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Abstract

This paper investigates the source of predictability of bond risk premia by means of long-term forward interest rates. We show that the predictive ability of forward rates could be due to the high serial correlation and cross-correlation of bond prices. After a simple reparametrization of models used to predict spot rates or excess returns, we find that forward rates exhibit much less predictive power than previously recorded. Furthermore, our economic value analysis indicates that there are no economic gains to mean-variance investors who use the predictions of these models in a stylized dynamic asset allocation strategy.

JEL classification: G0; G1; E0; E4.

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1 Introduction

The predictability of bond risk premia has occupied the attention of financial economists for many years. Several studies have reported consistent evidence that empirical models based on forward rates or forward spreads are able to generate accurate forecasts of future bond risk premia (or excess returns) Since forward rates represent the rate on a commitment to buy a one-period bond in a future date, it is natural to hypothesize that they incorporate information that is useful for predicting future bond risk premia (or excess returns). In support of this conjecture Fama and Bliss (1987, henceforth FB) find that the forward-spot spread has predictive power for the change in the spot rate and its forecasting power increases as the forecast horizon lengthens. This evidence is confirmed by Campbell and Shiller (1991), who show that bond yield changes can be forecast by means of yield spreads. Recently, Cochrane and Piazzesi (2005, henceforth CP) extend FB's original work by proposing a framework in which bond excess returns are forecast by initial forward rates. They find that their specification is able to account for more than 30 percent of the variation of bond excess returns one to five years ahead over the period January 1964 - December 2003.[1] Furthermore, their specification, in contrast with FB, is able to explain about 19 percent of the one-year-ahead change in the one-year bond yield.[2] The predictability recorded in these studies strongly corroborates the well-known empirical failure of the Expectations Hypothesis of the term structure of interest rates (Fama, 1984; Stambaugh, 1988; Bekaert *et al.*, 1997, 2001; Sarno *et al.*, 2007) and it is generally assumed to be the consequence of the slow mean reversion of the spot rate toward a time-invariant equilibrium anchor that becomes more evident over longer horizons (Fama 1984, 2003, and the references therein).

Although the proposed theoretical and empirical rationales are intuitively appealing, the predictive ability of forward rates is not immune from criticism. In fact, two key statistical properties of bond yields data, namely their high serial correlation and the similarly high correlation across maturities, pose serious econometric problems when estimating and evaluating the predictive performance of empirical models based on forward rates. If both regressors and regressands exhibit a high serial correlation, the predictive regressions based

[1] Cochrane and Piazzesi (2009) confirm these results using the dataset constructed by Gürkaynak *et al.* (2007), which includes a larger set of maturities.

[2] However, not all studies are supportive of the predictive power of forward rates. In fact, Hamburger and Platt (1975), Fama (1984), and Shiller *et al.* (1985) find weak evidence that forward rates predict future spot rates.

on forward rates may suffer from a spurious regression problem (see Ferson *et al.*, 2003a,b, and the references therein).[3]. This argument is echoed in Dai *et al.* (2004) and Singleton (2006) who show that the predictive regressions, such as CP and FB, are affected by a small-sample bias which causes the R^2 statistics to be substatially higher than their population values. By the same token, Duffee (2002) demonstrates that it is very difficult to improve upon the performance of forecasts obtained by bond yields that follow random walks.[4]

In this paper, we investigate the predictive ability of empirical models based on forward rates and innovate on two fronts. First, we propose a statistical framework that mitigates the spurious effect generated by the high serial correlation of individual bond prices and their similarly high cross-correlation among various maturities. Second, we move beyond a purely statistical perspective and provide evidence on whether the predictive information in forward rates is also economically significant. It is well known that statistical significance does not necessarily imply economic significance (Leitch and Tanner, 1991; Fleming, *et al.*, 2001; Della Corte *et al.*, 2008a,b,c). Moreover, it is unclear whether statistical tests of predictability are powerful enough to discriminate among competing predictive variables or models (Inoue and Kilian, 2004, 2006).

To preview our main results, we show that bond yields, forward rates, holding period returns, and excess returns used in these models are all simple linear functions of bond prices. Consequently, the same prices appear on both sides of the equations routinely used to evaluate the predictive power of forward rates. Since bond prices (and hence bond yields) are highly correlated over time and across maturities, it is possible that some of the explanatory power of forward rates is spurious. This spurious regression problem is mitigated by rewriting the relevant equations in an observationally equivalent form where variables that appear on the left-hand side of the equations are not present on the right-hand side. A statistical assessment of these reparametrized equations suggests that forward rates have some predictive power, but much less than previously recorded in the literature. Moreover, this predictive power is present only at longer maturities and only recorded by some statistics. The economic assessment of these models suggests that there are no

[3]The evidence of the near unit-root nature of bond yields is strenghtened by other studies that record that the slow mean reversion of the spot rate toward a constant is no longer valid after 1986 (Fama, 2003) and its dynamics are better approximated by a mean-reverting process that is anchored to a nonstationary central tendency that stochastically changes over time (Balduzzi *et al.*, 1998).

[4]Duffee and Stanton (2007) also show that the high persistence of interest rates has important implications for the preferred method used to estimate term structure models.

economic gains to an investor who allocates capital between the one-period and the n-period bonds simply using the predictions implied by models based on forward rates.

The reminder of the paper is as follows: Section 2 defines the variables used in the empirical analysis and outlines the empirical framework. Section 3 discusses in detail the reparametrization of the FB and CP regressions in terms of bond prices and explores the behavior of their coefficients of determination, R^2, under a variety of data generating processes (DGPs) encompassing different assumptions about the time-series properties of bond yields. Section 4 assesses the predictive ability of both FB and CP regressions from a statistical perspective. In Section 5, we outline the framework for measuring the economic value in a mean-variance setting and report the results of using economic value measures to assess the predictive power of forward rates. Section 6 concludes.

2 The empirical framework

Define the log yield of a n-year bond as

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}, \tag{1}$$

where $p_t^{(n)}$ is the log price of an n-year zero-coupon bond at time t, i.e., $p_t^{(n)} = \ln P_t^{(n)}$, where $P^{(n)}$ is the nominal dollar-price of zero coupon bond paying \$1 at maturity. A forward rate with maturity n is then defined as

$$f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)}. \tag{2}$$

The excess return of an n-year bond is computed as the log holding period return from buying an n-year bond at time t and selling it at time t+1 less the log return on a 1-year bond at time t,[5]

$$rx_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)}. \tag{3}$$

FB estimate a linear regression of the change in the 1-year bond yield on the forward-spot spread,

$$y_{t+1}^{(1)} - y_t^{(1)} = \mu + \varphi(f_t^{(n)} - y_t^{(1)}) + \upsilon_{t+1}^{(1)}. \tag{4}$$

[5] It is instructive to note that with monthly data the one-year excess return on a n-year bond is computed as $rx_{t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)} - y_t^{(n)}$. However, for comparability purposes, the notation adopted throughout the paper follows the one in CP and FB.

They also estimate a related excess return equation of the form

$$rx_{t+1}^{(n)} = \alpha + \beta(f_t^{(n)} - y_t^{(1)}) + \upsilon_{t+1}^{(n)}. \tag{5}$$

Note that equation (5) is equal to equation (4), with opposite sign and $\beta = (1 - \varphi)$, when $n = 2$.

CP estimate a modified version of equation (5) as follows:

$$rx_{t+1}^{(n)} = \beta_0 + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + ... + \beta_5 f_t^{(5)} + \varepsilon_{t+1}^{(n)}. \tag{6}$$

Because the log yields, forward rates, holding period yields, and excess returns are all calculated solely from bond prices, equation (6) can be rewritten as

$$p_{t+1}^{(n-1)} - p_t^{(n)} + p_t^{(1)} = \beta_0 + \beta_1(-p_t^{(1)}) + \beta_2(p_t^{(1)} - p_t^{(2)}) + ... + \beta_5(p_t^{(4)} - p_t^{(5)}) + \varepsilon_{t+1}^{(n)}. \tag{7}$$

Following the same line of reasoning, FB's equations (4) and (5) can be also rewritten as

$$-p_{t+1}^{(1)} + p_t^{(1)} = \mu + \delta(p_t^{(n-1)} - p_t^{(n)} + p_t^{(1)}) + \upsilon_{t+1}^{(1)} \tag{8}$$

$$p_{t+1}^{(n-1)} - p_t^{(n)} + p_t^{(1)} = \alpha + \beta(p_t^{(n-1)} - p_t^{(n)} + p_t^{(1)}) + \upsilon_{t+1}^{(n)}. \tag{9}$$

We begin by estimating equations (7), (8), and (9). The equations are estimated over both CP's sample period, January 1964 - December 2003, and FB's sample period, January 1964 - December 1985.[6,7] The estimates are presented in Table 1. Estimates of equation (7) are identical to CP's estimates of equation (6). In particular, the tent shape of the coefficients, noted by CP, is evident over both sample periods. The predictive power of forward rates is slightly higher at the short end and slightly lower at the long end over the shorter sample.

