Analysis of Electricity Market Rules and Their Effects on Strategic Behavior in a Non-Congestive Grid

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Abstract: Earlier work has discussed the potential for strategic bidding in deregulated electricity markets, and shown specifically how generators can take advantage of congestion in their strategy. We show that it is also possible for even mid-price suppliers to create congestion problems through gaming in a non-congestive system. Under auction mechanisms such as in the United Kingdom, this can be profitable, at the consumer's expense. The optimal auction prevents profitable gaming, but requires the simultaneous handling of market clearing and system dispatch, making it harder to ensure the neutrality of system operations.

Keywords: Auctions, Congestion, Economics, Game theory, Power transmission.

I. INTRODUCTION

Recent regulatory reforms in the power industry require the creation of markets for electricity. The monopoly utility had the necessary information to operate the system at least cost. The intent of the new structure is to make generation competitive while transmission system operation remains a regulated function. The question arises as to whether the interaction of profit-maximizing suppliers and an independent system operator will lead to efficient outcomes given the constraints unique to electrical systems – in particular, the possibility of transmission congestion and the fact that power flows are determined by electrical laws. When players behave strategically, there may be trade-offs between different measures of efficiency.

Important features of restructured systems are described in [1], focusing on Argentina and Chile. Game theory is used in [2] in dealing with transmission losses. In [3], players are "strategic" in the sense of not being myopic about the risks related to future price changes. The authors of [4] treat the transmission provider itself as a potentially strategic player. A genetic algorithm is used in [5] to allow players' optimal strategies to evolve. In [6] and [7], players are explicitly allowed to bid away from their marginal cost. The same is true in [8], which also mentions the possible role of congestion constraints but does not provide an analysis. Similarly, references [9] and [10] focus on congestion management, but do not consider strategic behavior (gaming) within that context.

Evidence from the U.K. shows that some auction participants were able to command significant mark-ups [11]. While the authors of [12] attribute this to the small number of competitors, the physics of transmission networks can also play a role. Economists have long been aware that applying their knowledge from other markets to the electricity market can be misleading [13]. An important example is in [14], which combines a Cournot game-theory approach with players that specifically take advantage of congestion for strategic advantage. This system has the economically unusual result that market power can be exercised by selling more output rather than less.

Reference [14] implicitly handles market clearing and system dispatch simultaneously, by deriving the solution through the maximization of an objective function combining system constraints and the cost of obtaining power. But as advocated in [10], some systems such as that in the United Kingdom separate these functions to a degree, in order to ensure the neutrality of system operation.

In this paper we first consider a market mechanism similar to the one in the UK, as the grid belongs to one company that handles market clearing first and then has the authority to curtail generators when congestion occurs. We then examine an optimal auction in which the two functions are handled simultaneously. We model a system that is non-congestive, and find that even a supplier who is not the cheapest can profitably *create* congestion in the initial market outcome by the same mechanism as in [14], i.e. by selling more power than in the efficient (least-cost) solution. In this paper, we also demonstrate that this is only true under "UK" type systems; the optimal auction prevents this behavior by removing the separation of the market-clearing and system-dispatch functions.

II. PROBLEM DEFINITION

Our analysis is performed for 3 generators owned by 2 firms in a N-bus power system. For illustration, we examine a 3-bus system first. The N-bus derivation is given in the appendix. The terminology is described in the following.

P_i:	Generation by unit i, in MW (i=A, B or C)
L:	Load, in MW
C_1:	Capacity of line 1, in MW

α:	Fraction of A's generation flowing on line 1
β:	Fraction of B's generation flowing on line 1
K_i	Capacity of generator i, in MW
S_i	Unit cost of generator i, in \$/MW
S_{MAX}	Maximum price allowed, in \$/MW
b_i	Bid of generator i, in \$/MW
X_A	A's critical generation for congestion, in MW
P_i*	Pre-curtailment schedule for generator i, in MW
ξ	Excess flow on line 1, in MW
OC_k	Total operating cost under mechanism k, in \$
PC_k	Total pool cost under mechanism k, in \$
$\underline{b_C}$	C's lowest bid for profitable gaming, in \$/MW
π_i	Generator i's total profit, in \$

There is generation at all 3 buses, and a load of L. For a lossless system, $P_A + P_B + P_C = L$, where P_i is power supplied by generator i, i = A, B, C.

