SIMON FRASER UNIVERSITY

Department of Economics

Discussion Papers

07-03

Competition in Law Enforcement and Capital Allocation

Nicolas Marceau and Steeve Mongrain

May 2007

SFU Economics

Competition in Law Enforcement and Capital Allocation⋆

Nicolas Marceau
Université du Québec à Montréal and CIRPÉE

Steeve Mongrain
Simon Fraser University, RIIM, and CIRPÉE

May 2nd, 2007

Abstract: This paper studies interjurisdictional competition in the fight against crime and its impact on occupational choice and the allocation of capital. In a world where capital is mobile, jurisdictions are inhabited by individuals who choose to become workers or criminals. Because the return of the two occupations depends on capital, and because investment in capital in a jurisdiction depends on its crime rate, there is a bi-directional relationship between capital investment and crime which may lead to capital concentration. By investing in costly law enforcement, a jurisdiction makes the choice to become criminal less attractive, which reduces the number of criminals and makes its territory more secure. This increased security increases the attractiveness of the jurisdiction for investors and this can eventually translate into more capital being invested. We characterize the Nash equilibria — some entailing a symmetric outcome, others an asymmetric one — and study their efficiency.

1. Introduction

Security matters when it comes to investment decisions. Indeed, capital owners prefer to invest where crime rates are low because in such places, the likelihood that they will be deprived of the return on their investment is lower.[1] Local authorities, responding to those preferences, invest in crime deterrence. In this context, adjacent jurisdictions may compete in law enforcement, to lower their respective crime rates and to make their jurisdiction relatively safer than others for investors. Understanding the mechanics of such competition and the choice of law enforcement chosen by adjacent jurisdictions is the focus of this paper.

In the United States, there are many cases of "twin" cities, with similar characteristics, which nevertheless exhibit very different crime rates. For example, the crime rate against properties is 60% higher in Minneapolis than in St-Paul, 100% higher in Tampa than in St Petersburg, and 46% higher in Oakland than in San Francisco.[2] In the literature, there are potential explanations for those differences in crime rates and for the concentration of criminal activities. For example, Freeman, Grogger and Sonstelie (1996) suggest that congestion in enforcement can explain these phenomena: Because of the technology of enforcement, more criminals translate into a lower probability of capture, which makes criminality a more attractive choice, and therefore leads to the concentration of crime. Zenou (2003) on the other hand, argues that social interactions could explain the concentration of crime: With social interactions, the choice of a criminal life is more attractive the larger the proportion of criminals in the community, so more individuals choose to become criminals in such a case, implying that the community then experiences a high concentration of crime. In another paper, Verdier and Zenou (2004) explains the concentration of

---

[1] Besley (1995) provides direct empirical evidence confirming that security matters using micro-data on investment in Ghana. More indirect evidence is found in a number of studies showing that in a given neighbourhood, residential property values are negatively affected by higher crime rates. For example, Schwartz *et al.* (2003, p.102) conclude that "falling crime rates are responsible for six percentage points of the overall 17.5 percent increase in property values that New York City experienced from 1994 to 1998." Similarly, Bowes and Ihlanfeldt (2001) found that property values in a given neighbourhood are negatively affected by the presence of a rail station with a parking because such stations make it easy for criminals to access the neighbourhood.

[2] Other examples of "twin" cities that exhibit very different property crime rates include Kansas City (Missouri) and Kansas City (Kansas), East St-Louis (Illinois) and St-Louis (Missouri), or Los Angeles and Anaheim.

criminal activity within a group of the population by building a model with endogenous wages and racial discrimination. Discrimination against a group implies lower wages, which makes honest work less attractive and crime more attractive, so that more individuals of that group become criminals and are, eventually, discriminated against. Albeit interesting, these papers abstract from the interaction between crime, enforcement, capital location, and wages, which is the focus of our paper.

In fact, when security levels differ, capital owners will invest in different amounts in different jurisdictions. In other words, it is possible that when crime becomes more concentrated, capital also becomes more concentrated, although obviously in different locations. Further, an important feature of our analysis is that individuals make the occupational choice of becoming workers or criminals.[3] For an individual, this occupational choice largely depends on the amount of capital — a complement in production — in the jurisdiction in which he resides: more capital increases the wage a worker can earn, but more capital also translates into a higher reward for criminal activities; and since investment in capital in a jurisdiction depends on the crime rate in that jurisdiction, there is a complex bi-directional relationship between capital investment and crime. We demonstrate this by adding occupational choice to the otherwise classic problem of capital location, we can create agglomeration effects, both for crime and capital. Note that as in the case of the concentration of crime, the reasons underlying the concentration of capital are still debated in the literature.[4]

The key mechanism we highlight in our analysis can be explained as follows. In standard models without occupational choice and in which capital must be allocated between competing jurisdictions (or uses), the unit return of capital in a given jurisdiction is a decreasing function of the stock of capital located in it. With occupational choice, it is not necessarily so because an extra unit of capital may lead to more individuals choosing to become workers (rather than criminals), and this in turn can make capital more productive. It follows that if an extra unit of capital sufficiently

---

[3] The interaction between crime and occupational choice has been examined in a number of papers, e.g. Baumol (1990), Murphy *et al.* (1993), Acemoglu (1995), Baland and Francois (2000), İmrohoroğlu *et al.* (2000), and Lloyd-Ellis and Marceau (2003). However, none of those papers account for capital investment and inter–jurisdictional competition.

[4] For example, Glaeser *et al.* (1992) argue that knowledge spillover can explain the rise of large cities. However, their argument is less compelling when it comes to understanding the variation of capital and crime within a city.

increases the number of workers (and decreases the number of criminals), then the unit return of capital may be an increasing function of the stock of capital located in a jurisdiction. Of course, whether the unit return of capital is an increasing or a decreasing function of capital affects the allocation of capital in an important way. Intuitively, if the unit return declines with the stock of capital, then capital will tend to be equally distributed between jurisdictions. On the other hand, if the unit return of capital increases with the stock of capital, then capitalists will find it advantageous to concentrate their capital in a single jurisdiction.

The nature of the law enforcement game between jurisdictions is also very different depending on whether the per unit return of capital a decreasing or an increasing function of the investment. The equilibria we characterize are symmetric but they can result in very different outcomes for *ex ante* identical jurisdictions. For the case of increasing per unit return on capital, we show that all the capital locates in the same jurisdiction, and this jurisdiction experiences low criminality, high output and a large working population, while the other jurisdiction attracts no capital, experiences high criminality and very low outputs.

Since Becker's (1968) seminal work on law enforcement, few economists have paid attention to the multi-jurisdictional nature of crime deterrence.[5] This may explain why economists have a limited understanding of the impact of law enforcement policies on criminal activities. In this paper, we explicitly account for the multi-jurisdictional nature of the interaction between criminals and governments and show it has important consequences. In a world without agglomeration effects, Marceau (1997) characterizes equilibria in which crime and capital are evenly distributed across jurisdictions — a non-realistic feature. In this context, he demonstrates that the "laissez-faire" equilibrium features a level of law enforcement greater than the socially efficient level. In this paper, by introducing occupational choice and by analyzing (symmetric) equilibria which result in asymmetric outcomes, we show that under deterrence is possible. We are able to show that the equilibria of the law enforcement game are generally inefficient, i.e. that the levels of enforcement chosen by the jurisdictions when they act independently differs from that which would be selected by a central authority maximizing the sum of the welfare of the two jurisdictions. Of course, since

---

[5] While much of the literature has focused on capital tax competition between jurisdictions –see the survey by Wilson (1999)–, the literature on competition in crime deterrence is extremely limited. An exception is Marceau (1997).

enforcement is inefficient, so is occupational choice within each jurisdiction.

This paper is organized as follows. In Section 2 we present a model with mobile capital and occupational choice. Private sector behaviour is described in Section 3 and the enforcement policies chosen independently by the jurisdictions are characterized in Section 4. We conclude in Section 5. All proofs can be found in the Appendix.

2. The Model

We examine the problem of competition in law enforcement when capital is mobile. Each jurisdiction is inhabited by a group of immobile individuals who have to choose between becoming workers or criminals. By investing in costly law enforcement, a jurisdiction makes the choice of becoming a criminal less attractive, which reduces the number of criminals and makes its territory more secure. This increased security increases the attractiveness of the jurisdiction for investors and can eventually translate into more capital being invested.

There are two jurisdictions, a and b. Each jurisdiction $i \in \{a, b\}$ is inhabited by a group of individuals who collectively own an aggregate production function $F(L^i, K^i)$, where L^i and K^i are the labour force and the capital in place in Jurisdiction i, respectively. The properties of F are standard: for the relevant range of (L^i, K^i), $F_K > 0$, $F_L > 0$, $F_{KK} < 0$, $F_{LL} < 0$, and $F_{LK} \geq 0$, where $F_h = \partial F(L^i, K^i)/\partial h$ and $F_{hj} = \partial^2 F(L^i, K^i)/\partial h \partial j$.

