Foreign Aid and Economic Growth: The Role of Flexible Labor Supply*

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Abstract

This paper examines the link between foreign aid, economic growth, and welfare in a small open economy. External transfers impinge on the recipient's macroeconomic performance by affecting resource allocation decisions and relative prices. The endogeneity of the labor-leisure choice and the adjustment of the real wage rate play a crucial role in the propagation of foreign aid shocks. Another crucial determinant of the efficacy of foreign aid is externalities associated with the public good that the aid helps finance. The impact of tied and untied aid on the recipient government's intertemporal fiscal balance is also considered. Finally, the transitional adjustment to a foreign aid shock is shown to depend crucially on the elasticity of substitution in production and the relative importance of the labor-leisure choice in utility.

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1. Introduction

Official development assistance, in the form of foreign aid or unilateral capital transfers, represents an important channel through which wealth is transferred from rich, developed nations to poorer, underdeveloped economies. Both the magnitude and the scope of these international transfers have increased significantly over the last four decades. For example, total flows of official development assistance from members of the OECD and OPEC countries have increased from about $6 million in 1965 to over $59 billion in 2002. By that time these funds had come to represent between 3-5 percent of the Gross National Income of the recipient low and middle income countries, and to finance between 10-20 percent of their gross capital formation.[1]

One issue of concern for both donors and recipients is how foreign aid should be spent in an economy with scarce resources. This has given rise to a long-standing debate, both in academic and policy circles, as to whether international transfers should be "tied" ("productive") or "untied" ("pure"). As Bhagwati (1967) points out, tied external assistance can take several forms. It may be linked to: (i) a specific investment project, (ii) a specific commodity or service, or (iii) procurement in a specific country. Recent studies by the World Bank point out that over time, a larger proportion of foreign aid has become "untied" with respect to requirements for procuring goods and services from the donor country, but it has become more "tied" in the sense of being linked to investments in public infrastructure projects (telecommunications, energy, transport, water services, etc). Between 1994 and 1999, for example, the proportion of official development assistance that was "untied" in the sense of not being subject to restrictions by donors on procurement sources rose from 66 percent to about 84 percent. At the same time, between two-thirds and three-fourths of official development assistance was either fully or partially tied to public infrastructure projects (see footnote 1).

The move toward tying more aid to public investment has been dictated mainly by the growing infrastructure requirements of developing countries. Most economists agree that investment in public infrastructure raises the productivity and efficiency of the private sector and, as a consequence, provides a crucial channel for economic growth, development, and higher living

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[1] World Bank (1994, 2004).

standards.[2] But financing public investment has proven to be a challenging task for developing economies. Most such countries have significantly restricted public borrowing after the debt-crisis of the early 1980s, while at the same time their infrastructure requirements have increased steadily. A 1994 World Bank study has estimated these requirements to be $200 billion a year. Facing binding fiscal constraints, governments in developing countries have turned to external financing, in the form of tied foreign aid programs, as a significant source of financing public investment.

The question of what form foreign aid should take has led to a growing empirical literature on the link between foreign aid and economic growth; see Burnside and Dollar (2000), Hansen and Tarp (2000), and Easterly (2003). Recently, Chatterjee, Sakoulis, and Turnovsky (2003) and Chatterjee and Turnovsky (2004) have developed a general equilibrium-growth framework within which this issue can be analyzed. Their results indicate that the consequences of tied and untied aid programs on economic growth and welfare depend crucially upon a number of key structural characteristics of the recipient economy. These include: (i) the costs associated with installing the publicly provided capital (intertemporal adjustment costs), (ii) the substitutability between public and private capital in production (intratemporal adjustment costs), (iii) the degree of access to the world financial market (financial adjustment costs), and (iv) the opportunities for co-financing infrastructure projects by domestic resources, like the domestic government or private sector.

This paper distinguishes itself from the existing theoretical literature on aid and growth by focusing on three critical aspects absent from previous analysis. First, and most important among these, is the relaxation of the restrictive assumption that labor supply in the recipient economy is inelastic. We show how the endogeneity of labor supply is a crucial transmission mechanism for foreign aid. Specifically, a given aid flow alters the marginal valuations of work and leisure, thereby impacting on the economy's productive capacity and resulting macroeconomic performance. Second, we focus on the interaction between labor and public capital in determining the productivity of private capital in production. A natural specification, consistent with sustaining an equilibrium of ongoing (endogenous) growth, is for labor to interact with public capital to yield "labor efficiency

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[2] Theoretical and empirical interest in the impact of public capital on macroeconomic performance originated with the work of Arrow and Kurz (1970) and more recently with Aschauer (1989a, 1989b); see Gramlich (1994) for a survey of the early empirical literature. Theoretical contributions include Futagami, Morita, and Shibata (1993), Baxter and King (1993), Fisher and Turnovsky (1998), Devarajan, Xie, and Zou (1998), Turnovsky (2004).

units", which then interact with private capital to produce final output in accordance with a production, which can conveniently be assumed to be of the constant elasticity of substitution (CES) form. Treating this specification as a benchmark, we further extend the model by introducing an additional general productivity enhancing role for public capital, whereby it mitigates congestion associated with aggregate capital. Our results show how the relative merits of tied and untied aid are quite sensitive to how these externalities impinge on production, as well as their magnitudes. Finally, motivated by Burnside and Dollar (2000), we show how our theoretical model enables us to assess the effects of tied and untied aid programs on the recipient government's fiscal deficit, which is an element of the "good" policy index in the Burnside-Dollar paper. The numerical simulations we conduct highlight the tradeoffs that exist between the impact of foreign aid on consumer welfare, on the one hand, and on the recipient government's financial position, on the other.

Given the complexity of the model, most of the analysis is conducted numerically. The ability of the representative agent to choose between work and leisure dramatically alters many of the results obtained by Chatterjee et al. (2003) and Chatterjee and Turnovsky (2004). Specifically, these earlier papers show that when labor supply is inelastic, even though a tied aid shock generates a dynamic adjustment, an untied aid shock has no dynamic consequences for the recipient economy, and leads only to instantaneous increases in consumption and welfare. In contrast, we show that with flexible labor supply, both types of aid generate dynamic responses, albeit dramatically different in nature. In the case of untied aid, the dynamic adjustment occurs primarily through the labor-leisure choice, and the effect this has on the marginal rate of substitution between consumption and leisure and on the real wage rate. Although the economy's current account and welfare improve in the long run, the reduced work effort and higher consumption leads to a decline in the equilibrium growth rate. In contrast, an aid program that is tied to investment in public capital generates a much more gradual dynamic adjustment, one that is precisely opposite in nature to that following an untied aid program. There are significant tradeoffs in welfare between the short run and the long run, as the agent increases his work effort and initially substitutes away from consumption toward investment. The benefits of this substitution are realized only gradually over time, as the investment in public capital enhances productivity in the recipient economy and thereby increases both growth and welfare. We further illustrate how the characteristics of the transitional dynamics and the long-run effects of tied and untied aid programs depend crucially upon the recipient's elasticity of substitution in production and the elasticity of leisure in utility.

All three aspects being emphasized – the endogeneity of labor supply, the extent of public good-production externalities, and the recipient economy's level of public spending – play crucial roles in determining the contrasting effects of tied versus untied aid programs. On the one hand, both the short-run and the long-run welfare gains from an *untied* aid shock are remarkably uniform and relatively insensitive to these characteristics, as well as to the elasticity of substitution in production. On the other hand, there is a sharp difference between the short-run and long-run welfare effects of *tied* aid, both of which are highly sensitive to these same characteristics.

These contrasting sensitivities of the welfare gains resulting from a tied and untied transfer, respectively, means that their relative merits, from a welfare standpoint, are also highly sensitive to these critical elements. For the benchmark case of a Cobb-Douglas production function, and for what we view as a plausible elasticity of leisure in utility, tied aid is marginally superior to untied aid from a long-run (intertemporal) welfare perspective. However, as this elasticity declines, untied aid becomes superior to tied aid, while as the elasticity increases, the superiority of tied aid is enhanced. But this comparison is very sensitive to even small variations in the elasticity of substitution, those that might lie within a statistical margin of error.

The economy's fiscal structure has interesting consequences for the choice between tied and untied aid. While *untied* aid always improves the government's intertemporal fiscal balance, *tied* aid always leads to a deterioration, thus suggesting a potential tradeoff between consumer welfare and government solvency in the latter case. The sensitivity of the welfare effects of tied transfers to the rate of domestic government investment in the recipient economy has further important policy implications. If this component of expenditure is extremely small, then tied aid is not only superior to untied aid, it is in fact optimal, while the reverse applies if it is large. In contrast, for intermediate rates of government investment, the optimal composition of a given flow of foreign aid requires that it be *partially* tied to public investment.

The remainder of the paper is structured as follows. Section 2 sets out the analytical framework and summarizes the macro-dynamic equilibrium. Since the basic framework builds on our previous work, our description can be brief. But we hasten to add that the introduction of labor is nontrivial, involving substantial technical details that are relegated to the Appendix. Section 3 conducts numerical simulations and considers the consequences of the elastic labor supply in assessing the relative merits of tied versus untied aid. Section 4 provides some sensitivity analysis, while Section 5 considers the consequences for fiscal balance. Section 6 concludes with some caveats and policy advice.

2. The Analytical Framework

We begin by spelling out the building blocks of the model.

2.1. Private Sector

We consider a small open economy populated by an infinitely-lived representative agent who produces and consumes a single traded commodity. The agent has a unit of time, a fraction l of which can be devoted to leisure, and the balance, $1-l$, to labor supply. Output, Y, is produced using the Constant Elasticity of Substitution (CES) production function

$$Y=\alpha\left(\frac{K_G}{\bar{K}}\right)^{\varepsilon}\left[\eta\left\{(1-l)K_G\right\}^{-\rho}+(1-\eta)K^{-\rho}\right]^{-1/\rho} \qquad \varepsilon\geq 0 \qquad \text{(1a)}$$

where K denotes the representative agent's stock of private capital, $\bar{K}$ is the average stock of private capital, and K_G denotes the stock of public capital. The production function comprises two components. In the first, public capital interacts with the agent’s labor supply to yield labor measured in efficiency units, $(1-l)K_G$ which in turn combines with private capital.[3] The second is the externality provided by public capital, incorporated in the term, $(K_G/\bar{K})^{\varepsilon}$. Here, K_G enhances general productivity by offsetting congestion effects associated with the aggregate private capital stock, $\bar{K}$; see e.g. Barro and Sala-i-Martin (1992), Eicher and Turnovsky (2000). The production function has constant returns to scale in both the private factors of production, K and $(1-l)$, and the accumulating factors, $K, \bar{K},$ and K_G, enabling it to support an equilibrium of ongoing (endogenous)

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[3] Thus $s\equiv 1/(1+\rho)$ is the elasticity of substitution between private capital and “efficiency units of labor” in production.

growth with both private factors being paid their respective marginal physical products.[4]

The agent consumes this good at the rate *C*, yielding utility over an infinite horizon represented by the isoelastic utility function:

$$U \equiv \int_0^\infty \frac{1}{\gamma}\left(Cl^\theta\right)^\gamma e^{-\beta t} dt \tag{1b}$$

where θ represents the relative importance of leisure in utility. The agent also accumulates physical capital, with expenditure on a given change in the capital stock, *I*, involving adjustment (installation) costs specified by the quadratic (convex) function

$$\Psi\left(I, K\right) = I + h_1 \frac{I^2}{2K} = I\left(1 + h_1 \frac{I}{2K}\right) \tag{1c}$$

Letting δ_K denote the rate of depreciation, the net rate of private capital accumulation is thus

$$\dot{K} = I - \delta_K K \tag{1d}$$

Agents have access to a world capital market, allowing them to borrow internationally. We assume that the creditworthiness of the economy influences its cost of borrowing from abroad. The world capital market assesses the economy's ability to service its debt costs, and views the country's debt-capital (equity) ratio as an indicator of its default risk. Accordingly, the interest rate a country is charged on the world capital market increases with this ratio. This leads to an upward sloping supply schedule for debt, which we express by assuming that the borrowing rate, $r(N/K)$, charged on (national) foreign debt, *N*, relative to the stock of private capital, *K*, is of the form:

$$r(N/K) = r^* + \omega(N/K); \quad \omega' > 0 \tag{1e}$$

where r^* is the exogenously given world interest rate, and $\omega(N/K)$ is the country-specific borrowing premium that increases with the nation's debt-capital ratio.[5] The homogeneity of the

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[4] Chatterjee and Turnovsky (2004) assume $\varepsilon = 0$ and we shall treat this as our benchmark case as well. We have also briefly considered the alternative specification: $Y = \alpha(K_G/\bar{K})^\varepsilon[\eta\{(1-l)\bar{K}\}^{-\rho} + (1-\eta)K^{-\rho}]^{-1/\rho}$, which simply augments the Romer (1986) model to include the "congestion-offsetting" externality effect of public capial; see footnote 18.