We are concerned with congestion on line 1, connecting buses 1 and 3, which has a transmission capacity of C_1. From the DC load flow model, we can determine the portions of each generator's contribution to the power flow on line 1. For simplicity, these are notated as α and β for A and B respectively.

To avoid complications arising with multi-unit auctions, we assume that each player bids its entire capacity at a single price. As a consequence, our problem is interesting only if A cannot supply the entire load.[1] There is no such restriction on B and C: In other words, $K_A < L$, $K_B \geq L$, $K_C \geq L$, where K_i is the capacity of generator i.

For the system to be non-congestive, both A and B must be unaffected by congestion when running alone, which means $\beta L < C_1$ and $\alpha K_A < C_1$. As in other research analyzing the strategic use of congestion, we assume that the strategizing player has more effect than other players on congestion. (See appendix.) The implication is that $\alpha > \beta$.

We examine a situation in which B is the cheapest supplier, A is intermediate, and C is the most expensive. We assume that the marginal cost of generating electricity is constant, and that, for various reasons[2], there is a maximum price S_{MAX} that can be charged by any of the 3 producers. If the cost of i's production is S_i per unit, we have $S_B < S_A < S_C < S_{MAX}$.

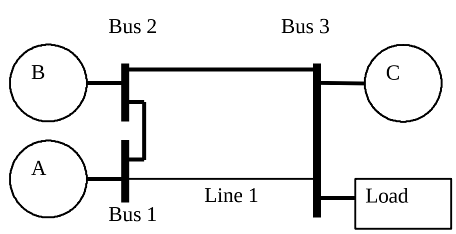

Fig. 1. Three-bus system with one potentially congested line.

[1] Alternatively, we could eliminate this assumption by allowing A to bid only a fraction of its capacity.

[2] This could be due to the existence of a competitive supply at that price at bus 3, as in [14], or the enforcement of a maximum price (as in the U.K. [15]) or even to suppliers' own choice given the elasticity of demand.

We look first at three different auction mechanisms that separate market clearing and potential congestion problems: uniform pricing, "UK" pricing, and discriminatory pricing, and compare them to the optimal auction, which is similar to nodal pricing. Under uniform pricing, the pool looks for congestion "along the way" while considering bids b_i. Thus if a generator's supply would cause congestion, some or all of that power is passed over in favor of another supplier who is more expensive but whose supply does not cause congestion. Previously accepted bids are not revised, even if that would lower the pool's cost. The highest bid accepted becomes the uniform price at which all power is purchased.[3]

Under "UK" pricing (so called because it is essentially the system used in the United Kingdom), the pool first purchases as much power as it needs to meet demand, and the highest bid it has accepted becomes the system marginal price (SMP). It then examines its purchases to see whether there are system security problems. If so, some suppliers must be "curtailed" or "constrained off," i.e., ordered to supply less power than the amount to be purchased originally. The resulting shortfall in supply is made up by "constraining on" other suppliers, i.e., ordering them to run their generation facility even though their power was not purchased in the original settling of bids. All power that is originally purchased and actually run is bought at the SMP. A supplier who is curtailed is compensated for possible lost profits by receiving a compensatory payment on the curtailed power; the price of this payment is calculated as the difference between the SMP and the curtailed bid. Any power constrained on is paid at the supplier's bid price.

Under discriminatory pricing, each supplier receives its bid for any power actually supplied. There is no SMP. The pool curtails suppliers to remove congestion but there are no compensatory payments for suppliers constrained off.

With both "UK" and discriminatory pricing, the pool must have some criterion for deciding which suppliers are curtailed. We consider two possible rules. (i) Under "merit-order curtailment," those purchased first (because of their low bids) are given preference, so that the last purchased are the first curtailed. (ii) With "least-curtailment," the pool tries to minimize the amount by which it asks suppliers to curtail.