In each jurisdiction, the population consists of a continuum of agents of measure 1, who each chooses to become a worker or a criminal. If L^i is the number of workers in Jurisdiction i, then the number of criminals in this jurisdiction is $C^i = 1 - L^i$.[6] An individual who chooses to become a criminal appropriates for himself some of the total return on capital. Denoted by $\alpha(d^i)$ is the proportion of the total return on the capital a criminal is able to steal. The proportion $\alpha(d^i)$ is a decreasing function of the level of law enforcement d^i chosen by the government of Jurisdiction i. Thus, an agent who decides to become a criminal obtains $\alpha(d^i)K^i F_K(L^i, K^i)$. Alternatively, if he chooses to become a worker, he is paid according to the marginal product of labour, which

---

[6] In reality, few individuals specialize solely in criminal activities. For a discussion on this topic, see Blumstein *et al.* (1986). However, to keep our model as simple as possible, we decided to assume that agents choose one of the two activities, as in Murphy *et al.* (1993).

amounts to a payoff given by $F_L(L^i, K^i)$.

A large number of atomistic capitalists endowed with a total of $\bar{K}$ units of mobile capital choose to allocate their capital between the two jurisdictions. The amount of capital invested in Jurisdiction a is denoted K_m^a, and $K_m^b = \bar{K} - K_m^a$ is that invested in Jurisdiction b. Capital is allocated by the owners after the choice of law enforcement by each government. The governments are assumed to be committed to their enforcement policy. Once capital is allocated, it becomes completely immobile. We also assume that in each of the two jurisdictions, some immobile capital is already in place. Denoted by K_o^i is the amount of capital already in place in Jurisdiction i. Without loss of generality, we assume that $K_o^a \geq K_o^b \geq 0$.

In Jurisdiction i, the government chooses the level of law enforcement, d^i, which it can buy at a cost of 1 per unit. As was mentioned above, a larger d^i negatively affects the proportion $\alpha(d^i)$ that is stolen by each criminal, i.e. $\alpha'(d^i) < 0$. The proportion $\alpha(d^i)$ is assumed to belong to the interval $[0, \bar{\alpha}]$, where $\bar{\alpha}$ is the maximum proportion that can be appropriated. Even in the absence of public enforcement, there are constraints on such a proportion. Private enforcement, which we do not consider in this paper, is a good example. We assume that governments finance their expenditures by use of a pure lump sum tax.

The objective function of the governments could take many forms. For example, governments could maximize total output. This would imply that criminals, workers and capital owners are all treated equally. Alternatively, governments could only care for workers — by maximizing wages —, or for capital owners — by maximizing the return on capital. We assume that governments maximize legal output — i.e. output minus what is appropriated by criminals — minus enforcement costs. This is consistent with governments caring for everyone but the criminals, and with the assumption that taxation is lump sum. Alternative objective functions as the ones mentioned above would generate slightly different tradeoffs, but the general results of the paper would qualitatively remain the same.

The timing is as follows. First, jurisdictions simultaneously choose their level of law enforcement. This investment is perfectly observable and is irreversible. Then, capitalists allocate their mobile capital between the two jurisdictions. Investments in capital are perfectly observable and

5

irreversible. The residents of each jurisdiction then make their occupational choice (worker or criminal). Finally, production takes place, theft takes place, and payments are awarded. The model is solved using backward induction.

3. Private Sector Behaviour

3.1 OCCUPATIONAL CHOICE

We solve for the occupational choice equilibrium of the residents of Jurisdiction i for given levels of enforcement d^i and capital $K^i = K_o^i + K_m^i$. Since agents choose the activity that generates the largest payoff, the equilibrium number of workers in Jurisdiction i, say $L^i(K^i, d^i)$, will be that which equates the return of the two occupations. Thus, $L^i(K^i, d^i)$ solves the following equation:

$$F_L(L^i, K^i) = \alpha(d^i)K^i F_K(L^i, K^i). \tag{1}$$

In other words, the number of workers must adjust so that the return to working, the wage, which is simply the marginal product of labour $F_L(\cdot)$, is equal to the return to criminal activity, $\alpha K F_K(\cdot)$. When $F_L(0, K) > \alpha K F_K(0, K)$, some individuals become workers ($L > 0$). Similarly, $F_L(1, K) < \alpha K F_K(1, K)$ is required for some individuals to become criminals ($L < 1$). We assume that both conditions are satisfied for the relevant range of K. Given these two conditions, and given that the left hand side of equation (1) is monotonically decreasing, while the right hand side is monotonically increasing with L, the solution to equation (1) is unique and denoted $L^i(K^i, d^i)$.

On one hand, an increase in K^i generates an increase in the wage a worker receives, provided that $F_{LK}(\cdot) > 0$. On the other hand, an increase in K^i translates into an increase in $K^i F_K^i$, the total return on capital.[7] Since the return to criminal activity is a proportion of this total return, an increase in K^i also leads to an increase in the return to criminal activity. The relative size of each effect determines whether an increase in K^i leads to more workers or to more criminals. To see this, note that from equation (1), we have:

---

[7] This follows from the assumption of strictly increasing marginal product ($F_K > 0$) and strict concavity ($F_{KK} < 0$).

$$\frac{\partial L^i(K^i, d^i)}{\partial K^i} = \frac{F_{LK}(L^i, K^i) - \alpha(d^i)[F_K(L^i, K^i) + K^i F_{KK}(L^i, K^i)]}{\alpha(d^i) K^i F_{LK}(L^i, K^i) - F_{LL}(L^i, K^i)} \tag{2}$$

The denominator of this last expression is positive, while the sign of its numerator is ambiguous. Thus, the impact of a change in the capital stock K on equilibrium employment L depends on the sign of $F_{LK} - \alpha[F_K + K F_{KK}]$. This implies that when $F_{LK} >$ (resp. $<$) $\alpha[F_K + K F_{KK}]$, then labour (resp. criminality) increases when capital increases. The incentive for a resident to participate in the legal sector increases only if the increase in wages due to additional capital is large enough. Note that for the particular case of $F_{LK}(\cdot) = 0$, an increase in capital leads to an increase in criminal activity for the recipient jurisdiction. An increase in law enforcement effort d^i unambiguously reduces the incentive to become a criminal, and consequently increases labour supply, i.e. $\partial L^i(K^i, d^i)/\partial d^i > 0$.

Consider now the following condition:

Condition I: $F_{LK}(L^i, K^i) \geq \bar{\alpha}[F_K(L^i, K^i) + K^i F_{KK}(L^i, K^i)], \quad \forall \ K^i \in [K_o^i, K_o^i + \bar{K}] \ \text{ and } \ \forall \ L_i \in [0, 1]$.

Condition I guarantees that $\partial L^i(K^i, d^i)/\partial K^i \geq 0$. Intuitively, Condition I requires that the increase in wages following the arrival of new capital dominates the increase in the return on criminal activities. A natural exercise would be to use the standard Cobb-Douglas production function to elaborate on Condition I. Unfortunately, since a Cobb-Douglas production function entails the ratio $F_L/K F_K$ being independent of K, it follows that L^i is also independent of K^i. However, we can show than Condition I is always satisfied for several alternative production functions, for example $F(L, K) = L K^\mu - L^2/2.$[8] We now turn to the characterization of the capital location choice, with particular attention paid to potential agglomeration effects.

3.2 CAPITAL LOCATION CHOICE AND AGGLOMERATION EFFECTS

The capitalists allocate their $\bar{K}$ units of capital between the two jurisdictions. Denoted by ρ^i is the per unit return on capital invested in Jurisdiction i. Since a proportion $\alpha(d^i)$ of the total return

---

[8] With this production function, $K_o^i > 1$ is required to guarantee a positive marginal return on labour.

on capital is stolen by each criminal, we have that $\rho^i = [1 - \alpha(d^i)C^i(K^i, d^i)]F_K[L^i(K^i, d^i), K^i]$. In a standard model of capital location with no crime, the per unit return on capital in a given jurisdiction decreases with the investment, because the marginal product of capital is itself a decreasing function of capital. Moreover, if both jurisdictions differ only in terms of their initial stock of capital, the jurisdiction with less capital will initially attract more mobile capital. In fact, provided there is enough mobile capital, and technologies are identical, marginal products and capital stocks will be equalized in the two jurisdictions. No agglomeration occurs in such a case.

In the literature, agglomeration effects are sometimes introduced directly by assuming that the technology exhibits increasing returns in capital, as in Boadway *et al.* (2004). Alternatively, agglomeration effects are introduced indirectly by assuming the presence of an externality, as in Glaeser *et al.* (1992), in which agglomeration in cities is the consequence of a transfer of knowledge externality. In the current framework, the return to capital in the two jurisdictions will differ because enforcement may differ between the two jurisdictions. More importantly, it will also differ because the number of criminals will vary relatively to the size of the investment in capital. Consider first the difference in enforcement between jurisdictions. Enforcement is good for capitalists because it reduces the amount of the return on capital that is stolen by criminals, and it is also good because it deters individuals from becoming criminals. *Ceteris paribus*, a jurisdiction with more enforcement will attract more capital. Consequently, two jurisdictions could end up with different levels of capital simply because of differences in their choice of enforcement. Of course, despite differences in capital allocation, no agglomeration effect is at work here. If $d^a > d^b$, Jurisdiction a will attract more capital, but capital will still be allocated to the point where the per unit return in one jurisdiction is equal to the per unit return in the other jurisdiction, provided the marginal return on capital is decreasing in capital.