Estimates of (8) and (9) are very similar to the estimates of equations (4) and (5) reported by FB over their sample period. Consistent with Cochrane and Piazzesi's (2004) findings, however, there is a marked decline in the predictive power of the FB equation (8) over the full sample compared with the shorter sample. For example, in the case of $n = 5$, the estimated $\overline{R}^2$ is 0.47 for the shorter sample, compared with only 0.11 for the full sample. The predictive power of equation (9) is fairly stable over the two sample periods, with $\overline{R}^2$ slightly higher over the longer sample across all maturities.

[6] We thank Monika Piazzesi for providing us with the dataset employed in Cochrane and Piazzesi (2004a, 2005).

[7] Fama and Bliss (1987) estimate the equation over different sample periods reflecting data lost in the construction of the variables. For example, when $n = 5$ the sample period is January 1964- December 1981. Our estimates over this period are very similar to theirs and our actual and predicted values are very similar to those reported by Fama and Bliss (1987, p. 687, Figure 2). As n is reduced by one, Fama and Bliss' sample period increases by 12 months. See also footnote 3.

3 Bond risk premia predictability: A simple example and Monte Carlo evidence

The reparametrization of equations (4), (5), and (6) into equations (7), (8), and (9) makes clear that some of the variables that appear on the left-hand side of these equations also appear on the right-hand side. This, coupled with the high serial correlation of individual bond prices, gives rise to the possibility that the predictability of bond risk premia reported by CP and FB may be affected by a spurious regression problem. This section explores this possibility. Specifically, we investigate the behavior of R^2 from these equations by assuming a variety of DGPs encompassing several assumptions regarding the time series properties of bond yields. We first consider the simplest possible DGP for bond yields, i.e., they are assumed *iid*. This assumption allows us to obtain a closed-form solution for the R^2 of the CP equation. We then relax the hypothesis of *iid* bond yields to carry out simple comparative exercises using more realistic DGPs based on the empirical estimates obtained from the zero coupon data used in this study.

Consider the reparametrized CP equation (7) under the assumption that bond yields are *iid*. It is easy to demonstrate that:[8]

$$R^2_{(n)} = \frac{var\left(y_t^{(1)}\right) + n^2 var\left(y_t^{(n)}\right)}{var\left(y_t^{(1)}\right) + (n-1)^2 var\left(y_t^{(n-1)}\right) + n^2 var\left(y_t^{(n)}\right)} \quad \text{for } n = 2, .., 5, \qquad (10)$$

where $R^2_{(n)}$ is the theoretical R^2 obtained from the CP predictive regression for maturity n. Equation (10) shows that $R^2_{(n)}$ is decreasing as n increases and the minimum $R^2_{(n)}$ attainable by the CP predictive regression equals 0.5.[9]

Of course, actual bond yields are not *iid*. In fact they are correlated both serially *and* cross-sectionally. However when more realistic bond yields' DGPs are employed, no closed-form solutions can be obtained. Hence, to illustrate the effect of different bond yields' correlation patterns on the distribution of $R^2_{(n)}$ from both CP and FB predictive regressions, we carry out a set of Monte Carlo exercises. Specifically, we assume that bond yields $y_t^{(n)}$

[8] Full details of the analytical derivations are reported in Appendix A.

[9] When the assumption of *iid* bond yields is relaxed by assuming that bond yields are serially correlated, the resulting R^2 increases toward the value of unity the larger the first-order correlation coefficient. Full details of the analytical derivations and a brief discussion of these results are reported in Appendix A.

follow a vector autoregressive process of order p (VAR(p)):

$$\mathbf{y}_t = \boldsymbol{\gamma} + \sum_{i=1}^{p} \Gamma_p \mathbf{y}_{t-p} + \Phi \mathbf{e}_t,$$

where $\mathbf{y}_t = \left[y_t^{(1)}, ..., y_t^{(5)}\right]'$ is a (5×1) vector of bond yields, $\boldsymbol{\gamma}$ is (5×1) vector of intercept terms, Γ and Φ are (5×5) matrices of parameters, and $\mathbf{e}_t \sim NIID(0, 1)$ is a vector of residuals. In what follows we impose the following assumptions:

- DGP 1: $\mathbf{y}_t = \boldsymbol{\gamma} + \Phi \mathbf{e}_t$, with Φ diagonal
- DGP 2 :$\mathbf{y}_t = \boldsymbol{\gamma} + \Gamma_1 \mathbf{y}_{t-1} + \Phi \mathbf{e}_t$, with Γ_1 and Φ diagonal
- DGP 3: $\mathbf{y}_t = \boldsymbol{\gamma} + \Gamma_1 \mathbf{y}_{t-1} + \Phi \mathbf{e}_t$, with Γ_1 and Φ full rank
- DGP 4: $\mathbf{y}_t = \boldsymbol{\gamma} + \sum_{i=1}^{p} \Gamma_p \mathbf{y}_{t-p} + \Phi \mathbf{e}_t$, with $p = 12$ and Γ_p and Φ full rank.

The DGPs 1 to 4 encompass a wide range of assumptions about bond yields. We consider the following: full independence in mean (DGPs 1 and 2), full independence in variance (DGPs 1 and 2), persistence in mean (DGPs 2 to 4) and dependence across yields in mean and variance (DGPs 3 and 4). We executed a battery of Monte Carlo experiments based on the DGPs 1 to 4 calibrated on the estimates of the individual DGPs on the data under investigation with *iid* Gaussian innovations. Initializing the artificial series at zero, we generated 5,000 samples of 959 observations and discarded the first 500, leaving 5,000 samples of 459 observations, matching exactly the total number of observations used in this study. The first 500 observations are discarded to reduce to the impact of the initialization.

For each generated sample and each individual DGP we estimate the CP predictive regression (7) and the FB equation (9). The average and the 5th and 95th percentile of the empirical distribution of $\overline{R}^2$ for all maturities are reported in Table 2. We find considerable predictive power for the forward rates when bond yields are assumed to be independent (i.e., DGP 1). The averages of the empirical distributions of $\overline{R}^2$ for the CP regression range between 0.613 and 0.829 and are very similar to the analytical estimates of R^2 discussed earlier in this section. Estimates relative to the FB equation are somewhat lower, ranging between 0.375 and 0.744.

When bond yields for all maturities are assumed to be highly serially correlated (DGP 2), but not cross-correlated, with an identical autoregressive root of about 0.98 across bond maturities, the average $\overline{R}^2$ increases across bond maturities for both CP and FB specifications. The results of these exercises suggest that the high R^2 recorded in Table 2 may be caused by a spurious regression problem.

If bond yields are assumed to follow a full VAR(1) (DGP 3), the results exhibit mean $\overline{R}^2$ which are in line with the values reported by CP and FB and similar to those reported in Table 1. The overall picture does not change when the assumption of a single lag entering the VAR model for bond yields is relaxed (DGP 4). In fact, CP argue that VAR(1) models may not be able to capture the predictability patterns recognizable at the annual frequency. In order to take this issue into account we estimated a DGP where the maximum lag length is set equal to 12.[10] The results reported in the last line of Table 2 exhibit average $\overline{R}^2$ values which are slightly higher than those obtained under DGP 3 with differences ranging between 2 and 10 percent.[11]

Cochrane and Piazzesi (2004b, p. 2) attempt to deal with the issue of the same variable appearing on both sides of the regression by noting that 'the forecasts work quite well with lagged right hand variables, in which case the same p_t is not on both sides of the regressions'. As a robustness exercise we carried out the full set of Monte Carlo experiments by estimating equations (7) and (9) using the variables on the RHS one-month lagged. The results of this exercise, not reported to save space, show that for DGP 2 - 4 the empirical distributions of $\overline{R}^2$ is qualitatevely and quantitatively similar to the ones reported in Table 2. However, when bond yields are assumed to be independent (DGP1) the average $\overline{R}^2$ across bond maturities and empirical specifications, are equal to zero. This evidence suggest once again that the high $\overline{R}^2$ recorded in the literature may be due to the fact that both regressors and regressands exhibit a high serial and cross-correlation.[12]

Overall, the results reported in this section corroborate with the early findings by Dai *et*

[10] More precisely, the employed DGP is a reduced version of a full VAR(12) where a general-to-specific procedure has been applied to avoid parameter proliferation. Nevertheless, in some equations the maximum significant lag length is equal to 12.