[5] The original formulation of this type of borrowing cost relationship is due to Bardhan (1967). Further discussion and rationalization of it, including variants of the specification and some empirical support, are discussed in our earlier work.

relationship is required to sustain a balanced growth equilibrium.

The representative agent chooses consumption, labor supply, and the rates of capital and debt accumulation, to maximize intertemporal utility, (1b), subject to the flow budget constraint

$$\dot{N} = C + r\left(N/K\right)N + \Psi\left(I,K\right) - (1-\tau)Y + \bar{T} \tag{2}$$

where N is the stock of debt held by the private sector, τ is the income tax rate, and $\bar{T}$ denotes lump-sum taxes.[6] In performing this optimization, the agent takes the borrowing rate, $r(.)$ as given. This is because the interest rate facing the debtor nation, as reflected in (1e), is a function of the economy's *aggregate* debt-capital ratio, which the individual agent knows he cannot influence.

The optimality conditions with respect to the individual's choices of C, l, and I are

$$C^{\gamma-1}l^{\theta\gamma} = \lambda \tag{3a}$$

$$\theta C^{\gamma}l^{\theta\gamma-1} = \lambda\left(1-\tau\right)\frac{\partial Y}{\partial\left(1-l\right)} \tag{3b}$$

$$1 + h_1\left(I/K\right) = q \tag{3c}$$

where λ is the shadow value of wealth in the form of internationally traded bonds, q' is the shadow value of the agent's private capital stock, and $q = q'/\mu$ is thus the market price of private capital in terms of the traded bonds. Equation (3a) equates the marginal utility of consumption to the shadow value of wealth, while (3b) equates the marginal utility of leisure to the shadow value of after-tax income foregone, where the marginal product of labor, $\partial Y/\partial\left(1-l\right)$, equals the equilibrium wage rate. The third equation equates the marginal cost of an additional unit of investment, inclusive of the marginal installation cost, $h_1 I/K$ to the market value of capital. Equation (3c) may be immediately solved to yield the following expression for the growth rate of private capital

$$\frac{\dot{K}}{K} \equiv \psi_K = \psi_K(q) = \frac{q-1}{h_1} - \delta_K \tag{3c'}$$

Applying the standard optimality conditions with respect to N and K leads to the usual

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[6] Since we are focusing on debtor nations, we assume $N > 0$. However, it is possible for $N < 0$ in which case the agent accumulates credit by lending abroad. For simplicity, interest income is assumed to be untaxed.

arbitrage relationships, equating the rates of return on consumption and investment in private capital to the costs of borrowing from abroad

$$\beta - \frac{\dot{\lambda}}{\lambda} = r\left(N/K\right) \tag{4a}$$

$$\frac{\alpha^{-\rho}\left(K_G/\bar{K}\right)^{-\varepsilon\rho}(1-\tau)(1-\eta)\left(Y/K\right)^{1+\rho}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2h_1 q} - \delta_K = r\left(N/K\right) \tag{4b}$$

Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must hold:

$$\lim_{t\to\infty}\lambda B e^{-\beta t} = 0; \ \lim_{t\to\infty} q' K e^{-\beta t} = 0. \tag{4c}$$

2.2 Public Capital, Foreign Aid, and National Debt

The resources for the accumulation of public capital come from two sources: domestically financed government expenditure on public capital, $\bar{G}$, and a program of capital transfers or foreign aid, *TR*, from the rest of the world. We therefore postulate

$$G \equiv \bar{G} + \phi TR \qquad 0 \le \phi \le 1 \tag{5}$$

where ϕ represents the degree to which the foreign aid is tied to investment in public infrastructure. If $\phi = 1$, the aid is completely tied to public investment and is therefore "productive"; if $\phi = 0$, aid is completely unrestricted and thus represent a "pure" transfer, of the traditional Keynes-Ohlin type.

Analogous to private capital, we assume that the gross accumulation of public capital, *G*, is also subject to convex costs of adjustment:[7]

$$\Gamma(G, K_G) = G\left(1 + \left(h_2/2\right)\left(G/K_G\right)\right).$$

Public capital stock depreciates at the rate, δ_G, so that its net rate of accumulation is

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[7] Note that there are different ways of specifying how aid is tied. The specification (5) relates aid to the accumulation of new public capital. An alternative formulation is to tie the aid to total investment costs, inclusive of installation costs. As noted by Chatterjee et al. (2003), the differences between these specifications are minor, and since there is no compelling evidence favoring one formulation over the other, we adopt (5), which turns out to be marginally simpler.

$$\dot{K}_G = G - \delta_G K_G \,. \tag{6}$$

To sustain an equilibrium of on-going growth, both domestic government expenditure on infrastructure, $\bar{G}$, and the flow of aid from abroad, *TR*, must be tied to the scale of the recipient economy. We do this by specifying them as a fraction of that country's level of income

$$\bar{G} = \bar{g}Y, \text{ and } TR = \sigma Y, \quad 0 < \bar{g} < 1, \ \sigma > 0, \quad 0 < \bar{g} + \sigma < 1$$

Substituting $\bar{G}$ and *TR* into (5) and then into (6), we can express the growth rate of public capital by

$$\frac{\dot{K}_G}{K_G} \equiv \psi_G = \left(\bar{g} + \sigma\phi\right)\frac{Y}{K_G} - \delta_G \,. \tag{6'}$$

The government sets its tax and expenditure parameters to maintain a balanced budget:[8] Expressing this in the form

$$\bar{T} = \Gamma(G, K_G) - \tau Y - TR \tag{7}$$

$\bar{T}$ determines the lump-sum tax necessary to balance the current budget, given by the right hand side of (7). The national budget constraint, (the current account) is obtained by combining (7) and (2),

$$\dot{N} = r\left(N/K\right)N + C + \Psi\left(I, K\right) + \Gamma\left(G, K_G\right) - Y - TR \,. \tag{8}$$

That is, the economy accumulates debt to finance its total expenditures on public capital, private capital, consumption, and interest payments net of output produced and transfers received.

2.3 Macroeconomic Equilibrium

The steady-state equilibrium of the economy has the characteristic that all real aggregate quantities grow at the same constant rate, and that the labor allocation, *l*, and the relative price of capital, *q*, are constant. We show in the Appendix how the equilibrium dynamics of the system can be conveniently expressed in terms of the following stationary variables, $z \equiv K_G/K$, $n \equiv N/K$, *l* and

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[8] Several aid programs call for co-financing by the government of the recipient economy, whereby it is required to match the foreign aid to some degree. This has been addressed in some detail in our previous work, where we also consider growth-maximizing and welfare-maximizing government responses. Since the main consequences of co-financing arrangements are only marginally affected by the elasticity of labor supply, we do not consider this aspect further here.

q. Assuming a symmetric equilibrium in which $K = \bar{K}$, we get:

$$\frac{\dot{z}}{z} = \left(\bar{g} + \sigma\phi\right)\frac{y}{z}\delta_G - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{9a}$$

$$\frac{\dot{n}}{n} = r\left(n\right) + \frac{1}{n}\left[c + \frac{q^2 - 1}{2h_1} + \left\{\left(\bar{g} + \sigma\phi\right) - \left(1+\sigma\right)\right\}y + \frac{h_2}{2}\left(\bar{g} + \sigma\phi\right)^2\frac{y^2}{z}\right] - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{9b}$$

$$\dot{q} = r\left(n\right)q - \frac{1}{\alpha^{\rho}z^{\varepsilon\rho}}\alpha\left(1-\tau\right)\left(1-\eta\right)y^{1+\rho} - \frac{(q-1)^2}{2h_1} + \delta_K q \tag{9c}$$

$$\dot{l} = \left(\frac{\left\{1+\Omega\right\}\left\{\beta - r\left(n\right)\right\} + \left(1-\gamma\right)\left\{\left[\Omega\left(1+\rho\right) - (1+\Omega)\varepsilon\right]\psi_K + \left[1+\rho+(\varepsilon-\rho)(1+\Omega)\right]\psi_G\right\}}{\left[\left\{\gamma\left(1+\theta\right)-1\right\}\left\{1+\Omega\right\} - \left(1-\gamma\right)\left(1+\rho\right)\Omega\left(\frac{l}{1-l}\right)\right]}\right)l \tag{9d}$$

where

$$\Omega \equiv \Omega(z,l) \equiv \left((1-\eta)/\eta\right)\left[(1-l)z\right]^{\rho} \tag{10a}$$

$$\frac{Y}{K} \equiv y = y(z,l) = \alpha z^{\varepsilon}\left[(1-\eta) + \eta\left\{(1-l)z\right\}^{-\rho}\right]^{-1/\rho} \tag{10b}$$

$$\frac{C}{K} \equiv c = c(z,l) = \frac{(1-\tau)}{\theta}\left(\frac{l}{1-l}\right)\left(\frac{1}{1+\Omega}\right)y \tag{10c}$$

$$r(n) = r^* + \omega(n) \tag{10d}$$

and the growth rates of the two types of capital are

$$\frac{\dot{K}}{K} \equiv \psi_K = \frac{q-1}{h_1} - \delta_K \tag{11a}$$

$$\frac{\dot{K}_G}{K_G} \equiv \psi_G = \left(\bar{g} + \sigma\phi\right)\frac{y}{z} - \delta_G\,. \tag{11b}$$

Equations (9a) – (9d) provide an autonomous set of dynamic equations in $z, n,$ $l,$ and q of which, two (k, n) are state variables, while the remaining two (q, l) are “jump” variables, free to respond instantaneously to new information as it becomes available. Once z and l are known, the output-capital ratio and the consumption-output ratio are determined by (10b) and (10c).

The economy reaches steady state when $\dot{z} = \dot{n} = \dot{l} = \dot{q} = 0$. Applying these conditions to (9a) – (9d) we can solve for the steady-state values $\tilde{z},\ \tilde{q},\ \tilde{n},$ and $\tilde{l}$. Given these quantities, (10b) – (10d) and either (11a) or (11b) determine $\tilde{y}, \tilde{c}$, the steady-state interest rate $\tilde{r}$, and the long-run growth rate $\tilde{\psi}$, respectively.[9] Linearizing (9a) – (9d) around the steady-state yields an approximation to the underlying dynamic system. This system (not reported) forms the basis for our dynamic simulations. To be saddlepoint-stable, we require that there be two unstable roots to match the two jump variables. For all plausible sets of parameter values our numerical simulations yield the required pattern of eigenvalues, namely two positive (unstable) and two negative (stable) roots, the latter being denoted by μ_1 and μ_2, with $\mu_2 < \mu_1 < 0$.

Equations (9) and (10) represent "core" dynamic equations from which other key variables, in particular the various growth rates, may be derived. In addition to the growth rates of the two capital goods reported in (11a) and (11b), the growth rates of consumption and output are given by

$$\frac{\dot{C}}{C} \equiv \psi_C = \frac{r(n) - \beta + \gamma\theta(1/l)\left[F(z,n,q,l)/G(z,l)\right]}{1-\gamma} \tag{11c}$$

$$\frac{\dot{Y}}{Y} \equiv \psi_Y = \frac{1}{1+\Omega(z,l)}\left[\Omega(z,l)\psi_K + \psi_G - \frac{\dot{l}}{1-l}\right] + \varepsilon\left[\psi_G - \psi_K\right] \tag{11d}$$

where $F(.), G(.)$ are defined in the Appendix. Although the growth rates diverge during the transition, they ultimately converge to the common equilibrium rate $\tilde{\psi}_K = \tilde{\psi}_G = \tilde{\psi}_C = \tilde{\psi}_Y = \tilde{\psi}$.