The effect of the stock of capital on the per unit return on capital in a given jurisdiction is much more interesting. The impact of a change in capital on this per unit return is given by:

$$
\begin{aligned}
\frac{\partial \rho^i}{\partial K^i} =& \alpha(d^i)F_K[L^i(K^i, d^i), K^i]\frac{\partial L^i(K^i, d^i)}{\partial K^i} \\
&+ [1 - \alpha(d^i)C^i(K^i, d^i)]\left[F_{KK}[L^i(K^i, d^i), K^i] + F_{LK}[L^i(K^i, d^i), K^i]\frac{\partial L^i(K^i, d^i)}{\partial K^i}\right]
\end{aligned}
\tag{3}
$$

The first term on the right-hand side of equation (3) shows that when K^i changes, the number of criminals changes; this change in the number of criminals will affect the proportion of the total return of capital that is stolen. The second term represents the more traditional impact of a change in K^i on the per unit return, but with one difference. When capital in Jurisdiction i increases, the marginal return on capital decreases; this is captured by $F_{KK}(\cdot) < 0$. However, when capital increases, the number of workers also changes, and so does the marginal return of capital through the cross effect $F_{LK}(\cdot)\partial L^i(\cdot)/\partial K^i$. Consequently, when more capital leads to more workers, the per unit return on capital invested in Jurisdiction i may be an increasing function of the stock of capital invested in i. Intuitively, because the labour supply and the crime rate both depend on the amount of capital located in a jurisdiction, it is possible for the per unit return on capital to increase when capital investment increases. More workers increases the marginal product of capital. Furthermore, when the number of workers increases, the number of criminals is reduced and this also leads to an increase in the total return on capital.

Below, we show that the sign of $\partial \rho^i/\partial K^i$ is a key determinant of the equilibrium allocation of capital. We focus on two simple cases: (a) $\partial \rho^i/\partial K^i < 0 \ \ \forall K^i$; and (b) $\partial \rho^i/\partial K^i > 0 \ \ \forall K^i$. We also briefly discuss the case in which the sign of $\partial \rho^i/\partial K^i$ varies with K^i. It should be obvious that when Condition I is not satisfied, the per unit return on capital decreases with the stock of capital.

Denoted by $K(d^a, d^b)$ is the equilibrium capital investment in Jurisdiction a, the equilibrium capital investment in Jurisdiction b is then given by $\bar{K} - K(d^a, d^b)$.

Proposition 1: *Suppose Condition I is not satisfied, implying that $\partial L^i(K^i, d^i)/\partial K^i \leq 0$ and $\partial \rho^i/\partial K^i < 0 \ \ \forall \ K^i \in [K_o^i, K_o^i + \bar{K}]$. Equilibrium capital investments $K(d^a, d^b)$ in Jurisdiction a and $\bar{K} - K(d^a, d^b)$ in Jurisdiction b are then the solution to:*

$$[1 - \alpha(d^a)C^a(d^a)]F_K[L^a(d^a), K_o^a + K(d^a, d^b)] \tag{4}$$
$$= [1 - \alpha(d^b)C^b(d^b)]F_K[L^b(d^b), K_o^b + \bar{K} - K(d^a, d^b)]$$

In such a case, $K(d^a, d^b)$ is an increasing function of d^a and a decreasing function of d^b.

Proposition 1 is easily understood by an examination of Figure 1. Capital owners prefer to invest in the jurisdiction in which the per unit return on capital is the highest. The more capital owners

invest in a given jurisdiction, the lower the per unit return on capital is. In equilibrium, capitalists allocate their capital so that the per unit return in both jurisdictions is equalized. Note that for a given level of enforcement chosen by the other jurisdiction, an increase in enforcement by a jurisdiction leads to an increase in capital invested on its territory. Consequently, both jurisdictions will compete to attract capital investment by offering a secure environment to the capitalists. Thus, this environment entails no agglomeration effects; it follows that Condition I is a necessary condition for the presence of agglomeration effects.

Furthermore, Condition I (i.e. labor supply being increasing in capital), is not a sufficient condition for the presence of increasing returns to investment. What is in fact required is that labour supply increases at a high enough rate. The following condition, Condition II, is the sufficient condition for the per unit return on capital to be increasing with the stock of capital:[9]

Condition II: $F_{KL}(L^i, K^i)[F_{KL}(L^i, K^i) - \bar{\alpha} F_K(L^i, K^i)] > F_{LL}(L^i, K^i) F_{KK}(L^i, K^i) \quad \forall \ K^i \in [K_o^i, K_o^i + \bar{K}] \ \text{and} \ \forall \ L_i \in [0, 1].$

As we intuitively already know, for Condition II to be satisfied, Condition I itself has to be satisfied. Further, for the per unit return on capital to be increasing with capital, labour must grow fast enough so that both the effects of a reduction in the number of criminals and the increase in the marginal product of capital out of complementarity are large enough. For those effects to be large, labour supply needs to be responsive enough to changes in capital, which is obtained when F_{LK} is large enough. Going back to the example presented before, Condition II will be satisfied whenever $(1 - \bar{\alpha}) K_o^\mu > (1 - \mu)/\mu$. Thus, the initial capital stock and/or the capital productivity parameter μ needs to be large enough so that wages increase fast enough with the addition of new capital. Also $\bar{\alpha}$ needs to be low enough so that the benefit of becoming a criminal is not too high. When both of these conditions are simultaneously satisfied, labour, and more importantly, criminality, becomes very responsive to the addition of new capital. Note that if $\partial L^i / \partial K^i > 0$, but F_{LK} is not large enough to ensure that $\partial \rho^i / \partial K^i > 0$ — i.e. Condition II is not satisfied — then the resulting equilibrium will be similar to that described in Proposition 1. To summarize, under Conditions II, the per unit rate of return on capital is increasing in capital: $\partial \rho_i / \partial K_i > 0$. Proposition 2, which

---

[9] The derivation of Condition II can be found in the Appendix.

we now introduce, deals with the possibility of increasing return on capital or agglomeration effects and describes an equilibrium in which all mobile capital is invested in a single jurisdiction.

Proposition 2: *When Conditions I and II are satisfied, there exists at least one equilibrium in which all mobile capital is invested in one jurisdiction.*

If the unit return ρ_i is an increasing function of capital for all levels of investment, then capitalists benefit from concentrating their capital in a single jurisdiction.

Which jurisdiction will obtain all the mobile capital in an equilibrium à la Proposition 2? Unfortunately, the answer is neither simple nor unique. Two types of problems arise. The first one is a coordination problem. Because there are a large number of capitalists who choose to invest their capital simultaneously, it is possible that they coordinate on the "wrong" jurisdiction, i.e. a jurisdiction in which total payoff is not maximized. For obvious reasons, we focus on the "right" equilibrium, that in which capitalists coordinate on the jurisdiction in which total payoff is maximized. Note that we could have ensured that the payoff maximizing jurisdiction is chosen by assuming a unique mobile capital owner or, alternatively, by maintaining the large number of capital owners assumption, but with the choice of location being made sequentially. The second problem is to identify the jurisdiction which is the most attractive for capital owners. As was discussed, both the enforcement effort and the initial capital influence the per unit return on capital. Enforcement effort has a positive effect on the per unit return on capital, and so does the initial capital stock when Condition II is satisfied. Consequently, we can derive the following result.

Corollary 1: *Suppose the two jurisdictions have the same initial endowment in capital ($K_o^a = K_o^b$). As established in Proposition 2, there is then an equilibrium in which all mobile capital is invested in Jurisdiction a ($K(d^a, d^b) = \bar{K}$) if $d^a > d^b$, or one in which all mobile capital is invested in Jurisdiction b ($\bar{K} - K(d^a, d^b) = \bar{K}$) if $d^a < d^b$. If $d^a = d^b$, then $K(d^a, d^b) = \bar{K}$ with probability p, and $K(d^a, d^b) = 0$ with probability $(1 - p)$ is an equilibrium allocation for any $p \in [0, 1]$; we arbitrarily assume that in such a case, $p = 1/2$.*

As can be seen in Figure 2, given equal initial capital, if the level of enforcement is larger in Jurisdiction a, then mobile capitalists prefer to concentrate their capital in that particular jurisdiction.

Naturally, all the capital is invested in Jurisdiction b if $d^a < d^b$. If both jurisdictions provide the same level of enforcement, the capitalists are indifferent between concentrating all their capital in one or the other jurisdiction. Again, to simplify, we assume that all capital owners pick Jurisdiction a with probability p.

We now know that a jurisdiction is more attractive for mobile capital when it exerts more effort in enforcement. However, initial capital endowment also plays a role in determining where mobile capital will locate.

Corollary 2: *Suppose the two jurisdictions have chosen to exert the same level of enforcement effort ($d^a = d^b$). As established in Proposition 2, there is then an equilibrium in which all mobile capital is invested in Jurisdiction a ($K(d^a, d^b) = \bar{K}$) if $K_o^a > K_o^b$, or one in which all mobile capital is invested in Jurisdiction b ($\bar{K} - K(d^a, d^b) = \bar{K}$) if $K_o^a < K_o^b$. If $K_o^a = K_o^b$, then $K(d^a, d^b) = \bar{K}$ with probability p, and $K(d^a, d^b) = 0$ with probability $(1 - p)$ is an equilibrium allocation for any $p \in [0, 1]$; we arbitrarily assume that in such a case, $p = 1/2$.*

Abstracting from possible differences in enforcement levels, the jurisdiction with more initial capital will attract all mobile capital. This is simply because the per unit return of capital is larger in the jurisdiction with more initial capital. In such an environment, agglomeration effects are at work. Not only does all mobile capital locate in the same jurisdiction, but it also does so in the jurisdiction which has the largest initial capital stock.

