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Federal Reserve Bank of Dallas

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Abstract

This paper conducts a theoretical comparison of the potential effectiveness, in terms of money stock controllability, of interest rate and reserve instruments. Whereas previous studies have been basically static, the present analysis is carried out in the context of a dynamic macroeconomic model with rational expectations. Particular attention is paid to the distinction between contemporaneous and lagged reserve accounting (CRA and LRA). The criterion employed is the expectation of squared deviations of the (log of the) money stock from target values that are reset each period. Analysis in the basic model suggests the following substantive conclusions. (1) With a reserve instrument, monetary control will be more effective under CRA than LRA. (2) With a reserve instrument and LRA, control will be poorer than with an interest rate instrument. (3) For a wide range of parameter values, control will be better with a reserve instrument and CRA than with an interest rate instrument.

I. Introduction

The purpose of this paper is to conduct a theoretical comparison of the potential effectiveness, in terms of money stock controllability, of interest rate and reserve instruments. Several studies with a similar objective have previously been conducted, some of the more notable including Pierce and Thomson (1972), Friedman (1975) (1977), Le Roy (1979), and Axilrod and Lindsey (1981).[1] These previous studies have all, however, been conducted in models that are essentially static and therefore neglect the distinction between real and nominal rates of interest. Our analysis, by contrast, is dynamic and presumes rational expectations throughout. In the present paper we employ as a policy criterion the unconditional expectation of the squared deviation of the (log of the) money stock from "target" values that are reset period by period in light of complete aggregate data from previous periods. Given this criterion, some of our conclusions are similar to those obtained in static models. Our approach permits the derivation of some new results, however, and provides a firmer understanding of the static-model conclusions. In addition, our discussion indicates how the analysis can be extended to cases in which the static-model results may not provide useful approximations.

Given the extent of professional and popular discussion of the basic "instrument choice" problem, it might seem surprising that no such analysis has previously been conducted. There is, however, a straightforward explanation for the absence of previous studies: until very recently it appeared that the values of all nominal magnitudes (including the money stock) are <u>indeterminate</u> in a rational expectations context if the monetary authority uses an interest rate as its operating instrument.[2] It has been shown in McCallum (1980a), however, that this indeterminancy does not prevail if an interest rate instrument is used but is set period by period in a manner that is designed to have some desired effect on the expected quantity of money in the upcoming period -- that is, if at least some weight is given to a money stock target.[3] Thus, it is in fact possible to conduct an analytical comparison of the type desired, even in a model entirely free of private sector money illusion and expectational irrationality.

In the context of the interest rate vs. reserve instrument comparison, there is a related choice to be made by the monetary authority that is of considerable significance. In particular, given that banks are required to hold reserves against their monetary liabilities, it is crucial to distinguish between a system featuring contemporaneous reserve accounting, which makes current monetary liabilities relevant for the determination of current required reserves, and one with lagged reserve accounting, which makes liabilities from a previous period relevant for current reserve requirements. This distinction is crucial because, with a reserve instrument, potential monetary control is likely to be poorer[4] under lagged reserve accounting ("LRA") than under contemporaneous reserve accounting ("CRA"). In fact, the combination of a reserve instrument with LRA tends to produce the poorest results, in terms of potential monetary control, of any of the four combinations defined by the two instruments and two accounting schemes. It is nevertheless the case that a reserve instrument with CRA will provide the best monetary control in our model for a large subset of plausible parameter values.

The relevance of these results for recent monetary experience in the United States is readily apparent. Since the Federal Reserve switched from CRA to LRA in 1968, any subsequent attempts to use a reserve instrument will have yielded poorer monetary control than would have been possible under CRA (or with an interest rate instrument). It would therefore be highly inappropriate to draw direct conclusions about the potential effectiveness of a reserve instrument on the basis of U.S. experience since the October 6, 1979, initiation of a period with increased emphasis on reserves.[5]

The analytical framework within which we shall demonstrate these results is a log-linear macroeconomic model similar to the IS-LM-Phillips Curve setups used by Sargent and Wallace (1975), Sargent (1979), and McCallum (1980a), with the addition of a relationship describing portfolio behavior of the banking system. Results are also obtained for a modification in which the Sargent-Wallace supply function, which permits prices to adjust freely within each period, is replaced with one that has prices set in advance. In both versions the model is highly simplified, of course, and is open to various criticisms.[6] The most basic of these is that the behavioral relations are not obtained from an analysis of optimizing behavior by individual agents, but are simply posited as plausible and orthodox relationships. As a result, the assumption that parameter values in the behavioral relations will be the same under various policy regimes is not well justified.[7] Thus, even with the incorporation of rational expectations, our analysis goes only part of the way toward the goal of a policy-invariant model.