[11] The results of DGP 1 and DGP 2 show that both CP and FB equations can generate evidence of high predictive power of forward rates even in the case where bond yields are *iid*. In the case of DGP 3 and DGP 4 we chose cross correlations in the means and variances that matched the ones of the historical data. Consequently, we cannot rule out the possibility that the estimates of $\overline{R}^2$ reported in Table 2 for DGP 3 and DGP 4 reflect the predictive power of forward rates embedded in the structure of the VAR used in our simulations.

[12] This finding reinforces the argument that although the same price (recorded at the same time) is not present on both sides of the regressions, it does it with a lag. Given the large serial correlation of exhibited by bond prices, it is not surprising that using lagged right hand variables do not make a big difference on the overall result.

al. (2004) and Singleton (2006). In fact these studies, along the lines of Bekaert *et al.* (1997) and Backus *et al.* (2001), argue that bond yields are highly persistent and therefore, the asymptotic distribution of the statistics of interest (e.g. $\overline{R}^2$) may be poor approximations for the actual small-sample distributions. In the next section we provide a reparametrization of these models that mitigates the spurious effect generated by the high serial correlation of individual bond prices.[13]

4 Evaluating the predictive power of forward rates: Statistical significance

The results of the previous sections suggest the possibility that because some of the same variables appear on both the left- and right-hand sides of the CP and FB predictive equations, their high values for $\overline{R}^2$ may be due to the fact that bond prices (and bond yields) are highly correlated over time and across maturities. In this section, we provide an alternative framework to evaluate the ability of forward rates to predict future bond risk premia.

To understand the procedure, we first consider CP equation written solely in terms of bond prices. When $n = 2$, $(p_t^{(1)} - p_t^{(2)})$ appears on both the right- and left-hand sides of equation (7), so it can be written as

$$p_{t+1}^{(1)} = \beta_0 + \beta_1(-p_t^{(1)}) + (\beta_2 - 1)(p_t^{(1)} - p_t^{(2)}) + ... + \beta_5(p_t^{(4)} - p_t^{(5)}) + \varepsilon_{t+1}^{(n)}. \qquad (11)$$

Note that (6), (7), and (11) are observationally equivalent, i.e., they have an identical standard error of the estimate (henceforth *s.e.*). However, the estimates of $\overline{R}^2$ are different. This is a consequence of the fact that equations (6) and (7), on the one hand, and equation (11), on the other hand, have different dependent variables and, consequently, different total sums of squares.[14]

Equation (11) can be seen as an AR(1) in $p_t^{(1)}$ with the addition of the forward rates, $f_t^{(2)}, ..., f_t^{(5)}$. Hence, the marginal contribution of the forward rates can be investigated by comparing the difference between the *s.e.* from (11) and that of a simple AR(1) in $p_t^{(1)}$.

[13] The framework introduced in Section 4 with the specification of alternative AR benchmarks for both CP and FB predictive regressions mitigates but not eliminates the spurious regression problem. In fact, as detailed in the next section, the AR benchmark allows the identification of the predictive ability of forward rates net of the high serial correlation associated with bond prices. The same benchmarks do not eliminate the potential bias associated with the correlation of bond prices across maturities.

[14] For an analytical treatment of the effects of normalization on parameter estimates and measures of statistical accuracy, see Koopmans (1953), Chow (1964) and Hamilton *et al.* (2007).

The above result is not limited to $n = 2$, but it applies to all bond maturities. This can be seen by noting that since $p_t^{(1)}$ and $p_t^{(n)}$ appear on both the right- and left-hand side of (7), a mechanical relationship holds for all n. Specifically, (6), (7), and

$$p_{t+1}^{(n-1)} = \delta_0 + \delta_1 p_t^{(1)} + \delta_2 p_t^{(2)} + ... + \delta_5 p_t^{(5)} + \varepsilon_{t+1}^{(n)} \tag{12}$$

are also observationally equivalent for any value of n, where $\delta_1 = (\beta_2 - \beta_1 - 1)$ for all n, $\delta_i = (1 - \beta_i + \beta_{i+1})$ for i equal to n, $\delta_i = (\beta_{i+1} - \beta_i)$ for $i \geqq n \neq 5$, $\delta_5 = \beta_5$ for $n \neq 5$, and $\delta_5 = (1 - \beta_5)$ for $n = 5$. Note that for any n, $p_{t+1}^{(n-1)}$ and $p_t^{(n-1)}$ appear on the left- and right-hand sides of (12). Given this fact and the observational equivalence of equations (6) and (12), the marginal contribution of forward rates for predicting excess returns can be obtained by comparing the difference between the *s.e.* from (12) and the corresponding AR(1) model[15]

$$p_{t+1}^{(n-1)} = \vartheta_0 + \vartheta_1 p_t^{(n-1)} + \omega_{t+1}^{(n-1)}. \tag{13}$$

It is important to reiterate that the information provided by the reduction in terms of *s.e.* improves upon conventional statistics of in-sample accuracy (such as R^2). This is due to the fact that the reduction in terms of *s.e.*, unlike R^2, is a unit-free statistics that does not suffer from a scaling problem caused by the different dependent variables, e.g. equations (7) and (12).

Estimates of equations (12) and (13) over the period January 1964 - December 2003 are presented in Table 3. The reduction in *s.e.* from equation (12) against equation (13) range from 5.9 percent for $n = 2$ to 12.4 percent for $n = 5$. These changes are reflected in improvements in the values of $\overline{R}^2$, in the range between 7 and 10 percent.[16]

The evaluation of the predictive power of bond yields for the two FB equations is more complex because neither the change in the sport rate equation (8) or the excess return equation (9) can be rewritten in an equivalent form as the CP's equation (7). Equation (8) can be rewritten as

$$-p_{t+1}^{(1)} = \mu + \theta_1 p_t^{(1)} + \theta_2 (p_t^{(n-1)} - p_t^{(n)}) + \upsilon_{t+1}^{(1)}. \tag{14}$$

[15] This can be easily seen by noting that equation (12) is observationally equivalent to equation (11), which is just a reparametrization of the original CP equation (6).

[16] We can also compute similar improvements when the dependent variables are expressed as excess returns. The magnitude of the improvements in $\overline{R}^2$ equals 9.2, 11.6, 15.4 and 19.9 percent, for $n = 2, 3, 4,$and 5.respectively.

This equation is identical to the FB equation (4) when the restriction $\theta_2 - \theta_1 = 1$ applies. Likewise, equation (9) can be written as

$$p_{t+1}^{(n-1)} = \alpha + \psi_1 p_t^{(n-1)} + \psi_2(-p_t^{(n)} + p_t^{(1)}) + \upsilon_{t+1}^{(n)}. \tag{15}$$

Again, equation (15) is identical to the FB equation (5) when the restriction $\psi_2 - \psi_1 = 1$ applies.

Equation (14) can be thought of as a simple AR(1) in $p_t^{(1)}$ with the additional term $(p_t^{(n-1)} - p_t^{(n)})$. Likewise, equation (15) can be thought of as an AR(1) in $p_t^{(n-1)}$ with the additional term $(-p_t^{(n)} + p_t^{(1)})$. Hence, if neither of the restrictions implied by FB's equations hold, the marginal predictive power of the forward rates can be evaluated by comparing the *s.e.* of equations (14) and (15) with those obtained from their corresponding AR specifications

$$-p_{t+1}^{(1)} = \zeta_0 + \zeta_1 p_t^{(1)} + \varpi_{t+1}^{(1)} \tag{16}$$

and

$$p_{t+1}^{(n-1)} = \varrho_0 + \varrho_1 p_t^{(n-1)} + e_{t+1}^{(n-1)}, \tag{17}$$

respectively.

The first step in evaluating the marginal predictive power of forward rates for FB equation is to test whether the restrictions imposed in the original study are consistent with the data. The results of these tests are reported in Table 4. Both restrictions are strongly rejected with p-values virtually close to zero. Moreover, estimates of θ_1 decline in absolute value and become statistically insignificant as n increases, while estimates of θ_2 get larger and become statistically significant when $n = 5$. In contrast, estimates of ψ_1 get larger, the larger is n and is always statistically significant, while the estimates of ψ_2 become smaller and it statistically significant only when $n = 4$ and 5.

