The Dynamics of Immigration Policy with Wealth-heterogeneous Immigrants

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Abstract

In this paper we consider a simple intertemporal economy in which immigrants, if admitted, bring heterogeneous amounts of capital. We show that under certain conditions there is a level of immigration which maximizes the economy's capital-labor ratio, and that this level of immigration is the preferred choice of a majority of the economy's citizens. We then characterize, in an overlapping generations setting, the dynamics of capital accumulation and immigration policy, which can include multiple steady state equilibria and a sensitivity of immigration levels to changes in the economy's technology growth rate.

1 Introduction

The objective of this study is to try to understand what motivates various countries—sometimes quite similar countries—to choose alternative levels of immigration. For example, Canada and the United States, two countries that are similar in many respects, appear to have very different criteria for admitting immigrants. Canada gives high priority to individuals who have valuable skills, or larger amounts of financial resources, whereas the US chooses to put more weight on the family relations of existing residents. By comparison, these countries are quite different from Australia—another industrialized country—which for most of the 20^{th} century has admitted few immigrants, those admitted being almost exclusively European. Since 1971, however, Australia has admitted a tremendous quantity of immigrants, and a large volume of those are from Asia.

Among European countries, through the 1980's France, Spain and Italy have admitted a very modest number of immigrants, whereas throughout this same period Norway, Denmark, Sweden and Belgium have admitted roughly three times as many immigrants, as a fraction of their respective populations.

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In this paper, we adopt a political economy approach to looking at the issue of immigration. In particular, we examine the economic determinants of natives' preferences over the level of immigration and—assuming a majority-voting framework—the translation of those preferences into policies. Our model is a dynamic one—households in the economy make consumption-savings decisions which, in conjunction with levels of immigration, determine the path of the economy's capital-labor ratio.

A key feature of the environment is that potential immigrants are heterogeneous in terms of their initial wealth, or capital. This heterogeneity turns out to have important consequences for the economy's policy decision. The preferences of natives in our economy are, in a sense, 'polarized'—depending on the extent to which a native is more or less reliant on either capital or labor income he or she will prefer admission policies which either minimize or maximize the economy's capital-labor ratio. When immigrants are homogeneous, this implies an 'all-or-none' policy choice—the economy will either admit everyone or no one, depending on whether the 'capital-rich' or 'capital-poor' are in the majority.[1] When immigrants are heterogeneous, the polarization of natives' preferences can still translate into an 'interior' policy—something between keeping everyone out and letting everyone in.

The paper is organized as follows. The following section lays out the basic features of the economy in a two-period example. We show that heterogeneity of immigrants in terms of their initial capital (together with a simple rule for determining the characteristics of a given quantity of immigrants to be admitted) can lead to an 'interior' equilibrium under majority voting. This result obtains under certain conditions on the distributions of immigrant and native wealth—loosely, under certain sets of demographics—and we try to provide some rough quantitative feel for the likelihood of the result obtaining in practice, for a country such as the US *vis-à-vis* the less-developed world. We also offer a comparison between our model and that of Benhabib [1], which is closely related to our work. As we argue, our work is both a simplification (along one dimension) and a generalization (along others) of Benhabib's; we also try to make a case that real-world demographics favor the direction in which we've simplified.

In the next section, we then extend our two-period intuition to an overlapping generations model with two-period-lived agents. In the context of this many-period model, we characterize the dynamics of immigration and capital accumulation, and illustrate that multiple steady state equilibria are possible. Also, the paths of immigration and capital accumulation depend on the growth rate of exogenous technological progress—with faster growth rates leading to larger immigrant inflows. This suggests the possibility that some of the variation in observed immigration to countries such as the US may be due, at least in part, simply to changes in the technology growth rate.

2 A two-period model

2.1 The basic set-up

In this section, we develop the intuition for the behavior of our model economy in the context of a simple two-period model in which natives decide how many immigrants to admit, then divide their first-period wealth between consumption and savings; immigrants, if admitted, arrive in the second period, at which time production takes place using labor and capital (savings of natives plus endowments of immigrants) as inputs. The key to our voting results is that when immigrants differ in their holdings

[1] And, depending on what ratio of capital to labor a typical immigrants brings.

of capital, and an economy follows a rule of accepting immigrants in descending order of wealth, there will (under some restrictions on the model's parameters) be a level of immigration which maximizes the second-period capital-labor ratio. This level of immigration turns out to be a local maximum or a local minimum of every native's lifetime utility. When the distribution of initial wealth among natives is such that the wealth of the median native lies below a particular critical value, then the level of immigration which maximizes the second-period capital-labor ratio is the unique preferred policy for at least half of the native population. We also provide some rough quantitative evidence suggesting that, at least for a country like the US, it's reasonable to expect the sufficient condition just described to hold, though—as will become clear—the condition is not simply about native demographics, but also about the demographics of the pool of prospective immigrants.

We will first lay out the basic features of the two-period model, then describe natives' preferences over alternative second-period capital-labor ratios. We then analyze the relationship between capital-labor ratios and immigration policies—in particular, describing conditions under which a capital-labor-ratio-maximizing policy exists—and then show how natives' preferences over capital-labor ratios translate into preferences over immigration policies.

We now proceed to describe the model. The economy in this section lasts for two periods. The population in the first period consists of a measure one of natives. Each native is endowed with some amount of initial capital, which may be consumed or saved for period two. Capital which a native saves for the second-period is rented to the economy's production sector at a competitively determined real rental rate r. Each native also has a unit of labor to supply in the second period, and that unit earns a competitively determined real wage of w. The production sector is characterized by a Cobb-Douglas production function $F(K,L) = AK^{\alpha}L^{1-\alpha}$. The second-period labor input, L, consists of the unit labor supplies of natives and immigrants, and the second-period capital stock consists of native savings plus capital brought by immigrants. Natives may differ in their initial capital-holdings, the total mass of natives is one, and $\bar{k}$ denotes average initial native capital.

We assume that all natives have the same time-additive, logarithmic preferences over consumption in the two periods. The problem faced by a native with initial capital k is

$$\max_{s} \log(k-s) + \beta \log(rs+w),$$

where $\beta \in (0,1)$ is the native's utility discount factor. The solution to the native's problem is characterized by the following savings decision-rule:

$$s(k; w/r) = \frac{\beta}{1+\beta}k - \frac{1}{1+\beta}\frac{w}{r}. \quad (1)$$

Aggregating savings over natives (at a given wage-rental ratio w/r) gives the total amount of capital supplied by natives, at the given prices, for production in the second period.

Immigrants, if admitted, arrive in the second period. Let M denote the number of immigrants admitted. Immigrants arrive with a unit of labor to supply plus some amount of capital. Let $\kappa(M)$ denote the total amount of capital among M immigrants, so aggregate native saving plus $\kappa(M)$ equals the total input of capital, K, in period two.

Implicitly, positing $\kappa(M)$ means assuming the economy has some rule for deciding on the order in which heterogeneous immigrants are to be admitted. We will assume that the economy follows a 'wealthiest-immigrants-first' rule, so that $\kappa'(M)$—the increment to total immigrant capital from a marginal immigrant—is decreasing in M.[2]

For a given value of M, describing the economy's equilibrium is straightforward. Let ξ denote the second-period capital-labor ratio. Given a period-two wage-rental ratio, the savings behavior of natives plus capital brought by immigrants implies a period-two capital-labor ratio

$$\xi = \frac{s\left(\bar{k}; w/r\right) + \kappa\left(M\right)}{1+M}.$$

Here, we have used the facts that, with a unit mass of natives, aggregate saving is identical to average saving, and, with a linear saving rule, average saving is the saving of the native with the average initial endowment of capital. On the other hand, from the Cobb-Douglas production technology, a capital-labor ratio of ξ implies a second-period wage-rental ratio of

$$\frac{w}{r} = \frac{1-\alpha}{\alpha}\xi. \tag{2}$$

Equilibrium, then, is characterized by a value of ξ such that

$$\xi = \frac{s\left(\bar{k}; \frac{1-\alpha}{\alpha}\xi\right) + \kappa\left(M\right)}{1+M}. \tag{3}$$

All other quantities, as wells as prices, then follow directly from the equilibrium ξ.

In this economy, the only effect of immigration on natives' welfare comes through the effect of immigration on the equilibrium capital-labor ratio, and the consequent effect this has on factor prices. Hence, before thinking about the majority-rule equilibrium level of immigration, it is useful to think about natives preferences in terms of the economy's second period capital-labor ratio.