It should be said at the outset that our emphasis on the controllability of money does not imply disagreement with the argument -- made by Kareken, Muench and Wallace (1973), Friedman (1975) (1977), and others -- that it is in principle inefficient for the monetary authority to rely on the "intermediate target" procedure. Certainly it would be better simply to focus upon ultimate target or "goal" variables such as unemployment and/or inflation rates, rather than upon a money stock target, if reliable models describing the effects of potential instruments upon such goal variables were available. In fact, however, it appears that such models are not available. Furthermore, the Federal Reserve System of the United States is significantly concerned with intermediate money-stock targets, and is charged to be so concerned by the U.S. Congress. Consequently, much professional and practical analysis presumes the use of monetary targets. Given this situation, our object is to advance professional understanding of the alternative control techniques potentially available.

The outline of the paper is as follows. In Section II we describe the model and in Section III obtain the main analytical results for the cases with contemporaneous reserve accounting. In Section IV comparable results for lagged reserve accounting are derived, and in Section V qualifications and ideas for future study are described. A brief conclusion and an appendix are also provided.

II. Analytical Framework

Let us begin by briefly describing the relationships that represent aggregate demand and supply behavior in our model economy, before turning to consideration of the banking sector. As in McCallum (1980a), we adopt slightly modified versions of the IS, LM, and aggregate supply (Phillips curve) relationships used by Sargent and Wallace (1975). Let p_t, y_t, and m_t denote logarithms of aggregate output, the price level, and the money stock (respectively) and let r_t be the nominal interest rate. Also, let u_t, v_t and η_t be stochastic disturbances. Then we have:

$$y_t = b_o + b_1[r_t - (E_{t-1}p_{t+1} - p_t)] + v_t \qquad b_1 < 0 \tag{1}$$

$$m_t - p_t = c_o + c_1 r_t + c_2 y_t + \eta_t \qquad c_1 < 0 < c_2 \tag{2}$$

$$y_t = a_o + a_1(p_t - E_{t-1}p_t) + a_2 y_{t-1} + u_t \qquad a_1 > 0, \quad 1 > a_2 \geq 0. \tag{3}$$

In (1), demand for consumption plus investment is negatively related to the real rate of interest. Here and elsewhere the operator E_{t-1} denotes the expectation of the indicated variable within the model and conditional upon values of all variables realized in periods prior to t. The precise specification of the real interest rate variable follows Sargent (1979) and is discussed in McCallum (1980a, pp.4-5).[8]

Equation (2) is a demand function for money, the asset used by the economy's agents as a medium of exchange. The demand for real balances is taken to be positively dependent upon the transaction "scale" variable y_t and negatively dependent upon the nominal rate of interest.

Finally, equation (3) is an aggregate supply function of the natural rate variety, rationalized by Lucas (1973) and utilized by numerous authors. Since some critics find the extent of price level flexibility provided by (3) to be excessive, we shall also consider in Section III an alternative specification that makes prices "sticky" within each period -- and which leads to a simplification of the instrument comparison expressions.

The stochastic disturbances in equations (1)-(3), u_t, v_t, and η_t, are assumed to be generated by mutually independent white noise processes. Each disturbance, moreover, is taken to be independent of (as well as uncorrelated with) past values of all disturbances and variables.

We now turn to the banking sector. The standard specification in the literature cited in the introduction relates the money stock positively to the nominal market rate of interest and to some reserve aggregate.[9] Let h_t denote the log of the relevant reserve aggregate -- e.g., total reserves, non-borrowed reserves, or the monetary base. Then a stochastic version of the standard relationship might be written as

$$(4) \qquad m_t = \nu_o + \nu_1 r_t + \nu_2 h_t + \zeta_t \qquad \nu_1, \nu_2 > 0$$

where ζ_t is a temporally independent, white noise disturbance that is also independent of u_t, v_t, and η_t. Relations of this type have been referred to as "money supply" functions by some writers and as "reserve demand" functions by others. The first of these terms is not generally appropriate, however, since such functions play no part in determining the value of m_t when an interest rate instrument is being employed. Consequently, we shall use the reserve-demand terminology in what follows.[10]

