closed economy.

6.1 Equilibrium Growth Rate and Preference Parameters

In general the range of mean equilibrium growth rates reported in Table 1, for given risk and return ($\sigma_y = 0.04, s = 0.08$) spans a broad range from nearly 6% to less than 0.5% as the risk and intertemporal preference parameters change. As noted, the benchmark parameters $R = 2.5, \varepsilon = 0.4$ yield a plausible equilibrium growth rate of 1.72%. The table yields the following patterns:

1. The equilibrium mean growth rate in a closed economy increases with the intertemporal elasticity of substitution, ε, at least for plausible values of the coefficient of relative risk aversion, *R*.[11] As *R* increases the sensitivity to ε decreases.

[11] An increase in ε has two offsetting effects on the mean growth rate, a positive effect that operates through the deterministic component $s-\rho$, and a negative effect that operates through the risk component $(1-\varepsilon)R\sigma_y^2/2$; see (5c”). For the chosen parameter values the first effect will dominate as long as $R < 25$.

2. The equilibrium growth rate in a closed economy increases with R if and only if $\varepsilon < 1$, as empirical evidence suggests.

3. The fact that the growth rate increases with both R and ε implies that employing the constant elasticity utility function, with its constraint $\theta \equiv R = 1/\varepsilon$, will certainly have inaccurate implications. For our chosen parameters, moving down the diagonal of Table 1, one will infer that the growth rate will decline as $\theta \equiv R = 1/\varepsilon$ increases through plausible values < 7; thereafter the growth rate increases uniformly.[12]

4. Starting from the benchmark parameterization $R = 2.5, \varepsilon = 0.4$, (the shaded cell) we see that doubling R to 5 will raise the growth rate by 0.12 percentage points. In contrast, the constant elasticity utility function erroneously implies that an increase in $R = 1/\varepsilon$ will reduce the equilibrium growth rate by 0.60 percentage points. It also understates the true reduction in the growth rate stemming from an equivalent decline in ε by 0.16 percentage points (0.60 rather than 0.76 decline).

5. As we move down the diagonal (in bold letters) we see that the biases of the constant elasticity utility function in describing the effects of R and ε move in opposite directions; the former declines, while the latter increases.

6.2 Changes in Risk and Return

These effects are reported in Panels A and B of Table 2. Focusing on the benchmark parameters in the shaded box, we see that an increase in σ_y from 0.04 to 0.05 raises the mean growth rate by 0.07 percentage points, reducing welfare by 1.79%. An increase in the rate of return on capital from 8% to 8.5% raises the growth rate by 0.20 percentage points, leading to a welfare improvement of over 8.1%. From the table, the following patterns in response to ε and R can be identified.

1. An increase in σ_y from 0.04 to 0.05 has a more positive effect on the mean growth rate as ε declines (and is negative if $\varepsilon > 1$). This reduces the adverse effect of the higher risk on welfare.

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[12] From (5c") one can show that the given $s, \rho, \varepsilon, R,$ and σ_y^2, the growth rate (as a function of θ) will be minimized where $\theta = \left[2(s-\rho)\right]^{.5} / \sigma_y$, which for the chosen parameters implies $\theta = 5\sqrt{2} \approx 7.07$.

2. The positive effects of an increase in risk on the mean growth rate increase with R if and only if $\varepsilon < 1$. However, as R increases, the adverse effects on welfare increase, despite the higher growth rate. For the plausible parameter combination, $\varepsilon = 0.10, R = 5$, this modest increase in risk reduces welfare by over 3%.

3. The impact of an increase in s declines with ε and is independent of R. The positive effects on welfare increase with increasing ε, but less so as R increases. The positive welfare effects increase with R if and only if $\varepsilon < 1$.

4. Suppose that the true economy is represented by the preference parameters $R = 5, \varepsilon = 0.4$. In this case the constant elasticity utility function $\theta \equiv R = 1/\varepsilon = 5$ will substantially **overstate** the positive effect of higher risk on the mean growth rate (0.180 vs. 0.135), and mildly **understate** its adverse welfare effect (-3.26% vs. –3.63%). It will **understate** the positive impact of a higher return to capital on the mean growth rate (0.10 vs. 0.20) and welfare (7.32% vs. 8.25%). By contrast, the constant elasticity utility function $R = 1/\varepsilon = 2.5$ will significantly **understate** both the positive effect of an increase in risk on the mean growth rate (0.068 vs. 0.135), and its adverse welfare effect (-1.79% vs –3.63%). It will **correctly predict** the positive impact of a higher return to capital on the mean growth rate (0.20), though weakly **understate** its positive welfare effect (8.10% vs. (8.25%).

