Macroeconomic Volatility and Income Inequality in a Stochastically Growing Economy

Cecilia García-Peñalosa
GREQAM and CNRS

Stephen J. Turnovsky
University of Washington

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Abstract: This paper employs a stochastic growth model to analyze the effect of macroeconomic volatility on the relationship between income distribution and growth. We initially characterize the equilibrium and show how the distribution of income depends upon two factors: the initial distribution of capital, and the equilibrium labor supply and find that an increase in volatility raises the mean growth rate and income inequality. We calibrate the model using standard parameter estimates, and find that it successfully replicates key growth and distributional characteristics. The latter part of the paper uses this framework to analyze the design of tax policy to achieve desired growth, distribution, and welfare objectives. Two general conclusions clearly emerge. First, increasing (average) welfare and the growth rate does not necessarily entail an increase in inequality. Second, fiscal policy often has conflicting effects on the distributions of gross and net income, with changes in factor prices resulting in greater pre-tax inequality but the redistributive effect of taxes yielding a more equal post-tax distribution.

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1. Introduction

Research on the macroeconomic determinants of income inequality has focused mainly on three aspects: growth, trade, and inflation. Studies of the impact of growth on distribution range from the analyses of the impact of structural change, such as the Kuznets hypothesis, to theories of skill-biased technical change. Based on Heckscher-Ohlin theory, international trade has been argued to be a major determinant of income distribution, and this aspect has recently acquired prominence in the debate on the increase in inequality in a number of industrialized countries. One of the most consistently supported empirical correlations is that between inflation and inequality, explained by the fact that because inflation is a regressive tax, it generates greater income inequality.[1] Our paper seeks to introduce a new and so far ignored factor, the degree of aggregate risk in the economy, into the analysis of inequality.

Empirical evidence suggests that macroeconomic volatility is potentially an important channel through which income inequality and growth may be mutually related. A striking difference when we compare Latin American economies with those of the OECD is that the former are associated with much greater income inequality. In 1990, the Gini coefficients of the distribution of income in Brazil, Chile, Mexico, and Venezuela ranged between 49-64%, while those for the OECD range between 24-44%. At the same time, the former were subject to much greater fluctuations in their respective growth rates than were the latter: during the 1980s, the standard deviation of the rate of output growth was, on average, 4.9% for the four Latin American economies, and 2.7% for the OECD. [2] In fact, using a broader set of data, Breen and García-Peñalosa (2004) obtain a positive relationship between a country's volatility (measured by the standard deviation of the rate of GDP growth) and income inequality.

Our objective in this paper is to model a mechanism through which aggregate risk -- which we shall attribute to production shocks -- has a direct impact on distribution. We employ an extension of the stochastic growth model developed by Grinols and Turnovsky (1993, 1998), Smith

[1] See Okun (1971) and Taylor (1981), and more recently, Albanesi (2003).
[2] See Breen and García-Peñalosa (2004) for a description of the data sources used for these calculations.

(1996), Corsetti (1997) and Turnovsky (2000b). This is a one-sector growth model in which, due to the presence of an externality stemming from the aggregate capital stock, equilibrium output evolves in accordance with a stochastic AK technology. Adopting this framework, aggregate production risk jointly determines the equilibrium growth rate, its volatility, and the distribution of income.[3]

Previous studies have been unable to analyze the impact of volatility on income distribution, as they either abstract from labor, or otherwise, assume that agents are identical in all respects. We introduce the assumption that agents are heterogeneous with respect to their initial endowments of capital, and allow for an elastic supply of labor. As a result, the labor supply responses to different degrees of risk will induce changes in factor prices and affect the distribution of income.

Our analysis proceeds in several stages. To start with, we derive the equilibrium balanced growth path in a stochastic growth model with given tax rates. We show how this equilibrium has a simple recursive structure. First, the equilibrium mean growth rate and labor supply (employment) are jointly determined to ensure that rates of return are in equilibrium and that the product market clears. These equilibrium quantities are affected by the degree of risk in the economy but are independent of the distribution of wealth. Second, the equilibrium labor supply, together with the given initial distribution of capital among agents, is shown to determine the volatility of the growth rate, on the one hand, and the degree of income inequality, on the other. We find that an increase in production risk raises the mean growth rate, its volatility, and the degree of income inequality.

The intuition for these results is straightforward. Because agents are sufficiently risk-averse, a greater variance of output has a strong income effect that induces them to increase their labor supply, increase their savings, and thus raise the growth rate. The increase in the labor supply raises the return to capital and lowers the real wage, thereby affecting the distribution of income. Since labor is more equally distributed than is capital, the income gap between any two individuals increases, and, for a given initial distribution of wealth, income distribution becomes more unequal.

The latter part of the paper uses this framework to analyze the effects of taxation. As is well-known, the externality associated with the capital stock implies that the competitive growth rate is

[3] Growth regressions have suggested that the mean growth rate is related to its volatility, although the results are not uniform. Kormendi and Mequire (1985) obtained a positive relationship, while Ramey and Ramey (1995) found a negative one.

too low. The first-best allocation can then be attained through suitable taxes and subsidies. When agents are heterogeneous, the use of growth-enhancing policies raises the question of the impact first-best policies have on the distribution of income. Two general conclusions emerge from our analysis. First, increasing (average) welfare and the growth rate does not necessarily entail an increase in inequality, as faster growth tends to be associated with lower post-tax income inequality. Second, we find that fiscal policy has conflicting effects on the distributions of gross and net income. First-best policies result in changes in factor prices that increase pre-tax inequality, but the direct redistributive effect of taxes tends to yield a more equal post-tax distribution. As a resul,t gross and net income inequality often move in opposite directions.

The paper contributes to the recent literature looking at the relationship between income distribution and growth.[4] It is related to Alesina and Rodrik (1994), Persson and Tabellini (1994), and Bertola (1993), who develop (non-stochastic) AK growth models in which agents differ in their initial stocks of capital. The first two papers have, however, a very different focus as they take initial inequality as given and argue that it has a negative impact on the rate of growth. In contrast to their results, this paper emphasizes that growth and distribution are jointly determined, and presents a possible mechanism that generates a positive relationship between these two variables in line with the evidence presented by Forbes (2000). Bertola (1993) is closer to our approach in that he emphasizes how technological parameters, specifically the productivity of capital, jointly determine distribution and growth. He also examines how policies directed at increasing the growth rate affect the distribution of consumption, although his assumption of a constant labor supply implies that the distribution of income is independent of policy choices. Our approach shares with these three papers an important limitation, namely, that the assumption that agents differ only in their initial stocks of capital coupled with an AK technology implies that there are no income dynamics.[5]

The paper closest to our work, at least in spirit, is Aghion, Banerjee, and Piketty (1999), who find that greater inequality is associated with more volatility. They show how combining capital

[4] See, among others, Stiglitz (1969), Bourguignon (1981), Aghion and Bolton (1997) and Galor and Tsiddon (1997), as well as the overview in Aghion, Caroli, and García-Peñalosa (1999).

[5] A more general study of heterogeneity and the dynamics of distribution in growth models can be found in Caselli and Ventura (2000).

market imperfections with inequality in a two-sector model can generate endogenous fluctuations in output and investment. In their model it is unequal access to investment opportunities and the gap between the returns to investment in the modern and the traditional sectors that cause fluctuations. We reverse the focus, examining how exogenous production uncertainty determines output volatility and income distribution.

The paper is organized as follows. Section 2 presents the model and derives the equilibrium growth rate, labor supply, and volatility. Section 3 examines the determinants of the distribution of income. Section 4 shows, analytically and numerically, that in the absence of taxation greater risk is associated with a more unequal distribution of income. Section 5 starts by obtaining the first-best optimum, and shows that the competitive growth rate is too low. It is followed by an analysis of first-best taxation, and a number of second-best policies. Numerical analysis is then used to illustrate the distributional implications of the various policies. Section 6 concludes, while technical details are provided in the Appendix.

2. The Model

2.1 Description of the economy

Technology and factor payments

Firms shall be indexed by j. We assume that the representative firm produces output in accordance with the stochastic Cobb-Douglas production function

$$dY_j = A(L_j K)^{\alpha} K_j^{1-\alpha}(dt + du) \tag{1a}$$

where K_j denotes the individual firm's capital stock, L_j denotes the individual firm's employment of labor, K is the average stock of capital in the economy, so that $L_j K$ measures the efficiency units of labor employed by the firm; see e.g. Corsetti (1997). The stochastic shock du is temporally independent, with mean zero and variance $\sigma^2 dt$ over the instant dt. The stochastic production function exhibits constant returns to scale in the private factors -- labor and the private capital stock.

All firms face identical production conditions and are subject to the same realization of an economy-wide stochastic shock. Hence they will all choose the same level of employment and capital stock. That is, $K_j = K$ and $L_j = L$ for all j, where L is the average economy-wide level of employment. The average capital stock yields an externality such that in equilibrium the aggregate (average) production function is linear in the aggregate capital stock, as in Romer (1986), namely

$$dY = AL^{\alpha}K(dt + du) \equiv \Omega(L)K(dt + du) \tag{1b}$$

where $\Omega(L) \equiv AL^{\alpha}$ and $\partial\Omega / \partial L > 0$.

We assume that the wage rate, z, over the period $(t, t + dt)$ is determined at the start of the period and is set equal to the expected marginal physical product of labor over that period. The total rate of return to labor over the same interval is thus specified nonstochastically by

$$dZ = zdt = \left(\frac{\partial F}{\partial L_j}\right)_{K_j = K, L_j = L} dt\ . \tag{2a}$$

where $$z = \alpha\Omega L^{-1}K \equiv wK\ .$$

The private rate of return to capital, dR, over the interval $(t,t+dt)$ is thus determined residually by

$$dR = \frac{dY - LdZ}{K} \equiv rdt + du_K \tag{2b}$$

where $$r \equiv \left(\frac{\partial F}{\partial K_j}\right)_{K = K, L = L} = (1 - \alpha)\Omega\text{, and } du_K \equiv \Omega du\ .$$

These two equations assume that the wage rate, z, is fixed over the time period $(t,t+dt)$, so that the return on capital absorbs all output fluctuations. The rationale for this assumption is that in industrial economies wages are usually fixed ex ante, while the return to capital is, at least in part, determined ex post and thus absorbs most of the fluctuations in profitability.[6] Differentiating the production function and given that firms are identical, we find that the equilibrium return to capital is independent of the stock of capital while the wage rate is proportional to the average stock of

[6] In the United States, for example, the relative variability of stock returns over the period 1955-1995 were around 32% per annum, while the relative variability of wages over that same period was only 2%.

capital, and therefore grows with the economy.[7] In addition, we have $\partial r/\partial L > 0$ and $\partial w/\partial L < 0$, reflecting the fact that more employment raises the productivity of capital but lowers that of labor.

Consumers

There is a mass 1 of infinitely-lived agents in the economy. Consumers are indexed by *i* and are identical in all respects except for their initial stock of capital, K_{i0}. Since the economy grows, we will be interested in the share of individual *i* in the total stock of capital, k_i, defined as $k_i \equiv K_i/K$, where K is the aggregate (or average) stock. Relative capital has a distribution function $G(k_i)$, mean $\sum_i k_i = 1$, and variance σ_k^2.

