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**Dual Provision of Public  
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# Dual Provision of Public Goods in Democracy

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## Abstract

This paper analyzes the provision of goods with consumption externalities (such as public policies) in hybrid settings: the ‘good’ is provided in a democratic process by majority vote, but each individual agent is free to contribute additional amounts before or after the political decision has been made. Prominent examples include policy making in federal states, charities, and dual provision of health care. We show that regardless of the timing of private and public actions, the results of the median voter theorem apply. A move from a purely public system to a dual system with private ex-ante contributions is shown to be unambiguously preferred by everybody in society. In contrast, establishing an ex-post contribution regime may be opposed by a minority of high-preference individuals. The paper also derives results for a scenario with endogenous timing of private contributions. Most importantly, this general regime is shown to be majority preferred not only to the systems with ex-post and the ex-ante contributions, but also to an institutional setting with private but no public provision.

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# 1 Introduction

Democratic policy decisions are often supplemented by private actions. Charities step in where, in the view of the donors, tax dollars do not guarantee a sufficient scale of services. In countries with a tax-financed public health care system, individuals often seek enhanced services from private doctors or hospitals. Various spending decisions in federal states are taken at a central level of government, but member regions are allowed to augment this funding at their own discretion. While some of these ‘private contributions’ have the characteristic of a private good whose benefits are exclusive to the contributor, in many cases there are positive consumption externalities from one individual - or region - to the other. A new or upgraded road or airport benefits visitors from other countries as well, in the same way as helping the City’s poor may reduce crime rates, thereby feeding back on other citizens. When a state government tightens federal regulation on vehicle or industry pollution, neighboring states will often benefit from this local policy measure.

The aim of the present paper is to explore situations of this type in a stylized model. A group of agents makes a decision to provide goods or services in a democratic fashion, using majority voting to determine the level of spending. Each agent can also enhance the consumption of this good, by privately buying additional amounts. Finally, public and private provision may give rise to consumption externalities across all agents in society. Our framework thus follows a line of research (for a brief review, see below) which combines two well known models in the literature: the democratic choice of policymaking under majority rule, and voluntary private contributions to (impure) public goods. Addressing these two aspects in a unified setting allows to tackle many questions of significant interest. The technical analysis, however, is challenging: the voting and contribution decisions of all agents are interrelated, and subject to strategic behavior. For this reason, it is a priori unclear whether the median voter theorem as a convenient tool to characterize political outcomes applies in the present setting.<sup>1</sup>

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<sup>1</sup>Stiglitz (1974) and Glomm-Ravikumar (1998) show that single-peakedness usually does not prevail in models where individuals can choose whether to consume a publicly provided private good such as schooling, or to consume a private alternative instead.

One contribution of the paper is to show that these difficulties can be overcome in various versions of the baseline model which is analyzed here. We first explore a setting where private contributions are made simultaneously and non-cooperatively in a first stage, before a political decision on the uniform financing of the public good is taken in a second stage. In this regime with ex-ante contributions (or for short ‘ex-ante regime’), individual preferences at the policy stage are single peaked for any arbitrary profile of initial contributions. The median voter theorem then applies and the individual with median preferences determines the policy outcome. In contrast to the familiar majority voting scenario, though, the identity of the median voter is endogenous: it depends on the vector of initial private contributions, specifically because an individual’s private buy decision in stage one generally reduces her stage two policy preferences relative to those of other individuals. Nevertheless, we establish that no ‘rank reversal’ will occur so that in equilibrium, the ‘natural’ median individual (the median voter under pure public provision without private contributions) always remains the pivotal decision maker. With this result, one can then immediately show that only high-preference individuals voluntarily contribute ex ante. These individuals are well aware of the crowding out effect of their contributions; with consumption externalities, equilibrium contributions will not only be smaller than those in absence of public provision, but they will be even smaller than the best responses to the policy level implemented ex post. Also, the equilibrium policy is strictly smaller while everybody’s public goods consumption is strictly larger than in a system with purely public provision where private topping up is infeasible.

We then turn to a scenario where the above timing is reversed (the ex-post regime). Most existing papers focus on this setting only: it allows for a policy commitment because democratic decisions are made first, before individual contributions can be made. In comparison to the ex-ante regime, the analysis is severely complicated by the fact that in presence of externalities, individual preferences over policies are not necessarily single peaked. To see this intuitively, notice that when comparing different policy levels, the vector of subsequent private contributions and, potentially, the set of contributors changes. An individual may be a contributor for a small but not for a large policy, making the shape of his utility function difficult to predict. One of the

central findings of the paper is that notwithstanding this problem, the results of the median voter theorem continue to hold. To demonstrate this outcome, we show that all individuals with natural preferences larger than the natural median individual prefer a larger policy over the one preferred by the median, while all individual with smaller natural preferences prefer a smaller policy. As a consequence, the policy preferred by the natural median cannot be defeated in majority voting, which makes her the democratic decision maker in society. Applying the median voter theorem then allows a precise characterization of equilibrium contributions and equilibrium policies. Among other things, equilibrium policies are shown to be smaller than in the ex-ante regime, while private contributions are larger.

A logical next step is the economic comparison of different institutional regimes. Both ex ante and ex post regime have in common that only a subset of high-preference agents make private contributions, while all other individuals including the median and all people with lower preferences do not.<sup>2</sup> However, private contributions in the ex-post setting are larger than in the ex-ante setting, while the reverse pattern characterizes the respective equilibrium policies. These findings can easily be understood in terms of the commitment capabilities assigned to the relevant agents. In the ex-post setting, the median voter as the pivotal player knows that implementing a relatively small policy will trigger large contributions from high-preference individuals. Reducing public provision boosts the median voter's private-goods consumption, while the associated loss in public good consumption is at least partially mitigated through enhanced private contributions of high-preference individuals. Conversely, in the ex-ante setting where private contributions are made first, each potential contributor knows that lowering his private contribution induces a larger public provision. Hence, high-preference individuals can partially free ride on public provision.<sup>3</sup> These different commitment

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<sup>2</sup>Hence, a majority of the population never makes a private contribution. In general (and in presence of externalities), even some agents with larger-than-median-preferences will not privately contribute because in contrast to the preferences revealed in the political process, a private topping-up decision does not involve cost sharing with other agents.

<sup>3</sup>For the polar case of pure public goods, it is also relatively easy to compare total public goods consumption in either regime: while it is larger in the ex-post regime when income effects are absent and if the set of contributors is non-empty in each regime, the opposite can happen otherwise.

opportunities induce equilibrium policies to be strictly larger in the ex-ante regime as compared to the ex-post regime. Moreover, equilibrium policies in both dual regimes are smaller than those in a pure public-provision setting: with normal preferences, the median voter's private goods consumption cannot be lower in a dual system where the contributions of higher-preference individuals raise public goods consumption. Hence, in a hybrid regime, she will choose a smaller policy.

From a political economy perspective, it is important to explore the relative support of different regimes by the citizens in society. For a regime change to happen, a well defined majority or super-majority of agents must prefer some alternative institution over the status quo. Again, our model allows to derive interesting and unambiguous results. A majority of citizens, comprising all individuals with low preferences for the public good, prefers the ex-post regime over both the ex-ante regime and pure public provision. However, a sizable minority of high-demand citizens may oppose a transition from the pure public to the system with ex-post contributions, in fear of exploitation by the majority that controls the political decision making. Strikingly, no such resistance arises in case of a design change from the pure public to the ex-ante contributions regime. In fact, we show that the population of agents unanimously supports this transition. This Pareto optimality result may provide some guidance for the direction of policy reforms in areas where the ex ante regime is likely to apply. It says that a hybrid public-private system can be unanimously desired if designed in the right way: after the public decision is taken, no additional private contributions should be allowed, while these contributions are encouraged prior to this decision.

As a final step, we endogenize the timing and allow agents to make private contributions before and after the policy is chosen. This setting is not only of independent interest but is the most natural framework in situations where governments are unable to control the amount and timing of private contributions. Immediate intuition may suggest that economic outcomes must coincide with those for the ex-post setting; after all, the median voter (who remains pivotal) can still choose the same public provision level at the policy stage, forcing high-demand citizens to privately contribute at the final stage. Perhaps surprisingly, though, we show that this intuition is misleading. In fact,

the endogenous regime will often dominate the ex-post setting at least for a majority of the population, including all high-demand agents. The explanation is based on an interesting reciprocal commitment argument. Specifically, the ex-ante contribution stage allows high preference individuals to commit to contributions larger than those made in the equilibrium of the ex-post regime. In response, the stage-2 median voter anticipates that, for a reasonable range of policy choices, there will be no additional ex-post contributions.<sup>4</sup> Large ex-ante contributions thus offset the public decisionmakers' interest in strategic underprovision, inducing her to vote in favor of increased public provision.<sup>5</sup> The interesting consequence is a 'crowding in' rather than the familiar 'crowding out' effect - private contributions and public provision are both larger than in the ex-post regime. Also, a majority of citizens prefers the system with endogenous timing over both alternative hybrid regimes, and over pure public provision. Under majority vote, this institutional design thus emerges as the only stable alternative.

## 2 Existing Literature

Our paper is part of a growing literature that explores the mechanics and the political economy implications of a dual private-public provision of goods and services. Most existing contributions confine attention to what we call the ex-post system, where the political decision is taken by majority rule, before individuals can privately contribute in a second stage. Epple and Romano (1996) pioneered the analysis of this scenario. They explore a dual health care system, with a public-provision element being funded through linear income taxes, as chosen in the democratic process. According to their main findings, the equilibrium provision of public services in the hybrid system is larger than in a purely public system, or a purely private system. In addition, a majority of citizens strictly prefers the dual regime. Combining theoretical analysis with empirical

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<sup>4</sup>This is true unless the median voter implements a very small policy, a choice she will generally find unappealing.

<sup>5</sup>The equilibrium policy is then a best response to the (and only the) ex-ante private contributions. In contrast, in the ex-post setting, the equilibrium policy choice is strictly lower than the best response to the equilibrium ex-post contributions: for strategic reasons, policy underprovision arises in order to stimulate subsequent private contributions.

testing, Fernandez and Rogerson (2003) compare a variety of school funding systems in a dual provision setting. For the cases of pure private, pure public, or the dual ex-post regime, their theoretical results largely coincide with those in Epple and Romano.<sup>6</sup>

In both of these papers, the relevant good is modeled as a private good, which imposes no consumption externalities on other agents in society. For this reason, the political choice is not implicated by strategic behavior on the voters' parts, which is shown to guarantee the single peakedness of individual preferences and the validity of the median voter theorem.<sup>7</sup> Epple and Romano (2003) shift the focus of their previous research, by considering the case of a pure public rather than a private good. In order to make the strategic effects of voting tractable, individuals are assumed to behave 'myopically' in the sense of disregarding the effect of their first-stage political votes on second-stage private contributions. Epple and Romano show that in this setting with not fully rational agents, equilibrium does not necessarily exist but if it does, the natural median voter is the pivotal individual. In a related theoretical setting but in the context of policymaking in federations, Hafer and Landa (2005) find that the lack of preference single peakedness generally prevents the applicability of the median voter theorem. Assuming Cobb-Douglas preferences, they derive sufficient conditions for equilibrium existence, and provide a variety of additional characterization results.

The contributions closest to the present work are those by Cremer and Palfrey (2000, 2006), and by Alesina et al. (2005). In line with the funding assumption that we impose, public provision is financed through uniform lump sum taxation. All these papers are set in the context of federal systems. Cremer and Palfrey investigate 'federal mandates', implemented by a central government via majority vote among individual regions. Essentially, a federal mandate imposes a minimum or maximum standard on a policy whose costs are incurred by individual states. These standards can subsequently be tightened by state legislation, which in our setting corresponds to a private contri-

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<sup>6</sup>Their paper also examines several other possible regimes, and provides a thorough empirical analysis.

<sup>7</sup>Note, though, that even in a setting where voters have identical preferences, the median voter need not coincide with the median income individual. Under income taxation, rich individuals may in fact favor lower taxes because they face a larger absolute tax burden than poorer agents.



bution. In Cremer and Palfrey (2000), policy choices and private contributions are not associated with external effects on regions, and again the median voter theorem applies. It is shown that equilibrium mandates are tighter than desired by a majority, and some high-demand states augment the mandate by imposing even more rigid standards. Cremer and Palfrey (2006) allow policies to cause positive externalities. According to their main finding, majority vote equilibria cannot be guaranteed to exist. To circumvent this problem, the paper characterizes the set of ‘local’ policy equilibria, which is the set of policies which is majority preferred to other policies in the vicinity.<sup>8</sup> Alesina et al. (2005) consider a framework similar to Cremer and Palfrey, but with homogenous agents within districts. Their paper explores not only the ex-post regime, but also the ex-ante regime. While Alesina et al. argue that a majority of agents prefers the ex-ante system over the pure public system, we show that in fact, the ex-ante system is preferred unanimously. Moreover, their paper acknowledges that all results require the validity of the median voter theorem, but do not prove its validity.<sup>9</sup>

Our paper goes beyond existing results in showing that in dual institutional systems, the median voter theorem often holds even in presence of consumption externalities, and with fully rational voters. Under income taxation, high-income individuals face a tradeoff between their income-driven higher demand, and the redistributive aspects of higher taxes. While one of the central arguments in our paper shows that no high-income individual will ever prefer a policy smaller than the one preferred by the median voter (and vice versa), this property cannot generally be expected to hold in a setting with income taxation, making it more difficult to obtain transparent results.<sup>10</sup> With regard to empirical relevance, both scenarios seem appropriate in different circumstances. Central funding in federations is sometimes (not always) based on the per capita wealth

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<sup>8</sup>There is one significant difference between the settings analyzed in Cremer and Palfrey, and our paper. While in Cremer and Palfrey (2006) each voter in each region participates in federal policy decisionmaking, our setting can be interpreted as one where only the district medians cast their federal votes. Our results suggest that this seemingly minor modification restores the existence of global majority vote equilibrium.