Note that the locational choice of mobile capital in the case in which a given jurisdiction has both more initial capital and exerts more enforcement effort is obvious, while that in which one jurisdiction dominates in one aspect and not in the other is more complicated. Nevertheless, agglomeration effects are still at work.

Propositions 1 and 2 deal with two simple cases in which the per unit return on capital investment is monotonically decreasing or increasing in capital. The resulting equilibria are either take the form of an interior solution in which some capital is invested in both jurisdictions, or a corner solution in which all the capital locates in a single jurisdiction. In fact, these two types of equilibria could also be obtained in other circumstances. For example, the per unit return on capital could be a

U-shaped, non-monotonic function of capital as in Figure 3. In the particular case of Figure 3, the capitalists will obviously find it profitable to invest all their capital in a. On the other hand, in Figure 4, where the per unit return on capital has an inverted U-shape, some mobile capital will be located in the two jurisdictions. While all those situations are interesting, the rest of the analysis will focus on the case where the per unit return of capital is a monotonic function of capital.

4. Enforcement Policies and Capital Allocation

We now examine the simultaneous choice of law enforcement by the two jurisdictions. Both jurisdictions are assumed to maximize legal output (i.e. output minus what is appropriated by criminals) minus enforcement costs. Such an objective implicitly assumes that lump sum taxation can be used to finance expenditures on enforcement. Thus, the problem of Jurisdiction a is given by:

$$\max_{d^a} \quad \tilde{F}^a(d^a, d^b) - [1 - \alpha(d^a)(1 - L^a(d^a, d^b))]K(d^a, d^b)\tilde{F}^a_K(d^a, d^b) - d^a \tag{5}$$

where $L^a(d^a, d^b) = L^a[K(d^a, d^b), d^a]$, and where $\tilde{F}^a_\ell(d^a, d^b) = F_\ell[L^a(d^a, d^b), K^a(d^a, d^b)]$ for $\ell \in \{\emptyset, L, K, LK, KK\}$. Similarly, the problem of Jurisdiction b is given by:

$$\max_{d^b} \quad \tilde{F}^b(d^a, d^b) - [1 - \alpha(d^b)(1 - L^b(d^a, d^b))](\bar{K} - K(d^a, d^b))\tilde{F}^b_K(d^a, d^b)] - d^b \tag{6}$$

where $L^b(d^a, d^b) = L^b[\bar{K} - K(d^a, d^b), d^b]$, and where $\tilde{F}^b_\ell(d^a, d^b) = F_\ell[L^b(d^a, d^b), \bar{K} - K(d^a, d^b)]$ for $\ell \in \{\emptyset, L, K, LK, KK\}$.

The resulting Nash equilibrium outcomes are strikingly different depending on whether Conditions I and II apply.

When Conditions I and II are not satisfied, the per unit return on capital is decreasing in capital, agglomeration effects are not present, and capital is allocated to the point where its per unit return is equalized in all jurisdictions, as stated in Proposition 1. This corresponds to the situation

described in Marceau (1997) in which each jurisdiction inefficiently exerts too much effort in enforcement. Such a result is reminiscent of those obtained in the literature on policy competition between governments.[10] By increasing enforcement, a region attracts some capital, but it imposes a negative externality on the other jurisdiction which loses some capital. Using the terminology of Eaton and Eswaran (2002) and Eaton (2004), the actions of the regions are then *plain substitutes*. In such a case, both jurisdictions will choose a level of enforcement larger than the efficient level (i.e. too much investment compare to what a central authority would select if it maximized the sum of both objective functions). Note that because enforcement is inefficient, so is occupational choices: in other words there are too few criminals in this world. Note however that the allocation of capital is efficient.[11]

When Conditions I and II are satisfied, all mobile capital locates in a single jurisdiction. In such an environment, the nature of the game between the jurisdictions is very different. For immediate purposes, denote by $\Omega[K^i, d^i]$ the value of a jurisdiction's objective function for a pair (K^i, d^i). As was explained in last section, differences in the initial endowment of capital complicate the analysis, but the forces at work remain the same, with or without these differences. Consequently, to focus on the mechanics of competition in enforcement between both jurisdictions, we assume from now on that both jurisdictions have the same initial capital stock. This assumption also has the advantage of allowing us to explain why two jurisdictions with identical initial conditions can evolve in drastically different directions. Also, from now on, and since the problem has been made symmetric, we simplify notation by dropping superscript $i \in \{a, b\}$ whenever possible.

Define $\Omega[K, d] = [1 - \alpha(d)(1 - L(K, d))][F(L(K, d), K) - KF_K(L(K, d), K)] - d$ as the payoff of a jurisdiction when K units of capital locate on its territory and when it invests d in enforcement. Let $d^*(\bar{K})$ denote the level of enforcement chosen by a jurisdiction when all mobile capital locates on its territory ($K = K_o + \bar{K}$): $d^*(\bar{K}) = \arg\max_d \Omega[K_o + \bar{K}, d]$. Note that we assume an interior solution ($d^*(\bar{K}) > 0$). Similarly, $d^*(0)$ is defined as the level of enforcement chosen by a jurisdiction when no capital is located on its territory ($K = K_o + 0$): $d^*(0) = \arg\max_d \Omega[K_o + 0, d]$. Notice that when Condition I is satisfied and for similar levels of enforcement, the jurisdiction that attracts no

---

[10] See, for example, Mintz et Tulkens (1986), Wildasin (1988), Wilson (1986, 1999), or Zodrow and Mieszkowski (1986).

[11] This would not hold if the supply of capital was elastic.

14

new capital ends up with more criminals. Obviously, a jurisdiction is better off when it receives all mobile capital, so $\Omega[K_o + \bar{K}, d^*(\bar{K})] > \Omega[K_o + 0, d^*(0)]$. Also, let $\hat{d}$ be the level of enforcement solving $\Omega[K_o + \bar{K}, \hat{d}] = \Omega[K_o + 0, d^*(0)]$. Clearly, it must be that $\hat{d} > d^*(\bar{K}) > d^*(0)$.

Note that the following chain of inequalities must hold:

$$\Omega[K_o + \bar{K}, d^*(\bar{K})] > \Omega[K_o + 0, d^*(0)] = \Omega[K_o + \bar{K}, \hat{d}] > \Omega[K_o + 0, d] \ \forall d \neq d^*(0), d > 0$$

Figure 5 depicts the payoffs of the jurisdictions in this law enforcement game. Recall that when $\partial \rho^i / \partial K^i > 0$ and $K_o^a = K_o^b = K_o$, the equilibrium we focus on entails that all mobile capital locates in Jurisdiction i if $d^i > d^j$. If $d^i = d^j$, then all the capital locates in Jurisdiction i with probability $1/2$, and in Jurisdiction j with probability $1/2$. For the game considered, a strategy for a jurisdiction is simply a level of enforcement d, and the strategy sets are the positive real numbers ($d \in [0, \infty]$). A strategy profile is a pair (d^a, d^b) consisting of a strategy for each jurisdiction.

We now present three useful lemmas.

Lemma 1: *When Conditions I and II are satisfied, the jurisdictions never choose a strategy $d > \hat{d}$.*

A jurisdiction will have no desire to invest more than $\hat{d}$ because attracting all mobile capital with $d > \hat{d}$ makes it worse off than investing nothing and having no capital.

Lemma 2: *When Conditions I and II are satisfied, the jurisdictions never choose a strategy $d < d^*(0)$.*

A jurisdiction will have no desire to invest less than $d^*(0)$ because welfare is strictly increasing in d for $d < d^*(0)$ and $K \in \{K_o + 0, K_o + \bar{K}\}$.

Lemma 3: *When Conditions I and II are satisfied,, the game has no pure strategy Nash equilibrium.*

There is no pure strategy equilibrium because if Jurisdiction i chooses an enforcement level $d^i < \hat{d}$, then Jurisdiction j will find it profitable to attract all mobile capital by choosing d^j such that

15

$d^i < d^j < \hat{d}$. As for $(d^i = d^*(0), d^j = \hat{d})$, it is not an equilibrium because $d^j = \hat{d}$ is not a best response to $d^i = d^*(0)$.

The main result of this section is as follows:[12]

Proposition: 3 *When Conditions I and II are satisfied, and when the two jurisdictions have the same endowment of immobile capital, the enforcement policy game has a symmetric mixed strategy Nash equilibrium in which the two jurisdictions play $d \in [d^*(0), \hat{d}]$ according to the continuous cumulative function $H(d)$ and density function $h(d) = H'(d)$ on $[d^*(0), \hat{d}]$. For $d \in [d^*(0), \hat{d}]$, the mixed strategy $H(d)$ is given by:*

$$H(d) = \frac{\Omega[K_o + 0, d^*(0)] - \Omega[K_o + 0, d]}{\Omega[k_o + \bar{K}, d] - \Omega[K_o + 0, d]}$$

In this equilibrium, the expected payoff of the two jurisdictions is $\Omega[K_o + 0, d^*(0)]$.

