

**Second-Best Optimal Taxation of Capital and Labor  
in a Developing Economy \***

Cecilia García Peñalosa  
CNRS and GREQAM

Stephen J. Turnovsky  
University of Washington

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**Abstract:**

As commercial integration reduces the reliance on foreign trade taxation, raising tax revenue has become a major concern for the governments of developing economies. This paper examines how the tax burden in a developing economy should be distributed between capital income and labor income. We study a two-sector model, where the traditional sector is “informal” and consequently cannot be taxed by the government. In this setup, we find that the optimal (second-best) tax structure in order to raise a certain amount of revenue requires to tax capital income at least as much as labor income, and possibly more.

**Key words:** endogenous growth, optimal taxation, informal sector, developing economies.

**JEL Classification:** E62, O17, O23.

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## 1. Introduction

Raising tax revenue is an important concern for the governments of developing economies. Not only are tax revenues small, but the structures of the tax systems differ substantially from we observe in industrial countries. For developing countries, indirect taxation is the main source of government revenue, representing in some cases up to 80% of total tax receipts, while personal and corporate taxes never account for more than 25%. By contrast, in OECD economies, personal and corporate income taxation provides over 40% of tax revenues, while indirect taxation is only 27%; see Tanzi (1987) and Mesere and Owens (1989). As developing countries grow, they need to generate larger tax revenues to finance the enhanced public services concomitant with a developed economy. Since indirect taxes are already at a high level, comparable to that in industrial countries, increasing tax revenue will require higher personal income tax rates, thus raising the question of the form that this increase in taxation should take.<sup>1</sup> This paper examines how the tax burden in a developing economy should be distributed between capital income and labor income.

An extensive literature on the optimal taxation of factor incomes in a dynamic setting has evolved. The main message to emerge from this is that in the long run, capital income should not be taxed, thus shifting the burden from factor income taxation toward labor; see Chamley, 1985, 1986; Judd, 1985, 1999, and Lucas, 1990. Indeed, in many developing countries interest income, if taxed at all, is taxed at a rate below the tax rate on labor income.<sup>2</sup> The standard optimal taxation result would imply that this is an efficient tax structure, although, being strongly regressive, it may not be desirable once equity considerations are taken into account.<sup>3</sup> In this paper we show that in contrast to the conventional view, taxing labor income more heavily than capital income may also be inefficient from a growth and welfare standpoint.

We study a two-sector economy with a modern and a traditional sector, in which agents

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<sup>1</sup> The importance of increasing direct taxation has been stressed by a number of studies on tax reform in developing countries, such as Ahmad and Stern (1991). This is particularly important since not only do indirect taxes account for a large fraction of total revenue, but also value added tax *rates* in developing countries are already at a level comparable to those in industrial countries; see Tait (1988).

<sup>2</sup> See Tanzi and Zee (2001).

<sup>3</sup> Jimenez (1986) documents that in most developing countries the tax system is highly regressive.

allocate their endowment of time and capital between the two sectors. Both sectors use private capital and labor, with the modern sector having a more capital-intensive technology. In addition, the aggregate capital stock provides an externality that is consistent with an equilibrium of ongoing growth, as in Romer (1986). Consumers are infinitely-lived and identical in all respects except for their initial endowment of capital. We derive a macroeconomic equilibrium in which the economy's growth rate, the sectoral allocation of resources and thus the relative size of the two sectors, and the distribution of income, all become jointly determined.

It is often argued that the production structure of the economy, and in particular the degree to which certain activities are commercialized as opposed to black-market or subsistence-oriented, is a major determinant of the capacity of governments to raise tax revenue. To capture this feature of developing economies we simply assume that all traditional sector activities are informal, and consequently non-taxable by the government. Depending on the country, estimates of the proportion of the male non-agricultural labor force that work in the informal sector range between 15 and 90%, and while the average for the OECD is 17%, it rises to 60% for less developed economies.<sup>4</sup> These figures indicate the importance of the black market economy in developing countries and hence of the fiscal constraints that it imposes on their governments.

A number of authors, such as Todaro (1989), have emphasized the “buffer function” of the informal sector, which absorbs the hours of work that individuals choose not to spend in the formal production sector. Hence we model labor supply decisions not as a tradeoff between work and leisure, but as the allocation of a fixed amount of time between formal sector employment and an informal productive activity. Empirical evidence on the elasticity of labor supply in developing countries is scarce due to the difficulty of having data on formal plus informal hours of work. But once the amount of time the individual devotes to informal/domestic production is taken into account in calculating total hours worked, existing evidence seems to support the hypothesis of a total fixed labor supply; see Skoufias (1996).

We assume that the only feasible fiscal instruments are proportional taxes on the capital and labor incomes generated in the formal sector. We also assume that the government fixes the amount

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<sup>4</sup> See Thomas (1992), United Nations (2000), Ihring and Moe (2000).

of revenue that it wants to raise, and obtain the (second-best) optimal tax structures under two possible scenarios. First, we suppose that the government redistributes the revenue raised from the capital-rich to the capital-poor, so that all revenue is rebated to consumers in lump-sum transfers. Our results are striking. On the one hand, we find that to maximize the growth rate, subject to the fixed revenue objective, requires capital and labor incomes to be taxed at the same rate. To understand this result note that both taxes are distortionary, as they shift capital and labor toward the informal sector. In fact, they generate two types of distortions: they affect both the allocation of factors *across* sectors and *within* sectors. Equalizing the tax rates eliminates the distortion within sectors. The capital-labor ratio in the formal sector adjusts to offset exactly the tax distortion, so that factor prices are those that would prevail in the absence of taxes, and growth is maximized. On the other hand, under the sectoral capital intensity assumption being made, maximizing welfare requires the capital income tax to *exceed* the tax on labor income. In addition to the above growth effect, taxes also have a level effect on welfare since too little capital and labor are employed in the formal sector, thus reducing the aggregate level of output. Now consider any given tax rate,  $\tau$ . The distortion arising from taxing capital income at rate  $\tau$  is equivalent to that of taxing labor income at the same rate. However, since the formal sector is capital-intensive, the capital income tax raises more revenue than does the labor income tax. It is therefore optimal to tax capital income more heavily in order to raise a given amount.

As an alternative scenario, we consider the case in which the government purchases some of the final good in order to provide the infrastructure required to operate the formal sector technology. The idea that the use of a modern technology requires the provision of public infrastructure – and consequently the raising of taxes – has been suggested to explain the existence of a large, low-productivity, informal sector in developing countries.<sup>5</sup> We use a simple version of this setup to highlight the differences between the previous case, in which public expenditure does not involve the purchase of goods, and the case in which it does. Public provision of infrastructure changes the aggregate budget constraint, and this has important implications. If the only use of government

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<sup>5</sup> See Dessy and Pallage (2002). See also Fortin, Marceau, and Savard (1997) for alternative explanations of the existence of an informal sector.

revenue is the provision of infrastructure, then taxing both capital and labor incomes at a rate equal to the infrastructure requirement equalizes the (static) private and social rates of return. In this case the level of output is optimal, implying that both growth and welfare are maximized when the tax rates on capital and labor income are the same and equal to the infrastructure requirement. But if some of the revenue is used for transfers, the resulting divergence between private and social rates of return would generate a level effect, and requires taxing capital income more heavily than labor income in order to maximize welfare.

A number of recent works have examined the circumstances under which optimal factor taxation may involve a non-zero tax rate on capital income. This literature has focused on two issues. First, the desirability of taxing capital can stem from suboptimal capital accumulation in a growing economy, caused by a technological externality associated with capital accumulation as in Turnovsky (1996), or by excessive savings when agents are credit rationed, as in Chamley (2001). The second reason is associated with restrictions on the taxation of factors. Correia (1996) shows that when a production factor is non-optimally taxed, a positive or negative tax on capital will be required, depending on whether the untaxed factors are complements or substitutes to capital. Jones, Manuelli, and Rossi (1997) find that the impossibility for the government to tax human capital and workers' time separately implies that the tax rates on both capital and labor incomes should be positive. Turnovsky (2000a) examines a setup with an elastic supply of labor and productive government expenditure. He shows that, if all other fiscal instruments are optimally chosen, the tax rate on capital income should be zero. But if government expenditures are not set optimally, then positive capital income taxation may be required. Cremer, Pestieau, and Rochet (2003) develop an overlapping-generations model with altruistic individuals. Under the assumption that inherited wealth cannot be taxed, it is optimal to tax or subsidize capital, in order to indirectly affect inherited wealth. All these papers can be seen as examples of the argument in Judd (1999) that it is the presence of constraints (for the government or the individual) or suboptimal expenditure choices that makes capital income taxation desirable. Hence, they are second-best results.

Our contribution to this literature is threefold. First, we explore an alternative scenario in which capital income taxation is desirable, one that is particularly relevant for developing countries,

namely the impossibility to tax a *sector* rather than a factor. Second, we illustrate that the use of tax revenue is crucial in determining the structure of taxes. Most of the literature assumes that all revenue is rebated in lump-sum transfers to consumers; see, for example, Chamley (2001). Our analysis shows that optimal tax rates depend on whether it is consumers or the government that spend the revenue, implying that the assumption of how the revenue is used is not innocuous. Third, although previous work has found that a non-zero tax rate on capital income may be desirable, in almost all cases the optimal rate remained well below that for labor income.<sup>6</sup> Our analysis provides a rationale for taxing capital income at least as much as, and possibly more heavily than, labor income.

The paper is structured as follows. Section 2 briefly sets out a basic one-sector model to serve as a benchmark against which we may compare our results. Section 3 then describes the two-sector economy, with the optimal tax structure being derived in Section 4. Section 5 supplements the analytical results with some numerical simulations, while Section 6 concludes, noting some caveats. Technical details are minimized throughout the text and relegated to an Appendix.

## 2. The Basic One-Sector Model

We begin by considering a conventional one-sector economy with a representative agent who we assume supplies a unit of labor inelastically. We shall derive the first-best and second-best optimal tax structures, which we will then compare to those of the two-sector economy.