The optimal auction (e.g., [16]) does not necessarily follow either of these curtailment rules, but instead evaluates the entire set of feasible schedules and chooses the one with the lowest cost to the pool. This requires that market clearing and system dispatch be handled simultaneously, as modeled in Section III.*D*, below.

We are interested in the strategic behavior of generators while bidding to the pool. We assume that the cheapest supplier, B, will always bid its marginal cost. For instance, B could represent a number of small producers with similar marginal costs. We will assume that A and C are two generators in the same company (Player 1) that can gain market share from B (Player 2).

Define X_A, the critical amount of generation from A such that if $P_A = X_A$ and $P_B = L - X_A$, line 1 is exactly at its capacity limit, that is:

[3] A common alternative is to choose the lowest rejected bid.

$C_1 = \alpha X_A + \beta(L - X_A)$. (1)

The implication of X_A is that if $P_A > X_A$, then it is impossible for A and B to supply the load without violating line 1's capacity limit. We will have $P_A + P_B < L$, and C will have to make up the difference.

Suppose now that quantities P_A* and P_B* have been scheduled, such that $P_A^* + P_B^* = L$ and $P_A^* > X_A$. There will be some amount ξ by which line 1 would exceed its capacity limit if these amounts were actually run, and this is the difference between the calculated flow in line 1 and the line's capacity, or,

$\xi = [\alpha \cdot P_A^* + \beta \cdot P_B^*] - C_1$.

Either A or B will have to be curtailed, i.e. ordered to run less than P_i^*, i = A, B. This same amount must be made up by C. Each unit of curtailment by A reduces the flow on line 1 by 1/α, thus the total curtailment (and P_C) must be ξ/α. By a similar argument, if B is curtailed, its total curtailment and P_C must both be ξ/β.

We compare different bidding structures and curtailment rules on the basis of operating cost (*OC*), pool cost (*PC*), and the ease of strategic behavior. *OC* is the cost of actually producing a given amount of power from certain producers, and thus represents the social cost of that production. *PC* is the cost to the pool of purchasing that power from the suppliers. Obviously if there is some net profit among the suppliers, *PC* is greater than *OC*. Ease of strategic behavior is measured by $\underline{b_C}$, which is C's minimum bid to give Player 1 positive profits. The higher $\underline{b_C}$ is, the more likely it is to be above S_{MAX}, making strategic bidding infeasible.

III. RESULTS OF ANALYSIS

A. Benchmark: no strategic behavior

According to standard auction theory [17], the promise of competitive bidding in a deregulated market is that sellers will bid their marginal costs. This means that $b_i = S_i,\ \forall\ i$, where b_i is generator i's bid.

We call this situation our benchmark since it involves no strategic behavior. Since B is the cheapest, and it has enough capacity to supply the entire load, it will be the sole supplier. A and C produce nothing. The system's operation will be described by the following equations.

$P_B = L$;

$P_A = P_C = 0$;

$PC_{BENCH} = OC_{BENCH} = S_B \cdot L$.

("BENCH" subscripts designate "benchmark.")

B has revenue and costs of $S_B \cdot L$, and thus zero profit.

B. Strategic behavior

We show that under the curtailment rules considered here, A can bid less than B (i.e., less than its own true cost) and cause congestion on the transmission line(s), which Player 1 can use to its advantage. This will sometimes entail a loss for A, in which case C's profit will have to be high enough to give A and C together positive profits. Recall that B is non-strategic; it always bids its marginal cost. That is, $b_B = S_B$ and $b_A < b_B = S_B < S_A$.

B.1. Uniform Pricing

The pool would buy all of A's electricity, because according to the bids it is the cheapest. Since A cannot supply the entire load, the pool would then turn to B to meet the remaining load, giving the schedule $P_A = K_A$, $P_B = L - K_A$.

As long as $K_A > X_A$, this will cause congestion in line 1. The pool checks for congestion as it purchases electricity. Since B's supply causes congestion, it will be scheduled at less than $L - K_A$, with C making up the difference. As a result, the actual schedule is $P_A = K_A$, $P_B = (C_1 - \alpha K_A)/\beta$, $P_C = L - K_A - (C_1 - \alpha K_A)/\beta$.

