The Adjustment of Prices and the Adjustment of the Exchange Rate

Charles Engel

University of Wisconsin and NBER

James C. Morley

Washington University

Preliminary draft: June 22, 2000

Abstract

The purchasing power parity puzzle relates to the adjustment of real exchange rates. Real exchange rates are extremely volatile, suggesting that temporary shocks emanate from the monetary sector. But the half-life of real exchange rate deviations is extremely large – 2.5 to 5 years. This half-life seems too large to be explained by the slow adjustment of nominal prices. We offer a different interpretation. We maintain that nominal exchange rates and prices need not converge at the same rate, as is implicit in rational-expectations sticky-price models of the exchange rate. Evidence from unobserved components models for nominal prices and nominal exchange rates that impose relative purchasing power parity in the long run indicates that nominal exchange rates converge much more slowly than nominal prices. The real puzzle is why nominal exchange rates converge so slowly.

Engel acknowledges research support from a National Science Foundation grant to the National Bureau of Economic Research.

Since the advent of floating exchange rates in 1973, real exchange rates among industrialized countries have been very persistent and much more volatile than economists predicted. There are two general classes of explanations for this outcome, but neither is entirely satisfactory. It is possible that real productivity shocks and real demand shocks to economies have been very persistent. But it is difficult to identify shocks that would lead to such great volatility of real exchange rates.

A second view builds on rational-expectations sticky-price (RESP) models of open economy in the tradition of Dornbusch (1976). Those models demonstrate that monetary shocks could lead to a high degree of real exchange rate volatility through the overshooting effect. Moreover, real exchange rates might be persistent because they adjust at the same rate as nominal prices adjust.

However, empirical studies of real exchange rate adjustment have found very long half-lives for transitory shocks to real exchange rates. Typically, the half-life of real exchange rates is estimated to be from 2.5 to 5 years.[1] That adjustment seems to be too slow to be explained by stickiness of nominal prices. Hence, we have the "purchasing power parity puzzle", as defined by Rogoff (1996):

> How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out? Most explanations of short-term exchange rate volatility point to financial factors such as changes in portfolio preferences, short-term asset price bubbles, and monetary shocks. Such shocks can have substantial effects on the real economy in the presence of sticky nominal wages and prices. Consensus estimates for the rate at which PPP deviations damp, however, suggest a half-life of three to five years, seemingly far too long to be explained by nominal rigidities. It is not difficult to rationalize slow adjustment if real shocks – shocks to tastes and technology – are predominant. But existing models based on real shocks cannot account for short-term exchange-rate volatility. (pp. 647-648.)

[1] See for example Frankel (1986), Lothian and Taylor (1996), Wu (1996), Papell (1997), Cheung and Lai (2000) and Murray and Papell (2000).

Earlier, Stockman (1984) also questions whether the slow convergence of real exchange rates can be explained by slow adjustment of nominal prices:

…This degree of persistence appears to be too large to explain on the basis of disequilibrium models that postulate sticky nominal prices. Many macroeconomists believe that sticky nominal prices play a major role in business cycles (though there is clearly controversies about this.) The length of time over which the economy recovers from recessions would provide a rough estimate of the time it takes the overall price level to adjust to its new equilibrium following a disturbance. This estimate would suggest a period of two to three years. In fact, because there are many reasons for business cycles to persist once they have begun, two to three years is probably an upper bound. Disequilibrium theories of exchange rates, based on sticky nominal goods prices, predict that real and nominal exchange rates should return toward their equilibrium levels when nominal goods prices do. This means that they predict systematic changes in real and nominal exchange rates that are not found in the data.

Here we offer one possible resolution to the purchasing power parity puzzle: nominal prices and exchange rates converge at different speeds. In fact, we find prices converge relatively rapidly. It is nominal exchange rates that move toward the purchasing power parity equilibrium very slowly. Why do Rogoff, Stockman and others mate the convergence speed of the real exchange rate with the convergence speed of prices? Probably it is because that is the sort of dynamics that arise from RESP models. In those models, prices, nominal exchange rates, and real exchange rates converge to the long run at the same rate. Such models restrict the dimension of deviations of exchange rates and prices from their long-run equilibrium values. These variables converge along a saddle path, which makes the deviation of the nominal exchange rate a linear combination of the deviation of domestic and foreign prices from their equilibrium values.

We raise a new puzzle: why does the nominal exchange rate converge so slowly? We do not provide an alternative theory that can explain why nominal exchange rates deviate from their long-run equilibrium value for such long periods of time. The model we present is empirical. Perhaps this new puzzle is related to the empirical failure of uncovered interest parity (UIP). In terms of the RESP model, the forward-looking behavior implicit in rational expectations modeling of the UIP condition is the key to the solution that puts exchange rates and prices on a saddle path, and reduces the dimensionality of the system. However, we do not attempt any theoretical modeling of an alternative to UIP. The UIP puzzle has been very resistant to theoretical explanations, so we leave that for future research.[2]

In section 1, we lay out the empirical model. Section 2 relates the model to RESP model directly, as a way to develop some restrictions that are helpful in estimation. (We build a model that nests a RESP model as a special case.) Section 3 reports results. Section 4 explores what happens when we relax the restrictions used in estimation. Section 5 compares our approach to other recent studies that have allowed different speeds of adjustment for exchange rates and prices. In section 6, we speculate on what type of economic behavior might produce the results we find.

1. Model

We propose an unobserved components (UC) model to examine price level and exchange rate adjustment. In our model, the log price levels and the log nominal exchange rate for a given pair of countries are subject to permanent and transitory shocks, but gravitate over time toward an unobserved equilibrium based on purchasing power parity (PPP).

[2] See Hodrick (1987) and Engel (1996) for extensive surveys.

In its most general form, our model has the observed log price levels and the log exchange rate adjust toward unobserved equilibrium values according to *k*th-order stationary autoregressive processes:

$$f_p(L)(p_t - \overline{p}_t) = v_t, \quad (1)$$

$$f_{p^*}(L)(p_t^* - \overline{p}_t^*) = v_t^*, \quad (2)$$

$$f_s(L)(s_t - \overline{s}_t) = v_t^s, \quad (3)$$

where, $f_j(L) = 1 - f_{1,j}L - f_{2,j}L^2 - ... - f_{k,j}L^k$ (L denotes the lag operator; e.g., $Lp_t = p_{t-1}$) for $j = p, p^*, s$ and the roots of $f_j(z) = 0$ lie outside the unit circle; p_t represents the domestic price level, p_t^* represents the foreign price level, and s_t represents the nominal exchange rate expressed as the price of the foreign currency in domestic currency units; $\overline{p}_t$, $\overline{p}_t^*$, and $\overline{s}_t$ represent the corresponding equilibrium values; and v_t, v_t^*, and v_t^s represent transitory shocks to domestic price level, foreign price level, and the exchange rate, respectively. Meanwhile, the first differences of the unobserved equilibrium log price levels adjust according to *k*th-order autoregressive processes:

$$f_{\overline{p}}(L)(\Delta\overline{p}_t - m) = \overline{v}_t, \quad (4)$$

$$f_{\overline{p}^*}(L)(\Delta\overline{p}_t^* - m^*) = \overline{v}_t^*, \quad (5)$$

where, $f_j(L) = 1 - f_{1,j}L - f_{2,j}L^2 - ... - f_{i,k}L^k$ for $j = \overline{p}, \overline{p}^*$ and the roots of $f_j(z) = 0$ lie outside the unit circle; m and m^* represent deterministic positive drifts in the domestic and foreign price levels, respectively; and $\overline{v}_t$ and $\overline{v}_t^*$ represent permanent shocks to the domestic and foreign price levels, respectively. The equilibrium exchange rate relates to equilibrium price levels according to PPP:

$$\bar{s}_t = \bar{p}_t - \bar{p}_t^* . \tag{6}$$

Finally, the permanent and transitory shocks have a joint Normal distribution:

$$\begin{bmatrix} v_t \\ v_t^* \\ v_t^s \\ \bar{v}_t \\ \bar{v}_t^* \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} s_p^2 & s_{p,p^*} & s_{p,s} & s_{p,\bar{p}} & s_{p,\bar{p}^*} \\ s_{p,p^*} & s_{p^*}^2 & s_{p^*,s} & s_{p^*,\bar{p}} & s_{p^*,\bar{p}^*} \\ s_{p,s} & s_{p^*,s} & s_s^2 & s_{s,\bar{p}} & s_{s,\bar{p}^*} \\ s_{p,\bar{p}} & s_{p^*,\bar{p}} & s_{s,\bar{p}} & s_{\bar{p}}^2 & s_{\bar{p},\bar{p}^*} \\ s_{p,\bar{p}^*} & s_{p^*,\bar{p}^*} & s_{s,\bar{p}^*} & s_{\bar{p},\bar{p}^*} & s_{\bar{p}^*}^2 \end{bmatrix} \right). \tag{7}$$