### 2.1 Technology and Returns

There is a mass 1 of firms, indexed by  $j$ . A representative firm produces output according to

$$Y_j = F(AL_j, K_j),$$

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<sup>6</sup> One exception is Fuest and Huber (2001), who using a static model, find that for some agents the optimal marginal tax rate on capital income is higher than that on labor income. Another is Koskela and Schöb (2002) who study optimal factor taxation in the presence of unemployment, resulting from union-firm bargaining when capital is internationally mobile but labor is immobile. Assuming that the government in setting taxes behaves as a Stackelberg leader toward the private sector, they also find that in the presence of unemployment capital should generally be taxed at a higher rate than labor. To the extent that unemployment is important in developing economies, these results are particularly relevant in the present context.

where  $K_j$  denotes the individual firm's stock of capital,  $AL_j$  are the efficiency units of labor employed by the firm, and  $F(\cdot)$  is assumed to have constant returns to capital and labor. All firms are identical and hence in equilibrium they will all choose the same level of employment and capital stock. That is,  $K_j = K$  and  $L_j = L$  for all  $j$ .

We further assume that there is an externality associated with the stock of capital, so that the efficiency of labor depends on the average stock of capital in the economy,  $K$ . In particular,  $A = K$ , such that aggregate output  $Y$  is linear in the stock of capital. That is,

$$Y = F(LK, K) \equiv Kf(L). \quad (1')$$

There is perfect competition in factor markets, so that wages and rates of return on capital are determined by the usual marginal productivity conditions,

$$r = \frac{\partial F}{\partial K_j} = f(L) - Lf'(L) \quad w = \frac{\partial F}{\partial L_j} = f'(L)K. \quad (2)$$

Since the labor supply,  $L$ , is constant, the interest rate,  $r$ , and factor shares,  $s_w \equiv f'(L)L / f(L)$  and  $s_K \equiv 1 - f'(L)L / f(L)$ , are all constant, while the wage rate grows at the same rate as does capital.

## 2.2 Consumer Optimization

The representative agent maximizes lifetime utility, taken to depend upon consumption,  $C(t)$ , as represented by the isoelastic utility function

$$\int_0^{\infty} \frac{1}{\gamma} C^\gamma e^{-\rho t} dt \quad -\infty < \gamma < 1 \quad (3a)$$

subject to the flow budget constraint

$$\dot{K} = r(1 - \tau_K)K + w(1 - \tau_w)L - C, \quad (3b)$$

where  $\tau_K$  and  $\tau_w$  are, respectively, the capital income and wage income taxes. The solution to this problem yields the equilibrium growth rate,  $\psi$ , together with the consumption-capital ratio  $c \equiv C / K$

$$\psi = \frac{(1 - \tau_K)(f(L) - Lf'(L)) - \rho}{1 - \gamma} \quad (4a)$$

$$c = f(L) - \psi \quad (4b)$$

Substituting (4a) and (4b) into (3a) the welfare of the representative individual along the equilibrium path can be expressed as

$$W = \int_0^{\infty} \frac{1}{\gamma} C^\gamma e^{-\rho t} dt = \frac{1}{\gamma} \frac{c^\gamma}{\rho - \gamma\psi} K_0^\gamma, \quad (5)$$

where  $\rho > \gamma\psi$  by the transversality condition, and  $\gamma W > 0$ .<sup>7</sup>

### 2.3. First- and Second-Best Optimal Taxation

It is well-known (see, for example, Romer, 1986) that because of the presence of the externality associated with capital, the competitive economy will not yield the socially optimal rate of growth. The socially optimal equilibrium takes into account the effect of the capital externality on the productivity of labor, which leads to the equilibrium growth rate

$$\psi^* = \frac{f(L) - \rho}{1 - \gamma}, \quad (4a')$$

and the corresponding consumption-capital ratio still given by (4b). Comparing (4a') to (4a), we see that the socially optimal growth rate can be achieved in the competitive economy by subsidizing the return to capital at the rate capital  $\tau_K^* = -s_W / (1 - s_W)$ . In the absence of any lump-sum taxation, the government's tax choices are restricted by its budget constraint  $\tau_W wL + \tau_K rK = 0$ . Substituting for  $\tau_K^*$  and the factor returns,  $w, r$ , this implies  $\tau_W^* = 1$ , so that the subsidy to capital must be financed by fully taxing labor income.

Suppose now that at each point in time the policy maker wants to raise a fixed fraction of output,  $\theta$ , as revenue using the capital income and labor income taxes only. We are not concerned with how this revenue is spent. The government budget constraint is then  $\theta Y = \tau_W wL + \tau_K rK$ , which

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<sup>7</sup>The transversality condition is  $\lim_{t \rightarrow \infty} \lambda K e^{-\rho t} = 0$ , where  $\lambda$  denotes the shadow value of capital.



can be expressed as  $\tau_w s_w + \tau_k s_k = \theta$ . We can now determine the second-best tax rates that would maximize the growth rate and welfare, given the target government revenue,  $\theta$ . Differentiating the expression (4a) for  $\psi$ , and since  $L$ , and therefore the return to capital, is constant, we obtain

$$\frac{\partial \psi}{\partial \tau_k} = -\frac{f(L)s_k}{1-\gamma} < 0 \quad .$$

The growth rate is maximized by setting the lowest possible capital income tax, that is, by setting the highest possible wage income tax. Since  $\tau_w$  is bounded above, the optimal policy is to set

$$\hat{\tau}_w = 1 \quad \text{and} \quad \hat{\tau}_k = \frac{\theta - s_w}{1 - s_w}, \quad (6)$$

which implies

$$\hat{\psi} = \frac{f(L)(1-\theta) - \rho}{1-\gamma}.$$

Note that  $\hat{\tau}_k$  can be positive or negative (i.e. a tax or a subsidy), depending on the size of the required government revenue. However, even when capital income is subsidized, the first-best growth rate cannot be obtained as long as  $\theta > 0$ .

Consider now the welfare-maximizing tax policy. Differentiating (5) we can show that

$$\frac{dW/d\tau_k}{\gamma W} = -f(L)^2(1-s_w) \frac{s_w + (1-s_w)\tau_k}{c(1-\gamma)(\rho - \gamma\psi)}.$$

Welfare is then maximized when capital is subsidized at the first-best tax rate  $\tau_k^* = -s_w/(1-s_w)$ . However, this implies a wage tax of  $\tau_w = (s_w + \theta)/s_w$ , which exceeds 1 and hence is infeasible. The second-best policy is then to set the wage tax as high as possible, i.e. to chose  $\hat{\tau}_w$  and  $\hat{\tau}_k$  in accordance with (6). Such taxes will simultaneously maximize growth and welfare. These results can be summarized in the following proposition.

**Proposition 1: A.** The first best optimum in the one-sector economy can be replicated by subsidizing capital at the rate:  $\hat{\tau}_k = -s_w/(1-s_w)$ , financed by fully taxing labor income. **B.** Consider the second-best optimum, where the objective is to

raise a fraction,  $\theta$ , of output from tax revenue. Fully taxing labor at the maximal rate  $\hat{\tau}_w = 1$  and capital at the rate  $\hat{\tau}_k = (\theta - s_w)/(1 - s_w)$  will maximize both the growth rate and welfare.

The clear message from the one-sector model is that for both objectives, the tax burden should be more heavily borne by the fixed factor, labor.

### 3. The Two-Sector Economy

We now modify the basic model in two dimensions. First, we assume that agents are heterogeneous and differ in their initial capital endowments, as in Bertola (1993). Second, we seek to capture an important feature of developing economies, namely the fact that much of the production takes place outside the formal sector, in a second sector, termed the informal sector. The latter, being less organized, is characterized by a lower capital intensity than is the formal sector. Also, being less structured, economic activities in the informal sector are less transparent to the government and thus can avoid all taxes.

We continue to maintain the assumption that aggregate labor is fixed, abstracting from the labor-leisure choice. While this assumption has the advantage of analytical convenience, it is not implausible for a developing economy. Given the low levels of consumption in such countries, it is unlikely that much leisure is consumed. Rather, what happens is that flexibility regarding hours of employment leads to variations in the labor supplied to the formal sector, with individuals then devoting the remaining time to informal productive activities, in the way that we model it. But given the importance of this assumption, in the concluding section we briefly discuss the modifications to our results when labor is supplied elastically, arguing how this basically reinforces our key findings.

#### 3.1. Technology and Returns

We shall denote the formal and informal sectors by 1 and 2, respectively. Output in each sector is produced by capital and labor in accordance with the production functions

$$Y_1 = F[L_1 K, K_1] \tag{7a}$$

$$Y_2 = G[L_2K, K_2] \quad (7b)$$

where  $K_1$  and  $K_2$  denote the capital stock of a representative firm in sector 1 and sector 2, respectively;  $K = K_1 + K_2$  is the economy-wide stock of capital, and  $L_1K$ ,  $L_2K$  measure the labor supply in each sector in efficiency units. We normalize the stock of labor so that  $L_1 + L_2 = 1$ .

Both production functions are assumed to exhibit constant returns to scale in the private factors, employment and the private capital stock. In addition, the aggregate stock of capital yields an externality such that in equilibrium, the production functions are linear in the accumulating stock of capital, as in Romer (1986). We further assume that the use of the formal technology requires the provision of infrastructure. The amount of infrastructure required is proportional to the level of output of that sector, so that in order for the economy to produce  $Y_1$  the government must spend an amount  $\phi Y_1$  on infrastructure, with  $\phi < 1$ .

The private returns to capital and labor are represented by their respective marginal physical products. Letting  $k_1 \equiv K_1/K$  and  $k_2 \equiv K_2/K$  denote the shares of aggregate capital employed in the formal and the informal sectors, respectively, and since

$$Y_1 = K_1 F\left(\frac{L_1K}{K_1}, 1\right) \equiv K_1 f\left(\frac{L_1}{k_1}\right) \quad \text{and} \quad Y_2 = K_2 G\left(\frac{L_2K}{K_2}, 1\right) \equiv K_2 g\left(\frac{L_2}{k_2}\right)$$

we can write factor payments as

$$r_1 = \frac{\partial Y_1}{\partial K_1} = f\left(\frac{L_1}{k_1}\right) - \frac{L_1}{k_1} f'\left(\frac{L_1}{k_1}\right); \quad w_1 = \frac{\partial Y_1}{\partial L_1} = K f'\left(\frac{L_1}{k_1}\right) \quad (8a)$$

$$r_2 = \frac{\partial Y_2}{\partial K_2} = g\left(\frac{L_2}{k_2}\right) - \frac{L_2}{k_2} g'\left(\frac{L_2}{k_2}\right); \quad w_2 = \frac{\partial Y_2}{\partial L_2} = K g'\left(\frac{L_2}{k_2}\right) \quad (8b)$$

### 3.2. Government Policy

The government is assumed to tax income from capital and labor in the formal sector, at rates  $\tau_K$  and  $\tau_W$ , respectively. There are two types of government expenditure. First, the government must finance the infrastructure requirement of the formal sector,  $\phi Y_1$ . Second, the government is assumed to be concerned about the distribution of income. An amount  $T$  is hence rebated as a lump-

sum transfer to all agents, and the policy-maker fixes the fraction of formal-sector output that is to be spent on transfers, so that  $T = \theta Y_1$  where  $\theta$  is given. The government budget constraint is then

$$\tau_w w_1 L_1 + \tau_k r_1 K_1 = (\theta + \phi) Y_1, \quad (9)$$

and we will term  $\theta$  the “transfer rate” and  $\phi$  the “infrastructure requirement.”<sup>8</sup>

### 3.3. Consumer Optimization

There is a mass 1 of infinitely-lived agents in the economy. Consumers are indexed by  $i$  and are identical in all respects except for their initial stock of capital,  $K_{i0}$ . Since the economy grows, we will be interested in the share of individual  $i$  in the total stock of capital,  $k_i$ , defined as  $k_i \equiv K_i / K$ , where  $K$  is the aggregate (or average) stock. Aggregating over the individual capital stocks,  $\sum_i k_i = 1$ , that is, the distribution of relative capital endowments has mean 1. In addition, we assume that the variability of the endowments across agents is given by the standard deviation,  $\sigma_k$  and the range is  $k \in [0, \bar{k}]$ .

