The Impact of Paying Interest on Reserves in the Presence of Government Deficit Financing

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Abstract

This paper re-examines the impact that paying interest on reserves has on price level indeterminacy, price level volatility, and overall economic well-being. Unlike previous papers which examined these issues, the model developed in this paper allows the return on reserves to equal the return on government securities, which is less than the prevailing return on storage. Equally important, this model also considers how deficit financing changes the impact that paying interest on reserves has on the economy. I show that the number of steady state equilibria are equal to, or greater than, the number that arise when no interest is paid on reserves. In other words, the level of economic indeterminacy is equal to or greater than in an economy without interest payments. When the level of indeterminacy is the same, then economic volatility is reduced with the introduction of interest payments. However, when there exists greater indeterminacy in the interest-on-reserves economy, then there also exists greater volatility. In addition, under certain conditions, paying interest on reserves can be welfare enhancing. When it is not, an appropriate expansionary open market operation can offset the welfare losses associated with interest payments. Finally, under a narrow set of conditions, unpleasant monetarist arithmetic may obtain.

1 Introduction

The issue of paying interest on reserves is not new and was introduced by Milton Friedman almost fifty years ago in *A Program for Monetary Stability*.[1] Friedman's original motivation was to make the 100% reserve requirement of the "Chicago Plan" more palatable to a banking system subject to only a fractional reserve system. The goal of the Chicago Plan and the proposal to pay interest on reserves was to establish greater price level stability and to reduce excessive fluctuations in the price level.

There has been considerable research regarding the implications of paying interest on reserves.[2] Three studies in particular, Sargent and Wallace (1985), Smith (1991), and Freeman and Haslag (1996), have examined in some detail whether Freidman's proposal would bring about the desired reductions in price level indeterminacy and volatility. In addition, these papers examined the welfare implications of switching from a system of not paying interest on reserves to one which did.[3] While the research cited above improved our understanding of those conditions under which paying interest will produce more or less economic volatility and greater welfare, it suffered from two specific limitations. First, it did not allow for multiple assets in a meaningful way. Second, it assumed that either the government ran a balanced budget or had a surplus.

These are important limitations. A lack of multiple assets results in the return on money balances (reserves) being equated to the return on storage (capital). Because storage is the only other asset and its return invariant, its real world counterpart would be the average long-term

[1] It was not until recently, however, that Congress has perennially introduced legislation which would allow the Federal Reserve to pay interest on reserves. Although this legislation has yet to pass, it has the support of many different constituencies, including the Federal Reserve itself. In particular see the recent testimony before Congress given by Kohn (2004) and the prior testimony by Meyer (2001).

[2] The focus of these studies fall broadly into three categories: the use of interest payments on reserves strictly as a policy tool, the role of paying interest on reserves on payment services policies, and the impact of this policy on general welfare, price level determinacy, and economic stability relative to the current system.

See Woodford (2003), Goodfriend (2002), Hall (2002), and Goodhart (2000), for examples of papers which examine the use of paying interest of reserves as an instrument of monetary policy. In particular, these papers discuss the role of paying interest on reserves in implementing monetary policy in a world without money or where the zero bound on interest rates is binding. See Toma (1999) and Lacker (1997) for discussions of interest on reserve payments in terms of the Fed's role as a clearinghouse for settlement of private payment systems.

[3] Sargent and Wallace (1985) highlighted two key facts. First, paying interest on reserves, combined with a 100% reserve requirement, would not necessarily lead to a deterministic price level and less fluctuations. Second, the method of financing interest payments could lead to real differences in economic outcomes. Smith (1991) showed that if the rate of return on reserves were tied to productive investment technologies, then the indeterminacies described in Sargent and Wallace (1985) disappear. However, interest financed via taxes resulted in a series of oscillating equilibria, and thus, might actually lead to greater economic fluctuations than when interest was not paid. In addition, Smith (1991) showed that there was no clear cut welfare justification for paying interest on reserves. Finally, Freeman and Haslag (1996) explored means by which paying interest on reserves could be Pareto improving. They showed that if an appropriate, accommodative open market operation was undertaken, then the initial old generation will be indifferent, while all future generations are better-off. See Guzman (2004) for a more in-depth review of these three papers.

return on capital. However, Friedman's proposal was for reserves to offer a return equal to that of short-term government bonds.[4] The fundamental idea here is that the return on reserves should be equal to the return on assets with similar maturity and risk structures.

In the previous literature, how interest payments were financed crucially impacted the likelihood for volatility to arise. However, this literature assumed that the budegt was either balanced or in surplus. This ignores the impact that deficit financing has on both the means for financing interest payments and also on the complications that arise from simultaneously attempting to finance a deficit and fix the real return on reserves. If the total sum of expenditures, interest paid on bond holdings, and interest paid on reserve balances exceeds tax revenue, then the role of financing interest payments on reserves via taxes or earnings on assets is not relevant. The appropriate concern now becomes how the mix of additional bond and money issues impacts the economy when the government simultaneously ties the return on reserves to other assets, such as bonds.

The goal of this paper is to correct for the two omissions cited above and to re-examine the impact of switching from a system where reserves earn no interest to one where they do. More specifically, I study the issues of indeterminacy of equilibria, economic volatility, and welfare gains in an economy where interest is paid on reserves. This is done in the context of a two period overlapping generations model with multiple assets and a government deficit that must be financed with either debt or seigniorage.

I am particularly interested in addressing three questions. First, in the presence of a government deficit and a return on storage that dominates all other rates of return, does paying interest on reserves reduce potential indeterminacy of equilibria? Second, under the same conditions does the amount of economic volatility increase or decrease? Third, are there any welfare justifications for switching to a system where reserves earn interest, without accompanying open market operations by the central bank? In addition, if paying interest on reserves is not welfare improving, then are the results of Freeman and Haslag (1996), namely that an accompanying, expansionary open market operation can provide a welfare justification, also relevant to this model? Finally, given the presence of both debt and seigniorage in financing of the deficit, does unpleasant monetarist arithmetic arise?

The key findings of this paper can be easily summarized as follows. When there exists an after-tax government deficit and reserves are paid a rate of return equal to that of bonds (and less than

[4] See Friedman (1960, Chapter 3, p75) for the lone paragraph devoted to the appropriate choice of the rate of return to be paid on reserve holdings.

the return on storage), the number of steady state equilibria (in terms of real money balances) are equal to, or greater than, the number that arise when no interest is paid on reserves. Thus, the level of economic indeterminacy is equal to or greater than in an economy without interest payments. This runs counter to what Friedman had envisioned and the results of Smith (1991). Second, when the number of steady state equilibria are the same in the interest and non-interest economies (i.e., the level of indeterminacy is equal), then economic volatility is reduced with the introduction of interest payments. However, when greater indeterminacy in the interest-on-reserves economy exists, then there also exists greater volatility.

Third, when there exists multiple (two) equilibria in both economies, then the equilibrium associated with low real money balances in the interest economy is welfare improving compared to the non-interest economy. The reverse is true at the high real money balance equilibrium. In this case, an appropriate expansionary open market operation can offset the welfare losses associated with interest payments on reserves. In addition, when there exists a unique steady state equilibrium in the non-interest bearing economy, then there is always a welfare loss associated with paying interest on reserves. In this case one may not be able to mitigate the welfare loss by undertaking appropriate open market operations. Finally, under a narrow set of conditions, unpleasant monetarist arithmetic may arise in steady state equilbrium.

The basic intuition behind these results is as follows. The government, in either economy, faces simultaneous, competing decisions that it must make regarding financing its deficit. It must decide which instruments to use, and their relative quantities, to finance its deficit while at the same time supplying quantities that are consistent with individual's wanting to hold all assets in equilibrium (i.e., no asset can have a negative return). Once it has decided the mix of money and bonds needed to finance its deficit, it must then choose between using a small seigniorage tax base and large seigniorage tax rate, or vice versa. This latter consideration gives rise to a Laffer curve and in its simplest case, two steady state equilibria.

However, in the economy with interest payments on reserves, because the returns on real balances are tied to the returns on bond holdings, this effectively constrains the range of the seigniorage tax base which is consistent with financing a given deficit. Thus, the set of real money balances which are consistent with equilibrium are smaller in the presence of interest payments on reserves. As a result, when there exist two equilibria in both economies, the variance (and hence volatility) of steady state outcomes will be less in the economy with interest payments.

