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Plant-Level Nonconvexities and the Monetary Transmission Mechanism

Roman Šustek*

Abstract

Micro-level empirical evidence suggests that plant managers adjust production by utilizing capital along nonconvex margins. Existing models of the monetary transmission mechanism (MTM), however, assume that production units adjust output smoothly. The objective of this paper is to determine whether such plant-level nonconvexities affect the MTM in a quantitatively significant way. To this end we replace the smooth production function in a prototypical model of the MTM with heterogeneous plants that adjust output along three nonconvex margins: intermittent production, shiftwork, and weekend work. We calibrate the model such that steady-state utilization of these margins is in line with U.S. data. We find that the nonconvexities dampen the responses of aggregate economic activity and prices to monetary policy shocks by about 50 percent relative to the standard model, thereby significantly reducing the effectiveness of the MTM. Due to heterogeneity and discrete choices at the plant level, monetary policy affects the output decisions of only "marginal" plants - those close to being indifferent between alternative production plans. In equilibrium the measure of such plants is rather small. In addition, contrary to popular belief, the quantitative effects of monetary policy shocks on aggregate output do not significantly change with the degree of capacity utilization over the business cycle. The effects on inflation, however, do change substantially over the business cycle when monetary policy shocks are persistent.

JEL Codes: E22, E23, E32, E52.Keywords: Asymmetries, heterogenous plants, monetary transmission mechanism, nonconvexities, nonlinear approximation.

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Nontechnical Summary

Micro-level empirical evidence suggests that plant managers in many manufacturing industries adjust production by utilizing capital along nonconvex margins. For example, when a plant manager wants to achieve a lower volume of output, he reduces the number of weeks the plant is scheduled to be open, drops weekend work, or reduces the number of shifts. Output adjustments at the micro level are thus lumpy.

Despite such observed nonconvexities, one characteristic of existing models of the monetary transmission mechanism (MTM) – the process through which monetary policy decisions are transmitted into the economy – is that production units have access to a smooth (aggregate) production function. The objective of this paper is to determine whether incorporating non-convex margins of output adjustment at the micro level into an otherwise standard model of the MTM affects the model's properties in a quantitatively significant way. In particular, we examine the extent to which the nonconvexities affect the responses of aggregate economic activity and prices to monetary policy shocks.

We construct an economy similar in many respects to the model of liquidity effects developed by Christiano and Eichenbaum (1992). Instead of assuming an aggregate production function, however, production in our model takes place at individual plants that differ in terms of productivity. The plant manager can adjust the plant's output by utilizing capital along three nonconvex margins. First, he can operate the plant or let it remain idle. Second, he can choose the number of straight-time shifts. Third, he can run overtime (weekend) shifts in addition to straight-time shifts.

We calibrate the economy such that the steady-state fraction of plants operating a given shift or using weekend work is in line with U.S. plant-level data. We further ensure that in a steady state the economy can be interpreted as a disaggregated version of the original Christiano and Eichenbaum economy, and that the cyclical behavior of output in the two economies is the same. This makes our model quantitatively comparable with Christiano and Eichenbaum's model, which we take as a benchmark for our experiments. We then compare how the two economies respond to monetary policy shocks.

We find that the nonconvexities dampen the responses by about 50 percent, thereby significantly reducing the effectiveness of the MTM. Due to heterogeneity and discrete choices at the plant level, in equilibrium monetary policy shocks affect the output decisions of only a small measure of "marginal" plants that are close to being indifferent between alternative production plans. We also explore the possibility that the measure of such plants can change over the business cycle. Some researchers suggest that monetary policy should be more effective in recessions than expansions because in recessions firms have more spare capacity and consequently can expand output more easily. We find that this is not the case. In our model, capacity utilization varies substantially over the business cycle, but the measure of the marginal plants that change production in response to monetary policy shocks stays roughly the same.

1. Introduction

Micro-level empirical evidence suggests that plant managers in many manufacturing industries adjust production by utilizing capital along nonconvex margins. For example, when a plant manager wants to achieve a lower volume of output, he reduces the number of weeks the plant is scheduled to be open, drops weekend work, or reduces the number of shifts. Output adjustments at the micro level are thus lumpy.

Despite such observed nonconvexities, one characteristic of existing models of the monetary transmission mechanism (MTM) – the process through which monetary policy decisions are transmitted into the economy – is that production units have access to a smooth (aggregate) production function. The objective of this paper is to determine whether incorporating non-convex margins of output adjustment at the micro level into an otherwise standard model of the MTM affects the model's properties in a quantitatively significant way. In particular, we examine the extent to which the nonconvexities affect the responses of aggregate economic activity and prices to monetary policy shocks.

We find that the nonconvexities dampen the responses by about 50 percent, thereby significantly reducing the effectiveness of the MTM. Due to heterogeneity and discrete choices at the plant level, in equilibrium monetary policy shocks affect output decisions of only a small measure of "marginal" plants that are close to being indifferent between alternative production plans. We also explore the possibility that the measure of such plants can change over the business cycle. Some researchers suggest that monetary policy should be more effective in recessions than expansions because in recessions firms have more spare capacity and consequently can expand output more easily. We find that this is not the case. In our model, capacity utilization varies substantially over the business cycle, but the measure of the marginal plants that change production in response to monetary policy shocks stays roughly the same.

The importance of nonconvex margins of capital utilization for output adjustments at the plant level has been well documented by empirical studies. For example, Bresnahan and Ramey (1994) find that plant managers in the automobile industry adjust output by closing the plant for a week at a time, by scheduling Saturday work, or by changing the number of shifts. According to the authors' estimates, these margins account for 80 to 90 percent of output volatility at the micro level. Bresnahan and Ramey's findings have been broadly supported by Hall (2000), also for the automobile industry, and Mattey and Strongin (1997) for manufacturing, especially for industries characterized by assembly production. Further, lumpy adjustments in plant-level employment have been well documented by Caballero and Engel (1993), Caballero, Engel, and Haltiwanger (1997), and Hamermesh (1989).¹

We construct an economy similar in many respects to the model of liquidity effects developed by Christiano and Eichenbaum (1992).² In their economy, the MTM consists of two effects: the liquidity effect and the output effect. Due to limited participation of households in the money market along the lines suggested by Lucas (1990), a monetary injection lowers both nominal and real interest rates (the liquidity effect). A lower real interest rate reduces produc-

¹ Nonconvex production margins are also a key element in the literature on inventories (Ramey, 1991; Cooper and Haltiwanger, 1992).

² Other popular models of the MTM include models with nominal rigidities (such as sticky prices or wages) and models with credit market imperfections. See Mishkin (1995) for a review of the literature.

tion costs of firms, which finance working capital through bank loans. Lower production costs then induce firms to increase demand for labor and expand output (the output effect).³

In our model we keep the mechanism behind the liquidity effect as in the Christiano and Eichenbaum (1992) paper and focus on the output effect. Instead of assuming an aggregate production function, production in our model takes place at individual plants that differ in terms of productivity. The plant manager can adjust the plant's output by utilizing capital along three nonconvex margins. First, he can operate the plant or let it remain idle. In this respect our model is similar to the one developed by Cooley, Hansen, and Prescott (1995) and Hansen and Prescott (2000). Second, the manager can choose the number of straight-time shifts.

This is similar to Burnside (2000), Halevy and Nason (2002), and Hornstein (2002). Finally, in the spirit of Hansen and Sargent (1988), the manager can run overtime (weekend) shifts in addition to straight-time shifts.⁴

We calibrate the economy such that the steady-state fraction of plants operating a given shift or using weekend work is in line with U.S. plant-level data. We further ensure that in a steady state the economy can be interpreted as a disaggregated version of the original Christiano and Eichenbaum economy, and that the cyclical behavior of output in the two economies is the same. This makes our model quantitatively comparable with Christiano and Eichenbaum's model, which we take as a benchmark for our experiments. We then compare how the two economies respond to monetary policy shocks. Output, employment, and the inflation rate increase in both economies following an unanticipated fall in the nominal interest rate. In the economy with nonconvexities, however, they increase about 50 percent less than in the benchmark economy.

Focusing on one particular margin, for example, shutting the plant down, the intuition behind the result is as follows. Consider a highly unproductive plant that is shut down. Other things being equal, an interest rate cut reduces the plant's potential losses, but does not make the plant profitable enough to induce the manager to operate it. Similarly, an interest rate increase reduces the profit of a highly productive plant, but not enough to induce the manager to shut it down. Only plants on the margin (i.e., close to the break-even point) change their output and employment decisions in response to interest rate movements. For parameter values consistent with U.S. data, the measure of such marginal plants, for any of the three nonconvex margins we consider, is rather small. Labor demand in the model with nonconvexities therefore increases less in response to a fall in the interest rate than in the standard model.

Most of the increase in labor demand in the model with nonconvexities, however, is attributed to higher demand by relatively more productive plants that can increase output further only by operating a night shift. Because households prefer daytime work to night work, in equilibrium the average wage rate in the model with nonconvexities increases more than the wage rate in

³ This monetary transmission mechanism has been further exploited by Altig, Carlstrom, and Lansing (1995), Chari, Christiano, and Eichenbaum (1995), Christiano and Eichenbaum (1995), and Fuerst (1992). Christiano, Eichenbaum, and Evans (1994, 1999) provide empirical evidence on the liquidity effect, while Barth and Ramey (2001) present evidence on the output effect.

⁴ The main difference between production in our economy and production in Halevy and Nason (2002), Hornstein (2002), and Hansen and Sargent (1988) is that these papers introduce shiftwork or overtime work directly into an aggregate production function.

the standard model. This effect further reduces the relative effectiveness of the MTM in the model with non-convexities.

We obtain this result for responses from a steady state. We therefore also explore the possibility that the measure of the marginal plants can substantially change, and thus that the magnitude of the responses to monetary policy shocks may be significantly different, when aggregate productivity shocks move the economy away from the steady state. We find that for productivity shocks of plausible magnitudes, the responses of output change only negligibly and the responses of the inflation rate change significantly only when monetary policy shocks are persistent. This is despite the fact that aggregate productivity shocks in our model generate fluctuations in aggregate output and capacity utilization (measured by the workweek of capital) of the same order of magnitude as in U.S. data.⁵

The importance of nonconvexities at the micro level for aggregate variables has been studied previously in the context of the business cycle. For example, in Hansen (1985) households can either work a fixed number of hours or not work at all. This nonconvexity has an important aggregate implication: the aggregate intertemporal elasticity of labor supply is much higher than the elasticity of the individual units being aggregated. On the other hand, Thomas (2002) demonstrates that lumpy plant-level investment has little effect on business cycle behavior of aggregate investment. We are not aware of any study, however, that evaluates the importance of micro-level nonconvexities for the MTM.

The paper proceeds as follows. Section 2 describes the two model economies, the benchmark Christiano and Eichenbaum (1992) economy and the economy with nonconvexities. Section 3 describes the models' calibration and Section 4 presents the quantitative findings. Section 5 concludes and provides suggestions for future research. The algorithm used to compute the equilibria is described in the Appendix.

2. Model Economies

In this section, we first describe the economic environment common to both economies. Then, we characterize the optimal plans for one part of the household's problem that is shared by both models. Finally, we introduce into the common framework the production side of each economy and the labor-leisure choice of the household associated with it.

2.1 The Economic Environment

The economies are populated by a representative household, firm, and financial intermediary that take all prices as given. Prices are flexible. In Economy 1, the benchmark economy, the representative firm operates just one (representative) production plant. In Economy 2, the economy with nonconvexities, the firm operates a continuum of (heterogeneous) plants. There is also a monetary authority that issues fiat money.

⁵ Álvarez (2003) finds a positive relationship between capacity utilization and the effectiveness of monetary policy. His model differs from ours in two main respects. First, in his economy demand shocks rather than productivity shocks drive variation in capacity utilization. Second, and more importantly, his model abstracts from plant-level nonconvexities.

The environment is nearly identical to the one in Christiano and Eichenbaum (1992).⁶ At the beginning of period t the household owns capital stock, k_t , and balances of fiat money, m_t . Each period the household is also endowed with one unit of time, which it allocates between leisure and labor.

After learning the current productivity z_t of capital and labor, the household decides how much money it will keep as cash, q_t .⁷ The remaining part of the household's money balances, $m_t - q_t$, is deposited with the financial intermediary. At the end of the period the household receives interest earnings $R_t(m_t - q_t)$, where R_t is the (gross) nominal interest rate set by the monetary authority. The nominal interest rate is revealed after the household has chosen q_t . (We discuss this assumption below.)

