#### Consumption Externalities, Production Externalities, and Efficient Capital Accumulation under Time Non-separable Preferences

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#### Abstract

The effects of consumption and production externalities on economic performance under time nonseparable preferences are examined both theoretically and numerically. We show that a consumption externality alone has long-run distortionary effects if and only if labor is supplied elastically. With fixed labor supply, it has only transitional distortionary effects. Production externalities always generate long-run distortions, irrespective of labor supply. The optimal tax structure to correct for the distortions is characterized. We compare the implications of this model with those obtained when the consumption externality is contemporaneous. While some of the long-run effects are robust, there are also important qualitative and quantitative differences, particularly along transitional paths.

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#### 1. Introduction

Externalities have engaged the attention of economists over a long period of time. Broadly speaking, they can be categorized as (i) consumption externalities, and (ii) production externalities. Recently, the former have been extensively studied in the context of models of "keeping up with the Joneses," and their implications for a range of important issues investigated. These include: asset pricing [Abel, 1990, Constantinides, 1990, Gali, 1994, Campbell and Cochrane, 1999], short-run macroeconomic stabilization policy [Ljungqvist and Uhlig, 2000], consumption [Dupor and Liu, 2003], capital accumulation and growth [Fisher and Hof, 2000, Liu and Turnovsky, 2005]. On the other hand, production externalities have been a key element in the recent endogenous growth literature. Empirical evidence on the importance of externalities is sparse, but several studies provide convincing support for the significance of consumption externalities, though less conclusive, is still quite compelling [Caballero and Lyons 1990, 1992, and Benarroch 1997].

A related and equally important issue concerns the specification of preferences themselves. The conventional intertemporal utility function is time-separable, with any consumption externality being introduced as *contemporaneous* economy-wide consumption, as in the references cited above. But a growing body of empirical evidence has confirmed the importance of *time non-separable* preferences, in which utility depends not only upon current consumption, but also on a benchmark or "habit" level of consumption determined from past behavior. In the case that this benchmark is defined in terms of the consumption of an external reference group it introduces a consumption externality (utility interdependence), but one that is tied to *past* consumptions. This formulation is often termed "*catching* up with", rather than "*keeping* up with", with the Joneses.<sup>1</sup> Empirical evidence supporting time non-separable utility specifications are provided by van de Stadt, Kapteyn, and van de Geer (1985), Osborn (1988) and more recently Fuhrer (2000).

In light of these bodies of evidence, the effect of consumption and production externalities on economic performance becomes important. To what extent do they introduce distortions into the

<sup>&</sup>lt;sup>1</sup>In the present case, where the benchmark involves the consumption of an outside reference group, agents are sometimes referred to as being "outward looking". In contrast, the benchmark may depend upon the agent's *own* past consumption. In that case agents are said to be "inward looking" and there is no consumption externality; see e.g. Carroll, et al (1997).

process of capital accumulation, and if so, what are the appropriate corrective policy responses? Liu and Turnovsky (2005) have addressed this question employing a standard time separable utility function. But given the evidence supporting the time non-separability of utility, it is important to re-examine the issue for this more general, and arguably more realistic, specification of preferences.

To do so is the objective of the present paper. More specifically, we introduce time nonseparable preferences, as originally specified by Abel (1990) in the context of asset pricing, into the "non-scale growth model" developed by Eicher and Turnovsky (1999). Previous applications of these preferences by Carroll, et al (1997) and others have typically imposed rigid production conditions of the simplest endogenous growth model.<sup>2</sup> But the interaction between preferences and production flexibility is important. This is shown by Alvarez-Cuadrado, et al (2004) who highlight the importance of combining more general preferences with the more flexible technology of the nonscale growth model to replicate certain observed behavior.

The paper proceeds in two main stages. The first part (Sections 2-6) develops the general theoretical model. The latter (Sections 7 and 8) supplements this with numerical simulations of both the steady-state equilibrium and the transitional dynamics in response to an increase in productivity. One general conclusion is that consumption externalities in isolation will have long-run distortionary effects on the economy if and only if labor supply is elastic. This is because such externalities affect the marginal valuation of consumption, which, if the leisure decision is endogenous, changes the optimal utility value of the marginal product of labor, thereby influencing long-run capital and output. Thus, with elastic labor supply, a negative consumption externality leads to long-run consumption, capital, labor supply, and output equilibria, all of which exceed their respective optima. But even if labor supply is fixed and consumption externalities alone have no long-run distortionary effects, they still distort the transitional dynamic path.

Production externalities alone always generate long-run distortions, irrespective of whether or not labor supply is fixed. Thus a positive production externality leads to a sub-optimally low capital stock, together with under-employment, under-production and under-consumption. In

<sup>&</sup>lt;sup>2</sup>There are exceptions, however. One is the pioneering work by Ryder and Heal (1973) who introduced habit formation into the basic neoclassical growth model; a more recent example is Alonso-Carrera et al. (2005).

addition, a consumption externality will affect the potency of the production externality on long-run activity through its impact on the labor-leisure choice and thus on the marginal product of capital.

We characterize an optimal tax policy to correct for the distortionary effects. It requires capital income to be taxed or subsidized at a constant rate that corrects for the production externality, while consumption should be taxed or subsidized at a time-varying rate that corrects for the divergence between the social and private benefits from the consumption externality as reflected in the evolving relative shadow value of habits to capital. The tax on labor income can be set at an arbitrary constant rate, allowing the policy maker to accommodate some other objective.

The simulations confirm and supplement these theoretical findings in several dimensions. First, the numerical impacts of the two externalities on steady-state equilibrium are assessed, and we find that both externalities of plausible magnitudes generate substantial long-run distortionary effects. For the consumption externality they are proportionate across key measures of economic activity, whereas for the production externality they are highly disproportionate.

Second, as a vehicle for examining the distortions along the transitional adjustment path, we introduce a 50% increase in productivity. Because of the non-scale technology, this shock affects the long-run measures of economic activity in both the decentralized and centrally planned economies by identical proportionate amounts. The differences in proportionate welfare gains between the two economies reflect differences along the transitional paths and are therefore small. But the actual magnitudes of the welfare differences can be substantial. We also trace out the ratios of key variables in the decentralized economy to their respective optima in order to determine how the sizes of the various distortions evolve over time. In our base simulations these ratios show relatively little intertemporal variation. But this finding is sensitive to a number of factors including: the relative weight of the benchmark consumption level in utility, the weight of past consumption in the construction of benchmark consumption, and, in some cases, the flexibility of labor supply.<sup>3</sup>

The paper is related to two recent studies, although it differs substantially from both in key ways. In addressing optimal fiscal policy it is related to Alonso-Carrera et al. (2005). But while

 $<sup>^{3}</sup>$  It is also dependent upon the underlying shock itself. In an expanded version of this paper we illustrate this by considering the time path of the distortion in capital following a 50% decrease in the rate of time preference.

they assume labor supply is inelastic, our analysis endogenizes labor supply. This is crucial for the consumption externality to generate permanent distortions, and therefore important for optimal tax policy. Second, (assuming discrete time) they specify the reference consumption level to be the previous period's economy-wide average consumption level. We allow for a much more gradual adjustment, which not only is important in reconciling the implications of this model with certain observed empirical phenomena [Alvarez-Cuadrado et al. 2004] but is also important for optimal policy. We parameterize the weight given to past consumption in the construction of the reference consumption level, with the contemporaneous externality emerging as a limiting case.

The paper can also be viewed as generalizing Liu and Turnovsky (2005) to more general preferences and extending it in several key dimensions. First, and most importantly, it presents a much more complete characterization of distortions along the dynamic adjustment path, a task for which numerical simulations become inevitable. Second, the numerical calibrations provide a sense of the plausible magnitudes of the distortionary effects, both in the long run and over time. Third, they enable us to assess the extent to which the specification of the reference stock is important. To accomplish this, throughout our analysis we compare the current approach, where the reference stock is based on past habit, to the case where the consumption externality is contemporaneous. As conjectured in the previous paper, the steady-state implications of that model are generally qualitatively similar to those summarized here in Propositions 1 and 2, suggesting some robustness of the previous results, although some modifications also arise. Moreover, the simulations highlight quantitative differences between the two formulations, which in some cases may be quite dramatic.

