Financial Risk Management in a Competitive Electricity Market

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Abstract
This paper proposes solutions for electricity producers in the field of financial risk management for electric energy contract evaluation. The efficient frontier is used as a tool to identify the preferred portfolio of contracts. Each portfolio has a probability density function for the profit. For important scheduling policies, closed form solutions are found for the amount of futures contracts that correspond to the efficient frontier. Production scheduling must consider resource constraints. It is found that, without resource constraints, the portfolio with the highest expected profit can be preferred - even for a risk-averse decision-maker. When resource constraints are present, portfolios not corresponding to the maximum expected profit criteria will more frequently be preferred.

Keywords: Power System Economics, Risk Management, Contract Decision Making, Open Access

I. INTRODUCTION

The energy markets have been, or are in the process of being, restructured in many parts of the world. This presents uncertainties to participants in the market that they did not have to face during the regulated era. The participants in the market must find ways to deal with these uncertainties when contracts are evaluated. Many risk factors can affect the profit of an electricity producer; examples are: varying electricity and fuel prices, varying demand, equipment malfunction and defaulted payments. This research is focused on the risks presented by fluctuating market prices.

An electricity producer usually has many contracts for buying and selling electricity. If the production is fuel-based, the producer also has contracts for the supply of fuel. These contracts define certain obligations that need to be met, e.g., supplying a certain amount of electricity and accepting delivery of an amount of fuel. Also, the contracts will affect how the producer operates in the spot markets for fuel and electricity. In addition, the producer has financial contracts such as futures and options that have some payoff decided by the future market prices.

Andrews [1] gives an overview of different approaches to risk management in resource planning. He focuses on different approaches to risk management for resource planning for the power industry. Kaye et al. [2] describe how forward contracts can be used for risk sharing in the electricity industry when a spot market is in operation. The paper discusses how forward contracts are affected by random generator failures and uses some scenarios to observe their effect. Gedra [3] considers two types of contracts: callable forward, i.e., interruptible load contracts, and puttable forward that can be interrupted at the delivery side. Fleten [4] proposes a method for risk management by applying stochastic optimization for hydro producers where the contracts enter as constraints. The approach follows the general formulation of stochastic optimization using scenario aggregation as proposed by Rockefellar [5].

II. PROBLEM FORMULATION

The set of all contracts is called the portfolio of contracts. Suppose a producer must choose from a set of different portfolios: \( S = \{S_1, S_2, ..., S_N\} \) (1)

Each portfolio \( S_k \) will generate some profit \( \Pi^k \). But this profit is uncertain at the time the portfolio is accepted. Suppose the producer has a mapping between the portfolio \( S_k \) and the probability density of the profit, \( \Pi^k \):

\[ S_k \rightarrow f_{\Pi^k}(\cdot) \] (2)

Which portfolio should a risk-averse decision-maker choose? If the densities are well approximated by the normal distribution, it will be sufficient to consider the means and variances of the profits corresponding to the portfolios. While one can not expect the densities to always be normal, the mean and variance can still be useful criteria. A small variance indicates that the profit is likely to be close to the expected value (Thchebycheff’s inequality).

III. EFFICIENT FRONTIER

The Efficient Frontier (EF) [6] is the Pareto optimal set between the goals of a high expected value, \( \eta \), and low standard deviation, \( \sigma \). Along the efficient frontier neither can be improved without sacrificing the other. The efficient frontier presents to the decision-maker a clear trade-off between risk and the expected profit. This is illustrated in Figure 1. The EF is the boundary of the feasible region, where portfolios satisfy the constraints such as financial and physical limitations. The stapled lines are the lines of indifference for some risk-averse decision-maker; along these

\[ \sigma^2 = \int \int f_{\Pi}(\tau) \cdot \sigma^2 \cdot d\tau \]

If the densities are not symmetrical, the downward semi-variance, \( \sigma'^2 = \int \int (\tau - \eta)^2 f_{\Pi}(\tau) d\tau \), could be used instead.
lines one is indifferent to the combinations of risk and return. A risk-averse decision-maker wants to move in the direction of higher mean and lower standard deviation. As a result, the preferred portfolio is represented by the tangent point between the EF and the line of indifference that intersects tangentially.

The tangent portfolio is derived from the theory of expected utility [6]. Let \( U(x) \) be the producer’s utility function that measures his or her utility of the monetary wealth, \( x \). The producer is said to be risk averse if \( U(x) \) is concave.