<sup>9</sup>Alesina et al. acknowledge that the theorem may not hold (See p. 614 of their paper).

<sup>10</sup>We conjecture, though, that under some assumption one the rank order of preferences often made in the literature, this results extends to the case of redistributive income taxation as well. Note also that in absence of income heterogeneity, lump sum taxation and income taxation are the same.

of individual member states; contributions to a state funded health care system are made lump sum in Canada, but through progressive taxes in many European countries. Federal mandates are usually imposed uniformly of all member states within a federation. Unless there are reasons to believe that implementation costs systematically differ across states, uniform taxation seems a good first pass on this scenario.

The remainder of the paper proceeds as follows. Section 3 below describes the baseline model. Section 4 explores some benchmark scenarios. Section 5 provides a general analysis of the systems with ex-ante and ex-post contributions, and Section 6 illustrates these results. Section 7 analyzes a system with endogenous contribution timing. Section 8 briefly discusses the switch from a private market regime, and Section 9 concludes.

### 3 The Model

Consider an economy with  $N \geq 3$  agents, where  $N$  is odd. Individual  $i$  derives his utility from the consumption of a private good  $x_i$ , and a public good  $G_i$  which will be defined more precisely below. We will assume that agent  $i$ 's utility function  $U^i(x_i, G_i)$  is quasiconcave, and that both  $x$  and  $G$  are normal goods according to  $i$ 's preferences. Moreover, we impose the standard Inada conditions and normalize the prices of both public and private good to one. Agents may differ in their exogenous incomes,  $y_i$ , and in their tastes for the public good.<sup>11</sup>

The agents play a version of the following general game. In a stage 1, they can simultaneously and non-cooperatively make private 'ex-ante' contributions  $\hat{g}_i$  towards the public good. These contributions become public information of all agents; they can alternatively be interpreted as individual  $i$ 's private purchase and consumption of the commodity in the marketplace.<sup>12</sup> In a subsequent policy stage, stage 2, all individ-

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<sup>11</sup>Throughout the paper, we assume that incomes are sufficiently large that each individual is able to pay his tax contribution in any equilibrium. At least implicitly, this assumption is shared by all papers in the literature.

<sup>12</sup>In both interpretations, the 'public good' may be offered by suppliers in a competitive market, or it may be produced in home production where all individuals have access to the same technology.

uals decide in a political decisionmaking process on a mandatory uniform lump-sum contribution  $g$  that is imposed on each agent. For concreteness, we follow the positive literature on public goods supply and assume that  $g$  is chosen by majority vote. Finally, in stage 3, individuals can simultaneously expand public-goods consumption by making additional ‘ex-post’ contributions,  $\tilde{g}_i$ . An agent’s consumption of the private numeraire good is then  $c_i = y_i - g - \hat{g}_i - \tilde{g}_i$ . As we have already emphasized in the Introduction, an analysis of private ex-ante and ex-post contributions allows us to explore the commitment effects of policy choices and of private consumption decisions, respectively.

Regarding the characteristics of commodity  $G$ , we will allow for pure and impure forms of public goods or services. The public goods consumption of each individual is composed of a uniform amount  $G^U$  which is provided to everyone through the political process, and of the private contributions of the individual himself, and of other individuals. In particular,  $G^U = g(1+\beta(N-1))$  where  $\beta \in (0, 1)$  indicates the degree of consumption externalities. For  $\beta = 0$ , there are no spillovers and  $G^U$  becomes a private good with uniform consumption requirement. For  $\beta = 1$ , we analyze a pure public good, while all interior specifications of  $\beta$  capture intermediate characteristics. The overall public good consumption of a citizen  $i$  is then denoted as  $G_i = G^U + [\hat{g}_i + \tilde{g}_i + \beta \sum_{j \neq i} (\hat{g}_j + \tilde{g}_j)]$ , where private contributions of individuals  $j \neq i$  again cause spillovers at a rate  $\beta$ . Note that unless  $\beta = 1$  where  $G_i = G$  for all  $i$ , individuals who contribute more towards the public good have a higher overall consumption.

Let  $g$  be a public policy and  $\hat{g}$  and  $\tilde{g}$  be the vectors of ex-ante and ex-post private contributions, respectively.<sup>13</sup> For subsequent reference it is useful to define

$$\Delta^i(g, \hat{g}, \tilde{g}) = \frac{U_G^i(c_i, G_i)}{U_c^i(c_i, G_i)}$$

as individual  $i$ ’s marginal rate of substitution between the private and the public good.

We impose

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<sup>13</sup>In what follows, we assume that  $g \in [0, \bar{g}]$  where the maximum policy  $\bar{g}$  is smaller than the income of the poorest individual in society. Alternatively, we could assume that an agent’s policy contribution is  $\min\{y_i, g\}$ , and close the model by imposing a sufficiently harsh punishment in case that the individual pays less than  $y_i$ . This latter specification would not alter any qualitative results but significantly complicate the exposition.

**Assumption 1. (Single crossing property)** *For any  $g$  and  $\hat{g}, \tilde{g}$  with the property  $\hat{g}_i = \hat{g}_j, \tilde{g}_i = \tilde{g}_j$  for all  $i, j$ , the rank order of the vector  $\Delta(g, \hat{g}, \tilde{g}) = \{\Delta^i\}_{i=1, \dots, n}$  remains unchanged. We say that an individual  $i = 1, \dots, n$  with a higher index exhibits ‘larger preferences’ for the public good.*

The single crossing assumption allows us to order individuals according to their ‘natural’ rank in terms of preferences towards the public good: for any arbitrary public-good level generated by identical funding from each individual, the ranking of the marginal rates of substitution across individuals remains the same. Notice that the definition is flexible enough to accommodate not only taste, but also income differences. For example, consider two individuals with identical homogenous preferences but different incomes. For identical individual contributions to the public good, their private consumption differs, and the lower-income individual displays a smaller  $\Delta(\cdot)$  and is considered a lower-preference individual. Another important class of preferences consistent with Assumption 1 are quasilinear utilities, where agent preferences and incomes are heterogenous:  $U^i = c_i + \alpha_i H(G_i)$ , with  $\alpha_i > 0$  being a preference parameter for the public good. In this representation, an individual with larger  $\alpha_i$  is characterized by a larger index  $i$ . In what follows, we will say that individual  $m$  is the ‘natural’ median individual according to the ordering described above.

## 4 Benchmark cases

Before going into a general analysis, we briefly investigate the two simple benchmark cases of pure public provision, and a situation where  $G$  is a private good.

### 4.1 Pure public Provision

As a benchmark, suppose public goods are provided exclusively through the political process, that is, no private consumption decisions can be made. Each individual  $i$  prefers a policy outcome  $g_i^{pp} = \arg \max_g U^i(y_i - g, g(1 + \beta(N - 1)))$ , which is implicitly defined by the first-order condition  $\Delta_i(\cdot) = U_G^i(c_i, G_i)/U_c^i(c_i, G_i) = 1/(1 + \beta(N - 1)) \equiv$

$1/z \leq 1$ . Note that under our previous assumptions, the single-peakedness requirement is satisfied so that the median voter theorem applies. Moreover, since  $\Delta^i$  and thus  $g_i^{pp}$  are increasing in the index  $i$ , the preferences of individual  $m$  with median bliss point represent the unique outcome of majority voting. We have

**Proposition 1.** *Consider pure public provision of  $G$ . The unique equilibrium policy  $g_m^{pp}$  is determined by the preferences of the natural median voter  $m$ , and is implicitly given by*

$$\Delta^m = \frac{U_G^m(y_m - g_m^{pp}, g_m^{pp}(1 + \beta(N - 1)))}{U_c^m(y_m - g_m^{pp}, g_m^{pp}(1 + \beta(N - 1)))} = \frac{1}{1 + \beta(N - 1)} = \frac{1}{z}. \quad (1)$$

Under the Inada conditions, the selected policy  $g_m^{pp}$  is interior. Moreover, in the special case where individuals differ only in their incomes, normality ensures the median-income individual to be the median voter  $m$  in the community.

## 4.2 Purely private goods

Before commencing a general analysis of institutional regimes with ex-ante and ex-post contributions, it is useful to consider the special case of a purely private good  $\beta = 0$ . This case has received most of the attention in the existing literature on dual institutions. Notice that since consumption externalities are absent, voting is not subject to strategic behavior, a feature that considerably simplifies the analysis. We can state

**Proposition 2.** *Suppose  $\beta = 0$ . In both the ex-ante and the ex-post regime, there exists a continuum of equilibria, characterized by equilibrium policies  $g^* \in [0, g_m^{pp}]$ . In addition,*

- a) *Each high-preference agent with rank  $i \geq m$  consumes the first best amount of public goods  $G_i^*$ , given by  $\Delta^i(G_i^*) = 1$ .*
- b) *For any equilibrium policy  $g^*$  satisfying  $g > g_1^{pp}$ , a subset of low-preference agents with ranks  $j < m$  is forced to overconsume,  $\Delta^j(G_j^*) < 1$ . The size of this subset strictly increases in  $g^*$ , and it comprises all individuals  $j < m$  if  $g^* = g_m^{pp}$ .*

Proof: Consider the ex-post regime and a policy  $g \leq g_m^{pp}$ . After any such policy has been implemented, each individual  $k$  finds it optimal to privately contribute  $\tilde{g}_k = \max\{g_m^{pp} - g, 0\}$  in stage 2. Hence, agent  $k$  overconsumes relative to the efficient amount if  $g > g_m^{pp}$ , while  $k$  is able to achieve his first best consumption  $g_m^{pp}$  otherwise. For this reason, any  $g \in [0, g_m^{pp}]$  can be supported as an equilibrium policy: each individual  $i \geq m$  and thus a majority of agents is indifferent between any policy from this set but strictly prefers each of them over any  $g > g_m^{pp}$ . Conversely, no  $g > g_m^{pp}$  is a candidate for policy equilibrium because it would be majority rejected.

Next, consider the regime with ex-ante contributions. In this setting, no  $g > g_m^{pp}$  can be the chosen equilibrium policy in stage 2, for two reasons. First, in absence of private contributions, any such policy is dominated by majority. Second, each individual's preferred policy is single-peaked, decreasing in his own contribution, and (since  $\beta = 0$ ) unaffected by the contributions of other agents. Conversely, each policy  $g < g_m^{pp}$  can be supported as a majority voting equilibrium. To see this, consider a contribution vector  $\hat{g}_{-k}$  for which without agent  $k$ 's participation, some policy  $g^* (< g_m^{pp})$  would be selected in majority vote. Note that for any such  $g^*$ , vector  $\hat{g}_{-k}$  always exists: each individual  $i \geq m$  can always contribute  $\hat{g}_i = g_i^{pp} - g^*$  so that  $g^*$  becomes  $i$ 's preferred stage 2 policy (note that if  $g^* < g_i^{pp}$ , a subset of lower-preference individuals  $j < m$  can do the same). Finally, for any  $\hat{g}_{-k}$ ,  $k$ 's best response is to contribute  $\hat{g}_k = g_k^{pp} - g^*$  in stage 1, and to prefer policy  $g^*$  as well. Accordingly, a majority of the population prefers policy  $g^*$  over any alternative policy. Hence, a majority including any  $i \geq m$  achieves a first-best outcome, while a subset of minority individuals (with size increasing in  $g^*$ ) does not privately contribute but is forced to overconsume relative to the first best whenever  $g^* > g_1^{pp}$ .<sup>14</sup>  $\square$

In the private-goods case, there is a continuum of equilibrium outcomes, which are the same irrespective of the timing of private contributions. All of these equilibria are efficient for individuals with larger than median preferences (including  $m$ ), in the

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<sup>14</sup>Note also that no individual  $i$  for which  $g_i^{pp} < g^*$  will overconsume or underconsume in an equilibrium with policy  $g^*$ : these individuals can always adjust their stage-1 contributions to a level where they prefer the induced equilibrium policy in stage 2.

sense of equating marginal rate of substitution and marginal rate of transformation. However, for equilibrium policies larger than the preferred consumption of the lowest-preference agent (which is identical to  $g_1^{pp}$ ), a subset of low-demand agents consumes and pays more than desired. In addition, the largest possible equilibrium policy,  $g_m^{pp}$ , coincides with the unique equilibrium under pure public provision as derived above. Both dual regimes thus Pareto dominate pure public provision: individuals  $i > m$  are strictly better off because they can enhance their consumption of the public good while, at the same time, some lower preference individuals enhance their utilities by reducing (or even avoiding) overconsumption.<sup>15</sup>

In contrast to our setting with uniform tax contributions, multiplicity of equilibria does not arise in a setting with income-dependent contributions.<sup>16</sup> For example, with linear income taxes as analyzed in Epple and Romano (1996) and Fernandez and Rogerson (2003), individuals have strict preferences over tax rates even if there are no externalities. While individuals with higher-than-average incomes strictly prefer zero public provision, lower-income individuals benefit from the tax contributions of richer individuals, and each of them favors a type specific positive tax level.<sup>17</sup> As we show in the following Sections, however, externalities generally restore uniqueness of equilibrium in our framework.