#### 2. Preferences and Technology: Consumption and Production Externalities

Consider an economy populated by N infinitely-lived identical households, where N grows at the constant exponential rate, n. The agent is endowed with a unit of time, part of which,  $L_i$ , can be supplied as labor input and the remainder,  $l_i \equiv 1 - L_i$ , consumed as leisure. At any instant of time households derive utility not only from their current consumption,  $C_i$ , and leisure, but also from the current level of a reference consumption stock, H, (habit) based on economy-wide consumption that the agent takes as given. Thus the agent's utility is represented by an iso-elastic function of the type employed by Abel (1990), Carroll, Overland, and Weil (1997), and others:

$$\Omega = \frac{1}{1-\varepsilon} \int_0^\infty \left( C_i H^{-\gamma} l_i^\theta \right)^{1-\varepsilon} e^{-\beta t} dt = \frac{1}{1-\varepsilon} \int_0^\infty \left( C_i^{1-\gamma} \left( \frac{C_i}{H} \right)^\gamma l_i^\theta \right)^{1-\varepsilon} e^{-\beta t} dt \quad \varepsilon > 0, \theta > 0, 1-(1-\varepsilon)(1+\theta) > 0$$
(1)

From the right hand side of (1), we see that agents derive utility from a geometric weighted average of absolute and relative consumption, these corresponding to  $\gamma = 0$ , and  $\gamma = 1$ , respectively. The agent's reference stock or consumption habit is specified by

$$H(t) = \rho \int_{-\infty}^{t} e^{\rho(\tau-t)} \overline{C}(\tau) d\tau \qquad \qquad \rho > 0 \qquad (2)$$

Thus (2) implies that the agent's reference stock is an exponentially declining weighted average of the economy-wide average consumption  $\overline{C}(\tau) \equiv \sum_{i=1}^{N} C_i / N$ . Differentiating (2) with respect to time yields the following rate of adjustment for the reference stock

$$\dot{H}(t) = \rho\left(\bar{C}(t) - H(t)\right) \tag{3}$$

The parameter,  $\rho$ , reflects the relative importance of recent consumption in determining the current reference (benchmark) stock. As  $\rho \to \infty$ ,  $H(t) \to \overline{C}(t)$ , the contemporaneous economy-wide average consumption, adopted by Gali (1994) and others, which obtains as a limiting case. With *H* determined by (2) [or (3)] the economy-wide consumption imposes an externality on the agent.<sup>4</sup>

Analogous to Liu and Turnovsky we shall impose the following restrictions on the size of the consumption externality, to ensure that its impact is dominated by the direct consumption benefits:

$$\gamma < 1$$
 (4a)

$$\varepsilon(1-\gamma) + \gamma > 0 \tag{4b}$$

Inequality (4a) is the non-satiation condition initially imposed by Ryder and Heal (1973), which asserts that a uniformly sustained increase in consumption level increases utility. The second

<sup>&</sup>lt;sup>4</sup> Most of the literature assumes  $\gamma \ge 0$ , implying that agents derive disutility from a *ceteris paribus* increase in the consumption reference stock. This expresses the idea of "catching up to the Joneses", as formulated by Carroll et al (1997), Ljungqvist and Uhlig (2000) and Alvarez-Cuadrado et al (2004). But we shall also permit  $\gamma < 0$ , thus characterizing an altruistic agent; see Dupor and Liu (2003).

condition restricts the externality so as to ensure that a uniformly sustained increase in consumption level across agents has diminishing marginal utility.<sup>5</sup>

The household has a production technology that is homogeneous of degree one in its private inputs, capital  $K_i$  and labor  $L_i$ , with both factors having positive but diminishing marginal physical product. In addition, output depends on the aggregate stock of capital, denoted by  $K = \sum_i K_i$ . Assuming a Cobb-Douglas production function, individual output is determined by

$$Y_i = \alpha L_i^{\sigma} K_i^{1-\sigma} K^{\eta}; \qquad 0 < \sigma < 1 \tag{5}$$

The externality generated by aggregate capital,  $\eta$ , may be positive, zero, or negative. In Romer (1986) the aggregate capital stock serves as a proxy for the level of knowledge and thus generates a positive production externality. However,  $\eta < 0$  may reflect adverse congestion effects of aggregate capital on the productivity of private capital; see e.g. Barro and Sala-i-Martin (1992).

The following restrictions, analogous to (4), are imposed on the production externality:

$$\sigma > \eta > -(1 - \sigma) \tag{6}$$

The right hand inequality ensures that the externality, if negative, is sufficiently small so that the social marginal product of capital remains positive [see (18c) below]. The left hand inequality imposes an upper limit on any positive externality generated by aggregate capital, in order for a uniformly sustained increase in capital stock to have diminishing marginal product.<sup>6</sup>

#### 3. Macrodynamic Equilibrium: Decentralized Economy

The individual in the decentralized economy chooses consumption, labor supply, and rate of capital accumulation to maximize the utility function (1) subject to the capital accumulation equation

$$\dot{K}_i = (r - n - \delta)K_i + wL_i - C_i \tag{7}$$

<sup>&</sup>lt;sup>5</sup> Expressed in terms of general utility functions these two conditions require:  $U_C(C_i, C_i, l_i) + U_H(C_i, C_i, l_i) > 0$ ;  $U_{CC}(C_i, C_i, l_i) + U_{CH}(C_i, C_i, l_i) < 0$ . In fact, for  $\varepsilon > 1$  and the constant elasticity utility function, (4a) implies (4b), although this is not true in general.

<sup>&</sup>lt;sup>6</sup>Turnovsky (2000) derives  $\sigma > \eta$  as a necessary and sufficient for stability in the basic one-sector non-scale growth model with conventional utility. It also turns out to be a necessary condition for stability for the present model.

where *r* denotes the gross return to capital, *w* denotes the wage rate, and  $\delta$  denotes the constant rate of depreciation of capital. In doing so, the agent takes the aggregate quantities  $\overline{C}$  and *K*, as well as the evolution of the reference consumption stock, (3), as given.

The first order conditions for an optimum  $are^7$ 

$$U_{C_i} \equiv C_i^{-\varepsilon} (l^{\theta} H^{-\gamma})^{1-\varepsilon} = \lambda_i$$
(8a)

$$U_{l} \equiv \theta l^{\theta(1-\varepsilon)-1} (C_{i} H^{-\gamma})^{1-\varepsilon} = \lambda_{i} w$$
(8b)

$$r - \delta - n = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{8c}$$

where  $\lambda_i$  denotes the private shadow value to agent *i* of an additional unit of capital, together with the transversality condition  $\lim_{t\to\infty} \lambda_i K_i e^{-\beta t} = 0$ . The interpretations of these equations are standard; (8a) equates the marginal utility of consumption to the *private* shadow value of capital taking into account that utility depends upon current consumption relative to the benchmark; (8b) equates the marginal utility of leisure to the private opportunity cost, the real wage valued at the shadow value of capital, while (8c) equates the marginal return to capital to the rate of return on consumption.

Aggregating (5) over the N identical agents, yields the aggregate production function

$$Y = \alpha \left( (1-l)N \right)^{\sigma} K^{1-\sigma+\eta}$$
<sup>(9)</sup>

where  $Y \equiv \sum_{i} Y_{i}$ . Total returns to scale,  $1 + \eta$ , are decreasing, constant, or increasing, according to whether the spillover from aggregate capital is negative, zero, or positive. The equilibrium gross real return to capital, *r*, and the real wage, *w*, are respectively:

$$r = \frac{\partial Y_i}{\partial K_i}\Big|_{K_i = K} = (1 - \sigma)\frac{Y_i}{K_i} = (1 - \sigma)\frac{Y}{K}; \ w = \frac{\partial Y_i}{\partial L_i}\Big|_{K_i = K} = \sigma\frac{Y_i}{L_i} = \sigma\frac{Y}{NL_i}$$
(10)

Substituting (10) into (7), the individual's rate of capital accumulation can be expressed as

$$\dot{K}_i = \alpha L_i^{\sigma} K_i^{1-\sigma} K^{\eta} - C_i - (n+\delta) K_i$$
(7')

<sup>&</sup>lt;sup>7</sup> Since all agents are identical and therefore allocate their time identically, we can drop the agent's subscript to l when setting out the equilibrium.

We define a balanced growth path as being one along which all quantities grow at a constant rate, except for the labor allocation, which is constant. With capital being accumulated from final output, along such a path the capital-output ratio, K/Y, remains constant. From the aggregate production function the long-run equilibrium balanced growth rate of output and capital,  $\hat{Y}$  and  $\hat{K}$ , is

$$\hat{Y} = \hat{K} = \frac{\sigma}{\sigma - \eta} n \equiv gn \tag{11}$$

Because of the non-scale nature of the production function, the equilibrium growth rate is determined solely by technological factors, summarized by the term  $g \equiv \sigma/(\sigma - \eta)$ , together with the population growth rate, and is independent of all demand characteristics, including the consumption externality; see Jones (1995). There is long-run per capita growth if and only if  $\eta > 0$ .

Following our definition of the balanced growth path, it is convenient to write the system in terms of the following stationary variables  $k \equiv K/N^g$ ,  $y \equiv Y/N^g$ ,  $h \equiv H/N^g$ ,  $c \equiv C/N^g$ , (where *C* also denotes the aggregate) which we term as being "scale-adjusted" per capita quantities, and which under constant returns to scale (g = 1) reduce to standard per capita quantities. Using this notation, the scale adjusted aggregate output (9) can be written as:

$$y = \alpha \left(1 - l\right)^{\sigma} k^{1 - \sigma + \eta} \tag{12}$$

We will focus on equilibrium paths along which all households are identical, so that  $C_i = C = \overline{C}$ ,  $K_i = K$ . We shall refer to such paths as "symmetric equilibria".

