The Transitional Dynamics of Fiscal Policy:

Long-Run Capital Accumulation and Growth*

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Abstract

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This paper analyzes the effects of fiscal policies in a non-scale growing economy with public and private capital. The equilibrium dynamics are characterized and we contrast the dynamic effects of government expenditure on investment and expenditure on consumption under four alternative modes of tax financing. Most of our attention focuses on the numerical simulations of a calibrated economy. The results emphasize the lengthy transition periods, which implies that policies have sizeable level effects, leading to substantial welfare effects, even though long-run growth rates are unaffected. The paper highlights the intertemporal dimensions of fiscal policy and the tradeoffs these involve for economic performance, especially growth and welfare.

1. Introduction

An initial attraction of endogenous growth models was that they assigned a key role to fiscal policy as a determinant of long-run economic growth. However, these models have several shortcomings, leading to a reassessment of their merits. In particular, they are frequently associated with "scale effects", meaning that the steady-state growth rate increases with the size (scale) of the economy, as indexed by say population. Empirical evidence by Backus, Kehoe, and Kehoe (1992) for the United States, and by Jones (1995b) for OECD economies does not support such scale effects. Moreover, Stokey and Rebelo (1995) and others find at best weak evidence for the effects of tax rates on the long-run growth rate, although Kneller, Bleaney, and Gemmell (1999) argue that these results are biased because of the incomplete formulation of the government budget constraint.

These considerations have stimulated the development of non-scale growth models; see e.g. Jones (1995a, 1995b). These models have the advantage that they are consistent with balanced growth under quite general production structures. Indeed, if the knife-edge restriction that generates endogenous growth models is not imposed, then any stable balanced growth equilibrium is characterized by the absence of scale effects. In this case the long-run equilibrium growth rate is determined by technological parameters and is independent of macro policy instruments.

But the fact that the equilibrium growth rate is independent of tax rates does not imply that fiscal policy is unimportant for long-run economic performance. In fact quite the contrary is true. Fiscal policy has important effects on the *levels* of key economic variables, such as the per capita stock of capital and output. Moreover, the non-scale model typically yields slow asymptotic speeds of convergence, consistent with the empirical evidence of 2-3% per annum; see Eicher and Turnovsky (1999).[1] This implies that policy changes can affect growth rates for sustained periods of time so that their accumulated effects during the transition from one equilibrium to another may therefore translate to potentially large impacts on steady-state levels. Thus, although the economy grows at the same rate across steady states, the corresponding bases upon which the growth rates compound may be substantially different.

These considerations suggest that attention should be directed to determining the impact of fiscal policy on the transitional dynamics. This is the focus of the present paper. The model we employ is of a one-sector economy in which output depends upon the stocks of both private and public capital, as well as endogenously supplied labor. Public capital introduces a positive externality in production, so that the complete production function is one of overall increasing returns to scale in these three productive factors. In addition to accumulating public capital, the government allocates resources to a utility-enhancing consumption good. These expenditures are financed by taxing capital income, labor income, and consumption, or by imposing non-distortionary lump-sum taxation. We set out the dynamic equilibrium of this economy and show how the stable adjustment is characterized by a two dimensional locus in terms of the two stationary variables, referred to as "scale-adjusted" per capita stocks of private and public capital.

The fact that the transitional paths are two-dimensional introduces important flexibility to the dynamics. Convergence speeds now vary over time and across variables, often dramatically, allowing different variables to follow different transitional paths; see Eicher and Turnovsky (1999). This characteristic is relevant to the empirical evidence of Bernard and Jones (1996), who find that while growth rates of output among OECD countries converge, the growth rates of manufacturing technologies exhibit markedly different time profiles. This contrasts with the standard one-sector neoclassical growth model, or the familiar two-sector endogenous growth model in which the stable locus is one-dimensional, thus constraining all variables to follow essentially identical time paths.

Our analysis focuses on two aspects. First, we briefly discuss the steady-state equilibrium and analyze the effects of various fiscal changes on the long-run labor-leisure allocation, the long-run changes in the capital stocks, and output. We compare the long-run effects of the two forms of government expenditure – investment versus consumption – and changes in the alternative tax rates.

But most of our attention is devoted to calibrating the model to a benchmark economy and assessing the numerical effects of various policy shocks, relative to this benchmark. To preserve comparability, the shocks are standardized in their impact on the current government deficit. Both long-run equilibrium responses and the transitional adjustment paths are considered. Particular attention is devoted to the welfare of the representative agent, both the time profile of instantaneous utility and his intertemporal welfare, as represented by the present value of the short-run benefits.

The implications for the government's intertemporal budget balance are also addressed.

The major contribution of the paper is to highlight the intertemporal dimensions of fiscal policy and the tradeoffs these involve for economic performance, especially growth and welfare. One general conclusion is that the relative merits of the two forms of public expenditure depend upon their sizes relative to their respective social optima. Calibrating the model to approximate the US, we find a sharp contrast between government expenditure on consumption and government expenditure on investment. For the basic case of lump-sum tax financing, the former has a gradual, but mild, uniform positive effect on welfare over time, with intertemporal welfare improving by about 2%. The latter involves a dramatic intertemporal tradeoff. Consumption and welfare decline in the short run, as resources are attracted toward government investment, but improve sharply over time as productivity is enhanced, and intertemporal welfare improves by nearly 8%.

Similar tradeoffs exist for standardized increases in distortionary taxes. Taxing capital yields the greatest immediate benefits in that it diverts resources away from capital accumulation and labor toward consumption and leisure. But in the long run, taxing capital is undesirable, being substantially worse than taxing labor income, which in turn is inferior to taxing consumption. These effects are reflected in the comparison of compensated standardized fiscal changes. Thus, for example, financing an increase in government consumption expenditure by taxing capital is contractionary in the long run, reducing long-run welfare by 1.34%, in comparison to the welfare improvement of 0.42% with a consumption tax and 1.97% with lump-sum tax financing.

There is an extensive literature analyzing fiscal policy on capital accumulation and growth and before proceeding we should briefly indicate the relationship of this paper to that literature. One strand is based on the AK models pioneered by Barro (1990) and Rebelo (1991). Government production expenditure in these models is usually introduced as a flow; see Barro (1990), Ireland (1994), Glomm and Ravikumar (1994), Bruce and Turnovsky (1999), and Turnovsky (2000). But to the extent that productive government expenditure represents infrastructure it is more appropriately treated as a stock. Futagami, Morita, and Shibata (1993) first adopt this approach in an AK model with fixed labor supply, showing how the transitional dynamics are described by a one-dimensional locus. Baxter and King (1993) analyze the dynamics of fiscal policy in a stationary Ramsey model, emphasizing the role of government capital on output. The present analysis also addresses this in a growing economy, though focusing on a broader set of fiscal policies and stressing the intertemporal welfare tradeoffs.

The remainder of the paper is structured as follows. Section 2 sets out the model, while its equilibrium dynamics are characterized in Section 3. Section 4 discusses some of the long-run comparative static properties. Section 5 calibrates the model and considers the numerical effects of a number of policy changes, while Section 6 concludes.

2. The Model

The economy consists of *N* identical individuals, with population growing exponentially at the steady rate $\dot{N} = nN$. Each representative agent has an infinite planning horizon and possesses perfect foresight. He is endowed with a unit of time that can be allocated either to leisure, l_i, or to labor, $(1-l_i)$, and produces output, Y_i, using the Cobb-Douglas production function

$$Y_i = \alpha(1-l_i)^{1-\sigma} K_i^{\sigma} K_G{}^{\eta} \qquad \sigma > 0,\ \eta > 0,\ 1 > \sigma + \eta \tag{1a}$$

where K_i denotes the agent's individual stock of private capital, and K_G is the stock of government capital, such as infrastructure. We assume that the services derived from the latter are not subject to congestion, so that K_G is a pure public good.[2] The producer faces constant returns to scale in the two private factors, and increasing returns to scale, $1+\eta$, in all three factors of production.

The representative agent's welfare is represented by the isoelastic utility function:

$$\Omega \equiv \int_0^{\infty} (1/\gamma)\left(C_i l_i^{\theta} H^{\phi}\right)^{\gamma} e^{-\rho t} dt; \quad \phi > 0,\ \theta > 0;\ -\infty < \gamma < 1;\ 1 > \gamma(1+\phi);\ 1 > \gamma(1+\theta+\phi) \tag{1b}$$

where C_i denotes the consumption of the individual agent at time *t*, *H* denotes the total consumption services of a government-provided consumption good, and the parameters θ and ϕ measure the impacts of leisure and public consumption on the agent's welfare.[3] The remaining constraints on the coefficients appearing in (1b) are imposed to ensure that the utility function is concave in C_i, l_i, and H. This specification is consistent with the allocation of time to labor and leisure being constant along the balanced growth path.

The agent's objective is to maximize (1b) subject to his capital accumulation equation

$$\dot{K}_i = \left[(1-\tau_k)r - n - \delta_K\right]K_i + (1-\tau_w)w(1-l_i) - (1+\tau_c)C_i - T_i \qquad (1c)$$

where r = gross return to capital, w = (before-tax) wage rate, τ_k = tax on capital income, τ_w = tax on wage income, τ_c = consumption tax, $T_i = T/N$ = agent's share of lump-sum taxes (transfers). Equation (1c) embodies the assumption that private capital depreciates at the rate δ_K, so that with the growing population the net after-tax private return to capital is $(1-\tau_k)r - n - \delta_K$.