All agents are endowed with a unit of time that can be allocated either to leisure, l_i or to work, $1-l_i \equiv L_i$. A typical consumer maximizes expected lifetime utility, assumed to be a function of both consumption and the amount of leisure time, in accordance with the isoelastic utility function

$$\max E_0 \int_0^\infty \frac{1}{\gamma}\left(C_i(t)l_i^\eta\right)^\gamma e^{-\beta t}dt, \quad \text{with } -\infty < \gamma < 1, \eta > 0, \gamma\eta < 1 \tag{3}$$

where $1-\gamma$ equals the coefficient of relative risk aversion. Empirical evidence suggests that this is relatively large, certainly well in excess of unity, so that we shall assume $\gamma < 0$.[8] The parameter η represents the elasticity of leisure in utility. This maximization is subject to the agent's capital accumulation constraint

$$\begin{aligned} dK_i = (1-\tau_k)rK_i dt + (1-\tau_k')K_i du_K + (1-\tau_w)w(1-l_i)Kdt \\ -(1+\tau_C)C_i dt + sE(dK_i) + s'(dK_i - Ed(K_i)) \end{aligned}$$

where $du_K = \Omega du$, and capital is assumed not to depreciate. The fiscal instruments used by the government are a subsidy on investment in physical capital, at rates s and s' for the deterministic and the stochastic component of investment, respectively; a consumption tax, τ_c; a wage tax, τ_w; a tax on the deterministic component of capital income, τ_k, and a tax on the stochastic component of capital income, τ_k'. Taking expectations of this expression and substituting back for $E(dK_i)$, we can

[7] Intuitively, in a growing economy, with the labor supply fixed, the higher income earned by labor is reflected in higher returns, whereas with capital growing at the same rate as output, returns to capital remain constant.
[8] Some of the empirical estimates supporting this assumption are noted in Section 4.2 below.

express this budget constraint as

$$dK_i = \frac{(1-\tau_k)rK_i + (1-\tau_w)w(1-l_i)K - (1+\tau_c)C_i}{1-s}dt + K_i\frac{1-\tau_k'}{1-s'}du_K \tag{4a}$$

It is important to observe that with the equilibrium wage rate being tied to the aggregate stock of capital, the rate of accumulation of the individual's capital stock depends on the aggregate stock of capital, which in turn evolves according to

$$dK = \frac{\big((1-\tau_k)r + (1-\tau_w)w(1-l)\big)K - (1+\tau_c)C}{1-s}dt + K\frac{1-\tau_k'}{1-s'}du_K \tag{4b}$$

where l denotes the average (aggregate) fraction of time devoted to leisure. The agent therefore needs to take this relationship into account in performing her optimization.

Government policy

The government balances the public budget each period, implying

$$sE(dK)dt + s'\frac{1-\tau_k'}{1-s'}Kdu_K = \big[\tau_c C + \tau_k rK + \tau_w w(1-l)K\big]dt + \tau_k' Kdu_K \tag{5}$$

Note that both expenditures and tax receipts have a deterministic and a stochastic component. Equating them respectively yields the following constraints required to maintain a balanced budget:

$$\tau_k' = s', \tag{6a}$$

$$\tau_k r + \tau_w w(1-l) + \tau_c\frac{C}{K} = s\psi, \tag{6b}$$

where ψ denotes the average growth rate.

Two points should be noted. First, some of the taxes may be negative, in which case they become subsidies, in addition to the investment subsidy. However, neither the deterministic nor the stochastic component of the two income taxes can exceed unity. Second, the assumption that the deterministic and stochastic components of income are taxed at different rates requires that the agent (and the tax authority) disentangle the deterministic from the stochastic components of income, something that may not be unlikely in practice. However, this assumption is mainly made for analytical simplicity. Taxing both components at the same rate, with the government using public debt in order to compensate any surplus or deficit, would not change any of our results, since as we will see below, the tax rate on the stochastic component of capital income does not affect any of the equilibrium relationships.[9]

2.2 Consumer optimization

The consumer's formal optimization problem is to maximize (3) subject to equations (4a) and (4b). The first-order conditions with respect to consumption and leisure yield

$$\frac{1}{C_i}\left(C_i l_i^{\eta}\right)^{\gamma} = \frac{1+\tau_c}{1-s} X_{K_i} \tag{7a}$$

$$\frac{\eta}{l}\left(C_i l_i^{\eta}\right)^{\gamma} = \frac{1-\tau_w}{1-s} wK X_{K_i} \tag{7b}$$

where $X(K_i,K)$ is the value function and X_{K_i} its derivative with respect to K_i (see Appendix).

In the Appendix we show that utility maximization implies that the dynamic evolution of the stock of capital of agent *i* is given by

$$\frac{dK_i}{K_i} = \left[\frac{r(1-\tau_k)/(1-s)-\beta}{1-\gamma} - \frac{\gamma}{2}\Omega^2\sigma^2\right]dt + \Omega du \equiv \psi dt + \Omega du\,, \tag{8}$$

where Ω and r are defined in equations (1) and (2). There we have expressed them as functions of equilibrium employment, L, but assuming that the aggregate labor market clears, yields

$$\sum\nolimits_j L_j = L = 1 - l = \sum\nolimits_i (1-l_i) \tag{9}$$

and we can equally well write Ω and r as functions of $(1-l)$.[10] From (8) we see that the rate of growth of capital -- and therefore of output -- has a deterministic and a stochastic component, so that the average growth rate, ψ, and its standard deviation, σ_ψ are respectively

[9] See Turnovsky (2000b) and García Peñalosa and Turnovsky (2002).

[10] Thus we may write $\Omega(l) = A(1-l)^{\alpha}$ and $r = (1-\alpha)\Omega(l)$, where $\Omega'(l) < 0$.

$$\psi = \frac{r\left(\dfrac{1-\tau_k}{1-s}\right)-\beta}{1-\gamma} - \frac{\gamma}{2}\Omega^2\sigma^2 \quad \text{and} \qquad \sigma_\psi = \Omega\sigma \tag{10}$$

Observe that the only difference between agents, namely their initial stock of capital, does not appear in this equation. Hence all individuals choose the same rate of growth of their stock of capital, ψ. This has two implications. First, the *aggregate* rate of growth of capital is identical to the *individual* rate of growth and unaffected by the initial distribution of endowments, hence

$$dK/K = \psi dt + \Omega du\,. \tag{8'}$$

Second, since the capital stock of all agents grows at the same rate, the distribution of capital endowments does not change over time. That is, at any point in time, the wealth share of agent *i*, k_i, is given by her initial share $k_{i,0}$, say.

Dividing equation (7a) by (7b), we obtain the consumption to capital ratio of agent *i,*

$$\frac{C_i}{K_i} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l_i}{k_i}, \tag{11}$$

Aggregating over the individuals and noting that $\sum_i k_i = 1$, $\sum_i l_i = l$, the aggregate economy-wide consumption-capital ratio is

$$\frac{C}{K} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}l \tag{11'}$$

In addition, the following transversality condition must hold

$$\lim_{t\to\infty} E\left[K_i(t)^\gamma e^{-\beta t}\right] = 0 \tag{12}$$

With $K_i(t)$ evolving in accordance with the stochastic path (8), (12) can be shown to reduce to[11]

$$\gamma\left[r\frac{1-\tau_k}{1-s} - \frac{\gamma}{2}(1-\gamma)\Omega^2\sigma^2\right] < \beta$$

which, when combined with (10), can be shown to be equivalent to the condition

[11] See Turnovsky (2000b).

$$r\left(\frac{1-\tau_k}{1-s}\right) > \psi \tag{13}$$

i.e. the equilibrium rate of return on capital must exceed the equilibrium growth rate. Dividing the aggregate accumulation equation, (4b), by K, this condition can also be shown to be equivalent to

$$(1+\tau_c)\frac{C}{K} > (1-\tau_w)w(1-l) \tag{13'}$$

implying that part of income from capital is consumed.[12]. Combining with (9'), this can be further expressed as

$$l > \frac{\eta}{1+\eta}. \tag{13''}$$

Recalling the individual budget constraint, (4a), we can write the individual's mean rate of capital accumulation as

$$\frac{E(dK_i / K_i)}{dt} = r\frac{1-\tau_K}{1-s} + w\frac{1-\tau_w}{1-s}\frac{1-l_i}{k_i} - \frac{1+\tau_c}{1-s}\frac{C_i}{K_i} \tag{14}$$

Together with equation (11), this expression implies that agent *i*'s supply of labor is

$$1-l_i = \frac{1}{1+\eta}\left[1-\eta\frac{1-s}{1-\tau_w}\frac{r(1-\tau_k)/(1-s)-\psi}{w}k_i\right] \tag{15}$$

Noting the transversality condition, $r(1-\tau_k)/(1-s) > \psi$, (15) implies that an increase in the agent's capital (wealth) has a negative effect on her labor supply; wealthier individuals chose to "buy" more leisure. In effect, they compensate for their larger capital endowment, and the higher growth rate it would support, by providing less labor and having an exactly offsetting effect on the growth rate.

Because the rate of growth is the same for all agents, individual labor supplies are linear in the wealth shares of agents. The aggregate labor supply, $1-l = 1-\sum_i l_i$, is then independent of the initial distribution of capital. Summing equation (15) over the agents and using the fact that $\sum_i k_i = 1$, we obtain the aggregate labor supply relation,

[12] This latter condition reduces to $C/K > 0$ in the original Merton (1969) model, which abstracted from labor income.

$$1-l=\frac{1}{1+\eta}\left[1-\eta\frac{1-s}{1-\tau_w}\frac{r(1-\tau_k)/(1-s)-\psi}{w}\right], \tag{15'}$$

and combining (15) and (15') we can derive the following expression for the "relative labor supply"

$$l_i-l=\left(l-\frac{\eta}{1+\eta}\right)\left(k_i-1\right) \tag{15''}$$

Again we see that the transversality condition, now expressed as (13"), implies a positive relationship between relative wealth and leisure. This relationship provides the fundamental mechanism whereby, given the initial distribution of capital endowments across agents, policy and risk are able to influence the distribution of income.