All agents supply a unit of labor inelastically. A fraction,  $L_{1i}$ , may be allocated to employment in the formal sector, with the remainder,  $L_{2i}$ , being spent in the informal sector, such that  $L_1 = \sum_i L_{1i}$ ,  $L_2 = \sum_i L_{2i}$  and  $L_{1i} + L_{2i} = 1$ . Similarly, his total stock of capital,  $K_i$ , is allocated between the two sectors. His objective is to select his portfolio of assets, allocation of labor time, and the rate of consumption to maximize lifetime utility, taken to depend upon consumption,  $C_i(t)$ , and represented by an isoelastic utility function. Formally, the problem is

$$\max \int_0^{\infty} \frac{1}{\gamma} C_i^\gamma e^{-\rho t} dt, \quad \text{with } -\infty < \gamma < 1 \quad (10)$$

subject to

$$\dot{K}_i = r_1(1 - \tau_k)K_{1i} + w_1(1 - \tau_w)L_{1i} + r_2K_{2i} + w_2L_{2i} + T - C_i \quad (11a)$$

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<sup>8</sup> Other types of public spending that involve the purchase of goods by the government – e.g. if formal sector production depended on the quality/quantity of infrastructure that the government provides, or if a public consumption good entered the consumers’ utility function- yield equivalent results. Such formulations, however, complicate the analysis as the optimal amount of government expenditure is to be endogenously determined.

$$L_{1i} + L_{2i} = 1 \quad (11b)$$

$$K_{1i} + K_{2i} = K_i \quad (11c)$$

The first-order conditions are

$$C_i^{\gamma-1} = \lambda \quad (12a)$$

$$\frac{\dot{\nu}_2}{\lambda} = -\frac{\dot{\lambda}}{\lambda} + \rho \quad (12b)$$

$$w_1(1 - \tau_w) = \frac{\nu_1}{\lambda} = w_2 \quad (12c)$$

$$r_1(1 - \tau_k) = \frac{\nu_2}{\lambda} = r_2 \quad (12d)$$

where  $\lambda$  is the shadow value of capital, and  $\nu_1$ , and  $\nu_2$  are the multipliers associated with the labor and capital allocation constraints, respectively.

Combining equations (8) with (12c) and (12d), we obtain the static allocation conditions for capital and labor,

$$(1 - \tau_k) \left[ f\left(\frac{L_1}{k_1}\right) - \frac{L_1}{k_1} f'\left(\frac{L_1}{k_1}\right) \right] = g\left(\frac{L_2}{k_2}\right) - \frac{L_2}{k_2} g'\left(\frac{L_2}{k_2}\right) \quad (13a)$$

$$(1 - \tau_w) f'\left(\frac{L_1}{k_1}\right) = g'\left(\frac{L_2}{k_2}\right) \quad (13b)$$

The first-order conditions (12a) and (12b), together with (12d), imply the rate of growth of consumption

$$\frac{\dot{C}_i}{C_i} = \frac{(1 - \tau_k) r_1 (L_1 / k_1) - \rho}{1 - \gamma} \quad (14a)$$

Observe that the only difference between agents, namely their initial stock of capital, does not appear in this equation. Hence all individuals choose the same consumption growth rate. This has two implications. First, the *aggregate* rate of growth,  $\psi$ , is identical to the *individual* rate of growth

and unaffected by the initial distribution of endowments, that is

$$\psi = \frac{(1 - \tau_K)(f(L_1/k_1) - L_1/k_1 f'(L_1/k_1)) - \rho}{1 - \gamma}, \quad (14b)$$

Second, since the capital stock of all agents grows at the same rate, the distribution of capital endowments does not change over time. That is, at any point in time, the wealth share of agent  $i$ ,  $k_i$ , is given by his initial share  $k_{i,0}$ .

Using the government budget constraint (9) to substitute for the transfer, and aggregating over the individuals, we simply have that the flow of physical goods in the economy to consumption and investment must satisfy the resource constraint

$$\frac{\dot{K}}{K} = (1 - \phi)k_1 f\left(\frac{L_1}{k_1}\right) + k_2 g\left(\frac{L_2}{k_2}\right) - c \quad (15)$$

where  $c \equiv C/K$  is the aggregate consumption-capital ratio. Lastly, the welfare of individual  $i$  is

$$W_i = \frac{1}{\gamma} \frac{(c_i k_i)^\gamma}{\rho - \gamma \psi} K_0^\gamma. \quad (16)$$

### 3.4. Macroeconomic Equilibrium

It is straightforward to show that the unique macroeconomic equilibrium in the two-sector economy is a balanced growth path determined by the following conditions:

#### *Resource Constraints*

$$L_1 + L_2 = 1 \quad (17a)$$

$$k_1 + k_2 = 1 \quad (17b)$$

#### *Equilibrium Factor Allocations*

$$(1 - \tau_w) f'\left(\frac{L_1}{k_1}\right) = g'\left(\frac{L_2}{k_2}\right) \quad (17c)$$

$$(1-\tau_K) \left[ f\left(\frac{L_1}{k_1}\right) - \frac{L_1}{k_1} f'\left(\frac{L_1}{k_1}\right) \right] = g\left(\frac{L_2}{k_2}\right) - \frac{L_2}{k_2} g'\left(\frac{L_2}{k_2}\right) \quad (17d)$$

### *Equilibrium Growth Rate*

$$\psi = \frac{g(L_2/k_2) - L_2/k_2 g'(L_2/k_2) - \rho}{1-\gamma} \quad (17e)$$

### *Goods Market Equilibrium*

$$c = (1-\phi)k_1 f(L_1/k_1) + k_2 g(L_2/k_2) - \psi \quad (17f)$$

The first four equations are the static efficiency conditions, and together they determine the allocation of labor and capital across sectors,  $L_1, L_2, k_1, k_2$ . Once capital and labor are allocated across sectors, the equilibrium growth rate follows. The last equation then determines the consumption-capital ratio as a function of sectoral allocations and the growth rate.

The effects of changes in the two tax rates on the allocation of factors across the two sectors and on the growth rate is examined in the Appendix. There it is shown that a critical factor determining some of these effects is given by the sign of the expression

$$M \equiv \left( (1-\tau_w) \frac{L_2}{k_2} - (1-\tau_K) \frac{L_1}{k_1} \right) \quad (18)$$

The assumption we are making, namely that the formal sector is more capital intensive, implies that in equilibrium  $L_2/k_2 > L_1/k_1$ . Given this assumption, if the tax rate on capital income is at least as high as that on wages, then  $M$  is certainly positive, an assumption we shall maintain.

The following qualitative effects of changes in the two tax rates on the allocation of factors across the two sectors are obtained:

$$\begin{aligned} \frac{\partial L_1}{\partial \tau_K} = -\frac{\partial L_2}{\partial \tau_K} < 0 ; & \quad \frac{\partial k_1}{\partial \tau_K} = -\frac{\partial k_2}{\partial \tau_K} < 0 \\ \frac{\partial L_1}{\partial \tau_w} = -\frac{\partial L_2}{\partial \tau_w} < 0 ; & \quad \frac{\partial k_1}{\partial \tau_w} = -\frac{\partial k_2}{\partial \tau_w} < 0 \end{aligned} \quad (19a)$$

As expected, since only the formal sector is taxed, an increase in either of the two tax rates shifts

capital and labor away from the formal sector. Consider the effect of an increase in  $\tau_w$ . The higher tax rate implies that the net wage in the formal sector is lower than the wage in the informal sector; hence labor moves to the latter. This tends to reduce the wage in the informal sector and increase that in the formal one. At the same time, the increase in  $L_2$  raises the marginal product of capital in sector 2 and reduces that in the formal sector. Capital must therefore flow from the formal to the informal sector in order to compensate this discrepancy and equalize again the (net) rates of return.

Moreover, for  $M > 0$ , we have

$$\begin{aligned} \frac{\partial(L_1/k_1)}{\partial\tau_K} < 0; & \quad \frac{\partial(L_2/k_2)}{\partial\tau_K} < 0 \\ \frac{\partial(L_1/k_1)}{\partial\tau_w} < 0; & \quad \frac{\partial(L_2/k_2)}{\partial\tau_w} < 0 \end{aligned} \tag{19b}$$

An increase in either of the two taxes shifts both capital and labor from the formal sector into the informal one. Under the assumption that the formal sector is more capital intensive than the informal one, the change in factor allocations also results in an increase in the capital-labor ratio in both sectors. The reason for this is that as resources move away from the capital-intensive sector and to the labor-intensive sector, the capital-labor ratios in the two sectors must rise to maintain full employment of capital and labor.

