

No. 7804

THE EFFECTS OF RECOGNITION AND IMPACT  
LAGS ON MONETARY RULE PERFORMANCE

by

Wallace H. Duncan

August 1978

# Research Paper



FEDERAL RESERVE BANK OF DALLAS

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The author wishes to thank Peter Formuzis for many helpful suggestions and comments.

## I. Introduction

In 1970, William Poole (9) published a theoretical analysis showing that in a comparative static framework, the superiority of an interest rate target or a money supply target as a monetary policy rule depended upon the source of disturbances to the system. Poole's work has had considerable impact upon the actual conduct of monetary policy. Under the leadership of the present Federal Reserve chairman, the Federal Open Market Committee has generally specified to the Trading Desk a monetary aggregates target constrained by an allowable range in the Federal funds rate. However, during times in which disturbances were perceived as originating primarily in the monetary sector, the Committee has reverted to a pure interest rate target. The rationale for target selection related to the source of disturbances follows directly from Poole's analysis.

This study examines the extent to which Poole's conclusions are applicable in a considerably more complex, dynamic macroeconomic system, rather than in his comparative static framework. Evidence is also offered as to how his results may be affected by changes in some of the more important parameters of the economy.

In the present study, alternative monetary rules are tested using computer simulation of the time path of a twelve-equation quarterly model of the economy, which is subjected to exogenous disturbances in either the expenditure or monetary sectors. The model is similar to that developed by Blinder and Solow (3) except that it incorporates an endogenous money supply, endogenous price determination, and both recognition and impact lags in monetary policy. The price function is an inflationary-expectations-augmented Phillips Curve of a type similar to that developed by Laidler (6).

Both constrained and unconstrained versions of money supply and interest rate rules are tested in different economic environments, including Keynesian and Monetarist interest elasticities, differing monetary policy impact lags, and zero or positive autocorrelation of disturbance terms. Two analysis of variance techniques are used to compare mean variances of real output and the price level in determining relative superiority among rules. All active rules are compared with each other and with a control policy representing a complete absence of open market operations.

The results of the simulation tests modify significantly Poole's comparative static conclusions. Due to the existence of recognition and impact lags, a rule recommended by Poole as appropriate for a particular disturbance may actually exacerbate the variance of real output. In such a case, imposing a constraint on the rule can change its effects from destabilizing to stabilizing.

Relative rule performance is found to be sensitive to the interest elasticities of private expenditures and money demand; to the presence or absence of positive autocorrelation of the disturbance terms; and to the source of the disturbances. Rule performance is not affected by the length of the impact lag. Effects of a given rule on real output and the price level are not always similar, in that some rules increase the variance of one while reducing the variance of the other.

## II. The Model and Monetary Rules

### A. Model Structure

The quarterly model used in this study (Table I) is a modified and extended version of the Blinder and Solow model. It contains expenditure and monetary sectors with wealth effects on both the demand for goods and

TABLE I  
THE MODEL

Expenditure Sector	( 1 )	$E/P = a + b \left( \frac{Y-T}{P} \right) + c \left( \frac{W}{P} \right)_{-2}^{-d} r_{-2} + \epsilon$
	( 2 )	$G/P = \bar{G} + B_{-1}/P$
	( 3 )	$T/P = s/P + t (Y/P)$
	( 4 )	$Y/P = E/P + G/P$
Monetary Sector	( 5 )	$Md/P = g/P + j \left( \frac{Y-T}{P} \right) + k \left( \frac{W}{P} \right) -$ $l \left[ r + 4e \frac{P_{-1}^{-P-2}}{P_{-2}} \right] + \epsilon$
	( 6 )	$Ms/P = h \left( \frac{H}{P} \right) + q \left[ r + 4e \frac{P_{-1}^{-P-2}}{P_{-2}} \right]$
	( 7 )	$Md/P = Ms/P$
Definition of Wealth	( 8 )	$W/P = \bar{W} + \frac{H}{P} + \frac{B}{P \cdot r}$
Gov't. Budget Constraint	( 9 )	$\frac{G}{P} - \frac{T}{P} = 4 \left( \frac{H-H_{-1}}{P} + \frac{B-B_{-1}}{P} \right)$
Monetary Policy	( 10 )	$\frac{H-H_{-1}}{P} = f \left( \frac{H_{-1}}{P} \right)$
Real Output	( 11 )	$\frac{X}{P} = \frac{Y}{P} - \frac{B_{-1}}{P}$
Price Formation	( 12 )	$4 \left( \frac{P-P_{-1}}{P_{-1}} \right) = 4u \cdot e \left( \frac{P_{-1}^{-P-2}}{P_{-2}} \right) + v \left[ \left( \frac{X}{P} \right)_{-1} - \left( \frac{X}{P} \right)^* \right]$

Note: The scale factor of 4 in Equations 5, 6, 9, and 12 makes discrete quarterly changes compatible with other variables expressed in annual flow rates.