3. The Dynamic Effects of Foreign Aid: A Numerical Analysis

Due to the complexity of the model, we will examine the dynamic effects of a foreign aid or a transfer shock using numerical simulations. We begin by calibrating a benchmark economy, using the following parameters representative of a small open economy, which starts out from an equilibrium without any transfers or aid from abroad.

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[9] The solution for the steady-state equilibrium is set out in the Appendix. Because this system is highly non-linear, it need not be consistent with a well-defined steady-state equilibrium with $\tilde{z} > 0, \tilde{c} > 0$. Our numerical simulations, however, yield well-defined steady-state values for all plausible specifications of all the structural and policy parameters.

The Benchmark Economy

Preference parameters:	$\gamma = -1.5$, $\beta = 0.04$, $\theta = 1$
Production parameters:	$\alpha = 0.6$, $\eta = 0.2$, $\varepsilon = 0$, $h_1 = 15$, $h_2 = 15$
Elasticity of substitution in production:	$s = 1$
Depreciation rates:	$\delta_K = 0.05$, $\delta_G = 0.05$
World interest rate:	$r^* = 0.06$,
Premium on borrowing:	$a = 0.15$[10]
Policy parameters:	$\tau = 0.15$, $\bar{g} = 0.05$
Transfers:	$\sigma = 0$, $\phi = 0$

Our choices of preference parameters β, γ, and depreciation rates, δ_K, δ_G, the world interest rate, r^* are standard, while α is a scale variable. The productive elasticity of public capital $\eta = 0.2$ is consistent with the empirical evidence (see Gramlich, 1994). But given the introduction of labor in efficiency units, this implies that the productive elasticity of labor is also 0.2, while that of private capital is 0.8.[11] Setting $s = 1$ yields the Cobb-Douglas technology, which serves as a reasonable benchmark. The borrowing premium $a = 0.15$ is chosen to ensure a plausible equilibrium debt-output ratio. The elasticity on leisure, θ, is the crucial determinant of the equilibrium labor-leisure allocation and has been set to ensure that this is empirically plausible. The tax rate is set at $\tau = 0.15$, while the rate of domestic government expenditure on public investment is assumed to be $\bar{g} = 0.05$. The choice of adjustment costs is less obvious and $h_1 = 15$ lies in the consensus range of 10 to 16.[12] Note also that the equality of adjustment costs between the two types of capital serves as a plausible benchmark. The public good externality parameter is set at $\varepsilon = 0$.

Substituting these base parameters into the steady state equations (A.7a) – (A.7f)m (11a) and the functional form for (10d), yields the following benchmark equilibrium values: $\tilde{z} = 0.253$, $\tilde{n} = 0.141$, $\tilde{q} = 1.997$, $\tilde{l} = 0.780$, $\tilde{y} = 0.337$, $\tilde{c} = 0.202$, $\tilde{r} = 0.0813$, and $\tilde{\psi} = 0.0165$.

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[10] The functional specification of the upward sloping supply curve that we use is: $r(n) = r^* + e^{an} - 1$. Thus, in the case of a perfect world capital market, when $a = 0$, $r = r^*$, the world interest rate.

[11] An inevitable feature of calibrating a Romer (1986) - type AK model is that keeping the size of the externality plausible, while maintaining the assumption of constant returns to scale in the private factors, imposes constraints on the elasticities on labor and private capital. In order to reconcile these elasticities with the empirical evidence on the income shares of labor and private capital, it is necessary to interpret K as an amalgam of physical and human capital, with $(1-l)$ describing "raw" unskilled labor; see Rebelo (1991).

[12] For example, Origueira and Santos (1997) choose $h_1 = 16$ on the grounds that it generates a plausible speed of convergence. Auerbach and Kotlikoff (1987) assume $h_1 = 10$, recognizing that this is at the low values of estimates, while Barro and Sala-i-Martin (1995) propose a value above 10.

Table 1, Row 1 summarizes these in a more convenient form. Thus the benchmark equilibrium yields a steady-state ratio of public to private capital of 0.25, a consumption-output ratio of 0.60, and a debt-output ratio of 0.42, yielding an equilibrium borrowing premium of 2.13 percent over the world rate of 6 percent. The capital-output ratio is 2.97, and 78 percent of the agent's time is allocated to leisure, consistent with empirical evidence, yielding a long-run growth rate of 1.65 percent. This equilibrium is a reasonable characterization of a small-medium indebted economy, experiencing a modest steady growth rate and having a relatively small stock of public capital.

This equilibrium is based on several specific assumptions and therefore it is important to conduct some sensitivity analysis. The critical parameters upon which we focus are: (i) the elasticity of substitution in production, s, (ii) the elasticity of leisure in utility, θ, (iii) the externality parameter, ε, and (iv) the domestic fiscal policy parameters, $\bar{g}, \tau$.

3.1. A Permanent Increase in the Flow of Foreign Aid: Long Run Effects

We now introduce a permanent foreign aid flow to the above benchmark economy. Specifically, the inflow of foreign aid is tied to the scale of the recipient economy, and increases from 0 percent of GDP in the initial steady-state to 5 percent of GDP in the new steady-state (an increase in σ from 0 to 0.05).[13] This aid may be tied to new investment in public capital ($\phi = 1$), representing a "productive" transfer, or it may be untied ($\phi = 0$), in which case it is a "pure" transfer. The long-run and short-run responses of key variables in the recipient economy are reported in Rows 2 and 3 in Tables 1a and 1b. In addition, the final columns in the tables summarize the effects on long-run welfare, $[\Delta W]$, and short-run welfare, $[\Delta W(0)]$, both measured by the optimized utility of the representative agent where C and l are evaluated along the equilibrium path. These welfare changes are measures of equivalent variation, calculated as the percentage change in the initial stock of capital necessary to maintain the level of welfare unchanged following the particular shock. The differences between the effects of the two types of transfer are dramatic.

We first consider the long-run effects of an increase in foreign aid (Table1a) and then discuss

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[13] While $\sigma = 0.05$ is arbitrary it is approximately the average rate of foreign aid offered by the European Union under its aid program to prospective members in the early 1990s and thus serves as a reasonable benchmark.

the short-run transitional dynamics generated by this shock (Table 1b and Figure 1).

3.1.1 Tied Aid

The long-run impact of a tied foreign aid shock is reported in Row 2 of Table 1a. Since the aid is tied directly to public investment, in the new steady state the ratio of public to private capital more than doubles, increasing from 0.25 to 0.54, as a consequence of the investment boom in infrastructure. The larger stock of public capital increases the marginal productivity of private capital and labor, leading to a positive, though lesser, accumulation of private capital, and increasing employment time from 0.220 to 0.232. Although the transfer stimulates consumption through a wealth effect, the enhanced productive capacity has a greater effect on output, leading to a decline in the long-run consumption-output ratio from 0.60 to 0.563. The higher productivity raises the long-run growth rate to 2.31 percent, while long-run welfare improves by 7.96 percent. The increased accumulation of both private and public capital lead to a higher demand for external borrowing as a means of financing the new investment in private capital and the installation costs of public capital. This results in an increase in the steady state debt-output ratio from 0.42 to 0.62, raising the borrowing premium to nearly 3.8 percent. However, this higher debt relative to output is sustainable since it is caused by higher investment demand rather than by higher consumption demand. The long run increase in the economy's productive capacity (as measured by the larger stocks of public and private capital, and output) ensures that the additional debt is sustainable.[14]

3.1.2 Untied Aid

A permanent untied aid shock, i.e., an aid flow not tied to any investment activity, has precisely the opposite qualitative effects, as illustrated in Row 3 of Table 1a. Apart from consumption and leisure, the changes are much smaller. Being untied, the transfer is devoted to debt reduction, thereby allowing an increase in consumption. The debt-output ratio declines to 0.396 and the consumption-output ratio rises to around 0.65. The increase in consumption raises the marginal utility of leisure, increasing the fraction of leisure time from 0.78 to 0.793. Since the aid no longer

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[14] This view has also been expressed by Roubini and Wachtel (1998).

favors public investment, the ratio of public to private capital remains virtually unchanged. With the shift toward more consumption and leisure, productivity of both types of capital decline and the equilibrium growth rate is marginally reduced from 1.65% to 1.60%, leading to an overall increase in welfare of around 7.71 percent, marginally less than for the tied transfer.

3.2 Transitional Dynamics

3.2.1 Tied Aid

The transitional adjustment paths following the increase in tied aid are illustrated in Fig. 1 for the benchmark economy. Fig. 1.1 illustrates the stable adjustment locus in *z*-*n* space, indicating how *z* and *n* both generally increase together during the transition.

The immediate effect of the tied aid shock is to raise the growth of public capital, to above 8 percent, thereby raising the productivity of both private capital and labor; see Table 1b, Row 2. Given the cost of borrowing, the higher return to capital causes an instantaneous upward jump in the shadow price of private capital, *q*, from its initial benchmark level of 2 to 2.04, thereby inducing a corresponding increase in private investment. At the same time, the higher productivity of labor induces an immediate, but slight, decline in leisure from 0.780 to 0.777. While the upward jump in *q* reduces the rate of return on private capital, the increase in labor raises the return. On balance, the former slightly dominates and immediately after its initial increase, *q* begins to drop slightly to around 2.03, after the first five periods. Leisure drops steadily toward its new equilibrium level of 0.768, so that after a few periods its positive productivity effect dominates, and *q* begins to rise monotonically toward its new equilibrium level of 2.10; see Figs. 1.2 and 1.3.

The introduction of the tied transfer leads to an initial short-run decline in the consumption-output ratio [Fig. 1.4]. This is because the short-run substitution from leisure to labor both increases output and reduces the marginal utility of consumption. Thereafter, as the larger capital stocks are reflected in more output, the consumption-output ratio continues to decline monotonically toward its new steady-state value. Also, note that leisure and the consumption-output ratio move together. The contrasting time paths of the four growth rates, ψ_K, ψ_G, ψ_Y, and ψ_C during the transition toward their common long-run growth rate of 2.31 percent are strikingly illustrated in Fig. 1.6. With public capital being directly stimulated by the transfer, its growth rate jumps initially to over 8.3 percent before gradually declining. By contrast, private capital increases only very gradually from 1.95 percent to 2.31 percent during transition, as the accumulation of public capital enhances its productivity. As a result, the ratio of public to private capital increases at a steady monotonic rate. The growth rate of output is a weighted average of the growth rates of the two capital stocks plus the temporary growth of labor and therefore immediately increases sharply to 3.5 percent with the transfer. On the other hand, the only influence on the initial growth rate of consumption is the effect that operates through the labor supply and the labor-leisure choice, raising its growth rate from 1.65 percent to 1.87 percent. Thereafter it responds only gradually, in response to the accumulation of assets in the economy. It always lies below the growth rate of output, so that C/Y is falling, as noted in Fig. 1.4. However, the level of consumption is still growing, albeit at a modest rate.

The final aspect of the dynamics concerns the debt-output ratio. Starting at 0.42, the short-run increase in output leads to a slight initial decline in the debt-output ratio, after which it increases monotonically through time. This is because the accumulation of public capital raises the average productivity of private capital, while the accumulation of both types of capital raises the need to borrow from abroad to finance new investment and installation costs. But as noted in Section 3.1.1, the higher debt, being backed by higher productive capacity through the tied transfer, is sustainable.

3.2.2 Untied Aid

The transitional dynamics following an untied aid shock are illustrated in Figure 2 and three points should be made at the outset, which distinguish our results from earlier findings. First, the existence of transitional dynamics following an untied aid shock depends crucially upon the endogeneity of labor supply. If labor supply is inelastic, then untied foreign aid has no dynamic or growth effects and the economy moves instantaneously to its new steady-state via a once-and-for-all increase in the consumption-output ratio; see Chatterjee et al. (2003). Second, the dynamics in response to untied aid are in sharp contrast to those generated by tied aid, being more or less the reverse. This reflects the fact, noted in Table 2, that the long-run responses of the economy to the two types of aid are generally opposite in nature. Third, the dynamic adjustment generally occurs much more rapidly than in response to the tied aid shock.