It is important to note that the above allocative effects hold even if no capital is employed in the informal sector. In this case, the entire stock of capital is used by the formal sector, and the only static decision is the allocation of labor between the two sectors. An increase in either tax rate would shift labor to the informal sector, thus increasing the capital output ratio in the formal sector.<sup>9</sup>

Consider now the effects of taxes on the growth rate. Define

$$\alpha \equiv \alpha(L_1/k_1) = 1 - \frac{L_1 f'(L_1/k_1)}{k_1 f(L_1/k_1)},$$

so that  $\alpha$  and  $(1-\alpha)$  are, respectively, the capital and labor shares in the formal sector. The effects of changes in the two tax rates on the growth rate are then given by

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<sup>9</sup> As will be clear below, all our results hold in this case. For an example of an economy where the informal sector uses no capital, see García Peñalosa and Turnovsky (2004).



$$\frac{\partial \psi}{\partial \tau_K} = -\frac{L_2}{k_2} \frac{\alpha(1-\tau_W)f}{(1-\gamma)M} < 0 \quad (20a)$$

$$\frac{\partial \psi}{\partial \tau_W} = -\frac{L_2}{k_2} \frac{(1-\alpha)(1-\tau_K)f}{(1-\gamma)M} < 0 \quad (20b)$$

implying that raising either tax reduces the growth rate. That increasing the tax on capital income reduces growth is standard, since output growth is driven by the accumulation of capital. In our setup, taxing labor income also dampens growth because of the indirect impact that  $\tau_W$  has on the return to capital. Taxing wages results in a higher capital-labor ratio in both sectors; consequently the (gross) rate of return to capital, and the output growth rate, fall. This effect is also present when there is an increase in the capital income tax. There is hence a reduction in the gross interest rate as well as in the net interest rate, implying that the reduction in the growth rate is greater than if the government were unconstrained in its capacity to tax. To see this, we rewrite equation (20a) as

$$\frac{\partial \psi}{\partial \tau_K} = -\frac{\alpha f}{1-\gamma} \left( 1 + \frac{L_1}{k_1} \frac{1-\tau_K}{M} \right) < 0.$$

The first-term captures the standard effect of capital income taxes obtained in the benchmark model of section 2. The second term in brackets is new, and represents the reduction in growth due to the fall in the (gross) return to capital caused by a misallocation of factors across sectors.

### 3.5. The Distribution of Income

The heterogeneity of agents raises the consequences of tax policy for the distribution of income. To consider this, we note that the income of agent  $i$  is given by

$$Y(K_i) = r_1(1-\tau_K)K_{1i} + w_1(1-\tau_W)L_{1i} + r_2K_{2i} + w_2L_{2i} + T$$

With all individuals being identical, except for their initial endowments of capital, they will in fact allocate factors in the same proportions across sectors, so that  $K_{1i}/K_i = K_1/K = k_1$ ;  $L_{1i} = L_1$  (and analogously for sector 2).<sup>10</sup> Using equations (9) to substitute for the transfer, we can then write

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<sup>10</sup>We wish to clarify the following point regarding notation. For the most part we shall be dealing with aggregate quantities and shall let  $k_1, k_2$  shall refer to the shares of *aggregate* capital employed in sectors 1, 2, respectively. In our

$$Y(K_i) = r_1 k_1 K_i + w_1 L_1 + r_2 k_2 K_i + w_2 L_2 + r_1 \tau_K k_1 (K - K_i) - \phi k_1 fK .$$

Two things are worth noting here. First, agents receive a net subsidy if their capital is below the average, and pay a net tax otherwise. Second, only the fraction of the transfer  $\theta$  that is financed through capital income taxation actually entails direct redistribution. Since all agents supply the same amount of labor, the revenue raised through wage taxation and rebated has no distributional impact. Nevertheless, because both taxes change factor prices, both will have an indirect effect on an agent's relative income.

In order to examine the effects of taxation on the income distribution, we consider the relative income of an individual with capital  $k_i$ ,  $y_i \equiv Y(K_i)/Y(K)$ , which can be expressed as<sup>11</sup>

$$y_i = 1 + \omega(k_i - 1) \quad \text{where} \quad \omega \equiv \frac{r_2}{(1 - \phi)fk_1 + gk_2} \quad (21)$$

Equation (21) emphasizes that the distribution of income depends upon *two* factors, the initial (unchanging) distribution of capital, and the equilibrium allocation of factors across sectors, insofar as this determines factor rewards. Under plausible conditions  $\omega < 1$ , and the variability of income across the agents,  $\sigma_y$ , is less than their (unchanging) variability of capital,  $\sigma_k$ .<sup>12</sup> In fact,

$$\sigma_y = \omega \sigma_k . \quad (22)$$

#### 4. First- and Second-Best Optimal Taxation

We now consider the implications of the informal sector for optimal tax policy, considering the first-best and second-best policies in turn.

##### 4.1. The First-Best Optimum

As a benchmark, we suppose that the social planner has no redistributive goals, i.e.  $\theta = 0$ ,

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brief discussion of individual agents we shall denote the relative capital endowment of individual  $i$  by  $k_i$ . The intended meaning should be clear from the context.

<sup>11</sup>To derive (21) we make extensive use of the definitions of factor returns given in (8) and the equilibrium factor allocation conditions given in (17c,17d).

<sup>12</sup>A simple sufficient condition to ensure that  $\omega < 1$  is that  $\tau_K > \phi$ . In our simulations we find  $\omega$  to be 0.23 and 0.32.

and simply maximizes the utility of the individual with average capital holdings. The social planner's decision problem is then to choose average consumption, the rate of capital accumulation, and the factor allocations to

$$\max \int_0^{\infty} \frac{1}{\gamma} C^\gamma e^{-\rho t} dt, \quad \text{with } -\infty < \gamma < 1 \quad (10)$$

subject to

$$\dot{K} = (1 - \phi)F(L_1K, K_1) + G(L_2K, K_2) - C \quad (11a')$$

$$L_1 + L_2 = 1 \quad (11b')$$

$$K_1 + K_2 = K \quad (11c')$$

The macroeconomic equilibrium derived by the central planner comprises the resource constraints, (17a) (17b), the resource allocation conditions and equilibrium growth rate given by

$$(1 - \phi)f' \left( \frac{L_1}{k_1} \right) = g' \left( \frac{L_2}{k_2} \right), \quad (17c')$$

$$(1 - \phi) \left[ f \left( \frac{L_1}{k_1} \right) - \frac{L_1}{k_1} f' \left( \frac{L_1}{k_1} \right) \right] = g \left( \frac{L_2}{k_2} \right) - \frac{L_2}{k_2} g' \left( \frac{L_2}{k_2} \right), \quad (17d')$$

$$\tilde{\psi} = \frac{(g - L_2 / k_2 g' + g') - \rho}{1 - \gamma}, \quad (17e')$$

and the goods market equilibrium condition (17f). Comparing these to the equilibrium conditions (17) for the decentralized economy we immediately see that as long as only the formal sector can be taxed it is impossible to replicate the first best optimum. The efficient sectoral allocation requires  $\tau_w = \tau_K = \phi$  (with  $\tau_w = \tau_K = 0$  if there are no infrastructure requirements), in which case the growth rate in the decentralized economy is too slow.

By contrast, if the government were unconstrained and could tax both sectors, it would be possible to replicate the first best optimum. Denoting the wage taxes in sectors 1 and 2 by  $\tau_{w,1}$  and  $\tau_{w,2}$ , and the capital income taxes by  $\tau_{K,1}$  and  $\tau_{K,2}$ , the first-best optimum can be attained by setting

$$\frac{1 - \tau_{W,1}}{1 - \tau_{W,2}} = \frac{1 - \tau_{K,1}}{1 - \tau_{K,2}} = 1 - \phi \quad (23a)$$

$$\tau_{K,1} = \phi - \frac{(1 - \phi)f'}{f - L_1/k_1 f'}, \quad (23b)$$

where the tax rates must satisfy the government budget constraint, in this case

$$\tau_{W,1} f' K L_1 + \tau_{K,1} \left( f - \frac{L_1}{k_1} f' \right) K_1 + \tau_{W,2} g' K L_2 + \tau_{K,2} \left( g - \frac{L_2}{k_2} g' \right) K_2 = \phi f K_1 \quad (24)$$

Equations (23a) require the relative tax rates in the two sectors to ensure that the relative after-tax factor returns in the two sectors are socially optimal, while the capital income tax,  $\tau_{K,1}$ , must be set in such a way as to equate the private and social return to capital. Solving these four equations we can show that the optimal taxes on labor are  $\hat{\tau}_{W,1} = \hat{\tau}_{W,2} = 1$ . In the absence of infrastructure requirements capital income in both sectors should be subsidized at the common rate  $\hat{\tau}_K = -f'/(f - L_1/k_1 f')$ . If  $\phi > 0$ , then  $\hat{\tau}_{K,1} > \hat{\tau}_{K,2}$ , which may require capital income in the formal sector to be taxed if  $\phi$  is sufficiently large. We may summarize these results with

- Proposition 2: A.** If only the formal sector can be taxed, the first-best optimum equilibrium cannot be replicated by taxing labor income and capital income alone.
- B.** The first-best optimum can be attained if both sectors can be taxed. In this case, labor income in both sectors should be fully taxed and the revenue used to subsidize capital income.

The results in the second part of the proposition, requiring the full taxation of labor income used to finance a subsidy to capital income, are thus direct extensions of the one sector model.

## 4.2. Second-Best Taxation

### *Redistributive Government Expenditure*

In order to highlight the differences between the implications of the two types of government expenditure we analyze them separately. This section obtains the optimal tax structure when there is

no infrastructure requirement in the formal sector; the next section examines the case in which there is as well an infrastructure requirement.

Suppose that  $\phi = 0$ . The government budget constraint can be expressed as

$$(1 - \alpha)\tau_w + \alpha\tau_k = \theta \quad (25)$$

where the capital and wage shares,  $\alpha$  and  $1 - \alpha$ , are a function of  $L_1 / k_1$ . We need to consider how the two tax rates vary together in order to raise the necessary revenue. Differentiating (25),

$$\frac{d\tau_k}{d\tau_w} = -\frac{1 - \alpha}{\alpha} \left[ 1 + \frac{(1 - \sigma)(\tau_k - \tau_w) \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right)}{(1 - \tau_k)\sigma \frac{L_1}{k_1} - (1 - \tau_w) \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} (1 - \sigma) \right)} \right] \quad (26)$$

where  $\sigma$  is the elasticity of substitution between capital and labor in the formal sector. The first term captures the direct effect of changing the wage tax, and it is always negative since for a given allocation of factors, a lower wage tax requires a higher capital income tax. The second is the indirect effect stemming from the fact that changes in the tax rates affect the allocation of factors across sectors and hence change the revenue raised by a particular tax. The sign of this effect depends upon which tax is larger (i.e. which tax is already creating a larger distortion) and on whether capital and labor are substitutes or complements in the formal sector.