TABLE II  
LIST OF VARIABLES

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E/P = Real Private Expenditures

G/P = Real Government Expenditures

$\bar{G}$  = Real Exogenous Government Spending

B = Number of Government Bonds in Private Holdings (Each Bears a One Dollar Coupon)

T/P = Real Tax Receipts

Y/P = Real Private Income

W/P = Real Wealth

r = Real Rate of Interest

$e \left( \frac{P_{-1} - P_{-2}}{P_{-2}} \right)$  = Expected Rate of Inflation

Md/P = Demand for Real Cash Balances

Ms/P = Real Money Supply

H/P = Real Monetary Base (High-powered Money)

$\bar{W}$  = Real Physical Wealth

X/P = Real National Output

$(X/P)^*$  = Level of Real Output at Which Prices are Stable

P = General Price Level

$\epsilon$  = Stochastic Disturbance to Expenditure or Monetary Sector

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for money, a government budget constraint, endogenous price level determination, and a means to inject active monetary policy. The model is characteristically short run, with no capital accumulation.

The first four equations describe the market for goods and services, which is assumed to clear in each period. Equation 1 is aggregate private demand, which is a function of disposable income, lagged wealth, the lagged real rate of interest, and a stochastic disturbance term. Equation 2 specifies that government demand for goods and services consists of an autonomous component and interest on the stock of interest-bearing government debt held in private hands. Equation 3 is the tax function, and Equation 4 is the market clearance condition.

Equations 5, 6, and 7 describe the monetary sector. The demand for money is a function of disposable income (a proxy for private transactions), wealth, the nominal rate of interest, and a stochastic disturbance term. The supply of money is a function of the monetary base and the nominal rate of interest. An endogenous money supply implies that the monetary authority does not control the money supply exactly, but can, by forecasting the nominal rate of interest and adjusting the base, seek a money supply target. Alternatively, it can adjust the base to seek a market-determined interest rate target to its liking.

The monetary sector affects the goods and services sector through wealth and interest rate changes in Equation 1. Since these arguments are lagged, monetary sector effects on the demand for goods and services, and hence on aggregate output, are also lagged. On the other hand, expenditure sector changes affect the monetary sector without a lag, through disposable income.



Equation 8 defines privately-held real wealth as inclusive of "outside" or high-powered money, interest-bearing government debt, and a fixed stock of physical wealth.

Equation 9 is the government budget constraint, specifying that any budget deficits must result in issuing a like quantity of high-powered money or debt. Conversely, budget surpluses must result in retiring debt or money.

Equation 10 is the monetary authority's policy instrument. Monetary policy consists entirely of open market operations. By specifying "f", an increase or decrease in the stock of high-powered money is determined. Given the government surplus or deficit and the change in the stock of high-powered money (determined by monetary policy) the change in the stock of interest bearing government debt is fully determined by equation 9.

Equation 11 subtracts the transfer payment, interest on the government debt, from aggregate income to obtain aggregate output. In the model, the government pays interest only on debt held in private hands. If the monetary authority retires debt through its open market operations, such debt ceases to exist. In effect, the monetary authority is considered to be a part of the government.

Equation 12 specifies the price function, which is essentially a shifting Phillips curve. The rate of inflation is a function of the expected rate of inflation and the deviation of real output in the previous period from that level at which prices are stable. The function uses real output as a proxy for the rate of resource utilization.

The expected rate of inflation is some fraction,  $e$ , of last period's rate of inflation. The expected rate might also be specified as a weighted sum of past weights; but in the interest of simplicity, only the immediately

preceding period is considered. If  $u \cdot e = 1$ , the price function is simply the natural rate hypothesis in a no-growth economy, assuming a negative linear relationship between output and unemployment. If  $0 < u \cdot e < 1$  (as is assumed in the simulation tests), a trade off between inflation and unemployment persists in the long run, though on less favorable terms than in a short-run Phillips Curve.

### B. Stability

All tested versions of the model are stable under either money or debt financing of government deficits. However, if the interest elasticity of the monetary sector is reduced, the model becomes unstable with either debt or money financing of deficits.

This result is at variance with Blinder and Solow's conclusion, based on a similar model, that money financing of deficits always results in stability.<sup>1/</sup> The difference can be traced to Blinder and Solow's use of a simultaneous model in instantaneous time rather than a model in discrete time containing internal lags. In the latter case, even though the system may necessarily reach its new equilibrium level of output under money financing, the existence of internal lags causes overshooting. This introduces the possibility that the system may be unstable in cycles of ever-expanding amplitude about the new equilibrium.

Such instability was achieved in the present model by simply reducing the interest elasticity in the monetary sector, and suggests that Blinder and Solow's stability conclusions are strictly applicable only in a system without lags. Yet, this also implies that at least some of their stability conclusions are of academic interest only, since empirical studies indicate that the economy is replete with lagged effects.

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<sup>1/</sup> See Blinder and Solow (2), pp. 50-52 and (3) pp. 327-329.

