Low Frequency Movements in Stock Prices: A State Space Decomposition

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Abstract

Previous analyses have concluded that expectations of future excess stock returns rather than future real dividend growth or real interest rates are responsible for most of the volatility in stock prices. In this paper, we employ a state-space model to model the dynamics of the log price-dividend ratio along with long-term and short term interest rates, real dividend growth, and inflation. The advantage of the state space approach is that we can parsimoniously model the low frequency movements present in the data. We find that if one allows permanent changes, even though very small, in real dividend growth, real interest rates, inflation but not excess stock returns then expectations of real dividend growth and real interest rates become significant contributors to fluctuations in stock prices. However, we also show that stock price decompositions are very sensitive to assumptions about which unobserved market fundamentals have a permanent component. When we allow excess stock returns to have a permanent component but not real dividend growth, then excess stock returns becomes an important contributor to stock price movements while real dividend growth is not. Unfortunately, the data is not particularly informative about which of these alternative models is more likely.

Low Frequency Movements in Stock Prices: A State Space Decomposition

1. Introduction

Stock prices reached historically high levels in the late 1990s and early 2000s. Not only were stock prices remarkably high during this period, but these high prices have persisted for nearly a decade. One explanation that has been advanced is that investors expect relatively high dividend or earnings growth in the future. For example, the so-called New Economy with its revolution in information technology and higher labor productivity growth has been invoked to explain historically high stock prices (see Greenwood and Yorukoglu (1997), Greenwood and Jovanovic (1998, 1999), Browne (1999), and Hobijn and Jovanovic (2000)). Alternatively, others have argued that a decline in the rate at which investors discount expected future real dividends may have caused the dramatic increase in stock prices.[1] For example, Siegel (1999) has suggested that a decline in transaction costs and the availability of low-cost index funds, which has decreased the cost of holding highly diversified portfolios. Heaton and Lucas (1999) in turn have argued that increased diversification has resulted in a substantial decline in the equity premium.[2]

One can place the debate about stock prices in the 1990s within the larger context of the longstanding debate about sources of stock market volatility. Stock price valuation models (such as Gordon (1962)) provide a concise way to think about the factors that affect the fundamental value of stock prices. With stock prices equal to the present discounted value of expected future real dividends, stock prices increase when either expected future real dividend growth increases or when the expected future real discount rate falls. Most of the existing literature has assigned a relatively small contribution to real dividend growth. For example, Shiller (1981) and LeRoy and Porter (1981) argue that the observed dividend series is too smooth to justify the observed volatility of stock returns. More recent studies such as Campbell and Shiller (1988, 1989), Campbell (1991), Shiller and Beltratti (1992), Cochrane (1992), and Campbell and Ammer

(1993) decompose the variance of stock returns into contributions of real dividend growth and other factors. In particular, Cochrane (1992) and Campbell and Ammer (1993) break stock price movements (or more precisely stock returns) into contributions of dividend growth, real interest rates, and excess stock returns. They argue that most of the variability in stock returns is due to innovations in excess returns and not dividend growth or real interest rates.

Much of the above mentioned literature employs a vector autoregression (VAR) framework in order to estimate expectations of future market fundamentals. In this paper, we employ an alternative approach to decomposing stock price movements. We estimate the unobserved expectations of market fundamentals with a state-space model. One attraction of the state space framework is that it allows for a parsimonious specification of low frequency movements in market fundamentals; a VAR estimated in levels may have difficulty capturing low frequency movements in small samples. It is these low frequency movements that are most important for the decomposition of stock price movements.

Employing a state space model also forces us to confront the problem of identification of long-run expectations of market fundamentals. A key finding of this paper is that decompositions of stock price movements are very sensitive to what assumptions one makes about the presence of permanent changes in either real dividend growth or excess stock returns. When we model real dividend growth as containing both a permanent and transitory component but only allow excess stock returns to have a transitory component, real dividend growth explains more of the movement in stock prices than does excess stock returns. Our results are reminiscent of Barsky and DeLong (1993), who argue that actual stock price movements could be rationalized by permanent changes in dividend growth, only that our framework is more general and allows for other factors, in addition to dividend growth, to affect stock prices. When we reverse this assumption so that the excess stock returns is allowed to have a permanent and temporary component and real dividend growth is modeled with no permanent component, then it is excess stock returns that explains more of the movements in stock prices. Regardless, of which model is considered, the contribution of future real interest rates is substantial.

The remainder of this paper is organized as follows. In section 2, we review the log-linear approximation for stock prices that has been featured in many of the recent analysis of stock price volatility. This approximation provides a tractable way of writing current stock prices as a linear function of expectations of future market fundamentals. In section 3, we present a dynamic common factor model used to specify and later estimate the evolution of these market fundamentals. In section 4, we report empirical evidence suggesting that three permanent components are required in our state-space model in order to explain the long-run movements of the data. In section 5, estimation results, obtained from our state-space model in which we allow for permanent components in the short-term real interest rate, inflation, and real dividend growth, are presented. In section 6, we use the state space model to decompose movements in stock prices and interest rates in terms of movements in their market fundamentals. In section 7, we report results from an alternative model in which a permanent component is allowed in excess stock returns but not in real dividend growth. In section 8, we ask whether our specification of allowing for permanent components in real dividend growth or excess stock returns is plausible on statistical and economic grounds. We also speculate on why our results differ from much of the previous literature. Section 9 provides a summary and conclusion.

2. Log -linear approximation for log Price-Dividend Ratio

We start with the same log-linear approximation employed in much of the previous literature. Using the accounting identity and the definition of (real) returns yields:

$$(1) \qquad 1 = \frac{1}{R^{e}_{t+1}}\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right)$$

where R^{e}_{t+1} is the gross real return on equity, P_t is the real price of equity at the end of period t and D_t is the real dividend payment during period t. Rearranging yields

(2) $$\frac{P_t}{D_t} = \frac{1}{R^e_{t+1}} \frac{D_{t+1}}{D_t}\left(1 + \frac{P_{t+1}}{D_{t+1}}\right).$$

Using the familiar log-linear approximation employed by Campbell and Shiller (1988, 1989) we can rewrite equation (2) in terms of logarithms:

(3) $$p_t \approx d_{t+1} - r^e_{t+1} + \rho\, p_{t+1} + k,$$

where $p_t = \log(P_t/D_t)$, $d_{t+1} = \log(D_{t+1}/D_t)$, $r^e_{t+1} = \log(R^e_{t+1})$, $\rho = \exp(\log(P/D)^{avg})/(1+\exp(\log(P/D)^{avg}))$ with $\log(P/D)^{avg}$ being the average of the log price-dividend ratio over the sample and k is a constant.[3] We will find it convenient to break up log real gross returns on equity into real returns on short term bonds, r_t, and excess returns, e_t,

(4) $$r^e_t = r_t + e_t.$$

Ruling out explosive behavior for the log price-dividend and taking expectations and recursively substituting we obtain:

(5) $$p_t = k_e + \sum_{i=0}^{\infty} \rho^i\, E_t\left(d_{t+i+1} - r_{t+i+1} - e_{t+i+1}\right).$$

Thus, the log price-dividend ratio is a weighted average of expected future real dividend growth, real interest rates, and excess returns.[4, 5]

Equation (5) provides a nice way to think about alternative explanations of stock price movements. Stock price movements driven by movements of expectations about future profits or earnings would be reflected in expectations about future real dividend growth.[6] On the other hand, stock price movements due to changes in savings behavior, say due to demographic changes, will be reflected in changes in the real interest rate while movements that are the result of changes in the equity premium would be reflected in changes in expected future excess returns. Note that expected future inflation does not have a direct effect on the price-dividend ratio. Only if inflation is negatively correlated with real dividend growth (as might be the case when nominal dividends only partially respond to inflation), real interest rates, or excess returns, will a reduction in inflation expectations be associated with an increase in the price-dividend ratio.

We can make a similar decomposition for bonds.[7] The nominal return for a one-period bond is

$$i_{1,t} = E_t r_{t+1} + E_t \pi_{t+1} \tag{6}$$

where $E_t r_{t+1}$ is the ex-ante real interest rate on a one-period bond and $E_t \pi_{t+1}$ is expected inflation. For an n-period bond, we can write the yield to maturity as:

$$i_{n,t} = \frac{1}{n}\left(\sum_{i=1}^{n} E_t r_{t+i} + E_t \pi_{t+i} + E_t \tau_{n+1-i,t+i} \right), \tag{7}$$

where $\tau_{n,t}$ is the excess (one-period) return in time period t of an n-period bond over a one-period bond (with $\tau_{1,t} \equiv 0$). Thus, long-term interest rates are just the average of current and future short term rates plus a term premium.

3. A Dynamic Common Factor Model for Stock Prices and Interest Rates

The main difficulty for stock-price and interest rate decompositions is that asset prices depend on expectations of future market fundamentals and these are not observed. Much of the previous literature (Campbell (1991), Campbell and Ammer (1993), Lee (1998) among others) attempts to calculate these expectations by estimating a time series model, typically a VAR, and use that model to construct expectations. Most of this literature assumed that market fundamentals were stationary. Our approach in this paper will also be to estimate a time series model, but our model will take the form of a state space model. The advantage of employing a state space model is that the state space model will allow parsimonious specification of low frequency components in market fundamentals. Furthermore, as we show below, the state variables in the model lend themselves quite nicely to economic interpretation in terms of expectations about the long-run values of the market fundamentals. We estimate a dynamic common factor model from which we infer what these expectations are by exploiting common factor restrictions implied by the asset pricing equations.[8]

For each of the market fundamentals, expected real dividend growth, real interest rate, expected inflation, and excess returns on stocks and bonds, we assume an unobserved components model. We also allow for the possibility that these variables may be characterized by permanent and temporary components. For example, consider the following unobserved components model for real dividend growth:

$$d_t = d_t^p + d_t^a + w_t^d, \tag{8}$$

$$d_t^p = d_{t-1}^p + v_t^{d,p}, \tag{9}$$

$$d_t^a = \sum_{j=1}^{k} \theta_j^d d_{t-j}^a + v_t^{d,a}, \tag{10}$$

where d_t^p can be interpreted as the market's expectation of long-run real dividend growth. This long-run dividend growth variable is not observed directly by the econometrician, but as we show below can be inferred from actual real dividend growth and from the log price-dividend ratio. The temporary component, d_t^a, represents the adjustment of actual real dividend growth to long-run real dividend growth. We allow innovations in the temporary component, $v_t^{d,a}$, to be (negatively) correlated with innovations in long-run dividend growth, $v_t^{d,p}$, so that an increase in expected long-run real dividend growth need not be reflected in a one-for-one increase in current actual real dividend growth. As a result, one can think of d_t^p as information that the market has about future real dividend growth that is not necessarily reflected in current real dividend growth. However, as we show below, this information is reflected in current stock prices. The term w_t^d represents random measurement error which we will assume is uncorrelated with any other variables in the model. We can similarly write inflation as a function of a permanent, temporary, and noise component. For the real interest rate, excess stock return, and excess bond return components, we will infer them indirectly from the log price-dividend ratio, long-term interest rates, and short-term interest rates.

To illustrate how expectations about future real dividend growth, real interest rates, etc can be inferred from the model, we can write the model in state space form as:

(11) $$S_t = FS_{t-1} + V_t$$

where

$$S_t = (d_t^p, r_t^p, \pi_t^p, e_t^p, \tau_t^p, d_t^a, r_t^a, \pi_t^a, e_t^a, \tau_t^a, d_{t-1}^a, r_{t-1}^a, \pi_{t-1}^a, e_{t-1}^a, \tau_{t-1}^a, \ldots, \\ d_{t-(k-1)}^a, r_{t-(k-1)}^a, \pi_{t-(k-1)}^a e_{t-(k-1)}^a, \tau_{t-(k-1)}^a)'$$

$$V_t = (v_t^{d,p}, v_t^{r,p}, v_t^{\pi,p}, v_t^{e,p}, v_t^{\tau,a}, v_t^{d,a}, v_t^{r,a}, v_t^{\pi,a}, v_t^{e,a}, v_t^{\tau,a}, 0_{1x5(k-1)})'$$

with $Q = E(v_t v_t')$ where $v_t = (v_t^{d,p}, v_t^{r,p}, v_t^{\pi,p}, v_t^{e,p}, v_t^{\tau,p}, v_t^{d,a}, v_t^{r,a}, v_t^{\pi,a}, v_t^{e,a} v_t^{\tau,a})$.