The operating cost OC_U (subscript "U" for uniform pricing) must be more than $OC_B = \boldsymbol{S_B \cdot L}$**,** because the total power produced is still L units, but part of this is produced at costs S_A and S_C, both of which are greater than S_B.

The cost to the pool will be set by C's bid. Indeed, C's bid determines the SMP which all suppliers will receive. Under our assumptions, C has no reason to bid anything less than S_{MAX}, so we have $PC_U = S_{MAX} \cdot L$, which is also greater than PC_{BENCH} since $S_{MAX} \geq S_C > S_B$.

Notating *i's* profits as π_i, we have

$\pi_A = (S_{MAX} - S_A) \cdot K_A > 0$,

$\pi_B = (S_{MAX} - S_B) \cdot (C_1 - \alpha K_A)/\beta > 0$,

$\pi_C = (S_{MAX} - S_C) \cdot [(L - K_A - (C_1 - \alpha K_A)/\beta] > 0$.

By bidding under its costs, A was able to generate positive profits. This result, unique to the electricity market, is due to two reasons. First, A's power congested line 1. Because of the congestion, the pool had to call on a more expensive supplier. Second, the most expensive supplier determines the SMP paid to everyone. Because C does not have to offset any losses from A ($\pi_A > 0$), S_{MAX} will never limit Player 1's strategic behavior.

The analytical results for uniform pricing and other auction mechanisms are summarized in Table 1.

B.2. "UK" System

The "UK" system avoids some of the problems of uniform pricing by paying the highest price only to those dispatched "out of merit" due to congestion.

B.2.1. Merit-Order Curtailment

With merit-order curtailment, the first supplier constrained off is the last one purchased. So if $b_A < b_B$, B will be curtailed. The pool schedules $P_A^* = K_A$ and $P_B^* = L - K_A$, establishing SMP = b_B (= S_B), then curtails B by *x/b*, with C again making up the difference. This results in the identical dispatch as under uniform pricing, meaning that $OC_M = OC_U$, where the subscript "M" denotes merit-order curtailment.

But A is now losing money, because it receives a price of $SMP = b_B < S_A$. This means that C must bid $\underline{b_C} > S_C$, where $\underline{b_C}$ is defined by setting C's profit equal to A's loss. If $\underline{b_C} > S_{MAX}$, Player 1's gaming is not profitable. If $\underline{b_C} \leq S_{MAX}$, C will bid S_{MAX}, leading to PC_M in Table 1.

The total power purchased is still L, but some is now purchased at less than S_{MAX}, so $PC_M < PC_U$. B's profit has been driven to 0, since it only receives the SMP, which is the same as its cost. Also, Player 1's profit has been reduced by: $(S_{MAX} - S_B)\cdot K_A$.

So we can conclude that, while it results in the same inefficient system operation as uniform pricing, "UK" pricing with merit-order curtailment reduces the pool's losses. And while under uniform pricing Player 1 has a profitable strategy even if $b_C = S_C$, with merit-order curtailment, C must bid above its cost, leaving less room for profitable strategic bidding.

B.2.2. Least-Curtailment

If the pool's rule is to curtail the smallest amount of power possible, it will curtail A because $\alpha>\beta$ implies that $\xi/\alpha<\xi/\beta$. In order for its strategic bidding to be profitable, A must still bid $b_A < S_B$ (otherwise the pool buys L from B). As under merit-order curtailment, the pool schedules $P_A^*=K_A$ and $P_B^*= L-K_A$, establishing SMP = b_B (= S_B). It then curtails A by ξ/α, and purchases this same amount from C.

To prove that $OC_L < OC_M = OC_U$ (subscript "L" for least curtailment), we can show that the power supplied by relatively expensive A and C is decreased, while that supplied by inexpensive B is increased and the total is the same. These results follow from $\alpha > \beta$, $K_A > X_A$, and (1). But because we still have some production from A and C, it must be true that $OC_L > OC_{BENCH}$.