We can now consider the second-best policies of maximizing the growth rate and welfare. Writing  $\psi = \psi(\tau_w, \tau_k)$ , the effect of the constrained increase in the wage tax on the growth rate is:

$$\frac{d\psi}{d\tau_w} = \frac{\partial\psi}{\partial\tau_w} + \frac{\partial\psi}{\partial\tau_k} \frac{d\tau_k}{d\tau_w} \quad (27)$$

where the components appearing in (27) are obtained from (20a), (20b), and (26), respectively.

Substituting for these expressions, we obtain

$$\frac{d\psi}{d\tau_w} = \frac{(1 - \alpha)\sigma f L_2}{1 - \gamma} \frac{\tau_k - \tau_w}{k_2 M + (\tau_w - \tau_k)L_1 / k_1 (1 - \sigma)} \quad (27')$$

which implies that growth is maximized when  $\tau_K = \tau_W = \theta$ .<sup>13</sup>

The intuition for this result can be seen clearly from (27), where we see that the direct effect of an increase in the wage tax is to reduce the growth rate. At the same time, to the extent that the higher wage tax permits a lower capital income tax, this is offset by an indirect positive effect on the growth rate. If the two tax rates happen to be equal,  $\tau_K = \tau_W$ , then these two effects are exactly offsetting and the growth rate remains unaffected by the substitution. If  $\tau_W > \tau_K$  the direct negative effect of a higher wage tax on the growth rate more than exceeds the increase due to an associated reduction in the tax on capital income, implying that the growth rate can be increased by reducing  $\tau_W$  and correspondingly increasing  $\tau_K$ . The reverse holds if  $\tau_K > \tau_W$ .

We now consider the policy that maximizes the welfare of the individual holding the average capital stock. We could follow the approach used in much of the optimal taxation literature and assume a utilitarian welfare function. Since individual welfare is concave in consumption, and hence in individual capital, such a social welfare function would imply that some redistribution would be optimal. This would require positive capital income taxation. By focusing on average welfare, we abstract from such a reason for taxing capital, and consider the optimal choice of tax rates for a given revenue requirement.

The welfare of the individual with an average endowment can be expressed as  $W = c^\gamma / (\gamma(\rho - \gamma\psi))K_0^\gamma$ , in which case

$$\frac{dW/d\tau_W}{\gamma W} = \frac{dc/d\tau_W}{c} + \frac{d\psi/d\tau_W}{\rho - \gamma\psi} = \frac{1}{c} \frac{dz}{d\tau_W} + \frac{c + \gamma\psi - \rho}{c(\rho - \gamma\psi)} \frac{d\psi}{d\tau_W} \quad (28)$$

where we have used the goods market equilibrium condition,  $c = k_1 f(L_1/k_1) + k_2 g(L_2/k_2) - \psi$ , to substitute for consumption, and defined  $z$  as

$$z \equiv k_1 f(L_1/k_1) + k_2 g(L_2/k_2). \quad (29)$$

Equation (28) does not yield a tractable solution for the optimal tax rates. Note, however,

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<sup>13</sup> Taking the second derivative of (27) and evaluating it at the point  $\tau_W = \tau_K$  one can verify that for  $M > 0$   $d^2\psi/d\tau_W^2|_{\tau_K=\tau_W} < 0$ . Using (26), it is straightforward to show that this result holds when no capital is used in the informal sector so that  $k_1 = 1, k_2 = 0$ .

that when the two tax rates are equal, growth is maximized, and the second term in (28) disappears. Moreover, in the Appendix we show that for  $\tau_K = \tau_W$

$$\text{sgn}\left(\frac{dW/d\tau_W}{\gamma W}\right)_{\tau_W=\tau_K} = \text{sgn}\left(\frac{L_1}{k_1} - \frac{L_2}{k_2}\right). \quad (30)$$

As long as the capital intensities in the two sectors differ, (28) and (30) together imply that a homogeneous tax rate,  $\tau = \tau_K = \tau_W = \theta$ , will not maximize welfare. In the more plausible case where the formal sector is more capital intensive,  $L_1/k_1 < L_2/k_2$ , implying that  $dW/d\tau_W|_{\tau_W=\tau_K} < 0$ . Maximizing welfare then requires setting the wage tax below  $\theta$  and the capital income tax above  $\theta$ , that is,  $\tau_W < \theta < \tau_K$ . If it were the informal sector that had a greater capital intensity, then maximizing welfare would require the relative tax rates to be set in accordance with  $\tau_K < \theta < \tau_W$ , although as we note in Section 4.3 below the solution will in fact be a corner solution.

The intuition for these results is easily established. In the absence of policy constraints, that is, if both sectors could be taxed, it would be optimal to raise all revenue through a labor income tax, just as in the basic one sector model. Such a tax would not discourage capital accumulation, and with labor incomes in both sectors taxed at the same rate, it would not distort the allocation of labor across sectors. The inability to tax the informal sector means that imposing a wage tax on the formal sector reduces the labor supplied to that sector and consequently the marginal product of capital, and capital accumulation. Taxing only labor is hence no longer optimal.

Taxing the informal sector only has two distortionary effects. On the one hand, it distorts the allocation of factors *across* sectors; on the other it changes factor intensities *within* each sector. Setting  $\tau_K = \tau_W = \tau$  implies (see Appendix)

$$\frac{d(L_2/k_2)}{d\tau_W}\bigg|_{\tau_W=\tau_K} = 0, \quad \frac{d(L_1/k_1)}{d\tau_W}\bigg|_{\tau_W=\tau_K} = \frac{1}{1-\tau} \frac{f'}{f''}.$$

Clearly, one of the distortions is eliminated. Factor intensities in the informal sector are the same as they would be in the absence of taxes, while the capital-labor ratio in the formal sector increases so as to exactly offset the direct effect of the tax on factor returns. Net factor prices are hence unchanged by the introduction of taxes, and growth is maximized.

But equalizing the two tax rates will not maximize welfare. As well as a growth effect, taxes have a level effect on welfare, as they shift capital and labor toward the informal sector, and hence reduce aggregate output. The impact of the taxes on aggregate output will depend on relative factor intensities. Contrary to the one-sector model, a wage tax and a capital income tax have equivalent allocative distortions, as even a wage tax distorts sectoral allocations. However, if the formal sector is more capital-intensive, imposing a given tax rate on capital income raises more revenue than imposing the same rate on wages. It is therefore optimal to tax capital income more heavily.<sup>14</sup>

### *Infrastructure and the Second-Best Optimum*

Suppose now that a positive amount must be spent in infrastructure in order to operate the formal technology,  $\phi > 0$ . The government budget constraint is

$$(1 - \alpha)\tau_w + \alpha\tau_k = \theta + \phi, \quad (25')$$

and the aggregate budget constraint  $c = (1 - \phi)k_1 f(L_1 / k_1) + k_2 g(L_2 / k_2) - \psi$ , since some final good is used by the government to provide infrastructure.

As before, differentiating the growth rate and given that equation (26) still holds we have

$$\left. \frac{d\psi}{d\tau_w} \right|_{\tau_w = \tau_k} = 0$$

which implies that growth is maximized whenever  $\tau_k = \tau_w = \tau$ . Maximizing welfare now requires

$$\frac{dW / d\tau_w}{\gamma W} = \frac{1}{c} \frac{d\tilde{z}}{d\tau_w} + \frac{c + \gamma\psi - \rho}{c(\rho - \gamma\psi)} \frac{d\psi}{d\tau_w} = 0 \quad (28')$$

where

$$\tilde{z} \equiv (1 - \phi)k_1 f(L_1 / k_1) + k_2 g(L_2 / k_2). \quad (29')$$

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<sup>14</sup> The result that maximizing growth need not coincide with welfare maximization is not new. It was first obtained by Futagami et al. (1993) in a model in which the stock of public capital enters the production function. Turnovsky (2000b, Chapter 13) notes two other diverse contexts in which this result occurs. These include: (i) the introduction of adjustment costs in investment that depend upon productive government expenditure, and (ii) an economy subject to stochastic productivity shocks.



and

$$\left. \frac{dW / d\tau_W}{\gamma W} \right|_{\tau_W = \tau_K} = \frac{\tau - \phi}{(1 - \tau)^2} \frac{f'g}{cf''} \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right)^{-1} \quad (30')$$

Suppose now that the social planner has no redistributive goals,  $\theta = 0$ . In this case,  $\tau = \phi$ , implying

$$\left. \frac{dW / d\tau_W}{\gamma W} \right|_{\tau_W = \tau_K = \phi} = 0. \quad (31)$$

Welfare maximization thus requires not only that the tax rates be equal, but also that they be set equal to the infrastructure requirement. For  $\tau_K = \tau_W = \tau = \phi$ , growth and welfare are consequently maximized. Using the formal technology incurs a social cost which is not directly taken into account by individuals – the infrastructure requirement. Setting  $\tau_K = \tau_W = \tau = \phi$  implies that the (static) private returns to capital and labor in the formal sector are equal to the social returns. Static allocation decisions are hence optimal and output is maximized. Since growth is maximized whenever  $\tau_K = \tau_W$ , welfare is maximized.

When the redistribution rate is positive, however, taxing capital and labor incomes at the same rate will not maximize welfare. For  $\theta > 0$ , taxing both types of income at the same rate implies  $\tau_K = \tau_W = \tau = \theta + \phi$ . From equation (30') and assuming that the formal sector is more capital intensive, we have that  $dW / d\tau_W \big|_{\tau_W = \tau_K = \theta + \phi} < 0$ . Welfare maximization hence requires a higher tax rate on capital than on labor income,  $\tau_W < \theta + \phi < \tau_K$ .

We may summarize these second-best optimum results with

**Proposition 3:** Consider a government that wishes to raise a fixed amount of revenue by taxing the formal sector.

**A.** If it wishes to do so in a way that *maximizes the growth rate* then it should tax labor income and capital income equally, irrespective of how it intends to spend the revenue.

**B.** If it wishes to do so in a way that *maximizes average welfare*, the optimal taxes

will depend on how the revenue is spent. If it is spent on infrastructure, then equal taxation of labor and capital income is again optimal. If it is redistributed, then labor income should be taxed less than capital income as long as the formal sector is more capital intensive than is the informal sector.

It is important to note that the optimal tax structure as set out in Proposition 3 rests crucially upon the static allocation conditions (17c), (17d), which in turn assume the existence of an interior solution. As long as the production function is sufficiently flexible so that the “Inada conditions” are met, this is feasible, irrespective of the size of the informal sector. In the limit, either because the informal sector vanishes, or the production function is insufficiently flexible for (17c) and (17d) to hold, the allocation decision breaks down. The production factors are no longer free to move between the sectors and we are essentially back in the one-sector economy.