The Appendix describes the F matrix in the state equation in detail. The state equation (11) describes the evolution of the observed market fundamental components.

Using equation (5) we can write the log price-dividend ratio in terms of the state space model:

(12) $$p_t = k_e + \sum_{i=1}^{\infty} \rho^{i-1} H_1 (E_t \; S_{t+i})$$

where $H_1 = (1,-1,0,-1,0,1,-1,0,-1,0,0_{1x5(k-1)})$. Using the state space model to evaluate the expectations in (12), we obtain

(13) $$p_t = H_1 (I - \rho F)^{-1} F \; S_t.$$

Thus, the log price-dividend ratio can be written as a linear function of the unobserved state vector, S_t. For the case where the temporary components follow a first-order autoregressive processes, equation (13) simplifies to

(14) $$p_t = \frac{1}{1-\rho}(d_t^p - r_t^p - e_t^p) + \frac{\theta^d}{1-\rho\theta^d} d_t^a - \frac{\theta^r}{1-\rho\theta^r} r_t^a - \frac{\theta^e}{1-\rho\theta^e} e_t^a.$$

Note that the coefficients on the market's expectations of long-run real dividend growth (d_t^p), real interest rate (r_t^p), and excess returns (e_t^p) are larger than the coefficients on the temporary components, d_t^a, r_t^a and e_t^a respectively. This allows for a situation in which if d_t^p rises and d_t^a falls by the same amount, the log price-dividend ratio rises even though current real dividend growth is unchanged.

We can similarly write the yield to maturity on an n-period bond as a function of the current state vector, S_t:

(15) $$i_t^n = \frac{1}{n} H_2 (I - F^n)(I - F)^{-1} F S_t + \frac{1}{n} H_\tau (I - F^{n-1})(I - F)^{-1} F S_t,$$

where $H_2 = (0,1,1,0,0,0,1,1,0,0,0_{1x5(k-1)})$ and $H_\tau = (0,0,0,0,1,0,0,0,0,1,0_{1x5(k-1)})$. Again, if the temporary components are first-order autoregressions, then equation (15) simplifies to

(16) $$i_t^n = \pi_t^p + r_t^p + \frac{(n-1)}{n}\tau_t^p +$$

$$\left(\frac{1}{n}\right)\frac{(1-\theta^{\pi\, n})}{1-\theta^{\pi}}\theta^{\pi}\pi_t^a + \left(\frac{1}{n}\right)\frac{(1-\theta^{r\, n})}{1-\theta^{r}}\theta^{r} r_t^{\,a} + \left(\frac{1}{n}\right)\frac{(1-\theta^{\tau\, n-1})}{1-\theta^{\tau}}\theta^{\tau}\tau_t^a$$

The greater the maturity date of the bond, the greater relative weight is placed on the factors describing the market's long-run expectations of inflation, real interest rate, and excess bond returns. Note that the excess bond return factor is not present in the one-period bond.

Thus, we can relate observed asset prices, real dividend growth, and inflation to the unobserved states by

(17) $$Y_t = HS_t + W_t,$$

where $Y_t = (p_t^{obs}, i_t^{n\,obs}, i_t^{1\,obs}, d_t^{obs}, \pi_t^{obs})'$, and $W_t \sim N(0,R)$ with R is a diagonal matrix. The H matrix depends on the number and the nature of states describing the market fundamentals. Appendix A presents the H matrix for each of the models we examine below. The key insight from equations (14) and (16) above, is that the state space model imposes restrictions across state equations and the observation equations. These restrictions enable us to interpret the unobserved state variables in terms of long-run expectations of market fundamentals.[9] Thus, for example, movements in the unobserved real (one-period) interest rate factor will affect the log(P/D) ratio, the short-term interest rate and the long-term interest rate. Movements in the unobserved real dividend growth factor will affect log(P/D) and observed real dividend growth.

4. Specifying the number of permanent components in the state-space model.

We will examine a five variable system that includes log price-dividend ratio, long and short-term nominal interest rates, real dividend growth, and inflation. Our data is quarterly and runs from 1953:2 to 1999:1. The price-dividend ratio is the S&P 500 composite stock price index for the last month of each quarter divided by nominal quarterly dividend flow for the SP500 composite index.[10] In our empirical analysis we will consider the 10-year Treasury bond rate (i^n, n = 40 for quarterly data), and the 3-month Treasury bill yield (i^1) as our interest rate series. We include the short-term interest rate in the analysis to help us better distinguish between the long-run and transitory components of the real interest rate and inflation factors (note the factor loadings on r_t^a and π_t^a in equation (16) depend on n). Inflation is calculated as the growth in the Consumer Price Index over the quarter. Real dividend growth is nominal dividend growth less CPI inflation.

Figure 1 plots the log price-dividend ratio and the log price-earnings ratio for the post-war period up until early 1999. Evident in Figure 1 is that both the log price-dividend ratio and the log price-earnings ratio are characterized by long swings or substantial low frequency movements. In fact, standard Dickey-Fuller unit root tests (results reported in Table 1) find that the unit root null cannot be rejected. The variance ratios for the log price-dividend ratio and the log price-earnings ratio at a horizon of forty quarters are quite close to one which is the implied variance ratio for a random walk (see also Table 1). Thus, there is a sizeable low frequency component in the log price-dividend ratio and log price-earnings ratio. In subsequent analysis we will focus our attention on the log price-dividend ratio, noting that results are qualitatively the same when we replace the log price-dividend ratio with the log price-earnings ratio; this is also true when we replace real dividend growth with real earnings growth. Within the context of our model, these persistent movements in the price-dividend ratio requires persistent movements in market fundamentals: real dividend growth, real interest rates, excess stock returns, or combination of these.

Table 1 also contains unit root tests and variance ratios for long and short term interest rates, real dividend growth, and inflation. In addition, Table 1 contains unit root tests and variance ratios for ex-post real interest rate (short-term interest rate at t minus inflation at t+1), the spread between long and short-term interest rates, and actual excess stock returns. Table 1 indicates that nominal interest rates, inflation, and real interest rates fail to reject a unit root while, on the other hand, the interest rate spread, real dividend growth, and excess stock returns are much less persistent and, thus, reject the unit root null hypothesis. We also examine the long-run properties of log price-dividend ratio, long and short term interest rates, real dividend growth, and inflation when taken together as a system. Table 2 presents the Johansen (1991) test for the number of cointegrating vectors in a system that contains log price-dividend ratio, long-term interest rate, short-term interest rate, real dividend growth, and inflation. The $\lambda - \max$ test rejects the null of fewer than two cointegrating vectors at the 5% level, yet fails to reject the null of fewer than three cointegrating vectors. The Trace test rejects the null of fewer than two cointegrating vectors at the 5% level and fails to reject the null of fewer than three cointegrating vectors. Together, these results suggest the presence of two cointegrating vectors or alternatively three stochastic trends in our system. Because one can think of real dividend growth, excess stock returns, and the interest rate spread as describing cointegrating vectors in a systems approach, the single variable and system results are not consistent.[11] It is not our objective to sort out the finite-sample size and power properties of single equation versus system tests for the number of unit roots. Rather, we note that there is evidence of substantial persistence in our data, and that one can plausibly and parsimoniously capture the persistence by including three permanent components in our state space model.

Recall that the market fundamentals for the log price-dividend consist of expectations of future real dividend growth, real interest rate, and excess stock returns while for nominal interest rates they consist of the real interest rate, inflation, and a term premium. Our state space approach requires us to specify unobserved components models for each of these market fundamentals. Given the persistence in log price-dividend ratio, to parsimoniously capture low frequency movements in log price dividend ratio our model must contain at least one permanent component for dividend growth, real interest rates, and excess stock returns. Of these, only the real interest rate appears to have substantial persistence. Nonetheless, it is difficult to believe, given the behavior of actual real interest rates, that real interest rates alone could drive most of the low frequency movements in stock prices.[12] Thus, we start with a benchmark model in which there are permanent (and temporary) components in real interest rate, inflation, and in real dividend growth but only temporary components for excess stock returns and the term premium.

One may question whether real dividend growth (or the other market fundamentals for that matter) contains a permanent component since actual real dividend growth over the sample displays very little persistence. However, because we put no restrictions on the variance of innovations in d_t^p relative to innovations in d_t^a it is possible for the scale of d_t^p to be arbitrarily small (but not zero). In section 7, we consider an alternative model in which there is a permanent component in excess stock returns but not in real dividend growth.

5. Estimated state space model with a permanent real dividend growth component

In estimating our state space models, we start by estimating the model using the EM algorithm for 500 iterations and then switch to standard maximum likelihood to obtain the final estimates.[13] The EM algorithm is more robust to initial conditions than standard maximum likelihood but is notoriously slow to converge. The Kalman filter is used to provide an estimate of the unobserved state vector, $S_{t|t-1}$, and its covariance matrix, $P_{t|t-1}$, for data up through time t-1 and a given parameter vector.[14] We set the number of autoregressive terms in the temporary components equal to two.[15] Because one might expect the innovations in the real dividend growth factor, the real interest factor, and the inflation factor to be correlated, we do not put any restrictions on the variance/covariance matrix of innovations, Q, in our empirical analysis. We demean the data, so that we do not have to include constant terms in the state space model. Finally, we set the value of ρ equal to .99078.[16]

Table 3 presents estimates of the autoregressive parameters, θ, and the estimates of the R and Q matrices (with the implied correlation coefficients displayed above the diagonal) for the model that includes permanent components for real dividend growth, real interest rate, and inflation and only temporary components for excess stock returns and the term premium. From the covariance structure of Q, we observe that innovations in d_t^p and r_t^p are highly positively correlated, suggesting that shocks that increase the market's expectations of future real dividend growth also coincide with increases in the long-run real interest rate. This correlation is consistent with the notion that a permanent increase in real dividend growth results in an increase in future income relative to income today which, in turn, brings about an increase in the real interest rate as households try to borrow to smooth consumption. Also note that the correlation between innovations in d_t^p and π_t^p are highly negative. This implies that increases in expectations of long-run inflation typically coincide with decreases in expectations of long-run real dividend growth. This correlation is consistent with the results of Sharpe (1999) who found increases in inflationary expectations lowered expectations of real earnings growth. Finally, note that the correlation between d_t^p and d_t^a is also negative. As we pointed out above, a negative correlation would be consistent with a partial adjustment model for real dividend growth in which current real dividend growth only partially respond to innovations in long-run expectations of real dividend growth. Innovations in the excess stock returns factor, $v_t^{e,a}$, are positively correlated with innovations to long-run dividend growth but negatively correlated with innovations in temporary dividend growth. They are also positively correlated with the permanent component of real interest rates.

Table 3 also presents R^2 for the one-step-ahead forecasts implied by the Kalman filter, $Y_{t|t-1} = HS_{t|t-1}$, and the Q-statistic for serial correlation for one step ahead forecast errors, $W_{t|t-1} = Y_t - HS_{t|t-1}$. For comparison, we also report the R^2 and Q-statistics for a low order vector autoregression (VAR(2)) similar to those used in the VAR decomposition literature. The R^2 for the five equations are quite good given that there are in total only 51 free parameters (10 excluding the parameters in R and Q variance/covariance matrices). In contrast, the unrestricted VAR(2) has a total of 65 parameters of which only 15 are parameters in the variance/covariance matrix. While the VAR(2) has a lower Akaike Information Criterion (1572.2 vs 1587.7), the state-space model has a lower Schwartz-Bayesian Criterion (1751.7 vs 1781.2) suggesting that criteria which place relatively greater emphasis on parsimony will tend to prefer the state-space model over the VAR. Furthermore, the state space model's one-step-ahead forecast errors appear to display less serial correlation than does the VAR's. Thus, the estimated state space model appears to capture the true data generating process of the five observed variables reasonably well.

6. Stock price decomposition for model with permanent dividend growth component.

6.1 State Space Decomposition

Once the state space model is estimated, we use the estimated factors to assess their contribution to stock price movements. Denote $S_{t|t}$ as the model's estimate of the state vector given information up to time t. The contributions of the jth factor to the values of the observation variables are given as

$$Y_t(j) = HS_{t|t}(j), \quad (18)$$

where $S_{t|t}(j)$ zeros out the elements of $S_{t|t}$ except for those associated with the jth factor.