Performing the optimization, yields:

$$C_i^{\gamma-1} l_i^{\theta\gamma} H^{\phi\gamma} = \lambda_i(1+\tau_c) \qquad (2a)$$

$$\theta C_i^{\gamma} l_i^{\theta\gamma-1} H^{\phi\gamma} = \lambda_i w(1-\tau_w) \qquad (2b)$$

$$r(1-\tau_k) - n - \delta_K = \rho - \frac{\dot{\lambda}_i}{\lambda_i} \qquad (2c)$$

Equation (2a) equates the marginal utility of consumption to the individual's tax-adjusted shadow value of wealth, λ_i, while (2b) equates the marginal utility of leisure to its opportunity cost, the after-tax real wage, valued at the shadow value of wealth.[4] The third equation is the standard Keynes-Ramsey consumption rule, equating rate of return on consumption to the after-tax rate of return on capital. Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality condition must be imposed:

$$\lim_{t\to\infty} \lambda_i K_i e^{-\rho t} = 0 \qquad (2d)$$

Aggregating over the individual production functions, (1a), aggregate output, Y, is

$$Y = NY_i = \alpha\left[(1-l)N\right]^{1-\sigma} K^{\sigma} K_G^{\eta} \qquad (3)$$

where $K = NK_i$ denotes aggregate capital. The equilibrium real return to private capital and the real wage are thus respectively:

$$r = \frac{\partial Y}{\partial K} = \frac{\sigma Y}{K} = \frac{\sigma Y_i}{K_i}; \quad w = \frac{\partial Y}{\partial[N(1-l)]} = \frac{(1-\sigma)Y}{N(1-l)} = \frac{(1-\sigma)Y_i}{(1-l)} \qquad (4)$$

The government accumulates capital, which depreciates at the rate δ_G, in accordance with

$$\dot{K}_G = G - \delta_G K_G \tag{5}$$

where G denotes the gross rate of government investment expenditure. It finances its gross expenditure flows from aggregate tax revenues earned on capital income, labor income, consumption, or lump-sum taxes, subject to its flow constraint:

$$G + H = \tau_k rK + \tau_w w(1-l)N + \tau_c C + T \tag{6}$$

where $C \equiv NC_i$ denotes aggregate consumption. We assume that the government sets its current gross expenditures on the consumption good and the investment good as fixed fractions of output:

$$H = hY \tag{7a}$$

$$G = gY \tag{7b}$$

where h, g are chosen policy parameters. Using (7a) and (7b), together with the optimality conditions (4), we may express the government's flow budget constraint as

$$T = \left[g + h - \tau_k \sigma - \tau_w (1-\sigma) - \tau_c \left(C/Y\right)\right]Y \tag{6'}$$

so that T describes the lump-sum taxes necessary to balance the current budget.

Aggregating (1c) over the N agents and recalling (4 and (6) implies goods market clearing

$$\dot{K} = Y - C - G - H - \delta_K K \tag{8}$$

and substituting (7a), (7b) into (8), we may write the growth rate of private capital as

$$\frac{\dot{K}}{K} = \left(1 - g - h - \frac{C}{Y}\right)\frac{Y}{K} - \delta_K \tag{9a}$$

Likewise, substituting (7b) into (5), the growth rate of public capital may be written as:

$$\frac{\dot{K}_G}{K_G} = g\frac{Y}{K_G} - \delta_G . \tag{9b}$$

3. Equilibrium Dynamics

Our objective is to analyze the dynamics of the aggregate economy about a stationary growth path. Along such an equilibrium path, aggregate output, private capital stock, and public capital are assumed to grow at the same constant rate, so that the output-capital ratio and the ratio of public capital to private capital remain constant, while the fraction of time devoted to leisure also remains constant. Taking percentage changes of the aggregate production function, the long-run equilibrium growth rate of output, private and public capital, ψ, is:

$$\psi = \left(\frac{1-\sigma}{1-\sigma-\eta}\right)n \tag{10}$$

We shall show that one condition for the dynamics to be stable is that $\sigma + \eta < 1$, in which case the long-run equilibrium growth rate $\psi > 0$. As long as government capital is productive, (10) implies that long-run per capita growth is positive as well.

To analyze the transitional dynamics of the economy about its balanced growth path, we express the system in terms of the stationary variables: (i) the fraction of time devoted to leisure, *l*, and (ii) the scale-adjusted per capita quantities $k \equiv K/N^{((1-\sigma)/(1-\sigma-\eta))}$; $k_G \equiv K_G/N^{((1-\sigma)/(1-\sigma-\eta))}$; $y \equiv Y/N^{((1-\sigma)/(1-\sigma-\eta))}$. Using this notation, the scale-adjusted output can be written as:[5]

$$y = \alpha(1-l)^{1-\sigma}k^{\sigma}k_g^{\eta} \tag{11}$$

In an expanded Appendix (available on request) we show how the equilibrium dynamics can be expressed as the following system in the redefined stationary variables, k, k_g, l:

$$\frac{\dot{k}}{k} = (1-c-g-h)\frac{y}{k} - \delta_K - \psi \tag{12a}$$

$$\frac{\dot{k}_g}{k_g} = g\frac{y}{k_g} - \delta_G - \psi \tag{12b}$$

$$\dot{l} = F(l)\left\{\left((1-\tau_k)\sigma - [1-\gamma(1+\phi)]\left[\sigma(1-c-g-h) + \frac{\eta g k}{k_g}\right]\right)\frac{y}{k} - \delta_K\left(1-\sigma[1-\gamma(1+\phi)]\right)\right.$$

$$\left. + \delta_G\eta[1-\gamma(1+\phi)] - \left([(1-\sigma)[1-\gamma(1+\phi)]+\gamma]n + \rho\right)\right\} \tag{12c}$$

$$c = \left(\frac{1-\sigma}{\theta}\right)\left(\frac{l}{1-l}\right)\left(\frac{1-\tau_w}{1+\tau_c}\right) \tag{12d}$$

where[6] $c \equiv \frac{C}{Y}$; $F(l) \equiv \frac{l(1-l)}{(1-\gamma)-(1-\sigma)[1-\gamma(1+\phi)]l-\theta\gamma(1-l)}$

Equation (12d) is obtained by dividing the optimality conditions (2a) and (2b), while noting (4). It asserts that the marginal rate of substitution between consumption and leisure, $\theta C_i/l$, which grows with per capita consumption, must equal the tax-adjusted wage rate, which from (4) grows with per capita income. With leisure being complementary to consumption in utility, the equilibrium consumption-output ratio thus increases with leisure.

The steady state to this economy, denoted by "~" can be summarized by:

$$(1-\tilde{c}-g-h)\left(\frac{\tilde{y}}{\tilde{k}}\right) = \delta_K + \psi \tag{13a}$$

$$g\frac{\tilde{y}}{\tilde{k}_g} = \delta_G + \psi \tag{13b}$$

$$(1-\tau_k)\sigma\left(\frac{\tilde{y}}{\tilde{k}}\right) = \delta_K + \rho + [1-\gamma(1+\phi)]\psi + \gamma n \tag{13c}$$

together with the production function, (11), and (12d). These five equations determine the steady-state equilibrium in the following sequential manner. First, (13c) determines the output-capital ratio so that the long-run net return to private capital equals the rate of return on consumption. Having determined the output-capital ratio, (13a) determines the consumption-output ratio consistent with the growth rate of capital necessary to equip the growing labor force and replace depreciation, while (13b) determines the corresponding equilibrium ratio of public to private capital. Given $\tilde{c}$, (12d) determines the corresponding allocation of time, l. Having obtained y/k, k_g/k, l, the production function then determines k, with k_g then being obtained from (13b).[7]

Linearizing around the steady state, $\tilde{l}$, $\tilde{k}$, $\tilde{k}_g$, the dynamics may be approximated by:

$$\begin{pmatrix} \dot{k} \\ \dot{k}_g \\ \dot{l} \end{pmatrix} = \begin{pmatrix} -(1-\sigma)\dfrac{\tilde{y}}{\tilde{k}}[1-\tilde{c}-g-h] & \eta\dfrac{\tilde{y}}{\tilde{k}_g}[1-\tilde{c}-g-h] & \dfrac{-\tilde{y}}{1-\tilde{l}}\left[(1-\sigma)(1-\tilde{c}-g-h)+\dfrac{\tilde{c}}{\tilde{l}}\right] \\ \dfrac{g\sigma\tilde{y}}{\tilde{k}} & \dfrac{g(\eta-1)\tilde{y}}{\tilde{k}_g} & -\dfrac{g(1-\sigma)\tilde{y}}{1-\tilde{l}} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} k-\tilde{k} \\ k_g-\tilde{k}_g \\ l-\tilde{l} \end{pmatrix} \quad (14)$$

where $$a_{31} \equiv -\frac{F\tilde{y}}{\tilde{k}^2}\left\{G(1-\sigma)+[1-\gamma(1+\phi)]\frac{\eta g\tilde{k}}{\tilde{k}_g}\right\}; \quad a_{32} \equiv \frac{F\tilde{y}\eta}{\tilde{k}\tilde{k}_g}\left\{G+[1-\gamma(1+\phi)]\frac{g\tilde{k}}{\tilde{k}_g}\right\};$$

$$a_{33} \equiv \frac{F\left(\tilde{y}/\tilde{k}\right)}{1-\tilde{l}}\left\{-G(1-\sigma)+[1-\gamma(1+\phi)]\frac{\sigma\tilde{c}}{\tilde{l}}\right\}; \; G \equiv (1-\tau_k)\sigma-[1-\gamma(1+\phi)]\left[\sigma(1-\tilde{c}-g-h)+\frac{\eta g\tilde{k}}{\tilde{k}_g}\right]$$

We can readily establish that the determinant of the matrix has the same sign as $(1-\sigma-\eta)$, so that provided $\eta < 1-\sigma$ the determinant is positive, which means that there are either 3 positive or 1 positive roots. This condition imposes an upper bound on the positive externality generated by government capital. Due to the complexity of the system we cannot find a simple intuitive condition to rule out the explosive growth case of three positive roots, though in all of the simulations carried out over a wide range of plausible parameter sets, 1 positive and 2 negative roots were always obtained. Thus since the system features two state variables, k and k_g and one jump variable, l, we are confident that the equilibrium is generally characterized by a locally unique stable saddlepath. Henceforth we assume that the stability properties are ensured so that we can denote the two stable roots by μ_1, μ_2, with $\mu_2 < \mu_1 < 0$.