The Cobb-Douglas technology implies that the equilibrium rental rate on capital will be

$$r = \alpha A \xi^{\alpha-1}.$$

Substituting this expression, the expression for the wage-rental ratio (2), and the savings decision rule (1) into a typical native's utility function gives the following indirect utility function defined over $\xi \in (0, +\infty)$ for a native with initial capital k:

$$v_k\left(\xi\right) = \left(1+\beta\right)\log\left(\alpha k + \left(1-\alpha\right)\xi\right) - \beta\left(1-\alpha\right)\log\left(\xi\right).$$

It's straightforward to show that $v_k\left(\xi\right)$ is '∪-shaped', with

$$\lim_{\xi\to 0} v_k\left(\xi\right) = \lim_{\xi\to+\infty} v_k\left(\xi\right) = +\infty,$$

and attaining a global minimum at

$$\xi_k = \frac{\alpha\beta}{1+\alpha\beta}k.$$

[2] Benhabib [1] examines the case where there is a distribution of immigrant capital-holdings, say $F\left(k\right)$, with associated density $f\left(k\right)$. A policy in Benhabib's model is a choice of 'cut-off' values, l and $u \geq l$, such that the economy admits all immigrants with capital-holdings in the interval $[l, u]$. Our $\kappa\left(M\right)$ corresponds to a restriction $u = +\infty$. Below, we will argue that our 'wealthiest first' assumption is not particularly restrictive: in a more general setting, such as Benhabib's, the native population would be polarized between a 'poorest first' and 'wealthiest first' policy, with the relatively poorer part of the native population (and presumably a majority) preferring the 'wealthiest first' alternative.

Now, suppose that feasible immigration policies correspond to an interval $[\xi_*, \xi^*]$ of second-period capital-labor ratios. Hold off momentarily on the possibility that two policies lead to the same value of ξ—or, equivalently, imagine that ξ is the issue to be decided. It follows from the $\cup$-shape of natives' utilities over ξ that if a native has initial capital k such that his minimum point, ξ_k, lies below the lower bound ξ_*, then that native prefers the upper bound ξ^* to any other $\xi \in [\xi_*, \xi^*]$. Conversely, if k is such the native's minimum point lies above the upper bound ξ^*, then the native prefers the lower bound ξ_*. If k is such that ξ_k is in between the upper and lower bounds, *i.e.*, $\xi_* \leq \xi_k \leq \xi^*$, then the native's most preferred point could be either ξ_* or ξ^*. One can show that there is a critical value of k—call it k^c—such that natives with $k < k^c$ prefer the higher ξ^* while natives with $k > k^c$ prefer the lower ξ_*.[3] Consequently, if the *median* level of initial native capital is less than the critical value k^c, a majority of natives prefer the higher capital-labor ratio ξ^*.[4] Thus, if there is a unique level of immigration which produces the maximal capital-labor ratio ξ^*, that policy will be a majority-rule equilibrium.

What is going on here is that a native who is relatively poor (*i.e.*, one whose k is low) would prefer a high value of ξ, because this would imply a higher wage. Conversely, a native who is relatively rich (*i.e.*, one whose k is high) would prefer a low value of ξ, because this would imply a higher return to capital. If there are enough natives who are relatively poor, a majority of natives will favor a higher value of ξ.

2.2 The K/L-maximizing level of immigration

In this section, we turn to consider conditions on $\kappa(M)$ which imply that ξ^* or ξ_* are attained by unique policies. We assume that feasible values of M lie in an interval $[0, M_{\max}]$. Using the form of the savings rule (1), the expression (3) for the equilibrium value of ξ becomes

$$\xi = \frac{1}{1+M}\left\{\frac{\beta}{1+\beta}\bar{k} - \frac{1}{1+\beta}\frac{1-\alpha}{\alpha}\xi + \kappa(M)\right\}.$$

Solving for ξ gives:

$$\xi = \frac{\alpha\beta\bar{k} + \alpha(1+\beta)\kappa(M)}{1+\alpha\beta+\alpha(1+\beta)M} \equiv \xi(M). \tag{4}$$

In the appendix, we prove the following:

Proposition 1 *Suppose that $\kappa(M)$ is C^2, strictly increasing and strictly concave on the set $(0, M_{\max})$, with $\kappa(0) = 0$. If $\kappa(M)$ is such that $\kappa'(0) > [\alpha\beta/(1+\alpha\beta)]\bar{k} > \kappa'(M_{\max})$, then there exists a unique $M^* \in (0, M_{\max})$ which attains ξ^*. Also, if $\kappa(M_{\max})/M_{\max} \leq [\alpha\beta/(1+\alpha\beta)]\bar{k}$, then $M = M_{\max}$ attains ξ_*; otherwise, $M = 0$ attains ξ_*.*

The first part states that concavity of $\kappa(M)$, together with conditions on its slope at $M = 0$ and $M = M_{\max}$, guarantee existence of a unique maximizer M^*. The middle term in the inequality $\kappa'(0) > [\alpha\beta/(1+\alpha\beta)]\bar{k} > \kappa'(M_{\max})$ is simply the capital-labor ratio which would obtain absent immigration. Thus, the inequality will hold if the marginal capital brought by the wealthiest immigrant exceeds the

[3] See the appendix for details.

[4] As we will see below, assuming the median native prefers ξ^* to ξ_* amounts to a joint assumption on all the parameters of the model. We think that it is a reasonable assumption, however, at least for the case of a developed economy like the US vis-a-vis a large pool of potential immigrants who are, *on average*, relatively quite poor.

average amount per native person at $M = 0$, and if this average, in turn, exceeds the marginal capital which would be brought by the poorest immigrant, were he or she to be admitted. The second part of the proposition is simply about locating the minimizer of $\xi(M)$; since $\xi(M)$ turns out to be hump-shaped, this minimizer is at either $M = 0$ or $M = M_{\max}$, depending on the average amount of capital of the entire potential immigrant pool. In either case, let M_* denote the value of M, either 0 or $M_{\max}$, which attains ξ_*.

2.3 Natives' preferences over immigration

Now, we turn to consider natives' preferences over M. As noted above, in terms of the economy's next-period capital-labor ratio, the preferred point of any particular native will be either ξ^* or ξ_*, depending on whether that native's initial wealth lies below or above the critical value k^c. As we show in the appendix, the critical value of initial native wealth is given by

$$k^c = \left(\frac{1-\alpha}{\alpha}\right) \frac{\xi^* (\xi_*)^\eta - \xi_* (\xi^*)^\eta}{(\xi^*)^\eta - (\xi_*)^\eta}, \tag{5}$$

where $\eta = \beta(1-\alpha)/(1+\beta)$. Every native with $k < k^c$ prefers ξ^* to ξ_*—and so prefers ξ^* to *any* alternative capital-labor ratio ξ. Hence, every native with $k < k^c$ prefers immigration M^* to any other $M \in [0, M_{\max}]$. Correspondingly, every native with $k > k^c$ prefers immigration M_* to any other $M \in [0, M_{\max}]$, where M_* is whichever of the two values, either 0 or $M_{\max}$, attains ξ_*.

The majority rule outcome will depend on whether the *median* native capital-holding, k^m, falls below k^c or above k^c. In the former case, M^* will be a majority rule outcome, while in the latter case it will be M_*. By a 'majority rule outcome' we mean the Nash equilibrium of the standard two-candidate competition for votes with simultaneous announcement of platforms over the issue M. Natives' preferences violate single-peakedness, so that the median voter theorem doesn't apply here. However, the fact that natives can be separated into two groups, with natives in each group sharing a *common* most preferred point, implies that, for either candidate, announcing the policy preferred by the larger group is a best response to any platform announced by one's opponent, so that both candidates announcing the policy preferred by the majority forms a Nash equilibrium in dominant strategies. Excluding the knife-edge case where $k^m = k^c$, and assuming a continuous distribution of native capital, the equilibrium policy is also clearly unique.

We record the following proposition:

Proposition 2 *Let k^c be defined as in (5), and assume $\kappa(M)$ satisfies the same set of assumptions as in Proposition 1. Then, if $k^m < k^c$, M^* is a majority-rule equilibrium policy.*

Since k^c is such that

$$\xi^* \geq \frac{\alpha\beta}{1+\alpha\beta} k^c \geq \xi_*,$$

we can put a rough lower bound on the critical capital-holding k^c. In the appendix, we show:

Corollary 1 *Suppose that $k^m < \bar{k}$, and $\kappa(M)$ satisfies the same set of assumptions as in Proposition 1. If either (a) $\kappa(M_{\max})/M_{\max} > [\alpha\beta/(1+\alpha\beta)]\,\bar{k}$ or (b)*

$$\left(\frac{1+\alpha\beta}{\alpha\beta}\right)\left(\frac{\alpha\beta\bar{k} + \alpha(1+\beta)\kappa(M_{\max})}{1+\alpha\beta+\alpha(1+\beta)M_{\max}}\right) \geq k^m,$$

then M^* is a majority-rule equilibrium policy.

It's possible to get a feel for whether or not k^m would fall below the critical bound k^c in a reasonably parametrized model. To that end, let λ denote the ratio of average potential immigrant capital to native capital—*i.e.*, $\kappa(M_{\max})/M_{\max} = \lambda\bar{k}$. Then, case (a) from the corollary—in which simply having $k^m \leq \bar{k}$ is sufficient to give M^* as the majority's preferred policy—obtains if

$$\lambda > \frac{\alpha\beta}{1+\alpha\beta}.$$

A standard value for α, capital's share in the production function is 0.30. The choice of β depends implicitly on the length of period we are assuming. For example, a period length of ten years would correspond (roughly) to a β of 0.7. For those parameter values, we are in case (a) if the ratio of average potential immigrant capital to average native capital exceeds $0.21/1.21 \cong 0.174$.