The top panel of Figure 2 displays the estimated contribution of expected future real dividend growth and the expected future real interest rate to movements in the log(P/D) for the model allowing for permanent components in real interest rate, inflation and real dividend growth and allowing for only temporary components in excess stock returns and term premium. The striking feature about Figure 2 is that both the contribution of expected future real dividend growth and the contribution of the expected future real interest rate display substantial volatility relative to the actual log price-dividend ratio. The bottom panel of Figure 2 displays the actual log price-dividend ratio and the estimated total contribution of the expected future excess stock returns. Compared to the contributions of real dividend growth or the real interest rate individually, the contribution of excess return fluctuations to stock price fluctuations are relatively small.

Figure 2 suggest some interesting interpretations of recent episodes of large price-dividend movements. Although on net, in the 1970s, the log price-dividend ratio fell, the large swings in the contributions of expected real dividend growth and the real interest rate factor seen in the mid and late 1970s largely offset one another. Because of the positive covariance between the real interest rate and future real dividend growth, the actual log price-dividend ratio is not as volatile as the contribution of each factor individually. Another interesting feature is that starting in the early 1980s the contribution of expected future real dividend growth has jumped up and while fluctuating has remained relatively high. But during the 1980s and early 1990 the positive contribution from real dividend growth was largely offset by a negative contribution the real interest rate as during this period the long-run real interest was relatively high. However, the long-run real interest rate drifted back to near its sample average (around zero with demeaned data) by the mid-1990s; thus, the increase in stock prices that occurred since the mid-1990s is in this model attributed almost entirely to an increase in expected future dividend growth.

While our model does not say something specific about the source of increased optimism, our model does suggest that changes in expectations about inflation may have had an important role in the increased optimism seen in the early 1980s. This period saw a significant decline in inflation (the Volcker/Reagan Disinflation). CPI inflation fell from over 10 percent during the late 1970s-early 1980s to under 5 percent in 1983. Recall from Table 3 that innovations in d_t^p and π_t^p are negatively correlated; thus, it is not surprising that we estimate the long-run expected real dividend growth to increase around the same time that inflation fell dramatically. The negative correlations between long-run real dividend growth and long-run inflation and the coincident timing of the initial increase in expectations of long-run real dividend growth with the disinflation of the early 1980s are consistent with the empirical findings of Sharpe (1999) who found that inflation and earnings expectations are negatively related.

To help understand the behavior of the real interest rate, the top panel of Figure 3 displays the total contribution of the real interest rate factor and expected inflation to long-term interest rates while the bottom panel displays the contribution of the term premium. From Figure 3, one observes that prior to 1973, much of the movement in long-term interest rates can be attributed to expected inflation rather than to the real interest rate. After 1982, because actual and expected long-run inflation have been relatively steady, much of the movement in long-term rates are attributable to movements in the real interest rate and the term premium.

During the mid and late 1970s, our model implies substantial volatility in both the contribution of the real interest rate and expected inflation. The increases in inflation, first during the mid 1970s and again in the late 1970s, are inferred by our model to be largely permanent and, hence, have a large impact on expectations of future inflation. Expected inflation rose during these periods but without a concomitant increase in nominal interest rates. Consequently, the latent real interest rate factor must fall to offset the increase in expected inflation. The negative relationship between the real interest rate and inflation is also consistent with the real interest rate rising during the disinflation of the 1980s. However, after the disinflation of the 1980s was complete, the real interest rate appears to be driven more by autonomous changes not related to the behavior of inflation. Turning to the term premium, we find that the contribution of the term premium factor in the long-term interest rate equation is quite volatile. The dips in the term premium factor occur primarily around business cycles and may reflect the fact that the yield curve has historically been inverted just before or during the early stages of a recession.

6.2 Forecast Variance Decomposition.

We also assess the contributions to asset price fluctuations by examining forecast variance decompositions. With in the context of our state space model, the variance of k-horizon forecast errors is given by

$$\mathrm{var}(Y_{t+k} - E_t Y_{t+k}) \; = \; H\,\mathrm{var}(S_{t+k} - E_t S_{t+k})H' \; + \; R.$$

Thus, we can decompose the k-horizon forecast variance of log price-dividend into portions contributed by variances and covariances of the unobserved state vectors.[17] Innovations in the state vector are not orthogonal and, like the decompositions of Campbell and Ammer (1993), the covariance terms of the innovation matrix, Q, will affect the variance decomposition.

Table 4 presents a decomposition of forecast variance for a number of variables for forecast horizon of one quarter. For state variables with permanent and temporary components, we combine these into the contribution of a single factor. Because of the strong covariance between factors, it is difficult to disentangle the contribution of these factors individually; nonetheless, our results suggest a larger contribution to the variance of stock prices (and real stock returns) for real dividend growth and the real interest rate than in previous decompositions, such as Campbell and Ammer (1993).[18] [19] On the other hand, the contribution of the covariance of dividend growth and real interest rate innovations, the covariance of dividend growth and excess return innovations, and the covariance of real interest rate and excess returns are qualitatively similar (have the same sign) to those reported in Campbell and Ammer. For the long-term interest rate, the real interest rate, inflation, and term premium factors, are all important contributors to the variability of long-term interest rates, with the inflation variance being the single most important contributor. Again, the covariance terms have contributions qualitatively similar to those reported for long-bond returns in Campbell and Ammer.

For the short-term interest rate, the real interest rate and inflation factors are roughly equally important with the covariance between inflation and real interest rate having a negative contribution to the forecast variance of short-term interest rates. Our decomposition of nominal interest rate spread suggests that the real interest rate factor and term premium factor are relatively important but that the inflation factor is not. This is in contrast to Campbell and Ammer who found that inflation had a relatively large contribution while term premium did not. In our framework, changes in future expected inflation are persistent and, hence, affect long and short- term interest rates in a similar way; thus, inflation variability does not have a large effect on the interest rate spread variability.

Finally, in our model, real stock and real bond returns are positively correlated. The covariance of real dividend growth and real interest rate innovations and the covariance of real dividend growth and inflation innovations are the largest contributors to the the covariance between stock returns and long bond returns, albeit of opposite signs. Real interest rate innovations in addition to their covariance with dividend and inflation innovations also have strong direct impact on the stock/bond return covariance as the real interest rate factor shows up in both real stock and real bond returns. Summarizing, while our decompositions are qualitatively similar to those in Campbell and Ammer, quantitatively we generally attribute a greater role to real dividend growth and real interest rate innovations and a smaller role to excess return innovations than they do.

7. Model with a permanent excess stock return components but no permanent real dividend component.

As we suggested above, the results in section 6 are in contrast to much of the previous literature. The key identifying assumption in the model presented in section 6 is that long-run real dividend growth has a permanent component and excess stock returns does not. Suppose we reverse that assumption and allow excess stock returns to have a permanent component while real dividend growth does not.

Table 5 presents the parameter estimates and model diagnostics for the model with a permanent excess return component but no permanent real dividend growth component. Comparing Tables 3 and 5, the autoregressive and variance parameters of the temporary components for the two models are very similar. The covariance structure of the permanent excess stock return component displayed in Table 5 is strikingly similar to the covariance structure of the permanent real dividend growth component in Table 3 once one considers that excess returns have the opposite effects on stock prices than does real dividend growth. Furthermore, the model with a permanent excess stock return component yields nearly the same fit as the permanent dividend component model; although, the log-likelihood is slightly lower for the permanent excess stock returns component model than for the permanent dividend growth model.

Figure 4, panels A and B, present the decomposition of stock prices for the model with a permanent excess stock return component. Unlike the model presented in section 6, the contribution of real dividend growth, when only a temporary dividend growth component is included in the model, is very small while the contribution of excess stock returns is quite substantial. In fact, the implied contribution of excess stock returns in Figure 4, panel A, looks nearly identical to the contribution of real dividend growth in Figure 2, panel A. Interestingly, future excess stock returns has a large positive contribution to log price-dividend (that is, future excess stock returns are expected to fall) starting in 1983 and continuing more or less to the end of our sample. Note also that the contribution of the real interest factor to log price-dividend is nearly the same regardless of whether we assume a permanent real dividend growth component or a permanent excess returns component. Likewise, the decompositions of interest rates and inflation are very similar in the model with a permanent excess stock return component as compared to the model with a permanent real dividend component.[20]

Table 6 presents variance decompositions for the model with permanent excess returns component. Comparing Tables 4 and 6, the long and short-term nominal interest rates as well as the interest rate spread variance decompositions shown in Table 6 are essentially the same regardless of whether the model has a permanent excess return component or a permanent real dividend growth component. Not surprisingly, the model with a permanent excess returns component attributes much of the variability of the log price-dividend ratio (and real stock returns) to excess returns and real interest rates and very little to real dividend growth. On the other hand, contributions of the covariances between dividends and real interest rates and between real interest rates and excess returns are qualitatively different from those of the permanent dividend growth model (and Campbell and Ammer). The covariance between real interest rates and excess return contributions is negative in the model with a permanent excess returns component because the long-run components are highly negatively correlated (see Table 5). Note, however, that the total forecast variances and covariances are nearly identical in Table 4 and Table 6, reflecting how similarly the two models fit the data.

In sum, the two competing models fit the data equally well, and the implied stock price decompositions are very similar, except in one model real dividend growth has a significant contribution to stock price movements while in the other excess returns has a significant contribution. By reversing the assumption about whether real dividend growth or excess returns has the permanent component, we can reverse which factor is the more important contributor to movements in log price-dividend. The implication is that there is little in the data to distinguish between a model in which expectations of real dividend growth play an important role in stock market fluctuations and a model in which expectations about future excess returns play a crucial role. In other words, the nature of stock market decomposition depends crucially on what assumptions one makes about the long-run fluctuations in market fundamentals.

8. Discussion

The above results clearly suggest that a permanent component in real dividend growth and/or excess stock returns is important in explaining low frequency movements in stock prices. Much of the previous literature has claimed that excess stock returns are responsible for stock price movements. This stems in part from the fact that actual real dividend growth displays very little persistence. Yet, as we saw above, excess stock returns also shows little persistence and on a statistical basis it is difficult to choose a model with permanent excess stock returns to one with a permanent dividend growth. In this section we ask whether the specification of a permanent component for either real dividend growth or real excess returns is plausible on statistical or economic grounds. Second, we speculate on why our results differ from much of the previous literature.

8.1 Is a permanent component for dividend growth or excess stock returns plausible?

Is the assumption that there is a permanent component for either real dividend growth or excess stock returns plausible on statistical grounds? The top panel of Figure 5 displays actual real dividend growth and the long-run real dividend growth component, $\mathbf{d}_t^{\mathbf{p}}$, and its two standard deviation band from the model with a permanent real dividend growth component while the bottom panel displays actual excess stock returns and the long-run excess stock returns component (and its two standard deviation band) from the model with a permanent excess stock returns component.[21] The movements in the estimated long-run real dividend growth component or the long-run excess stock return component are not outside historical experience and are roughly consistent with the data. For example, our estimate of long-run real dividend growth increased around 1983 and while fluctuating somewhat has remained relatively high; average actual real dividend growth over this period has also been higher than that averaged over the previous part of the sample. Real dividend growth over the full period 1953:2-1999:1 is 1.36%, while over the period 1983:1-1999:1 is 2.20%. Alternatively, for the model that allows for a permanent component in excess stock returns, long-run excess returns is estimated to have fallen around 1983 (see bottom panel of Figure 5). While this is in dramatic contrast to actual excess stock returns over that period, it is exactly what one would expect if long-run excess returns drifted downward.[22] Note that the permanent excess returns model suggests that the long-run equity premium is close to zero at the end of our sample. This is consistent with results of Jagannathan, McGratten, Scherbina (2000) who find that the equity premium over the period 1926-1970 averaged 6.8% and decreased to 0.7 percentage points during the post-1970 period.