### C. Monetary Rules

Four active monetary policy rules are tested in the study. The first is simply a fixed money supply rule, which targets the nominal money supply at its steady state value.<sup>2/</sup> This rule in a zero growth economy is analogous to Friedman's rule of a constant rate of growth of money in a growing economy and consequently is labeled the "Friedman rule." The present test is simply a special case of his policy prescription, in which rates of growth for both the economy and the money supply are equal to zero.

The second rule targets the real rate of interest at its steady state value.<sup>3/</sup> It is tempting to identify this rule with a neo-Keynesian prescription, since this school of economic thought has in the past emphasized interest rate and credit market conditions as the appropriate measures of monetary policy. However, since they typically have not distinguished between real and nominal interest rates in the literature, a real rate target does not necessarily reflect their views.

The third and fourth rules are similar to those proposed by Pierce (8) and are simply constrained versions of the first two rules. The rule designated "Pierce-M" targets the nominal money supply at its steady state value, subject to the constraint that the real rate of interest may not change by more than a specified number of basis points in each period ( $|\Delta r| \leq 10$  basis points per quarter in the tests). The rule designated "Pierce-r" targets the real rate of interest at its steady state value, subject to the constraint that the nominal money supply may not change by more than a given percentage

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<sup>2/</sup> Balbach and Karnosky (1) argue convincingly that the real money supply is not an appropriate target for stabilization policy.

<sup>3/</sup> The nominal rate of interest is not an appropriate target, since it includes the expected rate of inflation. An increase in inflation would increase inflationary expectations and the nominal rate of interest. A policy of targeting the latter would necessitate increasing the money supply--further exacerbating the inflation, leading to a further increase in the money supply, in self-reinforcing dynamics.

in each period ( $|\frac{\Delta M_S}{M_S}| \leq$  an absolute annual rate of change of 2 percent). Both of these rules simply constrain the vigor with which the monetary authority can pursue its policy operations. They allow for the possibility that in a stochastic system in which the monetary authority does not have perfect foresight and does not have exact control over the targets, it may be preferable to pursue policy operations which will only partially close the gap between actual and desired levels in any given period.<sup>4/</sup>

Each of the four rules is tested and compared, not only with the others, but also with a rule of total inactivity, one in which  $f = 0$  in all periods, so that the nominal monetary base remains fixed at its steady state value. In a growing economy, such a rule would be analogous to the monetary authority's causing the nominal monetary base to grow continually at a constant rate, without regard to the money supply, interest rate, or other fluctuating economic conditions.

A recognition lag of one period is incorporated into the implementation of each of the four active rules. The monetary authority observes a deviation of the targeted variable from its desired level in the period just past, then calculates the change in high-powered money (a value for "f") needed to return the variable to its desired level in the current period. In the tests of constrained rules, the change in the high-powered money is limited by the constraint, whenever the latter is effective.

The incorporation of a recognition lag significantly improves the model's representation of the actual conduct of policy. Consequently, the

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<sup>4/</sup> The Federal Open Market Committee typically follows such a procedure with respect to money supply targets. For example, if the money supply grows too rapidly in any period, a policy is adopted which is designed to return it gradually to the targeted level over the course of several periods, rather than pursuing an immediate and complete correction.

results of the simulation tests should be of greater interest to policy makers.

The policy equations for calculating the desired change in high-powered money necessary to implement each of the rules are given in Equations 13, 14, and 14b.

For Targeting  $r$

$$13 \quad f = - \left[ \frac{4kB_{-1}}{(r^T + r_{-1})^2} + q + \ell \right] \frac{r^T - r_{-1}}{\left[ h - k \frac{r^T - r_{-1}}{r^T + r_{-1}} \right] H_{-1}}$$

For Targeting  $M_s$

$$14a \quad \pi = (M_s^T - M_{s-1}) \cdot \left[ 1 - \frac{q}{\frac{kB_{-1}}{(r_{-1})^2} + q + \ell} \right]^{-1}$$

$$14b \quad f = \frac{-1 + \sqrt{1 + 4\pi/hH_{-1}}}{2}$$

The equations are derived by taking the total differential of the monetary sector and combining it with the monetary policy equation of the model.<sup>5/</sup> Since these equations are derived from a differential and yet are used to approximate a discrete change of a nonlinear function, they are not exact. However, in a number of simulation tests to check their validity,

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<sup>5/</sup> Since the lagged variables in equation 1 isolate the real sector from current period changes in the monetary sector, the model is partially decomposable. Policy changes in the stock of high-powered money have no immediate impact outside the monetary sector; hence, the total differential of this sector is sufficient to solve for the desired change in the monetary base to target either the money supply or the rate of interest. Mathematics of the derivation are available from the author upon request.

the calculated value of "f" caused a change in the targeted variable which was within five percent of the desired change in all instances and was usually within two percent. For the Pierce-r and Pierce-M rules, the appropriate constraint was appended to the equations.