Equations (1) and (2) are price-adjustment equations. These might be considered structural equations that describe how an aggregate price index adjusts to its long-run equilibrium. These equations are very similar to price-adjustment equations in open-economy models presented by Obstfeld and Rogoff (1984) and Engel and Frankel (1984). The equilibrium prices, $\bar{p}_t$ and $\bar{p}_t^*$, are interpreted in those models as the price level that would prevail in each country if prices were perfectly flexible, given the current values (and history) of the exogenous variables.

Under this interpretation, equations (4) and (5) describe what the evolution of p_t and p_t^* would be if prices were perfectly flexible. Nominal prices are determined by the dynamics of money supply and money demand. Our model incorporates a unit root in these equilibrium prices, but does not require that they follow a random walk. For example, with fixed money demand, nominal prices could follow such a process if money supplies were exogenously generated as unit root processes.

Equation (6) imposes long-run purchasing power parity. Rogoff (1997) claims there is a growing consensus on this empirical regularity (however, see Engel (2000)).

Equation (3) indicates there are transitory deviations from purchasing power parity.

It is easy to relate this model to stochastic versions of the RESP model. In section 2 we discuss the relationship in detail. It is useful now to point out the main contrast between this model and the RESP models: those models have $f_p(L)$, $f_{p^*}(L)$, and $f_s(L)$ the same.

2. Estimation

To make estimation more tractable, we place three major restrictions on the general model presented in the previous section. First, for simplicity and transparency, we assume first-order autoregressive adjustment processes (i.e., $k = 1$). Second, we impose some zero restrictions, discussed below, on the covariance matrix of the permanent and transitory shocks. Third, since our main focus is on the difference between the speeds of adjustment for nominal prices and for nominal exchange rates, we impose that nominal prices adjust at the same speed in each country (i.e., $f_p = f_{p^*}$ and $f_{\bar{p}} = f_{p^*}$) and, also as discussed below, that the relationship between shocks is proportional across countries. In section 4, we explore what happens when we relax these restrictions.

The standard approach to restricting the covariance matrix is to assume that all of the underlying shocks are independent. It turns out, however, that such a strong assumption is not necessary to identify the model. Furthermore, independence would have the drawback of not nesting RESP-style dynamics. Appendix 1 presents a RESP model, and discusses the restrictions imposed by that model. In this section, we discuss those restrictions more informally and describe how they are accommodated in our estimation.

Consider equations (1) and (4), the price-adjustment equation for domestic prices and the equation determining the dynamics of equilibrium prices in the home country. In the RESP model, monetary and fiscal shocks move the equilibrium price level. The shock in equation (4), $\bar{v}_t$, can be considered a linear combination of the shocks that affect equilibrium nominal prices. Under the assumption of independent shocks, the error term in the price-adjustment equation (1), v_t, would not be correlated with $\bar{v}_t$. The implication is that any shock that pushes up $\bar{p}_t$ would push p_t up immediately by exactly the same amount. In equation (1), shocks to v_t determine the gap $p_t - \bar{p}_t$, but $\bar{v}_t$ shocks have no effect on this gap unless $\bar{v}_t$ is correlated with v_t.

But this kind of immediate proportional response of prices, p_t, to shocks that affect equilibrium prices, $\bar{p}_t$, is completely inconsistent with the price-stickiness assumptions of RESP models. In terms of our model, RESP models have negative correlation between v_t and $\bar{v}_t$. Indeed, a literal representation of predetermined nominal prices has these terms perfectly negatively correlated: $v_t = -\bar{v}_t$. Under this assumption, the price adjustment equation (1) can be written as:

$$p_t = (1 - f_p(L))(p_t - \bar{p}_t) + E_{t-1}\bar{p}_t .$$

In practice, there are a couple of reasons to assume that while v_t and $\bar{v}_t$ might be negatively correlated, the correlation is not perfect. The assumption of perfect negative correlation means that prices do not respond at all in the current period to shocks that affect $\bar{p}_t$. That is an impractical assumption in our empirical model. Our price data are sampled quarterly, so the assumption means that even after one full quarter prices show no response to $\bar{v}_t$ shocks. We find in our empirical work that prices actually adjust fairly quickly – generally more than half of the adjustment occurs within six months. Even if prices do not respond on impact to $\bar{v}_t$ shocks, we should allow for the possibility that some of the adjustment occurs within the first quarter. So, we want to allow for negative correlation of v_t and $\bar{v}_t$ so that we have some sluggishness in the response of prices, but we do not want to impose perfect negative correlation which would require no response of prices after a full quarter.

Another reason to allow less than perfect negative correlation of v_t and $\bar{v}_t$ is that there may be some shocks to the price-adjustment process that are not perfectly correlated with the underlying permanent shocks to $\bar{p}_t$. For example, $\bar{v}_t$ is a linear combination of a variety of demand shocks that could hit the economy. Not all prices adjust at the same speed. So, consider two different shocks to $\bar{v}_t$. They might affect individual prices of goods differentially. Monetary shocks might have larger effects on food prices, while fiscal shocks may have larger effects on housing prices. Two shocks that end up having identical effects on $\bar{v}_t$ may imply different speeds of adjustment for the aggregate price level because housing and food prices adjust at different speeds. So, the v_t shock might incorporate these composition effects on the speed of adjustment which are separate from the impact of the aggregate nominal shock, $\bar{v}_t$.

Appendix 1 presents price-adjustment equations that incorporate these features.

We accommodate them in our estimation by allowing for non-zero values of $s_{p,\bar{p}}$ and $s_{p^*,\bar{p}^*}$.

Another instance in which it is important not to assume independence of shocks is with the shocks to the exchange rate and to $\bar{p}_t$ and $\bar{p}_t^*$. A key feature of the RESP model is that exchange rates instantaneously reflect shocks that ultimately are reflected in goods prices. Monetary and other demand shocks that affect $\bar{p}_t$ and $\bar{p}_t^*$ also affect s_t. Of course there is no restriction that the exchange rate move the same amount as $\bar{p}_t$ and $\bar{p}_t^*$. There may be overshooting or undershooting. But, to accommodate this behaviour, we also allow for non-zero values of $s_{s,\bar{p}}$ and $s_{s,\bar{p}^*}$.

Then, since the shocks to the exchange rate equation, v_t^s, and the shocks to prices, v_t and v_t^*, are correlated with the shocks to equilibrium prices, $\bar{v}_t$ and $\bar{v}_t^*$, it is logical to allow v_t^s to be correlated with v_t and v_t^*. So, we also allow $s_{s,p}$ and s_{s,p^*} to be non-zero.

Meanwhile, we assume that v_t and $\bar{v}_t$ are uncorrelated with v_t^* and $\bar{v}_t^*$. This is a typical assumption in RESP models. It corresponds to an assumption that domestic monetary and fiscal shocks are uncorrelated with the corresponding foreign shocks.