4. Some Steady State Fiscal Effects

Table 1 summarizes the long-run effects of fiscal changes on key economic variables. By the nature of the non-scale model, the long-run growth rate is unaffected. The responses of the scale-adjusted per capita quantities, k, k_g, and y are, however, important because they describe the effects on the base levels on which the constant steady-state growth rates compound. They represent the accumulated effects on the growth rates during the transition, and as our numerical results shall indicate, they can turn out to be substantial. The responses reported in Table 1 assume that all changes are accommodated by lump-sum taxes. The main results are that an increase in government consumption leads to proportionate increases in the two capital stocks and output, whereas an increase in government investment leads to proportionate increases in private capital and output but leads to a larger relative increase in public capital.[8] An increase in the tax on capital reduces public capital and output proportionately, but leads to a larger relative decline in private capital. In contrast a higher wage or consumption tax reduces both types of capital and output proportionately.[9]

5. Numerical Analysis of Transitional Paths

Further insights into the effects of fiscal policy can be obtained by carrying out numerical analysis of the model. We begin by characterizing a benchmark economy, calibrating the model using the parameters representative of the U.S. economy summarized in Table 2.

The elasticity on capital implies that approximately 35% of output accrues to private capital and the rest to labor, which grows at the annual rate of 1.5%. The elasticity $\eta = 0.20$ on public capital implies that public capital generates a significant externality in production. It is smaller than the extreme value (0.39) suggested by Aschauer (1989) and lies within the range of the consensus estimates; see Gramlich (1994). The elasticity of 0.3 on government consumption implies that the optimal ratio of government consumption to private consumption is 0.3 and is close to the corresponding observed ratio for the US over the 1990s of around 0.28.

The value of $\gamma = -1.5$ implies an intertemporal elasticity of substitution of 0.4, close to that estimated by Ogaki and Reinhart (1998), and well within the standard range of estimates. The elasticity $\theta = 1.75$ accords with the standard value in the business cycle literature and yields an equilibrium fraction of time devoted to leisure of around 0.7, consistent with the empirical evidence.

Our benchmark tax rate $\tau_w = 0.28$ reflects the average marginal personal income tax rate in the United States. Given the complexity of capital income taxes, part of which may be taxed at a lower rate than wages, and part of which at a higher rate, we have chosen the common rate $\tau_k = 0.28$ as the benchmark. The benchmark assumes a zero consumption tax. Government expenditure parameters have been chosen so that the total fraction of net national production devoted to government expenditure on goods and services equals 0.20, close to the fraction in the United States during the 1990s. The breakdown between $h = 0.16$ and $g = 0.04$ is based on the US evidence, suggesting that approximately 80% of government expenditure is on consumption. The annual depreciation rates $\delta_K = 0.05$ and $\delta_G = 0.035$ approximate the average depreciation rates for public and private capital for the United States during recent years.[10]

These parameters lead to the benchmark equilibrium, reported in Table 2.B. The implied fraction of time allocated to leisure, l = 0.71, accords with the empirical evidence, as noted. Also, the consumption-output ratio = 0.66, the output-private capital ratio = 0.52, and the ratio of public to private capital = 0.37, are all plausible and generally consistent with the empirical evidence for the United States suggesting that during recent years, C/Y = 0.67, Y/K = 0.48 and K_g/K = 0.30.[11] The table also reports the two stable eigenvalues, which for the benchmark economy are approximately –0.030, and -0.096. These imply that per capita output and capital converge at the asymptotic rate of approximately 2.3%, consistent with much of the empirical evidence.[12] Finally, the steady-state growth rate, which by the non-scale nature of the economy is independent of policy, equals 2.17%.

Table 3 describes the consequences of various policy changes and includes three measures of economic performance: instantaneous (short-run) welfare, intertemporal (long-run) welfare, and intertemporal fiscal balance. Intertemporal welfare is the representative agent's optimized utility:

$$W \equiv \int_0^\infty Z(t) e^{-\rho t} dt \equiv \int_0^\infty \frac{1}{\gamma}((C/N)\, l^\theta H^\phi)^\gamma e^{-\rho t} dt \tag{15}$$

where $Z(t)$ denotes instantaneous utility (short-run welfare) and C/N, l, H are evaluated along the equilibrium path. The welfare gains reported are calculated as the percentage change in the flow of income necessary to equate the initial levels of welfare (both long run and short run) to what they would be following a policy change and formal expressions are reported in the Appendix.[13]

The other quantity is a measure of the intertemporal fiscal balance. Recalling equation (6'), T represents the amount of lump-sum taxation (or transfers) necessary to finance the primary deficit and is therefore a measure of *current* fiscal imbalance. Defining

$$V \equiv -\int_0^\infty T(t) e^{-\int_0^t s(u)(1-\tau_k)du} dt = -\int_0^\infty \left[g + h - \tau_k \sigma - \tau_w (1-\sigma) - \tau_c \left(C/Y\right)\right] Y(t) e^{-\int_0^t s(u)(1-\tau_k)du} dt \tag{16}$$

where $s(1-\tau_k) \equiv r(1-\tau_k) - \delta_K$ is the implied equilibrium rate of interest, V measures the present discounted value of the lump-sum taxes or transfers necessary to balance the government budget over time, and thus is a measure of the *intertemporal* fiscal imbalance, expressed as a surplus. For the assumed tax rates and expenditure parameters, current tax revenues exceed government expenditures on goods and services by around 8% of current income.[14] Evaluating (16) along the balanced growth path we find that in the benchmark case, V = 0.314 and thus implies an intertemporal fiscal surplus.[15] In interpreting this, we should bear in mind that our measure of T and therefore V is net of government transfers, which historically for the United States have been of the order of 12% of current income, implying an overall deficit. Table 3 reports percentage changes in the intertemporal fiscal surplus, with a negative change implying a reduction relative to the base.

5.1 Uncompensated Normalized Fiscal Changes

Table 3.A describes various basic policy changes from the benchmark economy. These are uncompensated, meaning that they lead to changes in the government's current fiscal imbalance, T. However, to preserve comparability, they have been standardized so that the two expenditure increases lead to the same decrease, and the three tax increases lead to the same increase, in the government's current surplus ($-T$ in equation (6')), respectively.[16] With expenditures being tied to the same measure (income) the changes in g and h are virtually identical, whereas with the taxes being applied to differential bases, the changes in the normalized tax rates are different. Examples of dynamic transition paths are provided in Figures 1 and 2.[17]

Increase in government consumption expenditure: An increase in h of 0.04 increases long-run private capital, public capital, and output proportionately by 6.6%, while crowding out the long-run private consumption–output ratio by 0.04, with a corresponding reduction in leisure of 1.3 percentage points. The additional government expenditure leads to a substantial deterioration in the government's long-run balance; the intertemporal surplus, V, is reduced by 50%.

The immediate effect of the lump-sum tax financed increase in expenditure is to reduce the private agent's wealth, inducing him to supply more labor, thereby raising the marginal productivity of both types of capital, and raising output. The dynamic responses of this shock are shown in the 4 panels of Fig. 1.A. The phase diagram (i) indicates that the two capital stocks accumulate roughly proportionately. This is also reflected in their growth rates, illustrated in (ii), which both jump initially to around 2.4% and gradually decline to their steady-state rates of 2.17%, as both types of capital accumulate and their respective rates of return decline. The initial increase in labor supply (which remains virtually unchanged thereafter), and the associated decline in consumption, are welfare deteriorating, though this loss is more than offset by the direct benefits from the increase in public consumption expenditure. The initial expenditure increase raises instantaneous welfare by 0.66% and this grows uniformly with the growth in the capital stocks and output, to an asymptotic improvement relative to the benchmark of over 4%, with (15) implying an overall intertemporal welfare gain of nearly 2%.

Increase in government investment expenditure: The normalized increase in g is 0.0396, and apart from the effects on employment and the consumption-output ratio, the effects are dramatically different from those resulting from the comparable increase in h. Long-run private capital and output increase by 44.6%, while public capital increases by 288%, doubling the long-run ratio of public to private capital. The reduction in the intertemporal fiscal surplus is slightly larger, due to the expanded effect on output, and thereby on government expenditure.