Fig. 2.1 illustrates the transitional adjustment paths for the two state variables, debt/private capital and public capital/private capital. We see that on receipt of the aid, these move in opposite directions, implying that on impact the debt-capital ratio begins to decline, while the public-private capital ratio begins to increase. Indeed, the untied transfer is initially applied primarily to debt reduction, which allows an immediate substantial increase in consumption, increasing the marginal utility of leisure, and thus inducing an immediate sharp reduction in labor supply.

The main impact of an untied transfer is on consumption, leisure, and debt reduction, as illustrated in Figs. 2.3, 2.4 and 2.5. Its initial impact is to raise the marginal utility of leisure causing a reduction in labor supply, and hence in the productivity of private and public capital, and in q. The receipt of the untied transfer has a slightly less adverse short-run effect on the growth rate of public capital, reducing it to 1.57 percent, slightly above that of private capital. As z increases, the productivity of public capital declines relative to private capital, causing their relative growth rates to reverse. After just over two periods the growth rate of private capital exceeds that of public capital and z begins to decline with n. The decline in q is partially reversed during the subsequent transition as the relative stock of public to private capital declines.

4. Sensitivity Analysis

The contrast between the effects of tied and untied foreign aid is striking. It is therefore important to determine how sensitive this comparison is to the chosen parameter values for the benchmark economy. This is explored in Tables 2 – 5, along the various dimensions noted earlier.[15]

4.1 Elasticity of substitution in production (s) versus flexibility in labor supply (θ)

Table 2 presents a grid summarizing the changes in key variables in response to equal amounts of tied aid and untied aid, respectively, as the elasticity of substitution in production, s,

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[15] In our earlier work, with fixed labor supply, we conducted sensitivity analysis with respect to the adjustment costs of public capital as well as the degree of capital market imperfections. We have also addressed these aspects here, but since the conclusions are basically unchanged from those obtained previously, we omit them from our discussion.

varies between 0.8 and 1.6, while θ varies between 0 and 2.[16] One interesting feature is that the effects of tied aid on the growth rate are highly sensitive to even minor deviations from the benchmark value of $s = 1$ (Cobb-Douglas). Thus, for example, if a researcher estimates $s = 1$ with a standard error of 0.1 – a tight estimate - and if $\theta = 1$, then, with 95 percent probability the implied increase of 0.66 percentage points in the growth rate could be as high as 0.98 or as low as 0.45. A sustained difference in the growth rate of half a percentage point accumulates to a substantial difference in economic performance. This is seen from the spread on the implied welfare gain of 7.96 percent, which is even larger, ranging as high as 21.1 percent and as low as 0.53 percent.

Looking though the two panels of Table 2, the following observations can be made.

(i) The tendency for tied and untied aid to have opposite long-run effects on economic activity is robust to variations in *s* and θ.

(ii) Tied aid has substantially greater long-run effects on variables involving asset accumulation [capital aid and foreign debt], than does untied aid. The effects on consumption and leisure are comparable in magnitude (though opposite in direction).

(iii) Increasing the elasticity of substitution, *s*, reduces the positive effect of tied aid on the growth rate, while reducing the negative effect on the consumption-output and capital-output ratios. On the other hand, a higher *s* primarily reduces the adverse effect of an untied aid shock on the debt-output ratio, while decreasing the positive effects on the capital-output and consumption-output ratios, the latter only mildly. The net effect is to reduce the adverse effect on the growth rate.

Intuitively, the larger the elasticity of substitution, the more the increased productivity of private capital resulting from the tied transfer induces substitution toward private capital. As a result, the Y/K ratio rises less, so that the increased productivity of private capital is reduced, thus reducing its rate of accumulation, and mitigating the fall in consumption. For untied aid, a higher elasticity of substitution means that the reduction in the productivity of private capital resulting from the reduction in labor supply is mitigated, so that the fall in the growth rate is moderated. Slower

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[16] We have extended the sensitivity analysis to include $\theta = 5$, but the pattern remains unchanged.

growth means less borrowing, lower borrowing costs, and thus a decline in the debt-output ratio.

(iv) Increasing the importance of leisure in utility, θ, reduces the positive effect of tied aid on the growth rate, and reduces the adverse effect on the consumption-output and capital-output ratios. It increases the adverse effect of untied aid on the growth rate, while reducing the positive effect on the capital-output ratio and the adverse effect on the debt-output ratio.

The intuition is as follows. The more the agent values leisure in utility, the less they are willing to reduce it in response to an increase in tied aid, the less the reduction in consumption, and the less the positive effect on the growth rate. In the case of untied aid, as θ increases, and agents enjoy more leisure, the productivity of capital and the return on capital decline, so the adverse effect on the growth rate increases. However, there are some offsetting effects. As leisure increases, because of its diminishing marginal utility, agents increase their leisure at a diminishing rate. This mitigates the adverse effect of the untied aid on the growth rate, for sufficiently large θ.[17]

4.2 Welfare Comparisons

The comparison of the overall intertemporal welfare gains for the two types of aid is particularly striking. Table 3 indicates that for the benchmark case, $s=1, \theta=1$, the net effects of the two types of aid on intertemporal welfare are more or less comparable; the gains from tied aid are 7.96 percent, while those from untied aid are 7.71 percent. But despite this similarity in the overall intertemporal welfare gains for the two forms of aid, the contrasting dynamic adjustments in the economy lead to sharp differences in the time-profiles of the benefits they provide. For tied aid, the commitment toward public investment involves initial consumption losses and less leisure, leading to a short-run welfare loss of 1.53 percent. Over time, as the economy becomes more productive, consumption increases rapidly. Welfare increases drasmaically, with subsequent gains dominating

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[17] One result in Table 2 worth noting is the contrast in the response of leisure to an increase in tied aid as *s* increases from 0.8 to 1.6. As already noted, if $s=1$, a tied transfer, by increasing labor productivity, encourages more work effort, an effect that is exacerbated as the elasticity of substitution increases beyond 1. For low *s*, however, this response is reversed. The intuition is seen most clearly by focusing on the polar case of the fixed coefficient production function, $s=0$. In this case, private capital, *K*, and labor in efficiency units, $(1-l)K_g$, need to change proportionately. Since tied aid leads to an increase in the relative stock, $z \equiv K_g / K$, this must be accompanied by a decrease in labor for $(1-l)z$ to remain constant and for production to remain efficient.

the initial losses, resulting in an overall intertemporal welfare gain. In contrast, the response to untied aid does not involve intertemporal trade-offs. Instead, it results in an immediate and an more almost constant increase in consumption, leisure, and therefore welfare, along the transition path.

Table 3 presents the sensitivity of the short-run and long-run welfare responses to the two types of aid shocks, for variations in s and θ. The following patterns emerge from the table:

(i) Both the short-run and the intertemporal welfare gains from an untied aid shock are remarkably insensitive to variations in both s and θ. For plausible ranges of the parameters, an untied aid flow equal to 5 percent of GDP leads to short-run welfare gains of between 7-9 percent and long-run gains over the range of 5-10 percent, both measured by an equivalent variation in the initial stock of capital. The long-run gains are typically within 1 percentage point of the short-run gains suggesting a gradual increase over time.

(ii) In contrast, both the short-run and long-run welfare gains from a tied aid of the same magnitude are highly sensitive to both parameters. For any given θ, the long-run welfare gains decline with *s*. On the other hand, welfare gains increase with θ for values of *s* less than 1. For high values of *s*, tied aid yields both short-run and long-run losses, the former being relatively independent of θ, and the latter increasing with θ. There is therefore a sharp contrast between the short-run and long-run welfare effects of tied aid.

Results (i) and (ii) from Table 3 are two key findings, and the following intuition may be provided. An untied aid flow has little effect on the stocks of public or private capital. The higher elasticity of substitution raises the level of output attainable from given stocks of capital, thereby raising consumption and welfare approximately uniformly. If the aid flow is tied, it increases the rate of investment in public capital. With a low elasticity of substitution this requires an approximately corresponding increase in private capital, leading to a large increase in output, consumption, and benefits. As the elasticity of substitution increases, the higher public capital is associated with a smaller increase in private capital, so that the increase in output, consumption, and welfare declines. This is exacerbated by the fact that for a high elasticity of substitution, the tied transfer generates a large increase in the real wage and its growth rate, leading to substantial substitution toward labor, which is further welfare-reducing.

The contrasting sensitivities of the welfare gains resulting from a tied and untied transfer, respectively, to changes in s and θ, means that the relative merits of the two forms of transfers, from a welfare standpoint, are also highly sensitive to these two critical parameters. To consider this, we shall focus on the benchmark case $s = 1$, and consider variations in θ. As we have already noted, for $\theta = 1$, tied aid is marginally superior to untied aid from a long-run (intertemporal) welfare point of view. But as θ declines and leisure becomes less important in utility, untied aid is superior to tied aid. Indeed in the limiting case of inelastic labor supply, $\theta = 0$, an untied aid shock generates a long-run welfare gain of 8.66 percent, while the corresponding gain from a tied aid shock is much lower, at 6 percent. The less (more) important is leisure in utility, the more (less) tied aid crowds out private consumption, thus decreasing (increasing) the benefits, relative to untied aid. This comparison is sensitive to even small variations in the elasticity of substitution. For example, if $s = 0.8$ tied aid dominates untied aid (intertemporally), irrespective of the importance of leisure (even for $\theta = 5$), while if $s = 1.2$ precisely the reverse is true.

4.3 Generalizations of the Production Function

Table 4 extends the comparison of the long-run welfare effects of tied and untied aid by allowing public capital to have the additional externality effect as introduced in (1a). The main message of these results is clear and unsurprising. While the benefits of untied aid are relatively insensitive to ε (being mildly negative), the benefits of tied aid are highly sensitive to this effect, so that the latter is heavily favored as ε increases. Take, for example, the benchmark case, $\theta = 1, s = 1$. Whereas tied aid is only marginally superior in the absence of this effect, it clearly dominates for $\varepsilon = 0.1$ (7.51% vs. 18.50%). Moreover, in cases where for $\varepsilon = 0$ untied aid dominates tied aid, and where the latter is welfare-deteriorating, tied aid may now not only be positive from a welfare standpoint but may also be superior to untied aid. An example of this arises if $s = 1.6, \varepsilon = 0.2$.[18]

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[18] We have also conducted sensitivity analysis using the generalized Romer production function in footnote 4. If $\varepsilon = 0$ in that model, tied aid is clearly undesirable, since it is obliging the recipient economy to devote the resources to an unproductive use. But if ε is sufficiently large (e.g. around 0.2) it is again the case that tied aid is not only beneficial, but also superior to untied aid.

4.4 Sensitivity of Transitional Dynamics

We have recomputed the transitional paths for both types of aid to determine their sensitivity to variations in s and θ, as well as to ε. For untied aid, the time profiles retain the general qualitative characteristics, illustrated in Fig. 2 for the benchmark $\theta = s = 1$, as θ, s, and ε are varied. Many of the qualitative characteristics of the transitional paths following a tied aid shock also remain as illustrated in Fig. 1, although there are some substantive differences which are illustrated in Figs. 3 and 4.

The transitional time paths for leisure and the consumption-output ratio following a tied aid shock are sensitive to variations in the elasticity of substitution (s), and Fig. 3 compares them for three values of $s = 0.5,\ 1,\ \text{and}\ 1.6$, while θ remains at its benchmark value of unity. As already observed, for the benchmark economy, l and C/Y move together. For a low elasticity of substitution ($s = 0.5$), leisure generally increases, for reasons discussed in Section 4.1. The initial increase in leisure increases the marginal utility of consumption, so that C/Y initially increases, after which it declines steadily. This implies that l and C/Y move in opposite directions throughout the transition. For a high elasticity of substitution, l initially declines and continues to decline during the transition, just as in the benchmark case. But in this case, the initial decline in l is sufficiently sharp to cause a sharp decline in initial consumption, $C(0)$. The C/Y ratio overshoots its long-run response, and thus rises during the transition, implying again that l and C/Y move in opposite directions throughout the transition.