5. Suppose that the true economy is represented by the preference parameters $R = 2.5, \varepsilon = 0.2$. In this case the constant elasticity utility function $R = 1/\varepsilon = 5$ will substantially **overstate** both the positive effect of higher risk on the mean growth rate (0.180 vs. 0.098), and its adverse welfare effect (-3.26% vs. –1.60%). It will **correctly predict** the positive impact of a higher return to capital on the mean growth rate (0.10) but weakly **overstate** its positive welfare effect (7.32% vs. 7.15%). By contrast, the constant elasticity utility function $R = 1/\varepsilon = 2.5$ will **understate** the positive effect of an increase in risk on the mean growth rate (0.068 vs. 0.098), and **overstate** its adverse welfare effect (-1.79% vs. –1.60%). It will **overstate** both the positive impact of a higher return to capital on the mean growth rate (0.20 vs. 0.10) and its positive welfare effect (8.09% vs. 7.15%).

6. Suppose the economist is restricted to a constant elasticity utility function. He would incorrectly infer that an economy having a higher intertemporal elasticity of substitution (ε increased from 0.2 to 0.4) would be significantly **less** adversely affected by an increase in risk (-

1.79% vs. –3.26%), whereas in fact it would be **more** adversely affected (-3.63% vs. –3.26%). Likewise, he would incorrectly infer that an economy having a lower degree of risk aversion (R reduced from 5 to 2.5) would be *more* positively impacted by more productive capital (8.09% vs. 7.32%), whereas in fact it would be *less* positively impacted (7.15% vs. 7.32%).

7. Numerical Results for Open Economy

We now turn to the equilibrium in the open economy reported in the three panels of Table 3 and note the following.

7.1 Equilibrium Growth Rate and Preference Parameters

1. The equilibrium growth rate depends upon the riskiness of the two assets. Holding the risk of the domestic asset constant, it is uniformly higher (lower) according to whether the foreign asset is more (less) risky than the domestic asset. For the benchmark preference parameters in the shaded boxes, we see that opening the economy to trade in a risky asset will lower the growth rate if the foreign asset is equally or less risky than the domestic, while it will raise the growth rate if the foreign asset is riskier; see Obstfeld (1994a).

2. The equilibrium mean growth rate in the open economy decreases with ε, exhibiting a pattern similar to that of a closed economy.

3. The responsiveness of the equilibrium growth rate to R depends upon the relative riskiness of the two assets. If the domestic and foreign assets are equally risky, the growth rate increases with R if and only if $\varepsilon < 1$, as in a closed economy. But the mean growth rate is uniformly lower than in the closed economy, due to diversification and lower equilibrium risk ($\sigma_{\psi} = 2.828$) if and only if $\varepsilon < 1$.

4. If the two assets are not equally risky (Panels A and C), then the equilibrium mean growth rate declines with R if $\varepsilon \geq 1$. For $\varepsilon < 1$ it declines with R for low degrees of risk aversion, R, and then increases uniformly. Its sensitivity to R increases when the foreign asset is more risky and more risk is introduced into the economy.

5. Equilibrium risk and portfolio shares are always independent of ε. They are also independent of R if domestic and foreign assets are equally risky, in which case $n_K = 0.5$,

$\sigma_\psi = 2.83\%$. Portfolio share of the more risky asset and equilibrium risk both decline with *R*; see Panels A and C.

6. Using the constant elasticity utility function one will incorrectly infer that the equilibrium growth rate declines with *R* uniformly, whereas it actually increases with *R* as long as $\varepsilon < 1$. One will correctly infer that it increases with ε, although it will marginally underpredict the response. One will also correctly conclude that the volatility of the growth rate in general declines with *R*, but incorrectly infer that it increases with ε, when in fact it is independent of ε.[13]

7.2 Effects of Changes in Risk

These results are reported in Table 4 and a general observation is that an increase in risk from 0.04 to 0.05 is highly sensitive to (i) the coefficient of relative risk aversion, *R*, (ii) the intertemporal elasticity of substitution, ε, and (iii) the relative riskiness of the two assets. More specifically, the following patterns emerge.