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<sup>15</sup>Notice that a purely private regime without any public provision would dominate a dual regime. The reason is simple: Since the good in question is purely private and the financing mode does not entail a redistributive element, private consumption decisions must be individually optimal. Certainly, this property does not extend to a scenario with consumption externalities.

<sup>16</sup>Cremer and Palfrey consider uniform taxation. They resolve the multiplicity issue by assuming that regional populations are heterogeneous, but no individual knows his place in the regional preference distribution. Voting in favor of his preferred consumption level is then dominant strategy at the policy stage.

<sup>17</sup>However, the preferences of below-average income agents are not necessarily monotone in income: while more affluent individuals within this group *ceteris paribus* favor larger consumption, poorer individuals pay a smaller unit tax price.

## 5 Analysis

### 5.1 Dual System with Ex-ante contributions

In this Section, we provide a general analysis for a situation where only ex-ante private contributions  $\hat{g}_i$  are feasible or relevant. The public good in question might be a museum or University construction project whose technical specifications (design, size etc.) are irreversibly determined in the stage-2 political process.<sup>18</sup> Alternatively, the ex-ante contributions scenario might represent situations in which individual agents - such as states in a federation, or charities - act as ‘leaders’ who commit to a contribution level before political decisions are made. For instance, the extraordinary endowment of the Bill and Melinda Gates foundation make this charity a strategic player in Third World aid.

Using subgame perfection as the appropriate equilibrium concept, we first analyze the stage-2 political equilibrium, before proceeding to the initial private-contributions stage. Two crucial issues need to be addressed. First, it has to be shown that individual stage-2 preferences are single-peaked so that majority voting equilibrium exists and the preferences of the (endogenously chosen) median voter prevail in political equilibrium. Second, upon establishing single-peakedness, we must find out whether in Nash equilibrium, the ‘natural’ median  $m$  or some other individual actually becomes the median voter. Since each individual’s policy preferences in stage 2 depend on the voluntary contribution profile of all agents, the answer to this latter question is not immediate.

We start with the following lemma.

**Lemma 1.** *For any arbitrary stage-1 contribution profile  $\hat{g} = \{\hat{g}_i\}_{i=1,\dots,n}$ , individual preferences in stage 2 over  $g$  are single-peaked. Hence, the choice of the individual  $M$  with stage-2 median preferences, say  $g^{M^*}(\hat{g})$ , prevails in majority vote.*

Proof: Consider an arbitrary contribution profile  $\hat{g}$ . For any such profile, an individual

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<sup>18</sup>The political decision on project design then essentially determines the scale of the public good, and additional private contributions would leave consumption levels of this good unaffected.



$i$ 's preferences over policies  $g$  are described by the utility function

$$U^i(y_i - \hat{g}_i - g, g(1 + \beta(N - 1)) + \beta \sum_{j \neq i} \hat{g}_j + \hat{g}_i).$$

Since preferences are quasiconcave by assumption, they are single-peaked in  $g$  and (if interior) the maximizer  $g^{i*}$  for individual  $i$  is described by the first-order condition

$$\Delta^i(\hat{g}_i, \hat{g}_{-i}, g^{i*}) \equiv \frac{U_G^i(\cdot)}{U_c^i(\cdot)} = 1/(1 + \beta(N - 1)) = 1/z.$$

The individual  $i = M$  with ex-post median preferences then determines the policy outcome in pairwise majority vote.<sup>19</sup>  $\square$

To be more specific about the equilibrium outcome, we now investigate the private-contributions game in stage 1. The analysis first yields

**Lemma 2.** *There exists an equilibrium in which the natural median  $m$  becomes the median voter. No individual  $j$  with index  $j \leq m$  contributes in stage 1, while some of the higher-demand individuals  $j > m$  may contribute.*

Proof: Consider the stage-2 median voter as induced by the stage-1 contribution profile  $\hat{g}$ . Call this individual  $M$  (again, notice she is not necessarily the ‘natural’ median  $m$ ).

The proof is by contradiction. Suppose an equilibrium with  $M = m$  exists, and no  $j \leq m$  contributes. Since the lowest-preference  $(n + 1)/2$  individuals do not invest in stage 1, their ranking of policies  $g$  corresponds to the natural order of their preferences, and individual  $m$  by definition exhibits the maximum bliss point  $g^{m*}(\hat{g})$  within this group. Accordingly, individual  $m$  is the median individual in stage 2 unless some agent  $i > m$  contributes  $\hat{g}_i$  in a way that  $g^{i*}(\hat{g}) < g^{m*}(\hat{g})$ .

We show that indeed no  $i > m$  will make such a contribution. To see this, note first that because of normality the preferred second-stage policy  $g^{i*}$  of any individual  $i$  is strictly decreasing in  $\hat{g}_i$ . For  $\hat{g}_i = 0$ , this implies  $g^{i*} > g^{m*}$ , irrespective of the contribution profile  $\hat{g}_{-i} = \{\hat{g}_1, \dots, \hat{g}_{i-1}, \hat{g}_{i+1}, \dots, \hat{g}_n\}$  of all other individuals. Suppose one individual

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<sup>19</sup>For sufficiently large initial contributions  $\hat{g}$ ,  $M$  may prefer the corner solution  $g = 0$ . In this case, she shares her preferences with more than half of the population and remains decisive in stage 2.

$i > m$  contributes in a way that  $g^{i*} < g^{m*}$  and  $g^{i*} \leq g^{M*}$  (note that in order to generate  $m \neq M$  there must be at least one such individual  $i$ ). The stage-2 optimality condition for  $i$  then implies

$$\Delta^i(\hat{g}_i, \hat{g}_{-i}, g^{M*}(\hat{g})) \leq \frac{1}{z}, \quad (2)$$

and holds with equality if  $M = i$ . We establish that for  $\beta > 0$ , this condition violates the optimality condition for  $i$ 's stage-1 investments. For contributions  $\hat{g}_i$  inducing  $g^{i*} < g^{m*}$ , agent  $i$ 's stage-1 first-order condition reads

$$\Delta^i(\hat{g}_i, \hat{g}_{-i}, g^{M*}) = 1 + \frac{1}{U_c^i(\cdot)} [U_c^i(\cdot) - U_G^i(\cdot)z] \frac{dg^{M*}}{d\hat{g}_i}. \quad (3)$$

The second term on the right-hand side reflects the effect of  $\hat{g}_i$  on the policy chosen by (and possibly, on the identity of) the median individual. Note that  $dg^{M*}/d\hat{g}_i < 0$  whenever  $\beta > 0$ : with normal preferences, raising  $i$ 's contribution lowers the preferred stage-2 policy of any individual, and thus, of the induced median voter.<sup>20</sup> In addition,  $dg^{M*}/d\hat{g}_i > -1$  because otherwise, a raise in  $\hat{g}_i$  would induce a lower public goods consumption for  $M$ , thus violating preference normality. From the stage-2 optimality condition (2), the term in square brackets in (3)  $U_c^i(\cdot) - U_G^i(\cdot)z$  is zero if  $M = i$ , yielding an immediate contradiction since (2) and (3) become incompatible. Otherwise, when  $i < M$  according to induced stage-2 preferences,  $dg^{M*}/d\hat{g}_i > -1$  implies  $\Delta^i(\cdot) > 1 - \frac{1}{U_c^i(\cdot)} [U_c^i(\cdot) - U_G^i(\cdot)z]$ . This condition simplifies to  $1 > z$  which is impossible. Taken together, the assumed behavior cannot be optimal for agent  $i$ , implying individual  $i > m$  will never contribute in a way as to switch stage-2 preferences with  $m$ .<sup>21</sup> To complete the argument, note that  $m$  will be the median voter if she does not contribute. Hence,  $m$  will not invest since she can always consume the same amount of public goods spending a smaller amount of private resources. Finally, since  $g^{j*} < g^{m*}$  for any  $j < m$ , none of these individuals will be a contributor which proves the lemma.  $\square$

<sup>20</sup>This is true because raising  $\hat{g}_i$  can alter the identity of the median voter in only one direction: an individual  $k$  with a smaller stage-2  $\Delta^k$  might become  $M$ . This indirect effect reinforces the direct effect of increasing  $\hat{g}_i$ .

<sup>21</sup>For  $\beta = 0$ , the last Section has shown that each individual  $i > m$  will invest in such a way that he shares his stage-2 preferences with the median-income individual. Again, Lemma 2 continues to apply because  $m$  remains pivotal in stage 2. See our discussion below.

**Lemma 3.** *The equilibrium characteristics as described in Lemma 2 are unique if  $\beta > 0$ .*

Proof: The proof of Lemma 2 immediately implies that as long as  $\hat{g}_m = 0$ , the natural median is the induced median individual in stage 2,  $m = M$ . Specifically, for any individual  $i > m$ , contributions with the property  $g^{i*}(\hat{g}) \leq g^{M*}(\hat{g})$  were shown to be incompatible with utility maximization. Suppose now there exists an additional equilibrium in which  $\hat{g}_m > 0$ . Two cases need to be distinguished. First, suppose that  $g^{m*} > g^{M*}$  in this assumed equilibrium. For this to apply, at least one individual  $i > m$  has contributed in a way that  $g^{i*} < g^{m*}$  (and  $g^{i*} \leq g^{M*}$ ). But invoking the arguments of Lemma 2, for any individual  $i > m$  contributions with this property cannot be the best response to contribution profile  $\hat{g}_{-i}$ . Second, suppose that  $g^{m*} < g^{M*}$  in the assumed equilibrium. Note that for any  $\hat{g}_{-m}$ , individual  $m$  can always generate stage-2 references  $g^{m*} \geq g^{M*}$  by not contributing in stage 1: for  $\hat{g}_m = 0$ ,  $m = M$  unless one individual  $i > m$  invests in a way that  $g^{i*} \leq g^{m*}$ . As a consequence, for  $\hat{g}_m = 0$ ,  $M$ 's stage-2 preferences in the assumed equilibrium would be smaller than those of individual  $m$ . However, contributing a positive amount so large that  $g^{m*} \leq g^{M*}$  cannot be  $m$ 's best response to any contribution profile  $\hat{g}_{-m}$  of other individuals. Analogously to the arguments in Lemma 2, for  $g^{m*} \leq g^{M*}$  to apply,  $m$ 's stage-2 references would have to satisfy  $\Delta^m(\hat{g}_m, \hat{g}_{-m}, g^{M*}(\hat{g})) \leq 1/z \leq 1/(1+\beta) \leq 1$ . But contribution  $\hat{g}_m$  then violates the stage-1 optimality condition (??) for  $i = m$  (at least) if  $\beta > 0$ , yielding a contradiction. Combining these results, there cannot exist equilibria where individual  $m$  contributes  $\hat{g}_m > 0$ ; as a consequence,  $M = m$  and the equilibrium characteristics given in Lemma 2 are unique.  $\square$

We have established that the natural median individual  $m$  will be pivotal for the political decision in equilibrium, and she will provide no private ex-ante contribution on her own. While the proof is quite intricate, there is an intuitive logic behind this result. No individual with larger natural preferences has an incentive to switch preference ranks with  $m$ : voluntarily moving ‘to the left of the median’ means that an agent contributed too much, which cannot be best response to the strategy profile of other individuals. But if no (high preference) individual wants to switch ranks,  $m$

has no incentive to contribute anything because she is the median voter anyway, which allows her to implement her preferred public goods consumption in the political process without spending a large amount of private resources.

We are now prepared to summarize the equilibrium outcomes.