Fig. 4 highlights the role played by the real wage rate, in the absorption and propagation of a tied aid shock. This turns out to be sensitive to elasticity of leisure in utility (θ) and we consider three values of θ = 0, 1, and 5, while maintaining s = 1. When θ = 0, the agent allocates all of his time to work, and as we have shown previously, the aid shock does not generate any adjustment in the work-leisure choice (Fig. 4.1). On the other hand, the resulting increase in the productivity of public capital causes the growth rate of the real wage to increase instantaneously and to overshoot its higher long-run equilibrium (Fig. 4.2). Since the agent devotes all his time to work, diminishing returns set in during transition, causing a gradual decline in the growth of the real wage rate, until it finally converges to the new steady-state growth rate of the economy. As θ increases (Figs. 4.2, 4.3, $\theta = 1$), the agent divides his time between work and leisure, and the effect of the tied aid shock on the real wage rate is altered. The agent now substitutes away from leisure toward longer work-hours. The positive opportunity cost of this substitution dampens the response of the growth rate of the real wage rate, and it undershoots its higher long-run equilibrium. Thereafter, the accumulation of public capital during transition increases the productivity of labor, causing a gradual increase in real wage-growth during transition. For a very high value of θ (Fig. 4.5), the instantaneous response of leisure is to jump up before declining towards its lower long-run equilibrium. The upward jump in the growth rate of the real wage is consequently much smaller.

5. Consequences for the Government Fiscal Balance

One issue of debate in the aid-growth literature concerns the relationship between the effectiveness of foreign aid and “good” government policy. In examining this relationship empirically, Burnside and Dollar (2000) construct an index of good policy, which includes the budget surplus as one factor. Our model enables us to address this issue in the following way.

Recalling equation (7), *T* represents the amount of lump-sum taxation (or transfers) necessary to finance the primary deficit and is therefore a measure of *current* fiscal imbalance. Defining

$$V \equiv \int_0^\infty \frac{\overline{T}(t)}{Y(t)} e^{-\int_0^\tau r(n)ds} d\tau = \int_0^\infty \frac{1}{Y(t)} \left[\Gamma(G, K_G) - \tau Y - TR\right] e^{-\int_0^\tau r(n)ds} d\tau$$

V measures the present discounted value of the lump-sum taxes per unit of current output necessary to balance the government budget over time, and thus provides a measure of the *intertemporal* fiscal imbalance; see Turnovsky (2004). Substituting for the appropriate quantities from Section 2, yields

$$V = \int_0^\infty \left[(\overline{g} + \phi\sigma)[1 + \left(h_2/2\right)(\overline{g} + \phi\sigma)\frac{y(t)}{z(t)} - (\tau + \sigma) \right] e^{-\int_0^\tau r(n)ds} d\tau \tag{12}$$

The issue then is the effect of tied and untied aid on this measure. Table 5 summarizes the effects of tied and untied aid on both intertemporal welfare, *W*, and the government’s intertemporal balance, *V*, for varying domestic fiscal configurations. A number of important insights emerge, where we focus first on Panel A.

(i) The welfare benefits from untied aid are relatively insensitive to substantial variations in both the tax rate and the rate of government spending, decreasing mildly with the former and increasing mildly with the latter. In addition, untied aid always improves the government's intertemporal fiscal balance and thus is beneficial from that standpoint.

(ii) The welfare benefits from tied aid increase slightly with the tax rate but decrease dramatically with the rate of government spending. This is because the benefits from tied aid depend critically upon the level of domestic government spending $\left(\bar{g}\right)$ on public capital relative to the socially optimal level which, in turn, depends upon the tax rate. For example, for $\tau = 0.15$, the socially optimal fraction of government spending in the absence of foreign aid is $\bar{g} = 0.098$.[19] If $\bar{g} = 0.02$, government investment is far below the optimum and clearly foreign aid tied to public investment is highly desirable; if $\bar{g} = 0.05$, the tied aid raises total public investment to 0.10, which is close to the social optimum and therefore still desirable. But if $\bar{g} = 0.10$, the rate of public investment is above the social optimum and any further tied aid is welfare-reducing.[20]

(iii) Tied aid always worsens the government's intertemporal deficit and is not desirable from that standpoint.[21] Thus for low rates of government expenditure tied aid involves a tradeoff in that increased wealth is accompanied by a higher intertemporal government deficit. For high rates of government expenditure, tied aid is unambiguously bad in that it is both welfare deteriorating and also deteriorates the government's intertemporal balance. In contrast, untied aid always has a positive effect on both targets.

(iv) The larger is the current government surplus, as parameterized by $\tau - \bar{g}$, the more (less) beneficial is tied (untied) aid. "Good" policy in the Burnside-Dollar sense thus favors tied aid.

(v) For extreme rates of expenditure optimal policy involves corner solutions. Thus for $\bar{g} = 0.02$ tied aid is not only superior but maximizes welfare, while for $\bar{g} = 0.10$, untied aid is optimal. For $\bar{g} = 0.05$, the optimal policy is an interior mix. Thus for $\tau = 0.15$, the optimal is for

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[19] This is obtained numerically. For $\tau = 0.1,\ 0.2$ the socially optimal expenditures are 0.091 and 0.106, respectively.

[20] Since the total rate of public investment is $\bar{g} + \phi\sigma$, the benefits of tied aid also depend upon its size, σ, since like $\bar{g}$, too large an increase in σ will take $\bar{g} + \phi\sigma$ beyond the social optimum rate of public investment. There is, in effect a tradeoff between $\bar{g}$ and σ insofar as the benefit of tied foreign aid is concerned.

[21] This result can also be established analytically by considering (12) in the case $\phi = 1$. An important element for this result is the fact that tied aid also imposes installation costs, the government needs to finance.

53% of the aid to be tied and the remainder untied. While this will take the economy to below the optimal rate of government investment, the losses from this are more than compensated by the fact that this is associated with a reduction in the government's intertemporal fiscal balance.

Panel B yields a similar pattern in the case of a positive externality $\varepsilon = 0.2$. The main difference is that the existence of this externality raises the productivity of public capital. In this case the socially optimal rates of public investment are 0.120, 0.130, and 0.138, corresponding to $\tau = 0.10,\ 0.15,\ 0.20$, respectively. Hence the tied transfer remains optimal even if $\bar{g} = 0.05$.

6. Conclusions

The link between foreign aid, economic growth, and welfare depends crucially on the mechanism through which a particular aid program, whether tied or untied, is absorbed by the recipient economy. In this paper, we introduce two crucial aspects of this mechanism that have been absent from previous work. First, we highlight the importance of the endogeneity of labor supply as an additional margin through which foreign aid may impact on macroeconomic performance. Second, we focus on (i) the role played by the interaction of labor supply and public capital, and (ii) externalities associated with public capital accumulation in determining an economy's response to a foreign aid shock. In addition, we are also able to relate our theoretical model to the recent empirical literature by examining the impact of an underlying aid program on the recipient government's intertemporal fiscal deficit. While recent work by Chatterjee et al. (2003) and Chatterjee and Turnovsky (2004) has highlighted the role played by an economy's structural conditions in the absorption of aid, we view the current paper as further enhancing our understanding of the aid-growth link by focusing on the role played by relative prices (of consumption and leisure), production externalities, and the government's fiscal balance.

We conclude with two final comments. First, our results carry some important policy advice. They suggest that when donors decide on whether a particular aid program should be tied to an investment activity, careful attention should be paid to the recipient's opportunities for substitution in production, the elasticity of labor supply, and production externalities. It is perfectly possible for a tied transfer to have a presumably unintended adverse effect on the recipient economy, if that economy is structurally different from what the donor perceived. Second, we have abstracted entirely from any political economy factors relating to rent-seeking or corruption, which are clearly relevant issues in any foreign aid discussion. Recent work by Acemoglu and Robinson (2000) and others show that the existence of “political elites” and powerful interest groups in poor economies may be a deterrent to investment, technological change, and economic development. Further, the lack of institutions may also inhibit the effects of aid on growth. Clearly, the consequences of these are significant considerations for determining both the nature and composition of foreign aid and are important directions for future research.

Table 1: Permanent Foreign Aid Shock

Benchmark Equilibrium: Cobb-Douglas production function (*s* = 1)

a. Long-run Effects

	$\tilde{K}_G/\tilde{K}$	$\tilde{r}$ %	$\tilde{l}$	$\tilde{C}/Y$	$\tilde{K}/Y$	$\tilde{N}/Y$	$\tilde{\psi}$ %	$\Delta(W)$ %
Benchmark Equilibrium $\sigma = 0, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	**0.253**	**8.13**	**0.780**	**0.602**	**2.969**	**0.416**	**1.65**	**--**
Tied aid $\sigma = 0.05, \phi = 1,$ $\bar{g} = 0.05, \tau = 0.15$	0.542	9.77	0.768	0.563	2.523	0.622	2.31	7.96
Untied aid $\sigma = 0.05, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.252	7.99	0.793	0.653	3.011	0.396	1.60	7.71

b. Short-run Effects

	$l(0)$	$\frac{C(0)}{Y(0)}$	$\psi_K(0)$ %	$\psi_G(0)$ %	$\psi_Y(0)$ %	$\psi_C(0)$ %	$\Delta(W(0))$ %
Benchmark Equilibrium $\sigma = 0, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	**0.780**	**0.602**	**1.65**	**1.65**	**1.65**	**1.65**	**--**
Tied aid $\sigma = 0.05, \phi = 1,$ $\bar{g} = 0.05, \tau = 0.15$	0.777	0.594	1.95	8.33	3.48	1.87	-1.53
Untied aid $\sigma = 0.05, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.7925	0.650	1.55	1.57	1.54	1.64	8.32

Table 2

A. Sensitivity of Permanent Responses to the Elasticities of Substitution (*s*) and Leisure (θ)

(i) Tied Aid Shock: σ increases from 0 to 0.05, ϕ = 1

	s = 0.8					***s* = 1**					*s* = 1.2					*s* = 1.6				
$\tilde{N}/Y$	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$
θ = 0	0	-0.065	0.194	-0.35	1.03	0	-0.047	0.103	-0.30	0.741	0	-0.036	0.053	-0.266	0.566	0	-0.024	0.003	-0.227	0.375
θ = 1	0.009	-0.054	0.419	-0.630	0.984	**-0.012**	**-0.039**	**0.206**	**-0.446**	**0.657**	-0.026	-0.029	0.101	-0.327	0.446	-0.041	-0.024	0.019	-0.192	0.223
θ = 2	0.007	-0.046	0.572	-0.814	0.939	-0.007	-0.035	0.254	-0.507	0.608	-0.015	-0.028	0.111	-0.327	0.385	-0.017	-0.025	0.017	-0.147	0.158

(ii) Untied Aid Shock: σ increases from 0 to 0.05, ϕ = 0

	s = 0.8					***s* = 1**					*s* = 1.2					*s* = 1.6				
	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$	$d\tilde{l}$	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d(\tilde{K}/Y)$	$d\tilde{\psi}$
θ = 0	0	0.05	0	0	0	0	0.05	0	0	0	0	0.05	0	0	0	0	0.05	0	0	0
θ = 1	0.014	0.052	-0.038	0.055	-0.069	**0.014**	**0.051**	**-0.020**	**0.042**	**-0.053**	0.013	0.05	-0.011	0.032	-0.039	0.010	0.049	-0.002	0.018	-0.020
θ = 2	0.009	0.052	-0.058	0.080	-0.073	0.008	0.051	-0.028	0.053	-0.055	0.007	0.05	-0.012	0.034	-0.037	0.004	0.050	-0.002	0.014	-0.015

Table 3

Sensitivity of Short-run and Long-run Welfare Responses to the Elasticities of Substitution (*s*) and Leisure (θ)

(i) Tied Aid: σ increases from 0 to 0.05, ϕ = 1

	s = 0.8		$\boldsymbol{s}$ **= 1**		s = 1.2		s = 1.6	
	$\Delta[W(0)]$	$\Delta(W)$	$\Delta[W(0)]$	$\Delta(W)$	$\Delta[W(0)]$	$\Delta(W)$	$\Delta[W(0)]$	$\Delta(W)$
$\theta = 0$	0.71	10.18	-3.97	6.00	-7.76	1.06	-13.04	-3.14
$\boldsymbol{\theta}$ **= 1**	5.14	21.08	**-1.53**	**7.96**	-6.71	0.53	-12.56	-6.82
$\theta = 2$	7.03	26.46	-0.69	9.15	-6.55	0.16	-12.52	-7.68