Note that given $H(d)$, we have that $H(d^*(0)) = 0$, $0 < H(d) < 1$ for $d \in]d^*(0), \hat{d}[$, and $H(\hat{d}) = 1$. The equilibrium described here is such that in expected terms, the two jurisdictions obtain a net surplus of zero. The intuition is simple. Suppose all mobile capital is invested in Jurisdiction i which has chosen $d^i > d^j = d^*(0)$ and that $\Omega^i[K_o + \bar{K}, d^i] > \Omega^j[K_o + 0, d^*(0)]$. Clearly, since the two jurisdictions are otherwise identical, this situation cannot be an equilibrium because Jurisdiction j has an incentive to deviate to a level of enforcement $\breve{d}^j = d^i + \epsilon$, with ϵ small. Indeed, if Jurisdiction j does deviate to $\breve{d}^j$, the capitalists will re-locate all their capital from i to j, and Jurisdiction j will now get a payoff of $\Omega^j[K_o + \bar{K}, \breve{d}^j] > \Omega^j[K_o + 0, d^*(0)]$. Such an incentive to deviate will be present as long as a jurisdiction has a positive net payoff. Therefore, in equilibrium, it must be that both jurisdictions obtain a net surplus of zero in expected terms.

The mixed strategy equilibrium described in Proposition 3 is inefficient unless $d^i = d^*(K_o + 0)$ and $d^j = d^*(K_o + \bar{K})$ are drawn, an event which occurs with probability zero. The equilibrium is inefficient for several reasons. First, the jurisdiction which obtains no new mobile capital spends $d > d^*(K_o + 0)$ on enforcement with probability approaching one (an obvious case of over–deterrence). Second, the jurisdiction which obtains all mobile capital spends too little or too much

---

[12] Note that the equilibrium described in Proposition 3 is reminiscent of the equilibria characterized in Levitan and Shubik (1972), Varian (1980), or Kreps and Scheinkman (1983).

in enforcement $(d \neq d^*(K_o + \bar{K}))$. Finally, because enforcement is inefficient, occupational choice is distorted. Note however that all capital locates in a single jurisdiction, which is efficient.

Consider the *ex post* implications of such an equilibrium. First, note that all mobile capital locates in the jurisdiction which offers the highest level of protection. This jurisdiction will benefit from a level of welfare larger than that it would get in the no capital / low enforcement situation $(\Omega[K_0 + 0, d^*(0)])$. In this jurisdiction, the proportion of criminals is low because there is a lot of capital. Moreover, since enforcement is larger, the proportion of capital that is stolen is lower. On the other hand, the jurisdiction which receives no new mobile capital obtains a level of welfare lower than that it would get in the no capital / low enforcement situation — because the marginal benefit of enforcement effort is lower than its cost, i.e. $d > d^*(0)$. Note that since there is no mobile capital in this jurisdiction, wages are lower. Consequently, more of its residents choose to become criminals. To summarize, the *ex post* realization of the symmetric mixed strategy Nash equilibrium entails a jurisdiction who receives all mobile capital, experiences relatively moderate crime rate and relatively large output and wages. The other jurisdiction receives no new mobile capital, experiences a high crime rate and very low output and wages. The simple model presented in this paper can therefore explain how two *ex ante* identical jurisdictions can experience drastically different evolutions.

5. Conclusion

This paper has shown that in an economy with occupational choice and with jurisdictions competing in enforcement to attract mobile capital, the symmetric Nash equilibria result in an even or an uneven distribution of crime and capital across space. These equilibria are always inefficient.

The creation of a central organization to coordinate law enforcement policies would likely be beneficial in such a context, depending on the constraints it faces and the strengths and weaknesses of centralization. For example, a central organization may be forced, by political constraints, to select a uniform level of enforcement in all jurisdictions. Also, it could be that a central agency is not as efficient at identifying criminals. To analyze the opportunity of creating such a central agency, our model would have to be extended to take these factors into account.

The current analysis assumes that labour is immobile. In our model, the prospects for individual residing in a jurisdiction with a low level of capital are not very attractive ones: in individuals can either obtain a relatively low wage or become a criminal. This can be partly justified if one considers jurisdictions as being inhabited by very different individuals, say low-skilled workers in one and high-skilled workers in the other, with segregated labour markets. With housing prices in the jurisdiction of high-skilled individuals that are simply not affordable for the low-skilled individuals for example. Nevertheless, if individuals were identical and labour was mobile, individuals would be able to move to a region in which the labour market is more attractive than in their own. This would open a whole new set of possibilities. That our results would hold in such a context is not obvious. This is clearly the next step in our research.

In the future, we would also like to study the political economy rationale for the observed frequent arrangements in which crime enforcement falls into the hands of local authorities. To our knowledge, why this is so has not been satisfactorily answered. Certainly, the phenomena we have described in the current analysis is likely to be taken into consideration by voters, lobby groups, and politicians, and they should therefore be explicitly incorporated into a political economy model of the appropriate degree of centralization of the fight on crime.

6. Appendix I: Proofs

Proof of Proposition 1: When $\partial L^i(K^i, d^i)/\partial K^i = 0$, $i = a, b$, it follows from equation (3) that $\partial \rho^i/\partial K^i < 0$, $i = a, b$. In such a case, mobile capital is allocated between the two jurisdictions until the per unit return is equalized. Consequently, $K(d^a, d^b)$ satisfies equation (4), which simply states that $\rho^a[K_o^a + K(d^a, d^b)] = \rho^b[K_o^b + \bar{K} - K(d^a, d^b)]$. Differentiating equation (4) yields that:

$$\frac{\partial K(d^a, d^b)}{\partial d^a} = \frac{\alpha'(d^a)C^a F_K(\cdot) - [\alpha(d^a)F_K(\cdot) + (1 - \alpha(d^a)C^a)F_{LK}(\cdot)]\partial L^a/\partial d^a}{[1 - \alpha(d^a)C^a]F_{KK}(L^a, K_o^a + K^a) + [1 - \alpha(d^b)C^b]F_{KK}(L^b, K_o^B + K^b)}$$

$$\frac{\partial K(d^a, d^b)}{\partial d^b} = \frac{-\alpha'(d^b)C^b F_K(\cdot) + [\alpha(d^b)F_K(\cdot) + (1 - \alpha(d^b)C^b)F_{LK}(\cdot)]\partial L^b/\partial d^b}{[1 - \alpha(d^a)C^a]F_{KK}(L^b, K_o^a + K^a) + [1 - \alpha(d^b)C^b]F_{KK}(L^b, K_o^b + K^b)}$$

The denominator of these two expressions is clearly negative. Consequently, $\partial K(d^a, d^b)/\partial d^a$ is positive since its numerator is negative, and $\partial K(d^a, d^b)/\partial d^b$ is negative since its numerator is positive.

Similarly, when $\partial L^i(K^i, d^i)/\partial K^i < 0$, $i = a, b$, it follows from equation (3) that $\partial \rho^i/\partial K^i < 0$, $i = a, b$. In such a case, mobile capital is allocated between the two jurisdictions until the per unit return is equalized. Consequently, $K(d^a, d^b)$ satisfies equation (4), which simply states that $\rho^a[K_o^a + K(d^a, d^b)] = \rho^b[K_o^b + \bar{K} - K(d^a, d^b)]$. Totally differentiating equation (4) yields that:

$$\frac{\partial K(d^a, d^b)}{\partial d^a} = \frac{\alpha'(d^a)C^a F_K(\cdot) - [\alpha(d^a)F_K(\cdot) + (1 - \alpha(d^a)C^a)F_{KL}(\cdot)]\partial L^a/\partial d^a}{\Delta},$$

$$\frac{\partial K(d^a, d^b)}{\partial d^b} = \frac{-\alpha'(d^b)C^b F_K(\cdot) + [\alpha(d^b)F_K(\cdot) + (1 - \alpha(d^b)C^b)F_{KL}(\cdot)]\partial L^b/\partial d^b}{\Delta},$$

where

$$\Delta = \alpha(d^a)F_K(\cdot)(\partial L^a(\cdot)/\partial K^a) + [1 - \alpha(d^a)C^a(\cdot)][F_{KK}(\cdot) + F_{KL}(\cdot)(\partial L^a(\cdot)/\partial K^a)] + $$
$$\alpha(d^b)F_K(\cdot)(\partial L^b(\cdot)/\partial K^b) + [1 - \alpha(d^b)C^b][F_{KK}(\cdot) + F_{KL}(\cdot)(\partial L^b(\cdot)/\partial K^b)].$$

The denominator of these two expressions is clearly negative. Consequently, $\partial K(d^a, d^b)/\partial d^a$ is positive since its numerator is negative, and $\partial K(d^a, d^b)/\partial d^b$ is negative since its numerator is positive.

Derivation of Condition II: For $\partial \rho^i/\partial K^i > 0$, the following must be satisfied:

19

$$\alpha F_K \frac{\partial L^i}{\partial K^i} + [1 - \alpha C^i] \left[F_{KK} + F_{LK} \frac{\partial L^i}{\partial K^i} \right] > 0.$$

This is equivalent to:

$$\frac{\partial L^i}{\partial K^i} > \frac{-F_{KK}}{\frac{\alpha}{1-\alpha C} F_K + F_{KL}}.$$

A more constraining requirement would be that $\partial L^i / \partial K^i > -F_{KK}/F_{KL}$. Therefore, using equation (2), we can show that $\partial \rho^i / \partial K^i > 0$ as long as:

$$F_{KL}[F_{KL} - \bar{\alpha} F_K] > F_{LL} F_{KK}.$$

Proof of Proposition 2: Inspection of equation (3) reveals that if Conditions I and II are both satisfied, then $\partial \rho^i / \partial K^i$ is positive for all values of K^i. In such a case, there is an equilibrium in which all capital locates in a single jurisdiction. Since the per unit return on capital is increasing in capital in the two jurisdictions, $\rho^a|_{(K^a = K_o^a + \bar{K})} \geq \rho^b|_{(K^b = K_o^b)}$ or $\rho^a|_{(K^a = K_o^a)} \leq \rho^b|_{(K^b = K_o^b + \bar{K})}$. Moreover, given that all capitalists invest in the same jurisdiction, it is optimal for a given capitalist to also invest in that jurisdiction.