**Proposition 3.** *Suppose individuals can make ex-ante contributions  $\hat{g}_i$  into public policies. In equilibrium, the ‘natural’ median-preference individual  $m$  becomes the median voter and chooses the equilibrium policy  $g^{m*}(\hat{g})$ . Moreover,*

- 1) *No individual  $j$  with natural preferences  $j \leq m$  contributes in stage 1. All these individuals consume the same amount of same amount of the public good,  $G_j^* = G_m^* = g^{m*}[1 + \beta(N - 1)] + \beta \sum_i \hat{g}_i$ .*
- 2) *Some highest-preference individuals  $i > m$  may make a private contribution in stage 1. Equilibrium contributions are rank-ordered, i.e.,  $\hat{g}_n \geq \hat{g}_{n-1} \geq \dots \geq \hat{g}_{m+1}$ , with strict inequality for positive contributions. Accordingly, the public goods consumption of individual  $i$  is  $G_i^* = G_j^* + \hat{g}_i(1 - \beta)$ , which yields  $G_n^* \geq G_{n-1}^* \geq \dots \geq G_{m+1}^*$  (with strict inequality whenever  $\beta < 1$  and  $\hat{g}_i > 0$  for one of two adjacent individuals).*

While most of these results have been established above before, the additional monotonicity properties are proved in the Appendix. In fact, monotonicity of contributions in type is very intuitive. Higher-preference individuals make larger private contributions, and therefore consume larger quantities of the public good in equilibrium.

It is instructive to compare the economic outcomes achieved in the hybrid regime with ex-ante contributions, to those of pure public system where no private contributions are allowed. This is done in

**Proposition 4.** *Consider the hybrid ex-ante system (superscript  $ea$ ). In comparison to a public system (superscript  $pp$ ), we have the following results:*

- 1) *public policy provision is characterized by  $g^{ea} \leq g^{pp}$ , with strict inequality if  $\beta > 0$  and if the set of contributors is non-empty.*

2) *public-goods consumption for any non-contributor  $j$  (including the median) is  $G_j^{ea} \geq G_j^{pp}$ , with strict inequality if  $\beta > 0$ , if the set of contributors is non-empty, and if consumption is strictly normal. Contributors consume even more of the public good whenever  $\beta < 1$ .*

Proof: To prove part 1), recall that individual  $m$  chooses public policy  $g$  in both regimes, and in a way that  $\Delta^m = 1/z$ . Suppose  $\beta > 0$  and let the set of contributors in the hybrid setting be non-empty. If public goods consumption is only weakly normal (as in a quasi-linear setting),  $m$ 's optimization leads  $G_m = gz + \beta \sum_{i \in C} \hat{g}_i$  to be constant across regimes. Accordingly,  $g^{ea} < g^{pp}$ . If consumption of both goods is strictly normal, the median voter's private consumption  $c_m$  in the hybrid regime must be strictly larger, which again implies a smaller equilibrium policy. Part 2) follows immediately: with strictly normal preferences and because  $c_m$  is larger in the dual-provision setting,  $G_m$  will also be larger in this regime.  $\square$

From a policy perspective, it is important to know which individuals support the hybrid over the pure public system, or vice versa. Our findings here are very strong.

**Proposition 5.** *All individuals unanimously prefer the hybrid ex-ante system over the public system. This preference is strict (a) in presence of externalities,  $\beta > 0$ , and (b) if the equilibrium set of private contributors is non-empty.*

Proof: Consider an individual  $j$  who does not contribute in the dual system. For any such individual, his public consumption is characterized by  $G_j^r = G_m^r$  in regime  $r \in \{ea, pp\}$ . By Proposition 3,  $G_j^{ea} \geq G_j^{pp}$ ,  $g_m^{ea} \leq g_m^{pp}$  (with strict inequality if  $\beta > 0$  and if the set of contributors is non-empty). Hence,  $c_j^{ea} \geq c_j^{pp}$  and individual  $j$  prefers the ex-ante system. Consider now an individual  $i$  who contributes  $\hat{g}_i > 0$ . If instead this agent had decided not to contribute, he would again receive a utility not smaller than in the public system: first, if he is the only contributor, a decision not to contribute would trigger identical equilibrium policies and consumption levels in either regime. Second, if there are other contributors,  $\hat{g}_i = 0$  would induce  $G_i^{ea} \geq G_i^{pp}$ , and  $g_m^{ea} \leq g_m^{pp}$  (with strict inequality if  $\beta > 0$ ). Hence,  $i$  would prefer the ex-ante system, and strictly

so if  $\beta > 0$ . By revealed preference, making a contribution must raise  $i$ 's utility even further. Hence, each equilibrium contributor  $i$  prefers the hybrid ex-ante system which completes the proof.  $\square$

The result of Proposition 4 is intriguing, and may have important policy implications. According to the Proposition, each agent prefers the flexible public-private system over public provision, and strictly so in the most plausible and relevant scenarios of positive spillovers and a non-empty set of private contributors. Hence, a policy reform from a pure public system to the ex-ante dual system is Pareto preferred, provided that individuals are allowed to expand their consumption of the public good before the policy decision is made. Our finding remarkably differs from all results in the existing literature on hybrid regimes. While this literature shows that some form of hybrid system is usually preferred by a simple majority, in reality a constitutional change to switch from a pure (public or private) to a hybrid system often demands the approval of some well-defined super-majority. Proposition 4 shows that a Pareto-improving institutional change is not necessarily infeasible.<sup>22</sup>

## 5.2 Ex-post Contributions: Policy Commitment

We now turn to a setting where the order of moves in a dual public-private system is reversed. To do so, suppose the policy  $g$  is determined in a first stage by democratic majority voting. In a subsequent second stage, each individual can boost the provision of the public good by making a private contribution  $\tilde{g}_i$ . As before, these ex-post contributions are made simultaneously and in a non-cooperative fashion. This setting has received most of the attention in the existing literature. In a federalism context,

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<sup>22</sup>Remember that most of the relevant literature exclusively focuses on settings with ex-post private contributions; see below. The only exception is the work by Alesina et al. (2005) who analyze basically the same setting (but with the restriction to quasi-linear preferences), but arrive at a different result. While according to Alesina et al. in general only a majority of voters prefers the dual ex-ante system, we show that the much stronger statement of Proposition 4 applies. Of course, Pareto optimality is not achieved in a setting with intra-regional heterogeneity as analyzed in Cremer and Palfrey's work. While all regional median voters (the agents in our setting) favor the ex ante system, a minority of low-preference individuals in high-preference districts may oppose it.

for example, it can be interpreted as imposing the strategic ‘leader’ role on the central government, and the ‘follower’ role on regions or lower-tier policy makers within a federation.

As has been emphasized in this previous literature, one cannot generally take the existence of a political equilibrium for granted if there are externalities, for the following reason. The set of second-stage contributors, as well as the size of their respective contributions, depend on the policy  $g$  that was implemented in the first stage. Single-peakedness of preferences at the policy stage is then elusive because the shape of individual utility profiles is affected by changes in the set of ex-post contributors. A change in policies may induce an individual to become a contributor or a non-contributor, respectively, altering not only his individual utility representation but also the best responses of other members of society.

An important finding of the present paper is to show that even though the single-peakedness requirement may not be satisfied, the result of the median voter theorem still applies in our setting. The only tax alternative which cannot be beaten in majority vote is the alternative proposed by the natural median individual,  $m$ . In addition, we will offer a precise characterization of this equilibrium, and provide a detailed comparison to the model with ex-ante contributions.

The central issue is the outcome of the policy choice stage. It is first established that each individual  $j < m$  with preferences smaller than the natural median will prefer the policy which is preferred by  $m$  over any larger policy level. We call this policy  $g_m^{ep}$ , where the abbreviation *ep* stands for ex post contributions. Second, we show that any individual  $i > m$  with preferences larger than the natural median, prefers the tax rate favored by  $m$  over any smaller tax rate. Combining these results allows to show that the median voter theorem must hold, the potential non-regularity of preferences notwithstanding. For subsequent reference, it is useful to define

- $g_i^T = g + \tilde{g}_i(g)$  as the total contribution of individual  $i$  towards the supply of public goods;
- $G^{-i} = G_i - g_i^T = g\beta(N-1) + \beta \sum_{j \neq i} \tilde{g}_j$  as the portion of  $i$ 's public goods consumption derived from contributions of all other individuals.

We now develop our main result in the following lemmas.

**Lemma 4:** Each individual  $j < m$  prefers  $g_m^{ep}$  over any other policy  $g > g_m^{ep}$ .

Proof: Suppose to the contrary that individual  $j$  strictly prefers some larger policy, say  $g_j^*$ , over  $g_m^{ep}$ . If this is true, the total contributions of all other individuals under policy  $g_j^*$  exceed those under policy  $g_m^{ep}$ , that is,  $G^{-j}(g_j^*) > G^{-j}(g_m^{ep})$ . Suppose not. Then,  $j$  could under policy  $g_m^{ep}$  always replicate his public-goods consumption under policy  $g_j^*$ , by choosing  $\tilde{g}_j(g_m^{ep}) > 0$  in a way that  $G_j(g_m^{ep}) = G_j(g_j^*)$ .<sup>23</sup> At the same time, his total public goods contribution  $g_j^T$  would be lower and his private consumption  $c_j$  would be higher under policy  $g_m^{ep}$ , yielding a contradiction. Hence, if  $j$ 's preferences are strict,  $G^{-j}(g_j^*) > G^{-j}(g_m^{ep})$ , and  $j$ 's total consumption of public goods under both policies regimes is characterized by  $G_j(g_j^*) \geq G_j(g_m^{ep})$  due to normality of preferences.

Consider now individual  $m$  who by definition prefers  $g_m^{ep}$  over  $g_j^*$ . Two cases need to be distinguished. First,  $G^{-m}(g_m^{ep}) > G^{-m}(g_j^*)$  may hold. If in addition  $G_m(g_m^{ep}) \geq G_m(g_j^*)$  (this needs not be true, see below),  $j$  can strictly prefer  $g_j^*$  and  $m$  strictly prefer  $g_m^{ep}$  only if the conditions  $G^{-j}(g_j^*) > G^{-j}(g_m^{ep})$  and  $G^{-m}(g_j^*) < G^{-m}(g_m^{ep})$  are simultaneously satisfied. Note that  $G_m(\cdot) = G_j(\cdot) + (1-\beta)[\tilde{g}_m(\cdot) - \tilde{g}_j(\cdot)] = G_j(\cdot) + (1-\beta)[(g_m^T(\cdot) - g_j^T(\cdot))]$ . Using this fact and the definition of  $G^{-i}(\cdot)$ , a necessary condition for the validity of these conditions is

$$g_m^T(g_j^*) - g_m^T(g_m^{ep}) > g_j^T(g_j^*) - g_j^T(g_m^{ep}).$$

A second set of conditions is  $G_m(g_m^{ep}) \geq G_m(g_j^*)$  and  $G_j(g_j^*) \geq G_j(g_m^{ep})$ . Again using the definition of  $G_m(\cdot)$ , the validity of these two inequalities requires  $g_m^T(g_j^*) - g_m^T(g_m^{ep}) \leq g_j^T(g_j^*) - g_j^T(g_m^{ep})$ , yielding an immediate contradiction.<sup>24</sup>

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<sup>23</sup>Note this is always feasible; the required stage-2 contribution (if positive) would be  $\tilde{g}_j = \tilde{g}_j(g_j^*) + (g_j^* - g_m^{ep}) - [G^{-j}(g_m^{ep}) - G^{-j}(g_j^*)]$ , and imply  $g_j^T(g_m^{ep}) < g_j^T(g_j^*)$ . Note also that  $\tilde{g}_j$  will in general not be  $j$ 's best response to policy  $g_m^{ep}$  and contributions  $G^{-j}(g_m^{ep})$  of other individuals. However,  $\tilde{g}_j$  defines  $j$ 's lower utility bound by revealed preference.

<sup>24</sup>This is true for any  $\beta < 1$ . For  $\beta = 1$  so that  $G_m(\cdot) = G_j(\cdot) \equiv G(\cdot)$ , satisfying conditions  $G_m(g_m^{ep}) \geq G_m(g_j^*)$  and  $G_j(g_j^*) \geq G_j(g_m^{ep})$  requires public goods consumption under both policies to be identical,  $G(g_j^*) = G(g_m^{ep})$ . To prefer  $g_j^*$  over  $g_m^{ep}$ ,  $j$ 's total contribution must then be characterized by  $g_j^T(g_j^*) < g_j^T(g_m^{ep})$  (note that as an implication,  $j$  makes a positive stage-2 contribution under policy  $g_m^{ep}$ ). But this is inconsistent: because of preference normality,  $j$  would reduce  $g_j^T(g_m^{ep})$  to a level where  $G_j(g_m^{ep}) < G_j(g_j^*)$ , yielding a contradiction.