The contrast with government consumption is reflected in the dynamics and is highlighted in Fig. 1.B. The initial claim on capital by the government crowds out private investment, so that the growth rate of private capital is reduced below the growth rate of population; the scale-adjusted stock of private capital therefore initially declines. By contrast, public capital initially grows at over 7% (see (ii)). The reduction in initial consumption and leisure, with no public expenditure benefits, leads to an initial reduction in welfare of 2.9% (Fig. 1.B(iv)). As the new public capital is put in place, its productivity raises the growth rate of private capital, and as output and consumption grow, so does welfare. Asymptotically, instantaneous welfare increases 35% above the base level, and the overall intertemporal welfare gain, derived from (15) is around 7.8%.

Increase in Tax on Capital: Row 3 raises the tax on capital income from 0.28 to 0.40. This has a dramatic long-run effect, reducing private capital by about 30% and public capital and output by around 16%, raising both the output-private capital and public capital-private capital ratios by 20%. The reduction in long-run capital reduces the productivity of labor, causing a long-run substitution toward more leisure and consumption. The higher tax revenues increase the government's intertemporal surplus by 80%.[18]

The dynamics are illustrated in Fig. 2.A. Upon impact, the higher capital tax reduces the growth rate of private capital almost to zero, so that the scale-adjusted per capita stock, *k*, begins to fall rapidly. The reduction in labor, all of which occurs virtually on impact, and the reduction in private capital reduces the growth rate of output, and the growth rate of public capital begins to fall as well, so that k and k_g follow the declining paths in Fig. 2.A.(i). As private capital increases in relative scarcity its productivity rises, inducing investment in private capital and thereby restoring its growth rate. This in turn raises the productivity of public capital, the declining growth rate of which is reversed after 15 periods. In the short-run the higher capital tax raises welfare by nearly 4%. This is because of the short-run increase in leisure, which induces an increase in consumption. However, the steady decline in relative consumption implies that over time, instantaneous welfare falls rapidly relative to the benchmark, declining asymptotically by about 15%. The decline in intertemporal welfare, (15), is equivalent to a 3.56% reduction in base income.

Increase in Tax on Wage Income and Consumption: Rows 4 and 5 consider an increase in the wage tax from 0.28 to 0.349, and the introduction of a consumption tax of 0.0638, respectively, both of which increase *T* by the same amount as did the capital income tax . These taxes are qualitatively similar and since in many respects the dynamics of the economy are similar to those of a decrease in *h*, they are not illustrated; in particular public and private capital follow similar contractionary paths.

The contrasts with the capital income tax are quite striking. Since both τ_w and τ_c fall more directly on consumers, they have a significantly greater negative impact on labor supply. Furthermore, although all three taxes raise the same impact on the current surplus, the capital income tax has a far more substantial effect on the intertemporal surplus. This is because of its more adverse effect on output, and thus the implied reduction in the level of government spending. The welfare effects differ in two respects. Whereas, the capital income tax leads to short-run gains followed by large long-run losses, wage and consumption taxes lead to smaller, and steadily increasing, welfare losses. Overall, the capital income tax is the least desirable, as has long been argued.[19]

5.2 Compensated Changes

Many policy discussions focus on compensated fiscal changes, meaning that the policy change is accompanied by some other accommodating change so that the current government deficit, *T*, remains unchanged.[20]. Table 3.B describes a number of such changes.

Tax Substitution: Row 1 introduces a 6% consumption tax, which permits income taxes to be reduced to 24.03% to maintain the initial deficit, *T*, unchanged. This change in the tax structure raises long-run output and public capital by 5% and private capital by 10.8%, relative to the benchmark economy. The shift from the wage tax to the consumption tax causes a slight reduction in long-run leisure, with a proportionately slightly larger reduction in the consumption-output ratio. The government's long-run fiscal surplus, *V*, deteriorates slightly.

The dynamics are illustrated in Fig. 2.B. The reduction in the tax on income favors private capital, the growth rate of which rises to over 2.8% on impact. This stimulates the productivity of public capital, the growth rate of which also rises in the short run, though more modestly. The tax switch causes an initial drop in consumption of 1% and an increase in labor supply of 1%, thus leading to an initial deterioration in welfare. However, the higher output level and resulting higher consumption throughout the transition causes current welfare to improve over time, doing so by 3% asymptotically, and a present value equivalent of nearly 1%.

Restructuring Government Expenditure: Row 2 describes the case where the government changes its expenditure priorities, while maintaining the initial deficit. Specifically, it increases its investment expenditure from 4% to 8% of output, and this requires it to reduce its expenditure on consumption from 16% to 11.9% of output. This switch in spending priorities has negligible effects on long-run employment and the consumption-output ratio. It raises long-run private capital and output by 35.9% and public capital by 271.7%. Long-run government fiscal surplus declines slightly, but intertemporal welfare improves by 3.8%. Space limitations preclude an illustration of the dynamics. But we may note that the initial switch toward government investment crowds out private investment, the growth of which is initially reduced to around 1%, and thus the private scale-adjusted capital stock declines. Public capital initially grows at over 6% and as its stock accumulates, the productivity of private capital is enhanced and the incentives to invest increase. Eventually, the decline in the private capital stock is reversed and it begins to accumulate

Alternatively-Financed Modes of Government Consumption Expenditure: Rows 3 – 5 describe the effects of increasing government consumption expenditure, h, from 0.16 to 0.20, under the three alternatives of (i) consumption tax-financing; (ii) wage tax-financing; and (iii) capital tax-financing, such that the initial deficit remains unchanged. These effects can also be compared with those of lump-sum tax financing summarized in Table 3.A, Row 2. The dynamics of the capital stocks and the corresponding time paths for instantaneous welfare are illustrated in Fig. 3.A

The striking contrasts in the effects of government consumption expenditure under alternative modes of finance is dramically illustrated in the phase diagrams in panel i. Suppose that the initial equilibrium is at point A. As discussed previously, a lump-sum tax financed increase in government consumption expenditure is expansionary, increasing private and public capital proportionately by 6.6% along AB. If it is consumption tax-financed, the contractionary effect of the tax just balances the expansionary effect of the expenditure and there is no net change; the economy remains at A.[21] If it is wage tax-financed, private and public capital again move proportionately. However, this time, the contractionary effect dominates the expansionary effect and both stocks decline by around 3% along the direction AC. Finally, if the expenditure is financed by a tax on capital, the contractionary effect on capital is more than proportionate, and the economy evolves along AD, with a larger contraction in both capital goods.

These responses are reflected in the time paths for welfare illustrated in panel ii. As noted before, under lump-sum tax financing there is a uniform steady improvement in current welfare from 0.66% to 4%, relative to the benchmark, which in present value terms is nearly 2%. Although the consumption tax has no effect on activity and therefore no induced impact on welfare, the higher government consumption expenditure being financed does have a direct uniform welfare benefit, the present value of which is 0.42%. Under wage-tax financing, there are small short-run benefits associated with the higher expenditure, but these are more than offset by the longer-run losses as the economy contracts, leading to an overall long-run small welfare loss of 0.46%. The intertemporal tradeoff in welfare is more dramatic when the expenditure is capital tax financed. In that case, the reduction in private capital accumulation and employment, together with the switch in private consumption and the higher government consumption, lead to a short-run welfare gain of about 4.8%. However, this quickly erodes as the economy contracts, leading to an asymptotic reduction of around 8%, and an overall intertemporal welfare loss in present value terms of nearly 1.34%.

Alternatively-Financed Modes of Government Production Expenditure: Rows 6 – 8 describe the corresponding effects of increasing government investment expenditure from 0.04 to 0.08. These can also be compared with those of lump-sum tax financing summarized in Table 2.A, Row 3. The dynamics of capital and instantaneous welfare are illustrated in Fig. 3.B.

While the time paths for the two capital stocks exhibit interesting differences across financing modes, these differences are less dramatic than in the previous case of government consumption expenditure. In all cases government capital increases steadily. Private capital also increases in the long run, although in the case of capital income tax-financing it is modest and follows a substantial temporary decline during the transition.

The time profiles for the instantaneous welfare changes contrast sharply with those of government consumption expenditure. In the short run welfare declines by about 3% for the three cases of lump-sum tax, consumption tax, and wage tax financing. This is because the government investment stimulates employment, leading to substitution from consumption, while providing no initial direct benefits. Over time, as the government investment yields its productive payoff and output rises, instantaneous welfare in all cases rises steadily. With lump sum tax financing it converges to a higher long-run level of 35% above that of the benchmark economy, with a present value of 7.85%, as shown in Table 3.A. With consumption tax, wage tax, and capital income financing the respective improvements in asymptotic welfare are 30%, 27%, and 18%, respectively, yielding the corresponding intertemporal welfare gains of 5.92%, 4.83%, and 3.77%, respectively.

5.3 Parameter Sensitivity Analysis

Two key parameters are ϕ, the elasticity of utility with respect to government expenditure, and η, the productivity of public capital. While the values we have chosen are plausible, the empirical evidence on them is sparse. The understanding of our results is therefore enhanced by subjecting them to some sensitivity analysis.