It is straightforward (but tedious) to express the equilibrium dynamics of the decentralized economy in terms of the redefined stationary variables, l,k,h,c,<sup>8</sup>

$$\dot{k}^{*} = \left(1 - \frac{c^{*}}{y^{*}}\right) y^{*} - \left(\delta + gn\right) k^{*}$$
(13a)

$$\dot{h}^* = \rho(c^* - h^*) + (1 - g)nh^*$$
(13b)

<sup>&</sup>lt;sup>8</sup> Details are available from the authors. Solving (13d) and (12) yields  $c^* = c^*(k^*, l^*)$ ,  $y^* = y^*(k^*, l^*)$ . Then substituting these solutions into (13a) – (13c) yields an autonomous dynamic system in  $\dot{k}^*, \dot{h}^*, \dot{l}^*$ . The numerical analysis conducted in Sections 7,8 is based on the linearized approximation to this system.

$$\dot{l}^{*} = F\left(l^{*}\right) \left\{ \left[ (1-\sigma) - \varepsilon(1-\sigma+\eta) \left(1 - \frac{c^{*}}{y^{*}}\right) \right] \frac{y^{*}}{k^{*}} - \rho \gamma \left(1 - \varepsilon\right) \left(\frac{c^{*}}{h^{*}} - 1\right) - \left[\beta + \delta \left(1 - \varepsilon(1-\sigma+\eta)\right) + n \left[1 - \varepsilon \left(1 - \sigma\right)\right] \right] \right\}$$
(13c)

$$\frac{c^{*}}{y^{*}} = \frac{C^{*}}{Y^{*}} = \frac{\sigma}{\theta} \frac{l^{*}}{(1-l^{*})}$$
(13d)

and

$$F(l^*) = \frac{l^*(1-l^*)}{\varepsilon(1-\sigma l^*) - \theta(1-\varepsilon)(1-l^*)} > 0$$
(13e)

together with the production function (12), where \* denotes the decentralized economy:

Equation (13d) is obtained by dividing the optimality conditions (8a) and (8b). It reflects the condition that the marginal rate of substitution between consumption and leisure  $(\theta C/l)$ , which grows with per capita consumption, must equal the wage rate  $(\sigma Y/(1-l))$ , which grows with per capita income. Notice that the equilibrium consumption-output ratio increases with leisure, this result reflecting the complementarity between leisure and consumption in utility.

Imposing the steady state condition,  $\dot{l}^* = \dot{k}^* = 0$ , we can solve (13) for the steady-state values of the relevant variables denoted by tildes, as follows,

$$\left(1 - \frac{\tilde{c}^*}{\tilde{y}^*}\right) \frac{\tilde{y}^*}{\tilde{k}^*} = \left(\delta + gn\right)$$
(14a)

$$\rho \left[ \frac{\tilde{c}^*}{\tilde{h}^*} - 1 \right] = (g - 1)n \tag{14b}$$

$$(1-\sigma)\frac{\tilde{y}^*}{\tilde{k}^*} - (n+\delta) = \beta + (g-1)[\varepsilon(1-\gamma) + \gamma]n$$
(14c)

$$\tilde{y}^* = \alpha (1 - \tilde{l}^*)^{\sigma} (\tilde{k}^*)^{1 - \sigma + \eta}$$
(14d)

$$\frac{\tilde{c}^*}{\tilde{y}^*} = \frac{\sigma}{\theta} \frac{\tilde{l}^*}{\left(1 - \tilde{l}^*\right)} \tag{14e}$$

First, (14c) yields the steady-state output-capital ratio, so that the long-run net private return to capital equals the rate of return on consumption, the latter being  $\beta$ , adjusted by the long-run utility

benefits derived from the production externality. Equation (14b) determines the equilibrium ratio of consumption to its reference stock. Having determined the output-capital ratio, (14a) determines the consumption-output ratio consistent with generating the growth rate of capital necessary to equip the growing labor force and replace the depreciating capital stock. Given the steady-state consumption to output ratio, (14e) determines the allocation of time to leisure,  $\tilde{l}^*$ , and given the steady-state values for  $\tilde{l}^*$  and  $\tilde{y}^*/\tilde{k}^*$ , the production function, (14d), then implies the corresponding value of capital,  $\tilde{k}^*$  and hence output,  $\tilde{y}^*$ .

We shall focus on three parameters:  $\gamma$  and  $\eta$ , which specify the two externalities, and  $\rho$ , the (exponentially declining) weight given to past consumption in the construction of the reference stock. From (14) we observe an important asymmetry between the two externalities. The consumption externality influences steady-state equilibrium in the decentralized economy if and only if there is a production externality (i.e.  $\eta \neq 0$ ); see (14c). In contrast, the production externality influences the equilibrium independent of any consumption externality. This is through its effect on the equilibrium growth rate, reflected in the right hand side of (14a).

If  $\eta = 0$  (i.e. g = 1), the parameters  $\gamma, \rho$ , characterizing the consumption externality both become irrelevant. In that case (14b) implies  $\tilde{c}^* = \tilde{h}^*$ , so that the stationary current and reference consumption levels coincide. Equation (14c) reduces to the standard modified golden rule condition, and the overall steady-state equilibrium reduces to that of conventional time-separable utility. If  $\eta \neq 0$ , the consumption externality,  $\gamma$ , affects the entire steady state, while  $\rho$  affects only the consumption-habits ratio. Thus the steady-state values for  $\tilde{k}^*, \tilde{L}^*, \tilde{y}^*, \tilde{c}^*$  are identical, whether the reference stock is formed contemporaneously, or as a weighted average of past consumption.

Suppose  $\eta > 0$ , and assume  $\varepsilon > 1$ . Then, as  $\gamma$  increases, so that the externality imposed by aggregate consumption becomes more negative, the long-run rate of return on consumption [the right hand side of (14c)], and thus the output-capital ratio, declines. For goods market equilibrium to prevail, the consumption-output ratio must also decline, inducing less leisure, i.e. more labor supply. This enhances the productivity of capital, thus inducing a larger long-run level of capital stock, output, and consumption. The responses are reversed if the production externality is negative.

In Section 8 below, we shall illustrate the dynamic response of the economy by introducing

an increase in productivity,  $\alpha$ . From (14), we see that the long-run responses are

$$\frac{d\tilde{k}^*}{\tilde{k}^*} = \frac{d\tilde{y}^*}{\tilde{y}^*} = \frac{d\tilde{c}^*}{\tilde{c}^*} = \frac{1}{\sigma - \eta} \frac{d\alpha}{\alpha}$$
(15)

The capital stock, output, and consumption change proportionately, implying that the output-capital ratio, consumption-output ratio, and hence leisure (labor) remain unchanged.<sup>9</sup>

#### 4. Macrodynamic Equilibrium: Centrally Planned Economy

In deriving his optimum, the individual agent neglects the externalities present in both consumption and production. As a consequence, the macroeconomic equilibrium generated by the decentralized economy may diverge from the social optimum. To derive the optimal resource allocation of the economy, we consider a social planner who chooses quantities directly to maximize the intertemporal utility of the representative agent, (1), while taking both externalities into account.

Specifically, the central planner internalizes the aggregation relationship  $K = NK_i$ , thus perceiving the individual's resource constraint (7') as

$$\dot{K}_i = \alpha L_i^{\sigma} K_i^{1-\sigma+\eta} N^{\eta} - C_i - (n+\delta) K_i$$
(7")

He further perceives that the consumption reference stock depends upon the economy-wide average consumption level, [which equals the consumption of the representative agent], and thus internalizes the impact of the agent's current consumption decision on the future evolution of the reference stock, in accordance with (3). Performing the maximization, the optimality conditions become

$$C_i^{-\varepsilon} (l^{\theta} H^{-\gamma})^{1-\varepsilon} + \rho \lambda_{2i} = \lambda_{1i}$$
(16a)

$$U_{l} \equiv \theta l^{\theta(1-\varepsilon)-1} (C_{i} H^{-\gamma})^{1-\varepsilon} = \lambda_{1i} \sigma \frac{Y_{i}}{(1-l)}$$
(16b)

$$(1 - \sigma + \eta)\frac{Y}{K} - \delta - n = \beta - \frac{\dot{\lambda}_{1i}}{\lambda_{1i}}$$
(16c)

<sup>&</sup>lt;sup>9</sup> That these responses are independent of the consumption externality, but dependent upon the production externality, is a manifestation of the non-scale production technology; see Eicher and Turnovsky (1999).

$$-\frac{\gamma}{\lambda_{2i}}(C_iH^{-1})\left[C_i^{-\varepsilon}(l^{\theta}H^{-\gamma})^{1-\varepsilon}\right] - \rho = \beta - \frac{\dot{\lambda}_{2i}}{\lambda_{2i}}$$
(16d)

where,  $\lambda_{1i}$ ,  $\lambda_{2i}$ , denote the social shadow values associated with the capital stock,  $K_i$ , and the reference consumption stock, H, respectively, together with the transversality conditions

$$\lim_{t \to \infty} e^{-\beta t} \lambda_{1i} K_i = \lim_{t \to \infty} e^{-\beta t} \lambda_{2i} H = 0$$
(16e)

There are several key differences from the corresponding conditions for the decentralized economy. First, (16a) equates the utility of an additional unit of consumption, adjusted by its impact on the future reference stock, to the social shadow value of capital. Second, (16c) equates the social rate of return to capital to the social rate of return on consumption. Third, (16d) is an intertemporal allocation condition equating at the margin, the rate of return on habits – which consists of its direct utility benefits less costs through its impact on future accumulation – to the rate of return on consumption, both evaluated in terms of the shadow value of habits.