Our model generalizes the models of Mussa (1982), Obstfeld and Rogoff (1984) and Engel and Frankel (1984) in two ways. The first is relatively trivial. As we discussed above, we do not impose the restriction that shocks to current and equilibrium prices in each country are perfectly negatively correlated. The second is crucial. The two-country model yields saddle-path dynamics in which prices and the exchange rate converge at the same speed. It has a linear restriction of the form:

$$s_t - \bar{s}_t = -h(p_t - \bar{p}_t) + h^*(p_t^* - \bar{p}_t^*)\,, \tag{8}$$

where h and h^* are constants. We do not impose such a restriction. Furthermore, the symmetric model implies $h = h^*$. That is, it yields the restriction that $f_p(L)$, $f_{p^*}(L)$, and $f_s(L)$ are the same. We do not impose that restriction. Instead, we allow prices to have one speed of convergence and the exchange rate to have another. Indeed, it is by jettisoning the restriction that $f_p(L)$, $f_{p^*}(L)$, and $f_s(L)$ are all the same that we move from a model in which we can speak meaningfully about the speed of adjustment of the real exchange rate to a model that focuses on the speed of adjustment of nominal prices and nominal exchange rates.

In light of the above discussion, we impose only four independent zero restrictions on the covariance matrix, instead of nine, as in the case when all of the shocks are independent. Thus, the joint Normal distribution for the permanent and transitory shocks given in (7) becomes

$$\begin{bmatrix} v_t \\ v_t^* \\ v_t^s \\ \bar{v}_t \\ \bar{v}_t^* \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} s_p^2 & 0 & s_{s,p} & s_{p,\bar{p}} & 0 \\ 0 & s_{p^*}^2 & s_{s,p^*} & 0 & s_{p^*,\bar{p}^*} \\ s_{s,p} & s_{s,p^*} & s_s^2 & s_{s,\bar{p}} & s_{s,\bar{p}^*} \\ s_{p,\bar{p}} & 0 & s_{s,\bar{p}} & s_{\bar{p}}^2 & 0 \\ 0 & s_{p^*,\bar{p}^*} & s_{s,\bar{p}^*} & 0 & s_{\bar{p}^*}^2 \end{bmatrix}\right). \tag{7′}$$

In addition to these four zero restrictions, we consider three proportionality restrictions for the non-zero off-diagonal elements of the covariance matrix. These proportionality restrictions hold for the symmetric RESP model, discussed in the appendix, and might well be expected to hold for our model given the assumption that nominal prices adjust at the same speed in each country.

The first proportionality restriction we impose is that, while the direction is opposite, the degree of exchange overshooting or undershooting should be the same in response to equal shocks to $\bar{p}_t$ and $\bar{p}_t^*$:

$$s_{s,\bar{p}} = g s_{\bar{p}}^2 , \tag{9a}$$

$$s_{s,\bar{p}^*} = -g s_{\bar{p}^*}^2 . \tag{9b}$$

The second restriction we impose is that the relationship between permanent and transitory price shocks is proportional in each country:

$$s_{p,\bar{p}} = -d s_{\bar{p}}^2 , \tag{10a}$$

$$s_{p^*,\bar{p}^*} = -d s_{\bar{p}^*}^2 , \tag{10b}$$

where if $d = 1$, we would have the perfect price-stickiness case mentioned above. The third restriction is that the relationship between transitory price shocks and transitory exchange shocks is proportional with opposite signs in each country:

$$s_{s,p} = -k s_p^2 , \tag{11a}$$

$$s_{s,p^*} = k s_{p^*}^2 . \tag{11b}$$

With the three additional proportionality restrictions, we limit the number of independent elements in (7′) from eleven to eight.

It is important to emphasize that the restrictions on the covariance matrix are not necessary to identify our model. Given $k = 1$, the structural UC model in (1)-(7) corresponds to a reduced-form model with enough parameters for identification.[3] In particular, the most general structural model has 22 parameters (excluding the normalizing initial values for the unobserved permanent components, which cannot be identified from a reduced-form model in first differences alone). Meanwhile, the same structural model implies that the first differences of the price levels and the exchange rate have reduced-form univariate ARMA(2,2) and ARMA(3,3) representations, which corresponds to 22 independent parameters.[4] Difficulties can arise in practice when certain autoregressive roots are similar and cancel each other out. The subsequent reduction in the dimension of the structural model can correspond to an even larger reduction in the dimension of the reduced-form model. Thus, the restrictions imposed here may be helpful in estimation.[5] However, in section 4, we explore what happens when we relax these restrictions.

Appendix 2 discusses the Kalman filter and maximum likelihood estimation of this model.

3. Results

We consider six country pairs based on the G7 countries, with the US always serving as the home country. The foreign country is represented by Canada, France, Germany, Italy, Japan, and the UK, respectively. We employ price level and exchange

[3] See Morley, Nelson, and Zivot (1999) for a discussion of identification of non-zero covariances in a UC model.
[4] Our structural model implies only two independent constants in the reduced-form model.

rate data from Datastream. The price levels for the home and foreign countries are represented by their respective consumer price indexes (not seasonally adjusted). The exchange rates are end-of-period prices of foreign currency expressed in US dollars. The original data are sampled at a monthly frequency. However, we sample the data at a quarterly frequency to simplify estimation. The data are converted into logarithms and multiplied by 100. The sample period is 1974Q1 to 1998Q2.

We employ the OPTMUM procedure for the GAUSS programming language to obtain maximum likelihood estimates. Numerical derivatives are used for estimation and the calculation of asymptotic standard errors. Estimates appear robust to a variety of starting values.

Table 1 presents the maximum likelihood estimates for our model and the country pairs a) US and Canada, b) US and France, c) US and Germany, d) US and Italy, e) US and Japan, and f) US and UK, respectively. The most important thing to notice about these estimates is that, for every country pair, the adjustment of prices to a transitory shock is much faster than the adjustment of the exchange rate. The half-lives of transitory price shocks are less than a quarter in the first three cases and less than two quarters in the remaining three cases. Meanwhile, the half-lives of transitory exchange rate shocks range from two years for the US/UK case, to as many as thirteen years for the US/Canada case.

Another notable result is that equilibrium inflation is very persistent for every country pair. Indeed, it seems unlikely that we would be able to reject a unit root in equilibrium inflation in any of the cases. However, if a unit root really were present,

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[5] It appears that, as the number of restrictions is reduced, the more sensitive estimates become to starting values.

accounting for it should only serve to strengthen evidence for fast adjustment of prices in response to transitory shocks. In particular, an ommitted nonstationary component from equilibrium prices would show up in the estimated gap between prices and equilibrium prices, thus putting an upward bias on our estimates of the persistence of transitory price shocks. We explore a related phenomenon in the next section when we allow for a one-time structural break in the equilibrium inflation process.

The next result to notice is that the transitory exchange rate shocks are an order of magnitude more volatile than the permanent and transitory price shocks. This is not too surprising given the relative volatility of observed prices and exchange rates, which is the main stylized fact RESP overshooting models try to account for. But, it is notable since it potentially explains why Cheung, Lai, and Bergman (1999) find that nominal exchange rates do most of the adjustment towards PPP, even if prices adjust more quickly. We discuss this point in further detail in section 5.

In terms of undershooting or overshooting behaviour, the point estimates of $\gamma = \sigma_{s,\bar{p}} / \sigma^2_{\bar{p}} = -\sigma_{s,\bar{p}^*} / \sigma^2_{\bar{p}^*}$ generally imply overshooting of exchange rates in response to permanent price shocks. In particular, the exchange rate appears to overshoot a permanent price shock by 570% in the US/France case, 230% in the US/Germany case, 300% in the US/Italy case, and 730% in the US/Japan case. The exchange rate does appear to undershoot by 70% in the US/Canada case and 65% in the US/UK case, although, even in these cases, the exchange rates still moves in the “correct” direction. However, it should be noted that the undershooting and overshooting estimates are not significant at conventional levels.