2.3. Macroeconomic equilibrium

The key equilibrium relationships can be summarized by

Equilibrium growth rate

$$\psi=\frac{r\left(\frac{1-\tau_k}{1-s}\right)-\beta}{1-\gamma}-\frac{\gamma}{2}\Omega^2\sigma^2 \tag{16a}$$

Equilibrium Volatility

$$\sigma_\psi=\Omega\sigma \tag{16b}$$

Individual consumption-capital ratio

$$\frac{C_i}{K_i}=\frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l_i}{k_i} \tag{16c}$$

Aggregate consumption-capital ratio

$$\frac{C}{K}=\frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}l \tag{16d}$$

Individual Budget Constraint

$$\psi = r\frac{1-\tau_k}{1-s} + w\frac{1-\tau_w}{1-s}\frac{1-l_i}{k_i} - \frac{1+\tau_c}{1-s}\frac{C_i}{K_i} \tag{16e}$$

Goods market equilibrium

$$\psi = \Omega - \frac{C}{K} \tag{16f}$$

Government budget constraint

$$\tau_k r + \tau_w w(1-l) + \tau_c \frac{C}{K} = s\psi \tag{16g}$$

Recalling the definitions of $r(l)$, $w(l)$, and $\Omega(l)$, and given k_i, these equations jointly determine the individual and aggregate consumption-capital ratios, C_i/K_i, C/K, the individual and aggregate leisure times, l_i, l, average growth rate, ψ, volatility of the growth rate, σ_ψ, and one of the fiscal instruments given the other three policy parameters. Note that the tax and subsidy on the stochastic components of investment and the return to capital, have no effect on the equilibrium variables and thus $\tau_k' = s'$ can be set arbitrarily.

Using (16a), (16d), and (16f), the macroeconomic equilibrium of the economy can be summarized by the following pair of equations that jointly determine the equilibrium mean growth rate, ψ, and leisure l:

RR $$\psi = \frac{(1-\alpha)\Omega(l)(1-\tau_k)/(1-s) - \beta}{1-\gamma} - \frac{\gamma}{2}\Omega^2\sigma^2, \tag{17a}$$

PP $$\psi = \Omega(l)\left[1 - \frac{\alpha}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l}{1-l}\right]. \tag{17b}$$

The first equation describes the relationship between ψ and l that ensures the equality between the risk-adjusted rate of return to capital and return to consumption. The second describes the combinations of the mean growth and leisure that ensure product market equilibrium holds.

2.4. The laissez-faire economy

It is convenient to examine the equilibrium in the absence of taxation. Setting all taxes and subsidies to zero, the equilibrium mean growth rate and leisure are determined by the following pair of equations:

RR: $$\psi = \frac{(1-\alpha)\Omega(l)-\beta}{1-\gamma} - \frac{\gamma}{2}\Omega(l)^2\sigma^2 ,$$

PP: $$\psi = \Omega(l)\left(1-\frac{\alpha}{\eta}\frac{l}{1-l}\right),$$

The laissez-faire RR and PP locuses are depicted in Figure 1, and their formal properties are derived in the Appendix.[13] First, note that equation PP is always decreasing in l, reflecting the fact that more leisure time reduces output, thus increasing the consumption-output ratio and having an adverse effect on the growth rate of capital. On the other hand, for RR we have

$$\frac{\partial\psi}{\partial l} = \left(\frac{1-\alpha}{1-\gamma} - \gamma\Omega(l)\sigma^2\right)\Omega'(l)$$

This expression is unambiguously negative for $\gamma < 0$, as the empirical evidence suggests, and the case that we shall assume prevails. Intuitively, a higher fraction of time devoted to leisure reduces the productivity of capital, requiring a fall in the return to consumption. This is obtained if the growth of the marginal utility of consumption rises, that is, if the balanced growth rate falls. Under plausible conditions, the two schedules are concave, and an equilibrium exists if

$$\alpha-\gamma+\frac{\beta}{A} > -\gamma\frac{1-\gamma}{2}A\sigma^2 .$$

We will see in our numerical calibrations that this condition is met for reasonable parameter values.

3. The Determinants of the Distribution of Income

In order to examine the effect of risk on income distribution, we consider the expected relative income of an individual with capital k_i. Her (expected) gross income is simply

[13] See also Turnovsky (2000b).

$E(dY_i) = rK_i + wK(1-l_i)$, while expected average income is $E(dY) = rK + wK(1-l)$. Using equation (15) to substitute for labor, we can express the relative (expected) income of individual *i*, $y_i \equiv E(dY_i)/E(dY)$, as

$$y_i(l,k_i) = k_i + \frac{w}{(1+\eta)\Omega}(1-k_i) = k_i + \frac{\alpha}{(1+\eta)(1-l)}(1-k_i) \tag{18}$$

which we may write more compactly as:

$$y_i(l,k_i) = 1-\rho(l)(1-k_i), \quad \text{where} \quad \rho(l) \equiv 1-\frac{\alpha}{(1+\eta)(1-l)}, \tag{18'}$$

Equation (18') emphasizes that the distribution of income depends upon *two* factors, the initial (unchanging) distribution of capital, and the equilibrium allocation of time between labor and leisure, insofar as this determines factor rewards. The net effect of an increase in initial wealth on the relative income of agent *i* is given by $\rho(l)$. As long as the equilibrium is one of positive growth, it is straightforward to show that[14]

$$0 < \rho(l) < 1 \tag{19}$$

Thus relative income is strictly increasing in k_i, indicating that although richer individuals choose a lower supply of labor, this effect is not strong enough to offset the impact of their higher capital income. As a consequence, the variability of income across the agents, σ_y, is less than their (unchanging) variability of capital, σ_k.

The second point to note is that we can rank different outcomes according to inequality without needing any information about the underlying distribution of capital. For a given distribution of capital, changes in risk or policy affect the distribution of income solely through their impact on relative prices, as captured by $\rho(l)$. Correia (1999) has shown that when agents differ only in their endowment of one good, there exists an ordering of outcomes by income inequality, as measured by

[14] Writing $\rho(l) = \frac{1}{(1+\eta)(1-l)}\left[\left(\eta(1-l)-\alpha l\right)+(1-\alpha)(1-l)\right]$. If the equilibrium is one of positive growth, (17b) implies that the first term in brackets is positive, thus ensuring that $\rho(l) > 0$. The fact that $\rho(l) < 1$ is immediate from its definition.

second-order stochastic dominance.[15] That ordering is determined by equilibrium prices, and is independent of the distribution of endowments.

The DD locus in the lower panel of Fig. 1 illustrates the relationship between the standard deviation of relative income, σ_y, our measure of income inequality and the standard deviation of capital endowments, σ_k, namely

$$\textbf{DD} \qquad \sigma_y = \rho(l)\sigma_k \tag{19c}$$

Given the standard deviation of capital, σ_k, the standard deviation of income is a decreasing and concave function of aggregate leisure time. This is because as leisure increases (and labor supply declines) the wage rate rises and the return to capital falls, compressing the range of income flows between the wealthy with large endowments of capital and the less well endowed. Thus, having determined the equilibrium allocation of labor from the upper panels in Fig. 1, (19c) determines the corresponding unique variability of income across agents.

Because taxes also have direct redistributive effects, we need to distinguish between the *before-tax* and *after-tax* distribution of income. We therefore define the agent's after-tax (or net) relative income as

$$y_i^{NET}(l,k_i,\tau_k,\tau_w) \equiv \frac{r(1-\tau_k)k_i + w(1-\tau_w)(1-l_i)}{r(1-\tau_k)+w(1-\tau_w)(1-l)} = 1-\rho^{NET}(l,\tau_w,\tau_k)(1-k_i) \tag{20a}$$

where, ρ^{NET} summarizes the distribution of after-tax income and is related to corresponding before-tax measure, $\rho(l)$, by

$$\rho^{NET}(l,\tau_w,\tau_k) = \rho(l) + \left(1-\rho(l)\right)(1-\alpha)\frac{(\tau_w-\tau_k)}{\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)} \tag{20b}$$

with the standard deviation of after-tax income given by

$$\sigma_y^{NET} = \rho^{NET}(l,\tau_w,\tau_k)\sigma_k \tag{20c}$$

From (20a) and (20b) we see that fiscal policy exerts two effects on the after-tax income distribution.

[15] Her results also require that the economy be amenable to Gorman aggregation, which is the case in our setup.

First, by influencing *gross* factor returns it influences the equilibrium supply of labor, l, and therefore the before-tax distribution of income, as summarized by $\rho(l)$. In addition, it has a direct redistributive effect, which is summarized by the second term on the right hand side of (20b). The dispersion of pre-tax income across agents will exceed the after-tax dispersion if and only if $\tau_k > \tau_w$. As we will see below, in most cases tax increases affect the before-tax and after-tax distributions in opposite ways.

Lastly, we compute individual welfare. By definition, this equals the value function used to solve the intertemporal optimization problem evaluated along the equilibrium stochastic growth path. For the constant elasticity utility function, the optimized level of utility for an agent starting from an initial stock of capital, K_{i0}, can be expressed as

$$X(K_{i0}) = \frac{1}{\gamma}\frac{\left((C_i/K_i)l_i^{\eta}\right)^{\gamma}}{\beta-\gamma\left(\psi+1/2(\gamma-1)\sigma^2\right)}K_{i,0}^{\gamma} \tag{21}$$

The welfare of individual *i* relative to that of the individual with average wealth is then

$$x(k_i) = \frac{\left(C_i/K_i\right)^{\gamma}}{\left(C/K\right)^{\gamma}}\frac{l_i^{\eta\gamma}}{l^{\eta\gamma}}k_i^{\gamma} = \left(\frac{l_i}{l}\right)^{(1+\eta)\gamma}, \tag{22}$$

where the second term has been obtained by substituting for the consumption-capital ratio. Using equations (15), we can express relative welfare as

$$x(k_i) = \left[1+\left(1-\frac{\eta}{1+\eta}\frac{1}{l}\right)(k_i-1)\right]^{\gamma(1+\eta)}. \tag{22'}$$

Consider now two individuals having relative endowments $k_2 > k_1$. Individual 2 will have both a higher mean income but also higher volatility. The transversality condition (13") implies that if $\gamma > 0$, then their relative welfare satisfies $x(k_2) > x(k_1) > 0$, while if $\gamma < 0$, $x(k_1) > x(k_2) > 0$. However, in the latter case absolute welfare, as expressed by (19) is negative. Thus in either case, the better endowed agent will have the higher absolute level of welfare, so that the distribution of welfare moves together with that of income.

4. The Relationship between Volatility and Inequality

We now turn to the relationship between volatility, growth, and the distribution of income, focusing on how these relationships respond to an increase the volatility of production, σ^2. In this section we examine the case of an economy without taxation. We discuss the relationship analytically and then supplement this with some numerical simulations.

4.1. Analytical properties

The effect of risk operates through its impact on the incentives to accumulate capital. An increase in σ^2 shifts the RR curve only, and for $\gamma < 0$, it shifts the RR curve upwards, as seen in Fig. 2. Given the fraction of time devoted to leisure, the shift in RR tends to increase the growth rate. The higher ψ increases the return to consumption, which raises the labor supply, and hence the return to capital relative to that of consumption, causing a further increase in the growth rate. Thus the increase in risk raises the mean growth rate and reduces leisure unambiguously, as the equilibrium moves from Q to Q' along PP. In addition greater risk increases the variance of the growth rate, $\sigma_\psi^2 = \Omega^2\sigma^2$, because of both the direct effect of σ^2 and the indirect impact of a lower l on Ω.