B's profit is again 0 because $SMP = S_B$. A's profit is more complicated, because of the non-zero compensation payment. It can be shown that $\partial\pi_A/\partial b_A < 0$, that is, the lower A's bid, the greater its profit. This makes intuitive sense: as A reduces its bid, it does not reduce the price for the power it will ultimately sell, because it will receive the SMP, which will remain at S_B. But it widens the gap between the SMP and its bid, which increases its "compensation" for being curtailed. So $b_A = 0$.

π_A may be either positive or negative; in other words, Player 1 might make profits directly from A, without considering C's contribution. However, we know that $\alpha K_A + \beta(L - K_A) - C_1 > 0$. Therefore, π_A is more likely to be positive when K_A is much larger than X_A (i.e., A can cause congestion by selling a small fraction of its capacity) or when A's costs are very close to B's costs (so S_B/S_A is close to 1). When $p_A < 0$, one can again find $\underline{b_C}$ by requiring C's profit to be equal to A's loss. If $S_{MAX} < \underline{b_C}$, strategizing is unprofitable. When $\pi_A > 0$, or $\pi_A < 0$ and $S_{MAX} > \underline{b_C}$, C will bid S_{MAX}, and we can calculate PC_L, including A's compensation payment. Comparing it to PC_M and PC_U, we cannot say definitively whether PC_L is higher. But the higher S_B is, the more likely we are to have $PC_L > PC_M$ and $PC_L > PC_U$.

TABLE I
RESULTS

Benchmark:

$OC_{BENCH} = S_B \cdot L$

$PC_{BENCH} = S_B \cdot L$

Uniform pricing

$OC_U = S_A K_A + S_B(C_1 - \alpha K_A)/\beta + S_C\{L - K_A - (C_1 - \alpha K_A)/\beta\}$

$PC_U = S_{MAX} \cdot L$

"UK", merit-order curtailment

$OC_M = OC_U$

$PC_M = S_{MAX} \cdot L - (S_{MAX} - S_B)\cdot\{K_A + (C_1 - \alpha K_A)/\beta\}$

$$\underline{b_C} = S_C + \frac{(S_A - S_B)\cdot\beta K_A}{\alpha K_A + \beta(L - K_A) - C_1}$$

"UK", least curtailment

$P_A = [C_1 - \beta(L - K_A)]/\alpha$

$P_B = (L - K_A)$

$P_C = K_A - \{[C_1 - \beta(L - K_A)]\}/\alpha$

$PC_L = S_B \cdot L + S_{MAX}[\alpha K_A + \beta(L - K_A) - C_1]/\alpha$

$$OC_L = S_A \frac{C_1 - \beta(L - K_A)}{\alpha} + S_B(L - K_A) + S_C\left(K_A + \frac{C_1 - \beta(L - K_A)}{\alpha}\right)$$

$\pi_A = S_A \cdot\{(S_B / S_A)\alpha K_A + \beta(L - K_A) - C_1\}/\alpha$

$\underline{b_C} = S_C - S_A + [(S_A - S_B)\cdot\alpha K_A]/[\alpha K_A + \beta(L - K_A) - C_1]$

Discriminatory, merit-order curtailment

$\underline{b_C}_{DM} = \underline{b_C}_M$

Dispatch: same as uniform and "UK", merit-order curtailment

$OC_{DM} = OC_M = OC_U$

$PC_{DM} = PC_M$

Discriminatory, least curtailment

Dispatch: same as "UK", least curtailment

$\underline{b_C} = S_C - S_A + S_B + [(S_A - S_B)\cdot\alpha K_A]/[\alpha K_A + \beta(L - K_A) - C_1]$

$OC_{DL} = OC_L$

$PC_{DL} = S_B \cdot L + (S_{MAX} - S_B)\cdot[\alpha K_A + \beta(L - K_A) - C_1]/\alpha$

The intuition of this is that if B is considerably cheaper than S_{MAX}, then uniform pricing and merit-order curtailment are relatively expensive for the pool, forcing it to purchase all or a large portion of its power at a price much higher than S_B. As S_B rises, this penalty shrinks, while the size of A's potential compensation payment under least-curtailment grows.