### 4.3 Factor Intensity Reversal

We have based our discussion on the relevant case for a developing economy, where the formal sector is relatively capital intensive. We now briefly comment on the case where the factor intensities are reversed,  $L_1/k_1 > L_2/k_2$ , and we shall assume further that  $M < 0$ . As before, an increase in either tax rate,  $\tau_K, \tau_W$  will shift productive factors from the formal to the informal sectors, so that the qualitative responses in (19) will remain unchanged. However, if  $L_1/k_1 > L_2/k_2$ , resources are now moving from the labor intensive to the capital intensive sector. Capital therefore increases in relative scarcity and thus its rate of return must increase for factor market equilibrium to prevail, thereby raising the growth rate. Consequently, an increase in either tax rate is associated with a higher growth rate, a fact that can also be seen directly from (20a), (20b).

Setting  $\tau_K = \tau_W$  again implies that  $d\psi/d\tau_W = 0$ , but this time it is growth-minimizing, rather than growth-maximizing. Instead, the growth-maximizing tax policy is now a corner solution and it is straightforward to verify that this is achieved by fully taxing labor income in the formal sector, setting  $\tau_W = 1$ , with the corresponding tax (or subsidy) on

capital being  $\tau_K = (\theta + \phi - (1 - \alpha)) / \alpha$ , just as in the one-sector economy. Maximizing welfare also leads to a corner solution. This can be most directly seen in the case in which the revenues are spent on infrastructure, when welfare maximization and growth maximization coincide. In this case (30') is welfare-minimizing and the optimum again is to set  $\tau_W = 1$  with the corresponding tax on capital satisfying  $\tau_K = (\phi - (1 - \alpha)) / \alpha$ .

## 5. Some Numerical Simulations

The result summarized in Proposition 3 calling for the tax on labor income to be reduced below that on capital income is striking. Table 2 provides some numerical results contrasting the implications of growth-maximizing fiscal policy with welfare-maximizing fiscal policy. These are based on the parameters summarized in Table 1.

These parameters are standard. The preference parameters include a rate of time preference of 4% and an intertemporal elasticity of substitution of 0.4. The production functions in the two sectors are taken to be CES production functions. For the formal sector the elasticity of substitution,  $\sigma = 1$ , making it Cobb-Douglas, while for the informal sector two values of  $\sigma = 1, 0.5$ , are considered. The production parameters are chosen to yield an equilibrium in which the informal sector is more labor intensive, as we have been assuming. The total rate of government expenditure is 20%, and the three cases where this is spent all on infrastructure, split between infrastructure and redistribution, and entirely redistributed, are considered. Base tax rates equal to 20% on both labor income and capital income are assumed.

Table 2A reports the optimal tax rates in the case where the production functions of both sectors are Cobb-Douglas. In the first part of the table we find that if the objective is to raise 20% of tax revenues, then setting  $\tau_W = \tau_K = 0.20$  will succeed in achieving this, while maximizing the growth rate, irrespective of the allocation of the expenditures between redistribution and infrastructure. Thus in all cases, for the base parameter set labor will be equally allocated between the two sectors, while nearly 86% of the capital stock will be employed in the formal sector producing 65% of the output. The overall growth rate will be around 2.1%. As the government shifts its expenditure allocation from infrastructure to redistribution, the share of income devoted to

consumption rises, while income inequality, as measured by  $\omega$ , declines.

The second part of Table 2A chooses the tax rates to maximize welfare, as measured by the compensating variation in the initial capital stock necessary to offset the change in the policy. As our results in Section 4 suggest, this is highly sensitive to the expenditure mix chosen by the government. In the first row, where all expenditure is devoted to infrastructure, welfare maximization coincides with growth maximization, and setting  $\tau_w = \tau_k = 0.20$  will thus meet both objectives. However, if instead, the government chooses to split its expenditure,  $\phi = 0.10$ ,  $\theta = 0.10$ , welfare will be maximized by reducing the tax on labor income to 15.6% and raising the corresponding tax on capital to 26.6%. This will increase the fraction of labor and capital employed in the more productive formal sector to 59% and 88% respectively, expanding the size of that sector to nearly 72%. The higher tax on capital income stimulates consumption, while reducing the growth modestly to 2.08%, leading to an overall welfare gain of 0.41% over what it would be if the growth-maximizing tax structure  $\tau_w = \tau_k = 0.20$  were chosen. Income inequality also declines. If the government spends its entire revenue on redistribution then the tax rates should diverge even more sharply;  $\tau_w$  should be reduced to under 12% and  $\tau_k$  raised to over 32%, leading to a welfare gain of around 1.45%, and a further decline in income inequality.

Table 2B repeats the analysis in the plausible case where the elasticity of substitution of the informal sector is only 0.5. The same pattern emerges, although the divergence between the growth-maximizing tax rates and the welfare-maximizing tax rates increases, leading to larger welfare gains. Indeed, one striking feature of these numerical results is that for plausible parameters the deviation between the growth-maximizing and welfare-maximizing tax structures are substantial, generating potentially significant welfare gains.

## 6. Concluding Comments

Much economic activity in developing economies occurs in an informal sector that is beyond the control of the government. In this paper we have developed a two-sector model of such an economy and shown how the presence of an informal sector, in addition to a more conventional formal sector fundamentally changes the way that the government should set the tax rates on factor

income. The conventional proposition that tax burdens should be borne more heavily by labor income and that capital income should not be taxed, or that it should even be subsidized, are dramatically altered, when the government is unable to tax one of the two sectors. In general, the inability to tax the informal sector makes it impossible to attain the first-best equilibrium.

Thus we have focused on second-best optima, where the government chooses to raise a certain set revenue in some optimal way. If the its objective is to maximize the growth rate, then labor income and capital income should be taxed at the same rate, irrespective of how the revenue is spent. If the objective is to maximize welfare, then how the revenue is used matters for the choice of taxes. When all revenue is devoted to the provision of infrastructure, both sources of income should be taxed at the same rate. However, as the government shifts its expenditure from infrastructure toward redistribution, and as long as the formal sector is relatively capital intensive, then the tax rate on labor income should be reduced and that on capital income raised. This is a striking result, and numerical simulations support that for plausible parameterization of the model, the divergence of tax rates from the growth-maximization case are large.

We conclude by noting some caveats of the model and offering several more general observations. First, a key, but not implausible, assumption is that total labor supply is inelastic, with the agent's work decision being the allocation of his time across the two sectors. To obtain some intuition into the robustness of our results, we have examined the consequences of endogenizing labor supply in the basic one-sector model. Not surprisingly, an elastic labor supply calls for a reduction in the tax on labor income and an increase in the tax on capital relative to those obtained with a fixed labor supply, thus reinforcing the responses we have been discussing. Hence we are confident that endogenizing labor supply will preserve our main findings regarding the relative magnitudes of the optimal taxes on labor and capital. But it would be interesting to investigate the numerical sensitivity of these optimal income tax rates to the elasticity of labor supply.

Second, we have focused our attention solely on taxes on labor income and capital income. We have seen that these suffice to replicate the first best optimum in the one-sector model, but not in the two-sector economy. Moreover, a further instrument becomes necessary to attain the first-best optimum in the one-sector economy when labor supply is endogenized. In either case the

introduction of a consumption tax can help achieve the optimum, although its implementation when some of the consumption is produced in the informal sector would need to be carefully considered. More generally, extending the analysis to an open economy would also introduce a potential role for tariffs and other foreign taxes that have traditionally been important in developing economies. But as discussed in the introduction, an important aspect of tax reform in developing economies is to try and reduce their reliance on indirect taxation.<sup>15</sup>

Third, our treatment of infrastructure spending is very simple in that it is assumed not to interact directly with either consumer utility or, more to the point, with productivity in the economy. While such extensions may be of interest, we do not believe that they would affect the major insight of this paper. Finally, although the motivation for our model is that of a developing economy, it can be given an alternative interpretation, more relevant for industrial economies. Indeed, the formulation we have adopted is also appropriate for studying an economy comprising a manufacturing and a service sector. The former is capital intensive, while the latter is labor intensive and is also more susceptible to tax evasion. Our setup assumes that there is complete tax evasion in the service sector. It is straightforward to parameterize the degree of tax evasion, by assuming that only a fraction of all revenues from the service sector can be taxed. Our conclusions are robust to this parameterization, indicating that even in industrial economies the constraints faced by governments when raising tax revenue may make it optimal to tax capital income more heavily than labor income.

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<sup>15</sup>Because of the homogeneity necessary to generate a balanced growth equilibrium, the endogenous growth literature is necessarily restricted to constant tax rates, a restriction that applies here as well. In a growing economy nonlinear tax rates would in general continually vary as the economy grows. This would in turn imply that the growth rate continually varies over time, incompatible with balanced growth.

**Table 1**  
Basic Parameters

Preference parameters:	$\gamma = -1.5, \rho = 0.04$
Formal sector technology:	$Y_1 = AK_1^\alpha (KL_1)^{1-\alpha}$ $\alpha = 0.4, A = 0.4, \varepsilon_1 = 1$
Informal sector technology:	$Y_2 = B(\beta K_2^{-\sigma} + (1-\beta)(KL_2)^{-\sigma})^{-1/\sigma}$ $\beta = 0.10, B = 0.3, \varepsilon_2 = 1, 0.5$
Expenditure rates:	$\phi + \theta = 0.20$
Base Tax rates	$\tau_K = 0.20, \tau_W = 0.20$

**Table 2**

**A.**  $\varepsilon_1 = 1; \varepsilon_2 = 1$

<b>Growth Maximization</b>												
	$\tau_W$	$\tau_K$	$L_1$	$L_1$	$k_1$	$k_2$	$Y_1$	$Y_2$	$\psi$	$C/Y$	$\omega$	$\Delta W$
$\phi = 0.20, \theta = 0$	0.20	0.20	0.500	0.500	0.857	0.143	0.652	0.348	2.105	0.814	0.280	---
$\phi = 0.10, \theta = 0.10$	0.20	0.20	0.500	0.500	0.857	0.143	0.652	0.348	2.105	0.879	0.260	---
$\phi = 0, \theta = 0.20$	0.20	0.20	0.500	0.500	0.857	0.143	0.652	0.348	2.105	0.945	0.243	---
<b>Welfare Maximization</b>												
	$\tau_W$	$\tau_K$	$L_1$	$L_1$	$k_1$	$k_2$	$Y_1$	$Y_2$	$\psi$	$C/Y$	$\omega$	$\Delta W$
$\phi = 0.20, \theta = 0$	0.20	0.20	0.500	0.500	0.857	0.143	0.652	0.348	2.105	0.814	0.280	0
$\phi = 0.10, \theta = 0.10$	0.156	0.266	0.586	0.414	0.881	0.119	0.716	0.284	2.079	0.875	0.254	0.4%
$\phi = 0, \theta = 0.20$	0.118	0.323	0.667	0.333	0.902	0.098	0.773	0.227	2.015	0.948	0.232	1.45%