From equations (18') and (20), we see that the effect of an increase in risk on the gross and net distributions of income are given by

$$\frac{d\sigma_y}{d\sigma^2} = \sigma_k \frac{\partial \rho}{\partial l}\frac{dl}{d\sigma^2} > 0,$$
$$\frac{d\sigma_y^{NET}}{d\sigma^2} = \sigma_k \left(\frac{1-\tau_w}{\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)} \right) \frac{\partial \rho}{\partial l}\frac{dl}{d\sigma^2} > 0$$

An increase in l raises both pre-tax and post-tax inequality. Greater volatility of the production shock, by reducing the amount of time devoted to leisure, increases income inequality, as measured by the standard deviation of relative incomes. Pre-tax inequality will increase more than post-tax inequality if and only $\tau_k > \tau_w$, that is, if and only if the initial pre-tax income inequality exceeds the initial post-tax inequality.

Risk will also increase measures of inequality other than the standard deviation. To see this it suffices to note that the effect of an increase in risk on the relative gross income of an agent with capital share k_i is given by

$$\operatorname{sgn}\left(\frac{dy(k_i)}{d\sigma^2}\right) = -\operatorname{sgn}\left(\frac{\partial \rho}{\partial l}\frac{dl}{d\sigma^2}(1-k_i)\right) = \operatorname{sgn}\left(k_i - 1\right)$$

An increase in risk raises the income share for those with a wealth share above the average, and reduces the income share of those with wealth below. Consequently, inequality rises.

The intuition for these results is as follows. Because agents are sufficiently risk-averse, a greater variance of output has a strong income effect that makes them increase savings. Consequently, the growth rate increases. Note from the PP locus that the allocation of labor is unaffected by σ^2 *for a given growth rate*. A higher growth rate, however, implies higher future wages, and hence higher consumption for any extra time spent at work. It therefore reduces leisure time and increases the labor supply. The change in the labor supply, in turn, affects the distribution of income. A higher labor supply increases the return to capital and lowers the wage rate. Since labor is distributed more equally than is capital, the income gap between any two individuals increases, and income inequality increases.

Note that with an inelastic supply of labor, risk would not affect relative incomes. In this case, the income of agent i would be given by $y_i = (w + rk_i)/(w + r)$. With the AK technology resulting in a constant wage and interest rate, this expression would be unaffected by risk. In our setup risk matters because it affects the growth rate, and this, in turn, impacts the labor supply and factor rewards.

4.2. Numerical examples

To obtain further insights into the impact of risk on the equilibrium, and in particular the relationship between growth and income inequality, we perform some numerical analysis. In order to do so we use the following, mostly conventional, parameter values:

Parameter Values

Production	$A = 0.75,\ \alpha = 0.60$
Taste	$\beta = 0.04,\ \gamma = -2,\ \eta = 1.75$
Risk	$\sigma = 0.05,\ 0.10,\ 0.20,\ 0.30,\ 0.40$

The choice of production elasticity of labor measured in efficiency units implies that 60% of output accrues to labor. One consequence of the Romer technology being assumed, is that whereas this value is realistic in terms of the labor share of output, it implies an implausibly large externality from aggregate capital which implies extreme solutions for the first-best fiscal policy, discussed below. The choice of the scale parameter A = 0.75, is set to yield a plausible value for the equilibrium capital-output ratio.

Turning to the taste parameters, the rate of time preference of 4% is standard, while the choice of the elasticity on leisure, $\eta = 1.75$, is standard in the real business cycle literature, implying that about 72% of time is devoted to leisure, consistent with empirical evidence. Estimates of the coefficient of relative risk aversion are more variable throughout the literature. Values of the order of $\gamma = -18$ (and larger) have sometimes been assumed to deal with the equity premium puzzle (see Obstfeld, 1994). However, these tend to yield implausibly low values of the equilibrium growth rate. By contrast, real business cycle theorists routinely work with logarithmic utility functions $(\gamma = 0)$. More recently, a consensus seems to be emerging of values between 2 and 5 (see Constantinides, Donaldson, and Mehra 2002) and our choice of $\gamma = -2$ is well within that range.

Our main focus is on considering increases in exogenous production risk, which we let vary between $\sigma = 0.05$ and $\sigma = 0.40$. The value $\sigma = 0.05$ is close to the mean for OECD countries considered by Gali (1994) and Gavin and Hausmann (1995). Gavin and Hausmann present estimates for a wide range of countries and $\sigma = 0.10$ corresponds to countries subject to medium production risk. For virtually all countries they find $\sigma < 0.20$ so that the values $\sigma = 0.30,\ \sigma = 0.40$ are beyond the bounds of plausibility and are reported only to broaden the sensitivity analysis.

Choosing the distribution of wealth is less straightforward, as data on the distribution of wealth are difficult to obtain. Moreover, income distributions are reported in terms of Gini coefficients (rather than standard deviations as employed in our theoretical discussion) and Table 1 reports some actual distributions. The first two lines are the distributions of income in the US and Sweden in 1991 and 1992, respectively (from Deininger and Squire, 1996). The third is our hypothetical distribution of wealth. The values assumed are consistent with the data. For example, in the US in 1992, the bottom 40% of the population held 0.4% of total wealth, while the top 20% owed 83.8% of the total.[16]

The last line reports the income distribution generated by the model, using the hypothetical wealth distribution for the case of low risk $\sigma = 0.05$. To obtain it we have assumed that the bottom income group has no wealth and no labor endowment (i.e, zero income). Otherwise, since wages are identical for all workers, we would have a very large group with the same income at the bottom of the income distribution. Our assumption implies that the income share of the two bottom groups is 18.9 and hence of a similar magnitude to that observed in the data (15.2 and 19.1 for the US and Sweden, respectively). The resulting Gini coefficient lies between those of these two countries.

Table 2 reports the impact of increases in the volatility of the output shock on the equilibrium labor supply, the average rate of growth and its standard deviation, the Gini coefficient of income, and on overall welfare. Welfare changes reported are calculated as the percentage equivalent variations in the initial stock of capital of the average individual necessary to maintain the level of utility following the increase in risk from the benchmark level $\sigma = 0.05$ reported in the first row.

Line 1 of the table suggests that treating $\sigma = 0.05$ as a benchmark case leads to a plausible equilibrium, having a 3.3% mean growth rate and 1.73% relative standard deviation, with 72.5% of time allocated to leisure and a capital-output ratio (not reported) of approximately 3.[17] The implied distribution of income is also plausible, as noted.

As risk increases from $\sigma = 0.05$, Table 2 indicates the following. The mean growth rate increases, as does its standard deviation. The net effect of the greater risk dominates the positive effect of the higher growth rate, so that the increase in risk reduces average welfare. It should be noted that for the plausible range of $\sigma < 0.20$, the welfare loss is relatively modest. This is a

[16] See Wolff (1998).
[17] The mean growth rate for OECD economies is around 2.2%, with a standard deviation also around 2.2%.

characteristic limitation of this class of model having only aggregate risk, and has been discussed elsewhere in the literature.[18] More to the point here, we see that greater risk is associated with a substitution toward more labor (less leisure), and an increase in income inequality -- as measured by the Gini coefficient -- consistent with the formal analysis presented in Section 3.1.

In terms of magnitudes, the effect of risk on the Gini coefficient is quite modest, at least for plausible degrees of risk. It is interesting to note that income inequality in the US increased by 2.5 Gini points between 1980 and 1990, and that this has been considered a sizeable increase. From Table 2 it is seen that for risk alone to generate a similar increase it would have had to increase from $\sigma = 0.05$ to around $\sigma = 0.4$, which is obviously implausible. Clearly other structural and policy changes are primarily responsible. However, small changes in risk may still play a significant role if they give rise to large policy responses.

5. Taxation

A familiar feature of the Romer (1986) model is that by ignoring the externality associated capital, the decentralized economy generates a sub-optimally low growth rate. This suggests that an investment subsidy that increases the growth rate, will move the equilibrium closer to the social optimum. With heterogeneous agents, two questions arise. First, how to finance this subsidy if the government is concerned about inequality as well as about average welfare. An investment subsidy raises the return to capital and will tend to favor those with large capital holdings. If the subsidy were financed by a lump-sum tax, the system would redistribute away from those with lower incomes to those with higher incomes. Are there ways in which this reverse redistribution can be avoided? Second, we want to know whether the use of first-best policies has any implication for the relationship between volatility and inequality. In this section we investigate these questions in some detail. We begin by deriving the first-best optimal rate of growth and allocation of labor.

[18] Most notably it is characteristic of Lucas's (1987) of the cost of business cycles, and it is also discussed at more length for a model closer to present by Turnovsky (2000b).

5.1. The first-best optimum

Given the externality stemming from the aggregate capital stock, finding the first-best optimums amounts to solving the following problem:

$$\max E_0 \int_0^{\infty} \frac{1}{\gamma}\left(C_i(t) l_i^{\eta}\right)^{\gamma} e^{-\beta t} dt, \tag{23a}$$

subject to

$$dK_i = \left(\Omega K_i - C_i\right) dt + \Omega K_i du \tag{23b}$$

In the Appendix we show that the solution to this problem is given by the equations

R'R' $$\tilde{\psi} = \frac{\Omega(\tilde{l}) - \beta}{1-\gamma} - \frac{\gamma}{2}\Omega(\tilde{l})^2 \sigma^2 \tag{17a'}$$

PP $$\tilde{\psi} = \Omega(\tilde{l})\left[1 - \frac{\alpha}{\eta}\frac{\tilde{l}}{1-\tilde{l}}\right] \tag{17b'}$$

DD $$\sigma_y = \rho(\tilde{l})\sigma_k \tag{17c}$$

$$\frac{\tilde{C}}{\tilde{K}} = \frac{\alpha\Omega(\tilde{l})}{\eta}\frac{\tilde{l}}{1-\tilde{l}} \tag{17d}$$

where the tilde denotes the first-best optimum. Note that the only difference with the solution to the competitive equilibrium in the absence of taxes is that the social rate of return to capital now takes into account the production externality and hence exceeds the private return.[19] The R'R' schedule lies above RR. Given that the PP schedule is steeper than RR, the upward shift of RR results in a higher growth rate, lower leisure, and therefore increases inequality, as can be seen from Figure 2.[20]

[19] The transversality condition (12) for the central planner's problem again reduces to (13) but is now automatically satisfied without further restrictions being imposed.

[20] The reason why the social planner chooses less leisure is that there are in fact two externalities in the model. On the one hand, a greater individual stock of capital increases the aggregate level of technology. On the other, a higher labor supply raises the marginal product of capital and induces greater accumulation of capital, thus increasing the level of technology.

5.2 First-best taxation

Comparing the first-best optimum, described by R'R', and PP with the decentralized equilibrium, RR, PP we can see that the tax-subsidy system can be used to attain the optimal growth rate, leisure time, and consumption/capital ratio by setting

$$\frac{1-\tau_w}{1+\tau_c}=1, \quad \text{i.e. } \tau_w=-\tau_c \ , \tag{24a}$$

$$(1-\alpha)\frac{1-\tau_k}{1-s}=1, \quad \text{i.e. } s=\alpha+(1-\alpha)\tau_k \ , \tag{24b}$$

$$\frac{1-\tau_k}{1-\tau_w}=\frac{1}{1-\alpha}\left[1-\eta\frac{1-\tilde{l}}{\tilde{l}}\right], \tag{24c}$$

where the last equation is obtained from the government's budget constraint, (16g). The first two equations represent intuitive optimality conditions. The first states that any wage tax should be offset with an equivalent consumption tax so as not to distort the leisure-consumption choice. Interpreting the tax on wage income as a negative tax on leisure, (24a) states that the two utility enhancing goods, consumption and leisure, should be taxed uniformly. The second condition simply ensures that the private rate of return on investment must equal the social return, and for this to be so the subsidy to investment must exceed the externality by an amount that reflects any tax on capital income.[21] Note from the third equation that unless consumption equals total income, (in which case there is zero growth), the replication of the first optimum requires differential taxes on wages and capital; $\tau_w \lessgtr \tau_k$ according to whether there is positive or negative growth.