1. If the two assets are equally risky (Panel B) the responses are qualitatively similar to those in a closed economy, although smaller in magnitude, due to the portfolio adjustments. For the benchmark economy $R = 2.5, \varepsilon = 0.4$, the impact of more domestic risk is to raise the mean growth rate by only 0.013 percentage points, rather than 0.068 in the closed economy, while increasing risk by nearly 0.30 percentage points. With a smaller fraction of total income being exposed to this risk, consumption declines less in the open economy, with the corresponding welfare loss being only 0.35% rather than 1.79%.[14]

2. If the domestic asset is riskier than the foreign asset, increasing its risk has a negative effect on growth, but reduces overall volatility due to the portfolio adjustment. Welfare losses are even smaller, being 0.13% for benchmark case. If the foreign asset is riskier, adding to the risk of the domestic asset has a positive effect on growth, but increases overall volatility due to portfolio adjustment. Welfare losses are now greater, being 0.73% for benchmark case.

[13] In the case where the two assets are equally risky it is independent of *R*.
[14] This can be seen by comparing (19c) with (11).

3. Increasing the coefficient of relative risk aversion, *R*: (i) reduces the adverse effect or increases the positive effect of a larger σ_y on the mean growth rate; (ii) reduces the stabilizing effect or increases the destabilizing effect on σ_ψ. These two effects have offsetting impacts on welfare. In the event that the foreign asset is riskier, and the portfolio share of the domestic asset is large, the volatility effect dominates and welfare declines more with *R*. In the event that the domestic asset is riskier and the portfolio share of the domestic asset is small, the growth effect dominates and welfare declines less with *R*, at least for moderate values.

4. Decreasing the intertemporal elasticity of substitution, ε, reduces the adverse effect or increases the positive effect on the mean growth rate, while leaving risk unaffected. The adverse effects on welfare thus decline. The sensitivity to ε is larger when the domestic asset is riskier. Sensitivity to *R* is larger when foreign asset is riskier. Sensitivity to ε increases with *R* if foreign assets are riskier; it decreases if the domestic asset is riskier.

5. Suppose that the true economy is represented by the preference parameters $R = 5, \varepsilon = 0.4$. The constant elasticity utility function $R = 1/\varepsilon = 5$ **overstates** the positive effect of an increase in risk on the mean growth rate in the case that the foreign asset is at least as risky as the domestic (Panels B and C) and **understates** the adverse effects when the domestic asset is riskier (Panel A). It always correctly predicts its effect on the volatility of the growth rate, but mildly **understates** its adverse welfare effect.

6. By contrast, the constant elasticity utility function $R = 1/\varepsilon = 2.5$ significantly **understates** the positive effect of an increase in risk on the mean growth rate in Panels B and C, but **overstates** its adverse effect in Panel A. It **overstates** its stabilizing effect in Panel A and **understates** its destabilizing effect in Panel C. It seriously **understates** its adverse overall welfare effect in Panels B and C but **overstates** it in case A.

7. Suppose that the true economy is represented by the preference parameters $R = 2.5, \varepsilon = 0.2$. The constant elasticity utility function $R = 1/\varepsilon = 5$ **overstates** the positive effect of an increase in risk on the mean growth rate if the foreign asset is at least as risky as the domestic (Panels B and C) and **understates** the adverse effects when the domestic asset is riskier (Panel A). It also **overstates**

its destabilizing effect in Panel C or understates its stabilizing effect in Panel A. It seriously overstates its adverse effect on welfare in Panel C and mildly understates it in Panel A.

8. By contrast, the constant elasticity utility function $R = 1/\varepsilon = 2.5$ will **understate** the positive effect of an increase in risk on the mean growth rate in cases B and C, and **overstate** its adverse effect in case A. It correctly predicts the effect on volatility and thus overstates the adverse welfare effect in all cases.

9. Suppose the economist is restricted to a constant elasticity utility function. He would incorrectly infer that an economy having a higher intertemporal elasticity of substitution (ε increased from 0.2 to 0.4) would be significantly less adversely affected by an increase in risk in Panels B and C whereas in fact it would be more adversely affected. In Panel A he would correctly infer that the economy would be more adversely affected, though would overestimate the magnitude.

7.3 Effects of changes in return on domestic asset

The effects of an increase in *s* are summarized in Table 5 and suggest the following.

1. If the two assets are equally risky (equal beta coefficients) the effects of a higher return on the mean growth rate exceed those in a closed economy, for low *R*, but are smaller for large *R*, due to the portfolio adjustments. The effects on volatility decline with *R*. For low *R* welfare gains exceed those of a closed economy, for large *R* they are smaller. For the benchmark economy $R = 2.5, \varepsilon = 0.4$, the welfare gain is 6.5% rather than 8.1% as in the closed economy.