Contrary to the simple RESP model discussed in the previous section, the point estimates of $d = -s_{p,\bar{p}} / s^2_{\bar{p}} = -s_{p^*,\bar{p}^*} / s^2_{\bar{p}^*}$ are always negative, except in the US/Canada case. Furthermore, these estimates are always significant. Negative estimates of d correspond to a positive correlation between permanent and transitory price shocks. This finding could be a result of prices actually overshooting in response to permanent price shocks. A more plausible story, though, is that the causality runs the other way, with monetary authorities partially accommodating supply shocks. Estimates of $k = -s_{s,p} / s^2_p = s_{s,p^*} / s^2_{p^*}$ appear to confirm this latter interpretation since they are always positive, except again in the US/Canada case. Positive estimates correspond to co-movement of the price gap and the exchange rate gap that is typically opposite to what would be implied by joint overshooting. Instead, the co-movement is more consistent with a situation in which exchange rates do not respond to temporary accommodation of supply shocks. Of course, we should be careful about interpreting the estimates of k too literally since they are not significant.

The remaining estimates in Table 1 are of the long-run inflation rates in each country and the normalizing initial values for the unobserved equilibrium prices and exchange rate. It is encouraging to note that the estimates for all of the parameters associated with US prices only (i.e., $s_p, m, \bar{p}_{-1}$) are robust across all country pairs. The speed of adjustment parameters for US prices are different across country pairs since they are constrained to equal the speed of adjustment parameters for foreign prices, which are evidently somewhat different for each country under consideration.

Table 2 presents the results for a formal likelihood ratio test of the hypothesis that prices and the exchange rate adjust at the same speed against the alternative of different speeds of adjustment. Except for the US/Italy and US/Japan cases, the likelihood ratio statistics are quite large, suggesting that the overall evidence for different speeds of adjustment is strong. Thus, the results for the likelihood ratio test generally support what the point estimates seem to suggest: prices adjust more quickly than exchange rates.

4. Specification Tests

In this section, we explore what happens when we relax the restrictions imposed on our model in estimation. We also test the robustness of our main findings to other model specifications.

Table 3 reports the results for a likelihood ratio test of our assumption of first-order autoregressive adjustment processes (i.e., $k = 1$) against the alternative of second-order autoregressive adjustment processes (i.e., $k = 2$). The second-order dynamics are uniformly significant, with $c^2(3)$ likelihood ratio statistics ranging from 23.102 for the US/France case to 49.728 for the US/Germany case. Of course, the apparent inability of the restricted model to capture all of the serial correlation of permanent and transitory price and exchange rate movements begs the question of whether the main finding of different speeds of adjustment is spurious. Table 4 reports the results given second-order adjustment processes for a likelihood ratio test of the hypothesis that prices and the exchange rate adjust at the same speed against the alternative of different speeds of adjustment. Compared to Table 2, the hypothesis of the same speed of adjustment can be more strongly rejected. Indeed, the point estimates for the AR(2) parameters generally suggest exchange rate adjustment that is as slow as in the AR(1) case, but price adjustment that is much faster, with prices actually displaying a negative partial autocorrelation at the second lag that is so large as to be more consistent with price overshooting than price stickiness.

Table 5 reports the results for a likelihood ratio test of the four independent zero restrictions in the covariance matrix (7’) against the alternative of no zero restrictions. Since we do not impose the proportionality restrictions for this test, we also do not impose that nominal prices adjust at the same speed in each country. The $c^2(4)$ likelihood ratio statistics are all significant, suggesting that our zero restrictions can be statistically rejected. However, we note that relaxing these restrictions makes estimation much more sensitive to starting values, with the likelihood surface providing multiple local maxima. Table 6 reports the results given no zero restrictions on the covariance matrix (7) for a likelihood ratio test of the hypothesis that prices and the exchange rate adjust at the same speed against the alternative of different speeds of adjustment. Compared to Tables 2 and 4, the results are weaker, although the $c^2(2)$ statistics are still significant for the US/France case and the US/Germany case. We note that there appears to be no pattern as to which individual elements of the covariance matrix are significant across country pairs. Thus, we argue that the zero restrictions in (7’) are reasonable on economic grounds (see the discussion in section 2).

Table 7 reports the results for a likelihood ratio test of the various symmetry restrictions (same speed of adjustment for nominal prices and proportionality restrictions on (7’)) against the alternative of no symmetry restrictions. The $c^2(5)$ likelihood ratio statistics are generally not significant. Only the US/Japan case is significant at the 10% level. Both the same speed of adjustment restriction and the proportionality restrictions are insignificant when tested for separately. Thus, the symmetry restrictions in our model appear to be justified, with estimates changing little when the restrictions are relaxed.

Table 8 reports the results for a likelihood ratio test of the hypothesis that all the shocks are independent against the alternative that the shocks have the covariance structure imposed in our model and given by (7') and (9)-(11). The $c^2(3)$ likelihood ratio statistics are not significant at conventional levels. Thus, we should probably not put too much emphasis on our interpretation of a, d, and k in the previous section. Again, however, we consider a model that allows non-zero elements in the covariance matrix to accommodate the possibility of RESP-style dynamics.

[Other specifications to be considered include i) allowing for a one-time structural break in 1980 in the unconditional means of the equilibrium inflation rates and ii) allowing for the possibility that prices adjust to disequilibrium in the nominal exchange rate by including the lagged exchange rate gap in the price adjustment equations.]

5. Discussion

It is notable that our main finding that prices adjust more quickly than exchange rates appears to contradict the results of other related studies. In this section, we discuss why in particular our findings appear so different to the vector error correction model (VECM) results reported in Cheung, Lai, and Bergman (1999). In the next section, we conclude by speculating on what type of economic behaviour might produce our results.

To understand Cheung, Lai, and Bergman's (1999) results, consider the following VECM for relative prices $(p_t - p_t^*)$ and the exchange rate s_t:

$$(p_{t+1} - p_{t+1}^*) - (p_t - p_t^*) = a_p (s_t - p_t + p_t^*) + u_{t+1}^p, \tag{12}$$

$$s_{t+1} - s_t = a_s (s_t - p_t + p_t^*) + u_{t+1}^s, \tag{13}$$

where a_p and a_s are error correction coefficients and u_t^p and u_t^s are stationary residuals.[6] Cheung, Lai, and Bergman find that a_s is always much larger in magnitude than a_p. That is, exchange rates adjust much more than relative prices in response to a deviation from PPP. Thus, Cheung, Lai, and Bergman conclude that "exchange rates actually adjust faster than prices."

How do we reconcile the VECM results with our findings? We argue that Cheung, Lai, and Bergman incorrectly interpret the coefficients a_s and a_p as relating to speeds of adjustment. The speed of adjustment is a measure of how fast a variable returns to some equilibrium. Thus, in the traditional PPP literature, the real exchange rate is assumed to converge to some constant level, $\bar{q}$, in the long run. We can measure the speed of adjustment by determining how much of the gap $q_t - \bar{q}$ is carried through to the next period in $q_{t+1} - \bar{q}$. In our model, we look at speeds of adjustment for p_t, p_t^*, and s_t individually. For example, the speed of adjustment for the nominal exchange rate is measured by the degree to which $s_{t+1} - \bar{s}_{t+1}$ has adjusted to the gap $s_t - \bar{s}_t$.

[6] Note that a finite-order VECM can only approximate the dynamics of the infinite-order vector MA representation that corresponds to our UC model of prices and the exchange rate.