**B.**  $\varepsilon_1 = 1; \varepsilon_2 = 0.5$

<b>Growth Maximization</b>												
	$\tau_W$	$\tau_K$	$L_1$	$L_1$	$k_1$	$k_2$	$Y_1$	$Y_2$	$\psi$	$C/Y$	$\omega$	$\Delta W$
$\phi = 0.20, \theta = 0$	0.20	0.20	0.550	0.450	0.781	0.219	0.675	0.325	2.549	0.797	0.320	---
$\phi = 0.10, \theta = 0.10$	0.20	0.20	0.550	0.450	0.781	0.219	0.675	0.325	2.549	0.865	0.296	---
$\phi = 0, \theta = 0.20$	0.20	0.20	0.550	0.450	0.781	0.219	0.675	0.325	2.549	0.932	0.276	---
<b>Welfare Maximization</b>												
	$\tau_W$	$\tau_K$	$L_1$	$L_1$	$k_1$	$k_2$	$Y_1$	$Y_2$	$\psi$	$C/Y$	$\omega$	$\Delta W$
$\phi = 0.20, \theta = 0$	0.20	0.20	0.550	0.450	0.781	0.219	0.675	0.325	2.549	0.797	0.320	0
$\phi = 0.10, \theta = 0.10$	0.142	0.287	0.716	0.284	0.860	0.140	0.799	0.201	2.486	0.856	0.288	0.81%
$\phi = 0, \theta = 0.20$	0.082	0.377	0.918	0.082	0.958	0.042	0.943	0.057	2.285	0.942	0.245	3.04%



## Appendix

This appendix derives several results concerning the two-sector model of Sections 3 and 4.

### A.1 Comparative Static Analysis

To examine the allocation effects of changes in taxes, write the system of equations (7) as

$$\begin{bmatrix} -(1-\tau_K)f''L_1/k_1^2 & (1-\tau_K)f''L_1^2/k_1^3 & g''L_2/k_2^2 & -g''L_2^2/k_2^3 \\ (1-\tau_w)f''/k_1 & -(1-\tau_w)f''L_1/k_1^2 & -g''/k_2 & g''L_2/k_2^2 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} dL_1 \\ dk_1 \\ dL_2 \\ dk_2 \end{bmatrix} = \begin{bmatrix} f - fL_1/k_1 & 0 \\ 0 & f' \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d\tau_K \\ d\tau_w \end{bmatrix}$$

The determinant of the coefficient matrix is

$$\Delta = -\frac{f''g''}{k_1k_2} \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right) \left( (1-\tau_w)\frac{L_2}{k_2} - (1-\tau_K)\frac{L_1}{k_1} \right) \quad (\text{A.1})$$

We assume that  $\Delta$  is negative, which is definitely the case when the two tax rates are equal, or when the two production functions are Cobb-Douglas and the formal sector is more capital intensive.

Let

$$\alpha \equiv 1 - \frac{L_1 f'(L_1/k_1)}{k_1 f(L_1/k_1)} \quad (\text{A.2})$$

Then we have

$$\begin{aligned} \frac{\partial L_1}{\partial \tau_K} = -\frac{\partial L_2}{\partial \tau_K} &= -\left[ (1-\tau_w)\frac{L_1}{k_1^2}f'' + \frac{L_2}{k_2^2}g'' \right] \frac{\alpha f}{\Delta} < 0 ; \\ \frac{\partial k_1}{\partial \tau_K} = -\frac{\partial k_2}{\partial \tau_K} &= -\left[ (1-\tau_w)\frac{f''}{k_1} + \frac{g''}{k_2^2} \right] \frac{\alpha f}{\Delta} < 0; \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \frac{\partial L_1}{\partial \tau_w} = -\frac{\partial L_2}{\partial \tau_w} &= -\left[ (1-\tau_K)\frac{L_1^2}{k_1^3}f'' + \frac{L_2^2}{k_2^3}g'' \right] \frac{f'}{\Delta} < 0 ; \\ \frac{\partial k_1}{\partial \tau_w} = -\frac{\partial k_2}{\partial \tau_w} &= -\left[ (1-\tau_K)\frac{L_1}{k_1^2}f'' + \frac{L_2}{k_2^2}g'' \right] \frac{f'}{\Delta} < 0; \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned}
\frac{\partial(L_1/k_1)}{\partial\tau_K} &= \frac{\alpha f}{f''M} < 0; & \frac{\partial(L_2/k_2)}{\partial\tau_K} &= \frac{(1-\tau_w)\alpha f}{g''M} < 0 \\
\frac{\partial(L_1/k_1)}{\partial\tau_w} &= \frac{(1-\alpha)f}{f''M} \frac{L_2/k_2}{L_1/k_1} < 0; & \frac{\partial L_2/k_2}{\partial\tau_w} &= \frac{(1-\tau_K)(1-\alpha)f}{g''M} < 0
\end{aligned} \tag{A.5}$$

Consider now the effect on the growth rate

$$\begin{aligned}
\frac{\partial\psi}{\partial\tau_K} &= -\frac{L_2}{k_2} \frac{g''}{1-\gamma} \frac{\partial(L_2/k_2)}{\partial\tau_K} = -\frac{L_2}{k_2} \frac{(1-\tau_w)\alpha f}{(1-\gamma)M} < 0 \\
\frac{\partial\psi}{\partial\tau_w} &= -\frac{L_2}{k_2} \frac{g''}{1-\gamma} \frac{\partial(L_2/k_2)}{\partial\tau_w} = -\frac{L_2}{k_2} \frac{(1-\tau_K)(1-\alpha)f}{(1-\gamma)M} < 0
\end{aligned} \tag{A.6}$$

## A.2 The First-Best Optimum

The social planner maximizes (10) subject to (11'). The solution to this problem is

*Resource Constraints*

$$L_1 + L_2 = 1$$

$$k_1 + k_2 = 1$$

*Equilibrium Factor Allocations*

$$(1-\phi)f'\left(\frac{L_1}{k_1}\right) = g'\left(\frac{L_2}{k_2}\right)$$

$$(1-\phi)\left[f\left(\frac{L_1}{k_1}\right) - \frac{L_1}{k_1}f'\left(\frac{L_1}{k_1}\right)\right] = g\left(\frac{L_2}{k_2}\right) - \frac{L_2}{k_2}g'\left(\frac{L_2}{k_2}\right)$$

*Equilibrium Growth Rate*

$$\tilde{\psi} = \frac{(1-\phi)(g(L_2/k_2) + (1-L_2/k_2)g'(L_2/k_2)) - \rho}{1-\gamma}$$

*Goods Market Equilibrium*

$$c = (1-\phi)k_1f(L_1/k_1) + k_2g(L_2/k_2) - \psi$$

### A.3. Redistributive Government Expenditure and Second-Best Taxation

We need to consider how the two tax rates vary together in order to raise a given revenue. Since  $\phi = 0$ , we can write the government budget constraint as  $(1-\alpha)(\tau_w - \tau_k) + \tau_k = \theta$ , and totally differentiate to get

$$(1-\alpha)\left(1 - \frac{d\tau_k}{d\tau_w}\right) - (\tau_w - \tau_k)\alpha' \left( \frac{\partial(L_1/k_1)}{\partial\tau_w} + \frac{\partial(L_1/k_1)}{\partial\tau_k} \frac{d\tau_k}{d\tau_w} \right) + \frac{d\tau_k}{d\tau_w} = 0 \quad (\text{A.7})$$

Using (A.5) to substitute for the partial derivatives, and defining

$$\sigma \equiv \frac{\partial F / \partial L_1 \cdot \partial F / \partial K_1}{\partial^2 F / (\partial L_1 \partial K_1) \cdot F} \quad (\text{A.8})$$

we obtain equations (26) and (27).

Now consider maximizing welfare,  $W = c^\gamma / (\gamma(\rho - \gamma\psi))$ . Differentiating and using the fact that  $c = (Y_1 + Y_2) / K - \psi$ , we have

$$\frac{dW / d\tau_w}{\gamma W} = \frac{1}{c} \frac{dz}{d\tau_w} + \frac{c + \gamma\psi - \rho}{c(\rho - \gamma\psi)} \frac{d\psi}{d\tau_w} \quad (\text{A.9})$$

where

$$z \equiv k_1 f(L_1 / k_1) + k_2 g(L_2 / k_2). \quad (\text{A.10})$$

An analytical expression for the maximum cannot be obtained. However, by evaluating this derivative at  $\tau_w = \tau_k = \tau$  we can tell whether maximizing welfare requires the wage tax to be greater or smaller than the capital income tax. At  $\tau_w = \tau_k$ , the second term in (A.9) is zero, so we only need to sign the first term. Now

$$\begin{aligned} \frac{dz}{d\tau_w} = & (f - g) \left( \frac{\partial k_1}{\partial \tau_w} + \frac{\partial k_1}{\partial \tau_k} \frac{d\tau_k}{d\tau_w} \right) + k_1 f' \left( \frac{\partial(L_1/k_1)}{\partial \tau_w} + \frac{\partial(L_1/k_1)}{\partial \tau_k} \frac{d\tau_k}{d\tau_w} \right) \\ & + k_2 g' \left( \frac{\partial(L_2/k_2)}{\partial \tau_w} + \frac{\partial(L_2/k_2)}{\partial \tau_k} \frac{d\tau_k}{d\tau_w} \right) \end{aligned} \quad (\text{A.11})$$