2. If the domestic asset is riskier, increasing its return has a positive effect on growth, and increases volatility due to portfolio adjustment. Welfare gains are increased to 7.1%, for the benchmark case. If the foreign asset is riskier, increasing the return has a negative effect on growth, but decreases volatility. Welfare losses are now smaller, being 5.8%, for benchmark case.

3. Increasing *R* reduces the positive effect of a higher return on the mean growth rate if the domestic asset is at least as risky as the foreign [Panels A and B], but has a positive impact if the foreign asset is riskier [Panel C]. It reduces the destabilizing effect in Panels A and B, but reduces the stabilizing effect in Panel C. These effects on the mean growth rate and its volatility have

offsetting effects on welfare. In Panels A and B welfare declines with R; in Panel C welfare increases with R initially, but ultimately declines for very large (implausible) degrees of risk aversion.

4. Decreasing ε reduces the positive effect or increases the negative effect on the mean growth rate, while leaving risk unaffected, causing the positive effects of the higher return on welfare to decline. The sensitivity to ε is larger when the domestic asset is riskier. Sensitivity to R is larger when the foreign asset is riskier. Sensitivity to ε increases with R if the foreign asset is riskier; it decreases if the domestic asset is riskier.

5. Suppose that the true economy is represented by the preference parameters $R = 5, \varepsilon = 0.4$. The constant elasticity utility function $R = 1/\varepsilon = 5$ **understates** the positive effect of an increase in return on the mean growth rate. It correctly predicts its effect on the volatility of the growth rate, and mildly **understates** its positive welfare effect.

6. By contrast, the constant elasticity utility function $R = 1/\varepsilon = 2.5$ will significantly **overstate** the positive effect of an increase in return on the mean growth rate in Panels A and B but **understate** its effect in Panel C. It seriously **overstates** its destabilizing effect in Panels A and B and overstates its stabilizing effect in Panel C. It also seriously **overstates** its positive overall welfare effect in Panels B and C, but **understates** it in Panel A.

7. Suppose that the true economy is represented by the preference parameters $R = 2.5, \varepsilon = 0.2$. The constant elasticity utility function $R = 1/\varepsilon = 5$ **understates** the positive effect of an increase in return on the mean growth rate in Panels A and B, and makes the wrong qualitative prediction in Panel C. It also **understates** its destabilizing effect in Panels A and B but **understates** its stabilizing effect in Panel C. As a consequence, it seriously **understates** its positive effect on welfare in Panels B and C and mildly **overstates** it in Panel A.

8. By contrast, the constant elasticity utility function $R = 1/\varepsilon = 2.5$ **overstates** the positive effect of an increase in return on the mean growth rate. It correctly predicts the effect on volatility and thus **overstates** the positive welfare effect in all cases.

9. Suppose the economist is restricted to a constant elasticity utility function. He would incorrectly infer that an economy having a higher intertemporal elasticity of substitution (ε

increased from 0.2 to 0.4) would be slightly less positively affected by an increase in return in Panel C, whereas in fact it would be more positively affected. In Panels A and B he would correctly infer that the economy would be more positively affected, though would seriously overestimate the magnitude.

8. Conclusions

Most intertemporal studies of risk are based on the constant elasticity utility function, which has the property that the elasticity of substitution and the coefficient of relative risk aversion are both constant, but are tightly linked to one another. With the diversity of empirical evidence suggesting that this constraint may or may not be met, it is important that studies of risk and growth decouple these two parameters, which as we have shown impinge on the equilibrium in very distinct, and in some respects, conflicting ways.

Our paper has provided both an analytical characterization as well as extensive numerical simulations of the equilibrium of a stochastically growing small open economy. The general conclusion to be drawn is that errors committed by using the constant elasticity utility function, even for small violations of the compatibility condition ($R = 1/\varepsilon$) within the empirically plausible range of the parameter values, can be quite substantial. While one certainly cannot rule out using the constant elasticity utility function, as a practical matter, our results suggest that it should be employed with caution, recognizing that if the condition for its valid use is not met, very different implications may be drawn.

The issues raised in this paper have applications to other areas. One concerns the extent to which the use of the restrictive constant elasticity utility function rather than the more general recursive preferences may yield misleading inferences with respect to the impact of fiscal policy on growth and welfare in a stochastic environment. The consequences of this for policy making may be serious and merit investigation. Another application is to relax the assumed constancy of the intertemporal elasticity of substitution and the coefficient of relative risk aversion, as suggested by the micro-based empirical literature. Rebelo (1992) indicates how this may be achieved in the simplest deterministic endogenous growth model by adding a constant subsistence consumption level to the utility function, thereby making it of the Stone-Geary form. This generates transitional dynamics and thus its introduction into a stochastic growth model is likely to be analytically intractable, precluding closed form solutions such as those derived in this paper. But it is an interesting aspect that also merits further consideration.