Next, suppose again that  $G^{-m}(g_m^{ep}) > G^{-m}(g_j^*)$ , but now let  $G_m(g_m^{ep}) < G_m(g_j^*)$ . Notice that this case can occur only if policy  $g_j^*$  forces  $m$  to make an excessive total contribution from his point of view, implying a stage-2 corner solution  $\tilde{g}_m(g_j^*) = 0$ . But then, agent  $j < m$  must also be a non-contributor (by definition, it is impossible to have  $\Delta^j(\cdot) > \Delta^m(\cdot)$  at  $\tilde{g}_j = \tilde{g}_m = 0$ ) and as a consequence,  $g_m^T(g_j^*) = g_j^T(g_j^*)$ , which implies  $G^{-m}(g_j^*) = G^{-j}(g_j^*)$ . Note now that under the rank-order assumption,  $g_j^T(g) \leq g_m^T(g)$  for any  $g$ . Hence,  $G^{-j}(g_m^{ep}) \geq G^{-m}(g_m^{ep})$ . But since we consider the case  $G^{-m}(g_m^{ep}) > G^{-m}(g_j^*)$ , and since  $G^{-m}(g_j^*) = G^{-j}(g_j^*)$ , this immediately implies  $G^{-j}(g_j^*) \leq G^{-j}(g_m^{ep})$ , a contradiction to our previous result.

Next, suppose to the contrary that  $G_m^{-m}(g_j^*) \geq G_m^{-m}(g_m^{ep})$ . In this case,  $m$ 's preferences are consistent only if policy  $g_j^*$  forces her to make a contribution  $g_j^*$  that  $m$  perceives as excessive. Accordingly, her private contribution is  $\tilde{g}_m(g_j^*) = 0$ . But if  $m$  does not contribute under policy  $g_j^*$ , individual  $j < m$  will not make a private contribution either by Assumption 1, quasiconcavity, and normality of preferences. Hence, we have  $G_m(g_j^*) = G_j(g_j^*)$  and  $g_m^T(g_j^*) = g_j^T(g_j^*)$ . Consider now policy  $g_m^{ep}$  and suppose  $j$ 's total contribution satisfies  $g_j^T(g_m^{ep}) = g_m^T(g_m^{ep})$ , so that both agents consume the same amount  $G_m(g_m^{ep}) = G_j(g_m^{ep})$ . Since  $G_m^{-m}(g_j^*) \geq G_m^{-m}(g_m^{ep})$  by assumption, normality then (generically) implies  $G_m(g_m^{ep}) < G_m(g_j^*)$ . Also, since  $\Delta^j(\cdot) < \Delta^m(\cdot)$  for identical total contributions of  $j$  and  $m$  (Assumption 1),  $j$  must prefer  $g_m^{ep}$  over  $g_j^*$ , given this is  $m$ 's preference ranking.<sup>25</sup> Moreover, by revealed preference, choosing  $g_j^T(g_m^{ep}) \neq g_m^T(g_m^{ep})$  can only raise  $j$ 's utility under policy  $g_m^{ep}$ , reinforcing this preference. This yields a contradiction and completes the proof.  $\square$

**Lemma 5:** Any individual  $h > m$  prefers  $g_m^*$  over any policy  $g < g_m^*$ .

Proof: The proof mirrors the proof of Lemma 4, and is therefore omitted.

Combining the statements in these lemmas leads to the following important result.

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<sup>25</sup>This is clearly true if both  $j$  and  $m$  have identical incomes. Suppose not and notice that total contributions under policy  $g_m^{ep}$  must be characterized by  $g_m^T(\cdot) \geq g_j^T(\cdot)$  under Assumption 1. Since their individual contributions under the alternative policy  $g_j^*$  are identical, switching from  $g_m^{ep}$  to  $g_j^*$  causes  $j$ 's private consumption to decline more. Invoking the single-crossing assumption then establishes the result.

**Proposition 6.** *In the dual regime with ex-post contributions, the preferred policy of the median individual  $m$ ,  $g_m^{ep}$ , beats all other policy alternatives  $g$  in pairwise comparison.*

Proof: By Lemma 3,  $m$  and all individuals  $j < m$  with smaller preferences for public goods than  $m$  prefer  $g_m^{ep}$  over any some larger policy  $g > g_m^{ep}$ . By Lemma 4,  $m$  and all individuals  $h > m$  prefer  $g_m^{ep}$  over any smaller policy  $g < g_m^{ep}$ . Combined, no other policy can beat  $g_m^{ep}$  under majority vote, which proves the result.  $\square$

The next step is to characterize the stage-2 contribution levels of all individuals. We will see that neither the median individual, nor any individual with smaller preferences, will make a private contribution. While this sounds intuitive, a rigorous proof is in fact needed. The core of the argument is to show that for the range of policies for which the median voter subsequently becomes a contributor,  $m$ 's first-period utility is strictly increasing in the policy level  $g$ . For this reason,  $m$ 's optimal stage-1 choice cannot be such that she is a contributor in the stage-2 Nash equilibrium.

**Proposition 7.** *Neither the median individual  $m$ , nor any smaller preference individual  $j < m$  will contribute in stage 2.*

Proof: see the Appendix.

Proposition 7 is perhaps best understood by the following heuristic argument. Consider a policy level  $g_m$  sufficiently low that  $m$  would contribute in the stage-2 continuation equilibrium. The median voter knows that raising  $g_m$  further forces some low-preference individuals (who are non-contributors in stage 2) to contribute additional resources for the supply of the public good. As a consequence, raising  $g_m$  in this range transfers income from low-preference individuals to high-preference individuals. This allows the median voter  $m$  and all other contributors to reduce their private contributions not only by the amount of their increased policy payment but also by the amount of this transfer, hence lowering their total contributions while leaving their public goods consumption unaffected. As a result, these individuals can raise their private consumption, and they increase their utilities because naturally, non-contributors cannot reduce their private supplements. It is then not surprising that the median voter will want to extend  $g$  at least to the level where she ceases to contribute subsequently.

We have shown that the median voter  $m$  and all lower-preference individuals do not make a private contribution. What about higher-preference individuals? Similar to the model with private ex-ante contributions, a subset of highest-preference individuals may want to enhance their public goods consumption. The Proposition below compares the ex-post regime and the ex-ante regime with respect to private contributions and equilibrium policies  $g_m^{ep}$  and  $g_m^{ea}$ , respectively. To simplify the exposition, we focus on a situation where the set of contributors in both scenarios is non-empty.<sup>26</sup>

**Proposition 8.** *With ex-post contributions, the equilibrium policy is strictly smaller than in the model with ex-ante contributions,  $g_m^{ep} < g_m^{ea}$ . Moreover,*

- a) *each non-contributor (including all individuals  $j \leq m$ ) prefers the ex-post regime over the ex-ante regime (and over the public regime); this is true even though their public goods consumption in the former regime may be lower.*
- b) *Private contributions in the ex-post regime are strictly larger, and the set of contributors may be larger. Contributors strictly prefer the ex-ante over the ex-post regime, while the comparison between ex post regime and public regime is ambiguous.*

Proof: see the Appendix.

Equilibrium policies in all three regimes are ranked as  $g_m^{ep} \leq g_m^{ea} \leq g^{pp}$ , with strict inequalities unless no agent privately contributes. The intuitive reason for this result is again due to a commitment effect: by adopting a smaller policy in the ex-post regime, the median voter can induce high-preference individuals to raise their private contributions. Clearly, all low-preference agents (including all agents  $j \leq m$ ) welcome this commitment device.<sup>27</sup> While this means that a majority of individuals prefers

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<sup>26</sup>If this set is empty in the ex-post regime, it will be empty in the ex-ante regime as well. Without private contributors, equilibrium policies can be identical, and private goods consumption can be larger in the ex-ante setting. See the example below.

<sup>27</sup>This is due to revealed preferences: the median voter could always choose the same policy than in the ex-ante setting, and raise his utility (and the utilities of all lower-preference agents) because private contributions go up in the ex-post setting. Intuitively, while these contributions are best responses to a given policy in the ex-post setting, they are smaller than the best responses to the same policy level in the ex-ante setting.

the ex-post setting over both alternative regimes, a minority of high-preference agents dislikes the ex-post setting, because low-preference individuals now provide less input into the public good than before. In fact, for these individuals, even the pure public setting may be preferred to the ex-post regime. With regard to the total public goods consumption in each institutional setting, results are generally ambiguous. One can show that, for a pure public good, public goods consumption coincides in the ex-post regime and under public provision- and exceeds consumption in the ex-ante regime - when preferences do not exhibit income effects; otherwise, public good consumption in the ex-ante regime can be more substantial.<sup>28</sup>

## 6 ‘Almost’ Private and Pure Public Goods

At this point, it is useful to briefly examine the polar cases of a pure private, and a pure public good. Let us start investigating an ‘almost’ private good  $G$  for which externalities disappear in the limit,  $\beta \rightarrow 0$ .<sup>29</sup> In both dual-provision and the pure public regime, the equilibrium policy is then the same. The reason for this result is simple. First, and as we have seen above, the median voter herself will never make a private contribution. Second, because externalities ‘almost’ disappear, there exists no strategic voting motive for the median individual or anybody else in society. Regardless of the regime, the median voter  $m$  will then select her policy in a way that  $\Delta^m(c_m, G_m) = 1$ . Accordingly, individuals  $j \leq m$  make no private contribution in the hybrid regimes, while each individual  $i > m$  contributes to satisfy  $\Delta^i(c_i, G_i) = 1$ . Regardless of the regime under consideration, agents with preferences lower than the median voter are forced into an excessive consumption of the public good. In fact, all these individuals suffer from the public provision of  $G$ , as compared to the alternative of a ‘market

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<sup>28</sup>Suppose  $\beta = 1$ . In absence of income effects, and as long as some individuals make a private contribution, total consumption is higher in the ex-post setting than in the ex-ante setting: the public goods consumption of the highest-preference individual  $h$  is characterized by  $\Delta^h(\cdot) = 1$  in the ex-post setting, but by  $\Delta^h(\cdot) > 1$  in the ex-ante setting. Conversely, in presence of income effects, public goods consumption can be larger in the ex-ante regime. This regime features a larger equilibrium policy, thus boosting the marginal utilities of private contributors.

<sup>29</sup>For  $\beta = 0$ , we have shown above that multiple equilibria exist, making comparisons useless.

system' in which each individual would be allowed to freely decide on its consumption level. Conversely, and as we have already established for the case  $\beta = 0$  in Section 3 above, all higher-preference individuals including  $m$  are indifferent between the two hybrid systems and the market system, and strictly prefer them over the public regime: voluntary contributions allow those individuals to expand their consumption beyond the publicly provided level, and to achieve the same utility as in a private setting.

In contrast, the choice of the dual institution matters in presence of externalities. To highlight the economic differences between the ex-ante and the ex-post regime, consider the polar case of a pure public good,  $\beta = 1$ . Standard results (Bergstrom et al., 1986) suggest that in absence of income effects, only the highest-preference individual will privately contribute. As we show now, his private contributions can be very large in an ex-post regime, while under otherwise identical circumstances they can be even zero in the ex-ante regime.

Consider quasi-linear preferences,  $U^i = c_i + \alpha_i H(G_i)$ , where  $\alpha_i$  is a preferences parameter and  $H(\cdot)$  some increasing and concave function. Under pure public provision, the equilibrium policy  $g_m^{pp} > 0$  yields a public goods consumption  $G^{pp}$  implicitly given by  $H'(G^{pp}) = 1/N\alpha_m$ , where  $N$  represents the number of agents and  $\alpha_m$  the median individual's taste parameter. Now suppose that  $N\alpha_m$  is 'sufficiently close' to the preference parameter of the highest-preference agent,  $\alpha_h$ .<sup>30</sup>

In the ex-post regime, the equilibrium policy  $g^{ep}$  is then zero. This is because in absence of public provision, only  $h$  will subsequently contribute in the quasi-linear setting, and he will do so in a way that  $H'(G^{ep}) = 1/\alpha_h$ . This generates a public goods provision similar (or even identical) to  $G^{pp}$  at no cost to any other agent: the capability to commit to a policy makes it optimal for  $m$  (and a majority of citizens) to free ride on  $h$ 's contributions. For comparison, consider now the ex-ante setting. Here,  $h$  is aware that his private contributions will crowd out public provision. When  $N\alpha_m$  is again close to  $\alpha_h$ , his optimal reaction is to forego any private contribution, inducing a public policy  $g_m^{pp}$  and associated public goods consumption  $G^{pp}$  in the second stage. For obvious reasons, a large majority of individuals is made worse off compared to

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<sup>30</sup>For example, let  $N = 3$ ,  $\alpha_1 = 1$ ,  $\alpha_2 = \alpha_m = 2$ , and  $\alpha_3 = \alpha_h = 6$ .

the ex-ante system, and all those individuals achieve the same utility than under pure public provision.<sup>31</sup> Overall, these arguments show that while public good consumption levels may be very similar or even identical in both dual regimes, those regimes can display extreme differences regarding the amount of private contributions, equilibrium policies, and the overall utility distribution when externalities are significant.