The tangent portfolio is found by optimizing the expected utility. The implication is that when the uncertainty of \( x \) is high the expected utility would be low. An example of this is \( U(x) = 1 - e^{-x} \), where \( x \) is a normal random variable. The expected utility is \( E[U(x)] = 1 - E[e^{-x}] = 1 - e^{-\frac{\mu^2}{2}} \). In Figure 1, the dotted lines represent indifference curves for this utility function, i.e., along each curve the utility is constant. These indifference curves are represented by \( \eta = \frac{c}{2} \sigma^2 + K \), where \( K \) is a dummy variable. The direction of increasing expected utility is in the direction of higher mean and lower standard deviation. The goal of the optimization problem is to choose the highest indifference curve while meeting the constraint (efficient frontier).

![Figure 1 Efficient Frontier](image1)

### IV. FINDING THE EFFICIENT FRONTIER

To determine the EF, one would need to find the probability distribution function of the profit for all portfolios. The components for the evaluation of new contracts are illustrated in Figure 2, where the directions of the arrows indicate the flow of information.

The arrows are bi-directional between the two right-hand-side components in Figure 2. In general, the probability density function has to be found by Monte-Carlo simulations, given the market uncertainties represented by the prices and demands. Suppose the scheduler has a consistent policy, i.e., for the same scenario (s)he will take the same action. Then the outcomes of the stochastic market processes together with the bidding / scheduling policy would decide the profit over a time horizon. One can then analyze a number of different scenarios of the outcomes generated from the statistics of the market and find the accumulated profit over the time horizon. It is desirable to determine a scheduling policy that is optimal in some sense, e.g., a policy that maximizes the expected profit. It is noted that the Monte-Carlo approach can be applied whether the policy is optimal or not.

The bidding / scheduling and contract evaluation could also be solved as an integrated optimization problem, if the producer can identify a concave utility function that describes the risk-aversion, as described in Section III, (s)he would then optimize the expected utility with respect to the schedule. The optimization problem would have to be solved for all portfolios under consideration, and the portfolio with the highest utility should correspond to the tangent portfolio in Figure 1. However, non-linearity of the objective function makes it difficult to solve using stochastic dynamic programming techniques.

While the general solution technique is one of great computational complexity, financial contracts are usually included only for hedging purposes in order to reduce the profit uncertainty. In this paper, some cases have been identified in which the financial contracts can be determined for a given scheduling policy. In the next section two relevant cases for the energy market are considered.

### V. HEDGING USING FUTURES CONTRACTS

A common financial contract is the futures contract: A legally binding agreement made between two parties to buy or sell a commodity or financial instrument, at an agreed price, on a specified date in the future (delivery date). The commodity is usually not physically delivered, but the monetary difference between the agreed upon price and the market valuation is settled.

The concept of minimizing the risk using futures contracts is well known [7]: Suppose A sells \( \alpha_1 \) futures contracts of type 1 at the price \( f_1 \). Upon the date of delivery A settles the contracts by buying the commodity of type 1 in the spot market at price \( f_1^* \). Let \( \Pi_a \) be the profit A would gain if A had not sold the futures contract. Then A’s profit with the futures contract added to the portfolio will be:

\[
\Pi = \Pi_a + \alpha_1 (f_1 - f_1^*)
\]

(3)

If the futures price is equal to the expected spot price, the expected value of the profit does not change if transaction...
costs are ignored. However, the variance does change as a result of the futures contract, i.e.,
\[
\text{var} [\Pi] = \text{var} [\Pi_0] + \alpha_i^2 \text{var} [f_i^*] - 2 \alpha_i C
\]
where \( C \) is the covariance of \( f_1^* \) and \( \Pi_0 \). The variance of the profit is minimized when:
\[
\alpha_i = \frac{C}{\text{var} [f_i^*]}
\]
A perfect hedge is constructed if the amount of electricity sold in the spot market is the same as that in the futures market, and the futures and spot market is for the same good.

In the following, this method is extended for the calculation of which futures contracts an electricity producer should choose to minimize the risk given by the variance.

**Case A: Selling a given amount at spot prices**

Consider the case of a producer that is selling a predetermined amount of electricity over a time horizon at a price determined by the spot market. Also, the producer will buy the gas needed to produce electricity from the spot market. It will be assumed that the producer has only gas-fueled generators.