Equations (24) indicate the existence of a degree of freedom in the optimal tax-subsidy structure. One instrument can be set arbitrarily and we shall take it to be *s*. In this case (24a) – (24c) imply the following first-best optimal tax rates:

$$\hat{\tau}_k=\frac{s-\alpha}{1-\alpha} \tag{25a}$$

[21]The optimal tax rates set out in (24) are similar to those obtained by Turnovsky (2000a) in a pure deterministic representative agent endogenous growth model.

$$\hat{\tau}_w = \frac{s-\eta(1-\tilde{l})/\tilde{l}}{1-\eta(1-\tilde{l})/\tilde{l}} = -\hat{\tau}_c \tag{25b}$$

from which we may conclude the following relative magnitudes, in an economy with positive growth $[1>(\alpha/\eta)(\tilde{l}\,/[1-\tilde{l}\,]);$ see (17b')]:

$$0<s<\alpha: \qquad \tau_w = -\tau_c < \tau_k < 0 < s$$

$$\alpha < s < \eta\left(\frac{1-\tilde{l}}{\tilde{l}}\right): \qquad \tau_w = -\tau_c < 0 < \tau_k < s$$

$$\eta\left(\frac{1-\tilde{l}}{\tilde{l}}\right) < s < 1: \qquad 0 < \tau_w = -\tau_c < \tau_k < s$$

There are two things to note about the optimal tax structure. First, is that for a sufficiently small investment subsidy, the optimal tax structure will call for subsidies to both wage income and capital income, all financed by the consumption tax. But as the subsidy increases, both forms of income should be taxed, with the revenues financing both the initial investment subsidy and now a subsidy to consumption. This pattern will be seen to be borne out by our simulations. Second $\hat{\tau}_k, \hat{\tau}_w$ are both highly sensitive to the (arbitrary) choice of s.

What is the impact of the first-best taxation system on distribution? Recall that the dispersion of gross income is given by (17c), where $\rho(l)$ is a decreasing function of leisure time. Since the policy increases the time allocated to labor, it will increase gross income inequality. The dispersion of net income in the decentralized economy that mimics the centrally planned equilibrium is obtained by substituting the tax rates, (25a), (25b), into (20b) to yield

$$\rho^{NET}(l;\tau_w,\tau_k) = 1 - \frac{\alpha}{(1+\eta)(1-l)}\frac{1}{1+\alpha-\eta\left((1-l)/l\right)} \tag{26}$$

The striking aspect about (26) is that the distribution of net income is *independent* of the (arbitrary) choice of fiscal instruments employed to achieve this objective. As long as the equilibrium is one with positive growth, the optimal tax requires $\hat{\tau}_w < \hat{\tau}_k$. Then $\rho^{NET} < \rho$, and net income is less dispersed than is gross income. In addition, in all of our simulations we find that the direct redistributive effect of taxation dominates the indirect effect of changes in factor prices so that the distribution of net income is less unequal than in the economy without taxes.

When the first-best tax system is implemented, the effect of risk on growth and leisure is equivalent to that in the laissez-faire economy, as can be easily verified from equations (17). Greater risk is therefore associated with a greater supply of labor and hence with more pre-tax inequality. The effect of risk on after-tax inequality is, however, ambiguous. Differentiating (26) with respect to l, we can see that there are two opposing effects. On the one hand, more leisure tends to reduce pre-tax inequality. On the other, and as long as the subsidy rate is less than 1, a lower labor supply implies that a higher wage tax is required in order to finance any given subsidy rate (see (25b)), making the fiscal system less progressive. Either effect can dominate, implying that greater risk need not result in a more unequal distribution of after-tax income.

5.3 Alternative policy responses

To attain the first-best equilibrium is likely to require the tax rates to assume extreme values, even for plausible parameter values; see Table 3 below. These will generate dramatic changes in the distribution of income that may render them politically infeasible. Indeed, our numerical analysis (see Table 3 below) implies differences between the gross and the net Gini coefficients of 14 to 20 Gini points, whereas actual differences in OECD countries range between 1.5 and 4 points. Thus, we now consider some less drastic policy responses, which nevertheless, as our simulations show, may still yield substantial welfare gains.

Subsidy to Investment Financed by a Tax on Capital Income

Suppose that the fiscal authority decides to finance the subsidy to investment with a tax on capital income, alone. Setting $\tau_w = \tau_c = 0$ in the government budget constraint (24c) the required tax on capital income is:

$$\tau_k = \frac{s}{(1-\alpha)}\left(1 - \frac{\alpha}{\eta}\frac{l}{1-l}\right) \tag{27}$$

From equations (17a) we see that this policy shifts the RR schedule upwards and leaves the PP schedule unchanged, increasing the growth rate and reducing leisure. The reduction in leisure increases the pre-tax degree of income inequality, $\rho(l)$. Recall that the net distribution of income was characterized by (18'). Then, taxing capital income ensures that $\rho^{NET}(l,\tau_w\tau_k) < \rho(l)$. If the redistributive effect dominates, as our simulations suggest, the after-tax inequality actually declines.

Subsidy to Investment Financed by a Tax on Wage Income

Alternatively, the subsidy may be fully financed by a wage tax

$$\tau_w = \frac{s}{\alpha}\frac{\left(1-\frac{\alpha}{\eta}\frac{l}{1-l}\right)}{\left(1-\frac{s}{\eta}\frac{l}{1-l}\right)} \tag{28}$$

In this case, both the RR and PP schedules shift up, resulting in a higher growth rate and greater or lower leisure, depending on the relative shifts. The reason for the ambiguous effect on leisure is that the wage tax tends to reduce the supply of labor, while the higher growth rate tends to increase it.

The ambiguous response of labor complicates the impact on the inequality of income. First, the increase/decrease in leisure time will reduce/increase the variance of gross incomes, as seen from (18'). However, the required (positive) wage tax implies taxing the factor that is more equally distributed, and for any given distribution of gross incomes this raises the variability of net incomes (see (20b) above). If the policy reduces leisure time, it would then unambiguously increase pre-tax and post-tax income inequality. However, when leisure time increases, the two effects work in opposite directions: there will be a reduction in the variability of gross income, while net income inequality may increase or decrease as compared to the equilibrium without taxes.[22]

Subsidy to Investment Financed by a Tax on Consumption

As a third example, the subsidy may also be financed by setting the consumption tax equal to

$$\tau_c = \frac{s\left(1-\frac{\alpha}{\eta}\frac{l}{1-l}\right)}{\frac{\alpha}{\eta}\frac{l}{1-l}-s} \tag{29}$$

[22] We can, however, see that when the subsidy rate matches the externality, $s=\alpha$, $\tau_w=1$ and $\rho^{NET}=1$ implying that the net income inequality is increased to that of the initial endowment of capital.

in which case $\rho^{NET}(l,\tau_w,\tau_k) = \rho(l)$. Again both schedules shift upwards, increasing the growth rate. In this case it can be shown that under the weak condition $\gamma < 0$, leisure increases, so that gross income inequality declines. Since the consumption tax has no direct redistributive effect, the gross and the net distributions of income are identical and hence net income inequality declines as well.

5.4 Numerical analysis

Tables 3 - 5 report the numerical effects of a number of different policies, using the parameter values described in section 4.2.

We begin with Table 3, which summarizes the first-best equilibrium in the centrally planned economy. It offers a number of insights that both reinforce and complement our analytical results. First, we see that the policy involves a substantial reduction in leisure time (between 10 and 12 percentage points), raising the growth rate enormously (by a factor of 3!), and only slightly increasing volatility.[23] ΔX is the increase in the welfare of the average individual, measured as the percentage variation over that in an economy with the same level of risk in the benchmark equilibrium (i.e. those in Table 2). First-best taxation increases the welfare of the average individual in the economy by over 20%.

The effects on income distribution are substantial. The large reduction in leisure time results in a large increase in pre-tax inequality. However, the redistributive effect is strong enough to offset this effect and yield an overall reduction in the Gini coefficient of net income. The reduction in post-tax inequality relative to the economy without subsidies is large, amounting to between 6 and 12 Gini points.

Table 3 also illustrates the analytical results that the first-best equilibrium can be replicated by a variety of tax/subsidy configurations, each of which leads to precisely the same post-tax distribution of income. The sensitivity of the tax regime to changes in the subsidy rate is also borne out. Consider for example the case of low risk, $\sigma = 0.05$. In the absence of a subsidy to investment, the first best equilibrium can be sustained if income form capital and labor are subsidized at the rates of 150% and nearly 600%, respectively, while consumption is taxed at nearly 600%! This is hardly

[23] The implied percentage increase in labor supply is much larger, being of the order of 20%.

a politically viable tax structure. But the first-best equilibrium can also be attained if, more reasonably, investment is subsidized at around 86%, being financed by a tax on capital income of around 64%, leaving consumption and labor income untaxed. Or, if investment and consumption are subsidized at 90% and 30% respectively, with taxes on labor income and capital income of 30% and 75%, respectively.

Table 3 also considers the effects of increasing risk. This is shown to reduce leisure, thereby increasing the gross income inequality. At the same time, the decrease in leisure increases the growth rate and reduces the redistributive effect, thus reducing the inequality of net income. In all our examples, gross and net income inequality move in opposite ways, with greater volatility increasing pre-tax and reducing post-tax inequality. Our numerical results highlight the fact that the divergence between pre- and post- tax inequality is greater the more risky the economy is. The reason for this is that risk has a strong distortionary effect on the labor supply. Greater risk, by raising the labor supply and hence the wage bill, requires a lower wage tax, thus making the tax system more progressive. The effect of the increased labor supply is to raise pre-tax inequality, the impact of the lower wage tax is to reduce post-tax inequality, and as a result the gap between gross and net inequality increases.

Table 4 examines a number of alternative non-optimal policies. The first two lines are the benchmark case of no intervention, for a low-risk economy ($\sigma = 0.05$) and a medium/high risk economy ($\sigma = 0.20$), respectively. The welfare gains are measured as percentage increases over the welfare levels in these two base economies. We consider in turn the effects of financing a fixed 30% investment subsidy through a capital income, wage tax or a consumption tax, respectively. Financing by a capital income tax generates the least positive impact on the mean growth rate and on welfare. It raises the pre-tax income inequality, but lowers the post-tax income inequality. Employing a wage tax has a significantly larger effect on the mean growth rate and on welfare. It also has the opposite impacts on income distribution, reducing the before tax inequality, but raising it after tax. The consumption tax has the greatest benefit on the average agent and the most beneficial effect on the mean growth rate, and it increases the degree of income inequality (both pre- and post-tax) slightly. Interestingly, all three policies have virtually no adverse effect on aggregate volatility and in fact, the wage tax, by increasing leisure actually permits a substantial increase in the growth rate to be accompanied by a small reduction in its volatility.