Cheung, Lai, and Bergman do not measure speeds of adjustment. For example, the parameter a_p is a measure of how relative inflation, $p_{t+1} - p_t - (p_{t+1}^* - p_t^*)$, responds to the real exchange rate gap, $q_t - \bar{q}$. (We can rewrite equations (12) and (13) so that the error correction term can be written as $q_t - \bar{q}$.) That may be an interesting parameter, but it is difficult to see how to interpret it as relating to a speed of adjustment of prices.

To understand why their a_p is so low, we point out two crucial differences in our UC model and the VECM model. First, the error correction term in (12) and (13) is not the same as the exchange rate gap $(s_t - \bar{s}_t)$ or the relative price gap $(\bar{s}_t - p_t - p_t^*)$ implicit in our UC representation of prices and the exchange rate, but is, instead, equal to their difference. So, our UC representation has prices adjusting only to the relative price gap, while the ECM representation imposes that prices adjust equally to both gaps. One reason the coefficient a_p is so low is that it measures the response of prices to a very large gap, $q_t - \bar{q}$, while we measure the response of prices to the smaller gaps, $p_t - \bar{p}_t$ and $p_t^* - \bar{p}_t^*$. Our measures capture how quickly prices are adjusting to their deviation from equilibrium, while the ECM parameter measures how much prices are responding to the price gap *and* the exchange-rate gap.

An example makes this clear. If relative prices follow a random walk, then by construction they would adjust to equilibrium instantaneously. There would be no relative price gap, only an exchange rate gap. However, since relative prices follow a random walk, they would not adjust toward the exchange rate gap at all, implying that a_p would actually be zero.

Another way to think about the VECM results is to make the careful distinction between the "size" of adjustment and the "speed" of adjustment to equilibrium. The fact that exchange rates adjust much more than relative prices in response to deviations to PPP does not necessarily imply that exchange rates adjust more rapidly to equilibrium. Instead, it appears from our findings that the main reason exchange rates adjust more than relative prices is because the exchange rate gap is much larger than the relative price gap. Specifically, we find transitory exchange rate shocks are always an order of magnitude more volatile than transitory price shocks. The best way to distinguish between the size of adjustment and the speed of adjustment, then, is to control for the size of the gaps by considering half-lives of any given one standard deviation transitory shock to the exchange rate or prices. Our estimates of the half-lives make it clear that prices adjust more quickly than the exchange rate.

A second difference between our UC modeling approach and the VECM approach concerns the left-hand-side variable. Consider, for example, the nominal exchange rate. In our UC model, we examine changes in the exchange rate relative to its equilibrium value: $s_{t+1} - \bar{s}_{t+1} - (s_t - \bar{s}_t)$. The left-hand-side variable in the VECM approach is simply $s_{t+1} - s_t$. It is, of course, an empirical question as to which modeling approach fits the data the best.[7] Our approach is easier to understand as a generalization of the RESP model, and it is easier to infer the "speed of adjustment" from our parameter estimates.

Thus, when one considers that the error correction term mixes exchange rate and price gaps or, alternatively, when one carefully distinguishes between the "size" and the

[7] However, the two models are not easily nested in a more general model. Model comparison based, for example, on out-of-sample forecasting ability would be one approach to compare the models, but is beyond the scope of this paper.

"speed" of adjustment, it becomes clear that our main findings do not contradict Cheung, Lai, and Bergman's (1999) results. Our approach emphasizes the speed of adjustment to unobserved equilibrium levels.

6. Conclusions

What could explain the result that prices converge fairly quickly in each country to their equilibrium levels, but the exchange rate moves only very slowly to the PPP value? Rogoff's (1997) speculation is apropos:

> One is left with a conclusion that would certainly make the godfather of purchasing power parity, Gustav Cassel, roll over in his grave. It is simply this: International goods markets, though becoming more integrated all the time, remain quite segmented, with large trading frictions across a broad range of goods. These frictions may be due to transportation costs, threatened or actual tariffs, nontariff barriers, information costs or lack of labor mobility. As a consequence of various adjustment costs, there is a large buffer within which nominal exchange rates can move without producing an immediate proportional response in relative domestic prices. International goods markets are highly integrated, but not yet nearly as integrated as domestic goods markets. This is not an entirely comfortable conclusion, but for now there is no really satisfactory alternative explanation to the purchasing power parity puzzle. (p. 667-668.)

Perhaps, in addition, when these frictions are present, there is more scope for herding behavior and bubbles. It is unlikely that a fully-specified model would take as simple a form as the one posited here. But bubbles and herding behavior might temporarily send the exchange rate off on disequilibrium paths that result in the appearance of slow convergence to the equilibrium. It is suggestive to note that our empirical model of exchange rates is consistent with the RESP model except in one respect: it implies uncovered interest parity will not hold. (See Appendix 1.)

The failure of uncovered interest parity is, in itself, a puzzle. The ex post change in the exchange rate is consistently opposite of the expected change implied by relative interest rates under uncovered interest parity. The literature has been strikingly incapable of explaining this failure (known as the "forward premium puzzle") by appealing to models of the foreign exchange risk premium.[8] On the other hand, Frankel and Froot (1987, 1990) argue that the forward premium puzzle is consistent with a model in which noise traders follow bandwagon behavior: buying a currency if it appreciated in the previous period, for example. This type of bandwagon speculation conceivably could also be responsible for the very slow adjustment of nominal exchange rates to their equilibrium level.

[8] See the surveys of Hodrick (1987) and Engel (1996).

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Appendix 1

The purpose of this appendix is to derive the behavior of real exchange-rate adjustment from a RESP model. The derivation helps understand the implicit restrictions that are usually put on price and exchange-rate changes, and where we differ.

Start with money demand equations in the home and foreign country, and interest parity (all constant terms will be suppressed for simplicity):

$$u_t - p_t = -l\, i_t \tag{A1.1}$$

$$u_t^* - p_t^* = -l^* i_t^* \tag{A1.2}$$

$$i_t - i_t^* = E_t(s_{t+1}) - s_t \,. \tag{A1.3}$$

Here, u_t (u_t^*) is the log of the money supply less money demand shifters in the home (foreign) country, and i_t (i_t^*) is the home (foreign interest rate.)

We define the equilibrium price, $\overline{p}_t$ $(\overline{p}_t^*)$ as the level that p_t (p_t^*) would equal given current value of u_t (u_t^*). Under flexible prices, real interest rates are assumed constant, so nominal interest rates are assumed to equal the expected rate of inflation (plus a constant.)

$$u_t - \overline{p}_t = -l\,(E_t(\overline{p}_{t+1}) - \overline{p}_t) \tag{A1.4}$$

$$u_t^* - \overline{p}_t^* = -l^*(E_t(\overline{p}_{t+1}^*) - \overline{p}_t^*) \tag{A1.5}$$

Each of (A1.4) and (A1.5) are univariate rational expectations difference equations. They have solutions of the form:

$$\bar{p}_t = A(L)u_t \tag{A1.6}$$

$$\bar{p}_t^* = A^*(L)u_t^* \tag{A1.7}$$

Here $A(L)$ $(A^*(L))$ is the lag-operator on money supply and money demand shocks in the home (foreign) country that solves equation (A1.4) (equation (A1.5)).

As in Engel and Frankel (1984), we posit that nominal prices in each country adjust slowly toward their equilibrium levels. But, we make two adjustments. First, only a fraction d of prices are sticky. A fraction $1-d$ adjust instantaneously. (In the foreign country, a fraction d^* of prices are sticky.) Second, we allow a purely transitory shock to hit prices, so that even when $d = 1$ there can be some deviation of the actual price level from its expected level:

$$p_{t+1} - p_t = -q(p_t - \bar{p}_t) + dE_t(\bar{p}_{t+1}) + (1-d)\bar{p}_{t+1} - \bar{p}_t + e_{t+1} \tag{A1.8}$$

$$p_{t+1}^* - p_t^* = -q^*(p_t^* - \bar{p}_t^*) + d^*E_t(\bar{p}_{t+1}^*) + (1-d^*)\bar{p}_{t+1}^* - \bar{p}_t^* + e_{t+1}^* \tag{A1.9}$$

Prices each period adjust part of the way toward their equilibrium value, under the assumptions: $0 < q < 1$ and $0 < q^* < 1$. There are also terms that account for drift in the equilibrium prices.