Let \( s \) (MWH) denote the vector of sales of electricity at each period from period 1 to \( N_s = (s_1, s_2, ..., s_N) \). Similarly, let \( p \) be the vector of electricity prices in the spot market, \( p = (p_1, p_2, ..., p_N) \). No discounting is assumed. The income (revenues) \( I \) from the sales will be:
\[
I = \sum s_i p_i = s \cdot p
\]

The notation for gas prices is \( c = (c_1, c_2, ..., c_N) \) and the amount of gas bought is \( b = (b_1, b_2, ..., b_N) \). As a result, the cost of production \( K \) is:
\[
K = \sum_i b_i c_i = b \cdot c
\]

In vector notion, the profit \((\$)\) can be written as:
\[
\Pi_0 = I - K = [s - b] \cdot [p - c]
\]

There is a relationship between the amount of electricity sold and the amount of gas used to produce the electricity. The producer will minimize the costs, given the production over a time horizon, i.e.,
\[
s \xrightarrow{\text{Cost Minimization}} b
\]

Since the producer only has gas-fueled production, this will be the same as minimizing the total amount of gas used over the time horizon. This implies that there is a deterministic mapping from \( s \) to \( b \). (For example, this mapping could represent the unit commitment solution for the sales.) Consequently, \( b \) is deterministic.

Eq. (8) can easily be extended to include futures contracts of type \( 1, ..., N \). Let \( \alpha = [\alpha_1, ..., \alpha_N] \) be a vector where \( \alpha_i \) is the number of futures contracts of type \( i \) that is sold.

Upon delivery each futures contract is settled by receiving the price of the futures contract when the transaction was made, \( f_i \), and paying the settlement price of \( f_i^* \). Note that under physical settlement the settlement price is derived from the spot market, by buying from the spot market to deliver to the futures contract. It can also be settled financially by a lump sum payment. A third way is to pay the difference in electricity spot price and the futures electricity price over the delivery period. Also, \( f_i^* \) can differ from \( p \) due to different geographical areas. Ignoring transaction costs, the total profit is now:
\[
\Pi^* = [s - b - \alpha] \cdot [p - c - (f^* - f)]
\]

As the first vector in this equation is deterministic, the expected value and the variance of the profit can be found as:
\[
\text{E}[\Pi] = [s - b - \alpha] \cdot [\text{E}[p], \text{E}[c], \text{E}[(f^* - f)]]
\]

The futures price is set to be the expected spot price \([7]\), \( \text{E}[f^*] = f \). As a result, Eq. (11) yields
\[
\text{E}[\Pi] = [s - b] \cdot [\text{E}[p], \text{E}[c]]
\]

The profit in Eq. (10) can be written as \( \Pi^* = x \cdot \gamma^T \), where \( x = [s - b - \alpha] \) and \( y = [p - c - (f^* - f)] \). Then:
\[
\text{var}[\Pi^*] = \text{E}[x \cdot y^T] = \text{E}[x] \cdot \text{E}[y] - \text{E}[x \cdot y^T] \]

Expanding the vectors \( x \) and \( y \) gives the following expression for the variance of the profit:
\[
\text{var}[\Pi^*] = \begin{bmatrix} C_{x \cdot c} & C_{x \cdot f} & C_{x \cdot f} \cdot f^* - b \end{bmatrix} \begin{bmatrix} s \end{bmatrix}
\]

The C-matrices are covariance matrices between the vectors \( p, c \) and \( f^* \). Minimizing Eq. (15) with respect to the vector \( \alpha \) one obtains:
\[
\alpha^2 = \left( C_{f \cdot f} - b^2 \right) \left( C_{f \cdot f} - b^2 \right)
\]

The vector \( \alpha \) will correspond to a point on the EF where the variance is minimized for the same expected profit value.

**Case B: Bidding in a spot market**

When selling to a spot market, a seller does not know how much electricity \((s)\) he will sell. Bids will be submitted to the spot market, where the system price and the contracts will be decided. In this case, how many electricity futures contracts should a producer sell to achieve the minimum variance? This question will be investigated for a producer with certain cost and no market power.

In an electricity spot market, a bid may include such parameters as marginal production cost, start-up cost, ramp-up cost and minimum up time. A simplified spot market where the only bidding parameters are marginal cost from the sellers and marginal willingness to pay from buyers will be considered in this section. The price is found by equating the demand side and supply side bids.

Let \( f_i(s_i) \) be the amount of fuel needed to produce \( s_i \). If the producer has no market power, then the price as a
function of \( s_i \) is not influenced by \( s_i \), i.e., \( \Pi(s_i) = p_i \).