In some instances it will be the case that when no interest is paid on reserves, some real money balances consistent with supporting a given deficit would result in negative returns on assets and thus not all assets would be held. These money balances are obviously not consistent with equilibrium. In this case, while there might be two candidate steady state levels of real money balances, only the larger one is consistent with financing the deficit without violating the requirement that assets earn a non-negative return. This situation, of a unique steady state equilibrium, is more likely to occur in the non-interest economy since the set of real money balances which could potentially support a given deficit is larger for the non-interest economy. Thus, under certain parameter settings, the interest economy will have two steady state equilibria while the non-interest economy only one. Obviously, in this case, both the level of indeterminacy and volatility will be greater on the interest bearing economy. Finally, under certain assumptions, the economies are both Samuelson-case economies where savings (and hence consumption) are strictly increasing in the level of real money balances. Thus, since the seigniorage tax base is larger in the non-interest economy, the steady state equilibrium values of real balances are higher (and lower when two equilibria exists) than those of the corresponding interest economy.

The basic economic model used in this paper is a variation of Bhattacharya *et al.* (1998) and simply augments it with interest payments on reserves. The structure of the economy is as follows. The economy consists of an infinite sequence of two-period lived, overlapping generations, where individuals across generations are identical in all dimensions. Consumers are endowed each period with a given amount of a consumption good which they either consume or invest. Individuals may invest their saving in any of three different assets. There is a storage technology, which pays the highest rate of return, government bonds, and money, whose return is dominated by all other assets. It is assumed that individuals cannot invest directly in the storage technology and that all investment in storage must be intermediate and is subject to a reserve requirement. Required reserves pay a rate of return equal to that of government securities. Thus, individuals save by purchasing bonds and depositing their savings with intermediaries.

In addition, there exists a government which must finance a constant per capita after-tax deficit while also paying interest on bonds and reserves. This deficit and interest payments are funded by some combination of money creation and new debt offerings. Finally, it is assumed that the government conducts policy by choosing (once and for all in the first period) a ratio of bonds to currency. Variations in this ratio can be thought of as permanent open market operations.

The remainder of the paper proceeds as follows. Section 2 describes the model economy, while Section 3 states conditions necessary for steady state equilibrium exist. The propensity for unpleasant monetarist arithmetic to arise is also discussed in this section. A comparison of steady state equilibria, of the issue of price determinacy and volatility, and of economic welfare between an economy without interest payments and one with interest payments are the topics of Section 4. Section 5 concludes and all proofs can be found in the Appendices.

2 The Model

The economy consists of an infinite sequence of two-period lived, overlapping generations, along with an initial old generation. Time is discrete and indexed by $t = 1, 2, ...$ At every date t a new generation, comprised of N identical members, is born. There exists a government that has a constant per capita real expenditure level of $g > 0$ in each period. The government levies no direct taxes, and so it must finance its deficit by issuing money and bonds.[5] Let M_t denote the per capita stock of money outstanding at the end of period t, and B_t denote the outstanding per capita supply of bonds (in nominal terms). All bonds are of one-period maturity, and are default free.

2.1 Consumers

Individuals are endowed with some of a single, non-produced good, which can either be consumed or stored. The endowment of a representative individual is given by $\omega_1 > 0$ when young and by $\omega_2 \geq 0$ when old. In addition, members of the initial old are each endowed with $\omega_0 \geq 0$ units of consumption, and with $M_0 > 0$ units of fiat currency. Consumption of a representative agent born at t is denoted by c_t^t when young and c_{t+1}^t when old. All individuals have the identical utility function $U\left(c_t^t, c_{t+1}^t\right)$, where U is assumed to be strictly increasing in each argument, to be twice continuously differentiable, and to be strictly quasi-concave.[6]

Young individuals can either store their endowment, sell it to old individuals in exchange for money, or sell it to the government in exchange for either money or bonds. All individuals are assumed to have access to a non-stochastic, constant returns to scale technology for storing their endowment. In particular, one unit stored at date t returns $R > 1$ units of consumption at date $t + 1$.[7] Let k_t denote the amount that an individual chooses to store at date t. In addition, let p_t

[5] Alternatively, one could imagine that the government levies some (fixed) lump-sum taxes. Then one would interpret the endowments received by individuals, $\omega_1$and ω_2, as after-tax endowments, and g as the deficit. In much of the literature, for example Smith (1991), Freeman and Haslag (1996), and Sargent and Wallace (1985) how the interest payments on reserves is financed is important to the outcome. However, given a positive after-tax per capita deficit implies that the financing scheme will not be important.

[6] The initial old, of course, value only old age consumption and desire as much of it as possible.

[7] If population growth were allowed, then the condition that R exceeds one plus the rate of population growth

denote the time t price level, let z_t denote the holdings of real balances by a young individual at t, and let b_t denote real bond holdings by a representative young agent at t. It is assumed that storage is subject to a reserve requirement,

$$z_t \geq \lambda k_t, \tag{1}$$

and that the government pays a gross rate of return x_{t+1} in period $t+1$ on the nominal balances which were obtained in period t. In addition to this reserve requirement, each young individual faces the following budget constraints at t:

$$c_t^t + z_t + k_t + b_t \leq \omega_1 \tag{2}$$

$$c_{t+1}^t \leq \omega_2 + Rk_t + \rho_{t+1} b_t + \left(\frac{p_t}{p_{t+1}} \right) x_{t+1} z_t, \tag{3}$$

where ρ_{t+1} is the gross real rate of return on government bonds between t and $t+1$.

The problem of a young individual at t is to maximize $U\left(c_t^t, c_{t+1}^t\right)$ subject to equations (1)-(3). If

$$R > x_{t+1} \frac{p_t}{p_{t+1}} \tag{4}$$

holds, then the reserve requirement is binding, and equation (1) holds as an equality. This situation is focused on throughout, in which case one can transform the young individuals' problem as follows. Let $d_t \equiv k_t + z_t = (1+\lambda) k_t$ denote storage plus reserves, which will be referred to as "deposits." In addition, let $\phi \equiv \frac{1}{1+\lambda}$, where ϕ denotes the fraction of deposits held in the form of storage, and $1-\phi$ can be thought of as the fraction of deposits required to be held as reserves. With this notation, the problem of a young individual at t can be rewritten as

$$\max U\left(c_t^t, c_{t+1}^t\right)$$

subject to

$$c_t^t + d_t + b_t \leq \omega_1 \tag{5}$$

$$c_{t+1}^t \leq \omega_2 + \left[\phi R + (1-\phi) x_{t+1} \left(\frac{p_t}{p_{t+1}} \right) \right] d_t + \rho_{t+1} b_t \tag{6}$$

would need to be imposed.

Obviously, if bonds and deposits are both to be held,

$$\rho_{t+1} = \phi R + (1-\phi)\, x_{t+1} \left(\frac{p_t}{p_{t+1}} \right) \ ; \ t \geq 1 \tag{7}$$

must hold. The right hand side of equation (7) is simply the weighted return on a portfolio consisting of storage and currency, with $1-\phi$ being the portfolio weight attached to currency. Equation (7) then requires that the return on government bonds equal the appropriately weighted return on storage and currency, which is — in effect — the rate of return on deposits. Finally, in keeping with Friedman's original idea that the rate of return on reserves be equal to the short-term yield on government securities, it is assumed that $\rho_{t+1} = x_{t+1}$.[8] Thus equation (7)can be rewritten as

$$\rho_{t+1} = \frac{\phi R}{1-(1-\phi)\left(\frac{p_t}{p_{t+1}}\right)} \ ; \ t \geq 1. \tag{8}$$

When equation (8) holds, the problem confronting young individuals can be even further simplified. Let S_t denote total savings by a young individual at t : i.e., $S_t \equiv d_t + b_t$. Then this individual can be viewed as choosing S_t to maximize $U\left[\omega_1 - S_t, \omega_2 + \rho_{t+1} S_t\right]$. Let

$$S\left(\rho_{t+1}\right) \equiv \arg\max U\left[\omega_1 - S_t, \omega_2 + \rho_{t+1} S_t\right], \tag{9}$$

then the function S summarizes an individuals's optimal savings behavior. The following conditions on S are assumed to hold throughout the remainder of the paper.