Proof of Corollary 1: Given $K_o^a = K_o^b$, then $\rho^a|_{(K^a = K_o^a)} > \rho^b|_{(K^b = K_o^b)}$ if and only if $d^a > d^b$. Consequently, there exist an equilibrium where the entire $\bar{K}$ is invested in a. Given that the per unit return on capital is increasing in capital, this equilibrium dominates any other allocation. When $d^b > d^a$, it must be that $\rho^a|_{(K^a = K_o^a)} < \rho^b|_{(K^b = K_o^b)}$, therefore an equilibrium exist where the entire $\bar{K}$ is invested in b. When $d^a = d^b$, then $\rho^a|_{(K^a = K_o^a)} = \rho^b|_{(K^b = K_o^b)}$. The capitalists are then indifferent between investing all their capital in a or b.

Proof of Corollary 2: The proof of Corollary 2 is identical to that of Corollary 1, but with varying K instead of varying d.

Proof of Lemma 1: Since $\Omega[K_o + 0, d^*(0)] = \Omega[K_o + \bar{K}, \hat{d}] > \Omega[K_o + \bar{K}, d] \; \forall d > \hat{d}$, a jurisdiction is better off when it chooses $d = d^*(0)$ and obtains no mobile capital ($K = K_o$) than if it chooses a level of enforcement larger than $\hat{d}$ ($d > \hat{d}$) and obtains all mobile capital ($K = K_o + \bar{K}$).

Proof of Lemma 2: We know that $d^*(0)$ is given by $d^*(0) = \arg\max_d \Omega[K_o + 0, d]$, which implies that the first order condition characterizing the choice of d is given by:

$$\left[\alpha(d)\frac{\partial L(K_o,d)}{\partial d} - \alpha'(d)(1 - L(K_o,d))\right][F(L(K_0,d),K) - K_o F_K(L(K_o,d),K_o)]$$
$$+ [1 - \alpha(d)(1 - L(K_o,d))][F_L(L(K_o,d),K_o) - K_o F_{KL}(L(K_o,d),K_o)] > 1, \quad \forall d < d^*(0)$$

Consequently, a jurisdiction with no mobile capital will never choose a level of enforcement $d < d^*(0)$. Since $d^*(K_o + \bar{K}) > d^*(0)$, the same argument applies for a jurisdiction with all mobile capital. Consequently, $d \geq d^*(0)$.

Proof of Lemma 3: We first show that there is no symmetric ($d^i = d^j$) pure strategy Nash equilibrium and then show that there is no asymmetric ($d^i > d^j$) pure strategy Nash equilibrium.

(i) There is no symmetric ($d^i = d^j$) pure strategy Nash equilibrium.

Consider a strategy profile (d, d), with $d \in [d^*(0), \hat{d}]$ from Lemma 1 and Lemma 2.

If $d < \hat{d}$, then the payoff of each jurisdiction is $\Omega^a = \Omega^b = \frac{1}{2}\Omega[K_o + \bar{K}, d] + \frac{1}{2}\Omega[K_o + 0, d]$. Clearly, this cannot be an equilibrium as any jurisdiction, say a, has an incentive to deviate to $d^{a'} = d + \epsilon$, causing all the capital to locate in a, and ensuring itself a payoff $\Omega^{a'} = \Omega[K_o + \bar{K}, d + \epsilon] > \Omega^a$ for ϵ small enough (i.e. $\epsilon < \hat{d} - d$).

If $d = \hat{d}$, then the payoff of each jurisdiction is $\Omega^a = \Omega^b = \frac{1}{2}\Omega[K_o + \bar{K}, \hat{d}] + \frac{1}{2}\Omega[K_o + 0, \hat{d}]$. Clearly, this cannot be an equilibrium as any jurisdiction, say a, has an incentive to deviate to $d^{a'} = d^*(0)$, and ensuring itself a payoff $\Omega^{a'} = \Omega[K_o + 0, d^*(0)] > \Omega^a$.

(ii) There is no asymmetric ($d^i > d^j$) pure strategy Nash equilibrium.

Consider any strategy profile (d^a, d^b), with $d^a < d^b \leq \hat{d}$ from Lemma 1.

If $d^a < d^b < \hat{d}$, then $\Omega^a = \Omega[K_o + 0, d^a]$ and a has an incentive to deviate to $d^{a'} = d^b + \epsilon$ to obtain a payoff of $\Omega^{a'} = \Omega[K_o + \bar{K}, d^b + \epsilon] > \Omega^a$ for ϵ small enough.

If $d^*(0) < d^a < d^b = \hat{d}$, then $\Omega^a = \Omega[K_o + 0, d^a]$ and a has an incentive to deviate to $d^{a'} = d^*(0)$ to obtain a payoff of $\Omega^{a'} = \Omega[K_o + 0, d^*(0)] > \Omega^a$.

If $d^*(0) = d^a < d^b = \hat{d}$, then $\Omega^b = \Omega[K_o + \bar{K}, \hat{d}]$ and b has an incentive to deviate to $d^{b'} = d^*(\bar{K})$ to obtain a payoff of $\Omega^{b'} = \Omega[K_o + \bar{K}, d^*(\bar{K})] > \Omega^b$.

This completes the proof.

Proof of Proposition: 3 We show that when j plays according to the mixed strategy $H(d)$, i has no incentive to deviate from $H(d)$.

Suppose j plays the mixed strategy $H(d)$. Then, when i plays d', it obtains all mobile capital $(K = \bar{K})$ with probability $H(d')$ and no capital $(K = 0)$ with probability $1 - H(d')$.

Before solving for the mixed strategies equilibrium, first note that there are no point masses in equilibrium. The intuition is simple: if a level of enforcement d' was to be played with positive probability, there would be a tie at d' with positive probability. Imagine then that Jurisdiction j decides to play $d' + \epsilon$ (instead of d') with the same probability. The cost of such a deviation would be of the order of ϵ, but if the two jurisdictions were to tie, then Jurisdiction j would gain a fixed positive amount. The formal proof of this is as follows. Imagine that Jurisdiction i plays d' with positive probability ω, and that Jurisdiction j deviates to $d' + \epsilon$ with the same positive probability. The payoff for Jurisdiction j will change by a factor of:

$$
\left\{ \Pr(d^i > d' + \epsilon)\Omega[K_o + 0, d' - \epsilon] - \Pr(d^i > d')\Omega[K_o + 0, d'] \right\}
$$
$$
+ \left\{ \Pr(d^i < d' + \epsilon)\Omega[K_o + \bar{K}, d' - \epsilon] - \Pr(d^i < d')\Omega[K_o + \bar{K}, d'] \right\}
$$
$$
+ \omega \left\{ \Omega[K_o + \bar{K}, d' - \epsilon] - \frac{\Omega[K_o + \bar{K}, d'] - \Omega[K_o + 0, d']}{2} \right\}
$$

The first terms in braces represent the difference between losing with an effort level $d' + \epsilon$, and losing with an effort level d'. As for the second terms in braces, they represent the difference between winning with an effort $d' + \epsilon$, and winning with an effort level d'. It is easy to see that the sum of these terms goes to zero when ϵ goes to zero. Now, the last terms in braces represent the difference between winning alone with $d' + \epsilon$, and sharing the win with d' (in expected terms). Since the sum of these terms is strictly positive when ϵ goes to zero, it pays to deviate to $d' + \epsilon$ when there is a probability mass at d'. This implies that $H(d)$ cannot have a probability mass, and because the cumulative function is continuous, cases in which the jurisdictions play $d^i = d^j$ (a tie) occur with probability 0.