Table 5 presents estimates of equations (16) and (17) over the sample period January 1964 - December 2003. A comparison of the *s.e.* from equation (16) with those from equation (14) presented in Table 4 show that virtually no increase in predictive power is associated with the inclusion of the forward rates for any n. However, there is some reduction in *s.e.* for longer maturities. A similar comparison for equations (15) and (17) shows that there is some marginal reduction in *s.e.* associated with the spread between long-term and short-term rates, especially for $n = 3$ and 4, where the marginal improvement in the standard error equals 2.2 and 3.2 percent, respectively.

The evidence reported so far suggests that there is some predictive power in forward rates for both CP and FB regressions; however, this predictive ability is much lower than previously thought. Because of the small predictive power associated with forward rates, it is important to investigate whether the improvements in *s.e.* reported in Tables 4 and 5 are statistically significant. We do this by a simple bootstrap simulation generating the empirical distributions of $\overline{R}^2_{(n)}$ and *s.e.* under the null hypothesis that forward rates contain no predictive power over and above the AR models (13), (16), and (17).[17]

The results of the simulation are reported in Table 6. The *s.e.* recorded for the CP predictive equation are found to be statistically significant at the 5 percent statistical level for all maturities. That is, the estimates of *s.e.* from equation (12) are statistically significantly smaller than those estimated from equation (13). However, none of the recorded $\overline{R}^2_{(n)}$ exceeds the 95th percentile of the empirical distribution of $\overline{R}^2_{(n)}$ under the null hypothesis.

The results for the FB equations are less encouraging. There is no evidence of statistically significant improvement in predictive power at any conventional significance level for either *s.e.* or $\overline{R}^2$.

To sum up, the statistical tests provide some evidence of for the CP specification but no evidence of predictive power for the FB equations. A legitimate concern therefore is whether the weak evidence in favor of models based on forward rates may be due to low power of the tests employed (e.g., Inoue and Kilian, 2004, 2006). More importantly, the outcome of statistical tests is not necessarily informative on the economic value of forward rates for an investor. Specifically, the tests employed until now are not designed to discriminate between the performance of a strategy that trades on the basis of the information in forward rates and one that relies purely on the AR benchmarks proposed in this section. In order to shed light on this issue, we proceed to an economic evaluation of the information content in forward rates.[18]

[17] Full details of the bootstrap procedure are reported in Appendix B.

[18] The economic assessment of model based on forward premia proposed in this paper is novel in this literature. Cochrane and Piazzesi (2004) carry out a simple real time forecast test based on trading rule profits. However they indicate that the economic assessment of models based on forward rates should 'follow an explicit portfolio maximization problem' (Cochrane and Piazzesi, 2004, p. 12). This is the goal of the economic value exercise carried out in the next section.

5 Evaluating the predictive power of forward rates: economic significance

In this section we outline the framework used to assess the economic value of the predictive regressions discussed in Section 2. We consider a classic portfolio choice problem where an investor optimally invests in a portfolio comprising two bonds denominated in the same currency but with different maturities: a riskfree one-period bond and a risky n-period bond. The resulting portfolio return $r_{p,t+1}^{(n)}$ is computed as follows:

$$r_{p,t+1}^{(n)} = y_{t+1}^{(1)} + w_t^{(n)} \left(rx_{t+1}^{(n)} \right), \tag{18}$$

where $y_{t+1}^{(1)}$ and $rx_{t+1}^{(n)}$ are one-period bond yield and the n-period excess return, respectively, as discussed in Section 2 and $w_t^{(n)}$ is the weight attached to the risky n-period bond within the portfolio p. In mean-variance analysis, the maximum expected utility strategy leads to a portfolio allocation on the efficient frontier. Specifically, consider the trading strategy of a risk-averse investor who constructs a dynamically rebalanced portfolio that comprises the riskfree bond and the risky n-period bond. The solution to the optimization problem delivers the following weight on the risky bond:

$$w_t^{(n)} = \frac{1}{\lambda} \frac{E_t \left(rx_{t+1}^{(n)} \right)}{Var_t \left(rx_{t+1}^{(n)} \right)}, \tag{19}$$

where $E_t \left(rx_{t+1}^{(n)} \right)$ is the conditional expectation of $rx_{t+1}^{(n)}$; $Var_t \left(rx_{t+1}^{(n)} \right)$ is the conditional variance of $rx_{t+1}^{(n)}$; and λ is the relative risk aversion (RRA) coefficient (e.g., Campbell and Thomson, 2008). The weight on the riskfree bond is $1 - w_t^{(n)}$. In this setting we can compute asset allocations on the basis of the predictive regressions (12) and (15) (henceforth $\mathcal{P}$) and compare them with the ones obtained from their relative benchmarks, equations (13) and (17), respectively, (henceforth $\mathcal{B}$).[19] Assuming quadratic utility and following West *et al.* (1993) and Fleming *et al.* (2001), Cheung and Valente (2008) and Della Corte *et al.* (2008a,b,c), we can consistently estimate the average realized utility, $\overline{U}(\cdot)$, for an investor with initial wealth W_0 as

[19] It is important to emphasize that a portfolio consisting only of a one-year bond and a n-year risky bond is unlikely to be a realistic portfolio managed by a US investor in the fixed income market. However, our objective is not to design a realistic (executable) asset allocation strategy, but to measure the economic significance of the information embedded in forward interest rates.

$$\overline{U}(\cdot) = \frac{W_0}{T}\sum_{t=0}^{T-1}\left\{R_{p,t+1}^{(n)} - \frac{\lambda}{2(1+\lambda)}\left(R_{p,t+1}^{(n)}\right)^2\right\}, \tag{20}$$

where $R_{p,t+1}^{(n)} = 1 + y_{t+1}^{(1)} + w_t^{(n)}\left(rx_{t+1}^{(n)}\right)$ is the period $t+1$ gross return on the portfolio. We standardize the investor problem by assuming $W_0 = 1$.[20]

We measure the economic value of the $\mathcal{P}$ strategy against the $\mathcal{B}$ strategy by equating their average realized utilities. Suppose, for example, that holding a portfolio constructed using the optimal weights based on the $\mathcal{B}$ strategy yields the same average utility as holding the portfolio implied by the $\mathcal{P}$ strategy. The latter portfolio is subject to management expenses $\Phi^{(n)}$, expressed as a fraction of wealth invested in the portfolio. Since the investor would be indifferent between these two strategies, we interpret $\Phi^{(n)}$ as the maximum performance fee the investor would be willing to pay to switch from the $\mathcal{B}$ to the $\mathcal{P}$ strategy. In general, this criterion measures how much a risk-averse investor is willing to pay for conditioning on the information in the forward rates, as modeled in the predictive regression (12) and (15).[21] To estimate the performance fee, we find the value of $\Phi^{(n)}$ that satisfies

$$\sum_{t=0}^{T-1}\left\{\left(R_{p,t+1}^{(n),\mathcal{P}} - \Phi^{(n)}\right) - \frac{\lambda}{2(1+\lambda)}\left(R_{p,t+1}^{(n),\mathcal{P}} - \Phi^{(n)}\right)^2\right\} = \sum_{t=0}^{T-1}\left\{R_{p,t+1}^{(n),\mathcal{B}} - \frac{\lambda}{2(1+\lambda)}\left(R_{p,t+1}^{(n),\mathcal{B}}\right)^2\right\}, \tag{21}$$

where $R_{p,t+1}^{(n),\mathcal{P}}$ denotes the gross portfolio return constructed using the predictions from regressions (12) and (15) and $R_{p,t+1}^{\mathcal{B}}$ is the gross portfolio return implied by their relative benchmarks (13) and (17). If there is no predictive power embedded in forward rates, then $\Phi^{(n)} \leq 0$; whereas, if forward rates predict bond risk premia, $\Phi^{(n)} > 0$. We calculate and report the performance fee $\Phi^{(n)}$ in annual basis points.

A commonly used performance criterion is the realized Sharpe ratio. However, the Sharpe ratio, like many other performance measures of active management, can be manipulated in various ways. Goetzmann *et al.* (2007, henceforth GISW) suggest a set of conditions under which a manipulation-proof measure exists. This manipulation-proof performance measure is essentially an estimate of the portfolio's premium return after adjusting

[20] West *et al.*(1993) first derive expression (20) under the restriction that RRA is constant. Alternatively, one could build a utility-based measure using the certainty equivalent return (CER), defined as the riskfree return that gives the investor the same utility as the average utility obtained from the trading strategy examined. It turns out that this measure is similar to the performance fee measure discussed below (see Abhyankar *et al.*, 2005; Han, 2006).