Assumption 1 *For all dates $t \geq 1$, the function S satisfies*

$$S\left[\min\left\{\phi R, 1\right\}\right] \geq 0. \tag{A.1}$$

Assumption 2 *For all dates $t \geq 1$, the function S satisfies*

$$S'(\rho) > 0, \ \forall \rho > 0. \tag{A.2}$$

Assumption (A.1) implies that $S(1) \geq 0$ holds, rendering this a "Samuelson case" economy, and (A.2) asserts that savings are increasing in the rate of return, thereby ruling out "large" income

[8] See section 2.1.1 for a discussion of the interpretation of equating the return on nominal money balances to the real return on bond holdings.

effects.[9] Finally, Assumption (A.1) implies that $\phi \geq S^{-1}(0)/R$ must be satisfied; in effect this imposes an upper bound on the level of the reserve requirement. When this bound is in effect, individuals are willing to save non-negative amounts, regardless of the rate of return on reserves.

2.1.1 Remarks

Some aspects of the individual's problem described above would benefit from great explanation. With respect to how reserve requirements are modeled, equation (1) is meant to be interpreted as a conventional reserve requirement. Consistent with Bhattacharya *et al.* (1998), Espinosa-Vega and Russell (1998), and Wallace (1984), individuals can be thought of as not being allowed to store their own goods. In other words, all storage must be intermediate, where intermediaries are required to hold a fraction of deposits — equal to $1-\phi$ — in the form of cash reserves. If there is free entry into intermediation, intermediaries will earn zero profits and hold a portfolio maximizing the utility of a representative depositor. In this case, equations (1)-(3) simply represent the consolidated balance sheets of banks and individuals.[10]

The definition and interpretation of the interest rate paid on reserves, x_{t+1} also deserves further attention. While Friedman (1960) does not spend a great deal of time discussing how to set the interest rate on reserves, he does briefly suggest that a viable option would be to set the rate equal to the average yield on short-term government bonds from the previous few quarters.[11] As a practical matter, it was suggested that this be done on a quarterly or semi-annual basis. Consistent with, although not identical to, this suggestion and the idea that the rate of return on reserves should be equal to the rate prevailing on assets of a similar maturity and risk level, in this model the return on nominal money holdings, x_{t+1}, has been equated with the real return on bonds, ρ_{t+1}. The basic idea is that the monetary authority can set the rate of return on nominal reserves, but because the government can finance its after-tax deficit with either seigniorage or debt, the central bank cannot control the real return on reserves with certainty. Thus, how the government chooses to finance its debt will determine the extent to which the real return on reserves is close to the real

[9] See Gale (1973).

[10] It should also be noted that it has been assumed that bond-holders do not face a reserve requirement. See comments throughout Bhattacharya *et al.* (1998) about the impact on the basic model when interest is not paid on reserves.

[11] See Friedman (1961, chapter 3, p75) for the lone paragraph devoted to the appropriate choice of the rate of return to be paid on reserve holdings.

return on bonds.[12,13]

2.1.2 The Government

The government must finance a real per capita deficit of g each period through the issue of money and bonds. The government's budget constraint is given by

$$g = \frac{M_t - M_{t-1}}{p_t} + b_t - \rho_t b_{t-1} - \frac{x_t M_{t-1}}{p_t} \ ; \forall t \geq 1 \tag{10}$$

Equation (10) asserts that the real value of money created in period t, $(M_t - M_{t-1})/p_t$, plus the real value of the bonds sold at that date, b_t, must equal the real value of the government budget deficit, g, plus the interest obligations on outstanding government debt, $\rho_t b_{t-1}$ and the interest obligations associated with reserves, $x_t M_{t-1}/p_t$. It is assume that the government conducts policy by choosing (once and for all in the first period), a ratio

$$\mu \equiv \frac{b_t}{z_t} \ ; \ t \geq 1 \tag{11}$$

of bonds to currency. Variations in μ can be thought of as permanent open market operations.[14] In addition, the government sets the reserve requirement $1-\phi$. The initial level of the money stock must satisfy $M_0 > 0$ and $B_0 = 0$ is assumed to be given as initial conditions.

Substituting equations (7), (11), and $z_t \equiv M_t/p_t = \lambda k_t = (1-\phi)\, d_t$ in equation (10), it is possible to rewrite the government budget constraint as[15]

$$z_t (1+\mu) = g + z_{t-1} \left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)\, \rho_t} \right] + \rho_t z_{t-1} \left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)\, \rho_t} + \mu \right] \ ; t \geq 2. \tag{12}$$

[12] This is in contrast to Sargent and Wallace (1985), Smith (1991), and Freeman and Haslag (1996), where there was only one asset, equivalent to the storage technology in this paper, to which the return on reserves was equalized. Although the two-period nature of the model does not allow short and long term rates (in the true sense of Friedman's proposal), the multiple asset aspect does allow for setting the return on reserves to a rate less than that obtained by storage (or capital).

[13] An alternative interpretation would be that x_{t+1} represents the real return on real money balances, z_t, in terms of the period t price level. In affect, the central bank would pay interest on reserves at the end of period t, based on the real balances possessed by individuals at the end of the period. Equating x_{t+1} to the real return on bonds that individuals will receive at the beginning of next period implies that the only costs associate with holding reserves is that associated with the government financing their deficit via seigniorage.

[14] Note that this definition of an open market operation differs from that in Freeman and Haslag (1996). Here it represents a shift in the composition of deficit financing instruments. In Freeman and Haslag (1996) it amounted to a purchase of an asset which was used to reduce the funds the bank needed to acquire to pay interest on reserves.

[15] The initial, $t = 1$, government budget constraint is $(1+\mu)z_1 = g + (1+x_1)\, M_0/p_1$. Once z_1 and x_1 are determined, then this government budget constraint gives us the initial price level.

Equation (12) can be interpreted as the government must issue enough liabilities at t, $z_t + b_t = (1+\mu) z_t$, to finance its current deficit plus the implied interest obligation on its inherited liabilities.

3 Equilibrium

In order for equilibrium to obtain, consumers must be maximizing their utility and the government budget constraint must hold. The first condition requires that the quantity of savings demanded must equal the quantity supplied. Given the definition of z_t, where $z_t \equiv M_t / p_t$ is the real value of the per capita money supply at t, $b_t + d_t = z_t \left(\mu + \frac{1}{1-\phi}\right)$ must hold in equilibrium. In addition, the supply of government bonds plus deposits must equal the savings of young individuals. Thus $b_t + d_t = S\left(\rho_{t+1}\right)$ must hold as well. Combining these two observations yields the following asset market clearing condition:

$$z_t \left(\mu + \frac{1}{1-\phi}\right) = S\left(\rho_{t+1}\right); \, t \geq 1. \tag{13}$$

Inverting equation (13) to obtain $\rho_{t+1} = S^{-1}\left\{z_t \left(\mu + \frac{1}{1-\phi}\right)\right\}$, and substituting the result into equation (12) yields the equilibrium law of motion for per capita real balances:

$$\begin{aligned} g \;=\;& z_t (1+\mu) - z_{t-1} \left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi) S^{-1}\left\{z_{t-1}\left(\mu + \frac{1}{1-\phi}\right)\right\}}\right] \\ & - z_{t-1} S^{-1}\left\{z_{t-1}\left(\mu + \frac{1}{1-\phi}\right)\right\} \left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi) S^{-1}\left\{z_{t-1}\left(\mu + \frac{1}{1-\phi}\right)\right\}} + \mu\right] \end{aligned} \tag{14}$$

It is now possible to solve for that equilibrium sequence of real balances, $\{z_t\}$. Once this is obtained, $S^{-1}\left\{z_t \left(\mu + \frac{1}{1-\phi}\right)\right\}$ gives the equilibrium rate of return on government bonds, while for a given ϕ

$$(1-\phi)\frac{p_t}{p_{t+1}} = \frac{\rho_{t+1} - \phi R}{\rho_{t+1}} = 1 - \frac{\phi R}{S^{-1}\left\{z_t \left(\mu + \frac{1}{1-\phi}\right)\right\}} \tag{15}$$

describes the gross rate of return on real balances (the inverse of the gross rate of inflation.)

There are, of course, a number of conditions that an equilibrium sequence $\{z_t\}$ must satisfy. First, it must satisfy (14) at each date. Second, $z_t \geq 0 \; \forall t \geq 1$ must also hold. Third, given the method of derivation, the reserve requirement must be binding at each date. And, finally, equation (15) must yield a non-negative gross return on real balances. These last two requirements can be written as

$$\phi R \leq S^{-1}\left\{z_t\left(\mu+\frac{1}{1-\phi}\right)\right\} < R;\ t \geq 1 \tag{16}$$

Equations (14), (16), and $z_t \geq 0$ constitute our equilibrium conditions.