(ii) Untied Aid: σ increases from 0 to 0.05, ϕ = 0

	s = 0.8		$\boldsymbol{s}$ **= 1**		s = 1.2		s = 1.6	
	$\Delta[W(0)]$	$\Delta(W)$	$\Delta[W(0)]$	$\Delta(W)$	$\Delta[W(0)]$	$\Delta(W)$	$\Delta[W(0)]$	$\Delta(W)$
$\theta = 0$	8.24	8.24	8.66	8.66	8.94	8.94	9.28	9.28
$\boldsymbol{\theta}$ **= 1**	7.49	6.47	**8.32**	**7.71**	8.94	8.55	9.70	9.52
$\theta = 2$	7.22	5.89	8.21	7.50	8.94	8.54	9.78	9.64

Table 4

Sensitivity of Long-run Welfare Responses to the Elasticities of Substitution (*s*), Leisure (θ), and the Public Capital Externality (ε)

(percentage changes in welfare, σ increases from 0 to 0.05)

(i) $\varepsilon = 0.02$

	s = 0.8		**s = 1**		s = 1.2		s = 1.6	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
θ = 0	8.21	12.10	8.63	6.33	8.91	2.68	9.25	-1.68
θ = 1	6.44	23.46	**7.67**	**9.89**	8.51	2.24	9.47	-5.31
θ = 2	5.86	29.11	7.46	11.16	8.50	1.88	9.59	-6.16

(ii) $\varepsilon = 0.05$

	s = 0.8		**s = 1**		s = 1.2		s = 1.6	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
θ = 0	8.16	15.12	8.58	9.05	8.86	5.20	9.19	0.60
θ = 1	6.39	27.84	**7.61**	**12.94**	8.43	4.91	9.39	-2.93
θ = 2	5.82	33.34	7.40	14.34	8.42	4.57	9.50	-3.78

(iii) $\varepsilon = 0.1$

	s = 0.8		**s = 1**		s = 1.2		s = 1.6	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
θ = 0	8.08	20.57	8.49	13.93	8.77	9.70	9.09	4.65
θ = 1	6.30	34.21	**7.51**	**18.50**	8.32	9.73	9.25	1.33
θ = 2	5.75	41.17	7.31	20.18	8.32	9.46	9.37	0.51

(iv) $\varepsilon = 0.2$

	s = 0.8		**s = 1**		s = 1.2		s = 1.6	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
θ = 0	7.94	33.43	8.33	25.31	8.59	20.11	8.90	13.89
θ = 1	6.15	51.30	**7.31**	**31.90**	8.10	21.19	8.99	11.32
θ = 2	6.37	60.63	7.14	34.36	8.12	21.11	9.12	10.59

Table.5: Sensitivity of Foreign Aid Shocks to Domestic Fiscal Structure
(Cobb-Douglas production function, σ increases from 0 to 0.05)

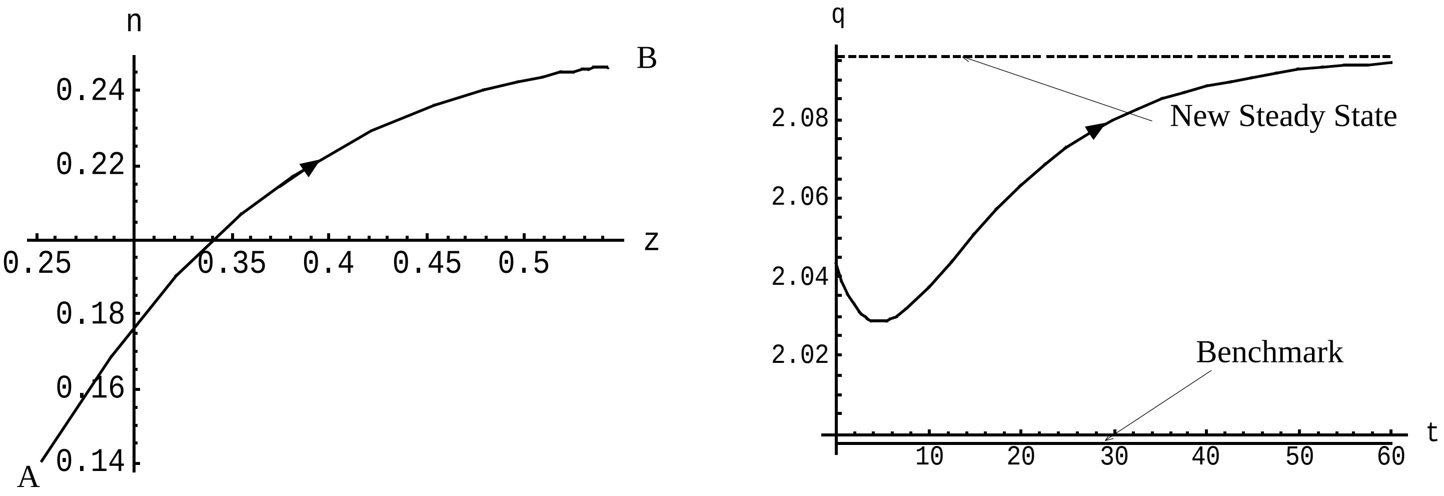
A. $\varepsilon = 0$

	$\bar{g} = 0.02$				$\bar{g} = 0.05$							$\bar{g} = 0.10$			
	Tied Aid ($\phi = 1$ is opt)		Untied Aid ($\phi = 0$)		Tied Aid ($\phi = 1$)		Untied Aid ($\phi = 0$)		Interior Optimal Mix			Tied Aid ($\phi = 1$)		Untied Aid ($\phi = 0$ is opt)	
	ΔW	ΔV	ΔW	ΔV	ΔW	ΔV	ΔW	ΔV	ΔW	ΔV	$\hat{\phi}$	ΔW	ΔV	ΔW	ΔV
$\tau = 0.10$	26.97	0.87	7.26	-0.76	7.16	0.57	8.23	-0.58	8.89	0.13	0.39	-2.68	0.45	10.35	-0.48
$\tau = 0.15$	27.73	1.09	6.87	-0.84	7.96	0.66	7.71	-0.64	8.92	0.07	0.53	-1.51	0.49	9.53	-0.52
$\tau = 0.20$	28.45	1.37	6.51	-0.94	8.70	0.77	7.25	-0.70	9.12	0.32	0.68	-0.50	0.54	8.82	-0.56

B. $\varepsilon = 0.10$

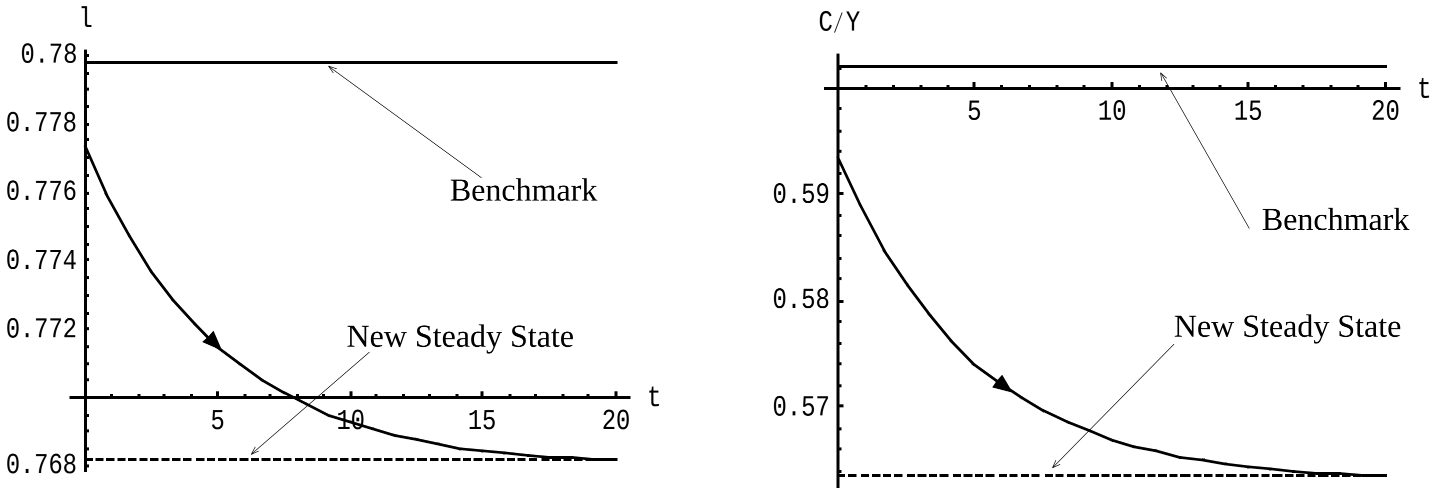
	$\bar{g} = 0.02$				$\bar{g} = 0.05$				$\bar{g} = 0.10$			
	Tied Aid ($\phi = 1$ is opt)		Untied Aid ($\phi = 0$)		Tied Aid ($\phi = 1$ is opt)		Untied Aid ($\phi = 0$)		Tied Aid ($\phi = 1$)		Untied Aid ($\phi = 0$ is opt)	
	ΔW	ΔV	ΔW	ΔV	ΔW	ΔV	ΔW	ΔV	ΔW	ΔV	ΔW	ΔV
$\tau = 0.10$	54.43	1.24	7.06	-1.14	17.69	0.63	7.98	-0.71	2.11	0.45	10.11	-0.51
$\tau = 0.15$	55.35	1.77	6.71	-1.28	18.50	0.79	7.51	-0.78	3.27	0.50	9.34	-0.56
$\tau = 0.20$	56.23	2.46	6.39	-1.45	19.26	0.99	7.08	-0.86	4.30	0.58	8.66	-0.61

Figure 1: Dynamic Responses toTied Aid Shock (Cobb-Douglas case)