### III. Simulation Procedure and Parameter Selection

#### A. Simulation

In the simulation tests, the system is initially at long-run equilibrium values. A random disturbance is then applied in each period, beginning with the first period. For any given test, the disturbance is applied to either the expenditure sector (equation 1) or the monetary sector (equation 5), is normally distributed with a mean of zero, and may have either zero or positive autocorrelation. The standard deviation of the disturbance term is varied across tests of different economic parameters in order to produce an approximately equal impact on the monetary sector regardless of the system parameters. Thus the monetary authority has the same magnitude of task in attempting to target the money supply or interest rate regardless of the specific parameters of the economic system being tested.

The disturbances push the economy off its long-run equilibrium path, and the monetary authority responds by changing the stock of high-powered money an amount calculated to return the targeted variable to its long-run equilibrium value. The monetary authority will not in general achieve its target, since unknown current period values of disposable income, the price level, and a new disturbance term will affect the monetary sector in addition to the policy operation.

Each simulation test is run for fifty periods; at the end of which, the variances of real output and the price level are computed. Each run is repeated fifty times for each economic environment (a given set of parameters), thus generating sets of 50 sample variances of real output and the price level for each environment.

#### B. Parameters

The rules are tested in sixteen different economic environments, corresponding to all combinations of varying four different parameters.

The four parameters are:

- (1) "Keynesian" or "Monetarist" interest elasticities. The Keynesian environment has a relatively high interest elasticity of money demand (-.8) and low interest elasticity of private expenditures (-.1). The "Monetarist" environment has low money demand interest elasticity (-.2) and relatively high private expenditure interest elasticity (-.4). Interest elasticities were chosen from among plausible values to provide a reasonable degree of contrast between the two environments.
- (2) Positive ( $\rho=.8$ ) or zero autocorrelation of the disturbance terms. Previous simulation tests, such as those of Cooper and Fischer (4), have not considered positive autocorrelation of disturbances. However, experience suggests that such is likely.
- (3) Disturbances in either the expenditure or monetary sectors. Poole's analysis concluded that a money supply target would be superior for expenditure disturbances, while an interest rate target would be superior for monetary disturbances.

- (4) An impact lag of the monetary sector on the goods and services sector of either two quarters or four quarters. Friedman has often suggested that long and variable impact lags may cause discretionary monetary policy to be destabilizing. While discretionary policy, per se, is not tested in the simulations; the effects of differing impact lags on the performance of monetary rules might prove to be interesting.

Values of parameters and steady state values of endogenous variables were selected from U.S. data and existing econometric studies to approximate the relationships in the U.S. economy. These values are listed in the appendix.

#### IV. Results

##### A. Criteria

Two criteria are considered in evaluating rule performance--the variance of real output and the variance of the price level. Each of the active policy rules is compared with the control policy of no change in high-powered money. If the variance of real output is smaller than that of the control policy at a statistically significant level of .95, then the active policy is considered to be stabilizing with regard to real output. Similarly, an active policy is stabilizing with respect to prices if the variance of the price level is smaller than that of the control policy at a significance level of .95. Conversely, a rule may also be inferior with respect to the control policy if the variance of real output or the price level is significantly larger.

Comparisons were also made between active policies. If one policy results in smaller price or output variance than all other policies at a



confidence level of .95, then such a policy is identified as the best for that criterion.

No attempt was made to develop an objective function which would combine the two criteria. Thus, it is possible for a rule to be stabilizing with regard to real output variance and inferior with regard to price level variance, or vice versa.

Two multiple analysis of variance techniques, developed by Dunnett (5) and by Tukey (10), are used for the statistical analysis. The Dunnett test simultaneously compares each active policy with the control policy, while the Tukey test compares active policies with each other. Prior to calculation of the appropriate confidence intervals, the sample variances are transformed into their natural logarithms in order to better satisfy certain assumptions. Naylor, Wertz, and Wonnacott (7) contains a useful discussion of these analysis of variance techniques and the data transformation.

## B. Comparative Rule Performance

Tables III and IV indicate the qualitative results of the tests. The appendix contains similar tables, providing the respective numerical values.

### 1. Effects on Real Output

Since Poole's analysis was conducted in a fixed-price model, only the effects on real output in the tables are comparable with his results. In the Keynesian economy (Table III), Poole's results are substantiated by the simulation tests only when the disturbance is positively autocorrelated and originates in the monetary sector. Under such circumstances, an unconstrained interest rate rule is clearly superior to all others, while a money supply target is actually worse than the control policy of doing nothing at all.

TABLE III  
KEYNESIAN ECONOMY

Autocorrelation of Disturbance Terms	Sector of Disturbance	Monetary Policy Lag	Effect on Real Output				Effect on Price Level				
			Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	
None	E	2									
None	E	4									
None	M	2	I					S*	I	S	I
None	M	4	I					S*	I	S	I
Positive	E	2									
Positive	E	4									
Positive	M	2	S*	I	S	I		S*	I	S	
Positive	M	4	S*	I	S	I		S*	I	S	I

Active policy rules are compared with a control policy of fixing the nominal monetary base (an absence of open market operations). "S" denotes stabilizing and means that the active policy produced a smaller real output or price level variance than the control policy. "I" denotes "inferior" and means that the active policy produced a larger real output or price level variance than the control policy. "S\*" denotes the best policy for a real output or a price level criterion in a particular economic environment.