The household derives utility from the consumption of goods produced by the firm, c_t , and leisure, l_t . (In Economy 2 the household also has preferences over the time when leisure is consumed.) Consumption expenditures must be financed with cash that comes from two sources: q_t and the current-period nominal labor income e_t , which the household earns in return for supplying labor services to the firm. Investment, i_t , does not have to be purchased with cash.

The household chooses c_t , i_t , and l_t after learning R_t . The period t consumption, investment, and leisure thus depend on z_t , R_t , k_t , and m_t , whereas cash balances depend on z_t , k_t , m_t , and R_{t-1} . This information structure, common to models with liquidity effects, captures the notion that at the time the central bank sets the interest rate, some agents, in this case the household, do not participate in the money market.⁸ As we discuss below, limited participation in the money market is what generates liquidity effects.

The firm rents capital and labor services from the household after observing both z_t and R_t . Its wage bill, equal to e_t , must be paid before the firm sells its output and is fully financed through a bank loan. At the end of the period, the loan is repaid (with interest) using the proceeds from sales, and profits, π_{Ft} , are distributed to the household.

The financial intermediary takes deposits from the household and issues loans to the firm. Intermediation is costless. Besides deposits the intermediary obtains funds from lump-sum injections X_t of fiat money from the monetary authority. Total loanable funds at the intermediary's disposal are thus equal to $m_t - q_t + X_t$.⁹ Free entry ensures that the interest rate charged for loans is equal to the interest rate paid on deposits. At the end of the period, after paying the household its interest earnings, the intermediary is left with net cash position in the amount of $R_t X_t$. This amount is distributed to the household in the form of profits π_{Bt} .

⁶ It differs from Christiano and Eichenbaum's framework only in two respects. First, in their model capital is owned by the firm whereas here it is owned by the household. The firm then rents capital services from the household. This modification has no effect on equilibrium allocations and prices. Second, in Christiano and Eichenbaum's model, the growth rate of fiat money is exogenous, and the nominal interest rate, the price that clears the money market, is determined endogenously. In our models this is reversed. The nominal interest rate is exogenous (set arbitrarily by the monetary authority) and the money growth rate is determined within the model. Such an institutional arrangement allows for a more straightforward discussion of our results.

⁷ We will be more specific about z_t when we describe the details of the firm's problem. In Economy 1 z_t is the Solow residual. In Economy 2 it is closely related to the Solow residual.

⁸ The theoretical justification for limited participation based on fixed costs of portfolio adjustment has been developed by Alvarez and Atkenson (1997), Alvarez, Atkenson, and Kehoe (1999), Alvarez, Lucas, and Weber (2001), and Occhino (2001).

⁹ Note that in this environment the intermediary does not create inside money.

The productivity shock z_t and the net nominal interest rate (R_t-1) follow stochastic processes

$$\log(z_{t+1}) = (1 - \rho_z) \log(\overline{z}) + \rho_z \log(z_t) + \xi_{t+1}$$
(2.1)

and

$$\log(R_{t+1} - 1) = (1 - \rho_R) \log(\overline{R} - 1) + \rho_z \log(R_t - 1) + \zeta_{t+1},$$
(2.2)

respectively, where ρ_z , $\rho_R \in (0, 1)$ and \overline{z} and \overline{R} are the nonstochastic steady-state levels of productivity and the gross interest rate, respectively. The innovations, ξ_t and ζ_t , are normally distributed with mean zero and standard deviations σ_{ξ} and σ_{ζ} , respectively.¹⁰

The money supply $(X_t + m_t)$ must be such that the money market clears at the interest rate the monetary authority wants to implement. That is,

$$m_t + X_t = e_t + q_t \tag{2.3}$$

must hold for a given R_t . Ceteris paribus, when the authority decides to reduce the interest rate, it must increase the supply of fiat money through X_t . Due to the restriction on the ability of the household to adjust its deposits, a fall in the interest rate is consistent with a money supply increase. Since the household does not participate in the money market at the time of the injection, the firm must hold the extra cash in the economy. It is willing to do so only if it is charged a lower interest rate on loans. Without restricting the household's participation in the money market, a monetary expansion would be fully reflected in a price level increase, and the interest rate of return on capital and the inflation rate).¹¹ Finally, the aggregate stock of fiat money evolves as

$$m_{t+1} = m_t + X_t. (2.4)$$

2.2 The (Partial) Household's Problem

Here we describe the representative household's problem of how much of its money stock to keep as cash and how much of its income to consume. These two problems are the same in the two economies and, for a utility function separable in consumption and leisure, can be solved independently from the labor-leisure choice. The labor-leisure choice is economy-specific and we describe it separately for each economy.

In both economies the preferences of the representative household are characterized by the utility function

$$E_t \sum_{t=0}^{\infty} \theta^t \left[\log(c_t) - v_t \right], \qquad (2.5)$$

¹⁰ Although it might be more realistic to assume the interest rate is dictated by a feedback rule, such as the Taylor rule, we use a simple exogenous process in order to eliminate the effect of the rule on the economy. This facilitates a clearer analysis of the transmission mechanism itself.

¹¹ Because the firm's loan is ultimately used to finance household consumption, a monetary injection does increase the price level to some extent even when liquidity effects are present. But the increase is in general smaller than in a model where the household faces no restriction on its ability to adjust deposits. The reason is that a lower interest rate reduces labor costs, which leads to higher output. And to the extent that consumption increases with an increase in output, the economy's money stock is used in a greater volume of transactions. Therefore, in an economy with liquidity effects, the price level (the price that clears the goods market) increases less than proportionally to the increase in the money stock.

where v_t is disutility from work in period $t, \theta \in (0, 1)$ is the discount factor, and the expectation operator E_t reflects the information structure introduced in the previous subsection.¹² We will refer to the expression in the square brackets as the "instantaneous utility function".

The household must obey three constraints. First, as mentioned above, it must obey the "cashin-advance" constraint

$$p_t c_t \le q_t + e_t, \tag{2.6}$$

where p_t is the price level. Second, it must obey the budget constraint

$$p_{t}c_{t} + p_{t}i_{t} + m_{t+1} \leq q_{t} + e_{t} + R_{t} (m_{t} - q_{t}) + p_{t}r_{t}k_{t} + \pi_{Bt} + \pi_{Ft}, \qquad (2.7)$$

where r_t is the real rental rate at which the household rents capital services to the firm. Finally, capital evolves according to the law of motion

$$k_{t+1} = (1 - \delta)k_t + i_t, \tag{2.8}$$

where $\delta \in (0, 1)$ is a depreciation rate.

Ignoring for the moment the household's labor-leisure choice, the household's problem is to choose contingency plans for c_t , i_t , k_{t+1} , q_t , and m_{t+1} in order to maximize (2.5) subject to (2.6)–(2.8).¹³ The first-order conditions for this problem are

$$E_t \left[\frac{1}{p_t c_t} \mid z_t, R_{t-1} \right] = \theta E_t \left[\frac{1}{p_{t+1} c_{t+1}} R_t \mid z_t, R_{t-1} \right]$$
(2.9)

and

$$p_t E_t \left[\frac{1}{p_{t+1}c_{t+1}} \mid z_t, R_t \right] = \theta E_t \left[\frac{1}{p_{t+2}c_{t+2}} p_{t+1} \left(1 + r_{t+1} - \delta \right) \mid z_t, R_t \right].$$
(2.10)

The first condition pertains to the optimal choice of q_t . It states that the household sets the expected marginal utility a dollar buys today when kept as cash equal to the expected marginal utility a dollar will buy tomorrow when deposited today. The second condition is associated with the household's optimal choice of i_t : the household trades off the expected marginal benefit of carrying over a unit of income from period t to period t+1 in the form of fiat money against the expected marginal benefit of saving in the form of capital.¹⁴

¹² Disutility from work will depend on the amount of labor supplied to the firm and, in Economy 2, in addition on the time when it is supplied.

¹³ Since from the stochastic process (2.2) it follows that $(R_t - 1)$ is always positive, constraints (2.6) and (2.7) will hold with equality.

¹⁴ Since at the time the household chooses i_t aggregate uncertainty in period t has been fully revealed, all prices are observed. We can therefore factor p_t out of the expectation operator on the left-hand side of equation (2.10).

2.3 Economy 1: Aggregate Production Function and a Representative Plant

The first economy abstracts from the plant-level nonconvexities. The economy is the same as the one in Christiano and Eichenbaum (1992) and serves as a benchmark for our experiments. We therefore describe it only briefly and refer the reader to the original paper for more details. The economy is characterized by the existence of an aggregate production function

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{2.11}$$

operated by a representative firm/plant unit. Here, y_t is aggregate output, n_t is aggregate employment, and $\alpha \in (0, 1)$. After observing z_t and R_t , the plant chooses k_t and n_t in order to maximize profits

$$\pi_{Ft} = p_t \left(y_t - R_t \omega_t n_t - r_t k_t \right), \qquad (2.12)$$

where ω_t is the period t real wage rate. The first-order conditions for this problem imply

$$n_t = (1 - \alpha)^{\frac{1}{\alpha}} (z_t)^{\frac{1}{\alpha}} (R_t \omega_t)^{-\frac{1}{\alpha}} k_t$$
(2.13)

$$r_t = \alpha A_t k_t^{\alpha - 1}, \tag{2.14}$$

which then give optimal output as

$$y_t = A_t k_t^{\alpha}. \tag{2.15}$$

Here

$$A_{t} \equiv z_{t} n_{t}^{1-\alpha} = (1-\alpha)^{\frac{1-\alpha}{\alpha}} (z_{t})^{\frac{1}{\alpha}} (R_{t}\omega_{t})^{-\frac{1-\alpha}{\alpha}} k_{t}^{1-\alpha}.$$
 (2.16)

At this point, defining A_t may seem redundant. Nevertheless, it will facilitate a better comparison of the two economies later in the text. Note that (ceteris paribus) a fall in the nominal interest rate reduces labor costs and thus increases aggregate employment and output. The nominal wage bill, and thus the firm's demand for loans, is given by

$$e_t = (p_t \omega_t) n_t. \tag{2.17}$$

The labor-leisure choice in this economy is the same as in Hansen (1985); that is, labor is indivisible and there is an insurance market where ex-ante identical households (of measure one) can fully insure against idiosyncratic employment risk. The representative household's instantaneous utility function is therefore

$$\log(c_t) - bn_t, \tag{2.18}$$

where b > 0 is a parameter.¹⁵ Note that this utility function is the same as the instantaneous utility function in (2.5) with $v_t \equiv bn_t$. The household chooses n_t in order to maximize (2.18),

¹⁵ The argument behind the utility function (2.18) is as follows. The households have instantaneous utility function $\log(c_t) + a \log(l_t)$, where a > 0 is the relative weight on utility from leisure. Each household can either work $h \in (0, 1)$ hours and consume $l_t = 1 - h$ units of leisure, or not work at all and consume $l_t = 1$ units of leisure. A lottery determines which households work and which do not. Since the households are ex-ante identical, and the instantaneous utility function is separable in consumption and leisure, the households want to have the same level of consumption regardless of their employment status. The existence of the insurance market makes such allocation possible. Since only a measure n_t of households are employed, the representative household's instantaneous utility function is as in (2.18), where $b \equiv -a \log(1 - h)$.

subject to the cash-in-advance constraint (2.6) and the budget constraint (2.7), with nominal labor income given by (2.17). The first-order condition for this problem is

$$\omega_t = bc_t. \tag{2.19}$$

The equilibrium is characterized by stochastic sequences of c_t , i_t , k_{t+1} , q_t , m_{t+1} , X_t , n_t , r_t , p_t , and ω_t that satisfy the household's first-order conditions (2.9), (2.10), and (2.19); the firm's first-order conditions (2.13) and (2.14); the money market equilibrium condition (2.3), where e_t is given by (2.17); the law of motion for m_t given by (2.4); and the constraints of the household's problem (2.6)–(2.8).

2.4 Economy 2: Nonconvexities and Heterogeneity at the Plant Level

Production

In the second economy output adjustment at the plant level is lumpy. A representative firm operates a continuum (of measure one) of production plants indexed by a pair of idiosyncratic productivity shocks (s, ε) . The shocks are independently and identically distributed across plants and across time. They are drawn independently of each other from normal distributions with density functions $f(s; z_t, \sigma_s)$ and $g(\varepsilon; \kappa z_t, \sigma_{\epsilon})$, where the logarithm of z_t follows the stochastic process (2.1) and $\kappa > 0$ is a ratio of the mean values of the two idiosyncratic shocks. The reason for introducing two shocks is related to the information structure of the plant manager we describe below.