The size of S_B also affects the relative $\underline{b_C}$. Comparing least-curtailment and merit-order, the closer S_B is to S_A, the more likely $\underline{b_C}_L < \underline{b_C}_M$, and thus the more likely it is to be under S_{MAX}, making the strategic bidding possible. Also, the higher A's cost, the more likely $\underline{b_C}_L > \underline{b_C}_M$, i.e., with a high S_A least-curtailment becomes less favorable to strategic behavior than merit-order.

C. Discriminatory Pricing

Under discriminatory pricing, supplies are again purchased in the order of increasing bids, but each supplier is paid exactly its bid. As a result, there is no SMP, and no compensation payment. Therefore if A bids strategically, such that $b_A < b_B = S_B < S_A$, its profits (actually losses) will be $\pi_A = (b_A - S_A)\cdot P_A < 0$. A clearly minimizes its losses by setting b_A as close to b_B as possible. In the limit, $b_A = b_B = S_B$. The pool uses curtailment to resolve any congestion problems.

C.1. Merit-Order Curtailment

With merit-order curtailment A's loss can be rewritten as $\pi_A = (S_B - S_A)\cdot K_A < 0$. This is identical to the situation under UK pricing with merit-order curtailment, so $\underline{b_C}$ carries over. Similarly, nothing changes when considering C's actual bid of S_{MAX}, so PC_M will likewise carry over, giving us $OC_{DM} = OC_M$, $PC_{DM} = PC_M$, where "DM" denotes "discriminatory pricing, merit-order curtailment."

C.2. Least-Curtailment

With least-curtailment, we have the same supply as with "UK" pricing, least-curtailment, so $OC_{DL} = OC_L$, where "DL" denotes "discriminatory pricing, least curtailment." But without the compensation payment, A's best strategy is no longer $b_A = 0$, but again $b_A = b_B = S_B$. The resulting $\underline{b_C}$ is clearly larger than that from the "UK" system, least-curtailment, having an additional S_B term in it. It can also be shown that $\underline{b_C}_{DL} > \underline{b_C}_M$, so discriminatory pricing, least-curtailment, pushes $\underline{b_C}$ to its highest level among the systems considered here. Turning to the pool's cost, assuming $S_{MAX} > \underline{b_C}$ for all systems, $PC_{DL} < PC_L$ and $PC_{DL} < PC_M$.

While it is still above the non-strategic benchmark, discriminatory pricing with least-curtailment is the least likely structure to allow strategic bidding and also minimizes the pool's costs from strategic behavior when it does occur. It eliminates A's possibility of ameliorating its losses or possibly even profiting from compensation payments; it holds B to its marginal price instead of receiving S_{MAX} as under uniform pricing; and it limits the amount of power bought from expensive C.

D. Optimal Auction

In the optimal auction, the pool solves a minimization problem that deals with market clearing and system dispatch simultaneously. If marginal costs were increasing and demand were price-sensitive, the resulting dispatch would be nodal pricing. However, the simplified system considered here has constant marginal costs and inelastic demand. Therefore, if a generator is offered less than its bid, it will supply nothing; if it is offered its bid or more, it is ready to supply up to its maximum capacity. Consequently, there is no reason to offer a generator more than its bid. Because B is not bidding strategically, we replace b_B with S_B immediately and get

$$\begin{aligned} L = \min\ & b_A\cdot P_A + S_B\cdot P_B + b_C\cdot P_C \\ \text{s.t.}\quad & P_A + P_B + P_C = L, \\ & \alpha P_A + \beta P_B \le C_1, \\ & P_A \le K_A, \\ & P_i \ge 0,\ i = \text{A, B, C.} \end{aligned}$$

In most rankings of bids, there will be no congestion, so either A and C are not dispatched, or only A is, and it is paid below its costs. Congestion only becomes an issue when we have $b_A < S_B < b_C$. Up to a load of K_A, it is cheapest for the pool to buy from A. For loads between K_A and $K_A + (C_1 - \alpha K_A)/\beta$, the pool buys from B, and there is still no congestion. For any power between $K_A+(C_1 - \alpha K_A)/\beta$ and L, the pool has two options. It can reduce P_A and increase P_B, at a cost of $\mu \equiv (\alpha S_B - \beta b_A)/(\alpha - \beta)$ per unit, or it can purchase from C at b_C, which it will only do if $b_C \le \mu$. There is thus a cap on what C can bid and still be dispatched, and this cap depends on A's bid.