We now solve for $H(d)$ knowing that it must be continuous on $[d^*(0), \hat{d}]$. When i plays the mixed strategy $H(d)$, its expected payoff is:

$$
\int_{d^*(0)}^{\hat{d}} [H(z)\Omega(K_o + \bar{K}, z) + (1 - H(z))\Omega(K_o + 0, z)] \, dH(z)
$$

For $(H(d), H(d))$ to be a mixed strategy Nash equilibrium, it has to be that all pure strategies played with positive probability yield the same payoff. We construct the equilibrium so that the expected payoff of the two jurisdictions is $\Omega[K_o + 0, d^*(0)]$. Therefore:

$$
H(d)\Omega[K_o + \bar{K}, d] + (1 - H(d))\Omega[K_o + 0, d] = \Omega[K_o + 0, d^*(0)] \ \forall \ d \in [d^*(0), \hat{d}]
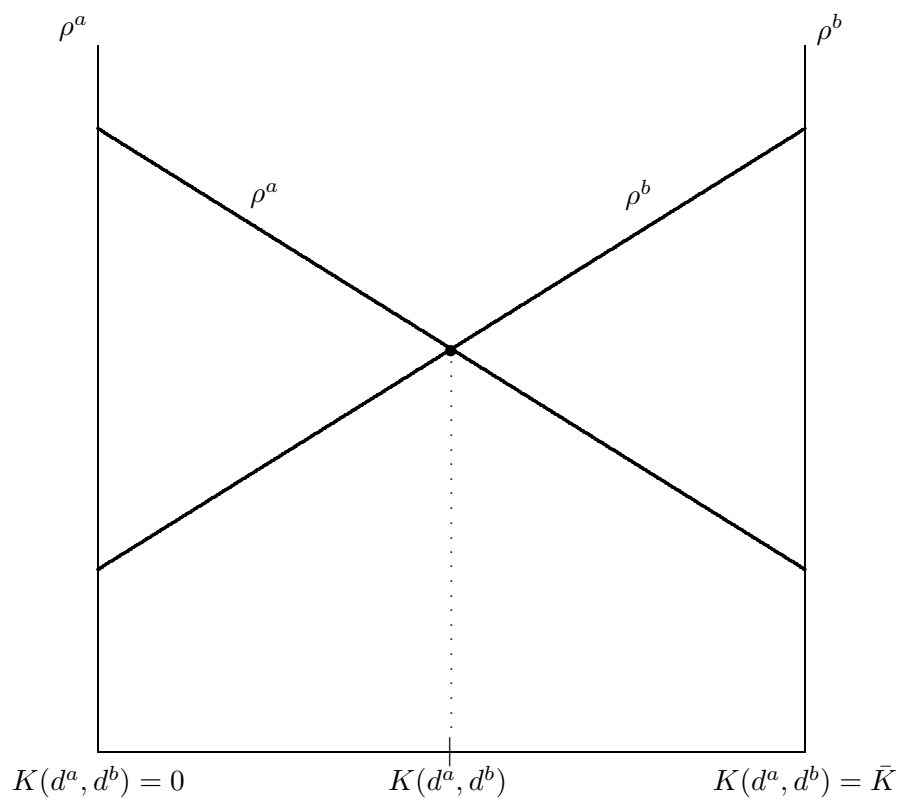$$

It follows that for $d \in [d^*(0), \hat{d}]$, the mixed strategy $H(d)$ is given by:

$$H(d) = \frac{\Omega[K_o + 0, d^*(0)] - \Omega[K_0 + 0, d]}{\Omega[K_o + \bar{K}, d] - \Omega[K_o + 0, d]}$$

When b plays the mixed strategy $H(d)$, a has no incentive to deviate from $H(d)$ because increasing the probability of playing any $d \in [d^*(0), \hat{d}]$ would not affect its payoff as all pure strategies are equivalent by construction.
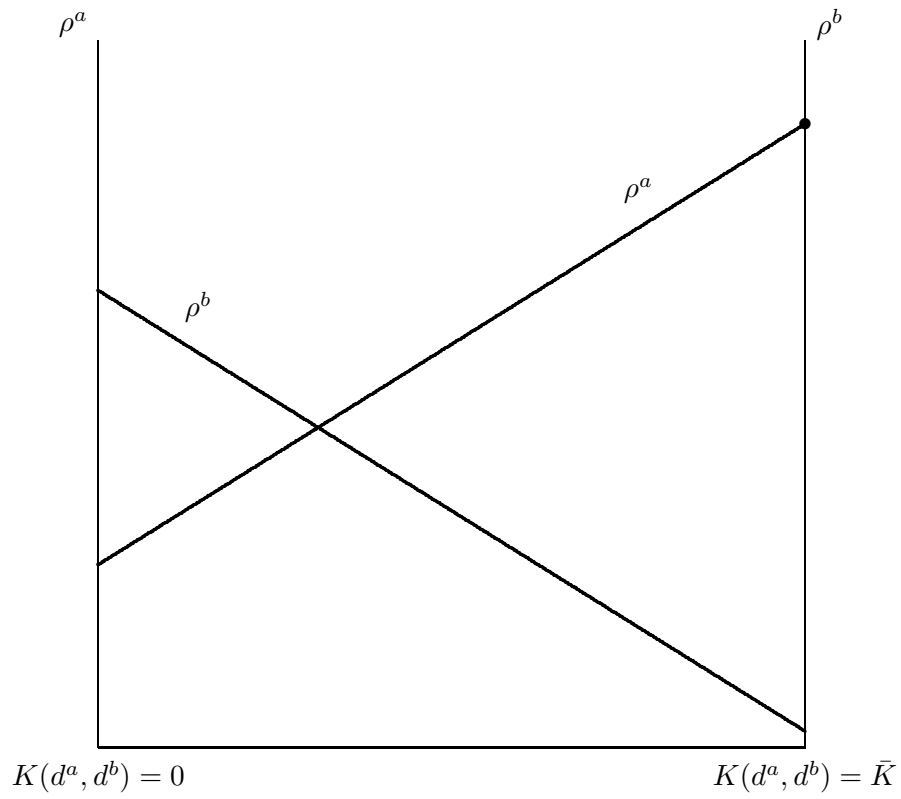
This completes the proof.

7. Appendix II: Figures

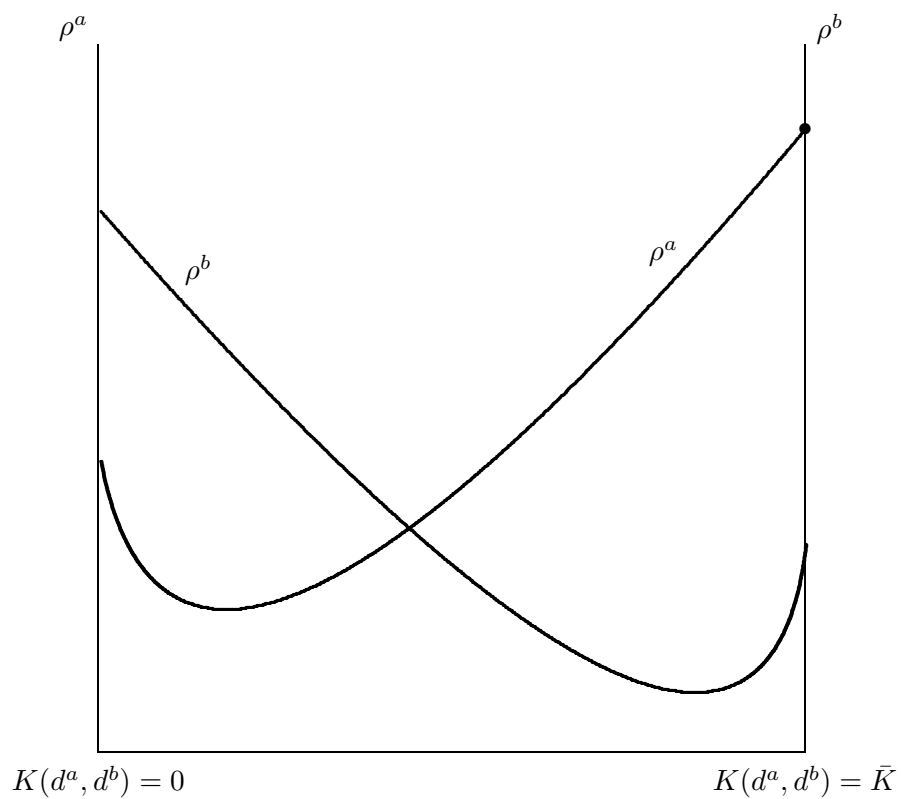$$\bullet \text{ is the equilibrium allocation of capital}$$

Figure 1

Case with $\partial \rho^i / \partial K^i < 0$ and $d^a = d^b$

$K(d^a, d^b) = 0$ $K(d^a, d^b) = \bar{K}$

• is an equilibrium allocation of capital

Figure 2

Case with $\partial \rho^i / \partial K^i > 0$ and $\rho(K_o^a, d^a) > \rho(K_o^b, d^b)$

ρ^a ρ^b

ρ^b ρ^a

$K(d^a, d^b) = 0$ $K(d^a, d^b) = \bar{K}$

$\bullet$ is an equilibrium allocation of capital

Figure 3

Case in which ρ^i is non-monotonic in K^i and U-shaped, $\rho(K^a_o, d^a) > \rho(K^b_o, d^b)$