Equations (A1.1), (A1.2) and (A1.3) imply

$$E_t(s_{t+1}) = s_t + \frac{1}{l}(p_t - u_t) - \frac{1}{l^*}(p_t^* - u_t^*) \tag{A1.10}$$

If long-run PPP holds, so $\bar{s}_t = \bar{p}_t - \bar{p}_t^*$, equations (A1.4) and (A1.5) yield:

$$E_t(\bar{s}_{t+1}) = \bar{s}_t + \frac{1}{l}(\bar{p}_t - u_t) - \frac{1}{l^*}(\bar{p}_t^* - u_t^*) \tag{A1.11}$$

Subtracting (A1.11) from (A1.10),

$$E_t(s_{t+1}) - E_t(\bar{s}_{t+1}) = s_t - \bar{s}_t + \frac{1}{l}(p_t - \bar{p}_t) - \frac{1}{l^*}(p_t^* - \bar{p}_t^*) \tag{A1.12}$$

Equations (A1.8), (A1.9) and (A1.12) can be written in matrix form as a three-equation homogenous system of difference equations:

$$\begin{bmatrix} E_t(p_{t+1} - \bar{p}_{t+1}) \\ E_t(p_{t+1}^* - \bar{p}_{t+1}^*) \\ E_t(s_{t+1} - \bar{s}_{t+1}) \end{bmatrix} = \begin{bmatrix} 1-q & 0 & 0 \\ 0 & 1-q^* & 0 \\ 1/l & 1/l^* & 1 \end{bmatrix} \begin{bmatrix} p_t - \bar{p}_t \\ p_t^* - \bar{p}_t^* \\ s_t - \bar{s}_t \end{bmatrix} \tag{A1.13}$$

Diagonalizing equation (A1.13) yields

$$\begin{bmatrix} E_t(p_{t+1} - \bar{p}_{t+1}) \\ E_t(p_{t+1}^* - \bar{p}_{t+1}^*) \\ E_t(z_{t+1}) \end{bmatrix} = \begin{bmatrix} 1-q & 0 & 0 \\ 0 & 1-q^* & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_t - \bar{p}_t \\ p_t^* - \bar{p}_t^* \\ z_t \end{bmatrix} \tag{A1.14}$$

where

$$z_t = p_t - \bar{p}_t - \frac{lq}{l^* q^*}(p_t^* - \bar{p}_t^*) + lq(s_t - \bar{s}_t)\,. \tag{A1.15}$$

Inspection of equation (A1.14) shows that imposing the condition that the system be expected to converge to the steady state requires $z_t = 0$. This is an important property of the RESP model, and the key difference between our model and the RESP model: that model makes $s_t - \bar{s}_t$ be a linear combination of $p_t - \bar{p}_t$ and $p_t^* - \bar{p}_t^*$. This is the requirement that the economy be on a stable saddle path. Our model does not impose that. As we discuss further below, our model is fundamentally different than the RESP model, even the version of the RESP model in which $q \neq q^*$.

If $q \neq q^*$, we will be unable to represent the dynamics of the real exchange rate only in terms of lagged values of the real exchange rate, because domestic and foreign prices converge at different speeds. But, if $q = q^*$ and $l = l^*$, we can use equations (A1.12), (A1.15) and the condition that $z_t = 0$ to get:

$$E_t(s_{t+1} - \bar{s}_{t+1}) = (1-q)(s_t - \bar{s}_t) \tag{A1.16}$$

Equations (A1.8), (A1.9) and (A1.16) show that domestic prices, foreign prices and the exchange rate all converge at the same speed (in expectations) when $q = q^*$ and $l = l^*$. Defining the real exchange rate as $q_t \equiv s_t + p_t^* - p_t$, we have:

$$E_t(q_{t+1} - \bar{q}_{t+1}) = (1-q)(q_t - \bar{q}_t) .$$

It may seem that merely relaxing the assumption of $q = q^*$ and $l = l^*$ yields a model in which domestic prices, foreign prices and exchange rates converge at different speeds. Clearly in this case, domestic prices converge at a rate of q and foreign prices converge at the rate q^*. The exchange rate equation could be written, for example, as:

$$E_t(s_{t+1} - \bar{s}_{t+1}) = (1-q)(s_t - \bar{s}_t) - \frac{1}{l^*}(1 - \frac{q}{q^*})(p_t^* - \bar{p}_t^*)$$

However, there is no unique way to write the exchange rate equation, because the condition that $z_t = 0$ implies that $s_t - \bar{s}_t$ is a linear combination of $p_t - \bar{p}_t$ and $p_t^* - \bar{p}_t^*$. That is, there are only two independent equations in the dynamic system (whether or not $q = q^*$) in the RESP model. The reduced dimension of the system is a result of the requirement that is imposed that the system converges to steady state. The exchange rate must jump in response to shocks so it is on the path that leads to the steady state.

So, our model can be thought of as generalizing the RESP model in two ways: we do not require that prices in both countries and the exchange rate converge at the same speed, and we allow for three independent equations for $s_t - \bar{s}_t$, $p_t - \bar{p}_t$, and $p_t^* - \bar{p}_t^*$.

To write the system of stochastic equations implied by the RESP model, note

$$\bar{p}_{t+1} - E_t(\bar{p}_{t+1}) = A_0 u_{t+1}, \tag{A1.17}$$

where A_0 is the first term in $A(L)$. Similarly:

$$\bar{p}^*_{t+1} - E_t(\bar{p}^*_{t+1}) = A_0^* u^*_{t+1}. \tag{A1.18}$$

We can use this to write equations for $p_{t+1} - \bar{p}_{t+1}$ and $p^*_{t+1} - \bar{p}^*_{t+1}$:

$$p_{t+1} - \bar{p}_{t+1} = (1-q)(p_t - \bar{p}_t) - dA_0 u_{t+1} + e_{t+1},$$

$$p^*_{t+1} - \bar{p}^*_{t+1} = (1-q)(p^*_t - \bar{p}^*_t) - d^* A_0^* u^*_{t+1} + e^*_{t+1}.$$

In terms of the notation of the text, then $v_{t+1} = -dA_0 u_{t+1} + e_{t+1}$, $v^*_{t+1} = -d^* A_0^* u^*_{t+1} + e^*_{t+1}$, $\bar{v}_{t+1} = A_0 u_{t+1}$, and $\bar{v}^*_{t+1} = A_0^* u^*_{t+1}$.

Then, because of the saddle path property that tells us $z_{t+1} = 0$, we have:

$$s_{t+1} - \bar{s}_{t+1} = \frac{-1}{lq}(p_{t+1} - \bar{p}_{t+1}) + \frac{1}{l^* q^*}(p^*_{t+1} - \bar{p}^*_{t+1})$$

Therefore, $$v^s_{t+1} = \frac{-1}{lq} v_{t+1} + \frac{1}{l^* q^*} v^*_{t+1}. \quad \text{(A1.19)}$$

Define $k \equiv 1/lq$ and $k^* \equiv 1/l^* q^*$. Then we can write the covariance matrix defined in equation (7) in the text as:

$$Var\begin{bmatrix} v_{t+1} \\ v^*_{t+1} \\ v^s_{t+1} \\ \bar{v}_{t+1} \\ v^*_{t+1} \end{bmatrix} = \begin{bmatrix} s^2_v & 0 & -ks^2_v & -ds^2_{\bar{v}} & 0 \\ 0 & s^2_{v^*} & k^* s^2_{v^*} & 0 & -d^* s^2_{\bar{v}^*} \\ -ks^2_v & k^* s^2_{v^*} & k^2 s^2_v + k^{*2} s^2_{v^*} & dks^2_{\bar{v}} & -d^* k^* s^2_{\bar{v}^*} \\ -ds^2_{\bar{v}} & 0 & dks^2_{\bar{v}} & s^2_{\bar{v}} & 0 \\ 0 & -d^* s^2_{\bar{v}^*} & -d^* k^* s^2_{\bar{v}^*} & 0 & s^2_{\bar{v}^*} \end{bmatrix} \quad \text{(A1.20)}$$