There are also several substantive points concerning equation (4) that require discussion. First, since m_t and h_t pertain to values for the same time period, the equation can be applicable only if CRA is in effect. If, on the other hand, LRA is operative, required reserves will be related to some past value of m_t. To keep matters as simple as possible, let us suppose that our "periods" are of the same duration as the lag in reserve accounting. Then under LRA we might have[11]

$$(4') \qquad m_{t-1} = \nu_o + \nu_1 r_t + \nu_2 h_t + \zeta_t \; .$$

where, in order to maintain comparability with (4), we suppose that the parameters on the right-hand side -- including the variance of ζ_t -- are precisely the same as under CRA. As that assumption is important in what follows, it is here that the aforementioned absence of individual optimization analysis is most serious.

Another issue that needs to be addressed is whether there is, in fact, a reserve aggregate that can be manipulated as an instrument. Our view, basically, is that a variable can appropriately be treated as an instrument -- presuming that it can be affected by the Fed -- if it can be observed "instantaneously". The idea, of course, is that if a variable can be observed instantaneously it can be "continuously" monitored and therefore kept on its chosen path by application of the requisite open-market stimulus. One conclusion, then, is that any variable that can be measured from the Fed's own balance sheet is in principle a feasible instrument: given today's computational and communicational facilities it is technically possible for the Fed to compile daily balance sheets. And in the context of a model with a two-week (or one-week) time period, daily observations on a variable make it effectively observable "instantaneously". Obviously this argument does not identify which specific variables are actually available as instruments in the U.S. economy -- that will depend on current regulations and institutions.[12] But it seems to provide adequate refutation of the occasionally-voiced notion that no reserve aggregate can be used as an instrument.[13]

Of course we recognize that under current arrangements precise control of reserve aggregates cannot be accomplished even with contemporaneous observability. One reason is that open-market operations cease before the discount window closes on the last day of each statement period -- on each Wednesday afternoon. Also we recognize that complete balance sheets are not compiled daily. Such practices are not, however, immutable. In a study of the potential for monetary control under alternative regimes, it seems inappropriate to presume that the Fed would fail to take feasible steps that would make potential instruments controllable. Our aim is not to predict what policy makers will do, but to understand the effects of what they could do under alternative, feasible arrangements.[14]

Given the point of view just expressed, we shall not specify which of the frequently-discussed reserve measures -- total reserves, non-borrowed reserves, or the monetary base -- is referred to by our reserve variable, h_t.[15] As there is comparatively little dispute about which actual interest rate should be emphasized, we shall occasionally refer to r_t as the "federal funds rate". Formally, however, it is simply "the" nominal interest rate in our aggregative model.

Finally, to complete the model, we need a relation describing policy behavior, one which determines either h_t or r_t on a period-by-period basis. In each case we assume that the instrument, h_t or r_t, is set according to a deterministic feedback rule that specifies h_t or r_t as a linear function of variables realized in period t-1 or before. And in each case, we assume that this linear function is chosen so as to make the expected value of m_t, that is, $E_{t-1}m_t$, equal to a target value denoted m_t^*. We conceive of m_t^* itself being set by a deterministic, linear feedback rule -- perhaps, but not necessarily, one that attempts to "lean against the wind" in some fashion. But since our concern will be how well the target values m_t^* (for t=1,2,...) are attained, we will not need to specify any particular rule for determining m_t^*. All we need note is that, since it is determined by a feedback rule, there are no one-period expectational errors: $E_{t-1}m_t^* = m_t^*$.

III. Monetary Control Under CRA

In this section we derive expressions for our monetary control criterion, the mean-squared error $E(m_t - m_t^*)^2$, under a regime of contemporaneous reserve accounting. First we find the value of $E(m_t - m_t^*)^2$ implied by the model when h_t is set at the beginning of period t to make $E_{t-1}m_t = m_t^*$, with r_t then determined in the marketplace. We find, that is, the minimum mean square control error when the reserve aggregate is used as the operating instrument. In this case, the relevant system of equations includes (1), (2), (3), (4), and the instrument setting

$$h_t = (m_t^* - \nu_o - \nu_1 E_{t-1} r_t)/\nu_2. \tag{5}$$

These equations determine values of y_t, p_t, m_t, r_t, and h_t.