Table 1: Closed Economy

Equilibrium Mean Growth Rates for Alternative Combinations of *R* and ε
(in percentages)

	R=2/3 ψ	R=1 ψ	R=2.5 ψ	R=5 ψ	R=10 ψ
$\varepsilon = 1.5$	**5.973**	5.960	5.900	5.800	5.600
$\varepsilon = 1$	4.000	**4.000**	4.000	4.000	4.000
$\varepsilon = 0.4$	1.632	1.648	**1.720**	1.840	2.080
$\varepsilon = 0.2$	0.843	0.864	0.960	**1.120**	1.440
$\varepsilon = 0.1$	0.448	0.470	0.580	0.760	**1.120**

Table 2: Closed Economy

A. Effects of an increase in σ_y from 0.04 to 0.05

	R=2/3 $d\psi$	R=2/3 $d\Omega$	R=1 $d\psi$	R=1 $d\Omega$	R=2.5 $d\psi$	R=2.5 $d\Omega$	R=5 $d\psi$	R=5 $d\Omega$	R=10 $d\psi$	R=10 $d\Omega$
$\varepsilon = 1.5$	**-0.015**	**-1.464**	-0.023	-2.170	0.056	-5.149	-0.113	-9.493	-0.225	-16.41
$\varepsilon = 1$	0	-0.747	**0**	**-1.119**	0	-2.774	0	-5.474	0	-10.65
$\varepsilon = 0.4$	0.018	-0.471	0.027	-0.708	**0.068**	**-1.785**	0.135	-3.626	0.270	-7.485
$\varepsilon = 0.2$	0.024	-0.419	0.036	-0.630	0.098	-1.595	**0.180**	**-3.260**	0.360	-6.812
$\varepsilon = 0.1$	0.027	-0.397	0.040	-0.598	0.101	-1.515	0.203	-3.103	**0.405**	**-6.519**

B. Effects of an increase in s from 0.08 to 0.085

	R=2/3 $d\psi$	R=2/3 $d\Omega$	R=1 $d\psi$	R=1 $d\Omega$	R=2.5 $d\psi$	R=2.5 $d\Omega$	R=5 $d\psi$	R=5 $d\Omega$	R=10 $d\psi$	R=10 $d\Omega$
$\varepsilon = 1.5$	**0.75**	**30.12**	0.75	29.88	0.75	28.85	0.75	27.28	**0.75**	**24.61**
$\varepsilon = 1$	0.50	13.32	**0.50**	**13.32**	0.50	13.32	0.50	13.32	0.50	13.32
$\varepsilon = 0.4$	0.20	7.974	0.20	7.995	**0.20**	**8.088**	0.20	8.248	0.20	8.588
$\varepsilon = 0.2$	0.10	7.034	0.10	7.055	0.10	7.152	**0.10**	**7.320**	0.10	7.679
$\varepsilon = 0.1$	0.05	6.642	0.05	6.664	0.05	6.761	0.05	6.930	**0.05**	**7.293**

Table 3: Equilibrium in Open Economy

A. Domestic Asset Riskier

	R=2/3	R=1	R=2.5	R=5	R=10
	$n_K = 0.944$ $\sigma_\psi = 3.779$	$n_K = 0.666$ $\sigma_\psi = 2.708$	$n_K = 0.333$ $\sigma_\psi = 1.633$	$n_K = 0.222$ $\sigma_\psi = 1.414$	$n_K = 0.166$ $\sigma_\psi = 1.354$
	ψ	ψ	ψ	ψ	ψ
$\varepsilon = 1.5$	**5.968**	5.932	5.883	5.853	5.829
$\varepsilon = 1$	3.994	**3.966**	3.934	3.926	3.920
$\varepsilon = 0.4$	1.627	1.609	**1.593**	1.599	1.622
$\varepsilon = 0.2$	0.837	0.823	0.813	**0.824**	0.857
$\varepsilon = 0.1$	0.442	0.430	0.423	0.437	**0.474**