Now the profit of the producer in period \( i \) is:

\[
\Pi_i = p_i s_i - K_i(s_i) = p_i s_i - c_i f(s_i)
\]

(17)

The producer wants to sell the amount that will maximize the profit

\[
\max_{s_i} \Pi_i = p_i s_i - K_i(s_i)
\]

(18)

\[
p_i = \frac{dK_i(s_i)}{ds_i} = c_i \frac{df(s_i)}{ds_i}
\]

(19)

In other words, the producer wants to sell an amount that makes the marginal cost of production equal to the system marginal price. How should the producer bid in the spot market to achieve this? Suppose the producer bids the actual marginal cost. Then, the last unit of energy that is acquired from the market to achieve this can be found from Eq.(20), i.e.,

\[
p_i = \frac{dK_i(s_i)}{ds_i} = c_i \frac{df(s_i)}{ds_i}
\]

(20)

Eq. (19) equals Eq.(20). Hence it is optimal for the producer to bid his true cost.

Assuming the producer has a quadratic cost-curve, \( f(s_i) = a_0 + a_1 s_i + a_2 s_i^2 \), the amount of energy the producer would supply can be found from Eq.(20), i.e.,

\[
a_1 + 2 a_2 s_i = \frac{p_i}{c_i} \Rightarrow s_i = \frac{1}{2 a_2} \left( \frac{p_i}{c_i} - a_1 \right)
\]

(21)

It will be assumed that the \( p_i, s_i \)'s are jointly normal, i.e. the prices follow a Gaussian process. As can be seen from Eq.(21) if \( p_i \) is normal, so is \( s_i \), which implies that there is a probability that \( s_i \) does not satisfy the \( production \ capacity \), or \( s_i \notin [0, Cap] \). It will be assumed that this probability is small enough to be ignored. If this were not the case, the density could be restricted on \( s_i = [0, Cap] \). Also, if \( s_i = Cap \) with a high probability, the expressions in Case A could be a good approximation. Inserting Eq.(21) into Eq.(17) gives the profit of:

\[
\Pi_i = k_2 p_i^2 + k_1 p_i + k_0
\]

(22)

\[
k_2 = \frac{1}{4 a_2 c_i}, \quad k_1 = \frac{a_1}{2 a_2}, \quad k_0 = (a_0 - \frac{a_1^2}{4 a_2}) c_i
\]

(23)

It is desirable to determine the amount of electricity futures contracts that should be bought to minimize the variance of the profit. The time horizon of the profit is \( (t_1, t_2) \), and the settlement price of the futures contract is found from the spot price over the delivery period, \( (t_3, t_4) \). The total profit is

\[
\Pi = \sum_{i=t_1}^{i=t_2} \Pi_i.
\]

The settlement price for one futures contract is

\[
\pi^* = \sum_{k=t_1}^{k=t_2} \pi^k.
\]

In appendix A, it is shown that the number of electricity futures contract that minimizes the variance is:

\[
\alpha = \sum_{i=t_1}^{i=t_2} \sum_{k=t_1}^{k=t_2} C_{i,k}^p \pi^* \frac{C_{i,k}^p \pi^*}{\sum_{i=t_1}^{i=t_2} \sum_{k=t_1}^{k=t_2} C_{i,k}^p \pi^*}
\]

(24)

The result of Eq.(24) is illustrated for cases that meet the following assumptions: (A1) The futures contract is based upon the same market as the spot market where the producer is selling electricity, or \( C_{i,k}^{p,p} = C_{i,k}^{p,p} \). (A2) The mean price of electricity is constant, or \( \pi = \pi \), and (A3) the time-horizon for the profit and the futures contract is identical, or \( (t_1, t_2) = (t_3, t_4) \).

Using the above 3 assumptions, Eq.(24) can then be simplified

\[
\alpha = 2 k_2 \pi + k_1 \sum_{i=t_1}^{i=t_2} \sum_{k=t_1}^{k=t_2} C_{i,k}^p \pi^k = 2 k_2 \pi + k_1
\]

(25)

Inserting \( k_1 \) and \( k_2 \) from Eq.(25) gives:

\[
\alpha = 2 k_2 \pi + k_1 = 2 \frac{1}{4 a_2 c_i} \pi - a_1\frac{a_1}{2 a_2} = 2 \frac{1}{4 a_2 c_i} \pi - a_1
\]

(26)

Comparing this with the amount of energy the producer expects to sell in each period and using the expected value of Eq.(21), it is seen that:

\[
E[s_i^2] = \frac{1}{2 a_2} (\pi - a_1) = \alpha
\]

(27)

Under the assumptions A1-A3, the amount of electricity bought in the futures contracts is equal to the expected sales.