Since our interest is in the expected square of $m_t - m_t^*$, let us begin by using (4) and (5) to obtain

$$m_t - m_t^* = \nu_1(r_t - E_{t-1}r_t) + \zeta_t. \tag{6}$$

Next we develop "innovation" versions of (1)-(3) by applying the operator E_{t-1} to each equation and, for each, subtracting the resultant from the original equation. The results are:

$$y_t - E_{t-1}y_t = b_1(r_t - E_{t-1}r_t) + b_1(p_t - E_{t-1}p_t) + v_t \tag{7}$$

$$m_t - m_t^* = p_t - E_{t-1}p_t + c_1(r_t - E_{t-1}r_t) + c_2(y_t - E_{t-1}y_t) + \eta_t \tag{8}$$

$$y_t - E_{t-1}y_t = a_1(p_t - E_{t-1}p_t) + u_t. \tag{9}$$

Elimination of the innovations for y_t, p_t, and r_t then leaves us with the desired expression for $m_t - m_t^*$. It is:

$$m_t - m_t^* = (1 - \psi_1)^{-1}[\eta_t + \phi_1 v_t + (c_2 - \phi_1)u_t - \psi_1 \zeta_t] \tag{10}$$

where $\phi_1 \equiv \dfrac{1 + a_1 c_2}{a_1 - b_1} > 0$

and $\psi_1 \equiv \left(\dfrac{\phi_1 b_1 + c_1}{\nu_1}\right) < 0$

Since we have assumed that the disturbances are mutually independent, the mean squared control error is then

$$E(m_t - m_t^*)^2 = \frac{\sigma_\eta^2 + \phi_1^2\sigma_v^2 + (c_2 - \phi_1)^2\sigma_u^2 + \psi_1^2\sigma_\zeta^2}{(1 - \psi_1)^2} \tag{11}$$

where σ_η^2 is the variance of η_t, etc.

Next we take the case in which the federal funds rate, r_t, is the operating instrument. Now the optimal setting, which makes $E_{t-1}m_t = m_t^*$, is

$$r_t = c_1^{-1}[m_t^* - c_o - E_{t-1}p_t - c_2 E_{t-1}y_t] \tag{12}$$

and $m_t - m_t^*$ is expressible as

$$m_t - m_t^* = p_t - E_{t-1}p_t + c_2(y_t - E_{t-1}y_t) + \eta_t. \tag{13}$$

Equations (9) and (7) -- with $r_t - E_{t-1}r_t = 0$ -- can then be used to eliminate the innovations for p_t and y_t. The resulting expression is

(14) $$m_t - m_t^* = \eta_t + \phi_1 v_t + (c_2 - \phi_1)u_t$$

where ϕ_1 is as before. The control criterion is, obviously,

(15) $$E(m_t - m_t^*)^2 = \sigma_\eta^2 + \phi_1^2\sigma_v^2 + (c_2 - \phi_1)^2\sigma_u^2 .$$

Given our specification, it is not clear whether expression (11) or (15) is the smaller. The variance of the banking sector disturbance, σ_ζ^2, does not appear on the right-hand side of (15), which tends to make the criterion smaller with the funds rate instrument. But the divisor, $(1 - \psi_1)^2$ in (11), is unambiguously greater than 1.0, which tends to make the reserve instrument preferable. Which tendency predominates depends upon the magnitudes of ψ_1 and the various variances.

One useful way of simplifying the expression for $E(m_t - m_t^*)^2$ is to note that, as the parameter a_1 grows in magnitude, ϕ_1 approaches c_2. This eliminates the term involving σ_u^2 from (11) and (15), and generates the implication that the reserve instrument h_t will be superior to the funds rate instrument r_t if

(16) $$\frac{\sigma_\eta^2 + c_2^2\sigma_v^2 + \psi_1^2\sigma_\zeta^2}{(1 - \psi_1)^2} < \sigma_\eta^2 + c_2^2\sigma_v^2 .$$

where $\psi_1 = \dfrac{c_2 b_1 + c_1}{\nu_1} < 0$. Rearranging, we obtain the condition

(17) $$\frac{\psi_1^2}{\psi_1^2 - 2\psi_1} < \frac{\sigma_\eta^2 + c_2^2\sigma_v^2}{\sigma_\zeta^2} .$$