## 7 Endogenous Timing of Contributions

How do these previous results fit into a general framework, where both ex-ante and ex-post private contributions are feasible, and can be freely chosen by each individual? At first glance, one may think that the outcome must coincide with the results for the ex-post system. Since public decisions still precede private contributions, adding an ex-ante contributions stage seems strategically irrelevant at first. Perhaps surprisingly, though, we will show that this intuition is incorrect. In fact, endogenizing the timing of private contribution will often strictly raise the wellbeing of a majority, or even benefit all members in society.

A full analysis of the general scenario is beyond the scope of our paper. However, central findings can already be obtained in a restricted setting. Specifically, we provide a characterization for the case of quasilinear preferences, and the special case where the number of agents is  $N = 3$ . Consider preferences of the form  $U^i = c_i + \alpha_i \ln G_i$  with  $\alpha_i$  increasing in  $i$ . Then,  $c_i = y_i - g - \tilde{g}_i - \hat{g}_i$  is the private consumption of individual  $i$  with income  $y_i$ , who contributes  $g$  to the public good in the political process, makes a voluntary contribution of  $\hat{g}_i$  in an ex-ante stage, and another ex-post contribution  $\tilde{g}_i$  after the policy  $g$  has been implemented. Accordingly,  $G_i = gz + (\tilde{g}_i + \hat{g}_i) + \beta \sum_{j \neq i} (\tilde{g}_j + \hat{g}_j)$  is  $i$ 's consumption level of public services.

To start the exploration, notice first that by extension of our previous results, only

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<sup>31</sup>For completeness, suppose  $N\alpha_m$  and  $\alpha_h$  are not of similar size. Then, results in both dual regimes coincide. For  $N\alpha_m$  sufficiently smaller than  $\alpha_h$ , the equilibrium policy is zero and  $h$ 's equilibrium contribution yields a public goods consumption given by  $H'(G) = 1/\alpha_H$ , which is larger than in the public provision regime. Conversely, for  $N\alpha_m$  sufficiently larger than  $\alpha_h$ ,  $h$  does not contribute and  $m$  implements a policy  $g^{pp}$ .

individual  $i = 3$  may in equilibrium provide voluntary contributions  $\tilde{g}_i$  and  $\hat{g}_i$ , respectively. To see this, note that after the policy stage, only the (now endogenous) individual with highest preferences will possibly contribute in stage 3. This individual is individual  $i = 3$  unless this agent made a rank-reversing contribution in stage 1. But for the same reasons as in the partial setting with ex-ante contributions, doing so can never be optimal: agent 3 can never benefit from an initial stage-1 contribution for which his stage-2 preferences for public goods are less than those of the natural median  $i = 2$ .<sup>32</sup>

Solving by backwards induction, we first study individual 3's ex-post contribution in stage 3, for given first-period contribution and given equilibrium policy  $g$ . Specifically, we have

$$\tilde{g}_3^* = \arg \max_{\tilde{g}_3} U^3(\hat{g}, g, \tilde{g}_3) = y_3 - g - \hat{g}_3 - \tilde{g}_3 + \alpha_3 \ln(gz + \tilde{g}_3 + \hat{g}_3) = \max\{\alpha_3 - gz - \hat{g}_3, 0\}. \quad (4)$$

As expected, the optimal ex-post contribution (if positive) decreases in both  $g$  and  $\hat{g}_3$ . For subsequent reference, (??) reveals that from the perspective of agent 3, the timing of private contributions is irrelevant as long as his stage-3 choice is interior (which in turn depends on  $g$ ).

Consider now the policy decision at stage 2. For given first-period contribution  $\hat{g}_3$ , the median individual 2 decides on  $g$ . Her goal is to maximize

$$U^2(g; \hat{g}_3) = y_2 - g + \alpha_2 \ln[gz + \beta(\hat{g}_3 + \tilde{g}_3^*(g, \hat{g}_3))]. \quad (5)$$

Taking individual 3's response into account, the median voter's utility function is piecewise defined. We first consider a range I where<sup>33</sup>

$$g \leq \bar{g}(\hat{g}_3) \equiv (\alpha_3 - \hat{g}_3)/z. \quad (6)$$

For policies weakly smaller than the threshold  $\bar{g}(\cdot)$ ,  $\tilde{g}_3^*$  is positive and the first-order

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<sup>32</sup>In absence of a rank-reversing contribution decision, then, our earlier findings immediately apply: only individuals with preferences larger than the median voter may possibly contribute in stage 1. For more on this, see the subsequent discussion.

<sup>33</sup>Note that  $\bar{g}(\hat{g}_3)$  is always non-negative because individual 3 will never invest more than  $\alpha_3$ , no matter how much the other individuals contribute.

derivative of (??) reads

$$-1 + \frac{\alpha_2}{gz + \beta\tilde{g}_3 + \beta\hat{g}_3} \left[ z + \beta \frac{d\tilde{g}_3^*}{dg} \right]. \quad (7)$$

Since  $d\tilde{g}_3^*/dg = -z$  ensures concavity of (??), an interior solution is obtained when (??) assumes a positive value at  $g = 0$ , and a negative value at  $\bar{g}$ . For an interior solution, the (local) policy optimum then is (using  $\hat{g}_3 + \tilde{g}_3 = \alpha_3 - gz$  from (??)),

$$g^+ = \alpha_2 - \frac{\beta}{z(1-\beta)}\alpha_3. \quad (8)$$

Of course, this policy (if positive) is exactly the equilibrium policy in the ex-post system. For the range of policies  $g$  where  $\tilde{g}_3$  is interior, the median voter's preferred policy is independent of  $\hat{g}_3$ , for obvious reasons: agent 2 knows that agent 3's total contributions  $\hat{g}_3 + \tilde{g}_3$  remain the same no matter how large his initial contribution is. Accounting for possible boundary solutions, the local optimum in range I yields

$$g_I^* = \min\{\max\{0, g^+\}, \bar{g}(\hat{g}_3)\}. \quad (9)$$

Next, consider range II which comprises the complementary interval of policies,  $g(\hat{g}_3) > \bar{g}(\hat{g}_3)$ . Now,  $d\tilde{g}_3^*/dg = 0$  and the first-order condition of (??) reads

$$-1 + \frac{\alpha_2 z}{gz + \beta\hat{g}_3} \leq 0. \quad (10)$$

An interior solution yields,

$$g^{++}(\hat{g}_3) = \alpha_2 - \frac{\beta}{z}\hat{g}_3, \quad (11)$$

so that the local optimum in range II is given as

$$g_{II}^* = \max\{g^{++}(\hat{g}_3), \bar{g}(\hat{g}_3), 0\}. \quad (12)$$

For future reference, note that  $g^{++} \geq g^+$  always holds (with strict inequality for  $g^{++} > 0$ ) because again,  $\hat{g}_3 \leq \alpha_3$  must be satisfied.<sup>34</sup> Collecting, we obtain the following preliminary results. Consider a given  $\hat{g}_3$ , as chosen by individual 3 in stage 1. Then, the

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<sup>34</sup>Note that agent 3 will never invest more than  $\alpha_3$ : this is not only his maximum contribution in any private-contributions game, but investing more would also reduce the continuation policy which cannot be in 3's interest.



median voter's local policy optima are  $g_I^*$  for policy choices from the interval  $g \leq \bar{g} = (\alpha_3 - \hat{g}_3)/z$ , and  $g_{II}^*$  for the complementary interval  $g > \bar{g}$  in which no additional ex-post contributions are made.

To identify global optima, suppose first agent 3's ex-ante contribution is at its maximum level,  $\hat{g}_3 = \alpha_3$ . For this initial contribution, the threshold investment  $\bar{g}(\cdot)$  is zero. Accordingly, range I collapses into the point  $g = 0$  and by continuity of  $U^2(\cdot)$ , the median voter adopts a policy  $g_{II}^* = \max\{0, g^{++}\}$  in stage 2. Two cases are to be distinguished. First, imagine that  $g^{++}(\alpha_3) \leq 0$ , which arises iff

$$z\alpha_2 \leq \beta\alpha_3. \quad (C1)$$

We show that if (C1) applies,  $\bar{g}(\hat{g}_3) \geq g^{++}(\hat{g}_3)$  for *any arbitrary*  $\hat{g}_3 \leq \alpha_3$ . We also show that as a consequence, the median voter's global optimum is  $g_I^*$ , and equilibrium results coincide with those in the ex-post contributions regime. To establish the first statement, notice that  $-d\bar{g}/d\hat{g}_3 \geq -dg^{++}/d\hat{g}_3$ . Hence,  $\bar{g}(\alpha_3) = 0 \geq g^{++}(\alpha_3)$  immediately implies  $\bar{0} < g(\hat{g}_3) \geq g^{++}(\hat{g}_3)$  for any  $\hat{g}_3 < \alpha_3$ . Range II thus features the boundary solution  $g_{II}^* = \bar{g}(\cdot)$ , and continuity of  $U^2(\cdot)$  in  $g$  ensures that  $g_I^*$  (which is either interior or zero) must be the equilibrium policy as chosen in majority voting. By definition of range I, agent 3 will then make an ex-post contribution  $\tilde{g}_3$  for which his total voluntary contributions equal those in the ex-post governance system.

Conversely, consider a situation in which the preferences of agents 2 and 3 are sufficiently close to each other that (C1) does not apply. Then,  $0 < g^{++}(\alpha_3) > \bar{g}(\alpha_3)$  and  $g^{++}(\hat{g}_3) > \bar{g}(\hat{g}_3)$  is satisfied for some non-empty set of contributions  $\hat{g}_3 < \alpha_3$ . We claim that agent 3 will in this situation choose some positive ex-ante investment level  $\hat{g}_3^*$  which is followed by an equilibrium policy  $g_{II}^* = g^{++}$ , and zero ex-post contributions. To verify this, start again from an initial contribution  $\hat{g}_3 = \alpha_3$  for which necessarily,  $\bar{g} = 0$  and  $g_3^{++} > 0$ . Note that for this contribution level, the stage-2 policy choice of agent 2 is  $g_{II}^* = g^{++}$ . Reducing  $\hat{g}_3$  below  $\alpha_3$ , the policy  $g^{++}$  increases in response. At the same time, reducing  $\hat{g}_3$  increases  $\bar{g}$  which (provided  $g^+$  is positive), at some point reaches  $g^+$ .<sup>35</sup> Again, the local boundary solution  $g_I^* = \bar{g}$  prevails in range I and by continuity of  $U^2(\cdot)$ ,  $g^{++}$  ( $> g^+$ ) will be the globally optimal policy reaction. Lowering

<sup>35</sup>Of course, this contribution range does not exist if  $g^+ = 0$ . Also, notice that if  $g^+$  is positive,

$\hat{g}_3$  further, the median voter utility function now displays the two interior local optima  $g_I^* = g^+$ , and  $g_{II}^* = g^{++}$ . At least for contribution levels where  $\bar{g}$  remains ‘sufficiently close’ to  $g^+$ ,  $g^{++}$  remains globally optimal.<sup>36</sup> But for some small  $\hat{g}_3$  where  $\bar{g}(\hat{g}_3)$  approaches  $g^{++}$ ,  $g^+$  becomes the global optimum. Denote the largest contribution level that triggers a switch in optima from  $g^{++}$  to  $g^+$  as  $\hat{g}_3^*$ . At  $\hat{g}_3 = \hat{g}_3^*$ , the subsequent equilibrium policy  $g$  exhibits a discontinuous downwards jump, necessarily reducing utility for agent 3. Recalling that for any  $\hat{g}_3 < \hat{g}_3^*$ , agent 3’s total contributions and his utility level (which is identical to the ex-post setting) remain unchanged, we can conclude that  $\hat{g}_3^*$  constitutes a lower bound on agent 3’s equilibrium contributions. The stage-2 equilibrium policy is thus  $g^{++}$ , and agent 3 does not provide any ex-post contribution. Finally, notice that by revealed preference agent 3 must be strictly better off than in the system with ex-post contributions; this is because he always has the option of investing nothing in the first period, thereby replicating his utility in the ex-post system.

The following figure illustrates these findings.

— Figure about here —

Our previous discussion allows us to state the following result.

**Proposition 9.** *Consider quasilinear preferences and  $N = 3$ . If  $z\alpha_2 \leq \beta\alpha_3$ , total voluntary contributions  $\hat{g}_3 + \tilde{g}_3$ , equilibrium policies, and overall economic outcome in the general system are identical to those in a system with only ex-post contributions. Conversely, if  $z\alpha_2 > \beta\alpha_3$ , agent 3’s ex-ante contribution  $\hat{g}_3$  is positive and larger than in the ex-post setting, but his ex-post contribution is zero. Individual  $m = 2$  is the median voter and implements a policy  $g_m^{gs}$  characterized by  $g_m^{ea} > g_m^{gs} > g_m^{ep}$ . Moreover,*

- 1) *Agent 2 and Agent 3 prefer the general system over the ex-post system (Agent 1’s*

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$\bar{g}$  can ‘reach’  $g^+$  only if  $g^+ < \bar{g}(\hat{g}_3 = 0)$ . Even if one of these properties are not satisfied, all of our subsequent arguments remain valid.