One might argue that our specifications in which either real dividend growth or excess returns contains a unit-root is wildly at variance with the data since both series display very little persistence. However, as we noted above, the Johansen test for cointegration suggests three stochastic trends in our five variable system which is consistent with our state space model specification. Second, our model imposes no restrictions on the relative variance of the permanent component of real dividend growth or excess stock returns. Indeed, the variance of the permanent component for either series is estimated to be quite small. Using the estimated variance/covariances, the ratio of the variance of innovations to the permanent component of real dividend growth to the variance of innovations in real dividends growth as a whole is only 0.02 and while the ratio for the model with a permanent excess stock returns component is only 0.001!.[23]

Given that the estimated permanent components of real dividend growth and real excess returns are quite small, our model is perfectly consistent with actual real dividend and excess returns showing very little persistence. The variance ratio implied by our models for either real dividend growth or excess stock returns are well within the confidence bounds for the variance ratio of actual real dividend growth or excess stock returns. Furthermore, it is well known (see Schwert (1987)) that standard Dickey-Fuller critical values can have substantial finite sample size distortions, when the series has a large negative moving average component (i.e. a small permanent component). When we calculate the finite sample size adjusted augmented Dickey-Fuller critical values for data generated by the two alternative state space models, we no longer reject the unit root null at the five percent level for either real dividend growth or excess stock returns.[24]

Thus, the data are not inconsistent with the presence of a small permanent component in real dividend growth and/or a small permanent component in excess stock returns. Yet, it is precisely the permanent component that has the largest impact on stock prices–recall the factor loadings in equation (14). The effect of a permanent shock to real dividend growth or excess stock returns is over 50 times larger than a shock to the temporary component.[25] As a result, a small permanent change in real dividend growth or excess returns has a much larger impact on the price-dividend ratio than a temporary change.

Are permanent changes in either real dividend growth or excess stock returns plausible on economic grounds? Changes in long-run real dividend growth are not likely to be sustained unless the long-run growth rate for the economy as a whole has experienced a similar change. Like real dividend growth, there is little statistical evidence of persistent changes in output growth, consumption growth, or productivity growth in the post-war period. Yet, one cannot entirely rule out the possibility of small changes in long-run growth; witness the discussion of the productivity slowdown during the late 1970s and 1980s and the subsequent resurgence of productivity growth in the late 1990s (see Oliner and Sichel (2000) and Gordon (2000)). As Barsky and DeLong (1993) suggest, it is plausible, in a world with changing technology and policy regimes, that investors may revise their expectations about long-run real dividend growth. As we pointed out above, it only takes a small change in expectations about long-run real dividend growth to have a dramatic affect on stock prices.[26] In fact, the possible increase in optimism about future real dividend growth that we estimated to have occurred in 1983 has been noted by Blanchard and Summers (1984). They argued that what were then considered high stock prices were in part due to optimism about future dividend and earnings growth. The possible reasons they list for this optimism included a general decrease in business taxation, reduction in factor prices, and increased profitability.

There are also several possible reasons for why future long-run excess stock returns or the equity premium might permanently change. Indeed, Blanchard (1993) seemed to have changed his earlier view about stock valuation by arguing that a declining equity premium was responsible for the increase in stock prices. Factors such as changes in investors attitudes towards risk or the changes in the perceptions about the riskiness of stocks versus bonds could lead to a change in the required excess return of stocks over bonds.[27] Similarly, changes in transactions costs can lower the costs investors face in constructing a diversified stock portfolio (see for example, Heaton and Lucas (1999) and Siegel (1999)). These all could change the required return for stocks relative to other assets. In fact, as our analysis is based on returns before transaction costs or taxes, a decline in either of these for stocks relative to other assets would result in a decline in required excess return for stocks (keeping required returns net of transactions costs and taxes constant) and, hence, would show up as a permanent decline in long-run excess stock returns.

Is it possible that the market's and, hence, our implied movements of either long-run real dividend growth or long-run excess stock returns reflects non-market fundamental behavior such as rational bubbles, fads, or irrational exuberance (Shiller 2000)? Our model takes as one of its identifying assumptions, market fundamentals based asset pricing and, hence, our results can not rule out the possibility of irrational behavior. However, our model does restrict the estimated state variables such as real dividend growth to be consistent with the dynamics of actual real dividend growth.

8.2 Dividends or excess stock returns, revisited?

On statistical grounds, as we argued above, we are unable to determine whether real dividend growth or excess stock returns are more important. In our model, the observation equations for interest rates and inflation end up tying down the long-run real interest rate. Because the real interest rate alone cannot explain the low frequency movements in the log price-dividend ratio, the remaining low frequency movements in the log price-dividend ratio (controlling for the contribution of the real interest rate) must be due to either expectations of future real dividend growth or excess returns. Both actual real dividend growth and actual excess stock returns have large temporary components and, hence, are very noisy indicators of changes in long-run values of these variables. Thus, the model gets most of the information about long-run real dividend growth or long-run excess returns from observations of the log price-dividend ratio. Unfortunately, it is not possible to infer from stock prices alone whether expected future real dividend growth or excess stock returns has the greatest impact on stock price variability. In fact, when we tried to estimate a model with both a permanent real dividend growth and a permanent excess stock return component, we were unable to achieve convergence. This is consistent with there not being sufficient information in our data to identify both a permanent real dividend component and a permanent excess stock return component.

As we showed above, our inference about the relative importance of expectations about future real dividend growth and future excess returns hinges on the assumption of which of these has a permanent component. This appears to run counter with much of the previous literature on stock price decompositions in that expectations of future real dividends typically were found not to be as important as expectations of future excess returns (see for example, Campbell and Shiller (1988a), Campbell (1991), Cochrane (1992), Campbell and Ammer (1993), and Lee (1998)).

In many respects, our approach and that of the VAR approach to stock market decompositions are quite similar. For example, Campbell and Ammer (1993) use a VAR with log price-dividend ratio, the ex-post real interest rate, the change in nominal interest rates, the spread between short and long-term interest rates, and excess returns on equity to capture the time series movements in these variables. They then use the estimated VAR to calculate the contribution of future dividend growth, real interest rate, and excess returns to stock prices. Thus, they use a similar information set as we do when calculating expectations of future market fundamentals.[28] Yet, they report no ambiguity about the relative role of real dividend growth or excess returns.

One important difference in our analysis is that we allow permanent components in some of the market fundamentals.[29] As we pointed out in the previous section, small permanent changes in real dividend growth might be overlooked in the data. It is these small but permanent changes in real dividend growth that are responsible for most of the contribution of real dividend growth to stock price movements. Note that we allow for permanent changes in the real interest rate and in the inflation rate as well; therefore, our approach was not preordained to ascribe such importance to real dividend growth or excess returns. Second, much of the previous literature does not include the recent runup in stock prices. For example, the Campbell and Ammer (1993) sample stopped in February 1987. The runup in stock prices during the 1990s has generally lowered the ability of log price-dividend to predict of stock returns, particularly at relatively low horizons (Campbell (2000)). Thus, these models are not likely to ascribe as much importance to excess stock returns as they previously did.

9. Summary and Conclusion

A large body of research has examined explanations for why stock prices fluctuate and the conclusion of this line of research is that most of the variability in stock returns is due to movements in expectations of future excess returns and not dividend growth. We show, however, that decompositions of stock price movements are very sensitive to what assumptions one makes about the presence of permanent changes in either real dividend growth or excess stock returns. When real dividend growth is allowed to have a permanent component but excess stock returns is allowed to have a transitory component, real dividend growth explains more of the movement in stock prices than does excess stock returns. When we reverse this assumption, the relative contributions of excess stock returns and real dividend growth are reversed as well.

In order to identify the relative importance of real dividend growth or excess stock returns for stock price variability other information needs to be used. For example, information on relative transactions costs and their effect on investors asset allocations (see Heaton and Lucas (1999)), information about the underlying determinants of a time varying equity premium (see for example, Campbell and Cochrane (1999)), or indicators of long-run economic growth could be employed to help one distinguish between changes in expectations of future real dividend growth and excess returns. Alternatively, one might attempt tying real interest rate and risk premium movements not only to the level of assets prices as done in this paper but to movement in the covariance structure of asset prices as well. Finally, one might formally incorporate prior information about the relative importance of dividend growth and excess returns by taking a Bayesian approach to stock price decompositions. We hope to explore some of these extensions in future research.

Appendix A. Alternative State Space Representations

The model with permanent and temporary components in dividend growth and temporary components in excess returns and the term premium can be described with the following state space model:

(A1a) $S_t = FS_{t-1} + V_t$

where

(A1b) $S_t = (d_t^p, r_t^p, \pi_t^p, d_t^a, r_t^a, \pi_t^a, e_t^a, \tau_t^a, d_{t-1}^a, r_{t-1}^a, \pi_{t-1}^a, e_{t-1}^a, \tau_{t-1}^a, \ldots,$
$\ldots, d_{t-(k-1)}^a, r_{t-(k-1)}^a, \pi_{t-(k-1)}^a, e_{t-(k-1)}^a, \tau_{t-(k-1)}^a)',$

(A1c) $V_t = (v_t^{d,p}, v_t^{r,p}, v_t^{\pi,p}, v_t^{d,a}, v_t^{r,a}, v_t^{\pi,a}, v_t^{e,a}, v_t^{\tau,a}, 0_{1x5(k-1)})',$

and

(A1d) $$F = \begin{pmatrix} I_{3x3} & 0_{3x5} & & \cdots & & 0_{3x5} \\ 0_{5x3} & \theta_1 & \theta_2 & \cdots & \theta_{k-1} & \theta_k \\ & I_{5x5} & 0_{5x5} & \cdots & 0_{5x5} & 0_{5x5} \\ \vdots & 0_{5x5} & I_{5x5} & \ddots & & \vdots \\ & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0_{5x3} & 0_{5x5} & \cdots & 0_{5x5} & I_{5x5} & 0_{5x5} \end{pmatrix} \text{ where } \theta_i = \begin{pmatrix} \theta_i^d & & & & \\ & \theta_i^r & & 0 & \\ & & \theta_i^\pi & & \\ & 0 & & \theta_i^e & \\ & & & & \theta_i^\tau \end{pmatrix}.$$

The observation equation in the state-space model is

(A2a) $Y_t = HS_t + W_t,$

where

(A2b) $$H = \begin{pmatrix} (H_1 + H_e)(I - \rho F)^{-1} F \\ \left(\frac{1}{n}\right)(H_2 (I - F^n)(I - F)^{-1} F + H_\tau (I - F^{n-1})(I - F)^{-1} F) \\ H_2 F \\ H_3 \\ H_4 \end{pmatrix}$$

with $H_1 = (1, -1, 0, 1, -1, 0, 0, 0, 0_{1x5(k-1)})$, $H_e = (0, 0, 0, 0, 0, 0, -1, 0, 0_{1x5(k-1)})$

$H_2 = (0, 1, 1, 0, 1, 1, 0, 0, 0_{1x5(k-1)})$, $H_\tau = (0, 0, 0, 0, 0, 0, 0, 1, 0_{1x5(k-1)})$

$H_3 = (1, 0, 0, 1, 0, 0, 0, 0, 0_{1x5(k-1)})$, and

$H_4 = (0, 0, 1, 0, 0, 1, 0, 0, 0_{1x5(k-1)})$.

The state space model with permanent and temporary excess returns components and temporary real dividend growth and term premium components is given by:

(A3a) $S_t = F S_{t-1} + V_t$

where

(A3b) $$S_t = (e_t^p, r_t^p, \pi_t^p, e_t^a, r_t^a, \pi_t^a, d_t^a, \tau_t^a, e_{t-1}^a, r_{t-1}^a, \pi_{t-1}^a, d_{t-1}^a, \tau_{t-1}^a, \ldots, \\ \ldots, e_{t-(k-1)}^a, r_{t-(k-1)}^a, \pi_{t-(k-1)}^a, d_{t-(k-1)}^a, \tau_{t-(k-1)}^a)',$$

(A3c) $V_t = (v_t^{e,p}, v_t^{r,p}, v_t^{\pi,p}, v_t^{e,a}, v_t^{r,a}, v_t^{\pi,a}, v_t^{d,a}, v_t^{\tau,a}, 0_{1x5(k-1)})'$

and the F matrix similar to that given in (A1d) except that θ_i^d and θ_i^e are exchanged.