Each period individual plants can adjust output along three margins: intermittent production (i.e., shutting the plant down), shiftwork, and overtime work. In particular, each plant can remain idle or operate one, two, or three shifts. (The shifts can be interpreted as a morning, afternoon, and night shift.) Provided a plant operates a shift during regular hours, it can also run that shift during overtime hours: in addition to a regular five-day workweek, the plant schedules Saturday work for that shift.¹⁶ The volume of output plant (s, ε) generates by running the *j*th shift, $j = \{1, 2, 3\}$, during regular hours in period *t* is

$$y_{jt}^{R}(s) = \begin{cases} \left(\frac{5}{7}h_{j}^{R}\right)sk_{t}^{\alpha}\overline{n}^{\beta} & \text{if } \eta_{jt} \ge \overline{n} \\ 0 & \text{otherwise.} \end{cases}$$
(2.20)

Here, $\alpha, \beta \in (0, 1)$ and $\alpha + \beta \in (0, 1)$, $h_j^R \in (0, 1)$ is the length of the shift during the regular workweek, and η_{jt} is the number of workers employed on that shift. The length of the period is normalized to one and the fraction 5/7 represents the number of days in the regular workweek. If Saturday work is also scheduled, the additional output of the shift is

$$y_{jt}^{o}(\varepsilon) = \begin{cases} \left(\frac{1}{7}h_{j}^{o}\right)\varepsilon k_{t}^{\alpha}\overline{n}^{\beta} & \text{if } \eta_{jt} \ge \overline{n} \\ 0 & \text{otherwise.} \end{cases}$$
(2.21)

Here, $h_j^o \in (0,1)$ is the length of the shift on Saturdays, and the fraction 1/7 represents the extra day that is added to the regular workweek. The shift lengths h_i^R and h_i^o are taken as

¹⁶ Modeling overtime work as running a shift on Saturdays is in line with empirical evidence, in particular for assembly manufacturing (Bresnahan and Ramey, 1994; Mattey and Strongin, 1997).

given.¹⁷ Total output of plant (s, ε) in period t is then

$$y_t(s,\varepsilon) = \sum_{i=1}^3 \left[y_{jt}^R(s) + y_{jt}^o(\varepsilon) \right].$$
(2.22)

The requirement that the number of workers on each shift must be greater or equal to \overline{n} if the shift is to generate positive output introduces a nonconvexity in the plant's choice set. This nonconvexity makes output adjustment at the plant level lumpy. Without such a minimum-staffing requirement, each plant would operate all three shifts and would adjust output smoothly by varying the number of workers on each shift.¹⁸ Since the marginal product of an additional worker beyond the threshold level \overline{n} is zero, whereas (in equilibrium) the marginal cost is positive, the plant will choose $\eta_{it} = \overline{n}$.

At the start of the period, after observing z_t and R_t but before observing (s, ε) of the individual plants, the firm rents capital from the household and allocates it to the plants. Once capital is assigned to a plant, it cannot be changed within the period. Since prior to the realization of plant-specific productivity shocks the plants are identical, the firm distributes capital across them equally. After that each plant learns its productivity shock s and decides whether it will operate that period, and, if it does, how many shifts it will run. Once the number of shifts has been chosen, the plant cannot change it within the period. Each plant then learns its productivity shock ε and decides whether to schedule Saturday work.

This timing captures in a simple form the behavior of establishments found in empirical studies:¹⁹ overtime is the most frequent margin of output adjustment, usually used in the short run for small changes in production volumes; intermittent production and shiftwork are margins used for medium-term, and rather significant, output adjustments; and changes in capital stock are long-term decisions about capacity.

When a plant runs the *j*th shift during the regular workweek, it incurs a fixed cost

$$R_t \left(\frac{5}{7} h_j^R\right) \omega_{jt}^R \overline{n}, \qquad (2.23)$$

where ω_{jt}^R is the real hourly wage rate for work on the shift during regular hours. When the plant runs the shift on Saturdays, the cost is

$$R_t \left(\frac{1}{7} h_j^o\right) \omega_{jt}^o \overline{n}, \qquad (2.24)$$

where ω_{jt}^{o} is the real hourly wage rate for overtime work.

¹⁷ The terms $sk_t^{\alpha}\overline{n}^{\beta}$ and $\varepsilon k_t^{\alpha}\overline{n}^{\beta}$ in the production functions (2.20) and (2.21) represent instantaneous production flows. The distinction between production flows and volumes is in the spirit of Lucas (1970) and the subsequent literature on the workweek of capital (e.g., Bils and Cho, 1994; Kydland and Prescott, 1988).

¹⁸ A minimum-staffing requirement is characteristic for assembly-type technology: a minimum number of workers around an assembly line is needed to operate the line and the marginal product of an additional worker beyond the critical number is small. Cooley, Hansen, and Prescott (1995), Hall (2000), and Hansen and Prescott (2000) use a production structure similar in this respect to ours.

¹⁹ Again, see Bresnahan and Ramey (1994), Mattey and Strongin (1997), and Shapiro (1986).

After learning ε , and conditional on operating the shift during regular hours, a plant schedules Saturday work on the *j*th shift only if output produced during overtime hours is greater or equal to the costs (2.24). Therefore, within plants operating the *j*th shift, plants that run the shift on Saturdays are characterized by

$$\varepsilon \geq R_t \omega_{jt}^o k_t^{-\alpha} \overline{n}^{1-\beta}$$

$$\equiv \phi_{jt}$$
(2.25)

and their conditional measure is

$$\widehat{\mu}_{jt}^{o} = \int_{\phi_{jt}}^{\infty} g\left(\varepsilon; \kappa z_t, \sigma_{\varepsilon}\right) d\varepsilon.$$
(2.26)

Output and profits generated from overtime work on the jth shift by all plants that run the shift during regular hours are therefore, respectively,

$$\widehat{y}_{jt}^{o} = \left(\frac{1}{7}h_{j}^{o}\right)k_{t}^{\alpha}\overline{n}^{\beta}\int_{\phi_{jt}}^{\infty}\varepsilon g\left(\varepsilon;\kappa z_{t},\sigma_{\varepsilon}\right)d\varepsilon$$
(2.27)

$$\widehat{\pi}_{jt}^{o} = \widehat{y}_{jt}^{o} - \widehat{\mu}_{jt}^{o} R_t \left(\frac{1}{7} h_j^{o}\right) \omega_{jt}^{o} \overline{n}.$$
(2.28)

After observing s, but before knowing ε , a plant opens the *j*th shift only if the shift makes nonnegative expected profit. Plants that operate shift *j* are therefore characterized by s that satisfies the inequality

$$\left(\frac{5}{7}h_j^R\right)\left[sk_t^{\alpha}\overline{n}^{\beta} - R_t\omega_{jt}^R\overline{n}\right] + \widehat{\pi}_{jt}^o \ge 0$$

or (after some manipulation)

$$s_{t} \geq k_{t}^{-\alpha} \left(R_{t} \omega_{jt}^{R} \overline{n}^{1-\beta} - \frac{7}{5h_{j}^{R}} (\overline{n})^{-\beta} \widehat{\pi}_{jt}^{o} \right)$$

$$\equiv \lambda_{jt}.$$
(2.29)

The measure of these plants in the economy is

$$\mu_{jt}^{R} = \int_{\lambda_{jt}}^{\infty} f(s; z_t, \sigma_s) ds$$
(2.30)

and their combined output and profits from operating the jth shift during the regular workweek are, respectively,

$$y_{jt}^{R} = \left(\frac{5}{7}h_{j}^{R}\right)k_{t}^{\alpha}\overline{n}^{\beta}\int_{\lambda_{jt}}^{\infty}sf\left(s;z_{t},\sigma_{s}\right)ds$$

$$(2.31)$$

$$\pi_{jt}^R = y_{jt}^R - R_t \left(\frac{5}{7}h_j^R\right) \omega_{jt}^R \overline{n} \mu_{jt}^R.$$
(2.32)

As we will see, in equilibrium, household preferences imply that the first (regular-time) shift is the least expensive to run. Therefore, the measure of plants that are shut down is equal to $(1 - \mu_{1t}^R)$.

The unconditional measure μ_{jt}^{o} of plants with overtime work on the *j*th shift is given by

$$\mu_{jt}^o = \mu_{jt}^R \hat{\mu}_{jt}^o, \tag{2.33}$$

and \widehat{y}^o_{jt} and $\widehat{\pi}^o_{jt}$ contribute to aggregate output and profits, respectively,

$$y_{jt}^{o} = \mu_{jt}^{R} \hat{y}_{jt}^{o}$$
 (2.34)

$$\pi_{jt}^o = \mu_{jt}^R \widehat{\pi}_{jt}^o. \tag{2.35}$$

The firm's profits are obtained by summing the profits from regular and overtime work on the three shifts less rental payments for capital services:

$$\pi_{Ft} = \sum_{j=1}^{3} \left(\pi_{jt}^{R} + \pi_{jt}^{o} \right) - rk_{t}.$$
(2.36)

At the start of the period the firm chooses k_t in order to maximize (2.36) subject to (2.26)–(2.28) and (2.30)–(2.35). The first-order condition for this problem then implies the equilibrium rental rate

$$r_t = \alpha \tilde{A}_t k_t^{\alpha - 1},\tag{2.37}$$

where \widetilde{A}_t is defined as

$$\widetilde{A}_{t} \equiv \overline{n}^{\beta} \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_{j}^{R} \right) \int_{\lambda_{jt}}^{\infty} sf_{t}(s) ds + \mu_{jt}^{R} \left(\frac{1}{7} h_{j}^{o} \right) \int_{\phi_{jt}}^{\infty} \varepsilon g_{t}(\varepsilon) d\varepsilon \right].$$
(2.38)

Aggregate output, y_t , is then given by

$$y_{t} = \sum_{j=1}^{3} (y_{jt}^{R} + y_{jt}^{o})$$

= $\widetilde{A}_{t}k_{t}^{\alpha}$. (2.39)

Note that the expressions for r_t and y_t have the same form as in Economy 1 (equations [2.14] and [2.15]). They only differ from their Economy 1 counterparts in the definition of \tilde{A}_t .

Aggregate employment n_t , the counterpart to n_t in Economy 1 given by equation (2.13), is obtained as

$$n_t = \overline{n} \sum_{j=1}^{3} \mu_{jt}^R.$$
 (2.40)

Out of the workers in the economy that work on the jth shift,

$$n_{jt}^R = \overline{n} \left(\mu_{jt}^R - \mu_{jt}^o \right) \tag{2.41}$$

work regular hours, and

$$n_{jt}^o = \overline{n}\mu_{jt}^o \tag{2.42}$$

work overtime, in addition to regular hours. Finally, the aggregate wage bill e_t , which equals the aggregate demand for loanable funds, is given by

$$e_{t} = \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_{j}^{R} \right) \omega_{jt}^{R} \left(n_{jt}^{R} + n_{jt}^{o} \right) + \left(\frac{1}{7} h_{j}^{o} \right) \omega_{jt}^{o} n_{jt}^{o} \right].$$
(2.43)

Note that through its effect on ϕ_{jt} and λ_{jt} (equations [2.25] and [2.29]), a fall in R_t (other things being equal) increases the measure of plants that operate any given shift or use overtime. This increases aggregate employment and output. Similarly, a positive shock to z_t increases employment by increasing μ_j^R and μ_j^o . The shock has two effects on aggregate output: first, it increases the measure of plants that operate any given shift or use overtime; and second, it increases their productivity.