3.1 Steady State Equilibria

Attention will now be turned to ascertaining the conditions under which there exist steady state equilibria. Setting $z_{t-1} = z_t$ in equation (14) and rearranging terms, one obtains the following steady state equilibrium condition:

$$g = z\left\{\left[1+\mu-\left(\frac{1}{(1-\phi)}-\frac{\phi R}{(1-\phi)}\right)\right]-\left(\frac{1}{(1-\phi)}+\mu\right)S^{-1}+\frac{\phi R}{(1-\phi)\,S^{-1}}\right\}. \tag{17}$$

Define $H(z,\mu,\phi,R)$ by

$$H(z,\mu,\phi) \equiv z\left\{\left[1+\mu+\frac{\phi R}{(1-\phi)}-\frac{1}{(1-\phi)}\right]-\left(\frac{1}{(1-\phi)}+\mu\right)S^{-1}+\frac{\phi R}{(1-\phi)\,S^{-1}}\right\} \tag{18}$$

The function $H(z,\mu,\phi,R)$ describes how much revenue — net of interest obligations — the government can raise in a steady state equilibrium if the per capita level of real balances is z, the bond-money ratio is μ, and the reserve requirement is $1-\phi$. In such an equilibrium, of course, the quantity of revenue raised must equal the government budget deficit g. However, in order for z to constitute a steady state equilibrium level of real balances, z must satisfy not only (17), but (16) as well.

To ascertain the conditions under which steady state equilibria exist, as well as their number, it will be useful to know more about the function H. Its properties are stated in the following lemma.

Lemma 1 *(a) $H(z,\mu,\phi,R)=0$ holds iff $z=0$ or*

$$z = z^{\dagger} \equiv S\left\{\frac{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]+\left\{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]^2+4\left[\frac{1}{(1-\phi)}+\mu\right]\frac{\phi R}{(1-\phi)}\right\}^{1/2}}{2\left[\frac{1}{(1-\phi)}+\mu\right]}\right\}\Bigg/\left(\mu+\frac{1}{1-\phi}\right) > 0.$$

(b) $H_1(0,\mu,\phi,R) > 0 > H_1\left(z^{\dagger},\mu,\phi,R\right)$ holds $\forall\ (\mu,\phi,R)$.

Lemma 1 is proved in Appendix A. For simplicity of exposition the following assumption is made

Assumption 3 *For all values of* μ, ϕ, R *and* $0 \leq z \leq z^{\dagger}$,

$$H_{11}(z, \mu, \phi, R) < 0. \tag{A.3}$$

Thus, $H(z, \mu, \phi, R)$ *is a concave function of* z*. The consequences of relaxing this assumption will be discussed later.*

Under Assumptions (A.1)-(A.3), equation (18) has the configuration depicted in Figure 1. Any values of z satisfying equation (18) are candidate steady state equilibria. As shown in the figure, if there are any such candidates, there will generically be exactly two.[16] Let z^- denote the steady state with lower real balance holdings and z^+ the steady state with higher holdings. As is evident from Figure 1 the lower level of real balance holdings occurs on the "bad" side of the Laffer curve and the higher level on the "good" side: i.e., $H_1(z^-, \mu, \phi) > 0 > H_1(z^+, \mu, \phi)$ must hold.

In addition, any candidate steady state equilibria must also satisfy (16), which can be rewritten as

$$\frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)} \leq z < \frac{S(R)}{\left(\mu + \frac{1}{1-\phi}\right)}. \tag{19}$$

We can now state the following result, which is proved in Appendix B.

Lemma 2 *Suppose that*

$$1 \geq \mu(\phi R - 1) \tag{A.4}$$

holds,[17] *then* $z^{\dagger}$ *satisfies*

$$\frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)} \leq z^{\dagger} < \frac{S(R)}{\left(\mu + \frac{1}{1-\phi}\right)}.$$

For the remainder of the paper Assumptions (A.4) is assumed to hold. In which case only the left-hand constraint in equation (16) can bind on the determination of a steady state equilibrium. There are three possibilities regarding whether z^- and z^+ constitute legitimate steady state equilibria.

Case 1 *If* $S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^-$*, then there are two genuine steady state equilibria.*

[16] If Assumption (A.3) is relaxed, there can be more than two candidate steady states. In general, these equilibria will occur in pairs.

[17] Assumption (A.4) holds for all values of μ if $\phi R \leq 1$. For an economy with a reserve requirement of 10 percent ($\phi = 0.9$), this condition will be satisfied if there is no asset with a safe, real rate of return in excess of 11.11 percent, which certainly seems empirically plausible. Of course if $\phi R > 1$ holds, then Assumption (A.4) places an upper bound on μ. In the third quarter of 2004, the outstanding gross public debt of the U.S. was \$7.38 trillion, while the monetary base was about \$749 billion. Thus, for the U.S., $\mu \approx 9.85$. In this case, Assumption (A.4) would hold so long as $\phi R < 1.102$.

Case 2 *If* $z^- < S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^+$, *then only* z^- *constitutes a legitimate steady state equilibrium. Thus, there exists a unique steady state equilibrium.*

Case 3 *If* $z^+ < S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right)$, *then there does not exists any steady state equilibria.*

Obviously this last case is not of particular interest, and thus remainder of the paper will focus on cases 1 and 2, which are represented by Figure 1 and Figure 2, respectively. An examination of the figures will indicate that case 1 is most likely to obtain for large values of g (given μ and ϕ), while case 2 *must* obtain for sufficiently small values of g (again, for given choices of μ and ϕ). Thus, to reiterate, for sufficiently small but positive values of g (where small means relative to μ and ϕ), there will exist a *unique* steady state equilibrium. This is true even though the function $H(z, \mu, \phi, R)$ exhibits all of the standard properties that give rise to "Laffer curve" phenomenon. The possibility that there is a unique steady state equilibrium, even in the presence of a Laffer curve, is a consequence of the binding reserve requirement faced by "depositors."

Finally, before comparing steady state equilibria with and without interest payments on reserves, it will be useful to examine the impact of open market operations on equilibrium values as well as whether unpleasant monetarist arithmetic obtains.

3.2 Comparative Statics

Of particular interest is how changes in the bond-money ratio, μ, affect the steady state equilibrium level(s) of real balances, and the rate of inflation. An increase in μ corresponds to a (permanently) higher bond-money ratio, and hence to a contractionary open market operation, as conventionally defined.

Straightforward differentiation of equation (18) yields that, for any candidate steady state equilibrium,

$$H_1(z, \mu, \phi, R) \frac{\partial z}{\partial \mu} = -H_2(z, \mu, \phi.R). \tag{20}$$

The following lemma (which is proved in Appendix C) is now established.

Lemma 3 *Suppose that* $\phi R / [1 + \phi(1-R)] > S^{-1}(z^\dagger)$ *holds, then* $H_2(z^+, \mu, \phi, R) < 0$ *and* $\partial z^+ / \partial \mu < 0$.

Lemma 3 asserts that under certain conditions, namely the return on bonds cannot be too large relative to the return on storage for a given reserve requirement, a contractionary open market operation necessarily reduces z^+.[18] Finally, what one would ultimately like to know is the effect of a change in μ on the steady state rate of return on real balances or, in other words, on the inverse inflation rate p_t/p_{t+1} . Appendix D establishes the following proposition.

Proposition 1 *The impact of a change in μ on p_t/p_{t+1} is given by*

$$(1-\phi)\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}=\frac{z\phi RS^{-1'}}{H_1\left(S^{-1}\right)^2}\left[\phi\left(R-1\right)-1+\frac{\phi R}{S^{-1}}\right]. \tag{21}$$

If $\phi R/\left[1+\phi\left(1-R\right)\right]>S^{-1}\left(z^{\dagger}\right)$ holds, as in Lemma 3, then $\partial\left(p_t/p_{t+1}\right)/\partial\mu<0$. When $H_1\left(z,\mu,\phi,R\right)<0$ and $\partial\left(p_t/p_{t+1}\right)/\partial\mu>0$ then $H_1\left(z,\mu,\phi,R\right)>0$.