In equation (A20), there are only eight independent elements to estimate: d, d^*, k, k^*, s^2_v, $s^2_{v^*}$, $s^2_{\bar{v}}$, and $s^2_{\bar{v}^*}$. Of course, the usual restriction that the lower and upper triangles be identical reduces the dimension of the matrix to fifteen. There are four additional zero restrictions that reduce the dimension to eleven. The other three restrictions come about because of the saddle-path restriction in equation (A19). Without that saddle-path restriction, there would be eleven elements to estimate:

$$Var\begin{bmatrix} v_{t+1} \\ v^*_{t+1} \\ v^s_{t+1} \\ \bar{v}_{t+1} \\ v^*_{t+1} \end{bmatrix} = \begin{bmatrix} s^2_v & 0 & s_{vs} & s_{v,\bar{v}} & 0 \\ 0 & s^2_{v^*} & s_{v^* s} & 0 & s_{v^*,\bar{v}^*} \\ s_{vs} & s_{v^* s} & s^2_s & s_{\bar{v}s} & s_{\bar{v}^* s} \\ s_{v,\bar{v}} & 0 & s_{\bar{v}s} & s^2_{\bar{v}} & 0 \\ 0 & s_{v^*,\bar{v}^*} & s_{\bar{v}^* s} & 0 & s^2_{\bar{v}^*} \end{bmatrix}. \quad \text{(A1.21)}$$

If $d = d^*$, and $k = k^*$, then we can derive the restrictions in equations (9) – (11).

Finally, as noted in the conclusions section, if we retain all of the equations of the RESP model (equations (A1.1), (A1.2), (A1.4)-(A1.9)), but do not assume uncovered interest parity (A1.3) and instead assume that exchange rates adjust to equilibrium at some rate $1-z$:

$$s_{t+1} - \bar{s}_{t+1} = (1-z)(s_t - \bar{s}_t) + v_{st},$$

we can solve to find that the uncovered interest parity condition does not hold:

$$E_t(s_{t+1}) - s_t = i_t - i_t^* - \frac{1}{l}(p_t - \bar{p}_t) + \frac{1}{l^*}(p_t^* - \bar{p}_t^*) - z(s_t - \bar{s}_t). \qquad \text{(A1.22)}$$

Appendix 2

For estimation given the restrictions, we cast the model in state-space form and apply the Kalman filter and maximum likelihood based upon the prediction error decomposition as discussed in Harvey (1990). The state equation, which represents the evolution of the unobserved components, is

$$b_t = \tilde{m} + F b_{t-1} + \tilde{v}_t, \tag{A2.1}$$

where

$$F = \begin{bmatrix} f_p & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_{p^*} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f_s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+f_{\bar{p}} & -f_{\bar{p}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+f_{\bar{p}^*} & -f_{\bar{p}^*} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$b_t = \begin{bmatrix} p_t - \bar{p}_t \\ p_t^* - p_t^* \\ s_t - \bar{s}_t \\ \bar{p}_t \\ \bar{p}_{t-1} \\ \bar{p}_t^* \\ \bar{p}_{t-1}^* \end{bmatrix}, \ \tilde{m} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ m(1-f_{\bar{p}}) \\ 0 \\ m^*(1-f_{\bar{p}^*}) \\ 0 \end{bmatrix}, \text{ and } \tilde{v}_t = \begin{bmatrix} v_t \\ v_t^* \\ v_t^s \\ \bar{v}_t \\ 0 \\ \bar{v}_t^* \\ 0 \end{bmatrix}.$$

Note that the covariance matrix for $\tilde{v}_t$, denoted $Q \equiv E[\tilde{v}_t \tilde{v}_t']$, is a simple linear transformation of (7′). Meanwhile, the observation equation, which relates the price levels and exchange rate to their unobserved components, is

$$y_t = A + Hb_t, \qquad (A2.2)$$

where

$$y_t = \begin{bmatrix} p_t \\ p_t^* \\ s_t \end{bmatrix},\ A = \begin{bmatrix} \bar{p}_{-1} \\ \bar{p}_{-1}^* \\ \bar{s}_{-1} \end{bmatrix},\ H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 \end{bmatrix}.$$

The inclusion of a separate initial value for the equilibrium exchange rate corresponds to relative, rather than absolute, PPP.[9] Meanwhile, we include initial values for the equilibrium price levels in A to address the lack of appropriate startup values for the Kalman filter. In particular, equilibrium prices follow unit root processes that have no unconditional expected values. By including initial values in estimation here, we are able to normalize the corresponding initial state variables to zero. Then, we estimate equilibrium prices by adding the estimated initial values to the filter output.[10]

The Kalman filter for this state-space model is given by the following six equations:

$$b_{t|t-1} = \tilde{m} + Fb_{t-1|t-1} \qquad (A2.3)$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q \qquad (A2.4)$$

$$h_{t|t-1} = y_t - Hb_{t|t-1} \qquad (A2.5)$$

$$f_{t|t-1} = HP_{t|t-1}H' \qquad (A2.6)$$

$$b_{t|t} = b_{t|t-1} + K_t h_{t|t-1} \qquad (A2.7)$$

$$P_{t|t} = P_{t|t-1} - K_t HP_{t|t-1} \qquad (A2.8)$$

[9] Since price data is in index form, only relative PPP is tenable.
[10] An alternative approach would be to make an arbitrary guess about the corresponding Kalman filter startup values and assign our guess an extremely large variance.

where $b_{t|t-1} \equiv E_{t-1}[b_t]$, for example, denotes the expectation of b_t conditional on information up to time $t-1$; $P_{t|t-1}$ is the variance-covariance of $b_{t|t-1}$; $h_{t|t-1}$ is a vector of the conditional forecast errors of the observed series; $f_{t|t-1}$ is the variance-covariance of $h_{t|t-1}$; and $K_t \equiv P_{t|t-1}H' f_{t|t-1}^{-1}$ is the Kalman gain.

Given arbitrary initial parameter estimates and initial values $b_{0|0}$ and $P_{0|0}$ based on unconditional expected values and the normalizations discussed above, we solve equations (A2.3)-(A2.8) recursively for $t = 1,\ldots,T$ to obtain filtered inferences about b_t conditional on information up to time *t*.

Then, as a by-product of the Kalman filter, we obtain $h_{t|t-1}$ and $f_{t|t-1}$, which allow us to calculate maximum likelihood estimates of the various parameters based on the prediction error decomposition (Harvey, 1990):

$$\max_{q}\left\{ l(q) = -\frac{1}{2}\sum_{t=1}^{T}\ln((2p)^3 |f_{t|t-1}|) - \frac{1}{2}\sum_{t=1}^{T} h'_{t|t-1} f_{t|t-1}^{-1} h_{t|t-1} \right\}, \qquad \text{(A2.9)}$$

where q is the vector of parameters.