[21] For studies following this approach see also Fleming, Kirby and Ostdiek (2003), Marquering and Verbeek (2004), Han (2006), Della Corte *et al.* (2008a,b,c), Cheung and Valente (2008).

for risk. Building on GISW, as a complement to the performance fee Φ, we calculate the risk-adjusted abnormal return of the $\mathcal{P}$ strategies relative to the $\mathcal{B}$ strategies as follows:

$$\Theta^{(n)} = \frac{1}{(1-\lambda)}\left[\ln\left(\frac{1}{T}\sum_{t=0}^{T-1}\left[\frac{R_{p,t+1}^{(n),\mathcal{P}}}{R_f}\right]^{1-\lambda}\right) - \ln\left(\frac{1}{T}\sum_{t=0}^{T-1}\left[\frac{R_{p,t+1}^{(n),\mathcal{B}}}{R_f}\right]^{1-\lambda}\right)\right]. \quad (22)$$

where $R_f = 1 + y_{t+1}^{(1)}$ denotes the gross yield on the riskfree one-period bond.

We compute the performance measures $\Phi^{(n)}$ and manipulation-proof performance measure $\Theta^{(n)}$ for all bond maturities $n > 1$. Furthermore we assume $\lambda = 3$ as in GISW and Campbell and Thomson (2008), we compute $Var_t\left(rx_{t+1}^{(n)}\right)$ as the variance of the n-period excess returns during the previous 12 month, and we assume that $-1 \leq w_t^{(n)} \leq 2$, which essentially allows for full proceeds of short sales (Abhyankar *et al.*, 2005).[22] The results of this exercise are reported in Table 7.

It is interesting to note that for all bond maturities and for both CP and FB specifications, the performance fees $\Phi^{(n)}$ and the manipulation-proof performance measures $\Theta^{(n)}$ are small in magnitude and negative in sign. These findings suggest that, from an economic perspective, an individual investor would not obtain any tangible economic gains by using the predictions of equations (12) and (15) instead of the ones generated by equations (13) and (17). The evidence reported in Table 7 complements and supports the statistical analysis discussed in Section 4. Overall, they indicate that there is very limited evidence of predictability associated with forward rates when one takes into account that bond yields (and hence bond prices) are highly autocorrelated over time and across maturities. Consistent with the existing literature, they once again confirm that it is very difficult to improve upon the predictive performance of simple autoregressive models of bond yields.

6 Conclusions

This study revisits the predictability of bond risk premia by means of forward rates. We note that the forward rates, holding period returns, and excess returns are simple functions of the same primitive bond prices. Hence, by construction, the same bond prices appear on both sides of equations commonly used in the literature to evaluate the predictive power of forward rates. Because bond yields (and hence bond prices) are highly correlated over time and across

[22] We have carried out the same exercise imposing no allowance for short selling $0 \leq w_t^{(n)} \leq 1$. The results, not reported to save space, are qualitatively and quantitatively similar to the ones reported in Table 7.

maturities, the fact that the same bond prices appears on both sides of these equations gives rise to the possibly that at least some of predictive power of forward rates reflected in the $\overline{R}^2$ for these equaitons may be spurious. We demonstrated this possibility analytically and through Monte Carlo simulations. We then show that these equations can be rewritten in a form that is observationally equivalent to an AR(1) in bonds prices with additional terms being linear functions of forward rates. When $\overline{R}^2$ is used as metric of evaluation of predictability, the results suggest that including forward rates as in the Cochrane and Piazzesi (2005) equation translates in a marginal improvement of the $\overline{R}^2$ on the order of a third to half of that reported in the original study. However these figures are not statistically significant at conventional statistical levels. When we used the percentage reduction in the standard errors of the regression as the performance metric, however, forward rates were found to be statistically significant. In the case of the Fama and Bliss (1987) predictive regressions, there is no marginal improvement associated with the forward rates using either metric. When economic criteria are used to assess the predictive performance of forward rates, we find there are no economic gains to an investor who invests in a portfolio consisting of a one-period bond and a n-period bond using the predictions implied by models based on forward rates.

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Table 1. Reparametrized Predictive Regressions

Panel A) and B) report the OLS estimates of the reparametrized Cochrane and Piazzesi (2005) equation $p_{t+1}^{(n-1)} + (p_t^{(1)} - p_t^{(n)}) = \beta_0 + \beta_1(-p_t^{(1)}) + \beta_2(p_t^{(1)} - p_t^{(2)}) + ... + \beta_5(p_t^{(4)} - p_t^{(5)}) + \varepsilon_{t+1}^{(n)}$. Panel C) and D) report the OLS estimates of the reparametrized Fama and Bliss (1987) equations $-p_{t+1}^{(1)} + p_t^{(1)} = \mu + \delta(p_t^{(n-1)} - p_t^{(n)} + p_t^{(1)}) + \upsilon_{t+1}^{(1)}$ and $p_{t+1}^{(n-1)} - p_t^{(n)} + p_t^{(1)} = \alpha + \beta(p_t^{(n-1)} - p_t^{(n)} + p_t^{(1)}) + \upsilon_{t+1}^{(n)}$. $\overline{R}^2$ denotes adjusted coefficient of determination, s.e. and mean dep are the standard errors of estimate and the average value of the dependent variable respectively. Values in parentheses are asymptotic standard errors.

Panel A) Cochrane and Piazzesi (2005), January 1964 - December 2003

	$n=2$	$n=3$	$n=4$	$n=5$
β_0	-0.0162	-0.0267	-0.0380	-0.0489
	(0.002)	(0.004)	(0.006)	(0.008)
β_1	-0.9816	-1.7811	-2.5700	-3.2083
	(0.126)	(0.227)	(0.308)	(0.385)
β_2	0.5917	0.5327	0.8680	1.2409
	(0.262)	(0.472)	(0.305)	(0.799)
β_3	1.2141	3.0736	3.6068	4.1080
	(0.217)	(0.391)	(0.529)	(0.662)
β_4	0.2877	0.3821	1.2849	1.2504
	(0.160)	(0.288)	(0.390)	(0.488)
β_5	-0.8860	-1.8580	-2.7285	-2.8304
	(0.135)	(0.244)	(0.331)	(0.414)
$\overline{R}^2$	0.313	0.333	0.364	0.338
s.e.	0.0160	0.0288	0.0390	0.0488
mean dep	0.0050	0.0083	0.0103	0.0099

(continued)

(continued Table 1)

Panel B) Cochrane and Piazzesi (2005), January 1964 - December 1985

	$n=2$	$n=3$	$n=4$	$n=5$
β_0	-0.0236	-0.0383	-0.0523	-0.0656
	(0.003)	(0.006)	(0.008)	(0.010)
β_1	-1.0194	-1.7755	-2.4836	-2.9925
	(0.155)	(0.276)	(0.373)	(0.472)
β_2	0.9620	0.9983	1.2300	1.3283
	(0.344)	(0.611)	(0.824)	(1.043)
β_3	1.2009	3.0742	3.6024	4.1283
	(0.245)	(0.436)	(0.589)	(0.745)
β_4	0.0085	0.1047	1.0414	1.1153
	(0.197)	(0.350)	(0.473)	(0.598)
β_5	-0.9404	-1.9562	-2.8140	-2.885
	(0.164)	(0.292)	(0.394)	(0.499)
$\overline{R}^2$	0.409	0.420	0.442	0.403
s.e.	0.0169	0.0300	0.0405	0.0512
mean dep	0.0014	0.0016	0.0009	-0.0010

(continued)

(continued Table 1)

Panel C) Fama and Bliss (1987), January 1964 - December 2003

	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
μ	-0.0007		-0.0035		-0.0069		-0.0078	
	(0.001)		(0.001)		(0.001)		(0.003)	
δ	0.0068		0.3335		0.6332		0.7925	
	(0.106)		(0.110)		(0.105)		(0.106)	
α		0.0007		-0.0013		-0.0040		-0.0008
		(0.001)		(0.001)		(0.002)		(0.003)
β		0.9932		1.3512		1.6122		1.2718
		(0.106)		(0.136)		(0.157)		(0.193)
$\overline{R}^2$	-0.002	0.156	0.017	0.172	0.073	0.182	0.111	0.082
s.e.	0.0177	0.0177	0.0252	0.0321	0.0282	0.0442	0.0298	0.0574
mean dep	-0.0007	0.0050	-0.0013	0.0082	-0.0018	0.0102	-0.0016	0.0099