TABLE IV  
MONETARIST ECONOMY

Autocorrelation of Disturbance Terms	Sector of Disturbance	Monetary Policy Lag	Effect on Real Output				Effect on Price Level				
			Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	
None	E	2	I		I			S	S		S
None	E	4	I	S			S		S		S
None	M	2	I	I	I			S*	I	S	I
None	M	4	I	I	I			S*	I	S	I
Positive	E	2	I	S			S	S	S	S	S
Positive	E	4	I	S				S	S	S	S
Positive	M	2	I	I			I	S*	I	S	I
Positive	M	4	I	I			I	S*	I	S	I

Active policy rules are compared with a control policy of fixing the nominal monetary base (an absence of open market operations). "S" denotes stabilizing and means that the active policy produced a smaller real output or price level variance than the control policy. "I" denotes "inferior" and means that the active policy produced a larger real output or price level variance than the control policy. "S\*" denotes the best policy for a real output or a price level criterion in a particular economic environment.

In most of the remaining Keynesian economic environments, no active policy rule performed significantly different from the control policy. The exception is an interest rate rule with non-autocorrelated disturbances in the monetary sector. While Poole's results indicate that such a rule should be the best choice to combat monetary disturbances, the simulations show that in this instance the rule actually performs worse than the control policy of complete inactivity. The monetary authority's efforts at stabilizing actually result in destabilizing, in the sense that the variance of real output is increased from what it would otherwise be in the total absence of monetary policy operations.

With regard to real output effects in the Monetarist economy (Table IV), the money supply rules, Friedman and Pierce-M, in general appear to have stabilizing effects when disturbances come from the expenditure sector. This is in agreement with Poole's analysis. However, with monetary sector disturbances, no active policy rule is able to have a stabilizing effect on the economy. In fact, the unconstrained interest rate rule, which Poole's analysis indicates is the preferred policy for money demand disturbances, actually has a destabilizing effect upon real output.

In attempting to track down the source of these perverse effects of the interest rate rule, it was noted that with positively autocorrelated monetary sector disturbances, the Pierce-r constrained interest rate rule provided results which were little different from the control policy. A plausible hypothesis is that the unconstrained interest rate rule identifies the correct target variable, but simply pursues it too vigorously. In the presence of recognition and impact lags, attempting to return the target variable to its desired level in one period may actually induce a greater real output variance.

To investigate this hypothesis, the simulation tests of monetary sector disturbances in a monetarist economy were rerun using a half-force interest rate rule. In employing this rule, the monetary authority changes the stock of high-powered money an amount calculated to close only one-half of the gap between actual and desired levels of the target variable. This constraint on policy is a stronger constraint than that employed in the Pierce-r rule.

The quantitative results of this simulation are contained in Table A-6 of the appendix. With positively autocorrelated monetary sector disturbances, the half-force interest rate rule has a stabilizing effect on real output. Thus, by constraining a Poole-recommended rule, destabilizing effects were transformed into stabilizing ones.<sup>6/</sup>

This suggests that Poole's conclusions provide only a first step toward devising a monetary rule to stabilize real output. Even if the monetary authority knows the source of disturbance, and therefore the appropriate target variable, pursuing a monetary rule may nevertheless act as a destabilizing force on the economy. In the presence of recognition and impact lags, it may be necessary to constrain policy to only partially close, in any one period, the gap arising between actual and desired levels. Identifying the appropriate speed with which to close the gap adds a difficult quantitative dimension to Poole's qualitative conclusions, even in the simple case of only a single source of disturbance to the economy.

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<sup>6/</sup> With non-autocorrelated disturbances, the half-force interest rate rule does not succeed in producing stabilizing effects, however. Throughout the tests, due to the recognition lag of policy, rule performance proved to be less effective when disturbances were not autocorrelated. It is probable that a more severely constrained interest rate rule would have been stabilizing even in this case, but no search for the appropriate degree of restraint was undertaken.

## 2. Effects on the Price Level

For price level stability, an unconstrained interest rate rule is significantly better than all other rules whenever the disturbance arises in the monetary sector. This is true regardless of whether the economy is Keynesian or Monetarist, whether there is zero or positive autocorrelation, and whether there exists a two-quarter or four-quarter impact lag.

Comparative rule performance with regard to price level stability is much less clear-cut for expenditure disturbances, however. In the Keynesian economy, no active rule yields results which are significantly different from the control policy; while in a Monetarist economy with auto-correlated disturbances, all active rules have a stabilizing effect relative to the control policy. In general, the Friedman Rule and its Pierce-M constrained version have stabilizing effects on the price level in the greatest number of economic environments for expenditure disturbances, although the interest rate rule is not far behind.