If $\lambda < \alpha\beta/(1+\alpha\beta)$, we could turn to evaluate the approximate bound in part (b) of the corollary. However, this bound—as we will see—turns out to be quite crude. Using λ to again denote the ratio of average potential immigrant capital to average native capital, the inequality in part (b) of the corollary may be written as

$$\left(\frac{1+\alpha\beta}{\alpha\beta}\right)\left(\frac{\alpha\beta+\alpha(1+\beta)\lambda M_{\max}}{1+\alpha\beta+\alpha(1+\beta)M_{\max}}\right) > k^m/\bar{k}.$$

In the US, the ratio of median wealth to average wealth is on the order of 0.3, while the ratio of median income to average income is on the order of 0.6.[5] Does the expression on the left side of the last inequality exceed 0.30 or 0.60 for reasonable parameters? While α and β are fairly straightforward to calibrate, the choices of $M_{\max}$ and λ—the characteristics of the pool of potential immigrants—are more difficult. For example, the population of the rest of the world is roughly 22 times the population of the US. World GDP per person is about one-fifth US GDP per person, so we might take $0.2^{\frac{1}{0.3}}$ as a rough estimate of λ.[6] For those numbers, the expression on the left-hand side of the last inequality is 0.1217, which is below the ball-park range $[0.30, 0.60]$ for $k^m/\bar{k}$.

But is $M_{\max} = 22$ a reasonable size for the US's pool of potential immigrants? The model is missing a number of features—notably land and possible congestion effects—which would surely come into play at levels of immigration that high. If we cut $M_{\max}$ in half, to $M_{\max} = 11$, keeping $\lambda = 0.2^{\frac{1}{0.3}}$, the bound increases only slightly to 0.1996—again less than 0.30.

The following table, however, records the *exact* critical values k^c at values of $M_{\max}$ ranging from 5 to 20, and values of λ ranging from 0.002 to 0.20. All fall above the 0.30 threshold (which is approximately the ratio of US median wealth to US mean wealth) and many fall above even the 0.60 threshold (which is approximately the ratio of US median income to US mean income).

[5] See Díaz-Giménez, *et al.* [2].

[6] This assumes identical production technologies for the US and the rest of the world, so that the ratio of per-person GDPs must equal the ratio of per-person capital stocks raised to the power $\alpha = 0.3$. Flipping this around, the ratio of per-person capital stocks must equal the ratio of per capita GDPs raised to the $\alpha^{-1} = 0.3^{-1}$.

		λ			
		0.0020	0.0680	0.1340	0.2000
	5	0.5868	0.8234	1.0807	1.3482
$M_{\max}$	10	0.4692	0.8106	1.1722	1.5424
	15	0.4084	0.8205	1.2510	1.6888
	20	0.3698	0.8367	1.3200	1.8097

The reason for looking at all at the crude bound of the corollary is that its calculation only uses values for λ and $M_{\max}$; the exact critical value k^c, on the other hand, requires calculation of M^*, which in turn requires that the entire $\kappa(M)$ function be specified. The numbers in the above table were calculated for a $\kappa(M)$ derived from a Pareto distribution with a ratio of median to mean capital of 0.30.

2.4 A comparison with Benhabib

Benhabib [1] examines a static model in which natives derive income from the supply of labor and capital to a neoclassical production sector. Natives' preferences over immigration derive from immigrants' impact on income. As noted above, Benhabib's immigrants are heterogeneous in their capital-holdings, with a distribution $F(k)$ of capital-holdings, with associated density $f(k)$. A policy in Benhabib's model is a choice of 'cut-off' values, l and $u \geq l$, such that the economy admits all immigrants with capital-holdings in the interval $[l, u]$. A policy implies both a level of immigration, $M(l, u) = F(u) - F(l)$, and a total amount of capital brought into the country, $K(l, u) = \int_l^u kf(k)\, dk$.

Our assumptions on $\kappa(M)$ would correspond to fixing $u = +\infty$ in Benhabib's framework, and considering only a choice of l. In terms of a distribution of immigrant capital, our $\kappa(M)$ could be defined as $\kappa(M) = \int_l^{+\infty} kf(k)\, dk$, where l is such that $M = F(+\infty) - F(l)$. In other words, if we denote $F(+\infty)$, the total number of potential immigrants,by $M_{\max}$, then

$$\kappa(M) = \int_{F^{-1}(M_{\max}-M)}^{+\infty} kf(k)\, dk.$$

Our assumption of strict concavity is natural in light of this construction, since

$$\kappa'(M) = F^{-1}(M_{\max} - M)$$

and

$$\kappa''(M) = -\frac{1}{f\left[F^{-1}(M_{\max} - M)\right]}.$$

In fact, in some of our numerical experiments, we construct $\kappa(M)$ in precisely this way. For example if F is a Pareto-type distribution, then $\kappa(M)$ has the form of a concave power function of M. If F is a log-normal distribution, then $\kappa(M)$ has the form $B[1 - N(\omega - \sigma)]$, where N is the standard normal C.D.F. and ω is such that $M/M_{\max} = 1 - N(\omega)$. The parameter σ is the standard deviation of the logarithm of immigrant capital, and B depends on $M_{\max}$, σ and the mean of the logarithm of immigrant capital.

In any case, our $\kappa(M)$ is obviously a restriction relative to Benhabib's more general policies. However, we do not believe that it is a serious one. As we have shown above, natives in our two-period economy prefer to either minimize or maximize the subsequent period's capital-ratio, with—under reasonable parameters—a majority preferring to maximize. If we took Benhabib's more general notion of a policy, the economy's policy choice would ultimately boil down to one between a minimizing policy of the form $[0, u]$, for a particular, common value of u, or a maximizing policy of the form $[l, +\infty)$, for a particular, common value of l. In general, for reasonable environments, we expect the latter type of policy to prevail.[7]

At the same time, the dynamic nature of our economy—in particular, the fact that agents are maximizing lifetime utilities rather than static income—gives rise to phenomena not possible within Benhabib's framework, namely a dependence of immigration policy on the growth rate of technology (which we incorporate below) and the potential for multiple steady state equilibria. While our exploration of the positive ramifications of these sorts of behaviors is still preliminary, both of these features seem *a priori* important to us, if models of this sort are to come close to explaining patterns we see in real-world policies. Our sense is that native demographics do not differ sufficiently, either across countries or over time, to fully explain differences in immigration policies (both across countries and over time). In our introduction, we noted the disparate immigration policies adopted by several otherwise similar countries. Thinking about variation over time in the policy of a single country, in a model such as ours—or that of Benhabib—policy doesn't vary smoothly with population characteristics; rather, rising or falling inequality has no effect on immigration policy until a critical threshold is passed, at which point an abrupt swing in policy occurs. Perhaps this is a feature of the real world (the Australian experience comes to mind in this respect); still, one would hope for a model which predicts some 'smooth' variation in policy, since not all changes in most countries' immigration policies take the form of a sudden opening or closing of the doors.[8]

3 An overlapping generations model

In this section, we extend the basic two-period model elaborated above to an overlapping generations setting with two-period-lived agents. One important difference from the model above is that production will take place in every period and, in contrast to most Diamond-type OLG models, we will assume our agents supply labor and capital to firms in both periods of their lives.

Time is discrete and indexed by $t = 0, 1, 2 \ldots$. At the start of each period t, a measure one of young natives is born. Each young native is endowed with 1 unit of labor and some amount of capital, k. The capital endowment is idiosyncratic, and there is a distribution μ with $\int \mu(dk) = 1$. We let $\bar{k}$ denote the average capital endowment of young natives. At each date t also, a measure M_t of immigrants arrive (where $M_t = 0$ is possible, depending on policy). Immigrants are two-period-lived as well, and are assumed to arrive young. Each new immigrant also has 1 unit of labor and some amount of capital, k, to supply. Capital endowments vary across immigrants, and we continue to let $\kappa(M)$ again denote the total amount of capital brought by immigrants when the number of immigrants admitted is M.[9] We assume that immigration is one-way, so that M_t young immigrants at t become M_t old citizens at $t+1$. The population at any date $t \geq 1$ is then $L_t = (1 + M_{t-1}) + (1 + M_t)$, where the first term consists of old agents at t and the second, young agents at t.[10]

[7] To be certain, however, our computational programs explicitly allow for the possibility that the K/L-minimizing choice is the majoritarian winner; this outcome never occured in the parametrizations which we examined.

[8] Even in regard to the last major round of 'door closing'—at the end of the first great wave of globalization which took place from the middle of the 19th century through to the early part of the 20th century—Timmer and Williamson [5] note that this closure was a gradual process, rather than an abrupt one. On the other hand, as an empirical matter, the main factor which they find to have prompted this closure was rising inequality in the main recipient countries of the New World.

[9] For now, we assume that $\kappa(\cdot)$, like $\mu(\cdot)$, is constant over time.

[10] The population at $t = 0$ consists of a mass of young agents, with some average capital holdings.

All young agents, whether natives or new immigrants, face the same decision problem. Young agents supply labor and capital to the production sector at competitively determined real wage and rental rates. Income from the supply of factors of production is divided between current consumption and savings (next-period's capital). In the subsequent period, the agent consumes his or her income from savings (*i.e.*, the rental on the agent's capital), plus income from the supply of labor.