The observation equation is given by:

(A4a) $Y_t = H S_t + W_t$,

where

(A4b) $$H = \begin{pmatrix} (H_1 + H_e)(I - \rho F)^{-1} F \\ \left(\frac{1}{n}\right)(H_2 (I - F^n)(I - F)^{-1} F + H_\tau (I - F^{n-1})(I - F)^{-1} F) \\ H_2 F \\ H_3 \\ H_4 \end{pmatrix}$$

with $\mathrm{H}_1 = (0, -1, 0, 0, -1, 0, 1, 0, 0_{1x5(k-1)})$, $\mathrm{H}_e = (-1, 0, 0, -1, 0, 0, 0, 0, 0_{1x5(k-1)})$

$\mathrm{H}_2 = (0, 1, 1, 0, 1, 1, 0, 0, 0_{1x5(k-1)})$, $\mathrm{H}_\tau = (0, 0, 0, 0, 0, 0, 0, 1, 0_{1x5(k-1)})$

$\mathrm{H}_3 = (1, 0, 0, 1, 0, 0, 0, 0, 0_{1x5(k-1)})$, and

$\mathrm{H}_4 = (0, 0, 0, 0, 0, 0, 1, 0, 0_{1x5(k-1)})$.

TABLE 1. Unit Root Test and Variance Ratios

variable	ADF t-stat: (lag length=4)	variance ratio (standard error)
log(P/D)	-2.08	0.92 (.30)
log(P/E)	-1.44	1.28 (.30)
interest rate (10 yr T-bond)	-1.93	1.50 (.39)
interest rate (3 mth T-bill)	-2.47	0.77 (.44)
real dividend growth	-4.33**	0.04 (.40)
inflation	-2.48	0.12 (.35)
real short-term interest rate	-2.61	0.09 (.34)
excess stock returns	-6.08**	0.04 (.35)
interest rate spread	-3.75**	0.23 (.40)

Note: ADF tests are for tests with a constant. Similar results are obtained when a time trend is included **-reject unit-root null at 5% nominal significance level (critical value =-2.88). Standard errors on variance ratio are adjusted for heteroscedasticity. The horizon for the variance ratio is 10 years. The interest rate spread is defined as the 10-year T-bond interest rate minus the 3-month T-bill interest rate.

Table 2. Johansen Test for Number of Cointegrating Vectors

System contains: i) log price-dividend ratio, ii) long-term interest rate, iii) short-term interest rate, iv) real dividend growth, and v) inflation.

$\lambda - \max$ Test

Null	Alternative	$\lambda - \max$ statistic	95% Critical Value	90% Critical Value
r = 0	r = 1	35.04	34.4000	31.7300
r <= 1	r = 2	31.21	28.2700	25.8000
r <= 2	r = 3	14.03	22.0400	19.8600
r <= 3	r = 4	8.89	15.8700	13.8100
r <= 4	r = 5	4.27	9.1600	7.5300

Trace Test

Null	Alternative	Trace statistic	95% Critical Value	90% Critical Value
r = 0	r >= 1	93.44	75.9800	71.8100
r <= 1	r >= 2	58.40	53.4800	49.9500
r <= 2	r >= 3	27.19	34.8700	31.9300
r <= 3	r >= 4	13.16	20.1800	17.8800
r <= 4	r = 5	4.27	9.1600	7.5300

TABLE 3.
Estimated Dynamic Common Factor Model with Permanent Dividend Growth Factor

Estimated coefficients from the state space model, with standard errors in parentheses:

θ_1^d	=	0.532 (.062)	θ_2^d	=	-0.020 (.063)
θ_1^r	=	0.339 (.036)	θ_2^r	=	0.286 (.028)
θ_1^π	=	-0.006 (.031)	θ_2^π	=	-0.140 (.029)
θ_1^e	=	0.269 (.092)	θ_2^e	=	0.318 (.098)
θ_1^τ	=	0.222 (.071)	θ_2^τ	=	0.630 (.070)

Estimated variance/covariance for the noise terms in the observation equation:

$R = \mathrm{diag}(1.45\mathrm{x}10^{-1}, 1.59\mathrm{x}10^{-14}, 3.01\mathrm{x}10^{-13}, 7.43\mathrm{x}10^{-10}, 3.78\mathrm{x}10^{-12})$.

Estimated variance\covariance matrix for the state equation innovations (Q matrix):

	v_t^{dp}	v_t^{rp}	$v_t^{\pi p}$	v_t^{da}	v_t^{ra}	$v_t^{\pi a}$	v_t^{ea}	$v_t^{\tau a}$
v_t^{dp}	$4.66\mathrm{x}10^{-2}$	0.91	-0.70	-0.38	-0.30	0.23	0.74	-0.02
v_t^{rp}	$2.99\mathrm{x}10^{-2}$	$2.34\mathrm{x}10^{-2}$	-0.54	-0.15	-0.42	-0.00	0.48	-0.12
$v_t^{\pi p}$	$-3.55\mathrm{x}10^{-2}$	$-1.95\mathrm{x}10^{-2}$	$5.51\mathrm{x}10^{-2}$	0.53	-0.46	-0.75	-0.59	-0.68
v_t^{da}	$-1.21\mathrm{x}10^{-1}$	$-3.38\mathrm{x}10^{-2}$	$1.86\mathrm{x}10^{-1}$	2.20	-0.31	-0.51	-0.32	-0.43
v_t^{ra}	$-4.53\mathrm{x}10^{-2}$	$-4.56\mathrm{x}10^{-2}$	$-7.67\mathrm{x}10^{-3}$	$-3.47\mathrm{x}10^{-1}$	$5.00\mathrm{x}10^{-1}$	0.70	-0.05	0.93
$v_t^{\pi a}$	$2.52\mathrm{x}10^{-2}$	$-1.65\mathrm{x}10^{-3}$	$-8.84\mathrm{x}10^{-2}$	$-3.78\mathrm{x}10^{-1}$	$2.49\mathrm{x}10^{-1}$	$2.53\mathrm{x}10^{-1}$	0.37	0.86
v_t^{ea}	1.25	$5.80\mathrm{x}10^{-1}$	-1.09	-3.71	$-2.72\mathrm{x}10^{-1}$	1.49	62.11	0.11
$v_t^{\tau a}$	$-4.99\mathrm{x}10^{-3}$	$-2.76\mathrm{x}10^{-2}$	$-2.32\mathrm{x}10^{-1}$	$-9.23\mathrm{x}10^{-1}$	$9.48\mathrm{x}10^{-1}$	$6.28\mathrm{x}10^{-1}$	1.20	2.09

Note: variances are along the diagonal, covariances below the diagonal, and correlations above the diagonal.

Log likelihood = -742.83

R^2 and Q-statistics for the observation equations of Dynamic Common Factor Model and a VAR(2):

	Dynamic Common Factor Model		VAR with 2 lags	
	R^2	Q-stat (p-value)	R^2	Q-stat (p-value)
log price-dividend ratio	0.93	38.9 (0.34)	0.94	43.5 (0.18)
yield on 10 year T-Bond	0.96	34.0 (0.57)	0.97	50.6 (0.05)
yield on 3 month T-Bill	0.94	30.7 (0.71)	0.94	74.8 (0.00)
real dividend growth	0.26	58.2 (0.01)	0.31	58.0 (0.01)
inflation	0.66	52.5 (0.04)	0.62	106.4 (0.00)

Table 4.

Forecast Variance Decompositions (horizon one quarter, k = 1) for Model with Permanent Real Dividend Growth Component

Contribution of: / to:	variance of log price-dividend ratio	variance of long-term interest rate	variance of short-term interest rate	variance of real stock returns	variance of interest rate spread	covariance of real stock returns and real bond returns
var(real dividend growth)	5.23x10^{2}	0.0	0.0	5.03x10^{2}	0.0	0.0
cov(div, real interest rate)	-6.79x10^{2}	0.0	0.0	-6.65x10^{2}	0.0	-1.18x10^{2}
cov(div, inflation)	0.0	0.0	0.0	0.0	0.0	1.41x10^{2}
cov(div, excess returns)	-3.66x10^{2}	0.0	0.0	-3.52x10^{2}	0.0	0.0
cov(div, term pr)	0.0	0.0	0.0	0.0	0.0	1.35x10^{1}
var(real interest rate)	2.61x10^{2}	2.05x10^{-2}	5.01x10^{-2}	2.56x10^{2}	4.43x10^{-2}	9.10x10^{1}
cov(real int. rate, infl.)	0.0	-4.54x10^{-2}	-9.20x10^{-2}	0.0	-3.96x10^{-4}	-9.06x10^{1}
cov(real int. rate, ex. ret.)	1.73x10^{2}	0.0	0.0	1.70x10^{2}	0.0	3.12x10^{1}
cov(real int. rate, term pr)	0.0	3.35x10^{-3}	0.0	0.0	-7.89x10^{-2}	-8.05
var(inflation)	0.0	5.57x10^{-2}	5.61x10^{-2}	0.0	1.82x10^{-6}	0.0
cov(infl, ex. ret.)	0.0	0.0	0.0	0.0	0.0	-5.98x10^{1}
cov(infl, term pr.)	0.0	-6.53x10^{-2}	0.0	0.0	4.71x10^{-4}	0.0
var(excess returns)	1.19x10^{2}	0.0	0.0	1.17x10^{2}	0.0	0.0
cov(ex ret, term premium)	0.0	0.0	0.0	0.0	0.0	9.23
var(term premium)	0.0	4.08x10^{-2}	0.0	0.0	4.08x10^{-2}	0.0
noise	1.45	1.59x10^{-14} 5	3.01x10^{-13}	2.87	3.17x10^{-13}	0.0
total	3.36x10^{1}	9.58x10^{-3}	1.42x10^{-2}	3.36x10^{1}	6.32x10^{-3}	9.73

TABLE 5.

Estimated Dynamic Common Factor Model with Permanent Excess Return Component and Stationary Dividend Growth

Estimated coefficients from the state space model, with standard errors in parentheses:

θ_1^d	=	0.528 (.065)	θ_2^d	=	-0.012 (.069)
θ_1^m	=	0.343 (.033)	θ_2^m	=	0.286 (.029)
θ_1^π	=	0.003 (.029)	θ_2^π	=	-0.130 (.029)
θ_1^e	=	0.333 (.149)	θ_2^e	=	0.338 (.127)
θ_1^τ	=	0.234 (.101)	θ_2^τ	=	0.625 (.099)

Estimated variance/covariance for the noise terms in the observation equation:

R = diag($5.51\text{x}10^{-1}$, $2.52\text{x}10^{-13}$, $3.51\text{x}10^{-14}$, $2.21\text{x}10^{-9}$, $5.02\text{x}10^{-10}$).

Estimated variance\covariance matrix for the state equation innovations (Q matrix):

	v_t^{ep}	v_t^{rp}	$v_t^{\pi p}$	v_t^{ea}	v_t^{ra}	$v_t^{\pi a}$	v_t^{da}	$v_t^{\tau a}$
v_t^{ep}	$4.83\text{x}10^{-2}$	-0.88	0.69	-0.80	0.28	-0.34	0.21	-0.00
v_t^{rp}	$-2.76\text{x}10^{-2}$	$2.03\text{x}10^{-2}$	-0.50	0.53	-0.43	0.08	-0.02	-0.12
$v_t^{\pi p}$	$3.52\text{x}10^{-2}$	$-1.64\text{x}10^{-2}$	$5.37\text{x}10^{-2}$	0.65	-0.50	-0.81	0.46	-0.71
v_t^{ea}	-1.13	$4.87\text{x}10^{-1}$	$-0.96\text{x}10^{-1}$	41.05	-0.04	0.44	-0.16	0.13
v_t^{ra}	$4.22\text{x}10^{-2}$	$-4.29\text{x}10^{-2}$	$-8.16\text{x}10^{-2}$	$-1.75\text{x}10^{-1}$	$4.95\text{x}10^{-1}$	0.68	-0.36	0.93
$v_t^{\pi a}$	$-3.85\text{x}10^{-2}$	$5.84\text{x}10^{-3}$	$-9.69\text{x}10^{-2}$	1.45	$2.45\text{x}10^{-1}$	$2.65\text{x}10^{-1}$	-0.50	0.86
v_t^{da}	$6.59\text{x}10^{-2}$	$-4.13\text{x}10^{-3}$	$1.52\text{x}10^{-1}$	-1.43	$-3.66\text{x}10^{-1}$	$-3.70\text{x}10^{-1}$	2.07	-0.45
$v_t^{\tau a}$	$-6.19\text{x}10^{-4}$	$-2.60\text{x}10^{-2}$	$-2.28\text{x}10^{-1}$	1.13	$9.06\text{x}10^{-1}$	$6.11\text{x}10^{-1}$	-0.89	1.92

Note: variances are along the diagonal, covariances below the diagonal, and correlations above the diagonal.