It is interesting that in some instances, a rule which has "inferior" or destabilizing effects on real output, may nevertheless have stabilizing effects on the price level. This paradox can be explained by examination of the price formation function. The variance of the price level is dependent not only on the variance of real output, but also on the degree of positive autocorrelation of the latter relative to its steady state value. A high degree of positive autocorrelation of real output permits the shifting Phillips Curve relationship to shift upward or downward quite far, thus increasing the variance of the price level. Thus, while a rule may increase the variance of real output, if it simultaneously decreases the degree of positive autocorrelation of real output relative to its steady state value, the net effect may be to reduce the variance of the price level.

## V. Conclusions

These simulation tests indicate that in the presence of recognition and impact lags, even a relatively passive, nondiscretionary monetary policy that simply targets either the money supply or the rate of interest according to the source of disturbance, may nevertheless be destabilizing. In a recent exchange with Modigliani, Friedman explicitly recognizes this problem when he suggests that recognition lags can cause even a money supply rule such as he favors to do more harm than good.

. . . a policy of discretionary movement in an instrument can lead to worse results than stability in it, if there is enough lack of correlation between the actions taken and the actions that should be taken, even though, on the average, those actions are in the right direction. . . . If you know there is a 10-percent decline in the quantity of money, you can offset it--of course! But what you really have to demonstrate is that, over time, you will in fact know enough about such changes and will be able to identify them soon enough, so that you can make adjustments which, on the average, will do more good than harm.<sup>7/</sup> (Emphasis added.)

While Poole's analysis relating the optimal choice of rule with the source of disturbance seems to be incontrovertible in a comparative static system, it may actually encourage destabilizing policy actions in a dynamic system containing recognition and impact lags. In the presence of lags, it is likely to be preferable for policy to aim at only partially closing the gap between actual and desired levels of the target variable in any one period.

Rule performance is not affected in these tests by the length of the impact lag. This, however, is not evidence that purely discretionary policy would be similarly unaffected. In the simulation tests, the impact

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<sup>7/</sup> Discussion by Milton Friedman of a paper by Franco Modigliani in "The Monetarist Controversy: A Seminar Discussion," Economic Review Supplement, Federal Reserve Bank of San Francisco, Spring 1977, pp. 25-26.

lag does not lie between a policy operation and its target (the money supply or interest rate); whereas, with purely discretionary policy, the impact lag does lie between a policy operation and its target (the rate of unemployment or inflation). In the latter instance, the length of the impact lag is likely to be important.

The simulation results do not provide overwhelming evidence in support of a Friedman-type money supply rule. In fact, only under circumstances of expenditure sector disturbances in an economy with characteristically Monetarist interest elasticities does such a rule prove to be stabilizing. In fairness to his position though, it should be noted that this is precisely the economic environment Friedman believes to exist in the United States.



## APPENDIX

TABLE A-1  
PARAMETER VALUES

	Value	Elasticity
$\left(\frac{X}{P}\right)^*$	1000	
$\bar{G}$	200	
$\bar{W}$	3600	
b	.6	$E_{(Y-T)} = .6$
c	.05	$E_{(W)} = .25$
e	.4	
h	2	$Ms_{(H)} = .8$
j	.125	$Md_{(Y-T)} = .4$
k	.04	$Md_{(W)} = .64$
q	1250	$Ms_{(r)} = .2$
s	-294	
t	.5	$T_{(Y)} = 2.39$
u	1	
v	$1 \times 10^{-4}$	

TABLE A-2  
DUAL PARAMETER VALUES

	<u>Keynesian</u>		<u>Monetarist</u>	
	Value	Elasticity	Value	Elasticity
a	200		400	
d	2000	$E_{(r)} = -.1$	8000	$E_{(r)} = -.4$
g	178		28	
z	4700	$Md_{(r)} = -.8$	950	$Md_{(r)} = -.2$

TABLE A-3  
EQUILIBRIUM VALUES OF VARIABLES

X = 1000	W = 4000
P = 1	B = 12
Y = 1012	H = 100
E = 800	$M_s = 250$
G = 212	$M_d = 250$
T = 212	r = .04

TABLE A-4  
KEYNESIAN ECONOMY

Autocorrelation of Disturbance Terms	Sector of Disturbance	Monetary Policy Lag	Effect on Real Output					Effect on Price Level				
			Control Policy	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	Control Policy	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule
None	E	2	9.40	9.43	9.41	9.42	9.36	9.33	9.27	9.07	9.22	9.11
None	E	4	9.36	9.42	9.46	9.42	9.37	9.28	9.39	9.05	9.13	8.98
None	M	2	2.50	3.25(I)	2.55	2.60	2.56	16.04	18.09(S)	15.50(I)	16.61(S)	15.64(I)
None	M	4	2.50	3.21(I)	2.55	2.60	2.53	16.13	18.11(S)	15.62(I)	16.54(S)	15.81(I)
Positive	E	2	9.25	9.28	9.28	9.32	9.22	7.37	7.53	7.21	7.19	7.34
Positive	E	4	9.24	9.18	9.28	9.25	9.26	7.24	7.60	7.24	7.42	7.35
Positive	M	2	2.38	1.65(S)	2.68(I)	1.96(S)	2.62(I)	14.28	18.02(S)	13.82(I)	15.93(S)	14.03
Positive	M	4	2.24	1.65(S)	2.73(I)	1.93(S)	2.63(I)	14.33	18.08(S)	13.72(I)	15.88(S)	13.86(I)

(S) = Stabilizing at a confidence level of .95.