Labor-Leisure Choice

As in Economy 1, there is a continuum of households of measure one that face idiosyncratic employment risk against which they can fully insure. A household that is employed on the jth shift receives instantaneous utility

$$\log\left(c_{jt}^{\tau}\right) + a_{j}\log\left(l_{j}^{\tau}\right),$$

where

$$l_{j}^{\tau} = \begin{cases} 1 - \frac{5}{7}h_{j}^{R} & \text{if } \tau = R\\ 1 - \frac{5}{7}h_{j}^{R} - \frac{1}{7}h_{j}^{o} & \text{if } \tau = o. \end{cases}$$

Here $a_j > 0$ is the relative weight on utility from leisure. A household that does not work gets

$$\log\left(c_{0t}\right) + a_0\log\left(l_0\right),$$

where $a_0 > 0$ and $l_0 = 1$. A lottery determines which households work on which shift (and whether they work overtime) and which households do not work. The probability of working only regular hours on the *j*th shift is n_{jt}^R ; the probability of working overtime, in addition to regular hours, is n_{jt}^o ; the probability of not working is then $1 - \sum_{j=1}^3 (n_{jt}^R + n_{jt}^o)$. An argument similar to the one in Hansen (1985), and outlined in footnote 15, implies that the representative household has instantaneous utility function

$$\log(c_t) - \sum_{j=1}^{3} \left[b_j^R n_{jt}^R + b_j^o n_{jt}^o \right], \qquad (2.44)$$

where $b_j^R \equiv -a_j \log(1 - h_j^R)$ and $b_j^o \equiv -a_j \log(1 - h_j^R - h_j^o)$. As we will see in the next section, U.S. data on shiftwork and labor market regulations imply

$$b_1^R < b_2^R < b_3^R$$

and

$$b_1^o < b_2^o < b_3^o$$
.

The household thus prefers morning shifts to afternoon and night shifts. Note that the instantaneous utility function is again the same as the instantaneous utility function in (2.5), but now with

$$v_t \equiv \sum_{j=1}^3 \left[b_j^R n_{jt}^R + b_j^o n_{jt}^o \right].$$

The representative household chooses $\{n_j^R, n_j^o\}_{j=1}^3$ in order to maximize the utility function (2.44) subject to the cash-in-advance constraint (2.6) and the budget constraint (2.7), where the nominal labor income e_t is given by (2.43). The optimal labor-leisure choice is characterized by the first-order conditions:

$$\omega_{jt}^R = \frac{7}{5} \left(\frac{b_j^R}{h_j^R} \right) c_t \tag{2.45}$$

$$\omega_{jt}^{o} = 7 \left(\frac{b_{j}^{o} - b_{j}^{R}}{h_{j}^{o}} \right) c_{t}$$
(2.46)

for $j = \{1, 2, 3\}$.

Equilibrium

The equilibrium is characterized by stochastic sequences of c_t , i_t , k_{t+1} , q_t , m_{t+1} , X_t , $\{n_{jt}^R, n_{jt}^o\}_{j=1}^3$, r_t , p_t , and $\{\omega_{jt}^R, \omega_{jt}^o\}_{j=1}^3$ that satisfy the household's first-order conditions (2.9), (2.10), (2.45), and (2.46); the firm's optimality conditions (2.37), (2.41), and (2.42); the money market equilibrium condition (2.3), where e_t is given by (2.43); the law of motion for m_t given by (2.4); and the constraints of the household's problem (2.6)–(2.8).

3. Calibration

Each model economy is calibrated using empirical estimates of steady-state relations among the model's variables and parameters. Measurements from plant-level studies and information from U.S. labor market regulations are also used to calibrate Economy 2. The steady-state values of the models' variables are summarized in Table 1 and the calibrated values of the parameters in turn in Table 2.[FOOTNOTE ON DATA HERE] The steady-state relations implied by (2.8), (2.9), (2.10), (2.14), and (2.15) are the same for the two economies. The values of the parameters obtained from them will therefore apply to both of them. We describe their calibration first and then explain how we calibrate parameters not shared by the two models.

Parameters Shared by Both Models

We interpret the length of the period as one quarter. The parameter α in the expression for output (2.15) equals the models' steady-state capital share of output and is set equal to 0.385. This is in line with estimates obtained for the

United States. We use a quarterly depreciation rate equal to 0.026, which is consistent with the U.S. long-run capital to output ratio of 8.519 and the share of investment in aggregate output equal to 0.223. Without loss of generality, we choose units so that steady-state output is one. For the capital to output ratio of 8.519, equation (2.15) then dictates a steady-state value of A equal to 0.438. The discount factor θ is set equal to 0.981, a value implied by the first-order condition (2.10) for the rate of return 0.045 given by the pricing function (2.14).

Economy 1

The parameter b in the utility function (2.18) is specific to the benchmark economy. As in Hansen (1985) we set the steady-state value of n equal to 0.31. The first-order condition (2.19) for the optimal labor-leisure choice then restricts b to be 2.516. The autocorrelation coefficient and the standard deviation of the innovation in the stochastic process for $\log(z_t)$ are set equal to 0.9 and 0.0067, respectively. These values come from a time series on the Solow residual for the period 1959 Q1–2000 Q4.

Economy 2

There are 17 new parameters in the second economy: \overline{n} , β , $\{h_j^R, h_j^o\}_{j=1}^3$, $\{b_j^R, b_j^o\}_{j=1}^3$, σ_s , σ_{ε} , and κ . Moreover, since in this economy z_t is not equivalent to the Solow residual, we need to parameterize ρ_z and σ_{ξ} in a different way than we did for Economy 1.

We set β equal to 0.58 as in Hall (2000). Because we do not have evidence that shift lengths differ across shifts and across regular-workweek and weekend work, we let $h_j^R = h_j^o = h$ for all j. We set h equal to 1/3, which implies that plants operate three eight-hour shifts.²⁰

For the following discussion, it is convenient to express the wage rates $\{\omega_j^R, \omega_j^o\}_{j=1}^3$ in terms of ω_1^R and overtime and shift premia. We define overtime premia $\{\Delta_j^o\}_{j=1}^3$ as

$$\Delta_j^o \equiv \left(\omega_j^o / \omega_j^R\right) - 1$$

and shift premia Δ_2^R and Δ_3^R as

$$\Delta_2^R \equiv \left(\omega_2^R / \omega_1^R\right) - 1$$

and

$$\Delta_3^R \equiv \left(\omega_3^R / \omega_1^R\right) - 1.$$

The Fair Labor Standards Act requires that a 50 percent premium be paid for hours in excess of 40 hours per week. We therefore set Δ_j^o equal to 0.5 for all j.

There is no legal requirement for shift premia. Using data from the Area Wage Survey (AWS), Shapiro (1986) estimates that for the period 1973–75, the average pay differential was 7.8

²⁰ A similar assumption has been made by Hall (2000). It is supported by King and Williams (1985), who report that three eight-hour rotating shifts are a common arrangement in U.S. manufacturing.

percent for work on the second shift and 10.3 percent for work on the third shift.²¹ He argues, however, that because most firms rotate shiftwork among their workforce, a large part of the premium needed to get workers to undertake it is built into the base wage rate. Shapiro (1995) takes this practice into account and obtains a premium of about 25 percent. Kostiuk (1990) finds that labor heterogeneity (such as union membership or firm size) also causes shift premia from AWS to be seriously underestimated.²² Due to this uncertainty about the true marginal cost of shiftwork to firms, we choose Δ_2^R and Δ_3^R , together with the standard deviations of the idiosyncratic shocks σ_s and σ_{ε} and the ratio of their mean values κ , so that the steady state of the model economy is in line with the observed organization of the workweek of capital in manufacturing.

Mattey and Strongin (1997) provide detailed analysis of the use of various margins of output adjustment in manufacturing based on plant-level data from the Survey of Plant Capacity (SPC). We use their findings for variable work-period industries (industries that primarily adjust production by varying the workweek of capital rather than production flows).²³ Mattey and Strongin find that 27.3 percent of plants that are open operate on average one shift, 40.4 percent operate two shifts, and 32.3 percent operate three shifts. Further, 19 percent of plants use weekend work.

Based on their estimate of the number of weeks per quarter plants are typically open, we calculate that the average plant is shut down for about 0.067 weeks per quarter. In our model this means that 0.067 plants are closed for the whole period. Given the values for overtime premia, we therefore choose values for Δ_2^R , Δ_3^R , σ_s , σ_ε , and κ such that in steady state, (i) the measure of plants ($\mu_1^R - \mu_2^R$) that operate one shift is equal to (1 - 0.067) * 0.273 = 0.255; (ii) the measure of plants ($\mu_2^R - \mu_3^R$) that operate two shifts is equal to 0.377; (iii) the measure of plants μ_1^R that operate three shifts is equal to 0.301; and (iv) the measure of plants μ_1^o that use weekend work is equal to 0.173. We obtain Δ_2^R equal to 0.79, Δ_3^R equal to 1.56, σ_s equal to 0.851, σ_ε equal to 0.802, and κ equal to 0.392.²⁴ The wage premium for the second shift is close to the value of 0.70 obtained by Hornstein (2002).

Using the observed values for overtime premia and our estimates of shift premia and of κ and σ_{ε} , we can calculate overtime work on the second and the third shift. We find that only 0.004 measure of plants use weekend work on the second shift, and $6.602 * 10^{-6}$ measure of them use weekend work on the third shift. The steady-state distribution of plants across the various margins of capacity utilization is summarized in Table 3.

We normalize the base wage rate ω_1^R and the minimum-staffing requirement \overline{n} such that Economy 2 can be interpreted as a disaggregated version of Economy 1. Note that the optimality conditions for labor-leisure choice for Economy 2 encompass the one for Economy 1: adding

²¹ King and Williams (1985) obtain similar values for 1984 for the manufacturing sector and Bresnahan and Ramey (1994) for the period 1972–83 for a panel of plants in the automobile industry.

²² Studies in Anxo et al. (1995) show that shift premia obtained by direct observation from wage data range from 5 percent in the United States to nearly 50 percent in Germany.

²³ For example, assembly manufacturing, such as transportation and machinery industries, belong in this group. Clark (1996) reports that assembly manufacturing accounts for about 20 percent of private sector output and Corrado (1996) claims that it accounts for a large part of cyclical variation in GDP.

²⁴ The steady state value of z_t implied by the model is 2.35. The standard deviations σ_s and σ_{ε} therefore imply coefficients of variation equal to 0.36 and 0.87, respectively. A plant thus faces relatively greater uncertainty about whether it will use overtime work than about how many shifts it will operate. This finding is in line with plant-level observations on the relative frequency of use of the two margins; overtime is used more frequently than adjustment in the number of shifts.

the first-order conditions (2.45) and (2.46) for Economy 2, after first multiplying both sides of the equations by $(n_j^R + n_j^o)$ and n_j^o , respectively, and then summing across the three shifts, we get the optimality condition (2.19) for Economy 1 where

$$b = \sum_{j=1}^{3} \left(b_j^R n_j^R + b_j^o n_j^o \right)$$

and

$$\omega = \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_j^R \right) \omega_j^R \left(n_j^R + n_j^o \right) + \left(\frac{1}{7} h_j^o \right) \omega_j^o n_j^o \right].$$
(3.47)

Equation (3.47) implies that in light of the economy with nonconvexities, the wage rate ω in the benchmark economy can be interpreted as the average weekly wage rate. We therefore choose ω_1^R so that ω in the economy with nonconvexities is equal to the value of ω for the benchmark economy. Similarly, setting n in the economy with nonconvexities equal to 0.31, the value used for the benchmark economy, we obtain \overline{n} from equation (2.40).

Using the values for ω_1^R and for shift and overtime premia, we can calibrate the utility parameters $\{b_j^R, b_j^o\}_{j=1}^3$ from the first-order conditions for labor-leisure choice (2.45) and (2.46). Their values are provided in Table 2. Finally, we assign values to the autocorrelation coefficient and the standard deviation of the innovation in the stochastic process for $\log(z_t)$ so that the volatility of output and its autocorrelation in Economy 2 are the same as in Economy 1. We obtain values: $\rho_z = 0.9$ and $\sigma_{\xi} = 0.009$. Whereas the autocorrelation coefficient for the two economies is the same, σ_{ξ} is larger for the second economy. Thus, nonconvex margins of output adjustment at the plant level somewhat dampen aggregate fluctuations. We discuss this finding in the next section.