Thus the optimization problem confronting the central planner requires the monitoring of two state variables. Letting  $q_i \equiv \lambda_{2i}/\lambda_{1i}$  denote the relative price of consumption habit to physical capital, after summing across households we can express the macrodynamic equilibrium of the centrally planned economy by the following fourth order system of the scale-adjusted variables:

$$\dot{k}^{o} = \left(1 - \frac{c^{o}}{y^{o}}\right) y^{o} - \left(\delta + gn\right) k^{o}$$
(17a)

$$\dot{h}^{o} = \rho \left( c^{o} - h^{o} \right) + \left( 1 - g \right) n h^{o} \tag{17b}$$

$$\dot{l}^{o} = F\left(l^{o}\right) \left\{ \left(1 - \sigma + \eta\right) \left[1 - \varepsilon \left(1 - \frac{c^{o}}{y^{o}}\right)\right] \left(\frac{y^{o}}{k^{o}}\right) - \gamma \rho \left(1 - \varepsilon\right) \left(\frac{c}{h} - 1\right) \right. \\ \left. - \left[\beta + \delta \left(1 - \varepsilon (1 - \sigma + \eta)\right) + n \left(1 - \varepsilon (1 - \sigma)\right)\right] \right. \\ \left. + \frac{\rho q^{o} \left(1 - \varepsilon\right)}{1 - \rho q^{o}} \left(\left(1 - \sigma + \eta\right)\frac{y^{o}}{k^{o}} + \gamma \frac{c^{o}}{h^{o}} \left(\frac{1 - \rho q^{o}}{q^{o}}\right) + \rho - \delta - n\right) \right\}$$
(17c)

$$\dot{q}^{o} = q^{o} \left\{ (1 - \sigma + \eta) \frac{y^{o}}{k^{o}} + \gamma \frac{c^{o}}{h^{o}} \left( \frac{1 - \rho q^{o}}{q^{o}} \right) + \rho - \delta - n \right\}$$
(17d)

where

$$\frac{c^{o}}{y^{o}} \equiv \frac{C^{o}}{Y^{o}} = \frac{\sigma}{\theta} \left[ \frac{l^{o}}{\left(1 - l^{o}\right)} \right] \left( \frac{1}{1 - \rho q^{o}} \right)$$
(17e)

o denotes the "optimum" or equilibrium in the centrally planned economy, F is defined as in (13e), and the production function is given by (12).

Imposing the stationary conditions,  $\dot{l}^o = \dot{h}^o = \dot{q}^o = k^o = 0$ , together with the production function, (12), and (17e), yields the optimal steady-state: <sup>10</sup>

$$\left(1 - \frac{\tilde{c}^{o}}{\tilde{y}^{o}}\right)\frac{\tilde{y}^{o}}{\tilde{k}^{o}} = \delta + gn \tag{18a}$$

$$\rho\left(\frac{\tilde{c}^{o}}{\tilde{h}^{o}}-1\right) = (g-1)n \tag{18b}$$

$$(1 - \sigma + \eta)\frac{\tilde{y}^{o}}{\tilde{k}^{o}} - (n + \delta) = \beta + (g - 1)[\gamma(1 - \varepsilon) + \varepsilon]n$$
(18c)

$$\tilde{y}^{o} = \alpha (1 - \tilde{l}^{o})^{\sigma} (\tilde{k}^{o})^{1 - \sigma + \eta}$$
(18d)

$$\frac{\tilde{c}^{o}}{\tilde{y}^{o}} = \frac{\sigma}{\theta} \left( \frac{\tilde{l}^{o}}{1 - \tilde{l}^{o}} \right) \left( \frac{1}{1 - \rho \tilde{q}^{o}} \right)$$
(18e)

$$\tilde{q}^{o} = \frac{-\gamma \left[1 + (g-1)n/\rho\right]}{\beta + (1-\gamma) \left[\rho + (g-1)\varepsilon n\right]}$$
(18f)

The parallels between these six equations and (14a) - (14e) for the decentralized economy are clear, and indeed, equations (18a), (18b), and (18d) remain unchanged. An important difference arises with regard to (18c), where the left hand side includes the effect of the production externality and thus measures the social rate of return to capital, while the right hand side coincides with that of (14c) and equals the private rate of return on consumption. In other words, the steady-state social and private rates of return on consumption coincide.

The consumption externality operates through two channels. In addition to its impact via the

<sup>&</sup>lt;sup>10</sup> The derivations actually involve some details that are spelled out in an Appendix available from the authors.

return to consumption [the right hand side of (18c)] it is also reflected in the social marginal rate of substitution between consumption and leisure, (18e). In contrast to the representative agent who evaluates the consumption-leisure choice in terms of the private marginal rate of substitution, (14e), the central planner also takes into account the social marginal value of the reference consumption stock through the term  $1/(1 - \rho \tilde{q}^{\circ})$ . Equation (18f) implies that unless there is an implausibly large negative production externality,  $\tilde{q}^{\circ} < 0$  if and only if the consumption externality is negative  $(\gamma > 0)$ .<sup>11</sup> In that case, an increase in the level of the reference stock, given current consumption, is welfare-reducing, so that its shadow value is negative. The opposite argument applies when  $\gamma < 0$ .

As a consequence of this second effect, a critical difference between (18) and (14) is that in the centrally planned economy the consumption externality  $\gamma$  has effects even in the absence of a production externality. Setting  $\eta = 0$ ,  $\tilde{y}^o/\tilde{k}^o, \tilde{c}^o/\tilde{y}^o$  are independent of  $\gamma$  and indeed are identical in the two economies. But when  $\eta = 0$ ,  $1/(1 - \rho \tilde{q}^o) = (1 - \gamma \rho/(\beta + \rho))$ , implying that an increase in  $\gamma$  reduces the relative valuation,  $1/(1 - \rho \tilde{q}^o)$ . Given  $\tilde{c}^o/\tilde{y}^o$ , this raises leisure (reduces labor supply), and given  $\tilde{y}^o/\tilde{k}^o$ , the reduction in labor supply reduces capital, output, and consumption.

It is further evident that an increase in  $\rho$  will increase  $1/(1-\rho\tilde{q}^{\circ})$  if  $\gamma > 0$ , in which case labor supply, capital, output, and consumption, all decline, with the opposite response occurring if  $\gamma < 0$ . Thus, in contrast to the decentralized economy, the steady-state values of  $\tilde{k}^{\circ}$ ,  $\tilde{L}^{\circ}$ ,  $\tilde{y}^{\circ}$ ,  $\tilde{c}^{\circ}$  in the centrally planned economy decrease (increase) if the reference stock is formed contemporaneously and there is a negative (positive) consumption externality. One further point is that the long-run responses in the centrally planned economy to an increase in productivity remain given by (15).

This significance of consumption externalities depends crucially upon the flexibility of labor supply. If labor supply is fixed, the optimality conditions for the labor/leisure decision [(14e) and (18e)] drop out, while the remaining equations are unchanged, with  $l^*$ ,  $l^o$  set at their inelastically fixed levels. In that case, as in the decentralized economy, the consumption externality will influence the equilibrium if and only if there is a production externality.

Our objective is to determine how closely the decentralized economy tracks the optimal time

<sup>&</sup>lt;sup>11</sup> More precisely, the constraint is  $\rho + (g-1)n > 0$ . The rate of adjustment of the reference consumption stock plus the economy's per capita rate of growth must be positive.

path, and to propose tax policies to correct for the distortions that may arise. Because of the complexity of the model we are forced to conduct the analysis of the dynamics numerically, but to aid in our understanding of these numerical simulations it is useful to examine first the steady states.

#### 5. Comparison of Steady-State Equilibria

We begin with the simple, but important, case where labor is supplied inelastically.

#### 5.1 Inelastic labor supply

In the decentralized economy the steady-state equilibrium values of  $\tilde{k}^*, \tilde{c}^*, \tilde{h}^*, \tilde{y}^*$  are determined by (14a) – (14d), while in the centrally planned economy, the corresponding steady-state values,  $\tilde{k}^o, \tilde{c}^o, \tilde{h}^o, \tilde{y}^o$ , are determined by (18a) – (18d). If labor is supplied inelastically at a common level,  $\bar{l}$ , in the two economies, then in the absence of a production externality,  $\eta = 0$ , these two sets of equations are identical, so that the steady-state equilibria in the two economies exactly coincide. Consumption externalities,  $\gamma$ , alone then have no effect on the steady state in either economy.

Thus the crucial factor is the presence of the production externality in the social return to capital in (18c). Assuming this to be positive, (18c) and (14c) together imply

$$\frac{\tilde{y}^*}{\tilde{k}^*} = \frac{1 - \sigma + \eta}{1 - \sigma} \frac{\tilde{y}^o}{\tilde{k}^o} > \frac{\tilde{y}^o}{\tilde{k}^o}$$
(19a)

which together with (14a) and (18a) immediately yields  $\tilde{c}^*/\tilde{y}^* > \tilde{c}^o/\tilde{y}^o$ . Combining (19a) with the production functions (14d) and (18d) implies

$$\frac{\tilde{k}^{*}}{\tilde{k}^{o}} = \left(\frac{1-\sigma}{1-\sigma+\eta}\right)^{1/(\sigma-\eta)} < 1; \quad \frac{\tilde{y}^{*}}{\tilde{y}^{o}} = \left(\frac{1-\sigma}{1-\sigma+\eta}\right)^{(1-\sigma+\eta)/(\sigma-\eta)} < 1$$
(19b)

and if, in addition, we (plausibly) assume that the centrally planned economy is dynamically efficient, we further obtain  $\tilde{c}^* < \tilde{c}^o$ .<sup>12</sup> We may summarize these results in

<sup>&</sup>lt;sup>12</sup>We define "dynamic efficiency" as usual to mean that the economy's equilibrium capital-labor ratio is less than the golden rule ratio. In this case the dynamic efficiency condition can be shown to be  $\beta + (g-1)(\varepsilon - 1)(1-\gamma)n > 0$  Given the restrictions  $\gamma < 1, \varepsilon > 1$ , this is certainly met unless the production externality assumes some implausibly large negative value which we effectively rule out. The relationship  $\tilde{c}^* < \tilde{c}^\circ$  can be formally established by expanding the decentralized equilibrium around the optimum, as set out in the Appendix to Liu and Turnovsky (2005).