The last two row of the table considers financing the subsidy through a combination of wage and consumption taxes. In particular, we set $\tau_w = -\tau_c$; that is, these two taxes are optimally set, although the subsidy is below the first-best level. The effect of this policy on the growth rate is stronger than in the previous three cases, the reason being that this policy does not distort the allocation of time between labor and leisure. Employing only a wage or a consumption tax tends to reduce the supply of labor, partially offsetting the effect of the subsidy. When both are used, this effect is absent. Since setting $\tau_w = -\tau_c$ results in faster growth than using only one tax, and since the volatility of growth is only slightly higher, this policy generates larger welfare gains than any of the pure policies. The effect on distribution is quite significant. In contrast to financing the investment subsidy by either a wage or a consumption tax alone, financing through a combination of a consumption tax and wage subsidy reduces substantially post-tax inequality.

The investment subsidy in Table 4 is arbitrary. Table 5 summarizes a number of second best policies, whereby the policy maker sets the optimal subsidy for each of the three modes of finance. In the case where it is financed with a tax on capital income, it is able to attain the first-best optimum. Focusing on $\sigma = 0.05$, setting s = 85.7% and $\tau_k = 64.2$ % improves welfare by over 20% and generates the distribution of income associated with the first-best optimum. Alternatively, setting s = 57.2%, financed with 26% tax on wages, or s = 60.2% financed with a 25.5% consumption tax yield second-best optima. The interesting aspect about these latter two alternatives is that they are fairly moderate policies, in contrast to the first-best, summarized in Table 3. In particular, the consumption tax yields the major portion of the welfare gains obtained in the first-best case (19% out of a total increase in welfare of 20.6%), while having only a minimally adverse impact on income distribution. Indeed, to a policymaker concerned with maximizing the welfare of the average agent, with minimum distortionary effect on the distribution of income, this policy may be particularly attractive.

6. Conclusions

Stochastic shocks are a major source of income disparities, and an extensive literature has explored how "luck" and the market's tendency towards convergence combine to create persistent inequality Yet this literature is concerned with idiosyncratic shocks that have no relation with aggregate shocks. The idea that aggregate uncertainty may also affect the distribution of income remains to be explored, and this paper is a first step in that direction.

We have used an AK stochastic growth model to show that, when agents differ in their initial stocks of capital, greater growth volatility is associated with a more unequal distribution of income. Greater risk tends to increase the supply of labor, reducing wages and raising the interest rate. If capital endowments are unequally distributed, while labor endowments are not, the change in factor prices raises the return to the factor that is the source of inequality, and the distribution of income becomes more spread.

The endogeneity of the labor supply also implies that policies aimed at increasing the growth rate will have distributional implications, and we have examined how these differ depending on the particular form of the policies. In particular, we have compared financing an investment subsidy through a capital income tax, a wage tax, or a consumption tax.

Our analysis yields two main conclusions. First, it is possible simultaneously to increase the growth rate and reduce net income inequality. In many instances, we find that polices that generate faster growth are associated with a reduction in the Gini coefficient of post-tax income, allowing the policymaker to attain both efficiency and equity goals. Second, it is often the case that fiscal policy has opposite effects on the distribution of gross and net income. These results highlight the fact that rather than the usual tradeoff between equity and efficiency, policymakers concerned with the distribution of income may face a tradeoff between pre- and post-tax inequality. Moreover, the divergence between pre and post tax inequality is greater the more risky the economy is. Understanding which type of inequalities agents and the social planner care about becomes essential, in particular in high risk economies, and it raises the question of whether a slightly more unequal distribution of both gross and net incomes may, in certain cases, be a more viable policy than a huge, but offset, increase in pre-tax inequality.

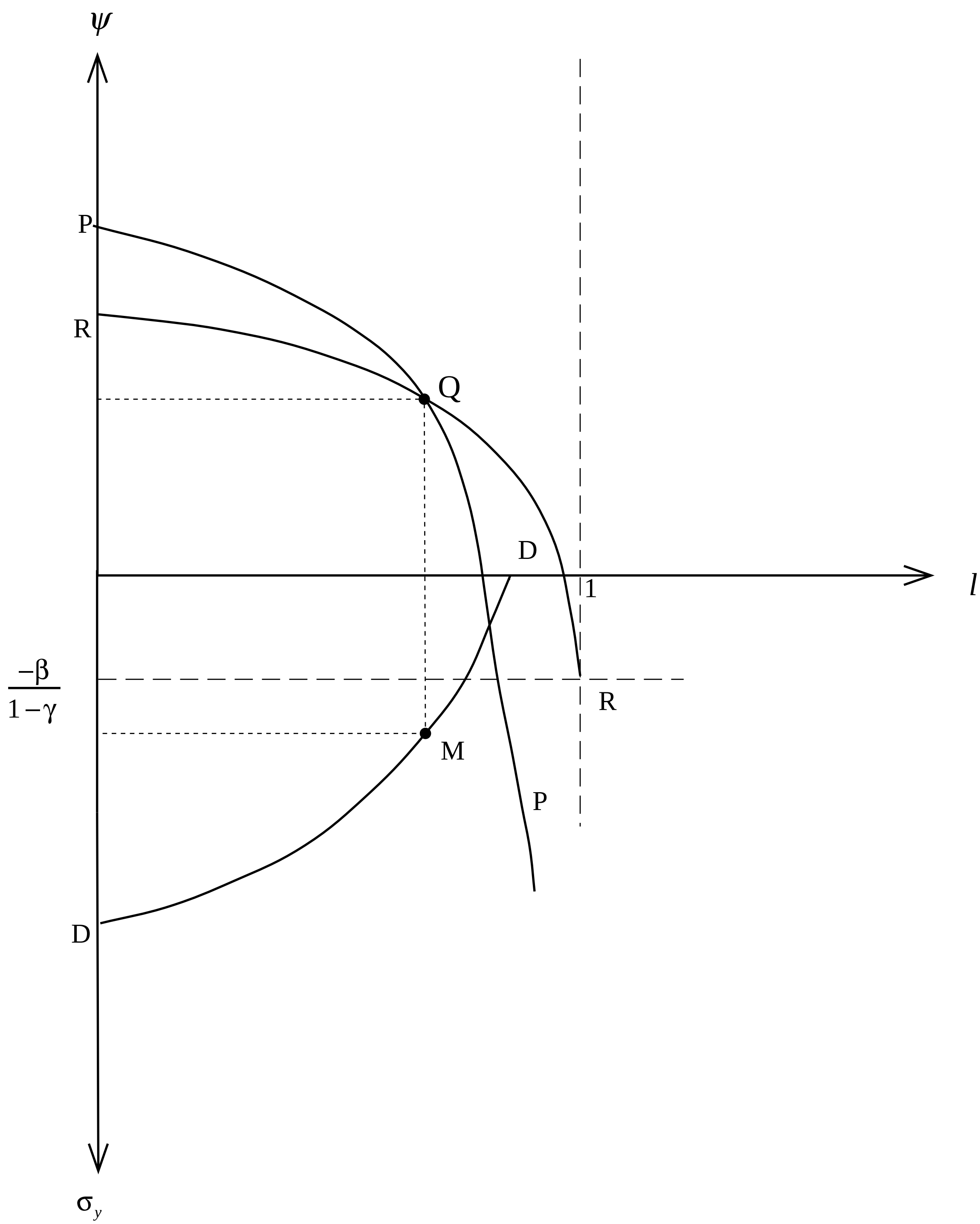

Fig 1: Equilibrium Growth, Employment, and Income Distribution

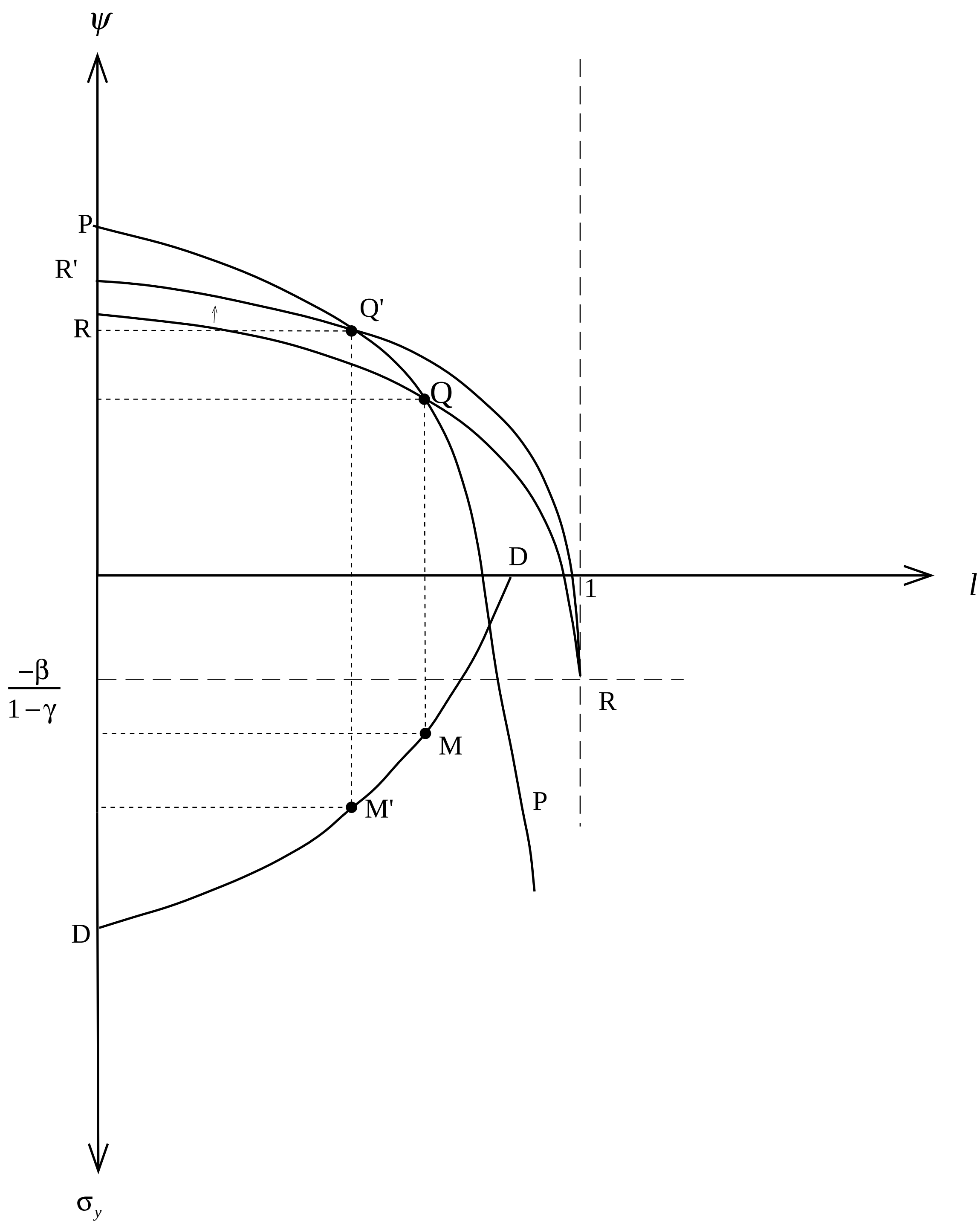

Fig 2: Increase in Production Risk

Table 1
The Distribution of Income and Wealth

	Q1	Q2	Q3	Q4	Q5	Gini
US: income shares	4.6	10.6	16.6	24.6	43.6	39.1
Sweden: income shares	6.3	12.8	19.2	24.8	36.9	31.1
Assumed wealth shares	0	0	1.2	12	86.8	74.2
Assumed wealth levels	0	0	0.06	0.6	4.34	
Simulated income shares ($\sigma = 0.05$)		18.9	19.1	21.8	40.2	33.30