The key issue concerns the actual size of the government, as represented by the expenditure parameters, g and h, relative to the socially optimal size, which depends crucially upon the elasticities ϕ,η. In the Appendix we show that the chosen benchmark parameters, $\phi = 0.3, \eta = 0.2$ imply the socially optimal values $\hat{g} = 0.0974$, $\hat{h} = 0.164$, and hence the optimal ratio of public to private capital, $\hat{k_g}/k = 0.645$. Thus whereas the actual fraction of output devoted to government consumption expenditure (0.16) is close to its social optimum, the fraction of output devoted to government investment (0.04) is well below its social optimum, implying that the actual ratio of public to private capital is also too small (for these assumed parameters). While underinvestment in public capital may indeed be characteristic of the United States, it is this expenditure imbalance that accounts for the relative desirability of government investment over government consumption.

Suppose instead, we reduce the productivity of government capital to $\eta = 0.1$, an assumption that may be consistent with substantial congestion in public capital. In this case, the socially optimal expenditure rates are $\hat{g} = 0.05$, $\hat{h} = 0.17$, implying $\hat{k_g}/k = 0.33$, and both forms of government expenditure are relatively close to their respective social optima. We now find that the intertemporal benefits of standardized increases in the two forms of government expenditures are approximately equal (about 1.7% in each case), although the time paths of instantaneous welfare follow the patterns of Figs. 1.A and 1.B. The reason that there are any welfare gains is because we are maintaining the income tax rates at their initial (non-optimal) levels. If instead, the tax rates are set optimally, then any deviation from the socially optimal government expenditure levels is welfare deteriorating.

For other (not necessarily plausible) combinations of ϕ and η, one may infer that increasing government consumption expenditure is preferable. For example, maintaining $\eta = 0.1$ and increasing ϕ to 0.45 we find that $\hat{g} = 0.05, \hat{h} = 0.24$ with $\hat{k_g}/k = 0.33$. Now it is government consumption expenditure that is below its social optimum, and that yields the greater benefits (4.33% versus 2.15% for government investment). In other extreme cases an increase in either form of expenditure is welfare deteriorating. For example, if $\phi = 0.15, \eta = 0.05$, so that neither form of government expenditure is particularly beneficial, standardized increases in g and h from their benchmark levels reduces welfare by 1.88% and 1.65%, respectively.

We may also note that even for the benchmark parameters, $\phi = 0.3, \eta = 0.2$, where both forms of government expenditure are relatively desirable, increases in g or h sufficiently above their respectively socially optimal values may be welfare deteriorating. The extent to which this occurs depends upon where tax rates are set relative to their respective optimal values, as set in footnote 24.

We have also conducted sensitivity analysis with respect to θ the elasticity of labor supply. Here we find that our conclusions with respect to the impact of government policy are largely robust with respect to this parameter, as also is the pattern of response of labor supply. The reason is that this parameter influences the equilibrium primarily through its impact on the equilibrium labor supply, which is subject to relatively small changes.

6. Conclusions

This paper has analyzed the effects of fiscal policies in a non-scale growing economy with public and private capital. We have characterized the equilibrium dynamics and have been concerned with contrasting two types of government expenditure – expenditure on an investment good and expenditure on a consumption good – under different modes of tax financing. Most of our attention has focused on the numerical simulations of a calibrated economy, which we have found to provide helpful intuition. We have obtained many results, of which the following merit comment.

1. Despite the fact that fiscal policy in such an economy has no effect on the long-run equilibrium growth rate, the slow rate of convergence implies that fiscal policy has a sustained impact on growth rates for substantial periods during the transition. These accumulate to substantial effects on the long-run equilibrium *levels* of crucial economic variables, including welfare.

2. As examples of the accumulated impacts of policy, an increase in government investment from 0.04 to 0.08 of output raises the long-run level of output by 44.6%. Raising the tax on capital income from 0.28 to 0.40 reduces long-run output by 16%.

3. For the calibrated economy, devoting a fixed fraction of output to government investment is better than allocating the same resources to government consumption. However, the time paths of the respective benefits are different. The benefits of (lump-sum tax-financed) government

consumption are uniformly positive; government investment involves short-run losses that are more than offset over time. But these comparisons depend upon the sizes of the two government expenditures, relative to their respective first-best optima and may be reversed in other cases.

4. The time paths and growth rates of private and public capital contrast sharply for policies such as g and τ_k, which impact on one or other capital stock directly; they move closely for those fiscal shocks -- h, τ_w, τ_c -- which do not impact directly on either form of capital. The most dramatic contrasts in the time paths for the two types of capital occur with respect to an increase in government consumption expenditure, under the four alternative modes of tax financing. Long-run stocks of both *increase p*roportionately under lump-sum taxes, *remain unchanged* under consumption tax-financing, *decrease proportionately* under wage tax-financing, and lead to a more than proportional decline in private capital under capital tax-financing.

5. Our numerical simulations suggest the following welfare ranking for the different modes of financing. For either form of expenditure, lump-sum tax financing dominates consumption tax financing, which in turn dominates wage tax financing and finally capital tax financing.

6. The analysis highlights the intertemporal welfare tradeoffs involved in policy changes. For example, both the substitution of a consumption tax for a uniform reduction in the income tax and a revenue-neutral switch from government consumption to government investment lead to a short-run welfare losses, which in both cases are more than offset by long-run welfare gains. This is a consequence of the growth generated during the subsequent transition.

7. The analysis also brings out tradeoffs between private intertemporal welfare gains and the government's long-run fiscal balance. Table 3 shows that long-run welfare gains are mostly (but not always) accompanied by a deterioration in the long-run government surplus and vice versa. The extent to which this occurs depends in part upon the actual fiscal mix relative to its social optimum. It may be possible to design fiscal policy that is welfare improving, while leaving the government's long-run fiscal balance unchanged.[22]

To conclude, it is useful to compare the numerical results we have obtained to the empircal results of Kneller, Bleaney, and Gemmell (1999). Specifically, they find that whereas productive government expenditure raises the growth rate (of output) and distortionary taxation reduces it, non-distortionary taxation and non-productive government expenditure have only weak statistically insignificant positive effects. These conclusions are mirrored by present results. By assumption, lumpsum taxation has no effects on the growth rate. From Panels (ii) in Figs 1 we see that that the short-run effects of government consumption is to raise the growth rate of output by about 0.10 percentage points over the first 5 years, whereas a comparable increase in productive government expenditure will raise the growth rate by over 1 percentage point over that same time period. From Fig 2.A we also find that a tax on capital income will reduce the growth rate of output by almost 0.5 percentage points for an extended period of time. Thus, while these authors focus on a narrower range of issues, their empirical results provide quite compelling support for the present model.

Finally, we may observe that the quantitative implications we have obtained for the growth rates and welfare along the transitional path in this non-scale model parallel those obtained in simple endogenous growth models in which the economy is always on its balanced growth path, and fiscal policy exerts permanent effects on the equilibrium growth rates; (c.f. Turnovsky (2000, Table 1). This suggests that to distinguish empirically between these two models will be difficult and will require a careful analysis of the dynamic relationship between fiscal variables and growth.

Appendix

A.1 Steady-State Equilibrium in the Centrally Planned Economy

To help understand the numerical results it is useful to set out the steady-state equilibrium for the centrally planned economy in which the planner controls resources directly. The optimality conditions for such an economy consist of equations (11), (13a), (13b), together with:

$$c \equiv \frac{C}{Y} = \left(\frac{1-\sigma}{\theta}\right)\left(\frac{l}{1-l}\right)\upsilon \tag{A.1a}$$

$$\upsilon\sigma\left(\frac{y}{k}\right) = \delta_K + \rho + \left[1-\gamma(1+\phi)\right]\psi + \gamma n \tag{A.1b}$$

$$\upsilon = 1 + (q-1)g + (\phi c - h) \tag{A.1c}$$

$$\upsilon\sigma\frac{y}{k} - \delta_K = \upsilon\frac{\eta}{q}\frac{y}{k_g} - \delta_G \tag{A.1d}$$

where υ denotes the shadow price of a marginal unit of output in terms of capital, q denotes the shadow price of public capital in terms of private capital, (and tildes are omitted). These equations determine the steady-state solutions for $c, y, l, k, k_g, \upsilon, q$ in terms of the arbitrarily set expenditure parameters g and h. Equations (A.1a) and (A.1b) are analogous to (12d) and (13c), with the after-tax prices being replaced by υ, determined in (A.1c). In the absence of government expenditure, $\upsilon = 1$. Otherwise, the social value of a unit of output deviates from the social value of capital due to the claims of government on output and the value this has for the consumer.[23] The final equation equates the long-run net social returns to investment in the two types of capital.

Choosing the expenditure shares g and h optimally, yields the additional conditions

$$\hat{h} = \phi c; \quad \hat{q} = 1, \text{ and hence } \upsilon = 1. \tag{A.1e}$$

The marginal benefit of government consumption expenditure should equal its resource cost, and the shadow values of the two types of capital should be equal. Substituting these conditions into (13b), (A.1b), and (A.1d), the optimal share of output devoted to government production expenditure is[24]

$$\hat{g} = \frac{\eta[\delta_G + \psi]}{\delta_G + \rho + [[1-\gamma(1+\phi)]\psi + \gamma n]} \tag{A.1f}$$

For the benchmark parameters the socially optimal values are $\hat{g} = 0.0974$, $\hat{h} = 0.164$.