Table 1
Maximum Likelihood Estimates for Country Pairs

Parameter	US/Canada	US/France	US/Germany	US/Italy	US/Japan	US/UK
$f_p = f_{p^*}$	0.273	0.478	0.480	0.681	0.641	0.569
	(0.201)	(0.128)	(0.114)	(0.244)	(0.163)	(0.120)
f_s	0.987	0.942	0.928	0.927	0.958	0.919
	(0.015)	(0.033)	(0.032)	(0.034)	(0.022)	(0.038)
$f_{\bar{p}} = f_{\bar{p}^*}$	0.955	0.965	0.926	0.938	0.962	0.935
	(0.026)	(0.020)	(0.029)	(0.028)	(0.019)	(0.030)
s_p	0.430	0.397	0.358	0.421	0.421	0.327
	(0.059)	(0.050)	(0.059)	(0.069)	(0.078)	(0.039)
s_{p^*}	0.365	0.235	0.396	0.359	0.497	0.783
	(0.058)	(0.037)	(0.049)	(0.100)	(0.068)	(0.032)
s_s	2.193	5.612	5.900	5.435	6.191	5.265
	(0.158)	(0.423)	(0.426)	(0.398)	(0.489)	(0.389)
$s_{\bar{p}}$	0.230	0.263	0.295	0.276	0.252	0.324
	(0.039)	(0.042)	(0.050)	(0.050)	(0.050)	(0.036)
$s_{\bar{p}^*}$	0.267	0.268	0.212	0.527	0.299	0.535
	(0.060)	(0.042)	(0.046)	(0.103)	(0.055)	(0.021)

Table 1 (Continued)

Parameter	US/Canada	US/France	US/Germany	US/Italy	US/Japan	US/UK
g	-0.286	5.776	2.285	3.036	7.319	-0.344
	(0.848)	(3.199)	(4.355)	(1.846)	(4.929)	(1.771)
d	1.364	-0.815	-1.171	-0.613	-1.049	-1.009
	(0.433)	(0.245)	(0.288)	(0.251)	(0.297)	(0.179)
k	-0.023	1.576	2.205	2.066	3.778	1.007
	(0.093)	(2.286)	(2.066)	(1.577)	(1.370)	(0.863)
m	1.296	1.356	1.331	1.318	1.339	1.338
	(0.442)	(0.616)	(0.363)	(0.396)	(0.552)	(0.447)
m^*	1.454	1.621	0.793	2.407	1.162	2.141
	(0.512)	(0.583)	(0.262)	(0.754)	(0.636)	(0.748)
$\bar{p}_{-1}$	355.708	355.352	355.055	355.424	355.187	354.854
	(0.986)	(1.055)	(1.084)	(1.202)	(1.172)	(1.130)
$\bar{p}^*_{-1}$	341.769	327.646	401.216	259.346	386.152	293.145
	(1.006)	(0.851)	(0.913)	(1.680)	(1.358)	(2.155)
$\bar{s}_{-1}$	-12.435	-153.173	-103.242	-637.282	-539.337	104.327
	(10.891)	(8.493)	(7.479)	(6.925)	(12.479)	(6.338)
Log likelihood	-404.190	-478.424	-502.276	-532.126	-529.959	-566.353

Note: Standard Errors are reported in parentheses.

Table 2

Likelihood Ratio Test of Same vs. Different Speeds of Adjustment I

Country Pair	$c^{2}(1)$ Statistic	*p*-value
US/Canada	5.585	0.018
US/France	11.633	<0.001
US/Germany	7.555	0.006
US/Italy	1.477	0.224
US/Japan	1.772	0.183
US/UK	3.665	0.056

Note: $H_0 : f_p = f_{p^*} = f_s$ vs. $H_1 : f_p = f_{p^*} \neq f_s$.

Table 3

Likelihood Ratio Test of First- vs. Second-Order Autoregressive Adjustment Processes

Country Pair	$c^{2}(3)$ Statistic	*p*-value
US/Canada	26.688	<0.001
US/France	23.102	<0.001
US/Germany	49.728	<0.001
US/Italy	28.780	<0.001
US/Japan	38.256	<0.001
US/UK	24.618	<0.001

Note: $H_0 : k = 1$ vs. $H_1 : k = 2$.

Table 4
Likelihood Ratio Test of Same vs. Different Speeds of Adjustment II

Country Pair	$c^2(2)$ Statistic	*p*-value
US/Canada	21.702	<0.001
US/France	16.532	<0.001
US/Germany	43.746	<0.001
US/Italy	14.794	<0.001
US/Japan	15.200	<0.001
US/UK	5.674	0.059

Note: $H_0: f_{1,p} = f_{1,p^*} = f_{1,s}, f_{2,p} = f_{2,p^*} = f_{2,s}$ vs. $H_1: f_{1,p} = f_{1,p^*} \neq f_{1,s}, f_{2,p} = f_{2,p^*} \neq f_{2,s}$.

Table 5

Likelihood Ratio Test of Zero Restrictions vs. Unrestricted Covariance Matrix

Country Pair	$c^{2}(4)$ Statistic	*p*-value
US/Canada	18.470	0.001
US/France	55.472	<0.001
US/Germany	18.442	0.001
US/Italy	21.764	<0.001
US/Japan	29.260	<0.001
US/UK	16.492	0.002

Note: $H_0 : s_{p,p^*} = s_{p,\bar{p}^*} = s_{p^*,\bar{p}} = s_{\bar{p},\bar{p}^*} = 0$ vs.
$H_1 : s_{p,p^*} \neq 0, s_{p,\bar{p}^*} \neq 0, s_{p^*,\bar{p}} \neq 0, s_{\bar{p},\bar{p}^*} \neq 0$.

Table 6

Likelihood Ratio Test of Same vs. Different Speeds of Adjustment III

Country Pair	$c^2(2)$ Statistic	*p*-value
US/Canada	0.818	0.664
US/France	14.382	<0.001
US/Germany	10.828	0.004
US/Italy	1.568	0.457
US/Japan	0.014	0.993
US/UK	1.980	0.372

Note: $H_0 : f_p = f_{p^*} = f_s$ vs. $H_1 : f_p \neq f_{p^*} \neq f_s$.

Table 7
Likelihood Ratio Test of Symmetry Restrictions vs. No Symmetry Restrictions

Country Pair	$\chi^2(5)$ Statistic	*p*-value
US/Canada	6.778	0.238
US/France	1.946	0.857
US/Germany	8.144	0.148
US/Italy	4.102	0.535
US/Japan	9.784	0.082
US/UK	3.838	0.573

Note: $H_0 : \phi_p = \phi_{p^*}, \phi_{\bar{p}} = \phi_{\bar{p}^*}, \frac{\sigma_{s,\bar{p}}}{\sigma^2_{\bar{p}}} = -\frac{\sigma_{s,\bar{p}^*}}{\sigma^2_{\bar{p}^*}}, \frac{\sigma_{p,\bar{p}}}{\sigma^2_{\bar{p}}} = \frac{\sigma_{p^*,\bar{p}^*}}{\sigma^2_{\bar{p}^*}}, \frac{\sigma_{s,p}}{\sigma^2_p} = -\frac{\sigma_{s,p^*}}{\sigma^2_{p^*}}$ vs.

$H_1 : \phi_p \neq \phi_{p^*}, \phi_{\bar{p}} \neq \phi_{\bar{p}^*}, \frac{\sigma_{s,\bar{p}}}{\sigma^2_{\bar{p}}} \neq -\frac{\sigma_{s,\bar{p}^*}}{\sigma^2_{\bar{p}^*}}, \frac{\sigma_{p,\bar{p}}}{\sigma^2_{\bar{p}}} \neq \frac{\sigma_{p^*,\bar{p}^*}}{\sigma^2_{\bar{p}^*}}, \frac{\sigma_{s,p}}{\sigma^2_p} \neq -\frac{\sigma_{s,p^*}}{\sigma^2_{p^*}}$.

Table 8
Likelihood Ratio Test of Independent Shocks vs. Reported Model

Country Pair	$c^2(3)$ Statistic	*p*-value
US/Canada	1.344	0.719
US/France	4.406	0.221
US/Germany	3.235	0.357
US/Italy	2.658	0.447
US/Japan	5.961	0.114
US/UK	1.542	0.673

Note: $H_0 : a = d = g = 0$ vs. $H_1 : a \neq 0, d \neq 0, g \neq 0$.