ρ^a

ρ^b

ρ^a

ρ^b

$K(d^a, d^b) = 0$ $K(d^a, d^b)$ $K(d^a, d^b) = \bar{K}$
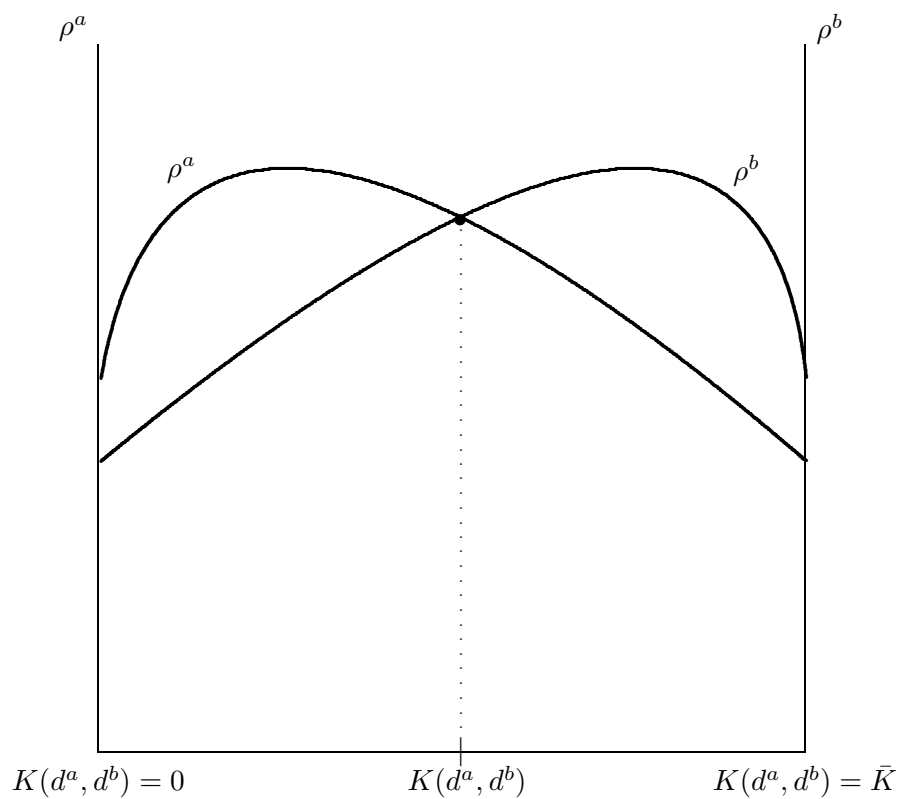
$\bullet$ is the equilibrium allocation of capital

Figure 4

Case in which ρ^i is non-monotonic in K^i and has an inverted U-shape, $d^a = d^b$
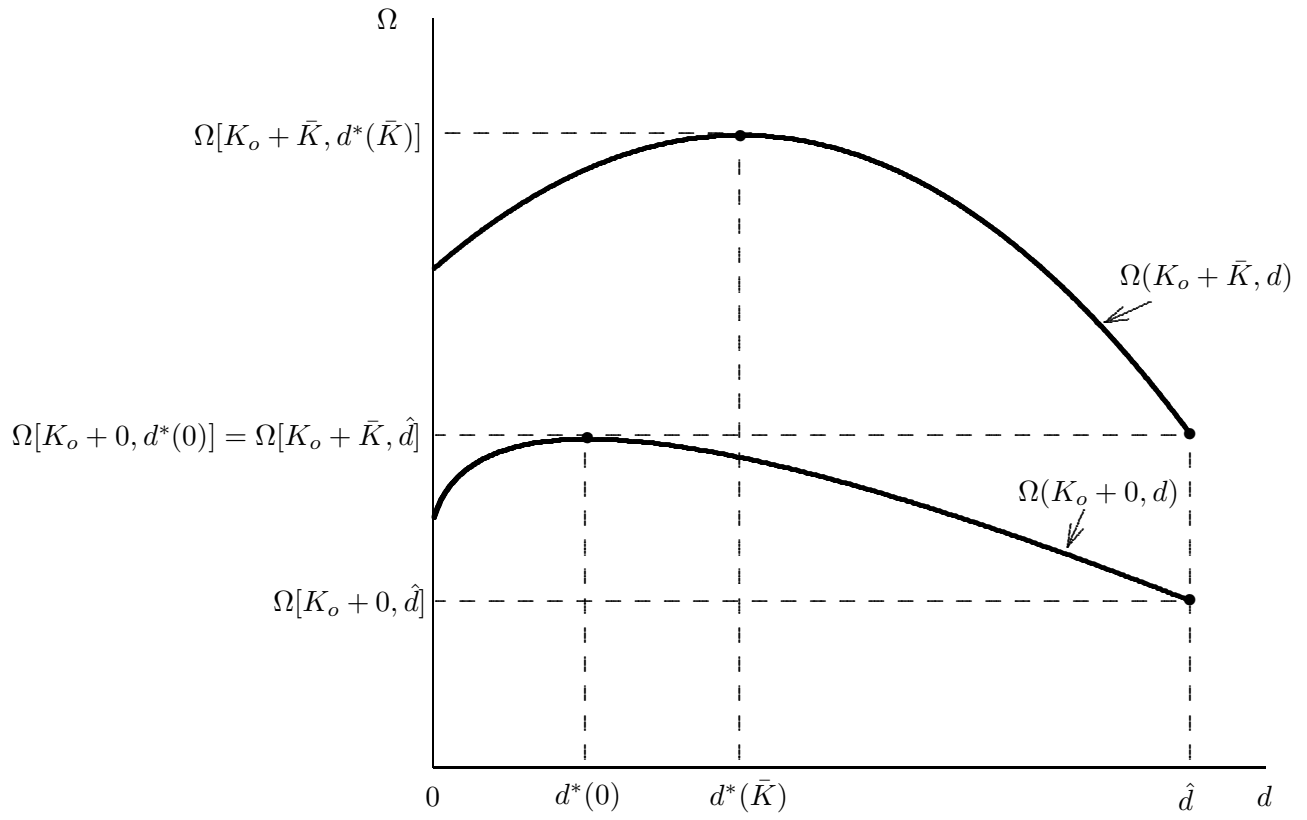
Figure 5

The Payoffs

8. References

Acemoglu, D. (1995), "Reward Structure and the Allocation of Talent," *European Economic Review* **39**, 17–33.

Baland, J.-M., and P. Francois (2000), "Rent-Seeking, Resource Booms and the Size of the Entrepreneurial Class: A Theoretical Analysis," *Journal of Development Economics* **61**, 527–542.

Baumol, W. (1990), "Entrepreneurship: Productive, Unproductive and Destructive," *Journal of Political Economy* **98**, 893–921.

Becker, G. (1968), "Crime and Punishment: an Economic Approach", *Journal of Political Economy* **73**, 169–217.

Besley, T. (1995), "Property Rights and Investment Incentives: Theory and Evidence from Ghana", *Journal of Political Economy* **103**, 903–937.

Boadway, R., K. Cuff, and N. Marceau (2004), "Agglomeration Effects and the Competition for Firms", *International Tax and Public Finance* **11**, 623–645.

Bowes, D.R., and K.R. Ihlanfeldt (2001), "Identifying the Impacts of Rail Transit Stations on Residential Property Values", *Journal of Urban Economics* **50**, 1–25.

Blumstein, A., J. Cohen, J.A. Roth, and C.A. Visher (1986), "Criminal Careers and Career Criminals," *National Academy Press,* Washington, DC.

Eaton B.C. (2004), "The Elementary Economics of Social Dilemmas", *Canadian Journal of Economics* **37**, 805–829.

Eaton B.C., and M. Eswaran (2002), "Noncooperative Equilibria in One-Shot Games: A Synthesis", in *Applied Microeconomic Theory*, Cheltenham, UK: Edward Elgar.

Freeman, C., J. Grogger, and J. Sonstelie (1996), "The Spatial Concentration of Crime," *Journal of Urban Economics* **40**, 216–231.

Glaeser, E.L., H.D. Kallal, J.A. Scheinkman, and A. Shleifer (1992), "Growth in cities," *Journal of Political Economy* **100**, 1126–1152.

İmrohoroğlu, A., A. Merlo, and P. Rupert (2000), "On the Political Economy of Income Redistribution and Crime", *International Economic Review* **41**, 1–25.

Kreps, D.M., and J.A. Scheinkman (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics* **14**, 326–227.

Levitan, R., and M. Shubik (1972), "Price Duopoly and Capacity Constraints", *International Economic Review* **13**, 111–122.

Lloyd-Ellis, H., and N. Marceau (2003), "Endogenous Insecurity and Economic Development", *Journal of Development Economics* **72**, 1–29.

Marceau, N. (1997), "Competition in Crime Deterrence", *Canadian Journal of Economics* **30**, 844–854.

Mintz, J., and H. Tulkens (1986), "Commodity Tax Competition Between Member States of a Federation: Equilibrium and Efficiency", *Journal of Public Economics* **29**, 133–172.

Murphy, K.M., A. Shleifer, and R.W. Vishny (1993), 'Why Is Rent-Seeking So Costly to Growth?,' *American Economic Review, Papers and Proceedings* **83**, 409–414.

Schwartz, A.E., S. Susin, and I. Voicu (2003), "Has Falling Crime Driven New York City's Real Estate Boom?", *Journal of Housing Research* **14**, 101–135.

Varian, H.R. (1980), "A Model of Sales", *American Economic Review* **70**, 651–659.

Verdier T., and Y. Zenou (2004), "Racial Beliefs, Location and the Cause of Crime," *International Economic Review* **45**, 731–760.

Wildasin, D.E. (1988), "Nash Equilibria in Models of Fiscal Competition", *Journal of Public Economics* **35**, 229–240.

Wilson, J.D. (1986), "A Theory of Interregional Tax Competition," *Journal of Urban Economics* **19**, 296–315.

Wilson, J.D. (1999), "Theories of Tax Competition," *National Tax Journal* **52**, 269–304.

Zenou, Y. (2003), "The Spatial Aspect of Crime," *Journal of the European Economic Association* **1**, 459–467.

Zodrow, G.R. and P. Mieszkowski (1986), "Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods," *Journal of Urban Economics* **19**, 356–370.