Letting c_t denote consumption of a young agent at date t, s_{t+1} savings by a young agent for period $t+1$, d_{t+1} consumption of an old agent at $t+1$, and w_t and r_t the wage and rental rates, a young agent at t, who is endowed initially with k units of capital, faces the following budget constraints:

$$w_t + r_t k = c_t + s_{t+1}$$

and

$$d_{t+1} = r_{t+1} s_{t+1} + w_{t+1}.$$

Agents are assumed to have preferences of the time-additive, logarithmic form

$$U(c_t, d_{t+1}) = \log(c_t) + \beta \log(d_{t+1}).$$

With these assumptions on tastes and opportunities, the behavior of a typical young agent is characterized by a savings rule of the form:

$$s_{t+1} = \frac{\beta}{1+\beta}(w_t + r_t k) - \frac{1}{1+\beta}\frac{w_{t+1}}{r_{t+1}}. \tag{6}$$

Then, aggregate capital in period $t+1$ will be the sum of savings by the young generation at t, plus the capital endowments of the new generation of natives born at $t+1$, plus the capital brought by the immigrants who arrive at $t+1$. In particular, we have:

$$K_{t+1} = \frac{\beta}{1+\beta}\left[w_t(1+M_t) + r_t\left(\bar{k} + \kappa(M_t)\right)\right] - \frac{1+M_t}{1+\beta}\frac{w_{t+1}}{r_{t+1}} + \bar{k} + \kappa(M_{t+1}).$$

As in our two-period model, production takes place within a representative firm operating a Cobb-Douglas technology. The period-$t+1$ wage-rental ratio, in equilibrium, is then

$$\frac{w_{t+1}}{r_{t+1}} = \frac{1-\alpha}{\alpha}\frac{K_{t+1}}{L_{t+1}},$$

where $L_{t+1} = (1+M_t) + (1+M_{t+1})$ is the total population at $t+1$. Substituting this expression for w_{t+1}/r_{t+1} in the equation describing K_{t+1} yields, after some rearranging,

$$\frac{K_{t+1}}{L_{t+1}} = \frac{\alpha\beta\left[w_t(1+M_t) + r_t\left(\bar{k} + \kappa(M_t)\right)\right] + \alpha(1+\beta)\left[\bar{k} + \kappa(M_{t+1})\right]}{(1+\alpha\beta)(1+M_t) + \alpha(1+\beta)(1+M_{t+1})}. \tag{7}$$

For convenience, let z denote the capital-labor ratio, and note that w_t and r_t can both be expressed in terms of z_t. The period-$t+1$ capital-labor ratio can then be seen to depend on M_{t+1}, z_t and M_t. Let $\xi(M_{t+1}; z_t, M_t)$ stand for the expression on the right-hand side of (7), when $w_t = (1-\alpha)Az_t^{\alpha}$ and $r_t = \alpha A z_t^{\alpha-1}$. Then, (7) may be expressed as:

$$z_{t+1} = \xi(M_{t+1}; z_t, M_t). \tag{8}$$

These young consist of natives plus some (possibly zero) number of 'original' immigrants.

Let

$$\pi(z, M) = \max_{M'} \{\xi(M'; z, M) : 0 \leq M' \leq M_{\max}\}$$

and

$$\phi(z, M) = \arg\max_{M'} \{\xi(M'; z, M) : 0 \leq M' \leq M_{\max}\}.$$

If immigration for period-$t+1$ is chosen to maximize the period-$t+1$ capital-labor ratio, given period t's capital-labor ratio and level of immigration, then the evolution of this economy over time has the following simple description:

$$z_{t+1} = \pi(z_t, M_t) \tag{9}$$

$$M_{t+1} = \phi(z_t, M_t). \tag{10}$$

One can show that the first-order conditions for choosing M' to maximize $\xi(M'; z, M)$ imply that

$$\kappa'(M^*) = \xi(M^*; z, M).$$

Thus, when z_t and M_t follow the dynamic system above, we will have at every date that

$$\kappa'(M_{t+1}) = \xi(M_{t+1}; z_t, M_t) = z_{t+1}.$$

Implicitly, the two-dimensional system above can be reduced to the one-dimensional system

$$M_{t+1} = \phi(\kappa'(M_t), M_t) \equiv \psi(M_t). \tag{11}$$

4 Numerical results

The dynamic system underlying this economy consists of a pair of non-trivial non-linear equations. As such, it is difficult to characterize, analytically, both the steady states and transitional dynamics of the economy. However, it is straightforward to utilize numerical methods to enhance our understanding of the model.

In what follows, we assume that $\kappa(M)$ has the form of a smooth concave power function—$\kappa(M) = BM^{\theta}$. This is in fact that the $\kappa(M)$ which would arise if immigrant capital had a Pareto distribution.[11] The examples below differ in their values of θ and B, but share a common set of core parameters: $\alpha = .3$, $\beta = .7$ and $\bar{k}$, the mean native capital endowment, equal to 1.

4.1 Steady states

4.1.1 An economy with a unique steady state equilibrium

In this economy, we set $B = 1.25$ and $\theta = .31$. The underlying Pareto distribution of immigrant wealth for these parameter values is one characterized by a ratio of median immigrant capital to mean immigrant capital of 0.50—*i.e.*, a moderate degree of inequality, comparable to the US—and a fairly low ratio of mean immigrant capital to mean native capital, assuming the potential immigrant population is large.[12] The

[11] We have obtained qualitatively similar results with the somewhat less tractable lognormal distribution.

[12] The underlying Pareto distribution with mass $M_{\max}$ has three parameters (including $M_{\max}$). The B and θ in $\kappa(M) = BM^{\theta}$ are functions of these underlying parameters. There's a one-to-one relationship between θ and the degree of inequality in the distribution (whether measured with the median-mean ratio or Gini coefficient), $\theta = 0.31$ corresponds to a median-mean ratio of 0.50. Given that ratio, a given value of B, however, corresponds to a locus of pairs of possible means and total masses for the distribution; so, for example, $B = 1.25$ is consistent with a plausible $M_{\max} = 11$ and a small mean capital-holding of 0.24 the size of the mean native caital-holding. It is also consistent, however, with a very small, very rich potential immigrant pool—*e.g.*, a pool of immigrants of half the size of the native population with twice the average wealth is also consistent with $B = 1.25$.

infinite horizon overlapping generations economy of the last section gives rise to a dynamic system that can be described succinctly by equations (9) and (10). The economy has two state variables in any period: the capital-labor ratio (z_t), and the level of immigration (M_t). This economy has a unique steady state, and it is straightforward to show that from any initial condition in the $z - M$ plane, that the economy converges to this unique steady state.

Figure 1 shows how the choice of next-period's level of immigration depends on the current state variables—*i.e.*, it is a graphical representation of equation (10). Generally, the level of immigration (M_{t+1}) is increasing in the level of immigration in the previous period (M_t). What this means is that, holding fixed the capital-labor ratio z_t (or, equivalently, the current factor prices w_t and r_t), high immigration in the previous period implies high immigration in the current period. In terms of the underlying maximization problem which M_{t+1} solves, the effects of M_t on the choice of M_{t+1} are quite complex. Intuitively, though, the marginal benefit of a bit more immigration is the extra capital it implies, while the marginal cost is the extra labor over which the capital stock must be spread. One can show mathematically that an increase in M_t has a clear negative effect on the marginal benefit of next-period's immigration—since the effect on the capital-labor ratio of a bit more capital is smaller the larger the population is. At the same time, an increase in M_t has an *a priori* ambiguous effect on the marginal cost of next-period's immigration—on the one hand, more people around to start with lowers the marginal impact on the capital-labor ratio of adding few more, but at the same time more people to start with means a bit more aggregate savings (and capital) to equip those few more with. Apparently, for the parameters in this experiment, an increase in M_t ends up lowering the marginal cost of M_{t+1}, and by more than it lowers the corresponding marginal benefit.

While the next-period's immigration decision is monotonic in current immigration, it is apparent from the diagram that the immigration decision is not monotonic in the current capital-labor ratio. If the current capital-labor ratio is relatively low, then the number of desired immigrants is *increasing* in the level of z_t. To see why, think of next period's capital-labor ratio as a convex combination of per capita savings by the currently young, plus per capita capital of next period's immigrants and next period's young generation; increasing next period's immigration, other things the same, reduces the weight placed on per capita savings by the current young. Now, when the capital-labor is sufficiently low, an increase in the capital-labor ratio will reduce the per capita savings of the current young, because it reduces the return to saving. This makes it worthwhile to increase next-period's immigration a bit (increasing the weight on the capital of the newcomers and reducing the weight on capital supplied by the current young). Conversely, if the current capital-labor ratio is high, increases in it will increase the per capita savings of the current young, because it raises wages. This makes it worthwhile to *decrease* next-period's immigration a bit. Hence, the decision rule for next-period's immigration is, at higher levels of z_t, *decreasing* in z_t.