B. Domestic and Foreign Assets Equally Risky

	R=2/3	R=1	R=2.5	R=5	R=10
	$n_K = 0.5$ $\sigma_\psi = 2.828$	$n_K = 0.5$ $\sigma_\psi = 2.828$	$n_K = 0.5$ $\sigma_\psi = 2.828$	$n_K = 0.5$ $\sigma_\psi = 2.828$	$n_K = 0.5$ $\sigma_\psi = 2.828$
	ψ	ψ	ψ	ψ	ψ
$\varepsilon = 1.5$	**5.987**	5.980	5.950	5.900	5.800
$\varepsilon = 1$	4.000	**4.000**	4.000	4.000	4.000
$\varepsilon = 0.4$	1.616	1.624	**1.660**	1.720	1.840
$\varepsilon = 0.2$	0.821	0.832	0.880	**0.960**	1.120
$\varepsilon = 0.1$	0.424	0.436	0.490	0.580	**0.760**

C. Foreign Asset Riskier

	R=2/3	R=1	R=2.5	R=5	R=10
	$n_K = 0.055$ $\sigma_\psi = 10.69$	$n_K = 0.333$ $\sigma_\psi = 7.659$	$n_K = 0.666$ $\sigma_\psi = 4.629$	$n_K = 0.777$ $\sigma_\psi = 4.000$	$n_K = 0.833$ $\sigma_\psi = 3.830$
	ψ	ψ	ψ	ψ	ψ
$\varepsilon = 1.5$	**6.943**	6.653	6.267	6.067	5.833
$\varepsilon = 1$	4.756	**4.533**	4.266	4.183	4.137
$\varepsilon = 0.4$	2.131	1.989	**1.867**	1.911	2.093
$\varepsilon = 0.2$	1.256	1.114	1.067	**1.116**	1.413
$\varepsilon = 0.1$	0.818	0.717	0.667	0.778	**1.073**

Table 4: Effects of an Increase in σ_y from 0.04 to 0.05

A. Domestic Asset Riskier

	R=2/3 $\beta_K = 1.545$ $\beta_F = 0.073$			R=1 $\beta_K = 2.000$ $\beta_F = 0.200$			R=2.5 $\beta_K = 2.273$ $\beta_F = 0.636$			R=5 $\beta_K = 1.852$ $\beta_F = 0.852$			R=10 $\beta_K = 1.471$ $\beta_F = 0.941$		
	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$
$\varepsilon = 1.5$	**-.0404**	**-0.587**	**-0.875**	-.0289	-0.351	-0.652	-.0153	-.0695	-0.405	-.0111	0.000	-0.358	-.0097	.0204	-0.398
$\varepsilon = 1$	-.0314	-0.587	-0.445	**-.0222**	**-0.351**	**-0.333**	-.0111	-.0695	-0.208	-.0074	0.000	-0.185	-.0056	.0204	-0.208
$\varepsilon = 0.4$	-.0208	-0.587	-0.280	-.0142	-0.351	-0.210	**-.0061**	**-.0695**	**-0.131**	-.0030	0.000	-0.117	-.0006	.0204	-0.132
$\varepsilon = 0.2$	-.0172	-0.587	-0.249	-.0116	-0.351	-0.186	-.0041	-.0695	-0.117	**-.0015**	**0.000**	**-0.104**	.0011	.0204	-0.118
$\varepsilon = 0.1$	-.0154	-0.587	-0.236	-.0102	-0.351	-0.177	-.0035	-.0695	-0.111	-.0007	0.000	-0.099	**.0019**	**.0204**	**-0.112**

B. Domestic and Foreign Assets Equally Risky

	R=2/3 $\beta_K = \beta_F = 1$			R=1 $\beta_K = \beta_F = 1$			R=2.5 $\beta_K = \beta_F = 1$			R=5 $\beta_K = \beta_F = 1$			R=10 $\beta_K = \beta_F = 1$		
	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$
$\varepsilon = 1.5$	**-.0029**	**0.295**	**-0.290**	-.0044	0.295	-0.433	-0.011	0.295	-1.062	-0.022	0.295	-2.058	-0.439	0.295	-3.875
$\varepsilon = 1$	0.000	0.295	-0.146	**0.000**	**0.295**	**-0.219**	0.000	0.295	-0.547	0.000	0.295	-1.093	0.000	0.295	-2.173
$\varepsilon = 0.4$	.0035	0.295	-0.092	.0053	0.295	-0.139	**0.013**	**0.295**	**-0.346**	0.0263	0.295	-0.698	0.0527	0.295	-1.421
$\varepsilon = 0.2$	.0047	0.295	-0.082	.0071	0.295	-0.124	0.018	0.295	-0.308	**0.0351**	**0.295**	**-0.623**	0.0702	0.295	-1.275
$\varepsilon = 0.1$	.0053	0.295	-0.077	.0080	0.295	-0.117	0.020	0.295	-0.293	0.0395	0.295	-0.592	**0.0790**	**0.295**	**-1.212**