Here the left-hand expression is positive and less than 1.0. Thus, even without knowledge of the magnitude of ψ_1, we can conclude that the reserve instrument will be superior provided that

$$(18) \qquad 1 < \frac{\sigma_\eta^2 + c_2^2\sigma_v^2}{\sigma_\zeta^2}$$

But this condition will obtain, whatever the magnitude of c_2, unless σ_ζ^2 is at least as large as σ_η^2. Furthermore, relatively small absolute values of ψ_1 -- reflecting relatively large values of ν_1 -- will decrease the left-hand side of (18) and make the sufficient condition less stringent. Consequently, it seems rather likely that the reserve instrument will be superior -- given contemporaneous reserve accounting! -- if a_1 is large.[16]

But why should that condition, a relatively large value for a_1, be considered likely? The answer, of course, is that a large a_1 value implies a high sensitivity of supply to the price level expectation error. Conversely, then, a large a_1 value corresponds to a situation in which actual inflation responds <u>weakly</u>, given inflationary expectations, to the value of y_t relative to normal -- i.e., to a relatively flat one-period Phillips relationship. That such a specification is empirically relevant is, of course, widely believed to be the case.

A bit of additional discussion may be warranted. Suppose we write (3) in the form just alluded to, as follows:

$$(3') \qquad p_t - p_{t-1} = \frac{1}{a_1}[y_t - a_2y_{t-1} - a_o] + E_{t-1}(p_t - p_{t-1}) - \frac{u_t}{a_1}\,.$$

In the limit, as $a_1 \to \infty$, this relationship degenerates to the condition $p_t = E_{t-1}p_t$. Given rational expectations, this implies that p_t is determined entirely by conditions prevailing in t-1 and is unaffected by shocks occurring in t. Beyond that, however, it does not specify aggregate supply or price behavior so some additional relationship must be included as a replacement for (3). One attractive specification, previously used by Barro and Grossman (1976), Mussa (1980), and McCallum (1980b), is as follows:

$$p_t - p_{t-1} = \alpha_1(y_{t-1} - \bar{y}_{t-1}) + E_{t-1}(\bar{p}_t - \bar{p}_{t-1}) \qquad \alpha_1 > 0$$

Here $\bar{y}_t$ is the capacity or "full employment" value of y_t while $\bar{p}_t$ is the value of p_t that would make aggregate demand equal to $\bar{y}_t$, given actual policy actions in t. Thus (19) is an accelerationist Phillips curve with the inflation rate determined by the previous period's level of excess demand and the expected rate of inflation of the full-employment price level, rather than the actual price level. Considerable discussion of such a relationship is provided by McCallum (1980b) and Mussa (1980). For present purposes, the main relevant feature of (19) is simply that it makes p_t predetermined and thus equal to $E_{t-1}p_t$, so that the $p_t - E_{t-1}p_t$ terms vanish from equations (7) and (8). Other supply functions with that property would also lead to the conclusion based on (18).

IV. Monetary Control Under LRA

We now turn to the derivation of expressions for the criterion $E(m_t - m_t^*)^2$ in regimes with lagged reserve accounting. In the case in which the instrument is the funds rate r_t, there are no new calculations to make: since banking behavior does not affect m_t when r_t is the instrument, the relevant criterion expression is again given by (15). In the case of the reserve instrument, however, a new calculation is needed. Indeed, the nature of monetary control with a reserve instrument is drastically different under LRA than it is under CRA. This can be seen from the banking sector behavioral equation (4'), in which the effect of h_t on m_t occurs only <u>indirectly</u>, by way of r_t, when LRA is in force.[17] The optimal setting for h_t is obtained from (4') only after r_t in (4') is replaced by the value of $E_{t-1}r_t$ that equates $E_{t-1}m_t$ to m_t^* in the money demand function (2), as follows:

$$(20) \qquad h_t = [m_{t-1} - \nu_o - \nu_1 c_1^{-1}(m_t^* - E_{t-1}p_t - c_o - c_2\, E_{t-1}y_t)] \;/\; \nu_2 \;.$$

Thus with LRA, monetary control with a reserve instrument amounts to an indirect method of exercising control with an interest rate! One would expect such a procedure to provide relatively poor monetary control, and such is the case -- as we shall now demonstrate.