Now from (A.3), (A.4), (A.5), and the definition of  $\alpha$  we have

$$\left. \frac{d\tau_K}{d\tau_W} \right|_{\tau_W=\tau_K} = -\frac{1-\alpha}{\alpha} \quad (\text{A.12})$$

$$\left. \frac{\partial k_1}{\partial \tau_W} \right|_{\tau_W=\tau_K} + \left. \frac{\partial k_1}{\partial \tau_K} \frac{d\tau_K}{d\tau_W} \right|_{\tau_W=\tau_K} = \frac{fk_1}{f''M} \quad (\text{A.13})$$

$$\left. \frac{\partial(L_1/k_1)}{\partial \tau_W} \right|_{\tau_W=\tau_K} + \left. \frac{\partial(L_1/k_1)}{\partial \tau_K} \frac{d\tau_K}{d\tau_W} \right|_{\tau_W=\tau_K} = \frac{f'}{f''M} \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right) \quad (\text{A.14})$$

$$\left. \frac{\partial(L_2/k_2)}{\partial \tau_W} \right|_{\tau_W=\tau_K} + \left. \frac{\partial(L_2/k_2)}{\partial \tau_K} \frac{d\tau_K}{d\tau_W} \right|_{\tau_W=\tau_K} = 0 \quad (\text{A.15})$$

This implies

$$\left. \frac{dz}{d\tau_W} \right|_{\tau_W=\tau_K} = \frac{f'}{f''M} \left( f - g + f' \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right) \right) \quad (\text{A.16})$$

using the equilibrium allocation equations (17c) and (17d) and substituting for  $M$ , we get

$$\left. \frac{dz}{d\tau_W} \right|_{\tau_W=\tau_K} = \frac{\tau}{(1-\tau)^2} \frac{f'g}{f''} \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right)^{-1} \quad (\text{A.17})$$

For  $L_2/k_2 - L_1/k_1 > 0$ , i.e. when the formal sector is more capital intensive, then  $dz/d\tau_W|_{\tau_W=\tau_K} < 0$  and hence  $dW/d\tau_W < 0$  at  $\tau_W = \tau_K$ . This implies that maximizing welfare requires  $\tau_W < \theta < \tau_K$ . When the informal sector is more capital intensive,  $L_2/k_2 - L_1/k_1 < 0$ , maximizing welfare would require  $\tau_W > \theta > \tau_K$ .

#### A.4. Infrastructure and Second-Best Taxation

We can then write the government budget constraint as  $(1-\alpha)(\tau_W - \tau_K) + \tau_K = \theta + \phi$ . Totally differentiating we get (A.7), which implies (26). Since the expression for the growth rate in the competitive economy is the same irrespective of the use to which government revenue is put, the analysis of the previous section holds and growth is maximized for  $\tau_W = \tau_K = \theta + \phi$ .

Now consider the welfare of the average individual,  $W = c^\gamma / (\gamma(\rho - \gamma\psi))$ . Differentiating and using the fact that now  $c = (1-\phi)k_1f(L_1/k_1) + k_2g(L_2/k_2) - \psi$ , we have

$$\frac{dW/d\tau_w}{\gamma W} = \frac{1}{c} \frac{d\tilde{z}}{d\tau_w} + \frac{c + \gamma\psi - \rho}{c(\rho - \gamma\psi)} \frac{d\psi}{d\tau_w} \quad (\text{A.9}')$$

where

$$\tilde{z} \equiv (1 - \phi)k_1 f(L_1/k_1) + k_2 g(L_2/k_2). \quad (\text{A.10}')$$

Differentiating,

$$\begin{aligned} \frac{d\tilde{z}}{d\tau_w} = & ((1 - \phi)f - g) \left( \frac{\partial k_1}{\partial \tau_w} + \frac{\partial k_1}{\partial \tau_K} \frac{d\tau_K}{d\tau_w} \right) + (1 - \phi)k_1 f' \left( \frac{\partial(L_1/k_1)}{\partial \tau_w} + \frac{\partial(L_1/k_1)}{\partial \tau_K} \frac{d\tau_K}{d\tau_w} \right) \\ & + k_2 g' \left( \frac{\partial(L_2/k_2)}{\partial \tau_w} + \frac{\partial(L_2/k_2)}{\partial \tau_K} \frac{d\tau_K}{d\tau_w} \right) \end{aligned} \quad (\text{A.11}')$$

From (A.12)- (A.15), we have

$$\left. \frac{d\tilde{z}}{d\tau_w} \right|_{\tau_w = \tau_K} = \frac{f'}{f''M} \left( (1 - \phi)f - g + (1 - \phi)f' \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right) \right). \quad (\text{A.16}')$$

Using the equilibrium allocation equations (17c) and (17d), and substituting for  $M$  we obtain

$$\left. \frac{d\tilde{z}}{d\tau_w} \right|_{\tau_w = \tau_K} = \frac{\tau - \phi}{(1 - \tau)^2} \frac{f'g}{f''} \left( \frac{L_2}{k_2} - \frac{L_1}{k_1} \right)^{-1}. \quad (\text{A.17}')$$

If there is no redistribution, the government budget constraint implies  $\tau_w = \tau_K = \phi$ , and

$$\left. \frac{d\tilde{z}}{d\tau_w} \right|_{\tau_w = \tau_K = \phi} = 0.$$

Hence, welfare is maximized when the two tax rates are set equal to the infrastructure requirement. For  $\theta > 0$ , and assuming the formal sector is more capital intensive than the informal one,

$$\left. \frac{d\tilde{z}}{d\tau_w} \right|_{\tau_w = \tau_K = \theta + \phi} < 0$$

and welfare is not maximized.

## References

- Aghion, P, E. Caroli, and C. García-Peñalosa, 1999. Inequality and economic growth: The perspective of the new growth theories. *Journal of Economic Literature* 37, 1615-1660.
- Ahmad, E. and N. Stern, 1991. *The Theory and Practice of Tax Reform in Developing Countries*, Cambridge University Press, Cambridge.
- Bertola, G., 1993. Factor shares and savings in endogenous growth. *American Economic Review* 83, 1184-1198.
- Bourguignon, F., 1981. Pareto-superiority of unegalitarian equilibria in Stiglitz's model of wealth distribution with convex savings function. *Econometrica* 49, 1469-1475.
- Chamley, C., 1985. Efficient taxation in a stylized model of intertemporal general equilibrium. *International Economic Review* 26, 451-468.
- Chamley, C., 1986. Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica* 54, 607-622.
- Chamley, C., 2001. Capital income taxation, wealth distribution and borrowing constraints. *Journal of Public Economics* 79, 55-69.
- Correia, I.H., 1996. Should capital income be taxed in the steady state? *Journal of Public Economics* 60, 147-151.
- Cremer, H., P. Pestieau, and J-C Rochet, 2003. Capital income taxation when inherited wealth is not observable. *Journal of Public Economics* 87, 2475-2490.
- Deaton, A., 1981. Optimal taxes and the structure of preferences. *Econometrica* 49, 1245-1260.
- Dessy, S. and S. Pallage, 2003. Taxes, inequality, and the size of the informal sectors. *Journal of Development Economics* 70, 225-233.
- Fortin, B., N. Marceau, and L. Savard, 1997. Taxation, wage controls and the informal sector. *Journal of Public Economics* 66, 293-312.
- Fuest, C., and B. Huber, 2001. Labor and capital income taxation, fiscal competition, and the distribution of wealth. *Journal of Public Economics* 89, 71-91.
- Futagami, K., Y. Morita, and A. Shibata, 1993. Dynamic analysis of an endogenous growth model with public capital. *Scandinavian Journal of Economics* 95, 607-625.

- García Peñalosa, C. and S.J. Turnovsky, 2004. Production risk and the functional distribution of income in a developing economy: Tradeoffs and policy responses. forthcoming *Journal of Development Economics*.
- Irhig, J. and K. Moe, 2000. The dynamics of informal employment. Federal Reserve Board, International Finance Discussion Paper no 664.
- Jiminez, E., 1986. The public subsidization of education and health in developing countries. *World Bank Research Observer* 1.
- Jones, L.E., R.E. Manuelli, and P.E. Rossi, 1997. On the optimal taxation of capital income. *Journal of Economic Theory* 73, 93-117.
- Judd, K.L, 1985. Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics* 28, 59-83.
- Judd, K.L, 1999. Optimal taxation and spending in general competitive growth models. *Journal of Public Economics* 71, 1-26.
- Kormendi, R. and P. Meguire, 1985. Macroeconomic determinants of growth: Cross-Country Evidence. *Journal of Monetary Economics* 16, 141-163.
- Koskela, E. and R. Schöb, 2002. Optimal factor income taxation in the presence of unemployment. *Journal of Public Economic Theory* 4, 387-404.
- Lansing, K.J., 1999. Optimal redistributive capital taxation in a neoclassical growth model. *Journal of Public Economics* 73, 423-453.
- Lucas, R.E. and N. Stokey, 1983. Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 22, 55-93.
- Lucas, R.E., 1990. Supply-side economics: An analytical review. *Oxford Economic Papers* 42, 293-316.
- Merton, R.C., 1969. Lifetime portfolio selection under uncertainty: The continuous-time case. *Review of Economics and Statistics* 51, 247-257.
- Messere, K. and J. Owens, 1989. "Long-term revenue trends and current tax reform issues in OECD Countries," in A. Chiancone and K. Messere (eds.), *Changes in Revenue Structures*, Wayne State University Press, Detroit.

- Obstfeld, M., 1994. Risk-taking, global diversification, and growth. *American Economic Review* 84, 1310-1329.
- Romer, P.M., 1986. Increasing returns and long-run growth. *Journal of Political Economy* 94, 1002-1037.
- Skoufias, E., 1996. Intertemporal substitution in labor supply: Microevidence from rural India. *Journal of Development Economics* 51, 217-237.
- Stiglitz, J., 1969. The distribution of income and wealth among individuals. *Econometrica* 37, 382-397.
- Tait, A.A., 1988. *Value Added Tax: International Practice and Problems*, IMF, Washington D.C..
- Tanzi, V., 1987. Quantitative characteristic of the tax systems of developing countries, in D. Newbery and N. Stern (eds.), *The Theory of Taxation for Developing Countries*, Oxford University Press, Oxford.
- Tanzi, V. and H. Zee, 2001. Tax policy for developing countries, IMF Economic Issues, No. 27.
- Thomas, J.J., 1992. *Informal economic Activity*, Harvester Wheatsheaf, New York.
- Todaro, M. 1989. *Economic Development in the Third World*, Longman, New York.
- Turnovsky, S.J., 1996. Optimal tax, debt, and expenditure policies in a growing economy. *Journal of Public Economics* 60, 21-44.
- Turnovsky, S.J., 2000a. Fiscal policy, elastic labor supply, and endogenous growth. *Journal of Monetary Economics* 45, 185-210.
- Turnovsky, S.J., 2000b. *Methods of Macroeconomic Dynamics*, 2nd Ed., MIT Press, Cambridge MA.
- United Nations, 2000. *The World's Women 2000: Trends and Statistics*, Department of Economics and Social Affairs, New York.