Panel D) Fama and Bliss (1987), January 1964 - December 1985

	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
μ	0.0019		0.0038		0.0047		0.0008	
	(0.001)		(0.001)		(0.001)		(0.001)	
δ	0.0652		0.5663		1.3609		1.7447	
	(0.151)		(0.148)		(0.152)		(0.125)	
α		-0.0019		-0.0057		-0.0094		-0.0104
		(0.001)		(0.002)		(0.003)		(0.003)
β		0.9348		1.2350		1.4100		0.7946
		(0.151)		(0.196)		(0.247)		(0.276)
$\overline{R}^2$	-0.003	0.129	0.053	0.132	0.258	0.111	0.471	0.028
s.e.	0.0198	0.0198	0.0255	0.0349	0.0262	0.0476	0.0236	0.0606
mean dep	0.0021	-0.0001	0.0054	-0.0014	0.0078	-0.0039	0.0121	-0.0071

Table 2. Monte Carlo Exercise

The Table reports averages of the empirical distributions of $\overline{R}^2$ obtained from the estimation of Cochrane and Piazzesi (2005) (equation 7) and Fama and Bliss (1987) (equation 9) predictive regressions under the various DGP reported in Section 3. The number of replications used in the Monte Carlo experiments is 5,000 and values in brackets are 95th and 5th percentile of the resulting empirical distributions.

Panel A) Cochrane and Piazzesi (2005)

	$n=2$	$n=3$	$n=4$	$n=5$
DGP 1	0.829	0.704	0.649	0.613
	$[0.849, 0.806]$	$[0.741, 0.664]$	$[0.691, 0.604]$	$[0.659, 0.566]$
DGP 2	0.942	0.889	0.875	0.859
	$[0.978, 0.886]$	$[0.959, 0.778]$	$[0.955.0.746]$	$[0.950, 0.718]$
DGP 3	0.182	0.176	0.193	0.186
	$[0.331, 0.055]$	$[0.326, 0.049]$	$[0.341, 0.063]$	$[0.336, 0.059]$
DGP 4	0.253	0.234	0.298	0.269
	$[0.398, 0.117]$	$[0.383, 0.098]$	$[0.458, 0.143]$	$[0.423, 0.121]$

Panel B) Fama and Bliss (1987)

	$n=2$	$n=3$	$n=4$	$n=5$
DGP 1	0.744	0.495	0.420	0.375
	$[0.777, 0.710]$	$[0.552, 0.436]$	$[0.481, 0.358]$	$[0.438, 0.311]$
DGP 2	0.930	0.864	0.847	0.829
	$[0.973, 0.866]$	$[0.948, 0.734]$	$[0.944.0.697]$	$[0.936, 0.663]$
DGP 3	0.068	0.062	0.074	0.061
	$[0.185, -0.002]$	$[0.178, -0.009]$	$[0.199, 0.001]$	$[0.174, -0.001]$
DGP 4	0.121	0.090	0.166	0.108
	$[0.249, 0.022]$	$[0.218, 0.005]$	$[0.327, 0.037]$	$[0.242, 0.009]$

Table 3. Cochrane and Piazzesi's (2005) Alternative Benchmark

The Table reports the OLS estimates of the equations $p_{t+1}^{(n-1)} = \delta_0 + \delta_1 p_t^{(1)} + \delta_2 p_t^{(2)} + \delta_3 p_t^{(3)} + \delta_4 p_t^{(4)} + \delta_5 p_t^{(5)} + \varepsilon_{t+1}^{(n)}$ and $p_{t+1}^{(n-1)} = \vartheta_0 + \vartheta_1 p_t^{(n-1)} + \omega_{t+1}^{(n-1)}$ discussed in the text. See notes to Table 1.

	$n=2$		$n=3$		$n=4$		$n=5$	
δ_0	-0.0162		-0.0267		-0.0379		-0.0489	
	(0.002)		(0.004)		(0.006)		(0.008)	
δ_1	0.5733		1.3137		2.4379		3.4492	
	(0.373)		(0.673)		(0.911)		(1.139)	
δ_2	1.6223		2.5409		2.7388		2.8671	
	(0.395)		(0.713)		(0.965)		(1.207)	
δ_3	-0.9263		-1.6915		-2.3218		-2.8576	
	(0.318)		(0.573)		(0.776)		(0.970)	
δ_4	-1.1736		-2.2400		-3.0134		-4.0807	
	(0.223)		(0.402)		(0.544)		(0.680)	
δ_5	0.8859		1.8579		2.7285		3.8304	
	(0.135)		(0.244)		(0.331)		(0.414)	
μ		-0.0118		-0.0218		-0.0314		-0.0426
		(0.002)		(0.004)		(0.006)		(0.008)
λ		0.8112		0.8331		0.8445		0.8461
		(0.030)		(0.028)		(0.027)		(0.026)
$\overline{R}^2$	0.653	0.607	0.698	0.645	0.734	0.670	0.755	0.682
s.e.	0.0160	0.0170	0.0288	0.0312	0.0390	0.0435	0.0487	0.0556
mean dep	-0.0658	-0.0659	-0.1363	-0.1363	-0.2097	-0.2097	-0.2851	-0.2851

Table 4. Fama and Bliss' (1987) Reparametrization and Parameters' Restrictions

The Table reports the OLS estimates of the equation $-p_{t+1}^{(1)} = \mu + \theta_1 p_t^{(1)} + \theta_2(p_t^{(n-1)} - p_t^{(n)}) + \upsilon_{t+1}^{(1)}$ and $p_{t+1}^{(n-1)} = \alpha + \psi_1 p_t^{(n-1)} + \psi_2(-p_t^{(n)} + p_t^{(1)}) + \upsilon_{t+1}^{(n)}$ respectively. Values in brackets denote p-values of the null hypothesis that the restriction $\theta_2 - \theta_1 = 1$ (equation 15) or $\psi_2 - \psi_1 = 1$ (equation 16) holds. 0 denotes p-values lower than 10^{-5}. See notes to Table 1.

	$n=2$		$n=3$		$n=4$		$n=5$	
μ	0.0137		0.0286		0.0327		0.0377	
	(0.002)		(0.003)		(0.004)		(0.004)	
θ_1	-0.9820		-0.6386		-0.2084		-0.0508	
	(0.101)		(0.101)		(0.095)		(0.095)	
θ_2	-0.1862		-0.0728		0.2581		0.3406	
	(0.106)		(0.110)		(0.101)		(0.102)	
α		-0.0137		-0.0324		-0.0459		-0.0510
		(0.002)		(0.004)		(0.006)		(0.008)
ψ_1		0.9820		1.4448		1.6588		1.3379
		(0.101)		(0.130)		(0.150)		(0.186)
ψ_2		0.1862		0.6555		0.8474		0.5061
		(0.106)		(0.136)		(0.153)		(0.189)
$\overline{R}^2$	0.609	0.609	0.285	0.661	0.152	0.689	0.103	0.686
s.e.	0.0170	0.0170	0.0230	0.0305	0.0253	0.0421	0.0262	0.0552
mean dep	0.0658	-0.0658	0.0664	-0.1363	0.0668	-0.2097	0.0673	-0.2851
$H_0: \theta_2 - \theta_1 = 1$	[0]		[0]		[0]		[0]	
$H_0: \psi_2 - \psi_1 = 1$		[0]		[0]		[0]		[0]

Table 5. Fama and Bliss' (1987) Alternative Benchmarks

The Table reports the OLS estimates of the equation $-p_{t+1}^{(1)} = \zeta_0 + \zeta_1 p_t^{(1)} + \varpi_{t+1}^{(1)}$ and equation $p_{t+1}^{(n-1)} = \varrho_0 + \varrho_1 p_t^{(n-1)} + e_{t+1}^{(n-1)}$. See notes to Table 1.