The distribution of plants across the various margins of capacity utilization in Table 3 can be used to calculate the steady-state workweeks of capital and labor. The workweek of capital (as a fraction of available time) is given as

$$h_{k} = \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_{j}^{R} \right) \mu_{j}^{R} + \left(\frac{1}{7} h_{j}^{o} \right) \mu_{j}^{o} \right]$$
(3.48)

and the workweek of labor (conditional on the worker being employed) as

$$h_{l} = \frac{1}{n} \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_{j}^{R} \right) n_{j}^{R} + \left(\frac{5}{7} h_{j}^{R} + \frac{1}{7} h_{j}^{o} \right) n_{j}^{o} \right].$$

Their steady-state values implied by the model are 0.464 and 0.243, respectively. In terms of hours, capital therefore works on average 77.9 hours per week and labor 40.7 hours per week. This is in line with U.S. experience.²⁵

²⁵ For example, using SPC data for the period 1974–92, Beaulieu and Mattey (1998) estimate the average workweek of capital in manufacturing to be about 97.0 hours. Based on the same data set, Shapiro (1996) estimates the workweek of capital for 2-digit SIC industries. In the transportation equipment industry (an industry characterized by assembly production), capital operates on average for 73.6 hours a week. Estimates based on AWS data give smaller values. For the period 1951–90 Shapiro (1996) reports an estimate of 54.5 hours for manufacturing. His estimate of the workweek of labor for manufacturing production workers is 40.4 hours. This is close to the steady-state value of 40.7 hours implied by the model.

4. Findings

Although we focus on the MTM, we start by examining the cyclical behavior of the two model economies. For reasonable values of the standard deviations of innovations in the stochastic processes (2.1) and (2.2), productivity shocks take on much greater importance than interest rate shocks in driving aggregate fluctuations in the two models. If the cyclical behavior of the economy with nonconvexities was too different from the cyclical behavior of the U.S. economy, and the cyclical behavior of the benchmark economy, there would be no point proceeding further with the analysis. The model of plant-level behavior in Economy 2 would be clearly inadequate. We find that despite some differences, the economies exhibit similar cyclical behavior, with the same strengths and weaknesses as standard real business cycle models. In addition, Economy 2 is broadly consistent with plant-level observations on the relative contribution of the various margins to output volatility.

After this test we compare the responses of key variables in the two economies to a 100 basis point serially uncorrelated shock to the nominal interest rate. We carry out this experiment under the assumption that the economies are initially in a steady state. For Economy 2, however, we also investigate how the responses of output and the inflation rate change when aggregate productivity shocks move the economy away from the steady state. Finally, we study the responses to highly autocorrelated interest rate shocks. Serially uncorrelated shocks generate large liquidity effects and thus can significantly affect aggregate output. Through inflation expectations, serial correlation reduces the impact of interest rate shocks on output.

4.1 Cyclical Behavior of the Model Economies

Because our main focus is on the responses of the two economies to nominal interest rate shocks that have potentially significant effects on aggregate economic activity, we study the cyclical behavior of the two economies for the case of serially uncorrelated interest rate shocks. $(\overline{R} - 1)$ in the stochastic process (2.2) is set equal to 0.014, the average Federal Funds Rate for the period 1959 Q1–2000 Q4, measured at a quarterly rate, and σ_{ζ} is set equal to 0.136, its estimate from Federal Funds Rate data for the same period. Summary statistics for the cyclical behavior of the model economies are presented in Tables 4 and 5 and those for the U.S. economy in Table 6.²⁶

As we explained in the previous section, the parameters ρ_z and σ_{ξ} of the stochastic process for log z_t for Economy 2 are chosen such that the standard deviation of output and its autocorrelation coefficient in Economy 2 are the same as in Economy 1 (subject to sampling error). Whereas ρ_z turns out to be the same for the two economies, σ_{ξ} in Economy 2 is larger than in Economy 1 (0.009 compared with 0.0067). This implies that the plant-level nonconvexities in Economy 2 somewhat reduce aggregate fluctuations driven by productivity shocks. The reason behind this finding is that employment in Economy 2 responds less to productivity shocks than in Economy 1, as can be seen from the standard deviations (measured relative to that of output) in Tables 4 and 5. Whereas in Economy 1 the relative standard deviation is 0.79, in Economy 2 it is only 0.57. As in a standard business cycle model, in response to a productivity shock, the representative plant in Economy 1 increases employment along the smooth demand curve (2.13). In Economy 2 this margin is not operative. Plants increase employment only if they find it profitable to increase the number of shifts they operate. An increase in aggregate

²⁶ Before computing the business cycle statistics, the artificial series are transformed by taking logarithms and filtered using the Baxter and King (1999) band-pass filter. The corresponding U.S. series are transformed and filtered the same way.

employment is then given by the measure of plants that do so. This measure, however, is not large enough to generate a response in employment of the same magnitude as in Economy 1.

The cyclical behavior of employment in both economies is nevertheless generally in line with U.S. experience.²⁷ The relative standard deviation of employment obtained from establishment survey data is 0.91, somewhat higher than in Economy 1. On the other hand, the relative standard deviation of employment obtained from household survey data is 0.63, which is in the ballpark of the value for Economy 2.

The cyclical behavior of output, consumption, and investment is nearly the same in the two economies and is generally consistent with U.S. experience. The cyclical behavior of the price level, on the other hand, is closer to the cyclical behavior of its U.S. counterpart (measured by a GDP deflator) in Economy 2 than in Economy 1. In Economy 1 the price level is too volatile and its negative contemporaneous correlation with output is much weaker than in the data.

An important aspect of Economy 2 is how well it accounts for the cyclical behavior of the workweek of capital. Unfortunately, quarterly data on the workweek of capital are not available. Findings by Beaulieu and Mattey (1998) based on annual SPC data suggest that the coefficient of variation of the capital workweek in manufacturing is 0.0293. In our model it is 0.0145, which is of the same order of magnitude as in the data, but clearly smaller. This finding suggests that the model does not capture all the dynamics driving the distribution of plants across the margins of capital utilization in response to productivity shocks.

Figure 1 displays the responses of the margins of capital utilization in Economy 2 to a 1-percent positive shock to z_t and their contribution to the increase in aggregate output.²⁸ In the top panel we see that most plants increase output by opening a second regular-time shift, a somewhat smaller measure of plants open the third shift, and a still smaller measure of them start to operate at least one shift. Overtime work is the least important margin. In terms of output, the contribution of the three shifts is about the same, each accounting for roughly one third of the increase in aggregate output. Shiftwork (i.e., adding the second or the third shift) thus contributes to the increase in output twice as much as intermittent production (i.e., operating the first shift). The contribution of overtime work is negligible. Bresnahan and Ramey (1994) find that in the automobile industry, out of the 71.7 percent of output adjustment purposes, 20.6 percent is due to intermittent production, 40.2 percent to shiftwork, relative to that of intermittent production, but underestimates the role of overtime in output adjustments.

4.2 Responses to an Interest Rate Shock from the Steady State

Figure 2 displays the responses of employment and output to an unanticipated 100 basis point fall in the nominal interest rate in a partial equilibrium setting. (The fall is measured at an annual rate, implying a fall at a quarterly rate of about 25 basis points.) Other things being equal, a fall in the interest rate reduces labor costs (the wage bill times the gross interest rate), and employment and output increase in both economies. In Economy 2, however, the increase in

 $^{^{27}}$ Except for employment data, the series are for the period 1959 Q1–2000 Q4. Household survey data on employment are for the period 1954 Q1–1991 Q2 and establishment survey data for the period 1964 Q1–2000 Q4.

 $^{^{28}}$ The response of output is shown as a percentage deviation from steady state and the responses of the measures as percentage-point deviations.

employment is 56 percent smaller and the increase in output 37 percent smaller than in Economy 1. Remember that in Economy 2 a fall in the interest rate leads to higher employment because it increases the measure of plants that operate the three shifts; output can also increase when the measure of plants using overtime increases. For the parameter values of idiosyncratic uncertainty and overtime and shift premia consistent with the observed organization of the capital workweek, the measure of such plants, however, is not large enough to generate responses in aggregate employment and output of the same magnitude as in Economy 1.

Figure 3 shows the responses of key variables to the fall in the nominal interest rate in a general equilibrium setting.²⁹ In general equilibrium the increase in labor demand leads to higher wages in both economies, which partly reduce the positive effect of a lower interest rate on output and employment. Because the partial equilibrium increase in employment is larger in Economy 1 than in Economy 2, we would expect wages to go up more in the former economy than in the latter. This, however, is not the case. As we can see in Figure 3, the average wage rate in Economy 2, given by (3.47), increases more than the wage rate in Economy 1. In general equilibrium, the quantitative differences in the responses of output and employment in the two economies are thus even larger than in partial equilibrium: employment in Economy 2 increases 60 percent less and output 43 percent less than in Economy 1.

The large increase in the average wage rate in Economy 2 can be explained by breaking down the response of aggregate output into the margins of capital utilization. The breakdown is shown in Figure 4. Compared with Figure 1 the use of the margins is now different. In particular, in terms of the measure of plants, regular-time work on the third shift is the most important margin, followed by regular-time work on the second shift and overtime work on the first shift. Regular-time work on the first shift is rather unimportant. This breakdown shows that the interest rate shock primarily affects the output decisions of relatively more productive plants. Because these plants can expand production further only by utilizing capital during times when leisure is most valuable to the household, the average wage rate in Economy 2 increases more than the wage rate in Economy 1, where such "distributional" effects (in terms of which plants are affected) of monetary policy shocks are absent.³⁰

In terms of their contribution to aggregate output, about two thirds of the output increase is due to the increase in the measure of plants operating the third shift, with nearly all of the remaining part attributed to the increase in the measure of plants operating the second shift.

Turning to the other variables in Figure 3, consumption, which is proportional to wage rates by the first-order conditions for the optimal labor-leisure choice, increases in both economies by nearly the same amount. As a result, investment in Economy 2 increases 60 percent less than in Economy 1. The response of the inflation rate, defined as $(p_t/p_{t-1} - 1)$, is also stronger in Economy 1 than in Economy 2. In Economy 1 the inflation rate increases by 5.5 percentage points, while in Economy 2 by only 3 percentage points. The reason for this is that demand for loanable funds in Economy 1 (due to space constraints not included in the figure) increases more than in Economy 2. The money market clearing condition (2.3) then requires that the negative shock to the interest rate be accompanied by a larger injection of money balances in

²⁹ The responses are shown as percentage deviations from steady state and the response of the inflation rate as percentage-point deviations at an annual rate.

³⁰ Plant heterogeneity and nonconvex margins are crucial for the large increase in wages. Preferences for daytime work are not sufficient. When we carry out the same experiment in an economy with the same preferences as in Economy 2, but with a representative plant that operates the three regular-time and overtime shifts, we obtain responses in output and employment similar to those in Economy 1.

Economy 1 than in Economy 2. Because consumption in both economies increases by about the same amount, the price level that clears the goods market must be higher in Economy 1 than in Economy 2.

4.3 Responses to an Interest Rate Shock Conditional on Productivity Shocks

In Figure 1 we saw that aggregate productivity shocks affect the measure of plants that utilize capital along any given margin. Aggregate productivity shocks therefore also affect the measure of the marginal plants that change their production in response to monetary policy shocks. The responses found in the previous experiment might therefore be sensitive to the state of aggregate productivity.

Empirical studies suggest that expansionary monetary policy has a larger (positive) effect on aggregate output in recessions than in expansions.³¹ The literature has proposed two explanations. According to one hypothesis this asymmetry arises due to credit constraints. According to the other, expansionary monetary policy should have a larger effect on output in periods of low economic activity because many firms operate below capacity. During periods of high economic activity, on the other hand, such policy would be mainly reflected in higher prices with little effect on output because capacity is tight.

Economy 2 provides a natural framework for assessing the latter hypothesis. An attractive feature of the economy is that the concept of capacity utilization is made operational by explicitly modeling the margins along which capacity is utilized at the plant level. Furthermore, the variation in the workweek of capital (a measure of capacity utilization in our model) over the business cycle is of the same order of magnitude as in the data.³²

We carry out two experiments. In the first experiment, Experiment A, the economy receives a productivity shock in period 1 followed by no other shocks in subsequent periods. We consider both positive and negative shocks. Three magnitudes of the shocks are considered: one, two, and three standard deviations of the innovation ξ_t in the stochastic process for z_t . In the second experiment, Experiment B, we consider a scenario where the economy is in a steady state characterized by z_t one, two, and three standard deviations away from \overline{z} in each direction. This scenario can be interpreted as the economy drawing a series of shocks that bring it to a low (high) productivity steady state.

In each experiment we study the responses of output and the inflation rate to an unanticipated 100 basis point fall in the nominal interest rate. (In Experiment A the interest rate shock occurs in the same period as the productivity shock.) The responses of output are measured as percentage deviations and the responses of the inflation rate as percentage-point deviations from the paths of output and the inflation rate, respectively, under no interest rate shock (but with the productivity shock present). For a more convenient presentation of the asymmetries,

³¹ Using mainly Markov switching models, such asymmetry has been documented for the United States by Weise (1999), Kakes (2000), Lo and Piger (2002), and Garcia and Schaller (2002), for a number of European countries by Kakes (2000), Peersman and Smets (2001), and María-Dolores (2002), and for the Eurozone by Peersman and Smets (2001) and María-Dolores (2002).