**Proposition 1:** In a decentralized economy with inelastic labor supply and a positive production externality, the steady-state equilibrium capital stock and output are below their respective optimal levels, while the equilibrium output-capital ratio is too high. The consumption-output ratio is also too high, although if the economy is dynamically efficient, the consumption level is too low. These comparisons are reversed if the production externality is negative. In the absence of any production externality, a consumption externality causes no long-run distortionary effects.

This proposition is analogous to Proposition 1 in Liu and Turnovsky (2005), although there is one significant difference. In their economy, which is stationary, consumption externalities *never* cause distortions when labor supply is inelastic. In the present growing economy, although consumption externalities alone cause no distortions, their interaction with production externalities does generate distortionary effects. This can be seen from (14c), (18c), together with (19b).

#### 5.2 Elastic labor supply

With endogenous labor supply, the consumption externality will now affect the steady state even in the absence of any production externality. This is because it affects the marginal valuation of consumption, which in turn changes the optimal utility value of the marginal product of labor. Thus, consumption distortion results in labor distortion and therefore creates production inefficiency.

The comparison now involves the complete sets of equations (14) and (18) and leads to the following. Consider first the absence of a production externality ( $\eta = 0$ ). In this case, comparing (14c) and (18c), (14b) and (18b), (14a) and (18a) implies the equality of the ratios

$$\frac{\tilde{y}^*}{\tilde{k}^*} = \frac{\tilde{y}^o}{\tilde{k}^o}; \quad \frac{\tilde{c}^*}{\tilde{y}^*} = \frac{\tilde{c}^o}{\tilde{y}^o}; \quad \frac{\tilde{h}^*}{\tilde{y}^*} = \frac{\tilde{h}^o}{\tilde{y}^o}$$

Noting these equalities in conjunction with (14d) and (18d) then implies  $\tilde{k}^*/\tilde{k}^o = \tilde{y}^*/\tilde{y}^o = \tilde{L}^*/\tilde{L}^o$ , and equating (14e) to (18e) yields further

$$\tilde{\varphi} \equiv \frac{\tilde{y}^*}{\tilde{y}^o} = \frac{\tilde{k}^*}{\tilde{k}^o} = \frac{\tilde{L}^*}{\tilde{L}^o} = \frac{\tilde{c}^*}{\tilde{c}^o} = \frac{\tilde{h}^*}{\tilde{h}^o} = 1 + \frac{(1 - \tilde{L}^*)\gamma\rho}{\beta + (1 - \gamma)\rho}$$
(20)

From (20) we see that all steady-state quantities in the decentralized economy deviate from their corresponding optimal quantities by the same proportionate amount,  $\tilde{\varphi}$ , which depends critically upon the nature of the consumption externality. If it is negative ( $\gamma > 0$ ) these ratios all exceed unity, implying that these levels all exceed their respective optima, and vice versa if  $\gamma < 0$ . They also increase with the fraction of time devoted to leisure in the decentralized economy.

As  $\rho$  increases, the magnitudes of these proportionate deviations from their respective optimum increase. Consequently the distortions with time non-separable preferences are smaller in magnitude than they are if the consumption externality is contemporaneous, when the right hand side of (20) becomes  $1+(\gamma(1-\tilde{L}^*))/(1-\gamma)$ . Moreover, the difference between the conventional formulation of contemporaneous consumption externality and the time non-separable specification adopted here increases with  $\gamma$ . The intuition is simply that the larger  $\rho$  the more rapidly current consumption is reflected in the reference consumption stock, the shadow value of which becomes more negative as  $\gamma$  increases. This accentuates the divergent responses of the central planner from the representative agent, who ignores these effects.

In the case of a positive production externality, but no consumption externality,  $(\eta > 0, \gamma = 0)$ , the comparisons made above in the case of inelastic labor supply continue to apply. In addition the fact that  $\tilde{c}^*/\tilde{y}^* > \tilde{c}^o/\tilde{y}^o$  implies  $\tilde{l}^* > \tilde{l}^o$ , or equivalently  $\tilde{L}^* < \tilde{L}^o$ , with the reverse applying if  $\eta < 0$ . In this case, it is clear that the relative weight,  $\rho$ , in the construction of the reference stock, is irrelevant insofar as the steady-state deviation from the optimum is concerned.

These results may be summarized by the following proposition relating the actual and socially optimal equilibria in response to consumption and production externalities.

**Proposition 2:** In an economy with endogenous labor supply, the steady-state equilibrium has the following properties.

1. In the absence of any production externality, a negative (positive) consumption externality causes the equilibrium capital stock, labor supply, output, and consumption all to exceed (fall short of) their respective long-run optima by the same proportionate amount. These deviations from the optimum increase in size as:

(i) The weight in the reference consumption stock is more heavily weighted

toward current consumption.

(ii) Utility is more heavily weighted toward relative consumption.

2. In the absence of any consumption externality, a positive (negative) production externality causes the equilibrium capital stock, labor supply, output, and consumption all to fall short of (exceed) their respective long-run optima. In this case, the construction of the reference consumption stock is irrelevant.

#### 6. Optimal Tax Policy

The fact that consumption and production externalities create distortions in resource allocation provides an opportunity for government tax policy to improve efficiency. Consider again the decentralized economy. Let  $\tau_k, \tau_w$ , and  $\tau_c$  denote the tax rates on capital income, labor income, and consumption, respectively, and let  $T_i$  be lump-sum transfers (taxes). The representative agent maximizes the utility function, (1), subject to the budget constraint, now modified to:

$$\dot{K}_{i} = \left[ (1 - \tau_{k})r - n - \delta \right] K_{i} + (1 - \tau_{w})wL_{i} - (1 + \tau_{c})C_{i} + T_{i}$$
(21)

The government maintains a balanced budget, rebating all tax revenues as lump sum transfers:

$$\tau_k r K_i + \tau_w w L_i + \tau_c C_i = T_i \tag{22}$$

The objective is to characterize a tax structure such that the decentralized economy mimics the dynamic equilibrium path of the centrally planned economy, (17a) - (17f). Two relationships are subject to distortions; the consumption-output ratio, (13d), and the evolution of leisure, (13c). In principle, the optimal time path can be replicated by the use of two, possibly time-varying, tax rates, which in fact can be chosen in different ways. Omitting details it is straightforward to establish

**Proposition 3:** The decentralized economy will replicate the optimal time path of the centrally planned economy if taxes at each instant of time are set in accordance with  $\hat{\tau}_k = -\eta/(1-\sigma)$ ,  $\tau_w = \overline{\tau}_w$ , and  $((1+\hat{\tau}_c)/(1-\overline{\tau}_w)) = 1-\rho q^o(t)$ , where  $\overline{\tau}_w$  is an arbitrarily fixed constant and  $q^o(t)$  evolves in accordance with (17d). In the limiting case  $\rho \to \infty$ , when the reference stock is contemporaneous,

 $((1+\hat{\tau}_c)/(1-\bar{\tau}_w)) = 1/(1-\gamma)$ , and the consumption tax is also constant over time.

The optimal tax on capital income corrects for the distortion resulting from the production externality. The government should subsidize (tax) capital income according to whether this externality is positive (negative).<sup>13</sup> Given the Cobb-Douglas technology, this tax is constant over time, although it would become time-varying for more general production functions; see Liu and Turnovsky (2005). The consumption tax corrects for the distortion caused by the consumption externality. This depends upon  $q^o$  and is therefore time-varying, as  $q^o$  evolves in accordance with (17d), and through which it depends upon the production externality,  $\eta$ . Once the two distortions are rectified, no labor income tax is needed and the arbitrarily fixed constant labor income tax can thus be set to zero. If  $\overline{\tau}_w = 0$ , private consumption should be subsidized if there is a positive consumption externality and should be taxed if there is a negative consumption externality.

Noting (18f), the consumption and labor income tax ratio converges to

$$\frac{1+\hat{\tau}_{c}}{1-\bar{\tau}_{w}} = 1 - \rho \tilde{q}^{o} = \frac{\rho + \beta + (g-1)(\varepsilon + \gamma(1-\varepsilon))n}{\rho(1-\gamma) + \beta + \varepsilon(g-1)(1-\gamma)n}$$
(26)

It is interesting to compare this result with the corresponding steady-state optimal tax result of Liu and Turnovsky (2005). Using a general utility function, with the consumption externality being contemporaneous (i.e.  $\rho \rightarrow \infty$ ), and abstracting from growth they obtain

$$\frac{1+\hat{\tau}_{c}}{1-\overline{\tau}_{w}} = \frac{U_{C}(C,C,l)}{U_{C}(C,C,l)+U_{H}(C,C,l)}$$
(27)

Setting n = 0 in (26) and using the fact that for the constant elasticity utility function in steady-state  $U_H/U_C = -\gamma$ , (26) can then be expressed in the analogous form

$$\frac{1+\hat{\tau}_{c}}{1-\overline{\tau}_{w}} = \frac{U_{c}(C,H,l)}{U_{c}(C,H,l) + \frac{\rho}{\rho+\beta}U_{H}(C,H,l)}$$
(27')

The difference arises because in the present model the benchmark consumption level, being an exponentially declining average of all past consumptions, is a stock, whereas in the Liu-Turnovsky

<sup>&</sup>lt;sup>13</sup> This result is a familiar one associated with the Romer (1986) technology.

model it is a current flow. We therefore must take account of both how rapidly the reference stock adjusts and the rate of time preference. Letting  $\rho \rightarrow \infty$ , (27') converges to (27).<sup>14</sup>

#### 7. Numerical analysis of some transitional paths.

To study the transitional dynamics we calibrate the model to reproduce some key features of actual economies.<sup>15</sup> Table 1 summarizes the parameters upon which our simulations are based.