Table 2
Growth and Distribution of Income and Wealth

	l	ψ	σ_ψ	ΔX	$Gini(y)$
$\sigma = 0.05$	72.5	3.30	1.73	--	33.30
$\sigma = 0.1$	72.4	3.40	3.46	-0.68	33.40
$\sigma = 0.2$	72.2	3.79	6.96	-3.41	33.82
$\sigma = 0.3$	71.8	4.45	10.53	-8.04	34.52
$\sigma = 0.4$	71.2	5.43	14.22	-14.70	35.51

Table 3
First-best Taxation

	s	τ_w $(=-\tau_c)$	τ_k	l	ψ	σ_ψ	*Gini*(*y*)	*Gini*(*ny*)	ΔX
$\sigma = 0.05$	0 30.0 60.0 85.67 90.0	-597.8 -388.5 -179.1 0 30.22	-150.0 -75.0 0 64.17 75.0	67.1	11.53	1.92	41.02	27.06	20.56
$\sigma = 0.1$	0 30.0 60.0 86.03 90.0	-615.9 -401.1 -186.4 0 28.40	-150.0 -75.0 0 65.08 75.0	67.0	11.66	3.85	41.13	26.94	20.63
$\sigma = 0.2$	0 30.0 60.0 87.53 90.0	-701.7 -461.2 -220.7 0 19.80	-150.0 -75.0 0 68.82 75.0	66.7	12.20	7.76	41.57	26.42	20.92
$\sigma = 0.3$	0 30.0 60.0 90.0 90.19	-919.1 -613.3 -307.6 -1.91 0	-150.0 -75.0 0 75.0 75.47	66.0	13.13	11.78	42.30	25.39	21.45
$\sigma = 0.4$	0 30.0 60.0 90.0 94.33	-1663 -1134 -605.1 -76.30 0	-150.0 -75.0 0 75.0 85.82	65.0	14.54	15.99	43.36	23.44	22.33

Table 4
Arbitrary Taxation

		τ_k	τ_w	τ_c	l	ψ	σ_ψ	*Gini*(*y*)	*Gini*(*ny*)	ΔX
s=0	$\sigma = 0.05$	0	0	0	72.5	3.30	1.73	33.30	33.30	--
	$\sigma = 0.2$	0	0	0	72.4	3.79	6.96	33.82	33.82	--
s=30	$\sigma = 0.05$	10.07	0	0	71.6	4.73	1.76	34.80	32.83	7.64
	$\sigma = 0.2$	10.96	0	0	71.4	5.18	7.09	35.26	33.13	7.65
s=30	$\sigma = 0.05$	0	7.63	0	72.8	5.24	1.72	32.79	34.39	9.66
	$\sigma = 0.2$	0	8.28	0	72.6	5.71	6.89	33.09	34.79	10.18
s=30	$\sigma = 0.05$	0	0	5.42	72.3	5.32	1.74	33.75	33.75	10.05
	$\sigma = 0.2$	0	0	5.98	72.0	5.80	6.98	34.11	34.11	10.54
s=30	$\sigma = 0.05$	0	-18.76	18.76	71.2	5.47	1.78	35.55	32.44	10.71
	$\sigma = 0.2$	0	-21.63	21.63	70.8	6.00	7.16	36.08	32.60	11.13

Table 5
Second Best

	s	τ_k	τ_w	τ_c	l	ψ	σ_ψ	*Gini*(*y*)	*Gini*(*ny*)	ΔX
$\sigma = 0.05$	0	0	0	0	72.5	3.30	1.73	33.30	33.30	--
$\sigma = 0.2$	0	0	0	0	72.4	3.79	6.96	33.82	33.82	--
$\sigma = 0.05$	85.7	64.17	0	0	67.1	11.53	1.92	41.02	27.06	20.56
$\sigma = 0.2$	85.5	68.82	0	0	66.7	12.20	7.76	41.57	26.42	20.92
$\sigma = 0.05$	57.2	0	25.96	0	74.1	9.07	1.67	30.20	36.56	16.79
$\sigma = 0.2$	57.4	0	27.39	0	74.1	9.54	6.66	30.20	36.97	17.72
$\sigma = 0.05$	60.2	0	0	25.52	72.0	10.40	1.75	34.19	34.19	18.87
$\sigma = 0.2$	60.1	0	0	27.02	71.9	10.87	7.01	34.39	34.39	19.72

Appendix

This appendix provides some of the technical details underlying the derivations of the equilibrium conditions (8) and (16a) to (16g).

A.1 Consumer optimization

Agent i's stochastic maximization problem is to choose her individual consumption-capital ratio and the fraction of time devoted to leisure to maximize expected lifetime utility

$$\max E_0 \int_0^\infty \frac{1}{\gamma}\left(C_i(t) l_i^\eta\right)^\gamma e^{-\beta t} dt, \qquad -\infty < \gamma < 1, \eta > 0, \gamma\eta < 1 \tag{A.1a}$$

subject to her individual capital accumulation constraint

$$dK_i = \frac{(1-\tau_k) r K_i + (1-\tau_w) w(1-l_i) K - (1+\tau_c) C}{1-s} dt + K_i dk \tag{A.1b}$$

and the aggregate capital accumulation constraint

$$dK = \frac{\left((1-\tau_K) r + (1-\tau_w) w(1-l)\right) K - (1+\tau_c) C}{1-s} dt + K dk \tag{A.1b'}$$

together with the economy-wide shock

$$dk = \frac{1-\tau_k'}{1-s'} \Omega du\ . \tag{A.1c}$$

Since the agent perceives two state variables, K_i, K, we consider a value function of the form

$$V\left(K_i, K, t\right) = e^{-\beta t} X(K_i, K)$$

the differential generator of which is

$$\Psi\left[V(K_i,K,t)\right] \equiv \frac{\partial V}{\partial t} + \left(\left[\frac{1-\tau_k}{1-s} r - \frac{1+\tau_c}{1-s}\frac{C_i}{K_i}\right] K_i + w(1-l)\frac{1-\tau_w}{1-s} K\right) V_{K_i}$$

$$+\left(\frac{1-\tau_k}{1-s} r - \frac{1+\tau_c}{1-s}\frac{C}{K} + w(1-l)\frac{1-\tau_w}{1-s}\right) K V_K + \frac{1}{2}\sigma_K^2 K_i^2 V_{K_i K_i} + \sigma_{K_i K} K_i K V_{K_i K} + \frac{1}{2}\sigma_K^2 K^2 V_{KK} \tag{A.2}$$

The individual's problem is to choose consumption, leisure, and the rate of capital accumulation to maximize the Lagrangian

$$e^{-\beta t}\frac{1}{\gamma}\left(C_i l_i^{\eta}\right)^{\gamma}+\Psi\left[e^{-\beta t}X(K_i,K)\right]. \tag{A.3}$$

In doing this, she takes the evolution of the aggregate variables and the externality as given. Taking the partial derivatives with respect to C_i and l_i, and cancelling $e^{-\beta t}$ yields

$$\frac{1}{C_i}\left(C_i l_i^{\eta}\right)^{\gamma}=\frac{1+\tau_c}{1-s}X_{K_i} \tag{A.4a}$$

$$\frac{\eta}{l}\left(C_i l_i^{\eta}\right)^{\gamma}=\frac{1-\tau_w}{1-s}wKX_{K_i} \tag{A.4b}$$

In addition, the value function must satisfy the Bellman equation

$$\max\left\{e^{-\beta t}\frac{1}{\gamma}\left(C_i l_i^{\eta}\right)^{\gamma}+\Psi\left[e^{-\beta t}X(K_i,K)\right]\right\}=0 \tag{A.5}$$

The Bellman equation is a function of two state variables, individual and aggregate capital, and hence it is a partial differential equation in these two variables. Using equations (A.1b) and (A.1b'), and given (A.2), the Bellman equation can be written as

$$\begin{aligned}&\frac{1}{\gamma}\left(C_i l_i^{\eta}\right)^{\gamma}-\beta X(K_i,K)+\frac{E(dK_i)}{dt}X_{K_i}+\frac{E(dK)}{dt}X_K\\&+\frac{1}{2}\frac{E(dK_i)^2}{dt}X_{K_iK_i}+\frac{E(dK_idK)}{dt}X_{K_iK}+\frac{1}{2}\frac{E(dK)^2}{dt}X_{KK}=0\end{aligned} \tag{A.6}$$

Next we take the partial derivative of the Bellman equation with respect to K_i, noting that l_i is independent of K_i, while C_i is a function of K_i through the first-order condition (A.4a),

$$\begin{aligned}&\frac{1}{C_i}\left(C_i l_i^{\eta}\right)^{\gamma}C_{i,K_i}-\beta X_{K_i}+\frac{E(dK_i)}{dt}X_{K_iK_i}+\left[r\frac{1-\tau_k}{1-s}-C_{i,K_i}\right]X_{K_i}+\frac{E(dK)}{dt}X_{K_iK}+K_iX_{K_iK_i}\sigma_K^2\\&+\frac{1}{2}\frac{E(dK_i)^2}{dt}X_{K_iK_iK_i}+\frac{E(dK_idK)}{dt}X_{K_iKK_i}+KX_{K_iK}\sigma_{K_iK}+\frac{1}{2}\frac{E(dK)^2}{dt}X_{K_iKK}=0\end{aligned} \tag{A.7}$$

Consider now $X_{K_i}=X_{K_i}(K_i,K)$. Taking the stochastic differential of this quantity yields:

$$dX_{K_i} = X_{K_iK_i}dK_i + X_{K_iK}dK + \frac{1}{2}X_{K_iK_iK_i}(dK_i)^2 + X_{K_iKK_i}(dK_i)(dK) + \frac{1}{2}X_{K_iKK}(dK)^2 \qquad \text{(A.8)}$$

Taking expected values of this expression, dividing by *dt,* and substituting the resulting equation along with (A.4a) into (A.7) leads to:

$$\left[r\frac{1-\tau_k}{1-s} - \beta\right]X_{K_i} + [K_iX_{K_iK_i} + KX_{K_iK}]\sigma_K^2 + \frac{E(dX_{K_i})}{dt} = 0\,, \qquad \text{(A.9)}$$