³² We recognize that the margins we consider in this paper are primarily used only in some manufacturing industries, such as durable goods industries. (See Shapiro (1996) for a complete list of industries at the 2-digit SIC level of disaggregation that utilize capacity along the margins considered in this paper.) Nevertheless, as reported in Clark (1996), these industries account for about 20 percent of private sector output and contribute substantially to its cyclical variation. Further, Shapiro (1996) finds that the workweek of capital explains about 70 percent of the variation in capacity utilization as published by the Federal Reserve Board.

we express the deviations relative to those from the nonstochastic steady state presented in Figure 3. Table 7 contains the results of the experiments.

In Experiment A, output increases relatively more in response to the interest rate fall in states where the economy receives a low productivity shock than in states where it receives a high productivity shock. The inflation rate, on the other hand, increases relatively more in high productivity states than in low productivity states. Thus, at least qualitatively, the results of Experiment A are consistent with the hypothesis. Quantitatively, however, the asymmetries are rather small. Looking at the extreme cases, on impact output increases only 4 percent less when ξ_t is equal to $3\sigma_{\xi}$, and only 4 percent more when ξ_t is equal to $-3\sigma_{\xi}$, relative to the response from the nonstochastic steady state. The asymmetries in the responses of the inflation rate are of the same order of magnitude as the asymmetries in the responses of output. In Experiment B, the asymmetries in both the responses of output and the responses of the inflation rate are qualitatively the same and of the same order of magnitude as in Experiment A. These experiments therefore suggest that capacity constraints are an unlikely source of significant asymmetries in the responses of economic activity to monetary policy shocks.

4.4 Autocorrelated Interest Rate Shocks

Figure 5 displays the responses of the key variables to a 100 basis point negative shock to the nominal interest rate when the shock is persistent. In particular, we set ρ_R equal to 0.96. Notice first that on impact consumption in both economies increases substantially more than when the shock is serially uncorrelated (for both economies the increase is more than twice as large). This increase can be explained by the response of the inflation rate. Remember that the restriction on the household's participation in the money market lasts for only one period. Liquidity effects are thus present only in the impact period. Therefore, as in the case of an uncorrelated shock, the fall in the interest rate is accompanied by a rise in the money supply, and the price level increases in the first period (although much less than in the case of uncorrelated shocks). In the subsequent periods the household can fully react to the interest rate shock by adjusting its cash balances, and the inflation rate is determined by Fisherian fundamentals. A lower interest rate thus implies a lower expected inflation rate. Because consumption must be financed with cash, a lower expected inflation tax on cash balances induces the household to increase its consumption expenditures as in Cooley and Hansen (1989). The large increase in consumption, which leads to proportional increases in wage rates, nearly overturns the positive effect of a lower interest rate on employment and output. As a result, employment and output in both economies increase much less than in the case of serially uncorrelated shocks. Due to the small increase in output, investment falls in both economies.

Note that on impact output increases in both economies by nearly the same amount. Serial correlation of the interest rate shocks thus reduces the effect of the nonconvexities on the MTM, but it also significantly reduces the effectiveness of the transmission mechanism in the benchmark economy: output increases by only 0.05 percent, compared with 0.26 percent in the case of serially uncorrelated shocks.

In Table 8 we present the asymmetries in the responses of aggregate output and the inflation rate. In Experiment A, the asymmetries in the responses of output are again small. But the asymmetries in the responses of the inflation rate are rather large. Looking at the extreme cases, on impact the inflation rate increases 29 percent more when ξ_t is equal to $3\sigma_{\xi}$, and 25 percent less when ξ_t is equal to $-3\sigma_{\xi}$, relative to the response from the nonstochastic steady state.

In Experiment B, the asymmetries in the responses of the inflation rate are even larger than in Experiment A. The asymmetries in the responses of output are again small, but reversed compared to Experiment A: output increases more in high productivity states than in low productivity states. Looking again at the extreme cases, when the economy is in a steady state characterized by z_t equal to $(\overline{z} + 3\sigma_z)$, on impact output increases 14 percent more and the inflation rate 83 percent more, relative to the responses from the steady state characterized by z_t equal to \overline{z} ; when the economy is in a steady state characterized by z_t equal to $(\overline{z} - 3\sigma_z)$, the increase in output is 9 percent smaller and the increase in the inflation rate 63 percent smaller compared to the responses from the steady state characterized by z_t equal to \overline{z} . Persistent interest rate shocks thus generate sizeable asymmetries in the responses of the inflation rate, but, as in the case of serially uncorrelated shocks, their effect on aggregate output does not significantly change with aggregate productivity.

5. Concluding Remarks

Micro-level empirical evidence suggests that production plants adjust output along nonconvex margins. In this paper we have attempted to evaluate the effect of such micro-level nonconvexities on one process through which monetary policy decisions are transmitted into the economy. To this end we have replaced the smooth aggregate production function in a prototypical model of the MTM due to Christiano and Eichenbaum (1992) with heterogeneous plants that adjust output along nonconvex margins. We have found that for parameter values consistent with U.S. data, these nonconvexities significantly reduce the effects of monetary policy shocks on aggregate economic activity and prices relative to the standard model. In addition, the quantitative effects of monetary policy shocks on aggregate output do not significantly change with the degree of capacity utilization over the business cycle.

These findings should be interpreted with three caveats in mind. First, the margins we considered are primarily used in manufacturing industries characterized by assembly production. Our model therefore overstates the extent to which they reduce the effect of monetary policy on output and employment in other sectors of the economy. Nevertheless, assembly manufacturing contributes significantly to U.S. private sector output and its volatility, which makes it an important sector for policies directed at stabilizing aggregate output and prices.

Second, the workweek of capital in our model is less volatile than in the U.S. economy and the role of overtime in output adjustment is underestimated. These deviations from the data suggest that our model does not capture all the dynamics in the distribution of plants across the margins of capital utilization following an aggregate productivity shock. The absence of such dynamics is likely to affect the conditional responses of the economy to monetary policy shocks, but not the responses from the steady state.

Finally, the measure of plants that are close to being indifferent between two alternative production schedules, and the relative size of these measures across the various cut-off points, are crucial for the quantitative assessment of the role of the nonconvexities in the transmission mechanism. Although there are data on the fraction of plants that operate a given shift or use overtime, we do not know the distribution of the "distance" of the plants' actual operations from their preferred work schedules. In the current framework the distribution was dictated by the distribution of idiosyncratic productivity shocks. Although this is a reasonable starting point, in the absence of independent measurement it would be interesting to see how our findings change when the distribution of the distance is generated as an outcome of the model. The current framework could be extended in two interesting ways. First, we have assumed that the nominal interest rate follows an exogenous stochastic process. We made this assumption in order to focus on the transmission mechanism rather than on monetary policy itself. It would therefore be interesting to assess the robustness of our findings to the introduction of a monetary policy feedback rule, such as the Taylor rule.

Second, in actual economies plant managers schedule overtime work or close the plant down for a week at a time more frequently than they change the number of shifts. We have tried to accommodate this notion by assuming that the manager observes the productivity of a straighttime shift before he finds out the productivity of an overtime shift. But he makes the decisions within the same period. The model could be extended to make the decisions about the utilization of the margins intertemporal. This could be done, for example, by introducing autocorrelated idiosyncratic shocks and fixed costs of adjusting output along the various margins. In such an environment plants will choose the utilization of the margins according to (S,s) decision rules. This modification could resolve two of the issues raised above: (i) generate more realistic dynamics at the micro level; and (ii) endogenously generate the distribution of the distance between actual and preferred work schedules.

Many decisions made by households and firms involve nonconvexities. For example, firm investment and household purchases of durable goods and housing investment are lumpy. Similarly, decisions by individuals about whether to join the labor force are discrete. The general equilibrium effects of micro-level nonconvexities have been studied previously in the context of the business cycle. But their implications for the aggregate effects of government policies have been largely unexplored. In this paper we have focused on one type of micro-level nonconvexities and studied their effect on the transmission of monetary policy. Our findings, however, illustrate a more general point: In the presence of micro-level nonconvexities, careful modeling of the micro behavior is crucial for the evaluation of the aggregate effects of government policies.

Figures and Tables













Note: The inflation rate is expressed at an annual rate.



Figure 4: Responses of Margins of Capital Utilization to a 100 Basis Point (Serially Uncorrelated) Negative Shock to the Nominal Interest Rate and Their Contribution to Aggregate Output





Note: The inflation rate is expressed at an annual rate.

| Symbol | Value | Data |
|----------------------------------|---------------|---|
| Data used for b | oth models | |
| k | 8.519 | Capital to output ratio |
| С | 0.777 | Consumption to output ratio |
| i | 0.223 | Investment to output ratio |
| n | 0.310 | Share of time spent in market activities |
| | | |
| Data for the mo | del with non- | convexities |
| $\{\Delta_{i}^{o}\}_{i=1}^{3}$ | 0.500 | Overtime premia |
| $\left(1-\mu_1^R ight)$ | 0.067 | Fraction of plants that are closed |
| $\left(\mu_1^R - \mu_2^R\right)$ | 0.255 | Fraction of plants operating one shift |
| $(\mu_2^R - \mu_3^R)$ | 0.377 | Fraction of plants operating two shifts |
| $\hat{\mu}_3^R$ | 0.301 | Fraction of plants operating three shifts |
| μ_1^o | 0.173 | Fraction of plants using weekend work |

Table 1: Long-Run Values of U.S. Data Used in Calibration

| Symbol | Value | Parameter |
|----------------------|----------------|---|
| Parameters | shared by both | economies |
| α | 0.385 | Capital share of output |
| δ | 0.026 | Capital depreciation rate |
| θ | 0.981 | Discount factor |
| Benchmark | model | |
| b | 2.516 | Parameter for disutility from work |
| ho z | 0.9 | Persistence in the productivity shock |
| σ_{ξ} | 0.0067 | Standard deviation of innovation |
| | | in the productivity process |
| Model with | non-convexitie | es |
| h | 1/3 | Shift length |
| eta | 0.580 | Share of labor in production flow |
| \overline{n} | 0.162 | Minimum-staffing requirement |
| κ | 0.38 | Ratio of the mean of ε to the mean of s |
| | | Parameter for disutility from work on: |
| b_1^R | 1.618 | first regular-time shift |
| b_2^R | 2.901 | second regular-time shift |
| b_3^R | 4.140 | third regular-time shift |
| b_1^o | 2.104 | first shift on Saturdays |
| b_2^o | 3.771 | second shift on Saturdays |
| b_3^o | 5.381 | third shift on Saturdays |
| σ_s | 0.851 | Standard deviation of the idiosyncratic shock s |
| $\sigma_{arepsilon}$ | 0.802 | Standard deviation of the idiosyncratic shock ε |
| $ ho_z$ | 0.9 | Persistence in the productivity shock |
| σ_{ξ} | 0.009 | Standard deviation of innovation |
| | | in the productivity process |

Table 2: Parameter Values for the Model Economies

| Symbol | Value | Variable |
|--------------|------------------------|--|
| | | Shift premium for work on |
| Δ_2^R | 0.79 | second shift |
| Δ_3^R | 1.56 | third shift |
| | | Measure of plants operating |
| μ_1^R | 0.933 | first shift |
| μ_2^R | 0.679 | second shift |
| μ_3^R | 0.301 | third shift |
| | | Measure of plants using Saturday work on |
| μ_1^o | 0.173 | first shift |
| μ_2^o | 0.004 | second shift |
| μ_3^o | 6.602×10^{-6} | third shift |
| h_k | 0.464 (77.9 hours) | Workweek of capital |
| h_l | 0.243 (40.7 hours) | Workweek of labor (conditional on |
| | | being employed) |