VI. HEDGING USING PRODUCTION

Hedging does not rely solely on futures contracts; a producer can hedge using the production scheduling. So far it is assumed that the financial contracts of the production does not affect physical scheduling. In many ways this can be a desirable feature of these contracts: It enabled us to divide the physical scheduling of generators and the evaluation of financial contracts into two separate sub-problems. However, from the general formulation of the interaction between contracts and the physical production in Section IV, it is clear that this division is not possible in general. This section will consider the fuel limitation if the producer is to incorporate scheduling in the calculation of the EF.

Case C: Unlimited Amount of Fuel

From Eq.(15) an expression for the variance of the profit from selling a predetermined amount of electricity at spot market prices and buying fuel from the fuel spot market is:

\[
\text{var}(\Pi) = s C_{i,k}^{p,p} s^T + b C_{i,k}^{c,c} b^T - 2 s C_{i,k}^{p,c} b^T
\]

Suppose an electricity producer sells electricity contracts that specify a certain profile of sales with prices linked to the spot market. Could this producer minimize the uncertainty by choosing the vector of sales \( s^* \)? According to Markowitz [8], this can be formulated as an optimization problem. For every
given value of expected profit, minimize the variance. The efficient frontier is found by solving the following optimization problem for a range of expected profits, \( E[\Pi] = \eta \). The minimization of the variance is formulated as an optimization problem, i.e.,

\[
\min_s \quad sC \cdot p \cdot s^T + bC \cdot e \cdot b^T - 2sC \cdot p \cdot b^T
\]

\[
\text{const.:} \quad b = f(s)
\]

\[
sE[p^T] - bE[e] = \eta
\]

\[
s_i \in [0, Cap]
\]

The relation \( b = f(s) \) follows from Eq.(9). An implicit assumption in this formulation is that the supply of fuel is unlimited, only the cost of fuel is uncertain. To simplify it is assumed that the cost of fuel is zero, which yields

\[
\min_s \quad sC \cdot p \cdot s^T
\]

\[
\text{const.:} \quad sp^T = \eta
\]

\[
s_i \in [0, Cap]
\]

Note that the zero fuel cost assumption does not affect the minimum variance solution from a fixed-cost formulation.

**Case D: Limited Amount of Fuel**

The formulation of the optimization problem must be modified if the supply of fuel is limited, as in the case of a hydro producer or a fuel-based bilateral contracts with a limited supply over a time horizon. In this case, every decision made in the current period will have consequences for the later decisions. However, given that a decision has been made to use a specified amount of fuel, \( F \), over some time horizon, one can determine the corresponding EF. The additional constraint of \( \sum f(s_i) = F \) must be added to reflect the fuel constraint. The constraint \( s_i \in [0, Cap] \) can be relaxed for a hydro producer by storing energy in the reservoir. Suppose a hydro producer already has fixed-price bilateral contracts to supply electricity. Then the producer can buy electricity in the market, supply the energy to the customers and save the water. In the next period the producer can produce more electricity, selling the stored energy. The corresponding optimization problem is (disregarding income from the bilateral contracts):

\[
\min_s \quad sC \cdot p \cdot s^T
\]

\[
\text{const.:} \quad sp^T = \eta
\]

\[
\sum_i f(s_i) = F
\]

\[
s_i \in [s_{min}, Cap]
\]

**Simulation Results for Case C and Case D**

To simulate the optimization problems for hedging using production, one would need the covariance matrix of the electricity prices. In other words, a stochastic model of the electricity market price is needed. Stock price models exist; however, they are not necessarily applicable to the electricity market. Since the electricity price modeling is an area yet to be established, the simulations in this study use simplified models that are judged to be meaningful.