Figure 2 illustrates how the capital-labor ratio in the subsequent period (z_{t+1}) is a function of the current capital-labor ratio (z_t) and the level of immigration this period (M_t)—*i.e.*, the figure plots equation (9). The optimal next-period capital-labor ratio is a decreasing function of the current level of immigration. From a purely formal standpoint, this property is equivalent to the positive dependence of M_{t+1} on M_t, which we described above. To see the connection, recall that the first-order condition for a maximizing choice of M_{t+1} can be written as $\kappa'(M_{t+1}) = \xi(M_{t+1}; z_t, M_t)$. Given that an increase in M_t leads to an increase in M_{t+1}—and that fact that

$\kappa'(M_{t+1})$ is a decreasing function of M_{t+1}—it follows that an increase in M_t must lower $\xi(M_{t+1}; z_t, M_t)$—*i.e.*, z_{t+1}—at the optimal choice of M_{t+1}. Intuitively, it must be the case that, for these particular model parameters, the positive impact which a higher value of M_t has on aggregate native savings is insufficient to offset the fact that capital must be spread over a larger number of persons.

Although it is not obvious in the diagram, the choice of z_{t+1} is not monotonic in z_t—for small values of z_t, the capital-labor ratio z_{t+1} is decreasing in z_t, while for larger values of z_t, z_{t+1} is increasing in z_t. As with the effect of z_t on M_{t+1}, the non-monotonicity here is due to the fact that increasing z_t will decrease the per capita savings of the current young when z_t is small, and increase the per capita savings of the current young when z_t is large. In particular, at low levels of the capital-labor ratio, an increase in this ratio in the current period reduces the level of domestic saving by the young because it lowers the rate of return to capital. This effect can only be partially offset through immigration of immigrants who are sufficiently wealthy, but the net effect is a lower capital-labor ratio next period. However, at high levels of the current period's capital-labor ratio, further increases in this ratio *raise* the subsequent period's capital-labor ratio. This is because of the fact that, while a rise in the current capital-labor ratio lowers the return to capital, this is offset by an increase in this period's wage, and some of this wage is then saved for the following period.

4.1.2 An economy with multiple steady states

In this section we consider slightly different parameter values, letting $\theta = .74$ and $B = .455$. The underlying distribution of immigrant capital associated with these parameters features a very low degree of wealth inequality and—assuming a large pool of potential immigrants—a fairly low mean relative to the average native capital-holding.[13] This economy has three steady states, labelled 'A', 'B' and 'C' in Figure 3. These correspond, respectively, to high, intermediate and low levels of immigration. The dynamics of this economy are quite interesting, since A and C are locally stable, or "sinks". The equilibrium at B, however, is saddle-path stable, though in an odd way. In particular, if the economy begins at points above or below the locus labelled XY, it converges to either A or C; if it begins at any point on XY, then it converges to point B—but in one step. What's going on? Let M_B denote the level of immigration associated with the equilibrium B; then, the locus of points XY is precisely the set of pairs (z, M) such that M_B solves $\max_{\tilde{M}} \xi(\tilde{M}; z, M)$.

Of course, these three equilibria are associated with different behavior. The steady state equilibrium at A features a very high level of immigration and a low capital-labor ratio while the opposite is true at C. Note that since equilibrium (z, M) pairs obey the first-order condition $\kappa'(M_{t+1}) = z_{t+1}$—and given the assumed concavity of $\kappa(\cdot)$—it will always be the case in this model that trajectories (and their limiting values) lie along a downward-sloping locus in $z - M$ space.[14]

While this example is only meant to be illustrative of the possible behavior of the model—in particular, we wouldn't consider our parametrization, at this stage, to constitute a careful calibration of the model's parameters—it's worth pointing out that, in terms of output per person, two otherwise identical economies which have ended up at A and C will not look *that* different. The capital-labor ratio at A is about half that at C, implying that per capita GDP at A is about 81% of per capita

[13] The main difference is the degree of inequality—the median-mean ratio in this example is 0.88 versus 0.50 previously. For an immigrant pool that encompasses the vertical axis in Figure 3—say, $M_{\max} = 20$—the immigrant-mean-to-native-mean ratio is somewhat smaller than that in the previous example (a mean-mean ratio of about 0.21 versus 0.24 in the previous example).

[14] Except possibly at $t = 0$.

GDP at C—about the standing of an Australia or Denmark relative to the US.[15]

If the initial conditions (z_0, M_0) are such that M_0 is 'optimal' given z_0 (or vice versa)—that is to say, if the first order condition $\kappa'(M_0) = z_0$ holds—then the dynamics of the economy can be expressed as one-dimensional, as in equation (11). That is, since the solution to $\max_{\tilde{M}} \xi(\tilde{M}; z, M)$ obeys the first-order condition $\kappa'\left(\tilde{M}\right) = \xi\left(\tilde{M}; z, M\right)$, if $z_0 = \kappa'(M_0)$, we have $M_{t+1} = \arg\max_{\tilde{M}} \xi(\tilde{M}; \kappa'(M_t), M_t) \equiv \psi(M_t)$ for all t.[16] The three steady states from Figure 3 can also be seen in Figure 4, which plots $\psi(M_t) - M_t$ (*i.e.*, $M_{t+1} - M_t$ plotted against M_t). One can also see in this representation the stability of the low and high steady states and the instability of the middle steady state.

As one might guess—given that the only difference between our unique and multiple steady state examples is the choice of θ and B—whether an economy has one or more steady states depends on these parameters of the $\kappa(M)$ function (which ultimately derives from a distribution of potential immigrants' capital). Figure 5 shows part of the parameter space, with values of B on the horizontal axis and values of θ on the vertical axis. The dark region corresponds to (B, θ) pairs for which the economy has three steady states, just as in the example of this section. Outside the dark region, the economy has a unique steady state. Along the knife-edge set of parameters which makes up the boundary of the dark shaded region, there are two steady states—at which either the 'low' and 'middle' equilibria or the 'middle' and 'high' equilibria collapse into a single point.

How can we interpret these results? The parameters θ and B of $\kappa(M) = BM^\theta$ have an interpretation in terms of an underlying distribution of capital among potential immigrants, in particular this form for $\kappa(M)$ arises when the distribution is of the Pareto type. Since the total mass of potential immigrants is $M_{\max}$, rather than one, the distribution has three parameters, including $M_{\max}$ itself. If we fix $M_{\max}$, then there is a one-to-one correspondence between B and θ on the one hand and the underlying distribution's mean and degree of inequality on the other. For example, specifying the Gini coefficient of the underlying distribution and the mean potential immigrant capital-holding implies a particular (θ, B) pair; likewise, specifying a θ and B implies a particular Gini coefficient and mean for the underlying distribution. Thus, we can map the region of multiple steady state equilibria shown in terms of θ and B in figure 5 into a region in the space of Gini coefficients and mean capital-holdings. Higher values of θ correspond to smaller degrees of inequality, independent of B. If we fix $M_{\max}$ at 10, for example, then the range of θ's in figure 5 for which there are multiple steady state equilibria ($\theta \cong .73$ to $\theta = .9$) corresponds to Ginis which run from about .16 down to about .05—in other words, a very low degree of inequality. The mean of the distribution is determined jointly by B and θ; fixing $M_{\max}$ again at 10, the range of means in the shaded area of Figure 5 is from (roughly) .07 to .15. Since the mean native capital endowment has been normalized to one, these values correspond to pools of potential immigrants whose mean capital-holding ranges from 7% to 15% of the average native's capital endowment. Figure 6 shows the region of multiplicity from figure 5, mapped into the space of Ginis and ratios of immigrant mean capital to native mean capital, assuming $M_{\max} = 10$.

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[15] The calculation assumes identical Cobb-Douglas technologies for the two countries, with capital's share, α, equal to 0.30.

[16] Even if z_0 and M_0 are not related in this way, (11) will govern the economy's dynamics *after* the initial period.

4.2 The effects of exogenous technological change

Consider modifying the model presented above to incorporate exogenous technological progress. What consequence would changes in the economy's technological growth rate have on the path of immigration? In particular, in this section we assume that the economy's production technology in period t is

$$F_t(K, L) = AK^{\alpha}(X_t L)^{1-\alpha},$$

where $X_t = (1+\gamma)^t$. In order to have balanced growth in this economy of two-period lived agents we also assume that agents' endowments of capital grow at rate γ as well—so that the average endowment of a native born at t is now $(1+\gamma)^t \bar{k}$ and the total capital brought by M_t immigrants is $(1+\gamma)^t \kappa(M_t)$. This leads to a minor modification of the equations describing the evolution of immigration, M_t, and the capital-labor ratio, z_t.