Log likelihood = -745.40

R^2 and Q-statistics for the observation equations of Dynamic Common Factor Model and a VAR(2):

	Dynamic Common Factor Model		VAR with 2 lags	
	R^2	Q-stat (p-value)	R^2	Q-stat (p-value)
log price-dividend ratio	0.93	39.1 (0.33)	0.94	43.5 (0.16)
yield on 10 year T-Bond	0.96	33.8 (0.57)	0.97	50.6 (0.05)
yield on 3 month T-Bill	0.94	31.9 (0.66)	0.94	74.8 (0.00)
real dividend growth	0.24	54.8 (0.02)	0.31	58.0 (0.01)
inflation	0.67	51.0 (0.04)	0.62	106.4 (0.00)

Table 6.

Forecast Variance Decompositions (horizon 1 quarter, k = 1) for Model with Permanent Excess Returns Component

To: / Contribution of:	variance of log price-dividend ratio	variance of long-term interest rate	variance of short-term interest rate	variance of real stock returns	variance of interest rate spread	covariance of real stock returns and real bond returns
var(real dividend growth)	2.31	0.0	0.0	8.66	0.0	0.0
cov(div, real interest rate)	2.22	0.0	0.0	4.26	0.0	1.60×10^{1}
cov(div, inflation)	0.0	0.0	0.0	0.0	0.0	-1.21×10^{1}
cov(div, excess returns)	-9.11	0.0	0.0	-1.75×10^{1}	0.0	0.0
cov(div, term premium)	0.0	0.0	0.0	0.0	0.0	1.08×10^{1}
var(real interest rate)	2.25×10^{2}	1.75×10^{-2}	4.89×10^{-2}	2.20×10^{2}	4.46×10^{-2}	7.79×10^{1}
cov(real int. rate, infl.)	0.0	-3.99×10^{-2}	-8.83×10^{-2}	0.0	8.36×10^{-4}	-7.70×10^{1}
cov(real int. rate, ex. ret.)	-4.27×10^{2}	0.0	0.0	-4.19×10^{2}	0.0	-7.34×10^{1}
cov(real int. rate, term pr)	0.0	3.64×10^{-3}	0.0	0.0	-8.06×10^{-2}	-7.79
var(inflation)	0.0	5.42×10^{-2}	5.31×10^{-2}	0.0	8.61×10^{-6}	0.0
cov(infl, excess ret.)	0.0	0.0	0.0	0.0	0.0	7.69×10^{1}
cov(infl, term pr.)	0.0	-6.80×10^{-2}	0.0	0.0	-1.03×10^{-3}	0.0
var(excess returns)	2.43×10^{2}	0.0	0.0	2.38×10^{2}	0.0	0.0
cov(ex ret, term premium)	0.0	0.0	0.0	0.0	0.0	1.27×10^{1}
var(term premium)	0.0	4.21×10^{-2}	0.0	0.0	4.21×10^{-2}	0.0
noise	5.51×10^{-1}	2.52×10^{-13}	3.51×10^{-14}	1.09	2.87×10^{-13}	0.0
total	3.69×10^{1}	9.65×10^{-3}	1.38×10^{-2}	3.67×10^{1}	5.94×10^{-3}	9.66

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ENDNOTES

1. For other studies that have argued that the equity premium has declined over time, see Cochrane (1997), Claus and Thomas (1999), Wadhwani (1999) and Fama and French (2001). For surveys of the equity premium puzzle see Kocherlakota (1996) and Siegel and Thaler (1997).

2. An additional explanation for the rise in stock prices has been offered by Robert Shiller and has been termed "irrational exuberance". According to Shiller (2000) stock prices have increased based on the expectation of further stock price increases, with little or no attention given to expected future earnings or dividend growth.

3. Campbell, Lo, and MacKinlay (1997) note that while the approximation misstates the average stock return, it captures the dynamics of stock returns well in high frequency data.

4. In a previous version of our paper we motivated an equation similar to (3) in terms of an equilibrium asset pricing model in which

$$1 = E_t \left[M_{t+1} R_{t+1} \right]$$

where R_{t+1} is the real return on the asset at t+1 M_{t+1}, called the asset pricing kernel, is the value investors place on the real return (see Campbell, Lo, and MacKinlay 1997, p. 295). In the standard consumption based asset pricing model of Lucas (1978), M_{t+1} is the marginal rate of substitution between consumption at time t+1 and consumption at time t. If the variables are jointly distributed log normal (conditional on information at time t), then we can approximate the log price-dividend ratio, p_t, as

$$p_t \approx E_t \log(M_{t+1}) + E_t d_{t+1} + \rho E_t p_{t+1} + k$$

$$+ \frac{1}{2} VAR_t(\log(M_{t+1})) + \frac{1}{2} VAR_t(d_{t+1} + \rho p_{t+1}) + COV_t(\log(M_{t+1}), d_{t+1} + \rho p_{t+1})$$

where $VAR_t(.)$ and $COV_t(.)$ are the variance and covariance at time t+1 conditional on information at time t. If the variances and covariance are constant, then in terms of the model in the text $E_t r_{t+1} = -E_t \log(M_{t+1})$ and expected excess returns, $E_t e_{t+1}$, is a constant.

5. While a rational bubble would yield an explosive price dividend ratio, rational bubbles are hard to support both empirically or theoretically (see Campbell, Lo, and Mackinlay (1997, pp 258-260).

6. In general equilibrium, its not clear that a permanent increase in real dividend growth will result in an increase in P/D ratio. Consider the case where households have power utility and consumption and real dividend growth are perfectly correlated, then

$$M_{t+j} \frac{D_{t+j}}{D_{t+j-1}} \propto \left(\frac{D_{t+j}}{D_{t+j-1}} \right)^{1-\gamma}.$$

If households have logarithmic utility $\gamma = 1$ then the P/D ratio is unchanged.

7. See Backus, Foresi, and Telmer (1998) for a recent survey of bond pricing models.

8. Fama and Gibbons (1982), Hamilton (1985), and Burmeister, Wall, and Hamilton (1986) employ dynamic factor models to infer market's expectations about expected inflation.

9. As long as $\sum_{i=1}^{k} \theta_i^j \neq I_{3x3}$ and $\theta_i^j \neq 0, \quad j = d, r, \pi, \quad i = 1, ..., k,$

then the above state-space model is identified (for demonstration see Technical Appendix which is available upon request).

10. A quarterly dividend series for the S&P 500 composite stock price index is computed from S&P 500 annualized monthly dividend yield and stock price level data both obtained from the DRI Pro database. We multiply the dividend yield by the monthly stock price and convert annualized dividend flow into a monthly dividend flow and then average the three monthly dividend series together to obtain an average dividend for the quarter.

11. For example, the log approximation of excess returns can be written in terms of the variables in our system as: $(\rho - 1)p_t + d_t - i_{1,t} + \pi_t + \Delta p_t + \Delta i_{1,t}$. If excess returns are stationary, then that implies that p_t, d_t, $i_{1,t}$, π_t are cointegrated. Similarly if the interest rate spread is stationary, then short term and long term interest rates are cointegrated. Finally, a stationary real dividend growth is the trivial cointegrating vector of just real dividend growth by itself.

12. In fact, both the Akaike Information and Schwartz Bayesian Criteria prefer the model that includes a permanent component for either real dividend growth or excess stock returns over a model that includes permanent components for only the real interest rate and inflation. The AIC (SBC) for three permanent component model is 1587.7 (1751.7) versus 1628.7 (1766.9) for the two permanent component model.

13. See Watson and Engle (1983) for application of the EM algorithm to estimation of a state space model.

14. We scaled up the data by 100 to reduce the effect of round-off error in the numerical calculations. The initial estimate of the state vector, $\mathbf{S}_{0|0}$, was set equal to zero. $\mathbf{P}_{0|0}$ was set equal to the

unconditional variance/covariance for the temporary factors, d_t^a, π_t^a, and r_t^a, and equal to 4900 for the permanent components, which corresponds to a flat prior for these nonstationary components.

15. The log price-dividend decompositions presented below do not change appreciably if we increase the number of lags in the temporary components to three.

16. This is based on the average log price-dividend ratio over the sample, where dividend flows are on a quarterly basis.

17. We examine the forecast variance decompositions for various forecast horizons because unlike unconditional variances these can be calculated for nonstationary variables. Like the variance decompositions undertaken within the VAR literature, we take the parameters of the state space model as known when calculating variances. Furthermore, we abstract away from uncertainty about the state (recall the state vector is itself estimated by the Kalman filter).

18. This particularly true at longer horizons. The longer horizon decompositions are available upon request.

19. Note that the 10-year forecast horizon for stock returns (and excess stock returns) is the variance of the one-period return, 10 years in the future. This is not the same thing as the forecast variance of the 10-year cumulative return. Note also that the one-quarter and 10-year horizon forecast variance decompositions are very similar for stock returns (and excess stock returns) which merely reflects the fact that price changes dominate the variance of returns rather than the level of dividends.

20. The variance decompositions for the model with a permanent excess stock returns component

(table available upon request) also reflect this dramatic decline in implied contributions for real dividend growth. However, the real interest rate component still has an important contribution to stock prices and returns.

21. The confidence interval is based on 1,000 simulations and takes into account both filter and parameter uncertainty as described in Hamilton (1994a,b).

22. In fact, Fama and French (2001) argue that high excess stock returns during the post-war period were in fact the result of a decline in the required return for stocks. Pastor and Stambaugh (2000) examine a Bayesian model that allows for structural change in the equity premium but includes a transition period as the market gradually adjusts to a change in beliefs about long-run excess returns.

23. These ratio are: $\mathrm{var}(v^{dp})/\mathrm{var}(v^{dp}+v^{da})$ and $\mathrm{var}(v^{ep})/\mathrm{var}(v^{ep}+v^{ea})$, respectively.

24. We generated 1000 samples from the state space model. For each sample, we ran a Dickey-Fuller regression (with a constant) and from these we calculated the size adjusted augmented (with 4 lags) Dickey-Fuller t-statistic. The 5th percentile Dickey-Fuller statistic for real dividend growth for the model with a permanent real dividend component was -5.28 while the 5th percentile ADF statistic for excess returns for the model with a permanent excess return component was -6.29.

25. The factor loadings are from the H matrix in our state space model.

26. Barsky and De Long (1990, 1993) advance a very similar argument to the one we make here. They argue that once one allows for small permanent changes in expectations of long-run dividend growth one can explain long swings in stock prices by expectations of future dividends. Barsky and DeLong point out, the presence of a small permanent component can result in the appearance of

excess volatility in that (log) stock prices react much more than one to one with (log) dividends. Like Barsky and DeLong, we have a small permanent component for dividend growth, but we also allow for time varying discount rates (real interest rate plus equity premiums) when estimating the permanent component of dividend growth.

27. More formally, a change in the covariance between the stochastic discount factor as described in footnote 3 and the returns on stocks would lead to a change in the equity premium.

28. In contrast to our model, Campbell and Ammer (1993) use stock returns rather than real dividend growth. However, one can replace in our framework actual real dividend growth with actual real returns as an observation equation. When we do so, we get essentially the same results.

29. We examined a model in which the observation equations for log price-dividend and interest rates contained no measurement error. This model yields essentially the same qualitative results as the model with measurement error. In a previous version of this paper, we also presented result for a model in which earnings replaced dividends, and also obtained similar result to those reported here. These results are available upon request.

(table available upon request) also reflect this dramatic decline in implied contributions for real dividend growth. However, the real interest rate component still has an important contribution to stock prices and returns.

21. The confidence interval is based on 1,000 simulations and takes into account both filter and parameter uncertainty as described in Hamilton (1994a,b).

22. In fact, Fama and French (2001) argue that high excess stock returns during the post-war period were in fact the result of a decline in the required return for stocks. Pastor and Stambaugh (2000) examine a Bayesian model that allows for structural change in the equity premium but includes a transition period as the market gradually adjusts to a change in beliefs about long-run excess returns.