Assume C bids exactly this cap, and is dispatched. The dispatch then matches our uniform pricing case. Using $b_C = \mu$, Player 1's profit can then be expressed as a function of b_A alone:

$$\pi_1 = (b_A - S_A)P_A + \left[(\alpha S_B - \beta b_A)/(\alpha - \beta)\right]P_C.$$

To find Player 1's optimal b_A we look at

$$\partial\pi_1/\partial b_A = P_A - [\beta/(\alpha - \beta)]P_C,$$

but by substituting in the actual values of P_A and P_C, this reduces to

$$\partial\pi_1/\partial b_A = (C_1 - \beta L)/(\alpha - \beta),$$

which is bounded below by 0, because $C_1 \ge \beta L$. Player 1's best choice is therefore $b_A = S_B$. μ then reduces to S_B, which in turn means that both A and C are bidding below their costs. Therefore Player 1 has no feasible strategic bid.

It appears that the optimal auction eliminates the kind of strategic bidding we have been examining. There are two reasons for this. First, it resembles discriminatory pricing in only paying each generator its bid. Second, compared to curtailment rules associated with a division between market clearing and system dispatch, the optimal auction gives the pool operator the flexibility to consider either a shift of generation from A to B, or the purchase of "fresh" power from C. This flexibility in turn puts a cap on what C can bid and still get dispatched. Because this cap is a decreasing function of A's bid, it squeezes Player 1, who can only cut its losses on A by sacrificing the gain on C.

One drawback should be noted, however. If A is truly the lowest cost supplier with the lowest bid and is curtailed due to congestion, A does not receive any compensation.

IV. SIMULATION RESULTS

We use the IEEE-30 model to simulate our results. We assume a limit of 21 MW on line 8, connecting buses 5 and 7. Our generator A is located at bus 13, B at bus 2, and C at bus 5. (The relevant line and buses are labeled in bold-face in Fig. 2.) Our benchmark case corresponds to B running 95 MW, supplying the load at bus 5, while A and C run 0; flow on line 8 is 19 MW. If A underbids B with a capacity of 60 MW, the pool would schedule $P_A{}^* = 60$ MW and $P_B{}^* = 35$ MW, but that would result in a flow of 27 MW on line 8. The actual dispatches under our different auction mechanisms are given in Table II, along with operating cost, pool cost, and C's

TABLE II
IEEE-30 RESULTS

System	**Dispatch**	$\underline{b_C}$	***OC***	***PC***
Bench-mark	$P_A = 0$, $P_B = 95$, $P_C = 0$	N/A	95	95
Uniform pricing	$P_A = 60$, $P_B = 18.5$, $P_C = 16.5$	N/A	204.5	760
UK, merit order	Same as Uniform	7.64	204.5	210.5
UK, least curtailment	$P_A = 48$, $P_B = 35$, $P_C = 12$	7	179	191
Discriminatory, merit order	Same as Uniform	7.64	204.5	210.5
Discriminatory, least curtailment	Same as "UK" least	8	179	179

minimum bid. These values are calculated with $S_A = 2$, $S_B = 1$, $S_C = 4$, and $S_{MAX} = 8$. (S_{MAX} has been chosen such that it will not limit C's necessary bid.)

With the costs assumed here, uniform pricing is very expensive for the pool, with S_{MAX} so much higher than the next lowest cost. "UK" least curtailment is somewhat worse for the pool than discriminatory pricing, least curtailment, but cheaper than the merit-order mechanisms. At the same time, A's ability to limit its losses makes strategic behavior slightly easier than in the merit-order schemes, as measured by C's minimum bid. Discriminatory pricing, least-curtailment is the cheapest for the pool, has the lowest operating cost, and leaves the least room for strategic behavior. The optimal auction would give the same results as the benchmark, since it makes strategic behavior unprofitable.