The stochastic process of prices is modeled as a sum of a mean value and a zero-mean normally distributed disturbance:

\[
p_s = \bar{p}_s + p_s^\Delta
\]

The mean goes through a one-week cycle:

\[
\bar{p}_s = \bar{p}_{s-7}
\]

The disturbance is modeled as an auto-regressive stationary time series dependent on the value yesterday and the value a week ago:

\[
p_s^\Delta(1 - a \Delta z^{-1})(1 - b \Delta z^{-2}) = h_0 e \quad e \sim N(0,1)
\]

The time-domain equation is:

\[
p_s^\Delta - ap_{s-1}^\Delta - bp_{s-7}^\Delta + abp_{s-8}^\Delta = h_0 e_{s-8} + h_1 e_{s-7} + h_2 e_{s-6}
\]

The following values are used: \( a = 0.8 \), \( b = 0.6 \), \( h_0 = 0.1 \) and \([\bar{p}_1, \ldots, \bar{p}_7] = [2.6, 2.5, 2.3, 1.8, 2.4, 2.5, 2.6] \). The covariance matrix of the prices is found from the Yule-Walker equations [9]. An example of Case C has a time-horizon of 17 periods. The plot of the EF is shown in Fig. 3.

![Figure 3 Efficient Frontier I](image-url)
Another example is to assign all capacity to bilateral contracts, and sell / buy spot market contracts to make use of the expected price differentials between different periods. The optimization problem stays the same, but the constraint of total usage of fuel sold to the spot market is zero, \( F = 0 \). Figure 5 gives the EF for the sales to the spot-market.

The EFs in both Figures 4 and 5 indicate saturation as the uncertainty increases. Consequently, there will not be a large family of risk-averse lines of indifference that produce a tangent portfolio at the maximal expected profit. The flat portion does not increase the expected profit significantly while risk increases. This implies that few risk-averse decision-makers would choose the portfolio with the maximal expected profit, which is different from Case C.

**VII. CONCLUSION**

This study finds solutions guiding choices of financial contracts for important scheduling policies. For the policy of selling electricity in the spot market, the effect of futures contracts is evaluated. If fuel is limited, scheduling over a time-horizon is an integral part of the contract evaluation.

This paper is focused on how contract evaluation and scheduling of physical resources are related. Future work might include the configuration of interconnected power systems and actions of independent system operators. This requires a more careful analysis of how the transfer of power could be constrained by nodal pricing or by curtailment of the load and / or generators. The specifics of how the market is cleared should be incorporated. Joint statistics between sales and prices of electricity at a location may have to be modeled for various locations. Related research being conducted includes bidding, pricing, market assessment and modeling.

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**IX. REFERENCES**


**X. BIOGRAPHIES**

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XI. APPENDIX A

The profit is $\Pi = \sum_{i=1}^{t_i} \Pi_i$. The settlement price is $f^* = \sum_{k=1}^{t_k} p^*_k$.

The co-variance between the two is:

$$C^{\Pi,f^*} = \sum_{i=1}^{t_i} \sum_{j=1}^{t_j} \mathbb{E}[\Pi_i p^*_j] - \mathbb{E}[\Pi] \mathbb{E}[p^*_j] = \sum_{i=1}^{t_i} \sum_{j=1}^{t_j} C^{\Pi,f^*}$$

(A1)

$$C^{\Pi,f^*} = \alpha \left( k_1 \mathbb{E}[p^*_1 p^*_i] - \mathbb{E}[p^*_1] \mathbb{E}[p^*_1] \right) + k_i C_{i,k}^{p,p'} \right)$$

(A2)

It can be shown that $\mathbb{E}[p^*_1 p^*_1] - \mathbb{E}[p^*_1] \mathbb{E}[p^*_1] = 2 \bar{p}_i C_{i,k}^{p,p'}$.

Inserting this into Eq.(A2), and Eq.(A2) into Eq.(A1) gives:

$$C^{\Pi,f^*} = \sum_{i=1}^{t_i} \sum_{k=1}^{t_k} \left( 2k_1 \bar{p}_i + k_i \right) C_{i,k}^{p,p'} \right)$$

(A3)

The variance of the settlement price is:

$$C^{f^*,f^*} = \sum_{i=1}^{t_i} \sum_{k=1}^{t_k} C_{i,k}^{p,p'}$$

(A4)

Inserting Eq.(A3) and Eq.(A4) into Eq.(6), the minimum variance is found:

$$\alpha = \frac{C^{\Pi,f^*}}{C^{f^*,f^*}} = \frac{\sum_{i=1}^{t_i} \sum_{k=1}^{t_k} \left( 2k_1 \bar{p}_i + k_i \right) C_{i,k}^{p,p'}}{\sum_{i=1}^{t_i} \sum_{k=1}^{t_k} C_{i,k}^{p,p'}}$$

(A5)