Proposition 1 states the familiar result about the "Laffer curve" and "unpleasant monetarist arithmetic." Under the conditions necessary for Lemma 3 to hold, $\partial\left(p_t/p_{t+1}\right)/\partial\mu$ increases on the upward sloping portion of $H\left(z,\mu,\phi,R\right)$, while decreasing on the downward sloping portion. Thus, a contractionary open market operation raises the steady state rate of inflation on the "good-side" of the Laffer curve and lowers inflation on the "bad-side." Consistent with Bhattacharya *et al.* (1998), this result does *not* require that ρ exceed the steady state rate of growth. All that is needed is for some asset whose real rate of return exceeds the economy's long-run rate of growth exist (in the model $R>1$.) The unpleasant monetarist arithmetic is the result of $\rho>x_{t+1}p_t/p_{t+1}$ holding. In this case, an increase in the bond-money ratio substitutes a more expensive for a less expensive financing instrument. Consequently, heavier use must be made of the inflation tax.[19] Finally, it should also be noted the set of interest rates, ρ, for which unpleasant monetarist arithmetic arises is potentially smaller when interest is paid on reserves than when it is not.[20] Thus, it is less likely to occur when the return on real balances is more closely tied to the real return on bonds.

Of course these remarks apply to *candidate* steady state equilibria [that is, to values of z satisfying equation (17)]. However, in this environment not all candidate steady state equilibria necessarily satisfy equation (16), and hence not all values of z satisfying equation (17) constitute legitimate steady states. There are two cases to consider in this respect.

[18] Based on Lemma 3 nothing definitive can be said as the what happens to z^- : it may rise or fall. In addition, if $\left[1+\phi\left(1-R\right)\right]<0$ or if $\phi R>1$, then for any $z\in\left[0,z^{\dagger}\right]$, when $H_1\left(z,\mu,\phi,R\right)<0$ it will be the case that $H_2\left(z,\mu,\phi,R\right)<0$.

[19] See Bhattacharya *et al.* (1998) for a more in-depth explanation of this result and also a discussion of the likelihood that it applies to the United States.

[20] For example, if $S^{-1}\left(z^+\right)>\phi R/\left[1+\phi\left(1-R\right)\right]>S^{-1}\left(z^-\right)$, then it would be the case that $\partial\left(p_t/p_{t+1}\right)/\partial\mu>0$ at both steady state equilibria. In this case unpleasant monetarist arithmetic would not arise. Finally, if $S^{-1}\left(z^-\right)>\phi R/\left[1+\phi\left(1-R\right)\right]$ then the results of Proposition 1 are reversed. This is in contrast to the non-interest bearing economy where unpleasant monetarist arithmetic is always present at the high real money balance steady state.

Case 4 *Let* $S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^{-}$, *both before and after the change in* μ.

Case 5 *Let* $z^{-} < S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^{+}$, *both before and after the change in* μ.

In Case 4, z^{-} and z^{+} are both legitimate steady state equilibria. The high real balance steady state is easily shown to be Pareto superior to the low real balance steady state. When Case 5 obtains there is a *unique* steady state equilibrium (z^{+}). If Lemma 3 holds, then unpleasant monetarist arithmetic prevails at the high real money balance equilibrium and *contractionary* open market activity *must* lead to a *higher* steady state inflation rate.

4 Comparison to Non-Interest Environment

It will now be useful to compare the steady state results in the interest-on-reserves economy to the case where interest is not paid on reserves. This latter case is described in detail in Bhattacharya *et al.* (1998), and hence only the results are presented here. However, as will be noted by the similarity to the results from this paper, the derivations of the non-interest results is completely analogous to the derivation of results in this paper.

When interest is not paid on reserves, then the consumer's budget constraints and the government's budget constraint are given by

$$c_t^t + d_t + b_t \leq \omega_1$$

$$c_{t+1}^t \leq \omega_2 + \left[\phi R + (1-\phi)\left(\frac{p_t}{p_{t+1}}\right)\right] d_t + \rho_{t+1} b_t$$

and

$$g = \frac{M_t - M_{t-1}}{p_t} + b_t - \rho_t b_{t-1} \ ; \forall t \geq 1,$$

respectively. Solving for the equilibrium law of motion for per capita real balances yields

$$H^{NI}(z, \mu, \phi, R) \equiv z\left\{1 + \mu + \left(\frac{\phi R}{1-\phi}\right) - \left(\mu + \frac{1}{1-\phi}\right) S^{-1}\left\{z\left(\mu + \frac{1}{1-\phi}\right)\right\}\right\} = g, \qquad (22)$$

which is very similar to equation (18). Equation (22) has the same basic hump shape as equation (18), and thus it remains to establish their relative positions. The following proposition states the relationship between $H(z, \mu, \phi, R)$ and $H^{NI}(z, \mu, \phi, R)$.

Proposition 2 *For $z = 0$, then $H(0, \mu, \phi, R) = H^{NI}(0, \mu, \phi, R)$. For all $z > 0$, then $H(z, \mu, \phi, R) < H^{NI}(z, \mu, \phi, R)$.*

The proof of this proposition follows from a straight foward comparison of $H(z, \mu, \phi, R)$ and $H^{NI}(z, \mu, \phi, R)$ and application of previous assumptions, and hence the proof is omitted. Figure 3 provides a generalized illustration of the relative positions of $H(z, \mu, \phi, R)$ and $H^{NI}(z, \mu, \phi, R)$.

4.1 Deficits and Inflation

The first thing to note is that for a given bond-money ratio, μ, and a given reserve requirement, ϕ, the set of sustainable government deficits is smaller when interest is paid on reserves. This is not surprising given that the total resources of the economy are the same in both economies and by paying interest on reserves, there are fewer resources available to sustain larger government deficits. In addition, one can compared the levels of inflation between the two economies.

In the non-interest economy, the inverse of the inflation rate is given by

$$(1 - \phi) \frac{p_t}{p_{t+1}}^{NI} = S^{-1} \left\{ z_t \left(\mu + \frac{1}{1 - \phi} \right) \right\} - \phi R \tag{23}$$

while for the interest-on-reserves economy it is given by equation (15). The following lemma states a sufficient condition under which the return on real balances in the interest economy is greater than in the corresponding no-interest economy.

Lemma 4 *Let $\hat{z}$ be the value of real balances such that $S^{-1}(\hat{z}) = 1$.*
(i) If $\hat{z} < z^-(NI)$, then $\frac{p_t}{p_{t+1}}^{NI} \left(z^{+(NI)}\right) > \frac{p_t}{p_{t+1}}(z^+)$.
(ii) If $z^- < \hat{z} < z^+$, then $\frac{p_t}{p_{t+1}}^{NI} \left(z^{-(NI)}\right) < \frac{p_t}{p_{t+1}}(z^-)$ and $\frac{p_t}{p_{t+1}}^{NI} \left(z^{+(NI)}\right) > \frac{p_t}{p_{t+1}}(z^+)$.
(iii) If $z^+ < \hat{z}$, then $\frac{p_t}{p_{t+1}}^{NI} \left(z^{-(NI)}\right) < \frac{p_t}{p_{t+1}}(z^-)$.

The proof of this lemma follows from Assumption A.2 and a comparison of equations (15) and (23). In part (i), the rate of return on real balances, for a given level of balances, is always larger in the non-interest bearing economy. Thus, at the high real balances steady state, inflation (the inverse of the return on money) will be higher in the interest-on-reserves economy. This is as expected because the seigniorage tax base is smaller in this economy too. In fact, in all three cases (parts (i) – (iii)) the economy with the lower level of real balances (when comparing high or low balance steady states across economies) always has the higher rate of inflation.[21]

[21] In parts (i) and (iii) it is not possible to compare the low or high real balance equilibria respectively. For example,

It is also possible, in steady state, to compare price level indeterminacy, volatility, and welfare between the economies with and without payment of interest of reserves.

4.2 Indeterminacy and Equilibrium

One of Friedman's primary concerns was eliminating price level indeterminacy and volatility. He felt that paying interest on reserves (and potentially combining it with a 100% reserve requirement) would achieve that goal. As is obvious from Figures 3 - 5, paying interest on reserves in the presence of a constant per capita government deficit at best maintains the level of price indeterminacy (in steady state) and at worst increases the indeterminacy when compared to an economy where reserves do not earn interest. The exact impact will depend on whether the lower bound on the interest rate paid on bonds is binding in equation (16).

Case 6 *Suppose that the following condition is satisfied*

$$\frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)} \leq z^{-}(NI).$$

In this case, the lower bound on real money balances does not bind in either the non-interest economy or the economy with interest payments on reserves. As is obvious from Figure 3, the number of steady state equilibria is equal in the two economies and thus interest payments on reserves do not impact indeterminacy of equilibria.