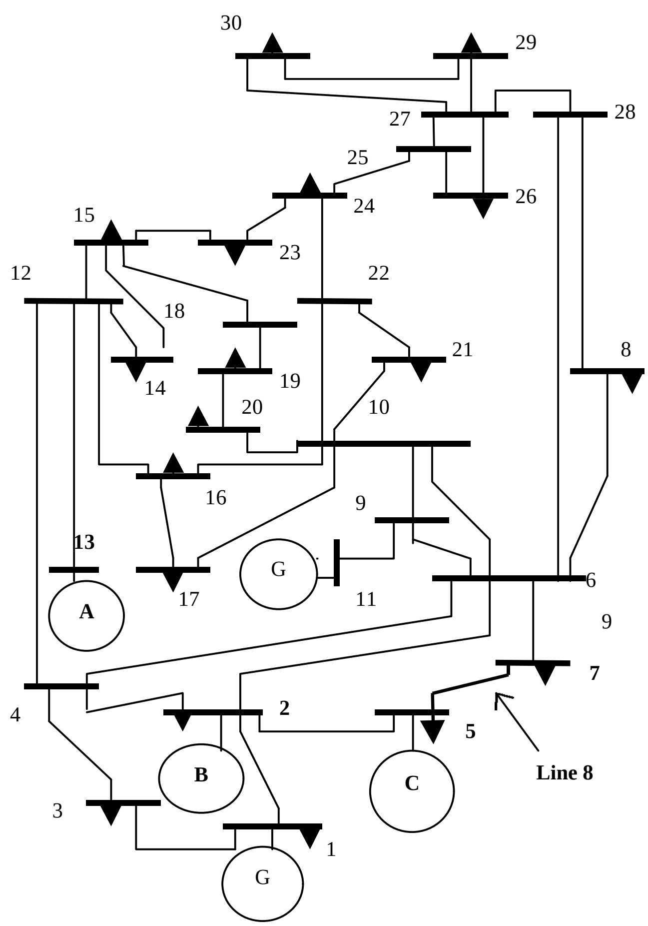

Fig. 2. IEEE-30 system

V. CONCLUSION

We have shown that intermediate-cost generators can use strategic bidding to profitably create congestion problems in non-congestive systems, when the market system separates the market-clearing and system-dispatch functions. The optimal auction prevents this behavior.

The use of constant rather than increasing marginal cost in the paper is a sacrifice of realism; however, it does allow an analytical approach rather than an iterative one. A task for future research is to test the insight presented here with increasing marginal cost.

A simple *n*-bus generalization is given in the appendix, but it does not address the issue of choosing the supplier to be constrained on. Future work will also relax the assumption that B does not behave strategically. As with increasing marginal costs, this will require a computational rather than analytical approach.

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VIII. APPENDIX

Denote by Y the mxm diagonal matrix of line susceptances, A the (n-1)xm reduced node incidence, B an (n-1)x(n-1) matrix relating the line angles to the bus injections in the DC load flow relation. Let **p** be a vector of injections at n-1 buses, then line flows on the system are

$\mathbf{f} = (YA^TB^{-1})\cdot\mathbf{p}$.

Then for line i we have

$$f_i = \sum_{j=1}^{n-1} D_{ij} p_j \ .$$

Separating the 3 buses, A, B and C, on line 1 of an n-bus system we have

$$f_1 = D_{11}P_A + D_{12}P_B + D_{13}P_C + \sum_{j=4}^{n-1} D_{1j} p_j \ ,$$

where C is the generator constrained on, $D_{11} = \alpha$, $D_{12} = \beta$, and $D_{13} = \gamma$. Let $\alpha^* = \alpha - \gamma$ and $\beta^* = \beta - \gamma$. In the system modeled in the paper, C is at the load, so $\gamma = 0$ and we are only concerned with α and β. In general, γ must be small or the congestion would not be relieved. α^* and β^* can then replace α and β throughout the paper and the analysis holds.

If there can be congestion when A and B are supplying the entire load between them, then it must be possible that

$\alpha P_A + \beta P_B > C_1$.

But it is also true that $P_A + P_B = L$, thus

$$(\alpha - \beta)P_A + \beta L > C_1, \qquad (2)$$

and since $\beta L < C_1$, (2) can only be true if $\alpha > \beta$.

IX. BIOGRAPHIES

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