23. These ratio are: $\mathrm{var}(v^{dp})/\mathrm{var}(v^{dp}+v^{da})$ and $\mathrm{var}(v^{ep})/\mathrm{var}(v^{ep}+v^{ea})$, respectively.

24. We generated 1000 samples from the state space model. For each sample, we ran a Dickey-Fuller regression (with a constant) and from these we calculated the size adjusted augmented (with 4 lags) Dickey-Fuller t-statistic. The 5^{th} percentile Dickey-Fuller statistic for real dividend growth for the model with a permanent real dividend component was -5.35 while the 5^{th} percentile ADF statistic for excess returns for the model with a permanent excess return component was -6.15.

25. The factor loadings are from the H matrix in our state space model.

26. Barsky and De Long (1990, 1993) advance a very similar argument to the one we make here. They argue that once one allows for small permanent changes in expectations of long-run dividend growth one can explain long swings in stock prices by expectations of future dividends. Barsky and DeLong point out, the presence of a small permanent component can result in the appearance of

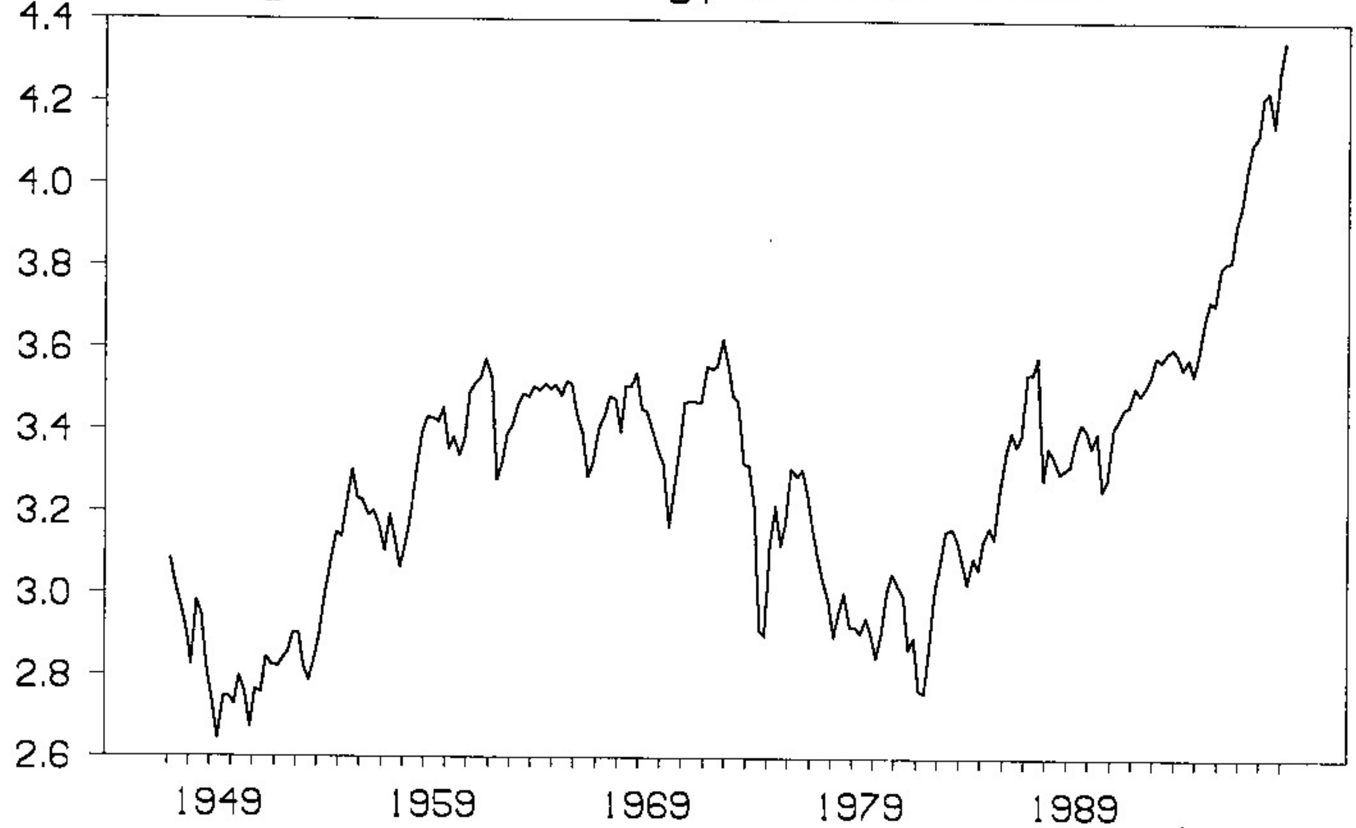
excess volatility in that (log) stock prices react much more than one to one with (log) dividends. Like Barsky and DeLong, we have a small permanent component for dividend growth, but we also allow for time varying discount rates (real interest rate plus equity premiums) when estimating the permanent component of dividend growth.

27. More formally, a change in the covariance between the stochastic discount factor as described in footnote 3 and the returns on stocks would lead to a change in the equity premium.

28. In contrast to our model, Campbell and Ammer (1993) use stock returns rather than real dividend growth. However, one can replace in our framework actual real dividend growth with actual real returns as an observation equation. When we do so, we get essentially the same results.

29. We examined a model in which the observation equations for log price-dividend and interest rates contained no measurement error. This model yields essentially the same qualitative results as the model with measurement error. In a previous version of this paper, we also presented result for a model in which earnings replaced dividends, and also obtained similar result to those reported here. These results are available upon request.

Figure 1. Panel A: log price/dividend ratio



Panel B: log price/earnings ratio

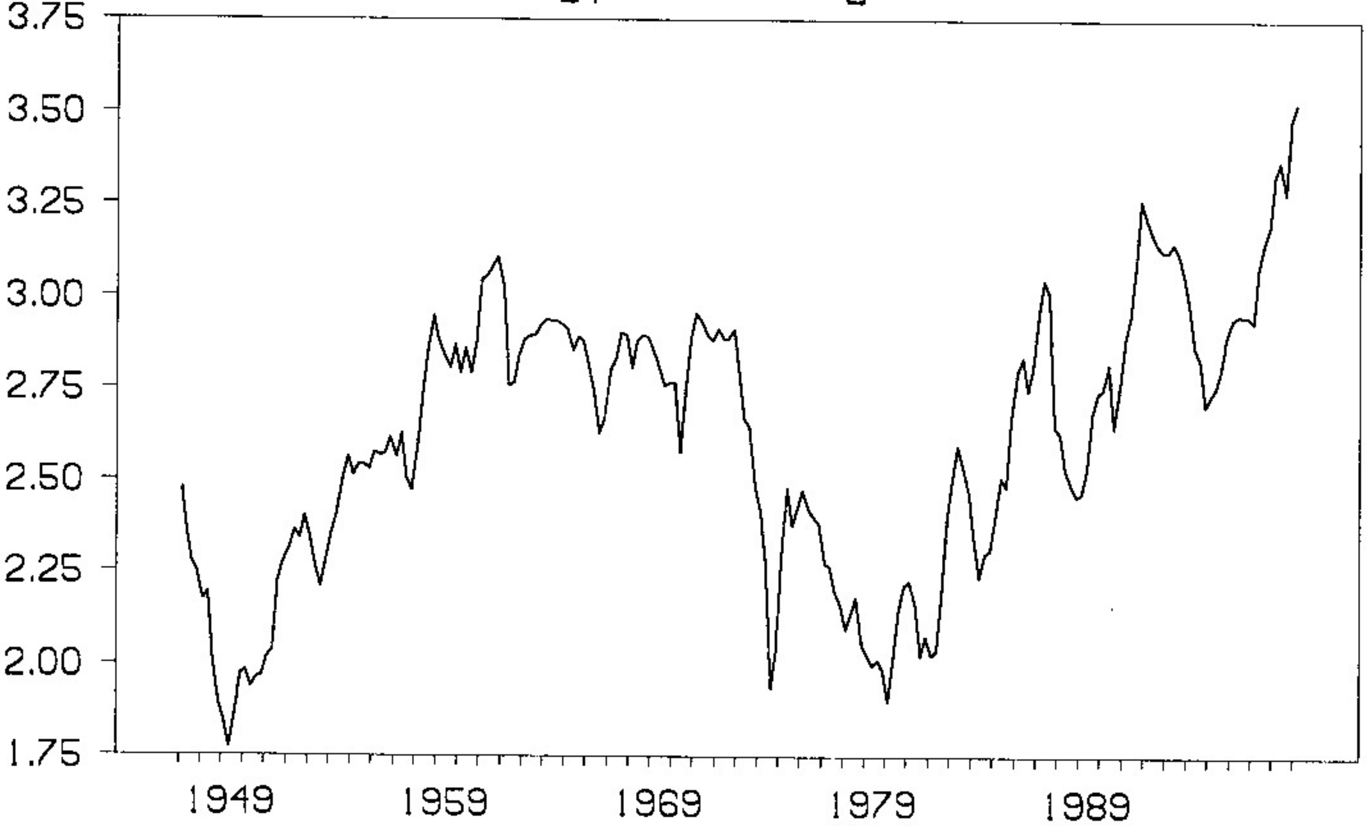
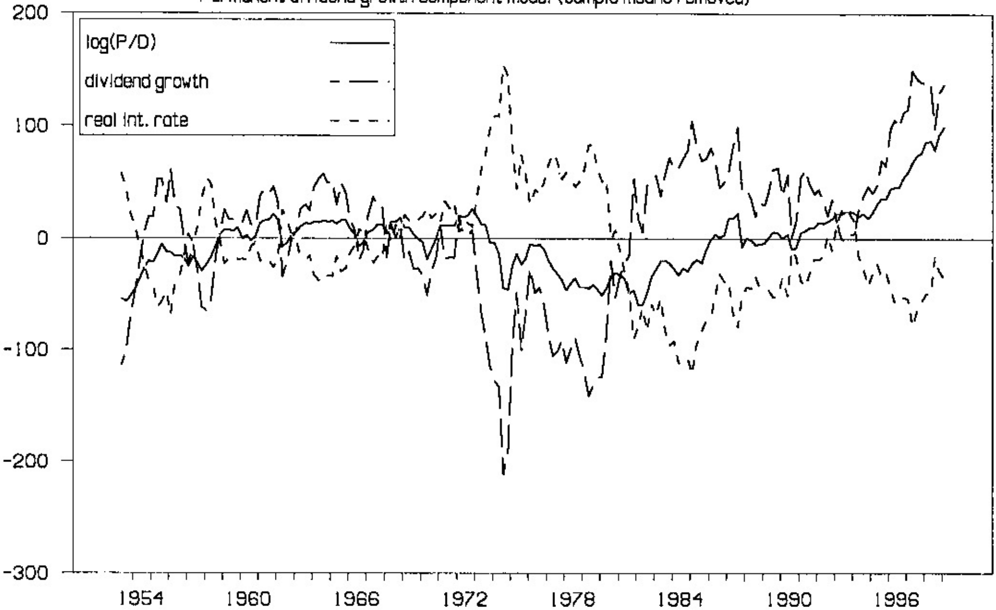


Figure 2.