Case 7 *Suppose that the following condition is satisfied*

$$z^{-}(NI) < \frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)} \leq z^{-}.$$

When this second case obtains, the introduction of interest payments on reserves actually increase the indeterminacy as depicted in Figure 4. This result follows from the fact that in the non-interest economy, the government is attempting to finance its deficit with a mix of more (bonds) or less (money) expensive financing options. This can be achieved either by means of a large tax base and small tax or vice versa. However, the government faces a lower bound on the return it can offer on bonds (due to the presence of the storage technology), while still insuring that individuals

in part (i), it is the case that $\frac{p_t}{p_{t+1}}^{NI}(z) > \frac{p_t}{p_{t+1}}(z)$ for all z. However, it is also true that $z^{-(NI)} < z^{-}$, and the returns on money balances are both increasing in the level of real money balances. Thus, it is not possible to determine the precise relationship between $\frac{p_t}{p_{t+1}}^{NI}\left(z^{-(NI)}\right)$ and $\frac{p_t}{p_{t+1}}\left(z^{-}\right)$.

hold money and bonds. This, in turn, results in a minimum level of the seigniorage tax base that must be maintained so that the fixed deficit, g, can be financed. However, in this case the steady state level of real balances is less than minimum level of the base needed to insure all assets are held, and consequently is not a viable level of real balances.

This is not the case when interest is paid on reserves because the range of tax base options is limited by the fact that the return on the less expensive financing option, money, is linked to the return on the more expensive financing option, bonds. Because the return on bonds and money are linked, this reduces the government's ability to choose the more or less expensive options to finance their deficit by limiting the trade-off between a large tax base or a large tax rate. Consequently, the small monetary balances equilibrium represented by $z^{-}(NI)$ is not an option in the interest economy, while z^{-} is sufficiently large to be consistent with a binding reserve requirement and positive money and bond holdings.

Case 8 *Suppose that the following condition is satisfied*

$$z^{-} < \frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)}.$$

As in the first case above, the number of steady state equilibria consistent with equation (16) are the same in both economies: a unique steady state equilibria. In this case, there is no indeterminacy regardless of whether interest is paid on reserves. Thus, it is either the case that paying interest on reserves does not affect the number of steady state equilibria and indeterminacy in the economy, or it increases the number of steady state equilibria and raise the level of indeterminacy – the opposite of what Friedman had envisioned.[22]

4.3 Volatility and Welfare

An examination of Figures 3-5 and the results of the previous section also bear upon the amount of volatility observed in the respective economies and the welfare implications of paying interest on reserves. Given Assumptions (A.1) and (A.2), which require that savings be a strictly increasing function of the rate of return on savings, it is straightforward to show that higher real money balances are Pareto superior to lower real balances. In addition, volatility will be defined as the

[22] This result is consistent with Sargent and Wallace (1985) and runs counter to Smith (1991). The difference between Smith (1991) and this paper is that the real return to holding money balance in terms of date $t+1$ is not fixed to the return on storage and the government must finance a deficit. If both of these conditions did not exist, the results of this paper would be consistent with those of Smith (1991).

variance over steady state equilibria. We now proceed to examine volatility and welfare under the following three conditions.

Case 6: In this case, there exist two steady state equilibria in both the interest bearing and non-interest economies, as depicted in Figure 3. With respect to volatility, there will be less volatility in the case where interest is paid on reserves. This results from the fact that by tying the rate of return on money balances to the real return on savings, the government's hands are tied regarding its ability to make a trade-off between a larger seigniorage tax base and the seigniorage tax rate. Consequently the range of real money balances which can sustain the per capital deficit g, is smaller when interest is paid on reserves. Thus, in this case interest payments have the desired impact of reducing economic volatility.

As for the welfare implications of paying interest on reserves, that depends on whether the high or low real money balance equilibria are compared. A comparison of low real money balance equilibria yields $z^{-}(NI) < z^{-}$. Consequently, welfare in the interest paying economy will be greater than when interest is not paid on reserves. In contrast, in the high real money balance equilibria, $z^{+} < z^{+}(NI)$ holds and welfare is decreased with the imposition of a requirement to pay interest on reserves. Consistent with Freeman and Haslag (1996), if part (a) of Lemma 3 holds, then the welfare loss associated with paying interest of reserves can be offset by an appropriate expansionary open mark operation (i.e. a decline in μ).

Case 7: In this case there exist two steady state equilibria in the interest bearing economy and only one in the non-interest economy, as depicted in Figure 4. As a consequence, paying interest on reserves both reduces welfare unambiguously and also increases volatility dramatically. The explanation as to why, is identical to that for the additional price indeterminacy observed in this case. Namely, because the range of real money balances which are consistent with equilibria is smaller, it is less likely that the lower bound for the range of returns on savings will be binding. As a result, it is more likely that multiple equilibria will exist. Consequently, there exists indeterminacy and volatility (associated with the indeterminacy) that are not present when a unique equilibria exists (as in the non-interest economy). In addition, since $z^{-} < z^{+} < z^{+}(NI)$ holds, the unique equilibrium associate with the non-interest economy is strictly better, in terms of welfare, than either steady state equilibrium which might prevail in the interest bearing economy.

Case 8: In this case there exist a unique, high real money balance equilibrium in both the interest bearing and non-interest economies, as depicted in Figure 5. Since there exists a unique equilibrium in both economies, there does not exist any volatility in either economy and paying interest on reserves neither helps nor hurts the economy in terms of reducing volatility. From a welfare perspective, since $z^{+} < z^{+}(NI)$ holds, welfare is worsened as a result of interest payments on reserves. This decline in welfare results from the fact that tying the rate of return on money balances and bonds together, reduces the government's freedom to choose a larger seigniorage tax base and accompanying smaller tax rate to finance its deficit. This restriction in the range of tax bases which can support a deficit of a given side is a doubled edged sword. It reduced the volatility in the case 6 economy described above, but in this case the same smaller tax base (as compared to a non-interest bearing economy) also reduces the benefits consumers might gain from having a smaller seigniorage tax.

5 Conclusion

Over the past few years, Congress has indicated an increased willingness to pass legislation allowing the Federal Reserve to pay interest on reserves and the likelihood of approval in the next few years is high. Although it will have taken over half a century since Milton Friedman first made his proposal for interest payments to become a reality, the likely impact on the economy is still not completely understood. Friedman's motivation for his proposal was to eliminate price level indeterminacy and volatility and thereby improve economic well-being. Although several authors have examined the impact of paying interest on reserves on these issues, their models omitted two important issues: equating the return on reserves to similar short-term assets and the impact of government deficit financing.

This paper has attempted to re-examine those issues of concern to Friedman in the context of a model where the rate of return on reserves equals that of government securities and where the government finances an after-tax deficit via debt and seigniorage. The model used is a standard three asset model (storage, bonds and money) where the return to money is dominated by the return on other assets. Storage must be intermediated and is subject to a reserve requirement, where the return on reserves is equated to the return on bonds. Finally, the government must finance an after-tax deficit, in addition to paying interest on bonds and reserves. This environment is similar to the previous models that studied this issue, although in my model the issue of how to finance interest payments is different.

I am able to demonstrate four basic results. First, the level of indeterminacy is equal to or greater than the level when interest is not paid on reserves. Second, when the level of indeterminacy is the same in the two economies, then economic volatility is reduced with the introduction of interest payments. However, when greater indeterminacy in the interest-on-reserves economy exists , then there also exists greater volatility. Third, when the level of indeterminacy is the same in the two economies, then the equilibrium associated with low real money balances in the interest economy is welfare improving compared to the non-interest economy. The reverse is true at the high real money balance equilibrium. In this latter case, an appropriate expansionary open market operation can offset the welfare losses associated with interest payments on reserves. Finally, under a narrow set of conditions, unpleasant monetarist arithmetic may apply to at most one of the steady state equilibrium. Some of these results run counter to what Friedman had envisioned and also to previous findings.

The key to these results is two-fold. First, by not pegging the return on reserves to the asset with the highest real return, the model allows for multiple equilibria (unlike for example Smith (1991) and Freeman and Haslag (1996) where equating the returns on storage and reserves eliminates indeterminacies). Second, by allowing for a government deficit, how that deficit is financed affects the overall impact of interest payments on the economy. When the return on reserves is linked to the return on bonds, this limits the options available to the government in terms of how it finances its deficit. The trade-off between higher cost funding (bonds) and lower cost funding (money) is diminished, as is the government's ability to make a trade-off between a large tax base and a high tax rate. This reduces the set of real money balances which can support a given deficit while still making bonds an attractive investment option.