	$n=2$		$n=3$		$n=4$		$n=5$	
ζ_0	0.0118		0.0272		0.0380		0.0453	
	(0.002)		(0.003)		(0.003)		(0.003)	
ζ_1	-0.8111		-0.5779		-0.4191		-0.3197	
	(0.030)		(0.042)		(0.048)		(0.050)	
ϱ_0		-0.0118		-0.0217		-0.0314		-0.0426
		(0.002)		(0.004)		(0.006)		(0.008)
ϱ_1		0.8111		0.8331		0.8445		0.8461
		(0.030)		(0.028)		(0.027)		(0.026)
$\overline{R}^2$	0.609	0.609	0.285	0.645	0.142	0.670	0.082	0.682
s.e.	0.0170	0.0170	0.0230	0.0312	0.0255	0.0435	0.0265	0.0556
mean dep	0.0658	-0.0658	0.0664	-0.1363	0.0668	-0.2097	0.0673	-0.2851

Table 6. Statistical Evaluation

The Table reports the results of the comparisons between the reparametrized Cochrane and Piazzesi's (2005) and Fama and Bliss' (1987) predictive regressions and their relative AR benchmark as discussed in Section 4. $\Delta\overline{R}^2_{(n)}$ denotes the increment in $\overline{R}^2_{(n)}$ obtained from predictive regression ($\overline{R}^2_{(n),predictive}$) with respect to ones obtained from their benchmark ($\overline{R}^2_{(n),bench}$) computed as $\Delta\overline{R}^2_{(n)} = \overline{R}^2_{(n),predictive} - \overline{R}^2_{(n),bench}$. $\Delta s.e.$denotes the percentage reduction in $s.e.$ obtained from predictive regression ($s.e_{predictive}$) with respect to ones obtained from their benchmark ($s.e._{bench}$) computed as $\Delta s.e. = 1 - \left(\frac{s.e._{predictive}}{s.e._{bench}}\right)$. $(\widetilde{x})_i$ denotes the $i-$th percentile of the distribution of the variable $x = \overline{R}^2_{(n)}$, $s.e.$ obtained by parametric bootstrap as discussed in Section 4 and detailed in Appendix B. $i = max$ denotes the maximum value of the distribution of the variable of interest x obtained in the bootstrap exercise.

Panel A) Cochrane and Piazzesi (2005)

	$n=2$	$n=3$	$n=4$	$n=5$
$\Delta\overline{R}^2_{(n)}$	0.046	0.053	0.064	0.073
$\Delta s.e.$	0.059	0.077	0.103	0.124
$\overline{R}^2_{(n)}$ Table 3	0.653	0.698	0.734	0.755
$\left(\widetilde{\overline{R}^2_{(n)}}\right)_{95}$	0.715	0.744	0.758	0.762
$\left(\widetilde{\overline{R}^2_{(n)}}\right)_{99}$	0.743	0.765	0.784	0.784
$\left(\widetilde{\overline{R}^2_{(n)}}\right)_{\max}$	0.772	0.803	0.825	0.816
$s.e.$. Table 3	0.0160	0.0288	0.0390	0.0487
$(\widetilde{s.e})_{95}$	0.0161	0.0295	0.0411	0.0525
$(\widetilde{s.e})_{99}$	0.0156	0.0287	0.0400	0.0511
$(\widetilde{s.e})_{\max}$	0.0146	0.0272	0.0383	0.0482

(continued)

(continued Table 6)

Panel B) Fama and Bliss (1987)

	$n=2$		$n=3$		$n=4$		$n=5$	
	eq. (14)	eq. (15)	eq. (14)	eq. (15)	eq. (14)	eq. (15)	eq. (14)	eq. (15)
$\Delta\overline{R}^2_{(n)}$	–	–	–	0.016	0.010	0.019	0.021	0.004
$\Delta s.e.$	–	–	–	0.022	0.008	0.032	0.011	0.007
$\overline{R}^2_{(n)}$ Table 4	0.609	0.609	0.285	0.661	0.152	0.689	0.103	0.686
$\left(\widetilde{\overline{R}^2_{(n)}}\right)_{95}$	0.720	0.714	0.522	0.744	0.386	0.760	0.288	0.761
$\left(\widetilde{\overline{R}^2_{(n)}}\right)_{99}$	0.743	0.740	0.558	0.766	0.429	0.785	0.334	0.782
$\left(\widetilde{\overline{R}^2_{(n)}}\right)_{\max}$	0.797	0.778	0.662	0.814	0.546	0.819	0.442	0.814
$s.e.$. Table 4	0.0170	0.0170	0.0230	0.0305	0.0253	0.0421	0.0262	0.0552
$(\widetilde{s.e.})_{95}$	0.0161	0.0161	0.0204	0.0295	0.0227	0.0411	0.0240	0.0525
$(\widetilde{s.e.})_{99}$	0.0156	0.0156	0.0198	0.0287	0.0220	0.0400	0.0231	0.0511
$(\widetilde{s.e.})_{\max}$	0.0146	0.0146	0.0184	0.0272	0.0198	0.0383	0.0207	0.0482

Table 7. Economic Assessment

The Table reports the results of the economic assessment between the reparametrized Cochrane and Piazzesi's (2005) and Fama and Bliss' (1987) predictive regressions and their relative AR benchmark as discussed in Section 5. The performance fee $\Phi^{(n)}$ denotes the amount that a risk-averse investor is willing to pay to switch from strategies based on the predictive regressions (12) or (15) in the main text to strategies based on equations (13) or (17) respectively. The manipulation-proof risk-adjusted abnormal return $\Theta^{(n)}$ measures the difference between the risk-adjusted portfolio's premium return of the strategies based on the predictability regressions (12) or (15) realtive to their benchmark (13) or (17). The measures are all computed by assuming a coefficient of relative risk aversion $\lambda = 3$ and they are expressed in annual basis points.

	Cochrane and Piazzesi (2005) Eq. (12) vs Eq. (13)		Fama and Bliss (1987) Eq. (15) vs Eq. (17)	
	$\Phi^{(n)}$	$\Theta^{(n)}$	$\Phi^{(n)}$	$\Theta^{(n)}$
$n = 2$	-6.15	-5.23	-0.87	-0.72
$n = 3$	-14.55	-11.84	-4.22	-3.85
$n = 4$	-19.78	-15.10	-4.47	-3.75
$n = 5$	-32.48	-24.12	-3.87	-3.47

A Appendix: Theoretical $\mathbf{R}^2$

A.1 *iid* bond yields

Let's consider the reparametrized Cochrane and Piazzesi's (2005) equation under the assumption that bond yields are *iid*

$$p_{t+1}^{(n-1)} + (p_t^{(1)} - p_t^{(n)}) = \beta_0 + \beta_1(-p_t^{(1)}) + \beta_2(p_t^{(1)} - p_t^{(2)}) + ... + \beta_5(p_t^{(4)} - p_t^{(5)}) + \varepsilon_{t+1}^{(n)} \tag{A1}$$

the variance of the LHS variable is

$$var\left[p_{t+1}^{(n-1)} + (p_t^{(1)} - p_t^{(n)})\right] = var\left(p_{t+1}^{(n-1)}\right) + var\left(p_t^{(1)} - p_t^{(n)}\right) \tag{A2}$$

the variance of the prediction on the RHS is

$$var\left\{\widehat{\beta}_0 + \widehat{\beta}_1(-p_t^{(1)}) + \widehat{\beta}_2(p_t^{(1)} - p_t^{(2)}) + \widehat{\beta}_3(p_t^{(2)} - p_t^{(3)}) + \widehat{\beta}_4(p_t^{(3)} - p_t^{(4)}) + \widehat{\beta}_5(p_t^{(4)} - p_t^{(5)})\right\} \tag{A3}$$

Equation (A3) can only be computed conditional upon the values of estimated parameters $\widehat{\boldsymbol{\beta}} = \left[\widehat{\beta}_1, ..., \widehat{\beta}_5\right]$. For simplicity we compute $\widehat{\boldsymbol{\beta}}$ under the assumption of yields being independent by simulation for all n. The results are as follows:

$$\begin{aligned} n &= 2 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \\ n &= 3 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 & 0.0 & 0.0 \end{bmatrix} \\ n &= 4 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 & 1.0 & 0.0 \end{bmatrix} \\ n &= 5 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix} \end{aligned} \tag{A4}$$

The results reported in (A4) allow us to define closed-form *formulae* for $R^2_{(n)}$ for all n. The general formula for the theoretical $R^2_{(n)}$ for all n can be written as follows

$$R^2_{(n)} = \frac{var\left(y_t^{(1)}\right) + n^2 var\left(y_t^{(n)}\right)}{var\left(y_t^{(1)}\right) + (n-1)^2 var\left(y_t^{(n-1)}\right) + n^2 var\left(y_t^{(n)}\right)} \tag{A5}$$

To investigate the behavior of equation (A5) at the limits (i.e. when $n = 1$ and, more importantly, when $n \to \infty$) we rewrite equation (A9) as follows:

$$
\begin{aligned}
\frac{1}{R^2} &= \frac{var\left(y_t^{(1)}\right) + (n-1)^2 var\left(y_t^{(n-1)}\right) + n^2 var\left(y_t^{(n)}\right)}{var\left(y_t^{(1)}\right) + n^2 var\left(y_t^{(n)}\right)} = \\
&= 1 + \frac{(n-1)^2 var\left(y_t^{(n-1)}\right)}{var\left(y_t^{(1)}\right) + n^2 var\left(y_t^{(n)}\right)}
\end{aligned} \tag{A6}
$$

when $n \to 1$, then $\frac{(n-1)^2 var\left(y_t^{(n-1)}\right)}{var\left(y_t^{(1)}\right)+n^2 var\left(y_t^{(n)}\right)} \to 0$ which causes $R^2 \to 1$.When $n \to \infty$ it is plausible to hypothesize that $n-1 \simeq n$. Let $var\left(y_t^{(1)}\right) = k$ constant independent from n, then

$$
n \to \infty : \frac{n^2 var\left(y_t^{(n)}\right)}{k + n^2 var\left(y_t^{(n)}\right)} \to 1
$$

This will causes $\frac{1}{R^2} \to 2$, or alternatively $R^2 \to 0.5$.