<sup>36</sup>This is because at a contribution level where  $\bar{g}(\hat{g}_3) = g_I^*$ , and since  $g^{++} > g_I^*$ ,  $U_2(\cdot)$  must necessarily be increasing at  $g = g_I^*$ .

*preference is ambiguous).*

- 2) *Agents 1 and 2 prefer the general system over the ex-ante system, and over the pure public system. Agent 3 is worse off compared to the ex-ante system, while his preference ranking with respect to the pure public system is ambiguous.*

Proof: Most of these results have been established before. In what follows, assume that (C1) does not apply. To show that agent 3 prefers the general system over the ex-post system, note that he could always choose  $\hat{g}_3 = 0$ , thus replicating results from the latter regime. When choosing  $\hat{g}_3 > 0$ , his utility must be higher by revealed preference. Next, the equilibrium policy  $g_m^{gs}$  in the general setting must satisfy  $g_m^{gs} > g_m^{ep}$ . If not, agent 3 would clearly prefer the ex-post system, because his private contribution  $\tilde{g}_3^{ep}$  is positive and a best response to  $g_m^{ep}$  in this regime. To show that agent 2 prefers the general system, note that she could always implement policy  $g_m^{ep}$  in stage 2. Under this policy, agent 3's total private contributions  $\tilde{g}_3 + \hat{g}_3$ , by (3) exceed those in the ex-post system. Hence, agent 2 must prefer the general system by revealed preference.<sup>37</sup>

Finally, let us show that agents 1 and 2 prefer the general system over the ex ante system, while agent 3 has reverse preferences. This ranking is immediate for agent 2, given that the ex-post system dominates the ex-ante system according to her preferences (refer Proposition 7). To see that agent 1 also prefers the general system, note that in the general system and in the ex-ante system, equilibrium policies are best responses to  $\hat{g}_3$ . But since  $\hat{g}_3^{gs} > \hat{g}_3^{ep} > \hat{g}_3^{ea}$ , we must have  $g_m^{ea} < g_m^{gs}$  which is preferred by agent 1 who has smaller policy preferences than the median agent. Conversely, agent 3 must prefer the ex ante system. While the actual equilibrium policy is best response to  $\hat{g}_3$  in both regimes, the general system requires agent 3 to operate under an additional constraint in stage 1, which leads to larger than optimal initial contributions from his point of view.  $\square$

Our results show that, for a wide range of parameter constellations, ex-post system and general system are not economically identical. In fact, the high-preference agent

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<sup>37</sup>For agent 1, though, the comparison is ambiguous: while agent 1 appreciates the larger private contributions made by agent 3, he dislikes the fact that the implemented policy may be larger than the one in the ex-post system.

may strictly prefer the general system for the following reason. By exerting a sufficiently large contribution ex ante, this agent generates a commitment not to make any additional private investment ex post. In contrast to the ex-post system, this deprives the median voter of her incentive to implement an artificially low policy, to lure the high-preference agent into making additional ex-post contributions. A simple argument also shows that if agent 3 takes advantage of this commitment device, the equilibrium policy  $g_m^{gs}$  in the general setting is always larger than in the setting with ex-post contributions. This follows from revealed preference: the high-preference agent can always replicate the outcome of the ex-post system, by providing no contribution in the first stage. Hence, investing ex ante (but not ex post) can be optimal only if the median voter's implemented policy exceeds the level in the ex-post system.

Importantly, not only the high-preference agent but also the median voter prefers the general system. The reason again relies on revealed preference: irrespective of agent 3's contribution in the first stage, the median voter cannot be made worse off compared to the ex-post system. After all, she can always replicate policy  $g_m^{ep}$ , inducing agent 3 to make a total private contribution at least as large as in the ex-post system. But this means that whenever agent 2 chooses another policy, it must make her better off. Overall, the general system is thus characterized by a larger equilibrium policy (to make agent 3 better off), and by larger private contributions (to make median agent 2 better off). Quite strikingly, the possibility to contribute ex ante, combined with the median voter's ability to punish initial contributions that she perceives as too small, allows (at least) a majority of the population to raise their utilities.<sup>38</sup> In particular, private contributions do not trigger the standard 'crowding out effect'; rather, a 'crowding in' arises in that initial private contributions boost public policies.

A comparison to the ex-ante system is simple but interesting. Clearly, private contributions in the general setting will again be larger. But this means that the equilibrium policy in the endogenous setting must be smaller, recalling that policy choice in both general and ex-ante setting are best responses to the stage-1 contributions. The welfare

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<sup>38</sup>Agent 1 may or may not prefer the general system: while he appreciates the larger contribution by agent 3 in the latter system, he dislikes the higher policy level.

comparison is now immediate. While all non-contributors prefer the general system over the ex-ante regime, the high-preference agent has the opposite ranking. From agent 3's point of view, his contribution is excessive and only made as a commitment device not to contribute ex post. Conversely, all other agents welcome this overinvestment.

Overall, our results suggest that equilibrium outcomes in the general system originate from a complex strategic interaction of both relevant players, the median voter and the high-preference individual. For the high-preference individual, allowing additional ex-ante contributions reduces the overall hold-up power of the majority, and makes him better off. Conversely, for the median voter, the general setting represents the best of all worlds: private contributions are large, while her equilibrium policy does not need to incorporate a downwards bias for incentive reasons.

Finally, our findings allow us to make a strong policy statement. Consider a scenario where citizens can choose the institutional structure by majority vote. Among all the systems that we analyzed, the regime with endogenous timing emerges as the Condorcet winner: it is majority preferred over the pure public system (at least by agents 1 and 2), over the ex-ante system (by agents 1 and 2), and over the ex-post system (at least by agents 2 and 3).

## 8 Switching from a Private Provision Regime

Our previous analysis explored a transition for pure public provision to one of various dual public-private systems. In many situations of empirical importance, though, the status quo institution is not one with public provision, but one where individuals privately provide those services. For example, large parts of the health care system are privately organized in many countries, some municipalities do not finance public transportation, and there are many areas of decentralized policymaking within federations.

When analyzing a market(or 'purely private') system, we should first note that the equilibrium in private contributions is identical to the continuation equilibrium in our

previous ex-post setting, for a stage-1 policy  $g = 0$ . In this purely private system, some subset of individuals with highest preferences invests. Their contributions are given by the system of first-order conditions  $\Delta^i(c_i, G_i) \leq 1$ , which hold with equality for any contributor. When externalities are sufficiently large, all lower-preference individuals then completely free ride on this private supply of public goods. As this argument suggests, non-contributing individuals may not support a switch from a private-provision regime to any system involving public provision. On the other hand, however, for some agents public provision is a device to alleviate or even mitigate the underprovision of public services that characterizes a private system.

The following Proposition addresses these questions.

**Proposition 10.** *Consider the transition from a private system to a dual public-private regime. Then,*

- 1) *regardless of  $\beta$ , adopting the dual ex-post regime is preferred by a majority of the population (strictly so if  $m$  chooses a positive policy), including any  $i \geq m$ . Conversely, a subset of lower-preference individuals may oppose.*
- 2) *adopting an ex-ante system may be opposed by a majority of individuals, including all low-preference individuals  $j \leq m$ .*

Proof: To establish part 1), notice that for a policy  $g = 0$ , the outcome is identical to the outcome in the private system. If the pivotal median voter  $m$  implements  $g^{ep} > 0$ , she must be better off by revealed preference. In addition, every individual  $i > m$  prefers  $g^{ep}$  over any smaller policy (Lemma 5), which validates the result. Finally, notice that for  $\beta \rightarrow 0$  where  $g^{ep} = g^{pp}$ , each individual  $j < m$  (and thus, a strong minority of citizens) opposes the regime change from private to ex-post system. To establish part 2), reconsider the quasilinear example analyzed in Section 6. If  $G$  is a pure public good, standard results suggest that only the highest-preference  $h$  individual will contribute to its provision in a decentralized setting, while all other individuals free ride. In the ex-ante system, the highest-preference agent  $h$  may not privately contribute, but rely on the public provision of  $G$  instead. As we have shown,  $h$  will always adopt this strategy if  $\alpha_m N$  and  $\alpha_h$  are close to each other, and  $m$  in response

implements a public provision level almost identical to the private provision setting. Both regimes are then characterized by a similar level of public goods consumption; however, the funding is provided entirely by  $h$  in the private regime, and uniformly by all society members in the ex-ante regime. Clearly, this makes everybody (other than  $h$ ) worse off in the ex-ante regime.  $\square$

When private provision represents the status quo, adopting a dual system does not find unanimous consent. A possibly surprising feature here is that it is the low-preference (or poor) individuals who often prefer a private system over one which ensures public provision. This finding is basically a consequence of two related arguments. The median voter's policy decision forces each individual with smaller preferences to pay more towards public goods consumption than privately desired.<sup>39</sup> In addition, and as has already been said above, the pure private system allows individuals to free ride on the contributions of high-preference individuals, in the extreme without providing any private contribution at all.<sup>40</sup> Both arguments push preferences of low-preference individuals in the same direction: their loss on private consumption in a dual system can be so severe that even a larger public goods consumption provides no sufficient compensation.

Remarkably, we find that the ex-post system - and by extension of our previous results the general system with endogenous timing - unambiguously dominates the private market system for a majority of agents. The preferences of these agents are strict (whenever  $g^{ep} > 0$ ) for any arbitrary degree of externalities.<sup>41</sup> The reason is simple,

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<sup>39</sup>A similar point is made in Cremer and Palfrey (2000).

<sup>40</sup>This argument essentially extends to a setting with income taxes: even though low-preference individuals may pay less taxes (if they are also the low-income individuals), their contribution is still positive.

<sup>41</sup>We have shown above that for  $\beta = 0$ , a majority of agents is indifferent. Slightly different results have been established in the literature for this case. In Cremer and Palfrey (2000), voters within districts are heterogenous and by assumption, the federal policy decision is made by the federal median individual. Then, a majority of voters in regions with median preferences below those of the federal median agent suffer from a transition to the ex-post system, because they would prefer a smaller provision level. Conversely, all other individuals and thus a majority of the overall population strictly welcome the transition. Epple and Romano (1996) also show that the majority of voters strictly prefers the dual system; all these voters benefit from the redistributive aspects of income

and relies on revealed preferences. If the median voter chooses a policy  $g = 0$ , she simply replicates the outcome in the private setting. Conversely, if she chooses  $g > 0$ , and since all individuals  $i > m$  prefer even larger policies, all those individuals must be better off.

In contrast, the ex-ante system is not necessarily preferred by a majority. In fact, since private contributions are made in the first stage prior to the policy decision, high-preference individuals have an incentive to reduce their own voluntary contributions, thus urging the median voter into a larger collective provision. This effect can lead a large majority to favor the private system over the ex-ante setting. Interestingly, there is a stark contrast to the case where public provision represents the status quo regime, where we found that adopting the ex-ante system is even preferred with unanimity.

Overall, our findings show that irrespective of the size of externalities, moving from a purely private regime to a dual system of provision is always majority preferred, as long as the system admits private ex-post contributions. This outcome is especially encouraging because, on the other hand, moving from a private system to a purely public system will often be rejected by a majority of the population. Adopting a dual institution thus relaxes political feasibility constraints, and it can also serve as a response to distributional concerns in society.

## 9 Conclusions

This paper offers an analysis of political decisionmaking in a framework where individuals can privately enhance their consumption of an impure public good, that is democratically provided by political choice. We explore different scenarios in which individuals can make their private provisions in a non-cooperative fashion, before and/or after the political outcome is decided by majority vote. Notwithstanding the relative complexity of the model a variety of strong and interesting results can be established. First and foremost, we find that while the feasibility of private contributions in presence of externalities may cause a break down of preference single peakedness, the outcome 

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taxation in their setting.



of the median voter theorem still applies: in equilibrium, the median voter's preferred provision level beats any other policy alternative in pairwise comparison. Relatedly, irrespective of the assumed timing, private contributions are never rank-reversing in the sense that some individual other than the 'natural' median (the median individual in a pure public setting) might become the political decisionmaker in equilibrium. We then use these insights to explore the characteristics of equilibria, and to provide a comparison of alternative regimes. When public goods are provided exclusively through the political process in the status quo, moving to a system with additional ex-ante contributions is beneficial for everybody in society. In contrast, unanimity is lost when moving to a system which allows only ex-post contributions; this latter regime is still preferred by majority but will in general be opposed by some high-preference individuals. Interestingly, allowing for private ex-post *and* ex-ante contributions finds the support of the majority of individuals, no matter what the status quo is: a majority of higher preference individuals prefer this endogenous timing regime over the ex-post regime (and by extension, over pure public provision); conversely, a different majority including all low-preference individuals prefers it over the ex-ante regime. Implementing this 'general' system may even generate unanimous consent. These latter findings are especially encouraging insofar that in many empirically relevant situations, individuals will likely have full discretion on the timing of their individual contributions. The paper also quantified and compared equilibrium policies and the equilibrium level of private contributions for each institutional setting.