Table 3: Implied Steady-State Values for Shift Premia, Capital Utilization,and the Workweek of Labor

| Variable x | | | | | | Correlatio | ins with c | utput (y_{H}) | ~ | | | |
|---|------------------|--------------|----------|------------|-----------------|------------|------------------------|-------------------|-----------|----------|-----------|-------------------|
| V ATONTTO L | std. dev. | x(t-5) | x(t-4) | x(t-3) | x(t-2) | x(t-1) | X (t) | x(t+1) | x(t+2) | x(t+3) | x(t+4) | x(t+5) |
| Output (y_t) | 1.33^{\dagger} | -0.16 | 0.00 | 0.28 | 0.61 | 0.89 | 1.00 | 0.89 | 0.61 | 0.28 | 0.00 | -0.16 |
| Employment (n_t) | 0.57 | -0.07 | 0.09 | 0.36 | 0.67 | 0.91 | 0.98 | 0.82 | 0.50 | 0.14 | -0.13 | -0.28 |
| Consumption (c_t) | 0.32 | -0.36 | -0.24 | -0.01 | 0.31 | 0.64 | 0.86 | 0.91 | 0.79 | 0.59 | 0.38 | 0.22 |
| Investment (i_t) | 3.95 | -0.09 | 0.08 | 0.35 | 0.66 | 0.91 | 0.99 | 0.83 | 0.52 | 0.17 | -0.11 | -0.26 |
| Price level (p_t) | 0.52 | 0.01 | -0.07 | -0.20 | -0.34 | -0.45 | -0.50 | -0.45 | -0.34 | -0.20 | -0.08 | 0.00 |
| Table 5: Cvclical Behavio | or of the Be | nchmark | Econom | w (Econ | (1 vmo | | | | | | | |
| | Ralativa | | | |) /- (| Correlatio | ne with c | intruit (ar. | | | | |
| Voriable v | otd dow | | | | | -011 -10 | ר וווז א נווי ייער) | $\frac{g_{t}}{2}$ | (C + 1) | | (1 1). | |
| Vallable X | siu. uev. | (C-1)X | X(l-4) | (c-1)x | (7-1)X | (1-1)X | X(L) | X(l+1) | (7+1)X | (c+1)x | X(l+4) | $(C+1)\mathbf{X}$ |
| Output (y_t) | 1.35^{\dagger} | -0.15 | 0.00 | 0.27 | 0.60 | 0.89 | 1.00 | 0.89 | 0.60 | 0.27 | 0.00 | -0.15 |
| Employment (n_t) | 0.79 | -0.04 | 0.11 | 0.38 | 0.68 | 0.92 | 0.97 | 0.79 | 0.46 | 0.10 | -0.18 | -0.31 |
| Consumption (c_t) | 0.31 | -0.40 | -0.29 | -0.08 | 0.23 | 0.56 | 0.80 | 0.88 | 0.81 | 0.64 | 0.45 | 0.30 |
| Investment (i_t) | 3.78 | -0.07 | 0.09 | 0.35 | 0.67 | 0.92 | 0.98 | 0.82 | 0.50 | 0.15 | -0.13 | -0.27 |
| Price level (p_t) | 0.88 | 0.10 | 0.05 | -0.02 | -0.11 | -0.19 | -0.24 | -0.25 | -0.23 | -0.18 | -0.14 | -0.09 |
| | | | | | | | | | | | | |
| Table 6: Cyclical Behavio | or of the U.S | 6. Econon | ıy, 1959 | Q1-200 | $0 \ Q4$ | | | | | | | |
| | Relative | | | | | Correla | tions with | n output | | | | |
| Variable x | std. dev. | x(t-5) | x(t-4) | x(t-3) | x(t-2) | x(t-1) | x (t) | x(t+1) | x(t+2) | x(t+3) | x(t+4) | x(t+5) |
| Output | 1.74^{\dagger} | 0.03 | 0.26 | 0.52 | 0.76 | 0.94 | 1.00 | 0.94 | 0.76 | 0.52 | 0.26 | 0.03 |
| Employment | | | | | | | | | | | | |
| Household survey* | 0.63 | -0.02 | -0.08 | 0.12 | 0.36 | 0.61 | 0.82 | 0.90 | 0.87 | 0.78 | 0.62 | 0.42 |
| Establishment | 0.91 | -0.17 | -0.03 | 0.16 | 0.38 | 0.63 | 0.83 | 0.88 | 0.80 | 0.65 | 0.46 | 0.25 |
| survey** | | | | | | | | | | | | |
| Consumption | 0.46 | -0.17 | 0.06 | 0.29 | 0.51 | 0.69 | 0.80 | 0.83 | 0.78 | 0.68 | 0.54 | 0.38 |
| Investment | 3.34 | 0.11 | 0.31 | 0.55 | 0.77 | 0.92 | 0.96 | 0.86 | 0.66 | 0.39 | 0.12 | -0.11 |
| GDP deflator | 0.46 | -0.39 | -0.58 | -0.72 | -0.81 | -0.84 | -0.79 | -0.69 | -0.53 | -0.35 | -0.14 | 0.05 |
| [†] Except for y_t standard devia *1954 Q1-1991 Q2. | ations are divi | ded by the | standarc | l deviatio | n of <i>y</i> . | | | | | | | |
| **1964 Q1-2000 Q4. Note: Both the U.S. and the | artificial seri | es are trans | sformed | bv taking | logarith | ms and fi | ltered usi | ng the Ba | uxter and | King (19 | 99) band- | pass filter |

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Table 7: Responses of Aggregate Employment and Output to an Unexpected 100 Basis Point (Serially Uncorrelated) Cut in the Nominal Interest Rate under Fixed Prices

| | Economy 1 | Economy 2 | Ratio |
|------------|-----------|-----------|-------|
| Employment | 0.62 | 0.27 | 0.44 |
| Output | 0.38 | 0.24 | 0.63 |

Table 8: Asymmetries in the Responses of Output and Inflation in Economy 2 to a 100 Basis Point Negative Shock to the Nominal Interest Rate; $\rho_R = 0$

| | Expe | eriment A | |
|---------------------------|------|------------------------------|------|
| $y_t(3\sigma_\xi)/y_t$ | 0.96 | $\pi_t(3\sigma_\xi)/\pi_t$ | 1.06 |
| $y_t(2\sigma_{\xi})/y_t$ | 0.97 | $\pi_t(2\sigma_{\xi})/\pi_t$ | 1.04 |
| $y_t(\sigma_{\xi})/y_t$ | 0.98 | $\pi_t(\sigma_\xi)/\pi_t$ | 1.02 |
| y_t | 0.15 | π_t | 3.00 |
| $y_t(-\sigma_{\xi})/y_t$ | 1.01 | $\pi_t(-\sigma_\xi)/\pi_t$ | 0.98 |
| $y_t(-2\sigma_\xi)/y_t$ | 1.03 | $\pi_t(-2\sigma_\xi)/\pi_t$ | 0.97 |
| $y_t(-3\sigma_{\xi})/y_t$ | 1.04 | $\pi_t(-3\sigma_\xi)/\pi_t$ | 0.95 |
| | | | |
| | Expe | eriment B | |
| $y_t(3\sigma_z)/y_t$ | 0.97 | $\pi_t(3\sigma_z)/\pi_t$ | 1.04 |
| $y_t(2\sigma_z)/y_t$ | 0.97 | $\pi_t(2\sigma_z)/\pi_t$ | 1.03 |
| $y_t(\sigma_z)/y_t$ | 0.98 | $\pi_t(\sigma_z)/\pi_t$ | 1.02 |
| y_t | 0.15 | π_t | 3.00 |
| $y_t(-\sigma_z)/y_t$ | 1.02 | $\pi_t(-\sigma_z)/\pi_t$ | 0.98 |
| $y_t(-2\sigma_z)/y_t$ | 1.06 | $\pi_t(-2\sigma_z)/\pi_t$ | 0.97 |
| $y_t(-3\sigma_z)/y_t$ | 1.10 | $\pi_t(-3\sigma_z)/\pi_t$ | 0.95 |

| | Fixed price | ces | |
|------------|-------------|------------|--------------------|
| | Economy 1 | Econ | omy 2 |
| | | (baseline) | $(\sigma_s = 0.5)$ |
| Employment | 0.62 | 0.27 | 0.32 |
| Output | 0.38 | 0.24 | 0.31 |
| - | | | |
| | | | |

Table 9: Responses Under a Smaller Dispersion of Idiosyncratic Shocks; $\sigma_s = 0.5$

| | General equil | ibrium | |
|----------------|---------------|------------|--------------------|
| | Economy 1 | Econ | omy 2 |
| | | (baseline) | $(\sigma_s = 0.5)$ |
| Employment | 0.42 | 0.17 | 0.15 |
| Output | 0.26 | 0.15 | 0.15 |
| | | | |
| Consumption | 0.08 | 0.09 | 0.12 |
| Investment | 0.90 | 0.37 | 0.21 |
| Wage rate | 0.08 | 0.16 | 0.20 |
| Inflation rate | 5.60 | 3.00 | 3.72 |

Table 10: Responses Under a Smaller Dispersion of Idiosyncratic Shocks; $\sigma_s = 0.5, b_2^R = 3.5, b_3^R = 3.9$

| | Fixed pri | ces | |
|----------------|--------------|------------|----------------------------|
| | Economy 1 | Econ | omy 2 |
| | | (baseline) | (σ_s, b_2^R, b_3^R) |
| Employment | 0.62 | 0.27 | 0.46 |
| Output | 0.38 | 0.24 | 0.44 |
| - | | | |
| | | | |
| | General equi | librium | |
| | Economy 1 | Econ | omy 2 |
| | | (baseline) | (σ_s, b_2^R, b_3^R) |
| Employment | 0.42 | 0.17 | 0.31 |
| Output | 0.26 | 0.15 | 0.29 |
| | | | |
| Consumption | 0.08 | 0.09 | 0.08 |
| Investment | 0.90 | 0.37 | 1.07 |
| Wage rate | 0.08 | 0.16 | 0.21 |
| Inflation rate | 5.60 | 3.00 | 9.38 |

| | | | | Per | riod | | | |
|--|-------|-------|-------|--------|--------|-------|-------|-------|
| t = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | | | | | | | | |
| | | | | Experi | ment A | | | |
| $y(3\sigma_{\xi})/y_t$ | 0.97 | 0.97 | 0.98 | 0.98 | 0.99 | 0.99 | 1.00 | 1.01 |
| $y_t(2\sigma_\xi)/y_t$ | 0.98 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 | 1.00 | 1.01 |
| $y_t(\sigma_{\xi})/y_t$ | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 1.00 | 1.00 | 1.00 |
| y_t | 0.05 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.02 | 0.02 |
| $y_t(-\sigma_{\varepsilon})/y_t$ | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 |
| $y_t(-2\sigma_{\varepsilon})/y_t$ | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.00 | 0.99 |
| $y_t(-3\sigma_{\epsilon})/y_t$ | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.01 | 1.00 | 0.99 |
| 3 | | | | | | | | |
| $\pi_t(3\sigma_{\xi})/\pi_t$ | 1.29 | 0.97 | 0.97 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 |
| $\pi_t(2\sigma_{\varepsilon})/\pi_t$ | 1.19 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 |
| $\pi_t(\sigma_{\epsilon})/\pi_t$ | 1.09 | 0.99 | 0.99 | 0.99 | 0.99 | 1.00 | 1.00 | 1.00 |
| π_t | 0.34 | -0.66 | -0.64 | -0.62 | -0.59 | -0.57 | -0.55 | -0.53 |
| $\pi_t(-\sigma_{\epsilon})/\pi_t$ | 0.91 | 1.01 | 1.01 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 |
| $\pi_t(-2\sigma_{\epsilon})/\pi_t$ | 0.83 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| $\pi_t(-3\sigma_{\varepsilon})/\pi_t$ | 0.75 | 1.03 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 |
| 3 | | | | л · | | | | |
| (2) | 1 1 4 | 1 1 4 | 1 1 4 | Experi | ment B | 1 1 4 | 1 1 4 | 1 1 4 |
| $y(3\sigma_z)/y_t$ | 1.14 | 1.14 | 1.14 | 1.14 | 1.14 | 1.14 | 1.14 | 1.14 |
| $y_t(2\sigma_z)/y_t$ | 1.09 | 1.09 | 1.09 | 1.09 | 1.09 | 1.09 | 1.09 | 1.09 |
| $y_t(\sigma_z)/y_t$ | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 |
| y_t | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.02 | 0.04 |
| $y_t(-\sigma_z)/y_t$ | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 |
| $y_t(-2\sigma_z)/y_t$ $y_t(-3\sigma_z)/y_t$ | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 |
| $g_t(-50_z)/g_t$ | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 |
| $\pi_t(3\sigma_z)/\pi_t$ | 1.83 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |
| $\pi_t(2\sigma_z)/\pi_t$ | 1.54 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |
| $\pi_t(\sigma_z)/\pi_t$ | 1.26 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| π_t | 0.34 | -0.66 | -0.64 | -0.62 | -0.59 | -0.57 | -0.55 | -0.53 |
| $\pi_t(-\sigma_z)/\pi_t$ | 0.76 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 |
| $\pi_t(-2\sigma_z)/\pi_t$ | 0.55 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 |
| $\pi_t(-3\sigma_z)/\pi_t$ | 0.37 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 |

Table 11: Asymmetries in the Responses of Output and Inflation in Economy 2 to a 100 Basis Point Negative Shock to the Nominal Interest Rate; $\rho_R = 0.96$

Appendix: Computation of the Equilibria

In this appendix we describe how we compute the equilibria of the two model economies.³³ For each economy we need to compute aggregate decision rules and pricing functions that generate stochastic sequences of allocations and prices that satisfy the economy's equilibrium conditions. Because we are interested not only in the "average" effects of monetary policy shocks on real activity and prices, but also in how these effects differ conditionally on aggregate productivity shocks, we need to use computational methods that preserve any potential nonlinearities in the decision rules and pricing functions. A (log)-linear approximation, probably the most popular computational tool in applied macroeconomics, is in our case inadequate. Suppose, for example, that an interest rate cut has a larger (positive) effect on aggregate output in states where aggregate productivity is low. A linear approximation to the decision rule for output, such as

$$y_t = a_0 + a_1 z_t + a_2 R_t + a_3 k_t + a_4 m_t,$$

would not pick up the state-dependent effects because it ignores the effect of z_t on the coefficient a_2 .