There is scope for extensions to this current work. The most obvious one would be to analyze the dynamics of the economy. Bhattacharya *et al.* (1998) demonstrated that under certain conditions a unique steady state equilibrium would always obtain. It would be interesting to derive those conditions under which a unique steady state equilibrium would obtain in an economy where interest is paid on reserves. In addition, it would be useful to know whether the economy would naturally gravitate to one or the other of the steady state equilibria.

A Proof of Lemma 1

a) That $H(z,\mu,\phi,R)=0$ iff $z=0$ or $z=z^{\dagger}$ follows immediately from the definition of $H(z,\mu,\phi,d)$, and the fact that $S(\rho)$ is an increasing function. Furthermore, $z^{\dagger}>0$ holds iff

$$\frac{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]+\left\{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]^{2}+4\left[\frac{1}{(1-\phi)}+\mu\right]\frac{\phi R}{(1-\phi)}\right\}^{1/2}}{2\left[\frac{1}{(1-\phi)}+\mu\right]}>S^{-1}(0).$$

However, Assumption (A.1).implies that $S^{-1}(0)<\min\{1,\phi R\}$ holds and thus this equation is satisfied.

(b) From the definition of $H(z,\mu,\phi,R)$, it follows that

$$\begin{aligned} H_1 \;=\; & \left\{1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]-\left[\frac{1}{(1-\phi)}+\mu\right]S^{-1}+\frac{\phi R}{(1-\phi)\,S^{-1}}\right\} \\ & -zS^{-1'}\left(\mu+\frac{1}{1-\phi}\right)\left\{\frac{1}{(1-\phi)}+\mu+\frac{\phi R}{(1-\phi)\left(S^{-1}\right)^{2}}\right\}. \end{aligned} \tag{a.2}$$

However $H_1(0,\mu,\phi,R)>0$ holds iff

$$1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]-\left[\frac{1}{(1-\phi)}+\mu\right]S^{-1}(0)+\frac{\phi R}{(1-\phi)\,S^{-1}(0)}>0.$$

As in part (a) above, this is guaranteed by Assumption (A.1). It is easy to verify that $H_1\left(z^{\dagger},\mu,\phi,d\right)$ is given by

$$H_1|_{z=z^{\dagger}}=-z^{\dagger}S^{-1'}\left(z^{\dagger}\right)\left(\mu+\frac{1}{1-\phi}\right)\left\{\frac{1}{(1-\phi)}+\mu+\frac{\phi R}{(1-\phi)\left(S^{-1}\left(z^{\dagger}\right)\right)^{2}}\right\}<0.$$

Thus, it is the case that $H_1\left(z^{\dagger},\mu,\phi,R\right)<0<H_1(0,\mu,\phi,R)$. ■

B Proof of Lemma 2

From the definition of $z^{\dagger}$, the claim follows if

$$\phi R<\frac{\left[(1+\mu)-\frac{1}{(1-\phi)}+\frac{\phi R}{(1-\phi)}\right]+\left\{\left[(1+\mu)-\frac{1}{(1-\phi)}+\frac{\phi R}{(1-\phi)}\right]^{2}+4\left[\frac{1}{(1-\phi)}+\mu\right]\frac{\phi R}{(1-\phi)}\right\}^{\frac{1}{2}}}{2\left[\frac{1}{(1-\phi)}+\mu\right]}<R. \tag{B.1}$$

The left-hand inequality in equation (B.1) follows from Assumption (A.4). The right-hand inequality is implied by $R > 1$. ■

C Proof of Lemma 3

Straightforward differentiation yields

$$H_2 = z\left\{1 - S^{-1} - z\left[\frac{1}{(1-\phi)} + \mu\right]S^{-1'} - \frac{\phi R S^{-1'} z}{(1-\phi)\left(S^{-1}\right)^2}\right\}. \tag{C.1}$$

Re-writing equation (a.2), one obtains

$$\begin{aligned} H_1 &= \left(\frac{1}{(1-\phi)} + \mu\right)\left\{\frac{1 + \mu - \left[\frac{1-\phi R}{(1-\phi)}\right]}{\frac{1}{(1-\phi)} + \mu} - S^{-1} + \frac{\frac{\phi R}{(1-\phi)S^{-1}}}{\frac{1}{(1-\phi)} + \mu}\right\} \\ &\quad - z\left(\frac{1}{(1-\phi)} + \mu\right)S^{-1'}\left\{\left[\frac{1}{(1-\phi)} + \mu\right] + \frac{\phi R}{(1-\phi)\left(S^{-1}\right)^2}\right\}. \end{aligned}$$

Thus, it is sufficient to show that

$$\frac{1 + \mu - \left[\frac{1-\phi R}{(1-\phi)}\right]}{\frac{1}{(1-\phi)} + \mu} + \frac{\frac{\phi R}{(1-\phi)S^{-1}}}{\frac{1}{(1-\phi)} + \mu} > 1. \tag{C.2}$$

If this equation holds, then $H_1\left(z^+, \mu, \phi, R\right) < 0$ implies that $H_2\left(z^+, \mu, \phi, R\right) < 0$ holds as well. However, equation (C.2) holds iff

$$\phi R > \left[1 + \phi\left(1 - R\right)\right]S^{-1}.$$

For $1 + \phi(1 - R) < 0$, this will obviously hold. In addition, if $1 + \phi(1 - R) > 0$ and $\phi R > 1$, then by equation (16) this will also always hold. If neither of these is the case, then given Assumptions (A.1) and (A.2), a sufficient condition to guarantee the above is

$$\phi R > \left[1 + \phi\left(1 - R\right)\right]S^{-1}\left(z^{\dagger}\right).$$

It then follows from equation (20) that $\partial z^+ / \partial \mu < 0$. ■

D Proof of Proposition 1

Differentiating equation (15) yields

$$(1-\phi)\frac{\partial \frac{p_t}{p_{t+1}}}{\partial \mu}=\frac{\phi R}{(S^{-1})^2}S^{-1'}\left[\left(\mu+\frac{1}{1-\phi}\right)\frac{\partial z}{\partial \mu}+z\right]. \tag{D.1}$$

In addition, equation (20), combined with equations (a.2) and (C.1) implies

$$\frac{\partial z}{\partial \mu}=-\frac{z\left\{1-S^{-1}-z\left[\frac{1}{(1-\phi)}+\mu\right]S^{-1'}-\frac{\phi R S^{-1'}z}{(1-\phi)(S^{-1})^2}\right\}}{\left[\frac{1}{(1-\phi)}+\mu\right]\left\{\frac{1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]}{\frac{1}{(1-\phi)}+\mu}+\frac{\frac{\phi R}{(1-\phi)S^{-1}}}{\frac{1}{(1-\phi)}+\mu}-S^{-1}-zS^{-1'}\left\{\left[\frac{1}{(1-\phi)}+\mu\right]+\frac{\phi R}{(1-\phi)(S^{-1})^2}\right\}\right\}}.$$

Substitution this equation into equation (D.1) and simplifying, one obtains

$$(1-\phi)\frac{\partial \frac{p_t}{p_{t+1}}}{\partial \mu}=\frac{z\phi R S^{-1'}}{H_1\,(S^{-1})^2}\left[\phi\,(R-1)-1+\frac{\phi R}{S^{-1}}\right].$$

Thus, if $\phi\,(R-1)-1+\phi R/\,S^{-1}>0$, then $\partial\frac{p_t}{p_{t+1}}\Big/\partial\mu$ has the same sign as H_1. However, given Assumption (A.2), if $\phi R/\left[1+\phi\,(1-R)\right]>S^{-1}\left(z^{\dagger}\right)$ holds, then $\phi\,(R-1)-1+\phi R/\,S^{-1}>0$ for all $z\in\left[0,z^{\dagger}\right]$.and $\partial\frac{p_t}{p_{t+1}}\Big/\partial\mu<0$ when $H_1<0$ and conversely. ■