Table 1Benchmark parameters

Production parameters	$\alpha = 1, \ \sigma = 0.65, \ \eta = -0.2, \ 0, 0.2, \ \delta = 0.05$
Preference parameters	$\beta = 0.04, \ \varepsilon = 2.5, \ \theta = 1.75, \ \rho = 0.2, \ \gamma = 0.5$
Population growth	n = 0.015

Most of these are standard and non-controversial. In this regard,  $\sigma = 0.65$ , the rate of time preference  $\beta = 0.04$ , the instantaneous intertemporal elasticity of substitution,  $1/\varepsilon = 0.4$ , population growth rate n = 0.015, and depreciation rate,  $\delta = 0.05$  are well documented. The benchmark value of the elasticity of leisure in utility,  $\theta = 1.75$ , implies an equilibrium fraction of time devoted to leisure of around 0.7, consistent with the empirical evidence; see e.g. Cooley (1995).<sup>16</sup>

The two key parameters upon which we shall focus are the consumption externality,  $\gamma$ , and the production externality,  $\eta$ . The absence of both externalities,  $\gamma = \eta = 0$ , serves as a natural benchmark. For the negative consumption externality, we consider  $\gamma = 0.5$ , the value taken by Carroll et al. (1997) as their benchmark. For symmetry, we choose  $\gamma = -0.5$  as the magnitude of an equivalent positive consumption externality. For the production externality, we consider the positive externality ( $\eta = 0.2$ ) [increasing returns] and negative externality ( $\eta = -0.2$ ) [decreasing returns], respectively. We set the speed of adjustment of the reference stock, at  $\rho = 0.2$ , also the benchmark value of Carroll et al.<sup>17</sup> Since we wish to compare the present case where the reference consumption

<sup>&</sup>lt;sup>14</sup> For the constant elasticity utility function (1), the optimal consumption tax, (27), reduces to the constant,  $\hat{\tau}_c = 1/(1-\gamma)$ .

<sup>&</sup>lt;sup>15</sup> The dynamics are studied by linearizing the relevant dynamic system, (13) or (17) about its respective steady state, (14) and (18). The details of this are cumbersome, but standard, and are available from the authors on request.

<sup>&</sup>lt;sup>16</sup> Being a non-scale model, the normalization  $\alpha = 1$  is unimportant.

<sup>&</sup>lt;sup>17</sup> Since information on both  $\gamma, \rho$  is sparse we have conducted some sensitivity analysis on these parameters. For example, we have run simulations using values  $\gamma = 0.8, 0.9$ , close to Fuhrer's (2000) estimates. We have also chosen

stock is formed as a weighted average of past average consumption with the conventional case where it is based on the contemporaneous current consumption, we also consider  $\rho \rightarrow \infty$ .

The three panels of Table 2 summarize the key steady-state production and consumption variables as  $\eta$  and  $\gamma$  vary across their respective specified values. The benchmark externality-free economy ( $\gamma = \eta = 0$ ) is indicated in bold face. Panel A summarizes the decentralized economy. The chosen parameter values imply an output-capital ratio of 0.3, consumption-output ratio of 0.78, and fraction of time devoted to labor of around 32% [68% devoted to leisure], all of which are plausible. These values are independent of  $\rho$ , as we have shown analytically. Panel B summarizes the first-best optimum, corresponding to our benchmark value  $\rho = 0.2$ , while Panel C provides the limiting case where  $\rho \rightarrow \infty$ . Overall, the numerical results reported in Table 2 reflect the analytical relationships summarized in Propositions 1 and 2. In particular, we note the following:

1. In the absence of both the production and the consumption externality the steady states of the decentralized and centrally planned economies coincide; see the bold face entries.

#### 2. **Consumption externality only**

(i) The steady-state equilibrium of the **decentralized** economy is independent of  $\gamma$ .

(ii) If the externality is **positive**  $(\gamma = -0.5)$ , then  $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$  in the **centrally planned** economy given in Table B(i) are all proportionately larger [relative to the benchmark,  $\gamma = 0$ ] by 24.9%; if is **negative**  $(\gamma = 0.5)$   $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$  are all proportionately smaller by 32.6%.

(iii) As a result,  $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$  in the decentralized economy are all 20% **below** their respective optima if  $\gamma = -0.5$ , while they are all 48.5% **above** if  $\gamma = 0.5$ .

#### 3. **Production externality only**

(i) If the externality is **positive**  $(\eta = 0.2)$ , then  $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$  in the **centrally planned** economy increase substantially, although non-uniformly, relative to the benchmark,  $\eta = 0$ . These range from a 212% increase in  $\tilde{k}$  to just a 10.2% increase in  $\tilde{L}$ , implying a 130% increase in  $\tilde{y}$ ,

larger values of  $\rho$ , as suggested by some of the applications of these models to the equity premium puzzle problem, as well as smaller values, which yield more plausible speeds of convergence. Generally, our main qualitative conclusions are robust to the parameter choice, although there are some differences as illustrated in Section 8.3.

while  $\tilde{c}$  increases by 98%. If the externality is **negative**,  $(\eta = -0.2)$   $\tilde{k}$ ,  $\tilde{L}$ ,  $\tilde{y}$ ,  $\tilde{c}$  decline by 68%, 9.6%, 32%, and 21%, respectively.

(ii) The response of the **decentralized** economy to the production externality is much milder.

(iii) As a result of these differential responses,  $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$  are substantially below their optima, by 68.5%, 10.1%, 50.6% and 42.1%, respectively if  $\eta = 0.2$ . For  $\eta = -0.2$ ,  $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$  exceed their optima by 194%, 11.3%, 25.9%, 7.9%, respectively.

4. Distortions arising from positive production externalities are generally reduced (increased) in the presence of negative (positive) consumption externalities. Distortions with respect to negative production externalities are generally increased in the presence of negative consumption externalities, but only slightly in the presence of positive consumption externalities.

5. Comparing Panels B(i) and B(ii) confirms the role of  $\rho$  noted in Prop. 2. Namely:

(i) The (uniform) ratio of  $\tilde{k}^*/\tilde{k}^o$ , etc. obtained in the absence of a production externality, [see (20)] increases from 1.48 to 1.68 for a positive consumption externality and decreases from 0.80 to 0.77 for a negative externality, as  $\rho$  increases from 0.2 to a contemporaneous externality ( $\rho \rightarrow \infty$ ).

(ii) In the absence of a consumption externality, the distortions induced by the production externality are independent of  $\rho$ 

These numerical results suggest that the distortions are relatively insensitive to substantial changes in  $\rho$ . However, this depends in part upon the choice of  $\gamma$ . If, we increase  $\gamma$  to 0.8, for example, we find that increasing  $\rho$  from 0.2 to  $\rho \rightarrow \infty$  has much more dramatic effects on the distortion. Assuming  $\eta = 0$ , we find that it increases the ratio  $\tilde{k}^*/\tilde{k}^o$  from 2.30 to 3.71.<sup>18</sup>

#### 8. Dynamic Adjustment

We now compare the dynamic adjustments of the decentralized economy to the optimal

<sup>&</sup>lt;sup>18</sup> We should point out that some aspects of our comparisons depend upon the elasticity of leisure in utility. While space limitations preclude exploring this aspect in detail, the following differences when labor supply is inelastic arise. First, changes in output and consumption due to both positive and negative production externalities are much less sensitive to the consumption externality, than when labor supply is elastic. Second, output and consumption in the centrally planned economy increase with  $\gamma$  in the presence of a positive production externality, in contrast to when labor supply is fixed.

adjustments by considering a 50% increase in productivity,  $\alpha$ . These are summarized in Tables 3 and 4 and illustrated in Figures 1 and 2. As a reference point, Table 3.A reports the proportionate long-run responses in the capital stock, output, labor, and consumption. Recalling (15), these responses are identical in both the decentralized and centrally planned economies, and increase with  $\eta$  but are independent of both  $\gamma$  and  $\rho$ . Consequently, the long-run relative distortions due to a consumption externality are unaffected by whether the externality is contemporaneous or lagged.

#### 8.1 Long-run Welfare

Panels B(i) and B(ii) report the changes in intertemporal welfare resulting from the productivity increase. These measure the changes in the representative agent's optimized utility function  $\Omega$  [given in (1)], in the two economies (decentralized economy and centrally planned). The welfare gains reported are equivalent variation measures, calculated as the percentage change in the permanent flow of consumption in the respective economies, necessary to equate the levels of welfare to what they would be following the increase in productivity.

In contrast to the measures of economic activity, the changes in welfare *do* depend upon the consumption externality,  $\gamma$ , as well as upon  $\rho$ . This is because with sluggish adjustment, current consumption exceeds the reference level during the transition. Accordingly, if  $\gamma > 0$ , the "catching up with the Joneses" individual derives additional utility, relative to an individual having conventional utility, due to the fact that he is doing well relative to the recent experience of his peers, as reflected in the current reference stock. For example, in the benchmark case  $\gamma = \eta = 0$ ,  $\rho = 0.2$ , the individual's intertemporal welfare increases by 69.7%, whereas if  $\gamma = 0.5$ , the welfare gain increases to 82.4%. An altruistic individual, however, enjoys only a 64.2% welfare gain.