A.2 Persistent bond yields

In the previous Section A.1 we have demonstrated that when bond yields are *iid* the Cochrane and Piazzesi' (2005) predictive equation implies theoretical R^2 higher than 0.5 and decreasing with the bond maturity n In this section, we relax the assumption of independence of bond yields and we allow the possibility that *at least* one bond yield is serially correlated. This will cause the $\widehat{\boldsymbol{\beta}}$ to be different from the one reported in (A4). More specifically, for simplicity, we assume that only the short-term bond yield $y_t^{(1)}$ is serially correlated. Also in this case we compute $\widehat{\boldsymbol{\beta}}$ by simulation for all n. The results are as follows:

$$
\begin{aligned}
n &= 2 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} \widehat{\beta}_{1,P}^{(2)} & 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \\
n &= 3 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 & 0.0 & 0.0 \end{bmatrix} \\
n &= 4 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 & 1.0 & 0.0 \end{bmatrix} \\
n &= 5 : \widehat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}
\end{aligned} \tag{A7}
$$

where $\widehat{\beta}_{1,P}^{(2)}$ is the estimate of $\widehat{\beta}_1$ obtained under the assumption that only the short-term bond yield $y_t^{(1)}$ is serially correlated The estimates reported in (A7) indicate that only when $n = 2$ the *plim* $\widehat{\beta}_1 \neq 0$. If we compute the equivalent of equation (A5), we obtain:

$$R^2_{(2),P} = \frac{var\left\{\widehat{\beta}_1(-p_t^{(1)}) + \widehat{\beta}_2(p_t^{(1)} - p_t^{(2)})\right\}}{var\left(p_{t+1}^{(1)}\right) + var\left(p_t^{(1)} - p_t^{(2)}\right)} = \frac{var\left\{\widehat{\beta}^{(2)}_{1,P}(y_t^{(1)}) + (-y_t^{(1)} + 2y_t^{(2)})\right\}}{var\left(-y_{t+1}^{(1)}\right) + var\left(-y_t^{(1)} + y_t^{(2)}\right)} =$$

$$= \frac{\left[1 + \left(\widehat{\beta}^{(2)}_{1,P}\right)^2\right] var\left(y_t^{(1)}\right) + 4var\left(y_t^{(2)}\right)}{2var\left(y_t^{(1)}\right) + 4var\left(y_t^{(2)}\right) + cov\left(y_{t+1}^{(1)}, y_t^{(1)}\right)} \tag{A8}$$

Equation (A8) is a convolution of the estimated parameter $\widehat{\beta}^{(2)}_{1,P}$,that is itself a complex function of variance and covariances of bond prices at different maturities. Note that (A8) also includes in the denominator a non-zero covariance $cov\left(y_{t+1}^{(1)}, y_t^{(1)}\right)$ which is due to the assumption of serial correlation in short-term bond yield. The theoretical $R^2_{(2)}$ cannot be computed analytically since it requires the empirical estimates of both $\widehat{\beta}^{(2)}_{1,P}$ and $cov\left(y_{t+1}^{(1)}, y_t^{(1)}\right)$. However equation (A8) does provide us with an insight about the effect of a serial correlation on $R^2_{(2)}$. When short-term bond yields are serially correlated the variance $var\left(y_t^{(1)}\right)$ is no longer equal to unity and it increases with the absolute value of the first-order autoregressive parameter. The higher the absolute value of the first-order autoregressive parameter, *ceteris paribus*, the larger is $var\left(y_t^{(1)}\right)$. When the $var\left(y_t^{(1)}\right)$ is sufficiently large, it will dominate the other variances and covariances reported in equation (A8). This will cause the R^2 obtained from equation (A8) to increase towards unity, since $var\left(y_t^{(1)}\right)$ is in both the numerator and denominator of (A8)[23]. If we assume that both $y_t^{(1)}$and $y_t^{(2)}$ are serially correlated (but not cross-correlated) with both first-order autocorrelation coefficients equal to 0.99, equation (A8) does not change[24]. However the variances of both $y_t^{(1)}$and $y_t^{(2)}$ are both no longer equal to unity and they are larger than one. This will cause the value of the $R^2_{(2)}$ to increase further towards the value of unity[25].

[23] A simple Monte Carlo simulation that replicates the above DGP generates an average value for $R^2_{(2)}$ of 0.90 that is higher than one computed for $n = 2$ when bond yields are assumed *iid*.

[24] This is due to the fact that, under the assumptions postulated in this exercise, autocovariances relative to bond yields with 2-year maturity are not present when $n = 2$.

[25] In fact the resulting distribution of $R^2_{(2)}$ from estimating equation (7) under the assumption of both $y_t^{(1)}$ and $y_t^{(2)}$ being serially but not cross-sectionally correlated, exhibits an average of 0.95.

B Appendix: Bootstrap

To evaluate the statistical significance of the marginal predictability of Cochrane and Piazzesi's (2005) and Fama and Bliss' (1987) equations over and above their relative AR benchmarks, we rely on a parametric bootstrapping procedure[26]. In this exercise we simply simulate artificial time series based on the estimates of the AR benchmarks and compute the empirical distribution of their adjusted coefficient of determination $\overline{R}^2_{(n)}$ and their standard error of estimate $s.e.$ This allows us to see how the values of $\overline{R}^2_{(n)}$ and $s.e$ recorded for the predictive regressions based on forward rates (reported in Table 3 and 4 of the main text) compare to a case in which there is no forward rates predictability.

For each maturity n we assume a null DGP where bond prices follow an AR process consistent with no predictive power from forward rates as outlined in Section 4. For the Cochrane and Piazzesi (2005) empirical framework is represented by

$$p_{t+1}^{(n-1)} = \vartheta_0 + \vartheta_1 p_t^{(n-1)} + \omega_{t+1}^{(n-1)} \tag{B1}$$

while for the Fama and Bliss' (1987) predictive regressions, their benchmark are represented by

$$-p_{t+1}^{(1)} = \zeta_0 + \zeta_1 p_t^{(1)} + \varpi_{t+1}^{(1)}, \tag{B2}$$

$$p_{t+1}^{(n-1)} = \varrho_0 + \varrho_1 p_t^{(n-1)} + e_{t+1}^{(n-1)}. \tag{B3}$$

We choose the parameters of the three DGPs to match the actual data and then construct bootstrapped distributions for the test statistics as follows:

1. Estimate the null DGPs described in equations (B1), (B2) and (B3).
2. Draw with replacement $1,500$ errors from the estimated null DGPs.
3. Use the errors to compute artificial time series of $p_{t+1}^{(n-1)}$ and $-p_{t+1}^{(1)}$ from equations (B1), (B2) and (B3). We discard the first 700 observations in order to minimize the impact of the initialization.
4. Estimate the AR regressions, to obtain values of $\overline{R}^2_{(n)}$ and $s.e$
5. Repeat 2. to 4. $5,000$ times.

[26] Parametric bootstrapping is a method of simulating the distribution of statistics with the distribution of actual errors estimated by the model - rather than pseudo-random errors form a normal (or other) distribution - under some assumption about the data generating process (DGP) of the data (Berkowitz and Kilian, 2000).

6. Obtain the bootstrapped distribution of $\overline{R}^2_{(n)}$ and $s.e.$ generated by the AR benchmark equations (B1), (B2) and (B3). Then compute the 95th, 99th percentiles of these distribution and the maximum value of the distribution recorded during the simulation.