In particular, equation (7) becomes

$$\frac{K_{t+1}}{L_{t+1}} = \frac{\alpha\beta\left[w_t(1+M_t) + r_t(1+\gamma)^t\left(\bar{k} + \kappa(M_t)\right)\right] + \alpha(1+\beta)(1+\gamma)^{t+1}\left[\bar{k} + \kappa(M_{t+1})\right]}{(1+\alpha\beta)(1+M_t) + \alpha(1+\beta)(1+M_{t+1})}.$$

With our new assumptions on the economy's technology, the wage and rental rate are given by

$$w_t = (1-\alpha)AK_t^{\alpha}(X_t L_t)^{-\alpha} X_t = (1-\alpha)A\left(\frac{K_t}{X_t L_t}\right)^{\alpha}(1+\gamma)^t$$

and

$$r_t = \alpha A K_t^{\alpha-1}(X_t L_t)^{1-\alpha} = \alpha A\left(\frac{K_t}{X_t L_t}\right)^{\alpha-1}.$$

Let $\tilde{z}_t \equiv K_t / X_t L_t$. Then,

$$\tilde{z}_{t+1} = \frac{\alpha\beta(1+\gamma)^{-1}\left[(1-\alpha)A\tilde{z}_t^{\alpha}(1+M_t) + \alpha A\tilde{z}_t^{\alpha-1}\left(\bar{k} + \kappa(M_t)\right)\right] + \alpha(1+\beta)\left[\bar{k} + \kappa(M_{t+1})\right]}{(1+\alpha\beta)(1+M_t) + \alpha(1+\beta)(1+M_{t+1})},$$

or

$$\tilde{z}_{t+1} = \tilde{\xi}(M_{t+1}; \tilde{z}_t, M_t),$$

where the function $\tilde{\xi}(M'; z, M)$ is otherwise identical to $\xi(M'; z, M)$ but for the $(1+\gamma)^{-1}$ in the first term in the numerator. As before, natives' utilities depend on immigration only through factor prices, which in turn depend on the capital-'effective labor' ratio $\tilde{z}$. The same arguments applied above imply that M_{t+1} will be chosen to maximize $\tilde{z}_{t+1}$, as M_{t+1} had been chosen previously to maximize z_{t+1}.

Note that, other things equal, an increase in γ will increase the $\tilde{z}_{t+1}$-maximizing choice of M_{t+1}, for given values of $\tilde{z}_t$ and M_t.[17] As a result, changes in the path of technological change can alter the path of immigration levels, with an increase in the

[17] $\tilde{\xi}(M_{t+1}; \tilde{z}_t, M_t)$, like $\xi(M_{t+1}; z_t, M_t)$, is hump-shaped, and has the form $(a + b\kappa(M_{t+1}))/(c + dM_{t+1})$; an increase in γ is the same as lowering the value of a in this expression. A lower value of a would increase the marginal incremement (or reduce the marginal decrement) in the capital-effective-labor ratio from a small increase in M_{t+1} at any value of M_{t+1} (and $\tilde{z}_t$ and M_t, as well). Note that this conclusion would still obtain even if total immigrant capital grew at a rate different from technology and mean native capital, though in that case $\tilde{\xi}$ would be time-varying, with either a growing or declining marginal benefit from immigration, depending on whether total immigrant capital was growing faster or slower than technology and capital in the native economy.

speed of technological advance leading to larger inflows of immigrants and slowdowns in the pace of technological change leading to smaller immigrant inflows. This is in contrast to the result in Benhabib's static model, in which immigration outcomes are invariant to any changes in the level of technology.

Figure 7 illustrates the effect of an increase in γ for an example economy. Here γ increases from 2% growth per year to 3% growth per year (a model period is taken to be ten years, reflected in the choice of the discount factor β). The pool of potential immigrants in this example has a size of $M_{\max} = 11$, and they are fairly poor on average (their mean capital-holding is 30% of natives' $\bar{k}$) and their distribution of wealth is fairly skewed (with a median-to-mean ratio of 30%). The effect of the increase in γ is fairly modest—resulting in a roughly 0.5 increase in the percent of foreign born agents in the economy. While this example is merely meant to be illustrative of one of the mechanisms at work in this model, dependence of the equilibrium immigration outcome on factors other than the demographics of the voting population is potentially quite important. One apparent drawback of voting models of immigration policy is the invariance of the policy outcome to all but very large changes in native demographics; moreover, when policy does change, the change is quite large, with an economy often swinging from admitting large numbers of immigrants to admitting very few or none.[18] While such models may be good for explaining episodes in which hithertofore 'closed' economies 'open their doors'—or conversely, 'open' economies 'shut their doors'—such episodes, while significant, are also quite infrequent. A responsiveness of policy to productivity growth may provide an explanation for the smaller, more frequent changes in policy which we observe in many economies.[19]

5 Conclusions

This paper has presented a simple dynamic model of the determination of immigration policy, under the assumptions that immigrants are heterogeneous in terms of wealth and that policy is decided by majoritarian means.

Depending on the distribution of wealth of the pool of potential immigrants—in our experiments, the degree of inequality in the distribution, the relation of its mean to the mean native capital-holding, and its total size—the model can yield a unique stable steady state equilibrium or multiple stable steady state equilibria. The model also predicts a positive relationship between the recipient country's rate of technological progress and the size of its immigrant inflows.

Obviously, much remains to be done. In particular, we have not attempted a careful calibration of the model's parameters to the situation of any particular country (and its corresponding pool of potential immigrants). While we have tried to argue that the conditions under which the main voting result hold plausibly fit the situation of the US *vis-à-vis* the less-developed world, this is no substitute for a careful empirical investigation of the model.

Also, the model only explores one dimension of heterogeneity, that being in terms of wealth, or capital. An important avenue for further work, in our view, is the incorporation of other dimensions of heterogeneity, particularly in terms of human capital, or skills.

[18] See our [3] for some results of this form.

[19] *E.g.*, the U.S.—and many other countries—experienced a long period of 'openness' up to the early part of the 20th century, followed by several decades of relative closure to immigration, with a return to openness again in recent decades. (See, for example, [4].) Between these major policy shifts, there has always been some minor policy change taking place—see the INS's website, in particular `http://www.ins.gov/graphics/aboutins/statistics/LegisHist/index.htm` for a summary of 142 legislative acts pertaining to immigration since 1790.

A Appendix

A.1 Derivation of the critical value k^c

A native with capital k has preferences over ξ given by

$$v_k(\xi) = (1+\beta)\log(\alpha k + (1-\alpha)\xi) - \beta(1-\alpha)\log(\xi).$$

For $\xi \in (0,+\infty)$, $v_k(\xi)$ is differentiable, with

$$v_k'(\xi) = \frac{(1+\beta)(1-\alpha)}{\alpha k + (1-\alpha)\xi} - \frac{\beta(1-\alpha)}{\xi}.$$

Clearly,

$$v_k'(\xi) \gtreqless 0$$

as

$$\xi \gtreqless \frac{\alpha\beta}{1+\alpha\beta}k \equiv \xi_k.$$

It follows that $v_k(\xi)$ has the '∪-shape' claimed above. Note that ξ_k is the capital-labor ratio which would obtain, absent immigration, if native k were the only, or average, individual in economy.

A native with capital k of section 2 prefers ξ^* to ξ_* if and only if $v_k(\xi^*) > v_k(\xi_*)$. Assuming that $\xi_* \neq 0$, a little algebra shows that $v_k(\xi^*) > v_k(\xi_*)$ if and only if

$$\frac{\alpha k + (1-\alpha)\xi^*}{\alpha k + (1-\alpha)\xi_*} > \left(\frac{\xi^*}{\xi_*}\right)^{\frac{\alpha(1-\beta)}{1+\beta}},$$

or

$$k < \frac{(1-\alpha)\left[\xi^*(\xi_*)^\eta - \xi_*(\xi^*)^\eta\right]}{\alpha\left[(\xi^*)^\eta - (\xi_*)^\eta\right]},$$

where $\eta \equiv \beta(1-\alpha)/(1+\beta)$. The expression on the right is the critical level of capital, k^c, described in section 2.

A.2 Proof of proposition 1

Suppose that $\kappa(M)$ is differentiable; then $\xi(M)$, the capital-labor ratio for a given M, is also differentiable, with $\xi'(M) > 0$ if and only if

$$\frac{\kappa'(M)}{\alpha\beta\bar{k} + \alpha(1+\beta)\kappa(M)} > \frac{1}{1+\alpha\beta+\alpha(1+\beta)M}.$$

Equivalently, $\xi'(M) > 0$ if and only if

$$\kappa'(M) > \frac{\alpha\beta}{1+\alpha\beta}\bar{k} + \frac{\alpha(1+\beta)}{1+\alpha\beta}\left[\kappa(M) - M\kappa'(M)\right]. \tag{12}$$

If $\kappa''(M) < 0$, the left-hand side of this inequality is continuous and strictly decreasing in M, while the right-hand side is continuous and strictly increasing in M. If

$$\kappa'(0) > \frac{\alpha\beta}{1+\alpha\beta}\bar{k} > \kappa'(M_{\max}), \tag{13}$$

and $\kappa(0) = 0$, it follows that there is a unique $M^* \in (0, M_{\max})$ with $\xi'(M^*) = 0$ and $\xi'(M) > 0$ for $M < M^*$ and $\xi'(M) < 0$ for $M > M^*$.[20]

[20] The first inequality in (13) means that the left-hand side of (12) exceeds the right-hand side at $M = 0$, while the second in equality in (13) implies the opposite at $M = M_{\max}$. To see this, note that $\kappa'(M) > 0$, $\kappa''(M) < 0$ and $\kappa(0) = 0$ together imply $\kappa(M) > M\kappa'(M) > 0$ for all $M > 0$. Thus $\lim_{M\to 0} M\kappa'(M) = 0$, which shows the sufficiency of the first inequality for its purpose, while $\kappa(\bar{M}) - \bar{M}\kappa'(\bar{M}) > 0$ shows the sufficiency of the second for its purpose.