Several features in Table 3.B(i) merit comment. First, the percentage utility *gains* are similar in magnitude for the decentralized and centrally planned economies with identical externalities, though generally somewhat larger for the latter. But since the decentralized economy usually has a non-optimal production and consumption structure, its initial welfare *level* is substantially below that of the centrally planned economy, so that the absolute utility gains are significantly smaller. Second, in some cases, utility gains are proportionately larger in the decentralized economy, though its welfare remains well below the optimum.

Table B(ii) presents the welfare for  $\rho \rightarrow \infty$ , enabling us to compare these long-run responses to those obtained when the consumption externality is contemporaneous. In general, we find that the utility gains for the time non-separable utility function exceed (are less than) those for conventional utility according to whether the consumption externality, is negative, (positive). This is because if  $\gamma > 0$ , for example, the additional utility the agent derives from having his current consumption exceed the economy-wide reference level, is enhanced by the slower adjustment of the reference level in the time non-separable case.

#### 8.2 Short-Run Effects

Table 4 reports the short-run responses to the productivity increase. We shall focus on the cases of a negative consumption externality ( $\gamma = 0.5$ ) and a positive production externality ( $\eta = 0.2$ ) for the case  $\rho = 0.2$  summarized in Col. 3, Row 3 of Panel (i).

Consider first the benchmark case where there are no externalities,  $\gamma = \eta = 0$ . In this case, the decentralized economy exactly mimics the centrally planned economy throughout the entire transitional path. A productivity increase of 50% immediately raises employment in both economies by 0.95 percentage points, raising output by 52.9% and consumption by 46.4%.

Negative consumption externality: Now suppose that there is a negative consumption externality ( $\gamma = 0.5$ ) but no production externality ( $\eta = 0$ ). In this case, the decentralized economy begins from an initial steady state in which consumption, capital, output, and employment are all 48.3% above their optimal values; see Table 2A, 2B. A 50% increase in  $\alpha$  raises output directly in both economies, and will raise consumption, though by a lesser amount, due an increase in the savings ratio. As a result, the consumption-output ratio falls initially in both economies. Since the representative agent in the decentralized economy ignores the negative consumption externality, he *overvalues* consumption, relative to its optimum, so that consumption increases more in the decentralized economy, causing the consumption-output ratio to decline by less in that economy. At the same time, by directly influencing final output, the productivity shock increases the ultimate scarcity of habit relative to capital, thereby making q more negative, so that the social value of the consumption-output ratio,  $(c/y)(1-\rho q)$  actually declines less. As a result, leisure in the centrally planned economy declines less [c.f. (13d) and (17e)] so that labor supply increases more in the decentralized economy, raising output by more in that economy, as well. Following their initial jumps, output, and consumption in both the decentralized and centrally planned economies increase monotonically toward their new steady states levels, while the capital stocks increase gradually from their respective initial levels. Since in the long-run labor allocations do not change, labor supply declines monotonically in both economies, back toward their respective original steady-state values.

Since we wish to focus on the distortions, Fig. 1.A plots the time paths of the ratios  $y^*/y^o$ ,  $c^*/c^o$ ,  $k^*/k^o$ ,  $L^*/L^o$ . Fig 1.A(i) illustrates the case  $\gamma = 0.5$ ,  $\eta = 0$ . All four ratios begin at the common value 1.483 and eventually converge back to that ratio after the shock. On impact,  $c^*/c^o$  rises to around 1.51,  $L^*/L^o$  to 1.495,  $y^*/y^o$  to 1.490, while the capital stock remains fixed instantaneously. The amount of over-consumption, over-production, and over-employment in the decentralized economy (relative to the optimum) all increase. The over-consumption in the decentralized economy means that capital begins to accumulate at a slower rate in the decentralized economy, so that  $k^*/k^o$  initially declines, although eventually it converges back to its equilibrium ratio. The interesting feature of these paths is that throughout the transition all four variables exceed their respective long-run optima, with some deviations in the degree during the early stages. These divergences in the paths reflect the fact that agents in the decentralized economy, by ignoring the impact of current consumption on the reference level generate a faster speed of convergence.

Positive production externality: Now suppose that there is a positive production externality  $(\eta = 0.2)$  but no consumption externality  $(\gamma = 0)$ . In this case the decentralized economy is one in which capital, labor, output, and consumption are all less than their respective optima, but to vastly different degrees  $(\tilde{k}^*/\tilde{k}^o = 31\%, \tilde{L}^*/\tilde{L}^o = 90\%, \tilde{y}^*/\tilde{y}^o = 49\%, c^*/c^o = 58\%)$ . Since the productivity shock affects all variables proportionately, and since the positive production externality has little effect on the relative speeds of convergence of the two economies, Fig. 1.A (ii) illustrates how the deviations from the optimum remain almost constant along the transitional paths.

Positive production externality and negative consumption externality: We now combine cases (i) and (ii) by assuming  $\eta = 0.2$ ,  $\gamma = 0.5$ . This combines elements of the two previous cases.

Whereas the capital stock, output, and consumption are all initially less than their respective optima  $(\tilde{k}^*/\tilde{k}^o = 53\%, \tilde{y}^*/\tilde{y}^o = 83\%, c^*/c^o = 99\%)$ , labor exceeds its optimum  $(\tilde{L}^*/\tilde{L}^o = 129\%)$ . On the one hand the negative consumption externality leads to over-employment, over-production, and over-consumption, on the other, the production externality has the opposite effect. As in Fig. 1.A(ii), there is almost no change in the deviation of the decentralized economy relative to its optimum over time. The most interesting feature about Fig. 1.A(iii) is that the effect of the negative consumption externality is to generate initial *over*-consumption by about 2%, which gradually declines over time and after about 20 years returns to *under*-consumption.

Table 4 (ii) provides the analogous effects in the case of the contemporaneous consumption externality, with Fig. 1.B illustrating the dynamic adjustments. The same generally qualitative responses can be seen, although there are some differences. Consumption initially jumps higher to 1.75 and quickly drops. The dynamic adjustments have generally the same qualitative properties as when  $\rho = 0.2$ , except that the convergence occurs more rapidly.

#### 8.3 Two Alternative Parameter Choices

The dynamic adjustments illustrated in Fig. 1.A exhibit two characteristics. First, while modest externalities can generate substantial long-run deviations of the decentralized economy from the optimum, these show relatively little variation over time. For the pure consumption externality the maximum distortions, which occur during the initial phase, are within 1.5% of the long-run distortions, and their time paths are even more uniform in the presence of a production externality. Second, while the magnitudes of the distortions are substantially larger when the externality is contemporaneous, the patterns over time are qualitatively similar to those obtained when  $\rho = 0.2$ .

In Figure 2 we illustrate two modifications to our parameter choice, which have the effect of accentuating the differences between the contemporaneous externality and the slowly adjusting externality. In the first case we increase  $\gamma$  to 0.9, so that utility is more influenced by relative rather than absolute consumption. This not only increases the (common) ratio of  $\tilde{k}^*/\tilde{k}^o$  etc. in the absence of a production externality from around 3 to over 7 as  $\rho$  increases from 0.2 to  $\infty$ , but it also reverses the patterns in the distortions,  $k(t)^*/k(t)^o$  during the early phase of the transition. With a

contemporaneous reference stock,  $k(t)^*/k(t)^\circ$  continues to initially fall and then rise, as in Fig. 1, but with a slowly evolving reference stock, this adjustment is now reversed.

One characteristic of our chosen parameterization is that it implies an asymptotic speed of convergence of around 10%. While this is consistent with some estimates [e.g. Caselli et al. (1996)] it exceeds the consensus values, which now range up to about 6%.<sup>19</sup> Calibration of this aspect of the model is improved by setting the consumption externality  $\gamma$  to 0.9 and slowing the adjustment of the reference stock to  $\rho = 0.1$ . In addition, assuming an inelastic labor supply enables us to bring out an interesting contrast from the result obtained by Liu and Turnovsky (2005) based on the conventional time separable utility function where the externality occurs contemporaneously. For the constant elasticity specification being assumed here, and inelastic labor supply, they find that the consumption externality causes no distortionary effects along the transitional path.

Fig. 2.II.A plots the deviations of the decentralized economy from the optimum, for the parameter set  $\eta = 0, \gamma = 0.9, \rho = 0.1, \theta = 0$ . The capital stock shows a highly non-monotonic adjustment path, which is mirrored in the other variables. During the first few years there is over-accumulation of capital by about 0.5%; during the next 85 years there is under-accumulation of capital, which reaches 5% below its optimum after around 40 years; and finally after about 95 years there is over-accumulation of capital. It is hard to provide a simple intuitive explanation for this pattern, except to note that it is a consequence of slower convergence, due to in part to less flexibility in labor supply, together with a slower adjusting reference stock. The cyclical element reflects the technical fact that the stable eigenvalues have quite substantial complex components. Increasing  $\rho \rightarrow \infty$ , Fig. 2.II.B shows that the dynamic paths of all the ratios simply converge to unity over time, implying the absence of distortionary effects, consistent with Liu and Turnovsky (2005).

#### 9. Conclusions

The theoretical and empirical importance of both consumption and production externalities are well documented. In addition, a growing body of empirical evidence supports the importance of time non-separable preferences as an alternative to the conventional time separable utility function.