A.2 Welfare Changes as Measured by Equivalent Variations in Income Flows

We assume that the economy is initially on a balanced growth path, which is growing at the equilibrium rate, ψ, implying the corresponding level of base intertemporal welfare

$$\begin{aligned}\frac{1}{\gamma}\int_0^\infty \left((C/N)_b l_b^\theta H_b^\phi\right)^\gamma e^{-\rho t} dt &= \frac{1}{\gamma}\int_0^\infty \left(c_b l_b^\theta h_b^\phi Y_0^{(1+\phi)}\right)^\gamma e^{[\gamma[(1+\phi)\psi - n]-\rho]t} dt \\ &= \frac{1}{\gamma}\frac{\left(c_b l_b^\theta h_b^\phi Y_0^{(1+\phi)}\right)^\gamma}{\rho - \gamma[(1+\phi)\psi - n]} \equiv W(c_b, l_b, h_b; Y_0) = W_b\end{aligned} \tag{A.2}$$

where c_b, l_b, h_b are constant along the initial balanced growth path, Y_0 is the level of income at time $t = 0$, and for simplicity we set $N_0 = 1$. Intertemporal welfare following a policy change is given by

$$\frac{1}{\gamma}\int_0^\infty \left(\left(C(t)/N\right)l(t)^\theta H(t)^\phi\right)^\gamma e^{-\rho t}dt = \frac{1}{\gamma}\int_0^\infty \left(c(t)l(t)^\theta h^\phi Y(t)^{(1+\phi)}e^{-nt}\right)^\gamma e^{-\rho t}dt \equiv W(c_a,l_a,h_a;Y_a) = W_a \quad \text{(A.3)}$$

where $c_a, l_a, h_a,\ Y_a$ denote the trajectories along the resulting transitional path and which in general are time-varying. As a means of comparing these two levels of utility, we wish to determine the percentage change in the initial income level, Y_0, (and therefore in the income flow over the entire base path), such that the agent is indifferent between (c_b, l_b, h_b) and (c_a, l_a, h_a). That is, we seek to find ζ such that

$$W(c_b,l_b,h_b;\zeta Y_0) = W(c_a,l_a,h_a;Y_a) = W_a \quad \text{(A.4)}$$

Performing this calculation yields

$$\frac{1}{\gamma}\int_0^\infty \left(c_b l_b^\theta h_b^\phi \left(\zeta Y_0\right)^{(1+\phi)}\right)^\gamma e^{[\gamma[(1+\phi)\psi-n]-\rho]t}dt = \frac{1}{\gamma}\frac{\left(c_b l_b^\theta h_b^\phi \left(\zeta Y_0\right)^{(1+\phi)}\right)^\gamma}{\rho-\gamma[(1+\phi)\psi-n]} = \zeta^{\gamma(1+\phi)}W_b = W_a$$

and hence $$\zeta - 1 = \left(W_a/W_b\right)^{1/\gamma(1+\phi)} - 1 \quad \text{(A.5)}$$

(A.5) determines the change in the base income level, and thus in the income level at all points of time, that will enable the agent's base level of intertemporal welfare to equal that following the policy change. Stating the comparison as in (A.4), highlights the fact that the fractions, c, l, and h are assumed to remain fixed at their initial base levels.

The short-run welfare gain is calculated analogously, by

$$\xi - 1 = \left(Z_a/Z_b\right)^{1/\gamma(1+\phi)} - 1 \quad \text{(A.6)}$$

where $Z_b \equiv \left(c_b l_b^\theta h_b^\phi Y_0^{(1+\phi)}\right)^\gamma$, $Z_a \equiv \left(c(0)l(0)^\theta h^\phi Y(0)^{(1+\phi)}\right)^\gamma$.

Footnotes

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[1] This benchmark was established in early work by Barro (1991), Mankiw, Romer, and Weil (1992). Subsequent studies suggest that the convergence rates are more variable and sensitive to time periods and the set of countries than originally suggested and a wider range of estimates have been obtained. For example, Islam (1995) estimates the rate of convergence to be 4.7% for non-oil countries and 9.7% for OECD economies. Evans (1997) obtains estimates of the convergence rate of around 6% per annum.

[2] This assumption is a polar one, since almost all public services are subject to some congestion. Eicher and Turnovsky (2000) develop a simple nonscale growth model in which productive public expenditure, introduced as flow (as in Barro 1990), is subject to two types of congestion. They discuss the effects of both types of congestion on the long-run growth rate, the speed of convergence and the equilibrium capital stock. Although they do not address the issue, their model also implies that congestion reduces the productivity of the public input. The reduction in the productivity parameter, η, briefly considered in Section 5.3 can be viewed as an initial attempt to take account of congestion in this model. Clearly, this is an important issue and is a dimension in which the present model of public capital could be fruitfully extended.

[3] Analogous to government capital, we assume that the government consumption good is a pure public good, not subject to congestion. The parameter γ is related to the intertemporal elasticity of substitution, *e* say, by $e = 1/(1-\gamma)$.

[4] Since all agents are identical, each allocates his time identically, and henceforth we can drop the agent's subscript to *l*.

[5] Under constant returns to scale, these expressions reduce to per capita quantities, as in the usual neoclassical model.

[6] We shall assume that $F(l) > 0$. Sufficient conditions that ensure this is so include: (i) $\gamma < 0$, and (ii) $\sigma > \phi$, both of which are plausible empirically, and imposed in our numerical simulations.

[7] Moreover, given the restrictions on utility and production this solution is unique, and economically viable in the sense of all quantities being non-negative, and in particular the fractions $0 < \tilde{c} < 1, 0 < \tilde{l} < 1$, if and only if $(1-g-h)/\sigma(1-\tau_k) > (\delta_K + \psi)/(\delta_K + \rho + [1-\gamma(1+\phi)]\psi + \gamma n)$. This condition holds throughout our simulations.

[8] The fact that output changes in proportion to the two capital stocks, despite the less proportionate change in employment, is a consequence of the overall increasing returns to scale of the production function in the three factors.

[9] The quantitative magnitudes of these effects corresponding to the discrete policy changes evaluated at the benchmark steady-state values for the model as calibrated in Section 5 are provided in Table 3.A; see in particular columns 1, 5, 6, and 7. The numerical magnitudes are discussed in conjunction with the simulations discussed below.

[10] Data on private and public stocks of capital have been obtained from the Table "Real Net Stock of Fixed Reproducible Tangible Wealth for the US" in the *Survey of Current Business*, May 1997. Data on gross public and private investment have been obtained from the *2002 US Economic Report of the President* (see Tables B2 and B21). Using these data we have computed the average annual depreciation rates on private and public capital over the period 1990- 1995 to be around 4.7% and 3.4%, respectively. A depreciation rate of 5% is a common benchmark in the real business cycle literature; see e.g. Cooley (1995). We also experimented with higher depreciation rates (double), with little effect on our conclusions, other than to yield a somewhat higher rate of convergence, but still consistent with the empirical evidence.

[11] These estimates have been computed from the sources cited in footnote 10.

[12] This estimate is based on the measure of convergence proposed by Eicher and Turnovsky (1999). An interesting feature of our results is that both stable roots are remarkably constant over all the fiscal exercises conducted. The unstable root is much larger, and much more variable across policies. This implies that the speeds of adjustments are fairly uniform across permanent fiscal changes, though they may vary across temporary policy changes. The speed of convergence is more sensitive to structural changes, such as changes in the productive elasticities.

[13] As an explicit example, the long-run welfare gain of 1.98% reported in Table 3.A, Row 1 is obtained as follows. The base welfare in the absence of any policy change evaluated using (A.2) yields $W_0 = -1932.62$, while the welfare following the increase in *h* from 0.16 to 0.20, evaluated in accordance with (A.3) yields $W_1 = -1860.24$. Applying (A.4) yields an equivalent variation of income welfare gain of 1.98%. The short-run welfare gain is computed analogously.

[14] This accords approximately with recent data on the surplus of government account on goods and services.

[15] Since both *V* and *Y* are proportional to α, set arbitrarily to 1, we interpret *V* as being measured in units of income.

[16] Given our choice of units, the base government surplus is 0.0188, which corresponds to 8% of GDP. Expenditure increases are standardized to reduce the surplus by 0.0091, thereby reducing the surplus-GDP ratio by 4%, while tax increases are standardized to increase the surplus by 0.0091, thereby increasing the surplus-GDP ratio by 4%.

[17] One striking pattern throughout all simulations is that the labor supply responds to any shock almost completely upon impact. This is because for plausible parameter values the elements a_{31}, a_{32} in the transitional matrix in (14) are both small relative to $a_{33} > 0$; there is little feedback from the changing stocks of capital to labor supply. The dynamics of labor can thus be approximated by the equation, $dl(t)/dt = a_{33}(l(t) - \tilde{l})$, which for stability requires that *l* jump instantaneously to steady state.

[18] Policy discussions have raised the possibility that in a growing economy *reducing* the income tax, particularly on capital, will stimulate growth and thereby improve the long-run government balance.

This possibility, known as 'dynamic scoring' has been investigated in an AK model by Bruce and Turnovsky (1999), who find that it may hold only in the unlikely case that the intertemporal elasticity of substitution exceeds unity. We have investigated this possibility in a number of simulations and never found it to obtain. This is because any positive effects on the growth rate occur only along the transitional path and are therefore only temporary.