(I) = Inferior at a confidence level of .95.

The control policy fixes the nominal monetary base. Values are the means of the natural logs of the variances of real output and the price level. All price level values have a negative sign, indicating that the mean variance is less than one.

TABLE A-5

## MONETARIST ECONOMY

Autocorrelation of Disturbance Terms	Sector of Disturbance	Monetary Policy Lag	Effect on Real Output					Effect on Price Level				
			Control Policy	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	Control Policy	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule
None	E	2	7.81	9.13(I)	7.70	8.05	7.74	10.74	11.13(S)	11.38(S)	10.74	11.21(S)
None	E	4	7.86	9.29(I)	7.66(S)	8.02	7.67(S)	10.80	10.96	11.24(S)	10.63	11.24(S)
None	M	2	5.35	7.60(I)	5.75(I)	6.10(I)	5.39	12.96	14.74(S)	12.55(I)	13.57(S)	12.40(I)
None	M	4	5.28	7.69(I)	5.68(I)	6.00(I)	5.48	13.19	14.78(S)	12.50(I)	13.81(S)	12.51(I)
Positive	E	2	7.60	7.99(I)	7.31(S)	7.72	7.21(S)	8.66	9.43(S)	9.42(S)	9.31(S)	9.52(S)
Positive	E	4	7.49	7.94(I)	7.25(S)	7.61	7.32	8.72	9.32(S)	9.63(S)	9.28(S)	9.86(S)
Positive	M	2	4.93	5.68(I)	5.85(I)	5.04	5.79(I)	11.39	15.32(S)	10.71(I)	14.50(S)	10.73(I)
Positive	M	4	5.01	5.82(I)	5.97(I)	4.91	5.83(I)	11.33	15.42(S)	10.70(I)	14.51(S)	10.86(I)

(S) = Stabilizing at a confidence level of .95.

(I) = Inferior at a confidence level of .95.

The control policy fixes the nominal monetary base. Values are the means of the natural logs of the variances of real output and the price level. All price level values have a negative sign, indicating that the mean variance is less than one.

TABLE A-6

## MONETARIST ECONOMY WITH INTEREST RATE RULE AT ONE-HALF

Autocorrelation of Disturbance Terms	Sector of Disturbance	Monetary Policy Lag	Effect on Real Output					Effect on Price Level					
			Control Policy	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	Control Policy	Interest Rate Rule	Friedman Rule	Pierce-r Rule	Pierce-M Rule	
None	E	2											
None	E	4											
None	M	2	5.35	5.93(I)				12.96	15.25(S)				
None	M	4	5.28	5.90(I)				13.19	15.23(S)				
Positive	E	2											
Positive	E	4											
Positive	M	2	4.93	4.58(S)				11.39	14.31(S)				
Positive	M	4	5.01	4.57(S)				11.33	14.32(S)				

(S) = Stabilizing at a confidence level of .95.

(I) = Inferior at a confidence level of .95.

The control policy fixes the nominal monetary base. Values are the means of the natural logs of the variances of real output and the price level. All price level values have a negative sign, indicating that the mean variance is less than one.

DERIVATION OF POLICY EQUATIONS

Derivation of Policy for Interest Rate Target

From the model, substituting Equations 5 and 6 into 7 and collecting terms:

$$(30) \quad h \left( \frac{H}{P} \right) = \frac{g}{P} + j \left( \frac{Y-T}{P} \right) + k \left( \bar{W} + \frac{H}{P} + \frac{B}{Pr} \right) - (q + \ell) \left[ r + 4e \frac{P_{-1} - P_{-2}}{P_{-2}} \right]$$

Open market operations do not affect  $Y-T$ ,  $P$ , or  $e \left[ \frac{P_{-1} - P_{-2}}{P_{-2}} \right]$  in the current period, hence these terms are considered as exogenously given for the current period total differential. Multiplying through by  $P$  and taking the total differential yields:

$$(31) \quad hdH = kdH + \frac{k dB}{r} - \frac{kB}{r^2} dr - (q + \ell) dr$$

In open market operations, since high-powered money is exchanged for bonds,  $dB/r = -dH$  for small changes. Substituting into (31) and collecting terms yields:

$$(32) \quad hdH = - \left[ \frac{kB}{r^2} + q + \ell \right] dr$$

At this point, it is useful to substitute the average level of bonds for  $B$  and the average interest rate for  $r$  in the above equation. This will make possible the evaluation of the differential at the midpoint of the discrete change rather than at the beginning point, and

will significantly improve accuracy. Since average  $B = B_{-1} + 1/2 dB$ , and  $dB/r \approx -dH$ , it follows that average  $B = B_{-1} - 1/2 rdH$ . Average  $r = \frac{r^T + r_{-1}}{2}$ . Substituting both of these into (32) and collecting terms yields:

$$(33) \quad \left( h - k \frac{dr}{r^T + r_{-1}} \right) dH = - \left[ \frac{4kB_{-1}}{(r^T + r_{-1})^2} + q + \ell \right] dr$$