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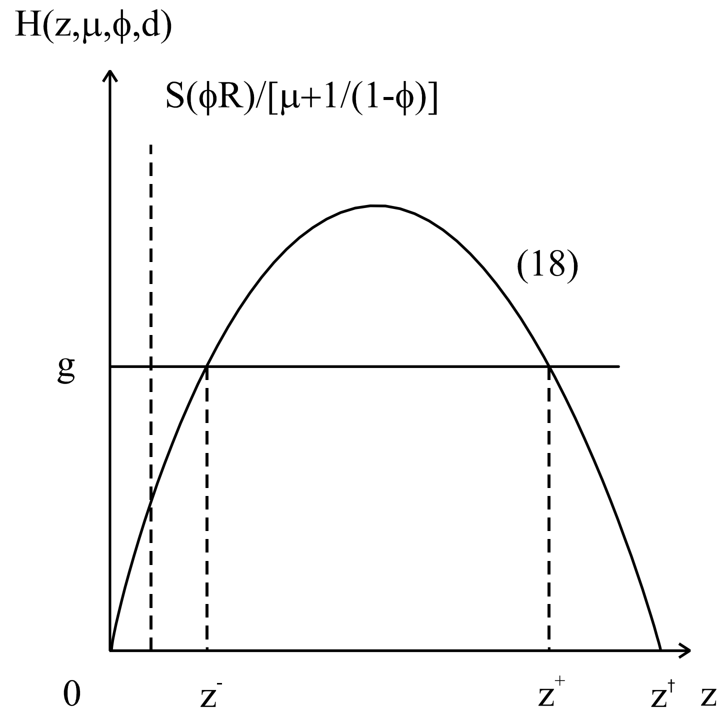

Figure 1: Case 1: Mutliple Steady State Equilibria

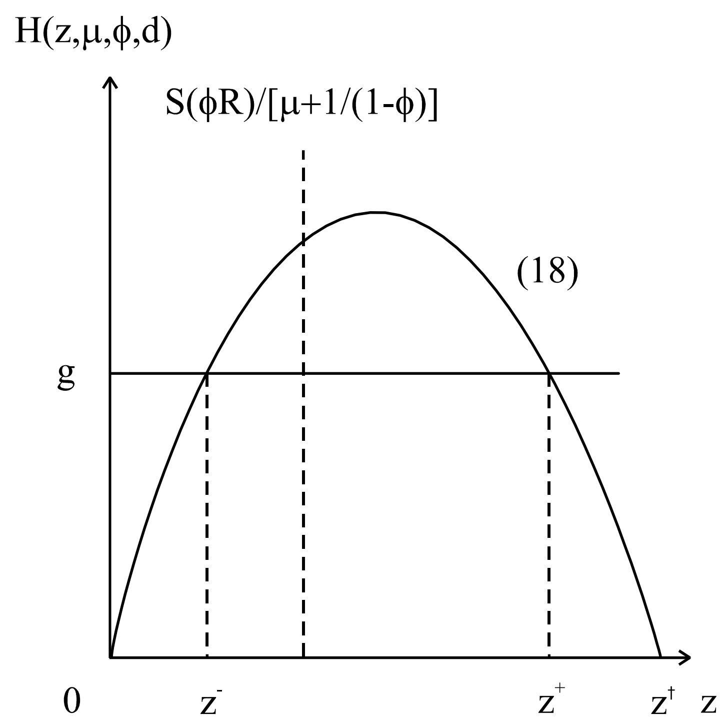

Figure 2: Case 2: Unique Steady State Equilibrium

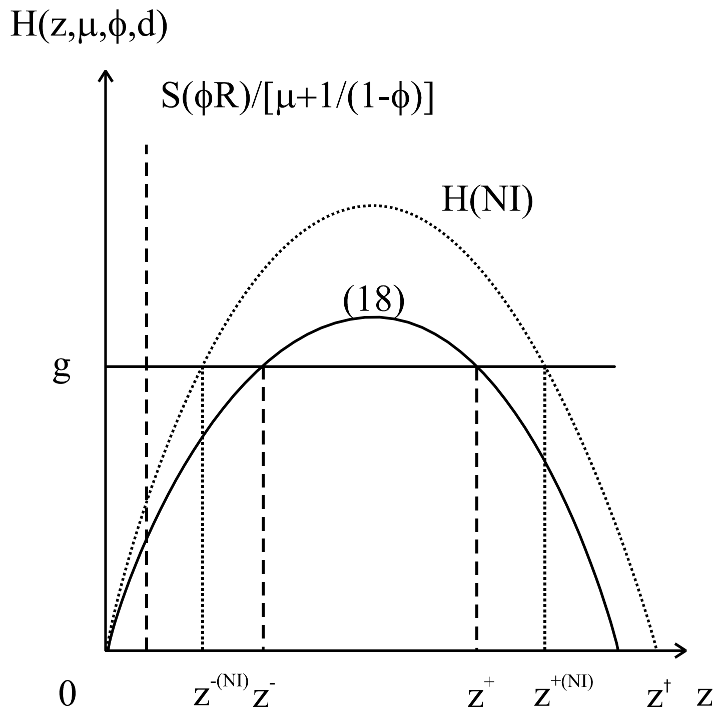

Figure 3: Case 6: Multiple Equilibria in the Interest and Non-Interest Bearing Economies

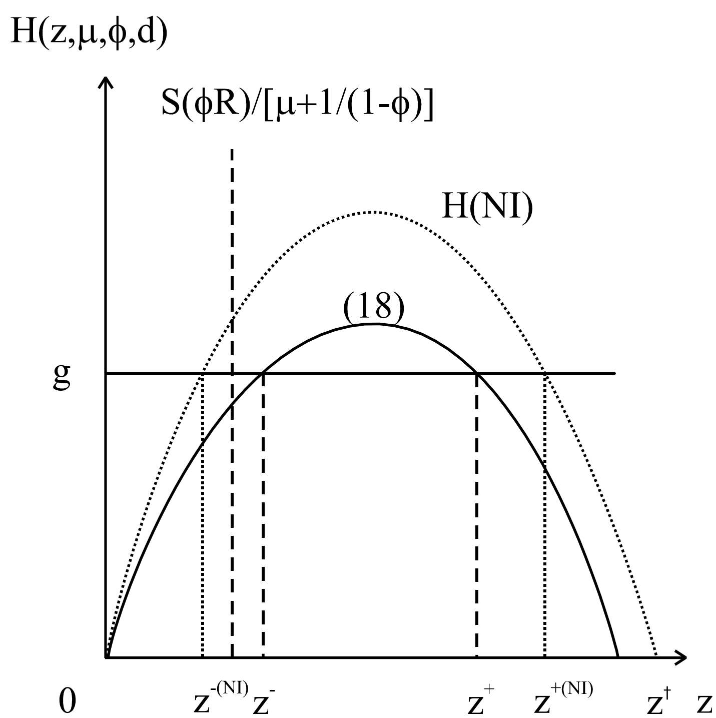

Figure 4: Case 7: Multiple Equilibria only in the Non-Interest Bearing Economies

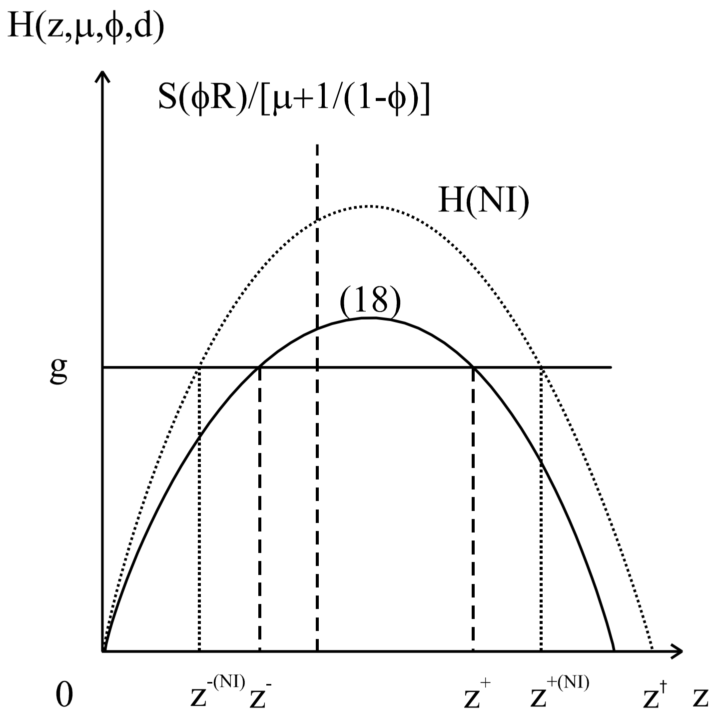

Figure 5: Case 8: Unique Equilibrium in the Interest and Non-Interest Bearing Economies