[19] Chamley (1986) originally showed that in the long-run the optimal tax on capital is zero. This result depends upon the absence of externalities and modifications of this proposition in the presence of externalities and growth has been addressed extensively in the recent growth literature.

[20] That is the deficit is held constant at its initial base level (see footnote 16). As we shall see this, has implications for the government's long-run fiscal position. An alternative exercise for which the present model is well-suited is to impose the discipline of long-run or intertemporal revenue neutrality on fiscal policy. While this may be more interesting from a theoretical standpoint, it is probably less relevant practically, in the sense that politicians are typically more focused on short-run constraints. But both aspects merit investigation.

[21] The fact that a consumption tax-financed increase in government consumption expenditure has no effect on (scale-adjusted) private capital, public capital, or output, can be shown analytically in an expanded version of this paper.

[22] The tradeoff between long-run fiscal balance and welfare for a simple static AK growth model has been discussed in detail by Bruce and Turnovsky (1999).

[23] With government expenditure tied to output, an increase in output diverts resources away from private consumption, leaving $1-g-h$ available to the agent. Offsetting this, public investment augments the stock of public capital, valued at qg, and provides utility benefits equal to ϕc making the overall value of output to capital as described in (A.1c).

[24] Provided $\gamma < 0$, (A.1f) implies $\hat{g} < \eta$. Equating (A.1a) to (12d) and (A.1b) with (A.1c) to (13c), the decentralized economy can replicate the steady state of the centrally planned economy if and only if the tax rates satisfy $1-\tau_k = (1-\tau_w)/(1+\tau_c) \; = s$. In the case that expenditures are set optimally, this condition simplifies to $\hat{\tau}_k = 0, \; \hat{\tau}_c = -\hat{\tau}_w$. That is, capital income should remain

untaxed, while the tax on consumption must be equal and opposite to that on wage income. The optimal tax must also be consistent with the government budget constraint. Given that the constraints on tax rates, $\tau_w < 1, \tau_c > 0$, this may well require the supplementation by lump-sum taxation in order to sustain the equilibrium.

Table 1

Long-run Fiscal Changes

(lump-sum Tax financed)

	g	h	τ_k	τ_w	τ_c
$\frac{dl}{(1-l)}$	$-\frac{l}{c}<0$	$-\frac{l}{c}<0$	$\frac{l(1-c-g-h)}{c(1-\tau_k)}>0$	$\frac{l}{1-\tau_w}>0$	$\frac{l}{1+\tau_c}>0$
$\frac{dk}{k}$	$\frac{\left[(1-\sigma)\frac{l}{c}+\frac{\eta}{g}\right]}{1-\sigma-\eta}>0$	$\frac{(1-\sigma)\frac{l}{c}}{1-\sigma-\eta}>0$	$-\frac{[c(1-\eta)+(1-\sigma)(1-c-g-h)l]}{c(1-\tau_k)(1-\sigma-\eta)}<0$	$-\frac{(1-\sigma)}{(1-\sigma-\eta)}\frac{l}{(1-\tau_w)}<0$	$-\frac{(1-\sigma)}{(1-\sigma-\eta)}\frac{l}{(1+\tau_c)}<0$
$\frac{dk_g}{k_g}$	$\frac{(1-\sigma)\left[\frac{l}{c}+\frac{1}{g}\right]}{1-\sigma-\eta}>0$	$\frac{(1-\sigma)\frac{l}{c}}{1-\sigma-\eta}>0$	$-\frac{[c\sigma+(1-\sigma)(1-c-g-h)l]}{c(1-\tau_k)(1-\sigma-\eta)}<0$	$-\frac{(1-\sigma)}{(1-\sigma-\eta)}\frac{l}{(1-\tau_w)}<0$	$-\frac{(1-\sigma)}{(1-\sigma-\eta)}\frac{l}{(1+\tau_c)}<0$
$\frac{dy}{y}$	$\frac{\left[(1-\sigma)\frac{l}{c}+\frac{\eta}{g}\right]}{1-\sigma-\eta}>0$	$\frac{(1-\sigma)\frac{l}{c}}{1-\sigma-\eta}>0$	$-\frac{[c\sigma+(1-\sigma)(1-c-g-h)l]}{c(1-\tau_k)(1-\sigma-\eta)}<0$	$-\frac{(1-\sigma)}{(1-\sigma-\eta)}\frac{l}{(1-\tau_w)}<0$	$-\frac{(1-\sigma)}{(1-\sigma-\eta)}\frac{l}{(1+\tau_c)}<0$

Tab1e 2

A. Base Parameter Values

Production parameters	$\alpha = 1,\ \sigma = 0.35,\ \eta = 0.20,\ \delta_K = 0.05,\ \delta_G = 0.035,\ n = 0.015$
Preference parameters	$e = 1/(1-\gamma) = 0.4,$ i.e.$\gamma = -1.5,\ \rho = 0.04,\ \theta = 1.75,\ \phi = 0.3$
Government Expenditure rates	$g = 0.04,\ h = 0.16$
Tax Rates	$\tau_k = 0.28,\ \tau_w = 0.28,\ \tau_c = 0$

B. Base Equilibrium

l	c	y/k	ψ	k_g/k	$\mu_1,\ \mu_2$
0.712	0.663	0.521	O.0217	0.368	-0.0296, -0.0960

Table 3

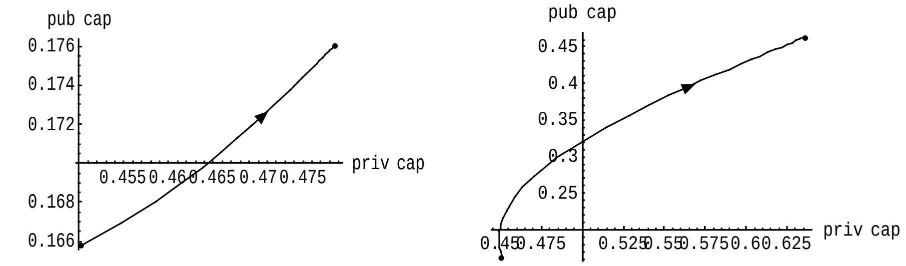
A. Uncompensated Standardized Changes

	Δl % points	$\Delta(c)$	$\Delta\left(\frac{y}{k}\right)$ %	$\Delta\left(\frac{k_g}{k}\right)$ %	$\Delta(k)$ %	$\Delta(k_g)$ %	$\Delta(y)$ %	$\Delta(V)$ %	Short-run welf. gains percent	Long-run welf. gains percent
1. Increase in *h* from 0.16 to 0.20	-1.29	-0.04	0	0	6.57	6.57	6.57	-49.7	0.66	1.98
2. Increase in g from 0.04 to 0.0796	-1.23	-0.0396	0	199.0	44.6	287.7	44.6	-51.3	-2.90	7.85
3. Increase in τ_k From 0.28 to 0.40	0.69	0.0229	20.0	20.0	-30.2	-16.2	-16.2	80.2	3.98	-3.56
4. Increase in τ_w From 0.28 to 0.349	2.02	0	0	0	-10.0	-10.0	-10.0	55.1	-0.81	-2.88
5. Increase in τ_c From 0 to 0.0638	1.25	0	0	0	-6.22	-6.22	-6.22	52.4	-0.45	-1.73

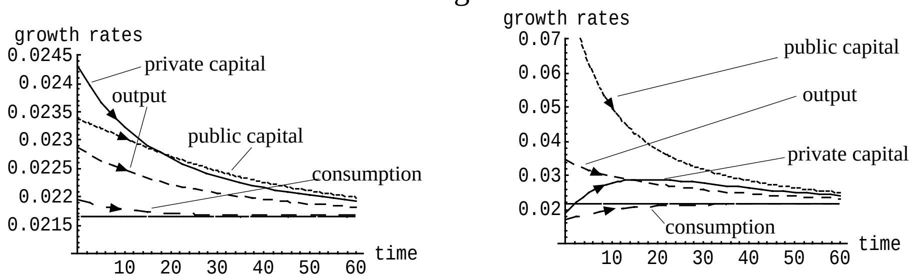
B. Compensated Standardized Changes

	Δl % points	$\Delta(c)$	$\Delta\left(\frac{y}{k}\right)$ %	$\Delta\left(\frac{k_g}{k}\right)$ %	$\Delta(k)$ %	$\Delta(k_g)$ %	$\Delta(y)$ %	$\Delta(V)$ %	Short-run welf. gains percent	Long-run welf. gains percent
1. Tax substitution $\tau_k = \tau_w = 0.2403$, $\tau_c = 0.06$	-0.14	-0.0076	-5.23	-5.23	10.8	5.01	5.01	-3.68	-1.25	0.94
2. Restructuring gov. expenditure $g = 0.08, h = 0.119$	0.03	0.0100	0	200.0	35.9	271.7	35.9	-4.47	-5.09	3.80
3. Incr. in *h*, financed by τ_c $h = 0.20, \tau_c = 0.0642$	-0.04	-0.0400	0	0	0	0	0	-0.04	0.36	0.36
4. Incr. in *h*, financed by τ_w $h = 0.20, \tau_w = 0.3433$	0.61	-0.0400	0	0	-3.03	-3.03	-3.03	1.24	0.16	-0.46
5. Incr. in *h*, financed by τ_k $h = 0.20, \tau_k = 0.3952$	-0.57	-0.0180	19.0	19.0	-24.6	-10.2	-10.2	16.2	4.77	-1.34
6. Incr. in *g*, financed by τ_c $g = 0.08, \tau_c = 0.0642$	0	-0.0400	0	200.0	36.1	272.2	36.1	-5.23	-3.17	5.92
7. Incr. in *g*, financed by τ_w $g = 0.08, \tau_w = 0.3449$	0.66	-0.0400	0	200.0	31.6	263.2	31.6	-4.03	-3.35	4.83
8. Incr. in *g*, financed by τ_k $g = 0.08, \tau_k = 0.3952$	-0.57	-0.0180	19.0	238.1	2.67	244.5	22.3	3.39	0.69	3.77