Panel A: log(P/D) and contribution of expected dividend growth and real interest rate factors

Permanent dividend growth component model (sample means removed)



Panel B: log(P/D) and contribution of excess equity return factor

Permanent dividend growth component model (sample means removed)

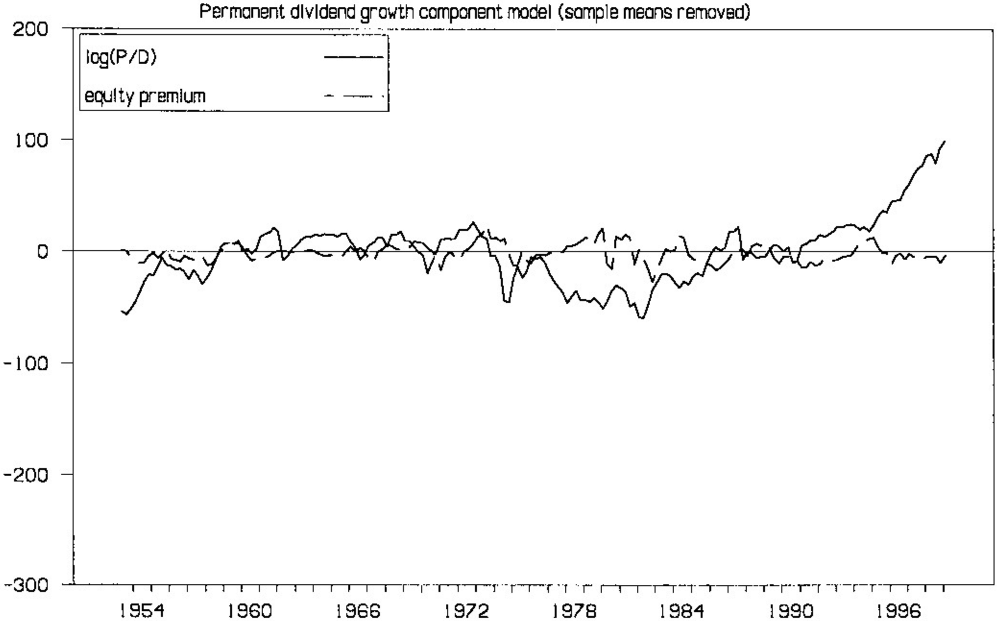
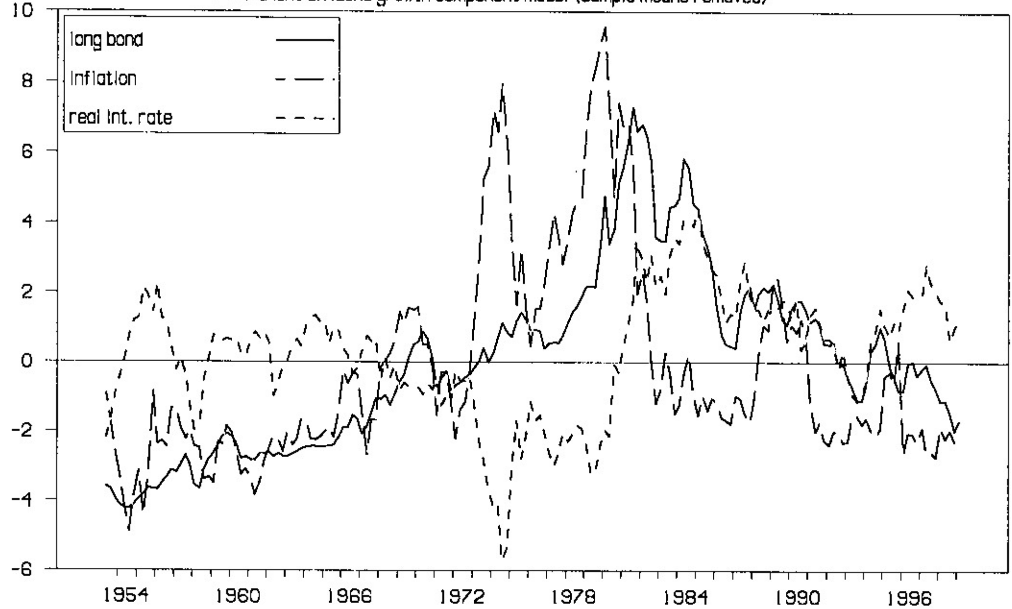


Figure 3.

Panel A: 10 year T-bond rate and contribution of expected future inflation and real rate

Permanent dividend growth component model (sample means removed)



Panel B: 10 year T-Bond rate and contribution of term premium

Permanent dividend growth component model (sample means removed)

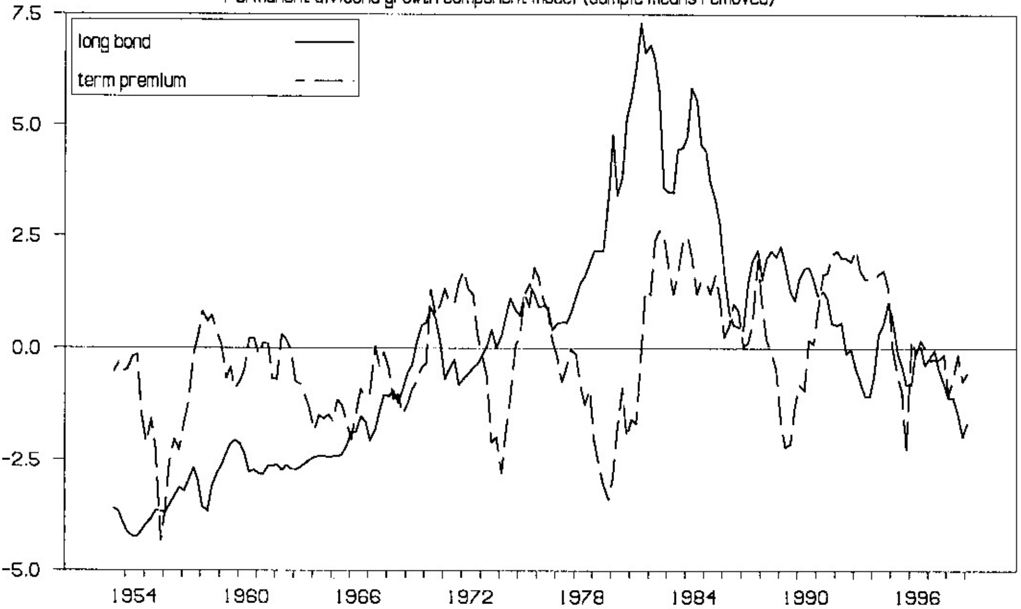
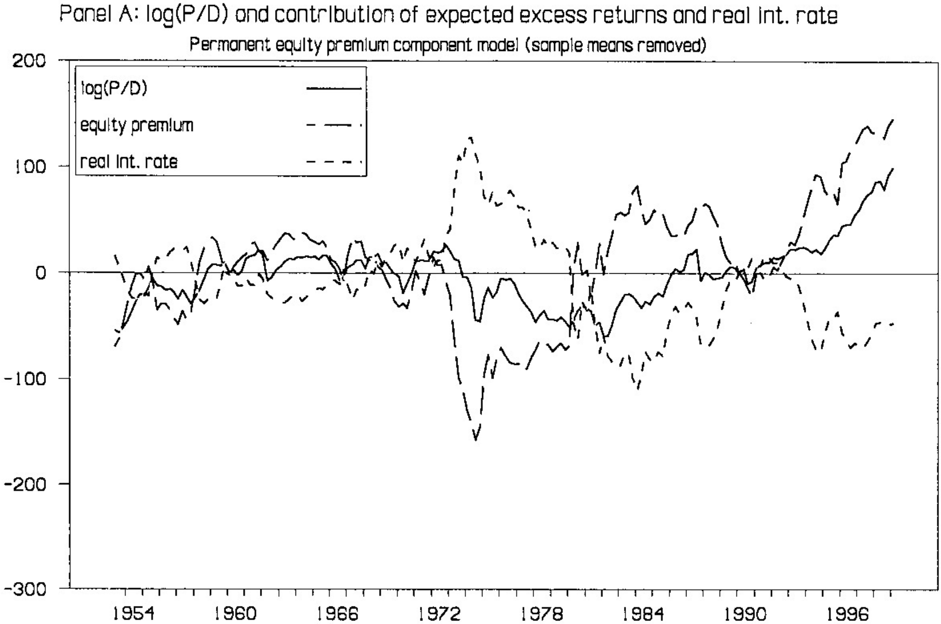


Figure 4.


Panel A: log(P/D) and contribution of expected excess returns and real int. rate
Permanent equity premium component model (sample means removed)
log(P/D)
equity premium
real int. rate
200
100
0
-100
-200
-300
1954
1960
1966
1972
1978
1984
1990
1996

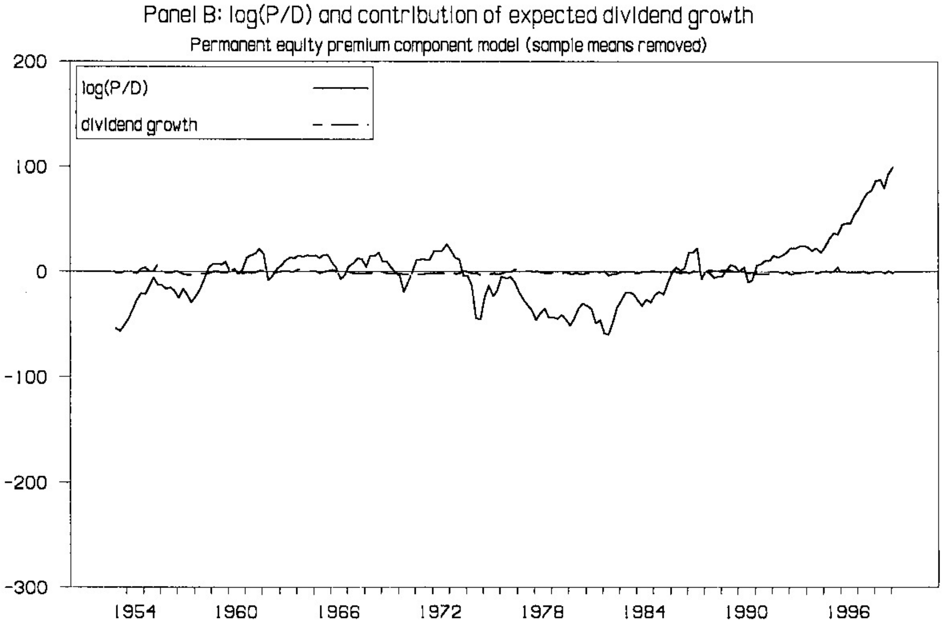
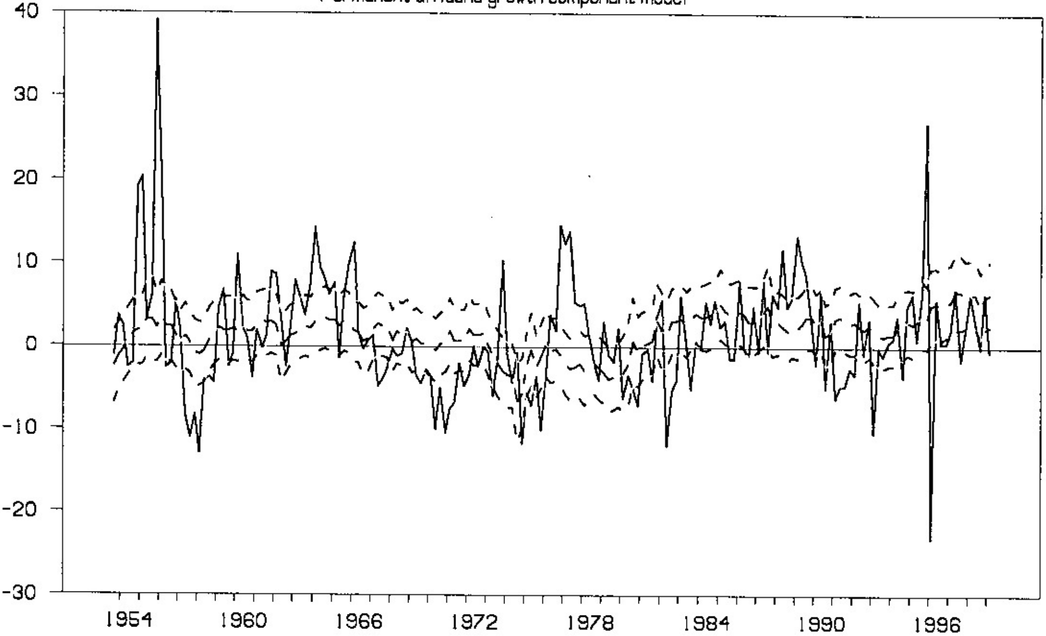

Panel B: log(P/D) and contribution of expected dividend growth
Permanent equity premium component model (sample means removed)
log(P/D)
dividend growth
200
100
0
-100
-200
-300
1954
1960
1966
1972
1978
1984
1990
1996

Figure 5

Panel A: Actual and long-run real dividend growth with 2 SE band

Permanent dividend growth component model



Panel B: Actual and long-run excess stock returns with 2 SE band

Permanent excess stock returns component model