Since $\xi(M)$ is 'hump-shaped', the value of M which minimizes $\xi(M)$—*i.e.*, which attains ξ_*—is either $M = 0$ or $M = M_{\max}$. Note that if $\kappa(0) = 0$, then

$$\xi(0) = \frac{\alpha\beta\bar{k} + \alpha(1+\beta)\kappa(0)}{1+\alpha\beta} = \frac{\alpha\beta}{1+\alpha\beta}\bar{k},$$

while

$$\xi(M_{\max}) = \frac{\alpha\beta\bar{k} + \alpha(1+\beta)\kappa(M_{\max})}{1+\alpha\beta+\alpha(1+\beta)M_{\max}}.$$

One can show that $\xi(M_{\max}) \leq \xi(0)$ if

$$\frac{\kappa(M_{\max})}{M_{\max}} \leq \frac{\alpha\beta}{1+\alpha\beta}\bar{k}, \tag{14}$$

while $\xi(0) \leq \xi(M_{\max})$ if the opposite inequality holds. It follows that $\xi(M_{\max}) = \xi_*$ when (14) holds, and $\xi(0) = \xi_*$ when the opposite inequality holds. At first glance, it might seem that if the pool of potential immigrants is large and, on average, relatively poor compared to natives, we would then expect (14) to hold readily—*i.e.*, the second-period capital-labor ratio would be minimized when all potential immigrants are admitted. However, if the period length is long (so that the discount factor β is correspondingly small) $[\alpha\beta/(1+\alpha\beta)]\bar{k}$ could be a quite small fraction of average native capital, in which case the opposite inequality might hold. If that is the case, ξ_* is attained at $M = 0$.

A.3 Proof of corollary 1

Since

$$k^c \geq \frac{1+\alpha\beta}{\alpha\beta}\xi_*,$$

if median native capital satisfied $k^m \leq [(1+\alpha\beta)/\alpha\beta]\xi_*$ we could conclude that a majority of natives preferred M^* to any other value of M.

This bound depends on the minimized capital-labor ratio ξ_*, and as Proposition 1 shows, this may correspond to either zero immigration or the maximum allowable, $M_{\max}$, depending on whether $\kappa(M_{\max})/M_{\max}$ is greater or less than $[\alpha\beta/(1+\alpha\beta)]\bar{k}$. When $\kappa(M_{\max})/M_{\max} > [\alpha\beta/(1+\alpha\beta)]\bar{k}$, ξ_* is attained at $M_* = 0$, and things turn out to be particularly simple. In that case, $\xi_* = \xi(0) = [\alpha\beta/(1+\alpha\beta)]\bar{k}$, so that the critical level of capital, k^c, obeys

$$k^c \geq \frac{1+\alpha\beta}{\alpha\beta}\xi_* = \bar{k}$$

—*i.e.*, the critical level of capital, below which everyone prefers M^*, is at least as big as the *average* level of native capital. Thus, in this case everyone from the mean on down prefers M^*. If, as seems a natural assumption, the distribution of native initial capital is right-skewed, with $k^m < \bar{k}$, it is immediate that a majority of natives share M^* as a unique preferred policy.

When $\xi_* = \xi(M_{\max})$, things are more complicated. In that case—recalling the expression for $\xi(M)$—the lower bound on k^c is given by:

$$\frac{1+\alpha\beta}{\alpha\beta}\xi_* = \left(\frac{1+\alpha\beta}{\alpha\beta}\right)\left(\frac{\alpha\beta\bar{k} + \alpha(1+\beta)\kappa(M_{\max})}{1+\alpha\beta+\alpha(1+\beta)M_{\max}}\right).$$

If the median native's capital holding were less than or equal to this quantity, we could again conclude that a majority of natives shared M^* as a common preferred point.

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Figure 1: Immigration decision rule