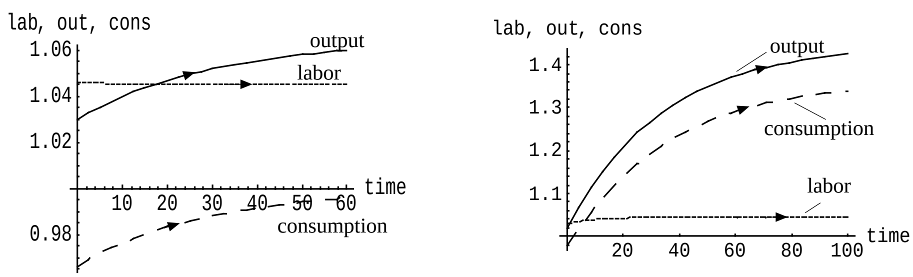
Fig. 1.A: Increase in Gov. Cons. Exp. Fig. 1.B: Increase in Gov.Inv. Exp.



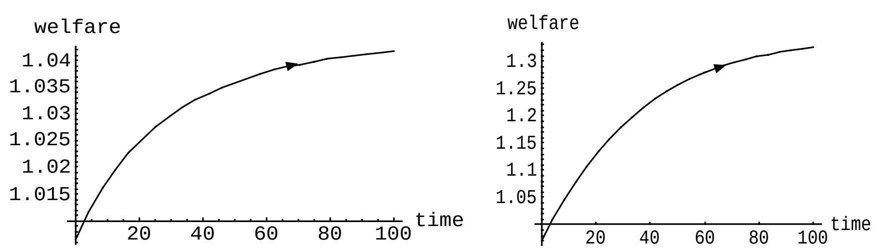
i. Phase Diagram



ii. Growth Rates

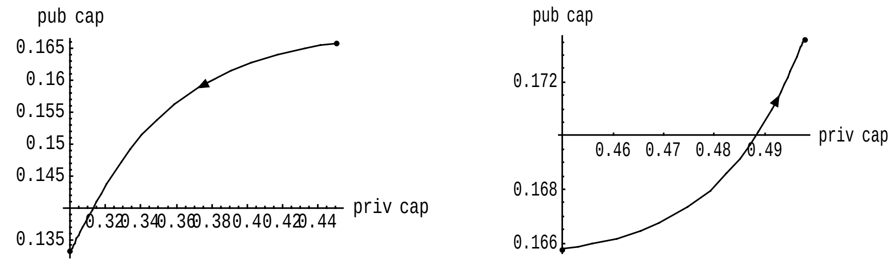


iii. Transition Paths, Rel to Benchmark Economy

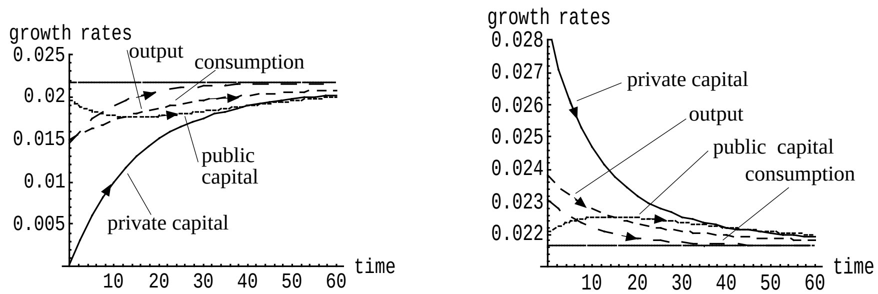


iv. Time Path for Instant. Welfare Gains

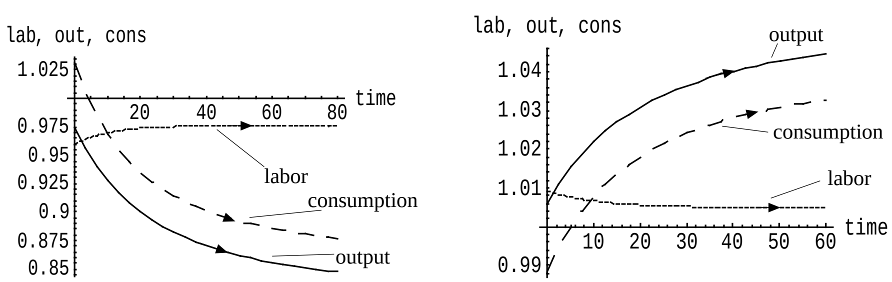
Fig. 2.A: Increase in Tax on Capital. Fig. 2.B: Subst. of Cons. Tax for Income Tax.



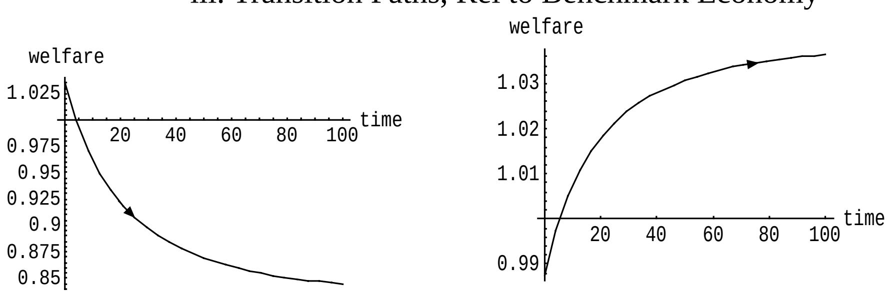
i. Phase Diagram



ii. Growth Rates

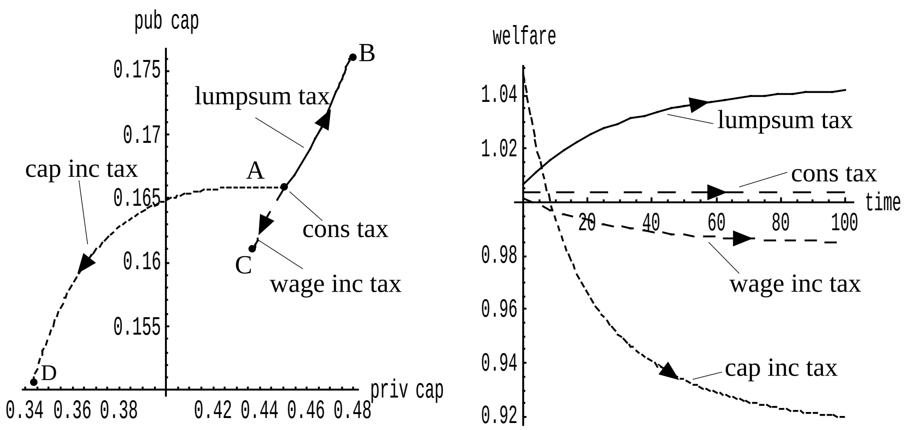


iii. Transition Paths, Rel to Benchmark Economy



iv. Time Path for Instant. Welfare Gains

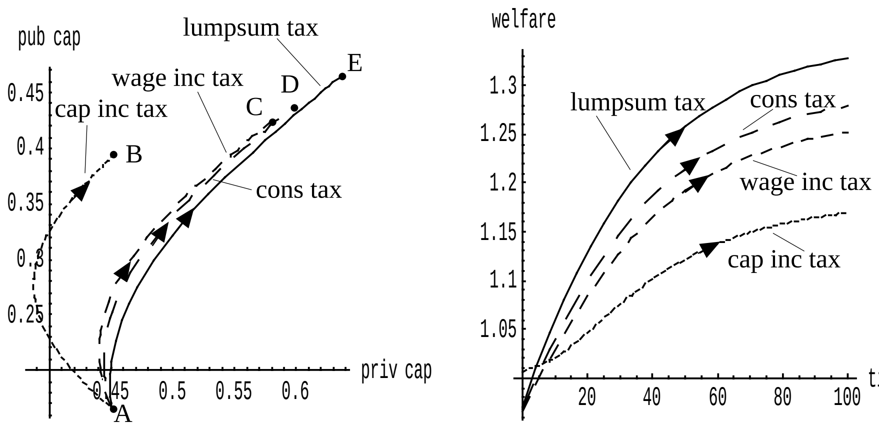
Fig. 3.A: Alternative Forms of Financing
Government Consumption Expenditure



i. Phase Diagrams

ii. Time Paths for Instant. Welfare Gains

Fig. 3.B: Alternative Forms of Financing
Government Investment Expenditure



i. Phase Diagrams

ii. Time Paths for Instant. Welfare Gains

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