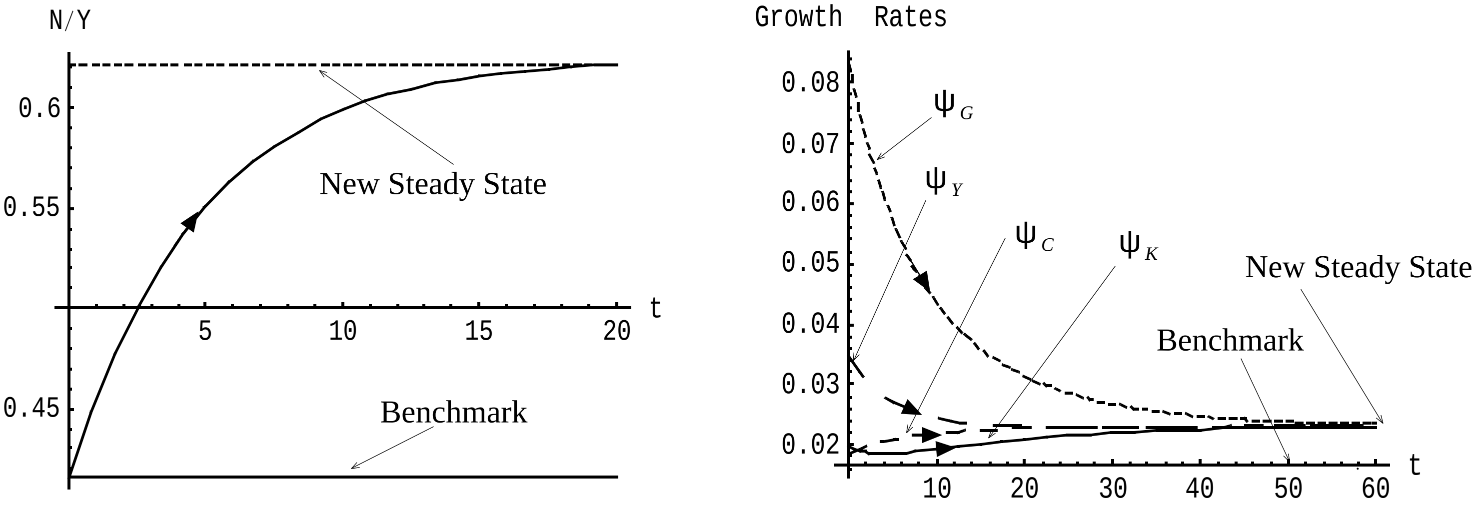
2.1 Phase Diagram

2.2 Market Price of Private Capital



2.3 Leisure

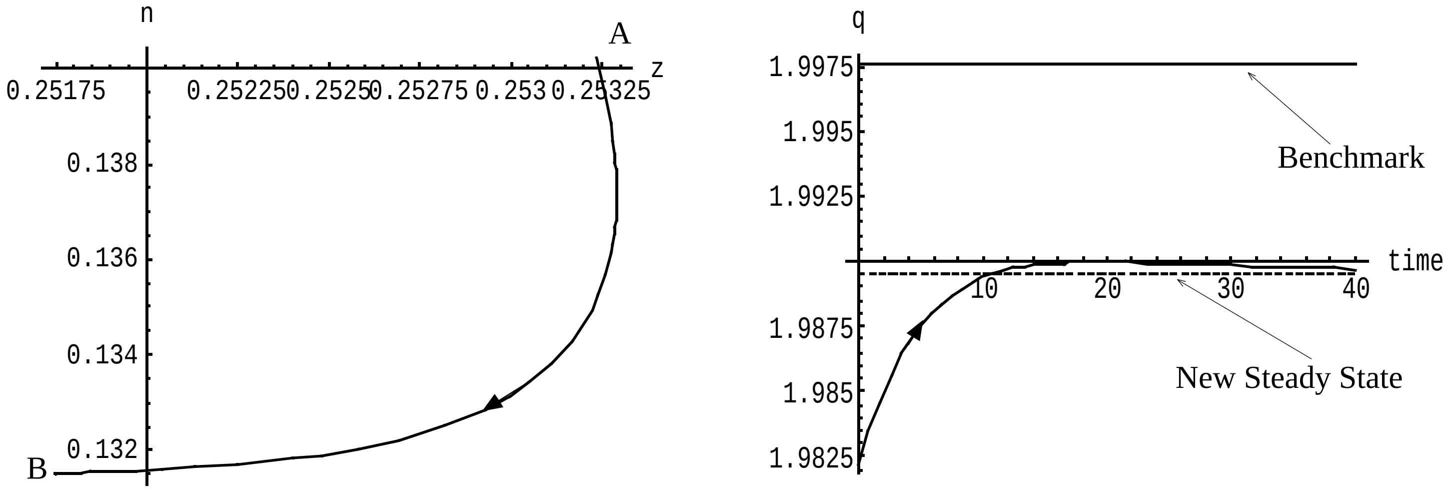
2.4 Consumption-Output Ratio



2.5 Debt-Output Ratio

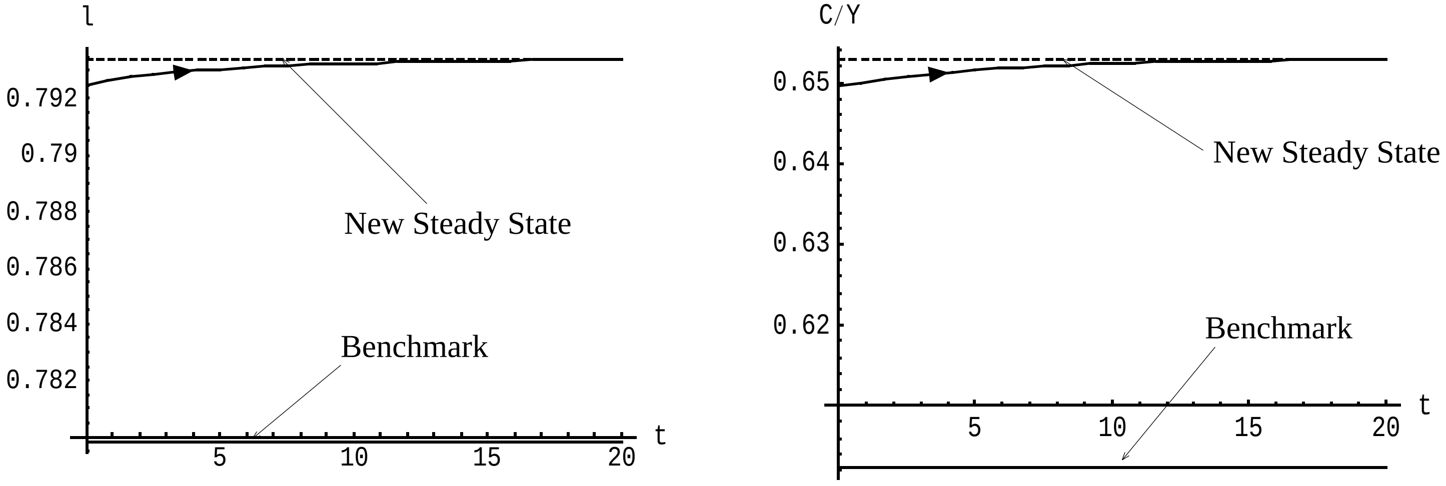
2.6 Growth Rates

Figure 2: Dynamic Responses to Untied Aid Shock (Cobb-Douglas case)



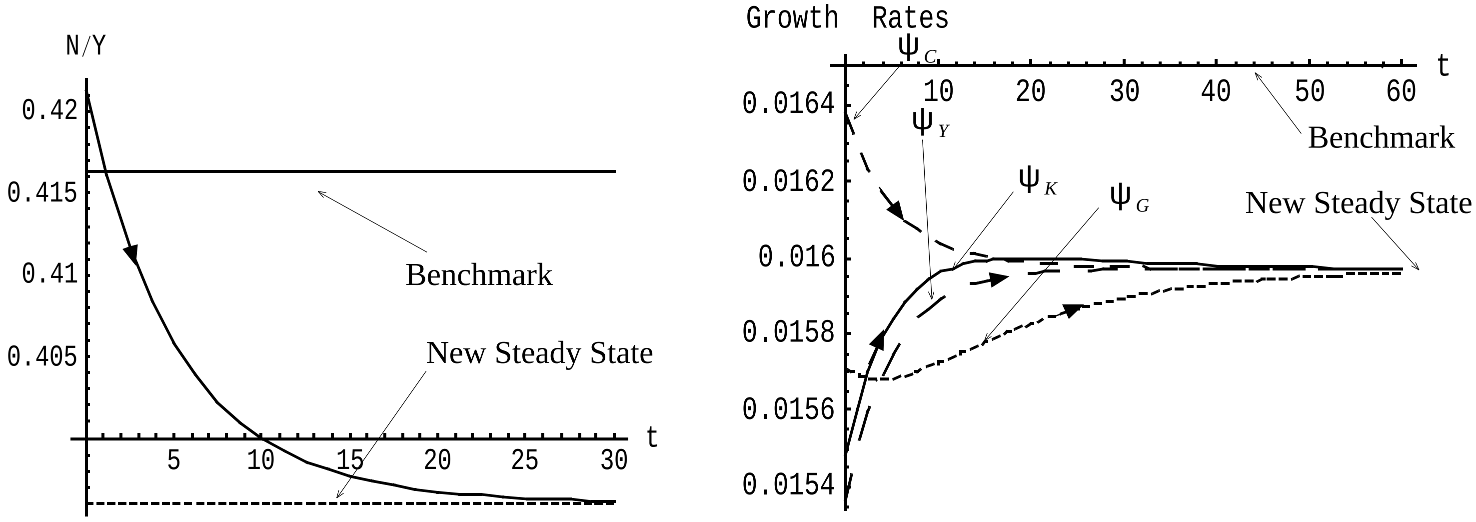
2.1 Phase Diagram

2.2 Market Price of Private Capital



2.3 Leisure

2.4 Consumption-Output Ratio



2.5 Debt-Output Ratio

2.6 Growth Rates

Figure 3: Sensitivity of Dynamics of Leisure-Consumption to Elasticity of Substitution (tied aid)

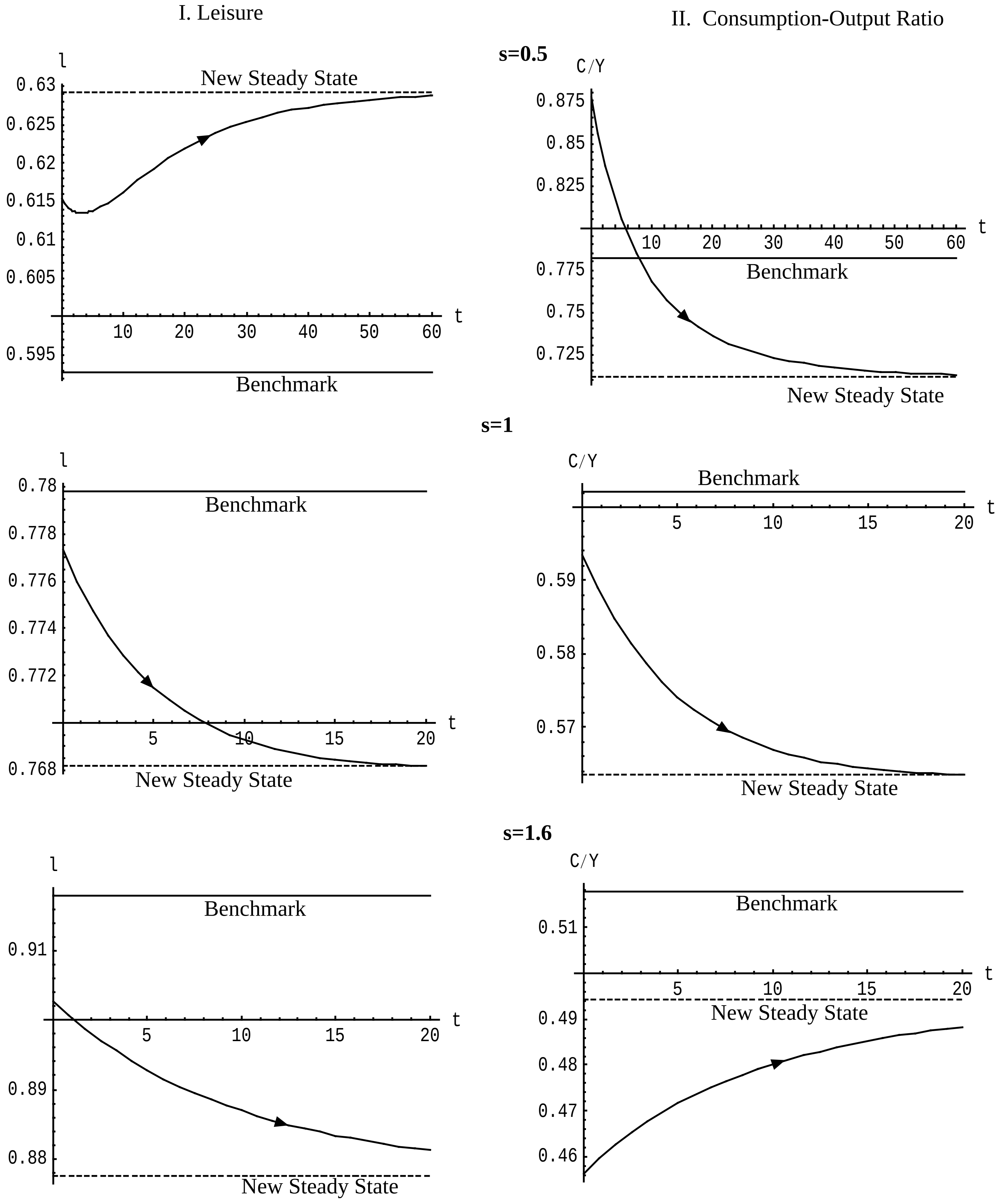
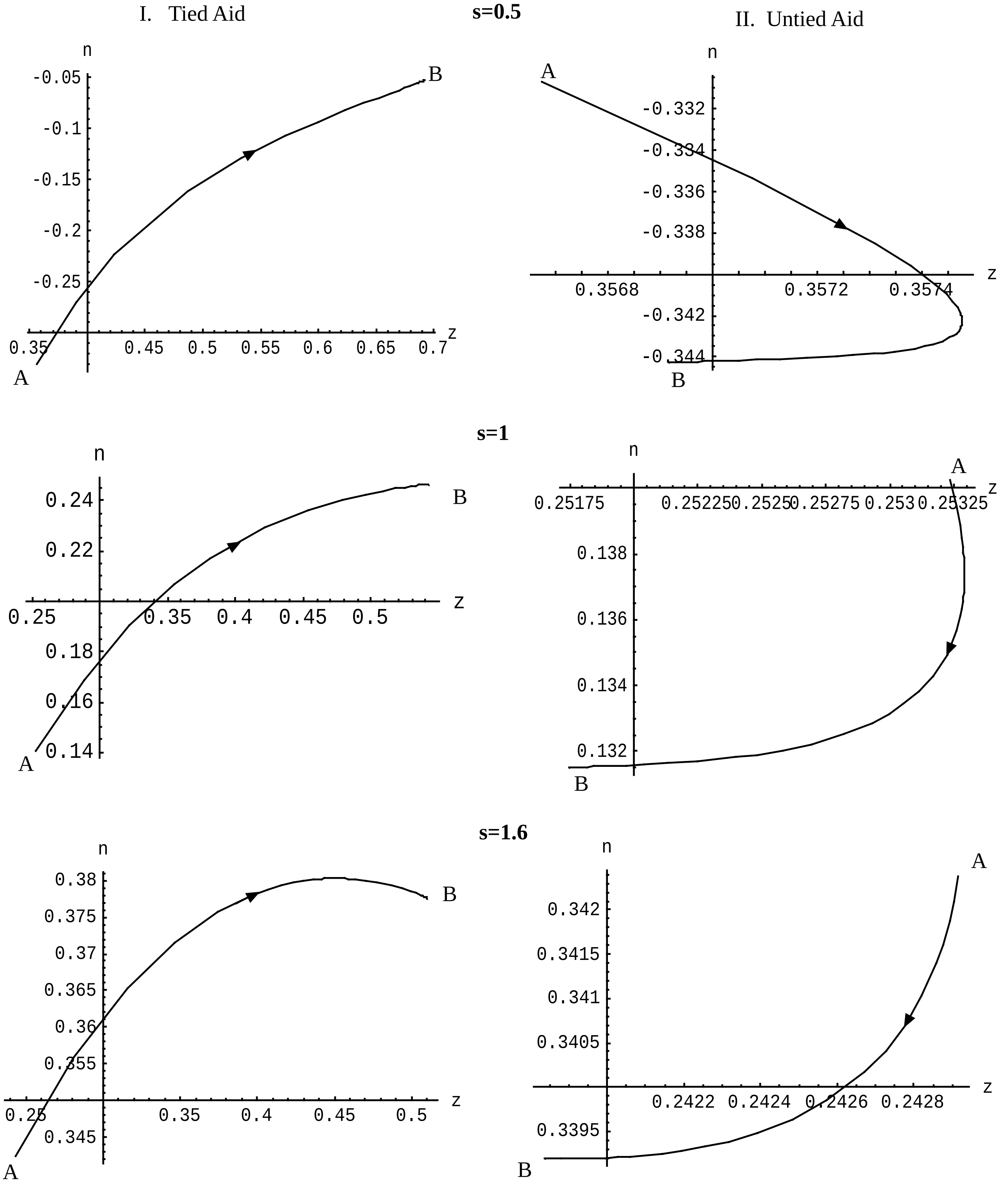