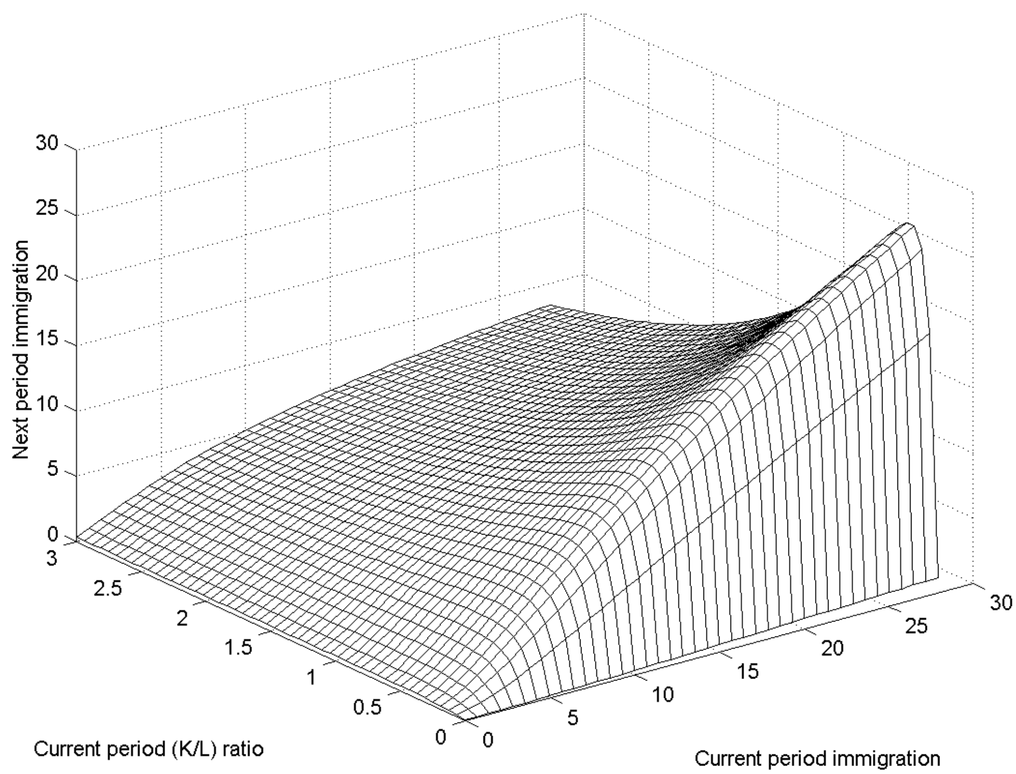

Figure 2: Capital-labor ratio evolution

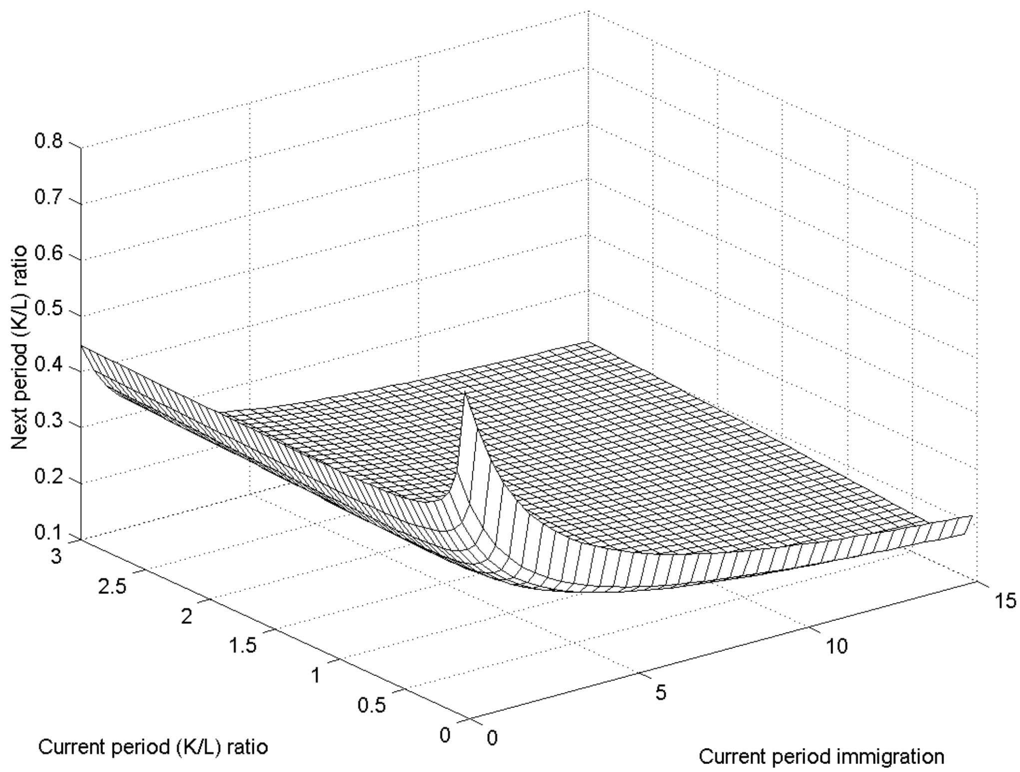

Figure 3: Multiple steady states

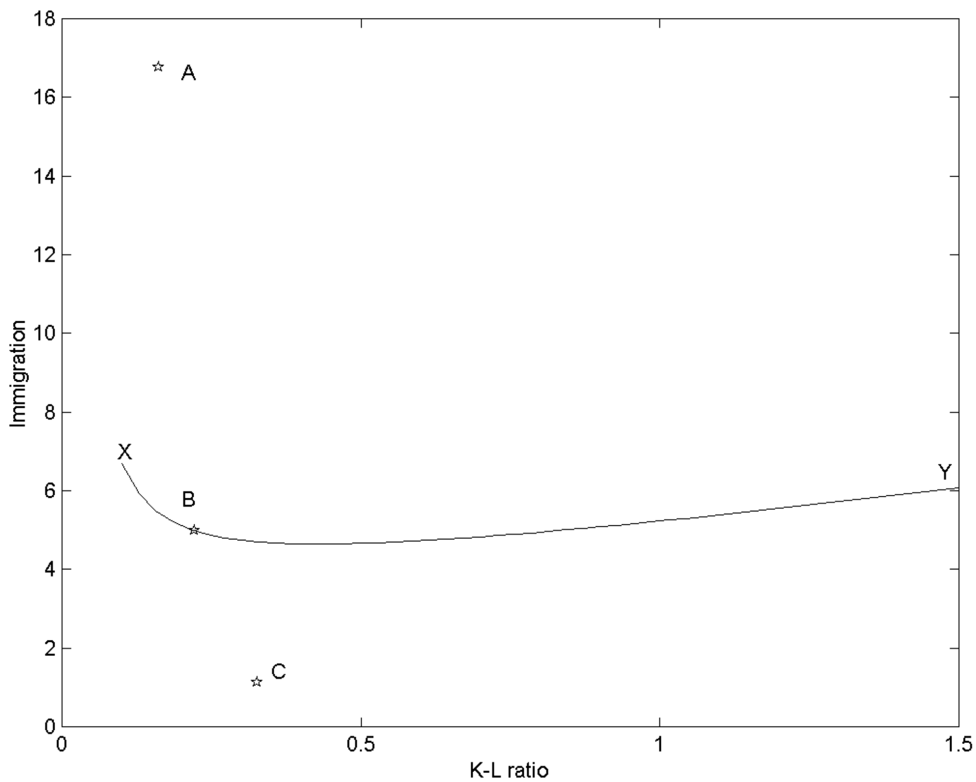

Figure 4: Multiple steady states, one-dimensional representation

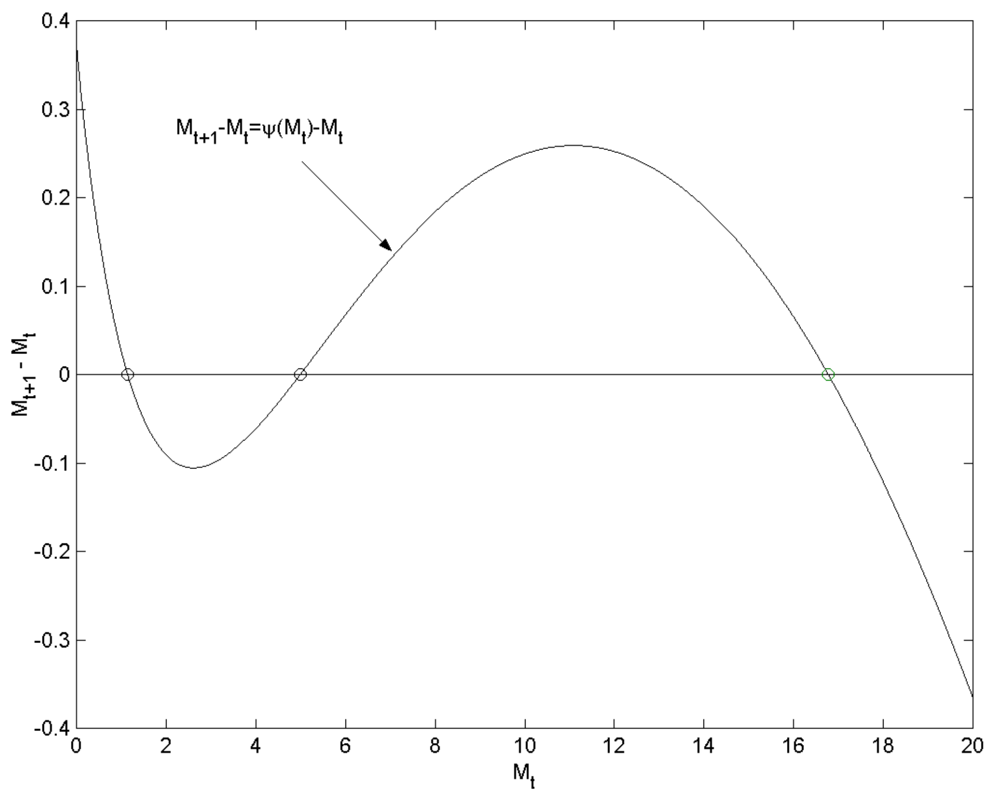

Figure 5: Regions of unique and multiple steady states

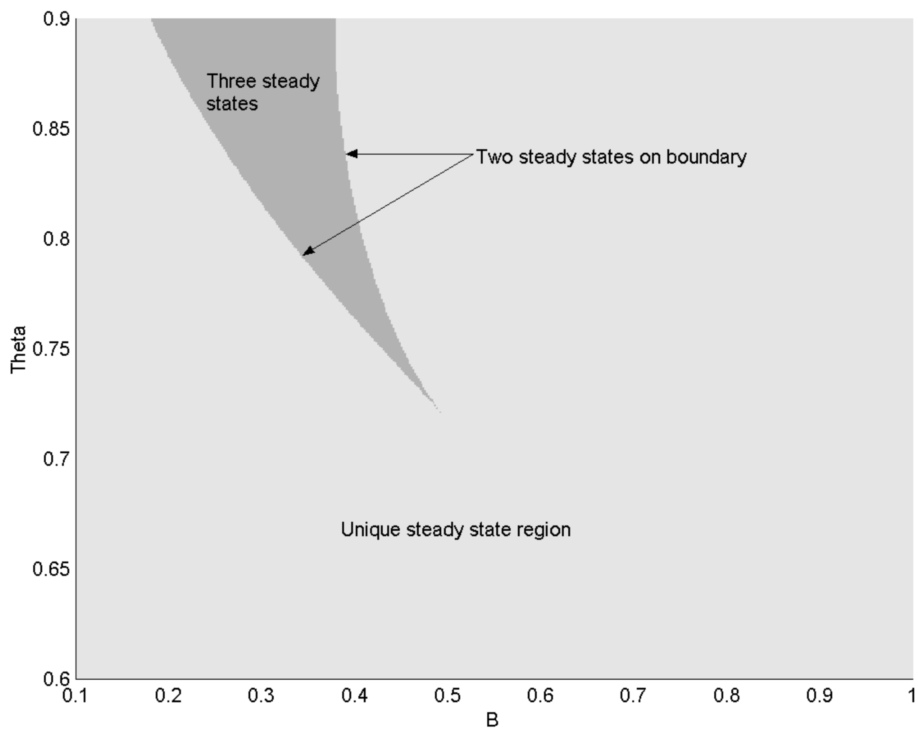

Figure 6: Parameters for multiple equilibria ($M_{\max} = 10$)

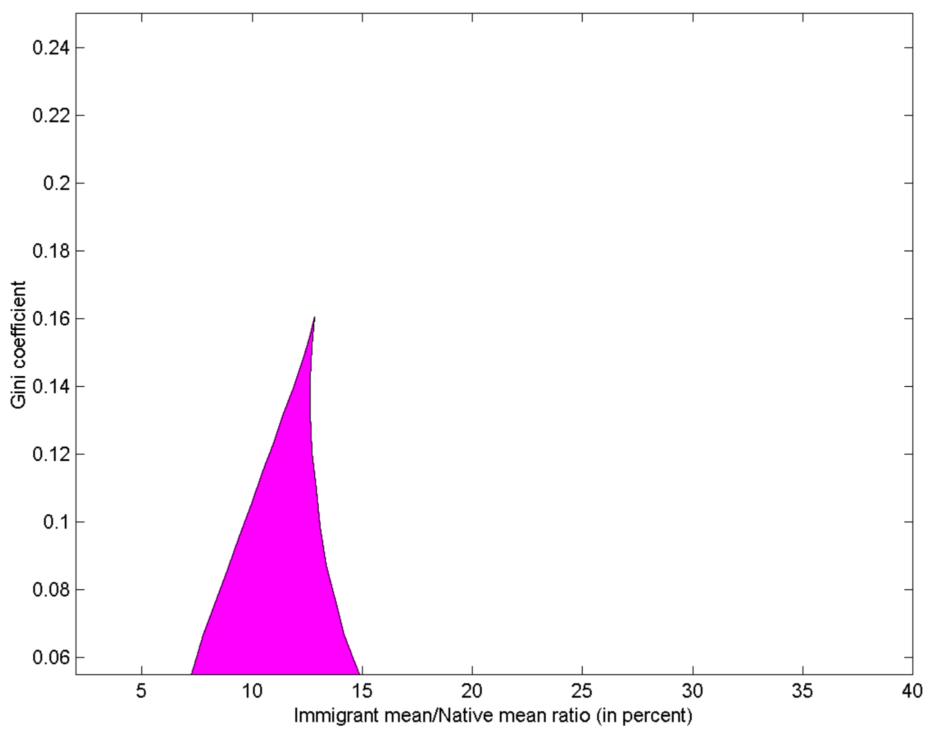

Figure 7: Effect of an increase in the technological growth rate (γ)