A suitable method for our purposes is the projection method (sometimes also referred to as the weighted residual method) described in Judd (1992). In particular, we use the collocation technique. The projection method allows us to compute approximations such as

$$y_t = \dots + a_i R_t + a_{i+1} z_t R_t + a_{i+2} z_t R_t^2 + a_{i+3} z_t^2 R_t + a_{i+4} z_t^2 R_t^2 + \dots$$

The nonlinear terms on the right-hand side of the equation (which can also include monomials of higher than second order) pick up nonlinear relations between output and the state variables which a linear approximation would miss.³⁴ Rewriting the right-hand side of the equation as

$$y_t = \dots + a_2 R_t + \dots ,$$

where

$$a_2 \equiv a_i + a_{i+1}z_t + a_{i+2}z_tR_t + a_{i+3}z_t^2 + a_{i+4}z_t^2R_t,$$

we see that the projection method allows us to compute apparently linear rules with statedependent coefficients.

Before we apply the projection method to our model, we reduce the model's size in two respects. First, we reduce the dimension of the state-space in order to mitigate the curse of dimensionality. There are four (continuous) state variables in our economies: z_t , R_t , k_t , and m_t . By appropriate normalization of nominal variables, we can eliminate m_t from this set: we divide p_t , m_t , q_t , and X_t in the equations that characterize the equilibria by m_t ; then we define new variables $\tilde{p}_t \equiv p_t/m_t$, $\tilde{q} \equiv q_t/m_t$, and $\tilde{x} \equiv m_{t+1}/m_t$.

Second, projection methods require the user to solve the nonlinear system of equations that characterize the equilibrium of an economy at a number of nodes in the state space. In order

³³ A Matlab code is available from the author on request.

³⁴ In the actual computation we use Chebyshev polynomials instead of the ordinary polynomials used in this example.

to simplify this step, we reduce the number of equilibrium conditions by substitutions. First, we eliminate \tilde{p}_t by substitution from the cash-in-advance constraint, which after normalization has the form

$$\widetilde{p}_t c_t = \widetilde{x}_t. \tag{A1}$$

This allows us to write the Euler equations (2.9) and (2.10), respectively, as

$$E_{t}\left[\frac{1}{\tilde{x}_{t}} \mid z_{t}, R_{t-1}\right] = \theta E_{t}\left[\frac{1}{\tilde{x}_{t}}R_{t}E_{t+1}\left[\frac{1}{\tilde{x}_{t+1}} \mid z_{t+1}, R_{t}\right] \mid z_{t}, R_{t-1}\right]$$
(A2)
$$\frac{1}{c_{t}}E_{t}\left[\frac{1}{\tilde{x}_{t+1}} \mid z_{t}, R_{t}\right] = \theta E_{t}\left[\frac{1}{c_{t+1}}\left(1 + r_{t+1} - \delta\right) \times E_{t+1}\left[\frac{1}{\tilde{x}_{t+2}} \mid z_{t+1}, R_{t+1}\right] \mid z_{t}, R_{t}\right].$$
(A2)

Further, we can write the money market equilibrium condition (2.3), after we have substituted for e_t , as

$$\widetilde{x}_t = \left[1 - \widetilde{e}_t\right]^{-1} \widetilde{q}_t. \tag{A4}$$

Here

$$\widetilde{e}_{t} = \begin{cases} b\left(1-\alpha\right)^{\frac{1}{\alpha}}\left(z_{t}\right)^{\frac{1}{\alpha}}\left(R_{t}bc_{t}\right)^{-\frac{1}{\alpha}}k_{t} & \text{for Economy 1} \\ \\ \overline{n}\sum_{j=1}^{3}\left(b_{i}^{R}\mu_{jt}^{R}+\left(b_{i}^{o}+b_{i}^{R}\right)\mu_{jt}^{o}\right) & \text{for Economy 2,} \end{cases}$$

where μ_{jt}^R and μ_{jt}^o are given by (2.30) and (2.33), with the wage rates eliminated by substitutions from the household's first-order conditions (2.45) and (2.46). Finally, we eliminate r_{t+1} from the Euler equation (A3) by substitution from the pricing function

$$r_t = \alpha A_t k_t^{\alpha - 1} \tag{A5}$$

shifted one period forward. Here A_t is given by equation (2.16) for Economy 1 and equation (2.38) for Economy 2.

In order to form expectations about the future state of the economy, the household uses the laws of motion (2.1) and (2.2) to forecast z_t and R_t , respectively, and the law of motion

$$k_{t+1} = A_t k_t^{\alpha} + (1 - \delta) k_t - c_t$$
(A6)

to forecast the capital stock. Here again, A_t is given by equations (2.16) or (2.38), depending on which economy we want to compute.³⁵

After these substitutions, we are left with just two Euler equations in two unknowns, c_t and \tilde{q}_t . The objects we need to compute are approximations to the decision rules $c_t = c(z_t, R_t, k_t)$ and $\tilde{q}_t = q(z_t, k_t, R_{t-1})$ that satisfy the two Euler equations. First, however, the Euler equations

³⁵ The law of motion for the capital stock (A6) is the goods market clearing condition. Although this equilibrium condition is not used in the definitions of the equilibria of our model economies, we know that it holds by Walras law.

themselves have to be approximated because the expectations in (A2) and (A3) do not have closed-form solutions. We approximate them using a Gauss-Hermite quadrature with seven nodes. In addition, in Economy 2 we need to approximate the measures

$$\widehat{\mu}_{jt}^{o} = \int_{\phi_{jt}}^{\infty} g\left(\varepsilon; \kappa z_{t}, \sigma_{\varepsilon}\right) d\varepsilon$$

$$\mu_{jt}^{R} = \int_{\lambda_{jt}}^{\infty} f\left(s; z_{t}, \sigma_{s}\right) ds$$

and the truncated means

$$\int_{\phi_{jt}}^{\infty} \varepsilon g\left(\varepsilon; \kappa z_t, \sigma_{\varepsilon}\right) d\varepsilon$$
$$\int_{\lambda_{jt}}^{\infty} sf\left(s; z_t, \sigma_s\right) ds.$$

Neither the measures nor the means have closed-form solutions. For the measures we use an approximation suggested by Bagby (1995):

$$\widehat{\mu}_{jt}^{o} \simeq \begin{cases} 0.5 - \Phi(\widehat{\varepsilon}_{t}) & \text{ if } \widehat{\varepsilon}_{t} < 0\\ 0.5 + \Phi(\widehat{\varepsilon}_{t}) & \text{ if } \widehat{\varepsilon}_{t} \ge 0, \end{cases}$$

where

$$\Phi(\widehat{\varepsilon}_t) \equiv 0.5 \left\{ 1 - \frac{1}{30} \left[7 \exp\left(-\frac{\widehat{\varepsilon}_t^2}{2}\right) + 16 \exp\left(-\widehat{\varepsilon}_t^2 \left(2 - \sqrt{2}\right)\right) + \left(7 + \frac{\pi}{4}\widehat{\varepsilon}_t^2\right) \exp\left(-\widehat{\varepsilon}_t^2\right) \right] \right\}^{\frac{1}{2}}$$

and

$$\widehat{\varepsilon}_t \equiv \left(\frac{\phi_{jt} - \kappa z_t}{\sigma_{\varepsilon}}\right).$$

The truncated mean of ε is then obtained using Bagby's approximation as

$$\int_{\phi_{jt}}^{\infty} \varepsilon g\left(\varepsilon; \kappa z_t, \sigma_{\varepsilon}\right) d\varepsilon \simeq \kappa z_t \left[1 - \Phi(\widehat{\varepsilon}_t)\right] + \frac{\sigma_{\varepsilon}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\widehat{\varepsilon}_t^2\right).$$

The same method is used to approximate μ_{jt}^R and the truncated mean of s.

The decision rules $c_t = c(z_t, R_t, k_t)$ and $\tilde{q}_t = q(z_t, k_t, R_{t-1})$ are then approximated by functions

$$\widehat{c}(z_{t}, R_{t}, k_{t}) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ijk} \Psi_{i}(z_{t}) \Psi_{j}(R_{t}) \Psi_{k}(k_{t})$$

$$\widehat{q}(z_{t}, k_{t}, R_{t-1}) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} b_{ijk} \Psi_{i}(z_{t}) \Psi_{j}(k_{t}) \Psi_{k}(R_{t-1})$$

where $\Psi_i(z_t) \equiv T_{i-1} \left(2\left(\left(z_t - z_m \right) / \left(z_M - z_m \right) \right) - 1 \right)$. Here T_{i-1} is the *i*th-order Chebyshev polynomial and z_m and z_M are the lower and upper bounds for z_t . $\Psi_j(R_t)$ and $\Psi_k(k_t)$ are defined similarly.³⁶

The unknowns of the computational procedure are the coefficients of the approximate decision rules $\{a_{111}...a_{ijk}...a_{nnn}\}$ and $\{b_{111}...b_{ijk}...b_{nnn}\}$. Using the collocation technique, the unknowns are obtained as a solution to a system of $2n^3$ equations in $2n^3$ unknowns: each Euler equation is evaluated on n^3 nodes in the state space. These nodes are the ordered pairs of n Chebyshev zeros in each dimension of the state space.

The solution is obtained in three steps. We start with n = 2. First, we make an initial guess about the coefficients. We choose them so that the decision rules are linear and pass through the steady state. In addition, we require that c_t and \tilde{q}_t are zero when either z_t or k_t is zero. Then, we carry out a couple of iterations using the Levenberg-Marquardt algorithm (see Judd, 1998, p. 119) in order to get "near" the solution. The solution is finally obtained with Powell's method (see Judd, 1998, p. 173), which takes the output of the Levenberg-Marquardt algorithm as its input. The solution for n = 2 is then used as an initial guess for n = 3. We know from the Chebyshev Approximation Theorem that as $n \to \infty$, $\hat{c}(z_t, R_t, k_t) \to c(z_t, R_t, k_t)$ and $\hat{q}(z_t, k_t, R_{t-1}) \to q(z_t, k_t, R_{t-1})$ uniformly. Further, as $n \to \infty$, the coefficients of the monomials with an increasingly higher order go to zero. For our economies, n = 2 is, in fact, sufficient; the marginal improvement in the precision in the decision rules when n is increased to 3 is near zero. The approximations to the decision rules we use thus have the form

$$c = a_{111} + a_{112}k + a_{121}R + a_{122}Rk + a_{211}z + a_{212}zk + a_{221}zR + a_{222}zRk q = b_{111} + b_{112}k + b_{121}R + b_{122}Rk + b_{211}z + b_{212}zk + b_{221}zR + b_{222}zRk.$$

³⁶ The lower and upper bounds for the state variables are chosen such that with 99 percent confidence the variables stay within the bounds.

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