C. Foreign Asset Riskier

	R=2/3 $\beta_K = 0.011$ $\beta_F = 1.055$			R=1 $\beta_K = 0.125$ $\beta_F = 1.400$			R=2.5 $\beta_K = 0.568$ $\beta_F = 1.727$			R=5 $\beta_K = 0.810$ $\beta_F = 1.519$			R=10 $\beta_K = 0.919$ $\beta_F = 1.294$		
	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$
$\varepsilon = 1.5$	**.0026**	**0.0378**	**-0.0048**	.0133	0.262	-0.250	0.078	0.636	-2.312	-0.028	0.753	-5.803	-0.108	0.789	-11.66
$\varepsilon = 1$	.0026	0.0378	-0.0022	**.0157**	**0.262**	**-0.118**	0.0313	0.636	-1.171	0.037	0.753	-3.155	0.039	0.789	-7.095
$\varepsilon = 0.4$	.0027	0.0378	-0.0013	.0185	0.262	-0.0719	**0.0596**	**0.636**	**-0.734**	0.114	0.753	-2.036	0.216	0.789	-4.822
$\varepsilon = 0.2$	.0027	0.0378	-0.0012	.0195	0.262	-0.0637	0.0690	0.636	-0.653	**0.139**	**0.753**	**-1.821**	0.275	0.789	-4.357
$\varepsilon = 0.1$	.0027	0.0378	-0.0011	.0199	0.262	-0.0602	0.0737	0.636	-0.619	0.152	0.753	-1.730	**0.304**	0.789	**-4.157**

Table 5: Effects of an Increase in s from 0.08 to 0.085

A. Domestic Asset Riskier

	R=2/3 $\beta_K = 0.181$ $\beta_F = -0.018$			R=1 $\beta_K = 0.273$ $\beta_F = -0.024$			R=2.5 $\beta_K = 0.684$ $\beta_F = -0.026$			R=5 $\beta_K = 1.273$ $\beta_F = 0.045$			R=10 $\beta_K = 1.882$ $\beta_F = 0.295$		
	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$
$\varepsilon = 1.5$	**3.729**	**17.48**	**154.8**	2.514	11.50	78.97	1.056	4.179	25.79	0.569	1.713	13.31	0.326	0.590	7.741
$\varepsilon = 1$	2.972	17.48	46.01	**2.000**	**11.50**	**29.29**	0.833	4.179	11.75	0.445	1.713	6.451	0.250	0.590	3.897
$\varepsilon = 0.4$	2.064	17.48	24.89	1.383	11.50	16.68	**0.567**	**4.179**	**7.108**	0.294	1.713	3.985	0.158	0.590	2.439
$\varepsilon = 0.2$	1.761	17.48	21.58	1.178	11.50	14.59	0.478	4.179	6.281	**0.244**	**1.713**	**3.535**	0.128	0.590	2.169
$\varepsilon = 0.1$	1.610	17.48	20.24	1.075	11.50	13.72	0.433	4.179	5.935	0.219	1.713	3.346	**0.113**	**0.590**	**2.055**

B. Domestic and Foreign Assets Equally Risky

	R=2/3 $\beta_K = \beta_F = 1$			R=1 $\beta_K = \beta_F = 1$			R=2.5 $\beta_K = \beta_F = 1$			R=5 $\beta_K = \beta_F = 1$			R=10 $\beta_K = \beta_F = 1$		
	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$
$\varepsilon = 1.5$	**1.840**	**10.73**	**59.26**	1.351	6.451	41.24	0.766	1.699	23.21	0.570	0.507	17.67	0.473	0.068	14.56
$\varepsilon = 1$	1.422	**10.73**	23.25	**1.031**	**6.451**	**17.37**	0.563	1.699	10.69	0.406	0.507	8.558	0.328	0.068	7.501
$\varepsilon = 0.4$	0.920	**10.73**	13.43	0.647	6.451	10.25	**0.319**	**1.699**	**6.489**	0.209	0.507	5.279	0.155	0.068	4.736
$\varepsilon = 0.2$	0.753	**10.73**	11.78	0.519	6.451	9.015	0.238	1.699	5.738	**0.144**	**0.507**	**4.682**	0.097	0.068	4.219
$\varepsilon = 0.1$	0.670	**10.73**	11.09	0.455	6.451	8.504	0.197	1.699	5.424	0.111	0.507	4.432	**0.068**	**0.068**	**4.000**