Solving for  $dH$  and converting to difference equation form yields:

$$(34) \quad H - H_{-1} = - \left[ \frac{4kB_{-1}}{(r^T + r_{-1})^2} + q + \ell \right] \frac{r^T - r_{-1}}{h - k \left[ \frac{r^T - r_{-1}}{r^T + r_{-1}} \right]}$$

From Equation 10 of the model,

$$(35) \quad H - H_{-1} = fH_{-1}$$

Substituting (35) into (34) and solving for  $f$  yields:

$$(36) \quad f = - \left[ \frac{4kB_{-1}}{(r^T + r_{-1})^2} + q + \ell \right] \frac{r^T - r_{-1}}{\left[ h - k \left[ \frac{r^T - r_{-1}}{r^T + r_{-1}} \right] \right] H_{-1}}$$

Equation (36) determines the value of "f" for targeting the real rate of interest,  $r^T$ .

#### Derivation of Policy for Money Supply Target

Taking the total differential of the money supply function of the

model and multiplying through by P, yields:

$$(37) \quad dM_s = h \, dH + q \, dr$$

Solving for dr in Equation (32) and substituting into (37):

$$(38) \quad dM_s = h \, dH - \frac{q \, h \, dH}{\left(\frac{kB}{r^2}\right) + q + \ell}$$

$$(39) \quad h \, dH = dM_s \left[ 1 - \frac{q}{\frac{kB}{r^2} + q + \ell} \right]^{-1}$$

Converting to difference equation form and substituting (35) into (39) yields:

$$(40) \quad hfH_{-1} = (M_s^T - M_{s_{-1}}) \left[ 1 - \frac{q}{\frac{kB_{-1}}{(r_{-1})^2} + q + \ell} \right]^{-1}$$

It is not feasible in this case to use midpoint values for r and B, since the midpoint of r is itself a function of "f" and any attempted substitution creates a very complex higher ordered equation. However, through trial and error it was found that substituting the end point H for  $H_{-1}$  in (40) significantly improved its accuracy. The substitution tends to compensate for the inaccuracies introduced by using beginning point rather than midpoint values for r and B. From Equation (35)

$$(41) \quad H = (1 + f) H_{-1}$$

Letting the right-hand side of (40) equal  $\pi$  and substituting the



value of  $H$  from Equation (41) into  $H_{-1}$  of Equation (40) (the latter is an ad hoc substitution, not a mathematical operation), yields:

$$(42) \quad hH_{-1} (1 + f) f = \pi$$

Solving for  $f$ :

$$(43) \quad f = \frac{-1 + \sqrt{1 + \frac{4\pi}{hH_{-1}}}}{2}$$

Equation (43) with  $\pi$  defined as the right-hand side of (40), determines the value of "f" for targeting the nominal money supply,  $M_s^T$ .

## LITERATURE CITED

1. Balbach, A. and Denis Karnosky. "Real Money Balances: A Good Forecasting Device and a Good Policy Target?" Federal Reserve Bank of St. Louis Review, 57: 11-15, September 1975.
2. Blinder, Alan S. and Robert M. Solow. "Analytical Foundations of Fiscal Policy," The Economics of Public Finance. Washington, D.C.: The Brookings Institution, 1974, pp. 45-57.
3. \_\_\_\_\_. "Does Fiscal Policy Matter?" Journal of Public Economics, 2: 318-337, November, 1973.
4. Cooper, J. and S. Fischer, "Simulation of Monetary Rules in the FRB-MIT-Penn Model," Journal of Money, Credit, and Banking, Vol. IV, No. 2 (May 1972), pp. 384-396.
5. Dunnett, C. W. "A Multiple Comparison Procedure for Comparing Several Treatments with a Control," Journal of the American Statistical Association, 50: 1096-1121, December, 1955.
6. Laidler, D. E. W., "The Influence of Money on Real Income and Inflation: A Simple Model with Some Empirical Tests for the United States, 1953-1972," Manchester School, 41, No. 4, (December 1973), 367-395.
7. Naylor, Th., K. Wertz, and Th. Wonnacott. "Some Methods for Evaluating the Effects of Economic Policies Using Simulation Experiments," Review of the International Statistical Institute, 36: 2, 1968, pp. 184-200.
8. Pierce, J. "Some Rules for the Conduct of Monetary Policy," Controlling Monetary Aggregates, Federal Reserve Bank of Boston (1969), pp. 133-144.
9. Poole, William. "Optimal Choice of Monetary Policy Instruments in a Simple, Stochastic Macro Model," Quarterly Journal of Economics, 84: 197-216, May, 1970.
10. Tukey, J. W. The Problem of Multiple Comparisons. Dittoed manuscript, Princeton University, 1953.