The solution to this equation is by trial and error. Given the form of the objective function, we propose a value function of the form:

$$X(K_i, K) = cK_i^{\gamma-\gamma_2}K^{\gamma_2} \qquad \text{(A.10)}$$

where the parameters c, γ_2 are to be determined. From (A.10) we obtain:

$$\begin{aligned} &X_{K_i} = (\gamma-\gamma_2)X/K_i; X_K = \gamma_2 X/K; \\ &X_{K_iK_i} = (\gamma-\gamma_2)(\gamma-\gamma_2-1)X/K_i^2; \\ &X_{K_iK} = (\gamma-\gamma_2)\gamma_2 X/K_iK; X_{KK} = (\gamma_2-1)\gamma_2 X/K^2. \end{aligned} \qquad \text{(A.11)}$$

We can now use equation (A.12) to re-express (A.11) as

$$\frac{E(dX_{K_i})}{X_{K_i}dt} = \beta - r\frac{1-\tau_k}{1-s} + (\gamma-1)\sigma_K^2 \qquad \text{(A.12)}$$

Now, returning to the first-order condition (A.4a), computing the stochastic differential of this relationship and taking expected values yields

$$\frac{E\left(dX_{K_i}\right)}{X_{K_i}} = (\gamma-1)\frac{E(dC_i)}{C_i} + \frac{1}{2}(\gamma-1)(\gamma-2)E\left(\frac{dC_i}{C_i}\right)^2 \qquad \text{(A.13)}$$

Along the balanced growth path, C_i/K_i is constant. Hence $dC_i/C_i = dK_i/K_i = \psi dt + dw$, and thus

$$\frac{E\left(dX_{K_i}\right)}{X_{K_i}dt} = (\gamma-1)\psi + \frac{1}{2}(\gamma-1)(\gamma-2)\sigma_K^2 \qquad \text{(A.14)}$$

As will be shown below (see equation (A.26)), the government's balanced budget implies that the stochastic component of the individual budget constraint is

$$dk = \Omega du \,. \tag{A.15}$$

Combining (A.13), (A.14), and (A.15) yields the mean growth rate of individual consumption

$$\psi = \frac{r\big((1-\tau_k)/(1-s)\big)-\beta}{1-\gamma} - \frac{\gamma}{2}\Omega^2\sigma^2 . \tag{A.16}$$

The labor supply is obtained from the first-order conditions (A.4a) and (A.4b), namely

$$C_i = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}Kl_i \,. \tag{A.17}$$

Dividing (A.17) by K we obtain the individual consumption to wealth ratio,

$$\frac{C_i}{K_i} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l_i}{k_i} , \tag{A.18}$$

and summing over all agents we have the aggregate consumption to wealth ratio,

$$\frac{C}{K} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}l \,. \tag{A.19}$$

From the individual budget constraint, the rate of growth is

$$\psi = r\frac{1-\tau_k}{1-s} + w\frac{1-\tau_w}{1-s}\frac{1-l_i}{k_i} - \frac{1+\tau_c}{1-s}\frac{C_i}{K_i} , \tag{A.20}$$

which using (A.18) and rearranging gives

$$1-l_i = \frac{1}{1+\eta} - \frac{\eta}{1+\eta}\frac{r(1-\tau_k)/(1-s)-\psi}{w}\frac{1-s}{1-\tau_w}k_i . \tag{A.21}$$

A.2 Macroeconomic equilibrium

Note that the growth rate is the same for all agents, irrespective of their initial wealth holdings. Equation (A.16) is hence the mean growth rate of the economy. The dynamic evolution of the aggregate stock of capital is given by

$$\frac{dK}{K}=\left[\frac{r\left((1-\tau_k)/(1-s)\right)-\beta}{1-\gamma}-\frac{\gamma}{2}\Omega^2\sigma^2\right]dt+\Omega du$$

and the standard deviation (volatility) of the growth rate is

$$\sigma_\psi=\Omega\sigma\,. \tag{A.22}$$

Summing (A.21) over all agents gives a relationship between the aggregate labor supply and the growth rate,

$$1-l=\frac{1}{1+\eta}-\frac{\eta}{1+\eta}\frac{\phi-\psi}{w}\frac{1-s}{1-\tau_k}. \tag{A.23}$$

Goods market equilibrium requires $dK=\Omega K(dt+du)-Cdt$, which taking expectations and dividing by K yields,

$$\psi=\Omega-\frac{C}{K}. \tag{A.24}$$

Equations (A.16), (A.22) (A,18), (A.19), (A.20), (A.24), and (16g) are the macroeconomic equilibrium conditions as specified in equations (16a) – (16g), respectively

In the absence of taxation, the equilibrium reduces to

$$\frac{C_i}{K_i}=\frac{w}{\eta}\frac{l_i}{k_i}, \tag{A.25a}$$

$$\psi=\frac{r-\beta}{1-\gamma}-\frac{\gamma}{2}\Omega^2\sigma^2, \tag{A.25b}$$

$$\psi=r+w(1-l)-\frac{C_i}{K_i}, \tag{A.25c}$$

$$\frac{C}{K}=\frac{wl}{\eta}, \tag{A.25d}$$

$$\sigma_\psi=\Omega\sigma, \tag{A.25e}$$

which jointly determine the consumption-capital ratio, the average growth rate, the labor supply, and the volatility of growth.

A.3 Existence of a balanced growth equilibrium

It suffices to focus on the economy without taxation; the introduction of taxes leads to minor modifcations and can be analyzed analogously. Differentiating the relations in (A.25) we obtain

$$\left.\frac{\partial \psi}{\partial l}\right|_{RR} = -\frac{\alpha\Omega(l)}{1-l}\left(\frac{(1-\alpha)}{1-\gamma} - \gamma\Omega(l)\sigma^2\right) < 0\,, \tag{A.26a}$$

$$\left.\frac{\partial \psi}{\partial l}\right|_{PP} = -\frac{\alpha\Omega(l)}{\eta(1-l)}\left(1+\eta+(1-\alpha)\frac{l}{1-l}\right) < 0\,, \tag{A.26b}$$

so that both schedules have a negative slope. Using the fact that $\Omega = A(1-l)^{\alpha}$, and under the assumption that $\alpha < 1/2$, both the (*PP*) and (*RR*) schedules can be shown to be strictly concave (see Turnovsky, 2000b, for more details). Also

$$\begin{aligned} &\psi_{RR}(l=0) = \frac{A(1-\alpha)-\beta}{1-\gamma} - \frac{\gamma}{2}A^2\sigma^2, \qquad &&\psi_{RR}(l=1) = -\frac{\beta}{1-\gamma} \\ &\psi_{PP}(l=0) = A, &&\psi_{PP}(l=1) = -\infty. \end{aligned}$$

A necessary and sufficient condition for the existence of a unique equilibrium is $\psi_{PP}(l=0) > \psi_{RR}(l=0)$. In this case, the (*PP*) schedule is below (*RR*) for $l=1$, and the two schedules only cross once. This condition is satisfied when

$$\alpha - \gamma + \frac{\beta}{A} > -\gamma\frac{1-\gamma}{2}A\sigma^2\,, \tag{A.27}$$

i.e. when risk is not excessively high. When equation (A.27) is not satisfied either an equilibrium does not exists or there are two.

Note also that the *PP* schedule is steeper than *RR* if and only if

$$\frac{1+\eta}{\eta} + \frac{1-\alpha}{\eta}\frac{l}{1-l} > \frac{1-\alpha}{1-\gamma} - \gamma\Omega\sigma^2\,. \tag{A.28}$$

Since at $l=0$, $l/(1-l)$ has its lowest and $\Omega(l)$ its highest possible value, *PP* is everywhere steeper than *RR* if and only if

$$\frac{1+\eta}{\eta}-\frac{1-\alpha}{1-\gamma}>-\gamma A\sigma^2 . \tag{A.29}$$

A.4 The centrally planned economy

The social planner's problem (23), leads to the following Bellman equation

$$\frac{1}{\gamma}\left(C_i l_i^{\eta}\right)^{\gamma}-\beta X(K_i,K)+\frac{E(\Omega K_i-C_i)}{dt}X_{K_i}+\frac{1}{2}\frac{E(dK_i)^2}{dt}X_{K_iK_i}=0 . \tag{A.6'}$$

Taking the partial derivative of this equation with respect to K_i then yields

$$\begin{aligned}&\frac{1}{C_i}\left(C_i l_i^{\eta}\right)^{\gamma}C_{i,K_i}-\beta X_{K_i}+\frac{E(dK_i)}{dt}X_{K_iK_i}+(\Omega-C_{i,K_i})X_{K_i}+K_iX_{K_iK_i}\sigma_K^2\\&+\frac{1}{2}\frac{E(dK_i)^2}{dt}X_{K_iK_iK_i}=0\end{aligned} \tag{A.7'}$$

and hence

$$\frac{E(dX_{K_i})}{X_{K_i}dt}=-(\Omega-\beta)+(\gamma-1)\sigma_K^2 , \tag{A.13'}$$

which together with (A.14) and (A.15) above yield (16a').

The first order conditions with respect to consumption and leisure, (16), together imply

$$\frac{C_i}{K_i}=\frac{wl_i}{\eta} , \tag{A.19'}$$

Goods market equilibrium is again given by equation (A.24). Using (A.19'), the equilibrium conditions can be expressed as

$$\psi=\frac{\Omega(l)-\beta}{1-\gamma}-\frac{\gamma}{2}\Omega(l)^2\sigma^2 , \tag{A.30}$$

$$\psi=\Omega(l)-\frac{C}{K}=\Omega(l)\left[1-\frac{\alpha}{\eta}\frac{l}{1-l}\right]. \tag{A.31}$$

(A.36) is strictly decreasing and concave in *l*. Differentiating (A.30), we obtain

$$\left.\frac{\partial\psi}{\partial l}\right|_{R'R'}=-\frac{\Omega(l)}{1-l}\left[\frac{1}{1-\gamma}-\gamma\Omega(l)\sigma^2\right]<0\,,\left.\frac{\partial^2\psi}{\partial l^2}\right|_{RR''}=-\frac{\alpha\Omega(l)}{(1-l)^2}\left[\frac{1-\alpha}{1-\gamma}-\gamma\alpha\Omega(l)(1-2\alpha)\sigma^2\right].$$

(A.30) is thus decreasing in l and a sufficient condition for concavity is $\alpha<1/2$. The necessary and sufficient condition for the existence of a unique equilibrium, $\psi_{PP}(l=0)>\psi_{RR''}(l=0)$, is now

$$-\gamma+\frac{\beta}{A}>-\gamma\frac{1-\gamma}{2}A\sigma^2\,. \tag{A.27''}$$

Note also that (A.31) schedule is steeper than (A.30) if and only if

$$\frac{1+\eta}{\eta}-\frac{1}{1-\gamma}>-\gamma A\sigma^2\,. \tag{A.29'}$$

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