Finally, we showed that if a completely decentralized (private-provision) regime is taken as the status quo, the institutional comparisons change drastically. While moving to the ex-post system (or a fortiori, the general system) is still preferred by a majority, a strong supermajority may now oppose any transition to the ex-ante system. Overall, our results suggest that when starting out from a pure public regime, there is always unanimous support for an institutional change to a dual system. Overwhelming support for a regime change is less likely when a private system represents the status quo. However, when only a majority is required for a regime change, a dual public-private system will be adopted regardless of the status in place; according to our preliminary analysis, endogenous contributions then emerge as the stable equilibrium institution.

Further research on these issues is clearly desirable to enhance our understanding of the evolution of institutions in setting with both dual private and public provision.

# Appendix

## Proof of Proposition 3

To establish the monotonicity properties as stated in the Proposition, we consider the private contributions equilibrium in stage 1. The utility-maximizing stage-1 contribution of an individual  $i$  is

$$\hat{g}_i^* = \arg \max_{\hat{g}_i} U^i(y_i - \hat{g}_i - g^{*M}(\hat{g}_{-i}^*, \hat{g}_i), g^{*M}(\hat{g}_{-i}^*, \hat{g}_i)[1 + \beta(N-1)] + \hat{g}_i + \beta \sum_{j \neq i} \hat{g}_j^*). \quad (13)$$

The set of contributors  $C$  must always include the highest-preference individuals. To see this, notice that the first-order derivative for individual  $i$ ,

$$\frac{dU^i(\cdot)}{d\hat{g}_i} = -U_c^i(\cdot)[1 + \frac{dg(\hat{g})}{d\hat{g}_i}] + U_G^i[1 + \frac{dg(\hat{g})}{d\hat{g}_i}(1 + \beta(N-1))],$$

is strictly increasing in index  $i$ : first,  $dg/d\hat{g}_i$  is uniform across individuals, and second, the marginal rate of substitution  $\Delta^i(\cdot) = U_G^i/U_c^i$  is increasing in  $i$ . Invoking normality, it is then impossible to have  $dU^l(\hat{g}_{-l}, \hat{g}_l = 0)/d\hat{g}_l \leq 0$  and at the same time  $U^k(\hat{g}_{-k}, \hat{g}_k = 0)/d\hat{g}_k > 0$  for any two individuals  $k, l$  with rank order  $l > k$ .<sup>42</sup>

For any contributor  $i$ , the first-order condition to (??) holds with equality and can be rewritten as

$$\frac{U_G^i(c_i, G_i)}{U_c^i(c_i, G_i)} = \frac{1 + \frac{dg(\hat{g})}{d\hat{g}_i}}{1 + \frac{dg(\hat{g})}{d\hat{g}_i}[1 + \beta(N-1)]}. \quad (14)$$

This system of first-order conditions defines the equilibrium contributions for any  $i \in C$ . To analyze these conditions, note first that for any  $\beta > 0$ , the marginal rate of substitution is not unity because any increase in private contributions triggers a smaller equilibrium public policy,  $dg(\hat{g})/d\hat{g}_i < 0$ . This feature is intuitive. With normal preferences, larger private contributions of some  $i > m$  cause the median voter  $m$  to raise her private consumption, with the consequence of a decrease in the public

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<sup>42</sup>Hence, the set of contributors is non-empty iff the first-order derivative for individual  $n$ ,

$$-U_c^n(\cdot)[1 + \frac{dg(\hat{g})}{d\hat{g}_n}] + U_G^n[1 + \frac{dg(\hat{g})}{d\hat{g}_n}(1 + \beta(N-1))],$$

is positive if evaluated at a contribution vector  $\hat{g} = 0$ .

policy  $g$ . In anticipation of this negative response, each contributor provides a smaller contribution than he would do otherwise. Specifically, contributions are kept below the level which equates (taken the subsequent public provision into account) marginal rate of substitution and marginal costs of provision. Finally, inspection shows that because of the symmetry property  $dg/\hat{d}g_i = dg/\hat{d}g_j$  for any two contributors  $i, j$ , equilibrium contributions must be strictly increasing in the index of natural preferences.<sup>43</sup> Hence, public-goods consumption is increasing in the natural preference index, and strictly so for the set of contributors and unless  $\beta = 1$ .  $\square$

### Proof of Proposition 6

For any private contributor  $i$ , the second-stage first-order condition  $\Delta^i(g_m^{ep}, \tilde{g}) = 1$  holds, which equalizes his marginal utilities from consuming public and private goods. In contrast,  $\Delta_j(g_m^{ep}, \tilde{g}_{-j}, \tilde{g}_j = 0) < 1$  for any non-contributor  $j$ . By standard arguments, the set of contributors in stage 2 is decreasing in  $g$  and at certain threshold levels  $g$ , the lowest-preference contributing individual becomes a non-contributor.

The proof proceeds in several steps. We consider the range of first-period policies  $g_m$  for which  $m$  contributes in the second stage,  $\tilde{g}_m(g_m) > 0$ . We show that within this range  $g \in [\underline{g}_m, \bar{g}_m]$ , an increase in  $g$  raises the total contributions  $G^T \equiv Ng_m + \sum_i \tilde{g}_i$  of all  $i \in N$  individuals. As an implication of this result, the total contributions  $G_m^{-m}(g)$  of individuals other than  $m$  are shown to increase in  $g$  as well. Since  $U^m(g_m)$  is increasing if and only if  $dG_m^{-m}/dg_m > 0$  (see Lemmas 1 and 2 above), the median voter  $m$  cannot be a stage-2 contributor in equilibrium. In other words,  $m$ 's globally optimal policy  $g_m^{ep}$  is (weakly) larger than  $\bar{g}_m$ , and therefore larger than any upper-boundary threshold  $\bar{g}_j$  for individuals indexed  $j < m$ . This proves the result.

Step 1:  $dG^T(g_m)/dg_m > 0$  whenever the number of contributors is  $\hat{N} < N$ , and if  $\beta < 1$ .

The proof is by contradiction. Suppose a raise in  $g_m$  from  $g'_m$  to  $g''_m$  would lower total contributions  $G^T(g_m)$  towards the public good. Then, there must be at least

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<sup>43</sup>Suppose not, and the lower-preference individual  $j$  contributes more in equilibrium. Then, the first-order conditions () and () cannot simultaneously hold for individuals  $i$  and  $j$ .

one individual  $k$  (necessarily, a contributor under policy  $g'_m$ ) who reduces his total contribution  $g_k^T$  under policy  $g''_m$ . We show that this behavior would be inconsistent with normality. To see this, suppose first that  $k$ 's public goods consumption  $G_k$  is smaller under policy  $g''_m$ . But since his private goods consumption must be larger under policy  $g''_m$  than under policy  $g'_m$ , this would contradict normality. Second, suppose other individuals compensate for the reduction in  $k$ 's total contribution in a way which allows  $k$  to (weakly) raise his public goods consumption  $G_k$  under policy  $g''_m$ . To see that this is impossible, notice that for any unit reduction in  $g_k^T$ , the total provision of all other individuals,  $G_k^{-k}$ , must increase by at least  $1/\beta > 1$  to avoid a reduction in  $G_k$ . But then, total public goods provision would be characterized by  $G^T(g''_m) > G^T(g'_m)$ , a contradiction.

Step 2:  $dG^T(g_m)/dg_m > 0$  implies  $dG_m^{-m}(g_m)/dg_m > 0$ .

Suppose not, and suppose instead  $G_m^{-m}(g''_m) - G_m^{-m}(g'_m) = -\epsilon$  for  $g''_m > g'_m$ , where  $\epsilon$  is a positive number. We show that  $m$ 's stage-2 contributions are characterized by  $\tilde{g}_m(g''_m) - \tilde{g}_m(g'_m) < \beta\epsilon$ . To see why, note that otherwise,  $m$  would raise her total public-goods consumption in state  $g''_m$  relative to state  $g'_m$ . Her private consumption would fall at the same time, which is inconsistent with the assumed normality of preferences. Since  $dG^T(g_m)/dg_m > 0$ , the difference in total public goods supply between both states would satisfy  $G(g''_m) - G(g'_m) \leq -(1 - \beta)\epsilon$ , which is negative for any  $\beta < 1$  and a contradiction to the result in Step 1.

Step 3:  $dG_m^{-m}(g_m)/dg_m > 0$  implies  $dU^m(g_m)/dg_m > 0$  for any  $g_m < \bar{g}_m$ .

For any  $g_m < \bar{g}_m$ ,  $m$  contributes a positive amount in stage 2. In the considered range  $g_m \in [\underline{g}_m, \bar{g}_m]$ ,  $m$  can always raise  $g_m$  in such a way that her total contribution  $g_m^T = g_m + \tilde{g}_m$  constant. Since  $G_m^{-m}(g_m)$  increases in  $g_m$ , this strategy would leave  $m$ 's private consumption constant while raising her public goods consumption. Accordingly,  $m$  always raises her utility by raising  $g_m$  to a level  $g_m \geq \bar{g}_m$  where she does not voluntarily contribute in continuation equilibrium. Finally, since  $\arg \max_{g_m} U^m(g_m) \geq \bar{g}_m$ , neither  $m$  nor any individual  $j < m$  contributes  $\tilde{g}_j > 0$  in stage 2: this is because  $\Delta^m(g_m^{ep}, \tilde{g}_m = 0, \tilde{G}_m^{-m}(g_m^{ep})) \leq 1$  implies  $\Delta^m(g_m^{ep}, \tilde{g}_j = 0, \tilde{G}_j^{-j}(g_m^{ep})) < 1$  by normality of preferences.  $\square$

### Proof of Proposition 7

For a given policy level  $g_m$ , private contributions are larger in the ex-post regime. To see this, notice that for each contributor  $i$ , the second stage optimality condition reads  $\Delta^i(c_i, G_i) = 1$ , as compared to the optimality condition  $\Delta^i(c_i, G_i) > 1$  that prevails in the ex-ante setting for commitment reasons. If in addition the equilibrium policies are characterized by  $g_m^{ep} \leq g_m^{ea}$  (this will be established below), the respective Nash equilibria unambiguously satisfy  $\tilde{g}_i > \hat{g}_i$  for each contributor  $i$ .

In what follows, we show that the equilibrium policy level in the ex-post regime is indeed smaller. To see this, note that in the alternative ex-ante setting,  $m$ 's optimal policy choice satisfies the first-order condition  $\Delta^m(c_m, G_m) = 1/z$ . In the ex-post scenario, the corresponding condition reads

$$\Delta^m(\cdot) = \frac{U_G(\cdot)}{U_c(\cdot)} = \frac{1}{z + \beta \sum_j \frac{d\tilde{g}_j}{dg_m}}.$$

By our previous arguments (See Step 2 in the Proof of Proposition 5), this implies  $\Delta^m(c_m, G_m) > 1/z$  whenever the set of contributors is nonempty. By way of contradiction, suppose now that  $g_m^{ep} \geq g_m^{ea}$ , i.e., the equilibrium policy in the ex-post setting is (weakly) larger. Since the median voter  $m$  does not privately contribute in either regime, her private consumption in the ex-post regime is then (weakly) smaller. But in order to satisfy the respective first-order conditions for equilibrium policy choice in each regime,  $m$ 's public goods consumption  $G_m$  in the ex-post regime would have to be smaller as well. As a consequence, her utility in this regime would be smaller, which is impossible: by our above results, implementing a policy  $g_m^{ea}$  would raise  $m$ 's public-goods consumption relative to the ex-ante regime (for any  $g_m$ , private contributions are larger in the ex-post regime) while leaving her private consumption the same, a contradiction. Hence,  $g_m^{ep} < g_m^{ea}$ .

By revealed preference, our previous arguments imply that any non-contributor is better off in the ex-post regime: the median voter could have chosen the same policy in each regime, rendering private consumption for non-contributors in both regimes identical. At the same time, public consumption in the ex-post setting had been larger

because of the higher level of private contributions (see above).<sup>44</sup>

Conversely, contributors  $i$  are worse off in the ex-post system: since the equilibrium policy is lower, achieving the same consumption  $G_i$  requires each of these these individuals to raise their total contributions in the ex-post regime. A contributor in the ex-post regime may do even worse compared to the pure public system: the equilibrium policy in the ex-post system is smaller because first, private contributions reduce  $m$ 's preferred policy. Second, the equilibrium policy in the ex-post setting is even smaller than the best response to private contributions for commitment reasons. Hence, non-contributors contribute less towards a contributor's public-goods consumption than in the pure public system, trading off the utility-enhancing effect of the feasibility of private contributions.  $\square$

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<sup>44</sup>Since  $g_m^{ep} < g_m^{ea}$  and since for commitment reasons,  $g_m^{ep}$  is smaller than the best response to the vector of subsequent private contributions, a comparison of public good consumption in both regimes is generally ambiguous. However, note that if  $\beta = 1$  and preferences are quasilinear, public goods consumption in the ex-post regime will be larger.

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