Figure 4: Sensitivity of Basic Dynamics to Elasticity of Substitution



Appendix

This Appendix provides the detailed derivations of the macrodynamic equilibrium

A.1 Derivation of the Equilibrium Relationships (9a) – (9d)

The production function we consider is

$$Y = \alpha\left(\frac{K_G}{\bar{K}}\right)^{\varepsilon}\left[\eta\left\{(1-l)\bar{K}\right\}^{-\rho} + (1-\eta)K^{-\rho}\right]^{-1/\rho} \tag{1a}$$

The marginal rate of substitution between *C* and *l* is given by:

$$\frac{C}{Y} = \frac{\eta(1-\tau)}{A^{\rho}\theta}\left(\frac{l}{1-l}\right)\left(\frac{Y}{1-l}\right)^{\rho} \tag{A.1}$$

where, $A = \alpha\left(K_G/K\right)^{\varepsilon}$. Next, we recall the definition of $\Omega(z,l) \equiv \Omega = \left((1-\eta)/\eta\right)\left[(1-l)z\right]^{\rho}$ given in (10a). Substituting this into the production function (1a), and into (A.1) we can express the output-capital ratio and consumption-capital ratios in the form

$$Y/K \equiv y = y(z,l) = \alpha z^{\varepsilon}\left[(1-\eta) + \eta(1-l)^{-\rho}\right]^{-1/\rho} \tag{A.2a}$$

$$\frac{C}{K} \equiv c = c(z,l) = \frac{(1-\tau)}{\theta}\left(\frac{l}{1-l}\right)\left[\frac{1}{1+\Omega}\right]y \tag{A.2b}$$

Then, differentiating the optimality condition, (3a), the marginal rate of substitution condition, (A.1), the production function (1a), all with respect to time, and recalling (4a), yields

$$(\gamma-1)\frac{\dot{C}}{C} + \gamma\theta\frac{\dot{l}}{l} = \frac{\dot{\lambda}}{\lambda} = \beta - r(N/K) \tag{A.3a}$$

$$\frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = \frac{\dot{l}}{l} + (1+\rho)\frac{\dot{l}}{1-l} + \rho\left(\frac{\dot{Y}}{Y} - \frac{\dot{K}}{K}\right) - \varepsilon\rho\left(\frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K}\right) \tag{A.3b}$$

$$\frac{\dot{Y}}{Y} \equiv \psi_Y = \frac{1}{1+\Omega}\left[\Omega\frac{\dot{K}}{K} + \frac{\dot{K}_G}{K_G} - \frac{\dot{l}}{1-l}\right] + \varepsilon\left[\frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K}\right] \tag{A.3c}$$

Combining these four equations together with (3c') and (6'), we can eliminate the growth rates, $\dot{C}/C, \dot{K}/K, \dot{K}_G/K_G, \dot{Y}/Y$, and $\dot{\lambda}/\lambda$ and express the dynamics of leisure by the differential equation:

$$\dot{l} = \frac{F(z,n,q,l)}{G(z,l)} \tag{A.4}$$

where

$$F(z,n,q,l) \equiv \left[\beta - r(\tilde{n}) + (1-\gamma)\left\{\varepsilon\left(\frac{\dot{K}_G}{K_G}\right) + (1-\varepsilon)\left(\frac{\dot{K}}{K}\right)\right\}\right] l \,,$$

$$G(k,l) \equiv \left[\{\gamma(1+\theta)-1\} - (1-\gamma)(1+\rho)\left(\frac{\Omega(z,l)}{1+\Omega(z,l)}\right)\left(\frac{l}{1-l}\right)\right],$$

and

$$\psi_K(q) \equiv \frac{\dot{K}}{K} = \frac{(q-1)}{h_1} - \delta_K \tag{A.5a}$$

$$\psi_G(z,l) \equiv \frac{\dot{K}_G}{K_G} = g\frac{Y}{K_G} - \delta_G = \alpha(\bar{g}+\sigma\phi)(1-l)z^{\varepsilon-1}\left[\eta\{1+\Omega(z,l)\}\right]^{-1/\rho} - \delta_G \tag{A.5b}$$

Using (A.3a) and (A.4) we can express the growth rate of consumption as

$$\psi_C \equiv \frac{\dot{C}}{C} = \frac{r(n) - \beta + \gamma\theta(1/l)\left[F(z,n,q,l)/G(z,l)\right]}{1-\gamma} \tag{A.5c}$$

while (9c) follows directly from the optimality condition (4b).

The equilibrium dynamics can now be represented by the following autonomous system in the stationary variables, z, n, q, l,

$$\frac{\dot{z}}{z} = \frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K} = (\bar{g}+\sigma\phi)\frac{y}{z} - \delta_G - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{A.6a}$$

$$\frac{\dot{n}}{n} = \frac{\dot{N}}{N} - \frac{\dot{K}}{K} = r(n) + \frac{1}{n}\left[c + \frac{q^2-1}{2h_1} + \{(\bar{g}+\sigma\phi)-(1+\sigma)\}y + \frac{h_2}{2}(\bar{g}+\sigma\phi)^2\frac{y^2}{z}\right] - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{A.6b}$$

$$\dot{q} = r(n)q - \frac{(1-\tau)(1-\eta)y^{(1+\rho)}}{\alpha^{\rho}z^{\varepsilon\rho}} - \frac{(q-1)^2}{2h_1} + \delta_K q \tag{A.6c}$$

$$\dot{l}=\frac{F(l)}{G(l)}=\frac{\left[\beta-r(\tilde{n})+(1-\gamma)\left\{\varepsilon\left(\frac{\dot{K}_G}{K_G}\right)+(1-\varepsilon)\left(\frac{\dot{K}}{K}\right)\right\}\right]}{\{\gamma(1+\theta)-1\}-(1-\gamma)(1+\rho)\left(\frac{\Omega(z,l)}{1+\Omega(z,l)}\right)\left(\frac{l}{1-l}\right)}l \tag{A.6d}$$

where, from above, $\Omega=\Omega(z,l)$, $y=y(z,l)$, $c=c(z,l)$, $\psi_K=\psi_K(q)$, and $\psi_G=\psi_G(z,l)$.

A.2 Steady-State Equilibrium

Steady-state equilibrium is attained when $\dot{z}=\dot{n}=\dot{l}=\dot{q}=0$, so that

$$\frac{\dot{C}}{C}=\frac{\dot{K}}{K}=\frac{\dot{K}_G}{K_G}=\frac{\dot{Y}}{Y}=\frac{\dot{N}}{N}=\tilde{\psi}.$$

$\dot{z}=\dot{n}=\dot{l}=\dot{q}=0$ in (A.6a) – (A.6d) and recalling (A.2a), (A.2b), (A.5a), (A.5b), and the definition of $\Omega(z,l)$, we can summarize the steady-state in the following form:

$$\left(\bar{g}+\sigma\phi\right)\frac{\tilde{y}}{\tilde{z}}-\delta_G=\frac{\tilde{q}-1}{h_1}-\delta_K \tag{A.7a}$$

$$r(\tilde{n})+\frac{1}{\tilde{n}}\left[\tilde{c}+\frac{\tilde{q}^2-1}{2h_1}+\left\{\left(\bar{g}+\sigma\phi\right)-(1+\sigma)\right\}\tilde{y}+\frac{h_2}{2}\left(\bar{g}+\sigma\phi\right)^2\frac{\left(1-\tilde{l}\right)\tilde{y}^2}{\tilde{z}}\right]=\frac{(\tilde{q}-1)}{h}-\delta_K \tag{A.7b}$$

$$r(\tilde{n})\tilde{q}-\frac{(1-\tau)(1-\eta)\tilde{y}^{(1+\rho)}}{\alpha^{\rho}\tilde{z}^{\varepsilon\rho}}-\frac{(\tilde{q}-1)^2}{2h_1}+\delta_K\tilde{q}=0 \tag{A.7c}$$

$$\frac{r(\tilde{n})-\beta}{1-\gamma}=\frac{(q-1)}{h_1}-\delta_K \tag{A.7d}$$

$$\tilde{y}=\alpha\tilde{z}^{\varepsilon}\left[(1-\eta)+\eta\left(1-\tilde{l}\right)^{-\rho}\right]^{-1/\rho} \tag{A.7e}$$

$$\tilde{c}=\frac{(1-\tau)}{\theta}\left(\frac{l}{1-l}\right)\left[\frac{1}{1+\left((1-\eta)/\eta\right)(1-\tilde{l})^{\rho}}\right]\tilde{y} \tag{A.7f}$$

These six equations can be solved for the steady-state values of $\tilde{z},\tilde{n},\tilde{l},\tilde{q},\tilde{c},\tilde{y},$ and consequently, the equilibrium growth rate, $\tilde{\psi}$.

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