<sup>&</sup>lt;sup>19</sup>Alvarez-Cuadrado et al. (2004) provide a detailed analysis of convergence speeds for time non-separable utility.

With this motivation, this paper has examined the effects of both types of externalities on economic performance assuming this more general specification of preferences and discussed the appropriate corrective taxes. In the light of previous research emphasizing the importance of the interaction between preferences and technology, our analysis has employed the more flexible non-scale production technology. The approach we have taken is to combine the theoretical analysis with numerical simulations based on the calibration of a plausible macroeconomic growth model.

We have drawn three main sets of theoretical conclusions. First, a consumption externality in isolation has long-run distortionary effects if and only if labor is supplied elastically. But even with fixed labor supply, consumption externalities will still have transitional distortionary effects, and they will generate long-run distortions through their interaction with production externalities. With elastic labor supply, a negative consumption externality leads to sub-optimally large long-run capital, labor supply, output, and consumption. Second, production externalities always generate long-run distortions, irrespective of whether or not labor supply is fixed. Thus a positive production externality leads to a sub-optimally low capital stock with under-production and under-consumption. Third, we have provided a simple characterization of optimal tax policy that enables the replication of the entire optimal path. It requires that capital income be taxed or subsidized at a constant rate that corrects for the production externality, while consumption should be taxed or subsidized at a time-varying rate that corrects for the divergence between the social and private benefits from the consumption externality as the economy evolves along its transitional path.

The simulations supplement these theoretical findings with important quantitative insights. One striking finding is the sharp contrast between the effects of consumption and production externalities on the deviations of the decentralized economy from the optimum along the transitional paths. Finally, throughout we have compared our results with those obtained under conventional time-separable preferences when the consumption externality is contemporaneous. While some of the long-run theoretical effects are fairly robust in this respect across steady states, there are also important qualitative and quantitative differences, particularly along transitional paths.

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#### Table 2: Impact of Externalities on Steady State Equilibrium

	$\gamma = -0.5$				$\gamma = 0$				$\gamma = 0.5$			
η	${ ilde k}^*$	$ ilde{L}^{*}$	$\tilde{y}^*$	$\tilde{c}^*$	${ ilde k}^*$	$ ilde{L}^*$	$\tilde{y}^*$	${ ilde c}^*$	${ ilde k}^*$	$ ilde{L}^{*}$	$ ilde{y}^*$	${ ilde c}^*$
-0.2	2.00	0.325	0.535	0.412	1.93	0.324	0.530	0.412	1.86	0.322	0.525	0.411
0	2.05	0.322	0.615	0.482	2.05	0.322	0.615	0.482	2.05	0.322	0.615	0.482
0.2	1.82	0.317	0.658	0.527	2.01	0.319	0.698	0.554	2.23	0.321	0.743	0.583

#### A. Decentralized Equilibrium (independent of $\rho$ )

#### **B.** Centrally Planned Economy (i) Slowly Adjusting Reference Stock ( $\rho = 0.2$ )

		$\gamma = \cdot$	-0.5		$\gamma = 0$				$\gamma = 0.5$			
η	$ ilde{k}^{o}$	$ ilde{L}^{o}$	$\tilde{y}^{o}$	${ ilde c}^o$	$ ilde{k}^{o}$	$ ilde{L}^{o}$	${ ilde y}^o$	$ ilde{c}^{o}$	$ ilde{k}^{o}$	$ ilde{L}^{o}$	$\tilde{y}^{o}$	$\tilde{c}^{o}$
-0.2	0.816	0.371	0.509	0.459	0.657	0.291	0.421	0.381	0.462	0.192	0.305	0.276
0	2.56	0.402	0.768	0.602	2.05	0.322	0.615	0.482	1.38	0.217	0.414	0.325
0.2	7.70	0.430	1.774	1.222	6.39	0.355	1.414	0.956	4.20	0.248	0.890	0.590

## B. Centrally Planned Economy (ii) Contemporaneous Reference Stock $(\rho \rightarrow \infty)$

		$\gamma = -$	-0.5		$\gamma = 0$				$\gamma = 0.5$			
η	$ ilde{k}^{o}$	$ ilde{L}^{o}$	$ ilde{y}^o$	$ ilde{c}^{o}$	$ ilde{k}^{o}$	$ ilde{L}^{o}$	$ ilde{y}^o$	$ ilde{c}^{o}$	$ ilde{k}^{o}$	$ ilde{L}^{o}$	$\tilde{y}^{o}$	$ ilde{c}^{o}$
-0.2	0.834	0.381	0.520	0.469	0.657	0.291	0.421	0.381	0.421	0.170	0.278	0.252
0	2.65	0.415	0.794	0.622	2.05	0.322	0.615	0.482	1.22	0.192	0.366	0.287
0.2	8.15	0.446	1.88	1.29	6.39	0.355	1.41	0.956	3.50	0.219	0.742	0.491

Table 3: Effect of 50% Increase in Productivity  $\alpha$  :

	$\Delta \tilde{y}/\tilde{y} = \Delta \tilde{k}/\tilde{k} = \Delta \tilde{c}/\tilde{c}$
$\eta = -0.20$	61.1
$\eta = 0$	86.6
$\eta = 0.20$	146.2

A. (Common) Long-run Effects (independent of  $\gamma, \rho$ )

#### **B.** Percentage Change in Intertemporal Welfare

(i) Slowly Adjusting Reference Stock ( $\rho = 0.2$ )

	Dec	entralized Econ	omy	Optimum				
η	$\gamma = -0.5$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = -0.5$	$\gamma = 0$	$\gamma = 0.5$		
-0.2	53.0	54.7	60.9	53.9	57.1	68.2		
0	64.2	69.7	82.4	64.2	69.7	85.7		
0.2	74.3	89.3	119.7	76.5	87.8	116.0		

## (ii) Contemporaneous Reference Stock $(\rho \rightarrow \infty)$

	Dec	entralized Econ	omy	Optimum				
$\eta$	$\gamma = -0.5$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = -0.5$	$\gamma = 0$	$\gamma = 0.5$		
-0.2	55.8	54.7	51.7	56.9	57.1	56.9		
0	70.1	69.7	66.9	69.5	69.7	69.5		
0.2	86.6	89.3	89.9	89.9	87.8	89.9		

		$\gamma = -0.5$				$\gamma = 0$		$\gamma = 0.5$		
		%pt	%	%	%pt	%	%	%pt	%	%
$\eta$		$\Delta L$	$\Delta y$	$\Delta c$	$\Delta L$	$\Delta y$	$\Delta c$	$\Delta L$	$\Delta y$	$\Delta c$
-0.2	Decentr.	-0.04	49.9	50.2	1.82	55.4	43.2	3.56	60.6	37.0
	Optimum	0.15	50.4	53.8	1.04	53.5	46.0	0.87	54.4	37.5
0	Decentr.	-0.71	47.9	52.7	0.95	52.9	46.4	2.62	57.8	40.3
	Optimum	-0.33	49.2	55.3	0.95	52.9	46.4	1.60	57.1	38.2
0.2	Decentr.	-0.87	47.3	53.4	0.29	50.9	47.7	1.61	54.8	44.0
	Optimum	-0.86	48.0	56.9	0.49	51.3	48.1	1.69	56.6	40.1

# Table 4. Short-run Effects(i)Slowly Adjusting Reference Stock ( $\rho = 0.2$ )

(ii). Contemporaneous Reference Stock  $(\rho \rightarrow \infty)$ 

		$\gamma = -0.5$				$\gamma = 0$		$\gamma = 0.5$		
		%pt	%	%	%pt	%	%	%pt	%	%
$\eta$		$\Delta L$	$\Delta y$	$\Delta c$	$\Delta L$	$\Delta y$	$\Delta c$	$\Delta L$	$\Delta y$	$\Delta c$
-0.2	Decentr.	-0.67	48.0	52.6	1.82	55.4	43.2	4.27	62.7	34.5
	Optimum	-0.27	49.3	62.6	1.04	53.5	46.0	1.94	61.0	30.6
0	Decentr.	-1.60	45.1	56.3	0.95	52.9	46.4	3.50	60.4	37.2
	Optimum	-0.52	48.8	63.0	0.95	52.9	46.4	2.11	60.6	31.3
0.2	Decentr.	-1.96	43.9	57.8	0.29	50.9	47.7	2.64	57.9	40.2
	Optimum	-0.97	47.9	64.1	0.49	51.3	48.1	1.90	58.3	33.5

- A. Slowly Adjusting Reference Stock ( $\rho = 0.2$ )
- B. Contemporaneous Reference Stock ( $\rho \rightarrow \infty$ )





(i)  $(\eta = 0, \gamma = 0.5)$ 



(ii)  $(\eta = 0.2, \gamma = 0)$ 



(iii)  $(\eta = 0.2, \gamma = 0.5)$ 



### Figure 2: Increase in Productivity $\alpha$ : Two Alternative Comparisons

I. Larger Weight to Reference Stock

- A. Slowly Adjusting Reference Stock ( $\rho = 0.2$ )
- B. Contemporaneous Reference Stock ( $\rho \rightarrow \infty$ )



 $(\eta = 0, \gamma = 0.9)$ 

- II. Inelastic Labor supply
- A. Slowly Adjusting Reference Stock ( $\rho = 0.1$ )
  - ratio dec /gpt
- B. Contemporaneous Reference Stock ( $\rho \rightarrow \infty$ )



(iii)  $(\eta = 0.0, \gamma = 0.9, \theta = 0)$ 



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