C. Foreign Asset Riskier

	R=2/3 $\beta_K = 0.326$ $\beta_F = 1.917$			R=1 $\beta_K = 0.532$ $\beta_F = 1.997$			R=2.5 $\beta_K = 0.847$ $\beta_F = 1.635$			R=5 $\beta_K = 0.934$ $\beta_F = 1.351$			R=10 $\beta_K = 0.972$ $\beta_F = 1.182$		
	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$	$d\psi$	$d\sigma_\psi$	$d\Omega$
$\varepsilon = 1.5$	**-0.050**	**-5.369**	**9.321**	0.189	-3.134	-0.250	0.476	-0.717	21.29	0.571	-0.196	22.43	0.618	-0.050	21.42
$\varepsilon = 1$	-0.128	-5.369	4.029	**0.063**	-3.134	**6.542**	0.292	-0.717	9.639	0.368	-0.196	10.70	0.407	-0.050	11.23
$\varepsilon = 0.4$	-0.223	-5.369	2.396	-0.090	-3.134	3.904	**0.071**	-0.717	**5.817**	0.124	-0.196	6.566	0.151	-0.050	7.141
$\varepsilon = 0.2$	-0.255	-5.369	2.111	-0.140	-3.134	3.441	-0.003	-0.717	5.138	**0.043**	**-0.196**	**5.818**	0.066	-0.050	6.369
$\varepsilon = 0.1$	-0.271	-5.369	1.992	-0.166	-3.134	3.248	-0.040	-0.717	4.854	0.003	-0.196	5.505	**0.023**	-0.050	**6.043**

APPENDIX: Biases in Open Economy

We report the expressions for the biases obtained when using the constant elasticity utility function instead of the recursive utility function in the open economy.

I. $\lambda = 1$

In this case we find

(i) $$\left(\frac{\partial \psi}{\partial s}\right)_C - \frac{\partial \psi}{\partial s} = \frac{\varepsilon n_K}{R}\left(\frac{1}{\varepsilon} - R\right) \quad \text{(A.1a)}$$

(ii) $$\left(\frac{\partial \psi}{\partial \sigma_y^2}\right)_C - \frac{\partial \psi}{\partial \sigma_y^2} = \frac{\varepsilon n_K^2}{2}\left(R - \frac{1}{\varepsilon}\right) \quad \text{(A.1b)}$$

(iii) $$\left(\frac{\partial \sigma_\psi^2}{\partial s}\right)_C - \frac{\partial \sigma_\psi^2}{\partial s} = 0 \quad \text{(A.1c)}$$

(iv) $$\left(\frac{\partial \sigma_\psi^2}{\partial \sigma_y^2}\right)_C - \frac{\partial \sigma_\psi^2}{\partial \sigma_y^2} = 0 \quad \text{(A.1d)}$$

II. $\lambda = 0$

(i) $$\left(\frac{\partial \psi}{\partial s}\right)_C - \frac{\partial \psi}{\partial s} = \varepsilon(1+\varepsilon)\left(R - \frac{1}{\varepsilon}\right)\left(n_K - \tilde{n}_K\right) \quad \text{(A.2a)}$$

(ii) $$\left(\frac{\partial \psi}{\partial \sigma_y^2}\right)_C - \frac{\partial \psi}{\partial \sigma_y^2} = \frac{1}{2}\left(\frac{1}{\varepsilon} - R\right)\left(\varepsilon(1+\varepsilon)R\left(n_K - \tilde{n}_K\right)^2 + (1-\varepsilon)\tilde{n}_K^2\right) \quad \text{(A.2b)}$$

(iii) $$\left(\frac{\partial \sigma_\psi^2}{\partial s}\right)_C - \frac{\partial \sigma_\psi^2}{\partial s} = \frac{2\varepsilon^2}{R}\left(R^2 - \frac{1}{\varepsilon^2}\right)\left(n_K - \tilde{n}_K\right) \quad \text{(A.2c)}$$

(iv) $$\left(\frac{\partial \sigma_\psi^2}{\partial \sigma_y^2}\right)_C - \frac{\partial \sigma_\psi^2}{\partial \sigma_y^2} = \varepsilon^2\left(\frac{1}{\varepsilon^2} - R^2\right)\left(n_K - \tilde{n}_K\right)^2 \quad \text{(A.2d)}$$

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