To determine the value of $E(m_t - m_t^*)^2$ with LRA and the h_t instrument, we use equations (7), (8), and (9) and the innovation version of (4'), which is

$$(21) \qquad 0 = \nu_1(r_t - E_{t-1}r_t) + \zeta_t \;.$$

Elimination of innovations in y_t, p_t, and r_t from these four equations then yields

(22) $$m_t - m_t^* = \eta_t + \phi_1 v_t + (c_2 - \phi_1)u_t - \psi_1 \zeta_t$$

where ϕ_1 and ψ_1 are as defined above. From (22), then, we immediately obtain

(23) $$E(m_t - m_t^*)^2 = \sigma_\eta^2 + \phi_1^2\sigma_v^2 + (c_2 - \phi_1)^2\sigma_u^2 + \psi_1^2\sigma_\zeta^2 .$$

And from the latter we are able to draw some quite definite conclusions. First, since $\psi_1 < 0$, $(1 - \psi_1)^2$ is greater than 1.0, so a comparison of (23) with (11) shows that the mean squared control error with the h_t instrument is unambiguously larger under LRA than under CRA.

The quantitative magnitude of this effect may not, moreover, be small. To see this, suppose that $c_1 = -\nu_1$; i.e., that the interest rate semi-elasticities of money demand and supply are equal in magnitude. Then with $\phi_1 > 0$ and $b_1 < 0$, the value of $\psi_1 = (\phi_1 b_1 + c_1) / \nu_1$ is negative and greater than 1.0 in absolute value. Consequently, $1 - \psi_1$ exceeds 2.0 and $(1 - \psi_1)^2$ is greater than 4.0. The magnitude of the mean squared control error in (23) is therefore over four times as great as in (11).

Furthermore, since $(-\psi_1)^2 > 0$, the mean squared error in (23) is also greater than that in (15), which holds for the r_t instrument with either CRA or LRA. Thus, the h_t - LRA combination provides the poorest monetary control among the four possibilities considered.[18]

Finally, since some analysts have suggested that excess reserves are highly insensitive to interest rate movements, let us briefly consider results that obtain as $\nu_1 \to 0$. This implies, since $\psi_1 = (\phi_1 b_1 + c_1)/\nu_1$, that $\psi_1 \to \infty$. Of course, the behavior of the control error is independent of reserve demand parameters when r_t is used as the instrument, a conclusion that is verified by expression (15). With h_t used as the instrument, however, $E(m_t - m_t^*)^2$ approaches σ_ζ^2 and ∞ under CRA AND LRA systems, respectively: see expressions (11) and (23). Thus the results in this limiting case do not alter the main conclusions.

V. Generalizations and Qualifications

It is important to recognize that the foregoing results are robust to model specification in some significant respects. One such respect involves distributed-lag modifications of the behavioral equations. Suppose, for example, that the money demand equation (2) was replaced with a distributed-lag version such as the following:

$$m_t - p_t = c_o + \sum_{j=0}^{J} c_{1j} r_{t-j} + \sum_{j=0}^{J} c_{2j} y_{t-j} + \eta_t . \tag{24}$$

Here it is assumed that past, as well as present, values of r_t and y_t influence money demand in period t, with the number of relevant past values, J, arbitrarily large. It is important that generalizations of this type be considered, because with a two-week definition of the time period, large values of J will presumably be required for a realistic specification. But this extension has no effect on the foregoing analysis, for it is the <u>innovation</u> version of the relationship that enters in the various calculations of $E(m_t - m_t^*)$. And the innovation version of (24) is simply

$$m_t - m_t^* - (p_t - E_{t-1}p_t) = c_{10}(r_t - E_{t-1}r_t) + c_{20}(y_t - E_{t-1}y_t) + \eta_t, \tag{25}$$

which is identical in form to (8). All that is needed in making this generalization is to interpret the a, b, c, and ν parameters as those applicable to the first-period response in a distributed-lag formulation. Thus, for example, c_1 in (8) must be interpreted as c_{10} in (24). Clearly, the same type of argument would apply if, instead of (24), we had a money-demand specification of the partial adjustment type often used in empirical work, as follows:

$$m_t - p_t = c_o + c_1 r_t + c_2 y_t + c_3(m_{t-1} - p_{t-1}) + \eta_t \ . \tag{26}$$

Furthermore, the stochastic disturbances in the behavioral equations do not need to be white noises. If η_t in (2) were, for instance, of the form

$$\eta_t = \rho\eta_{t-1} + \xi_t \tag{27}$$

with ξ_t white, then a familiar transformation would convert the equation to one with lagged values of r_t, y_t, and $m_t - p_t$ on the right-hand side in which the disturbance is the white noise ξ_t . Consequently, expressions such as (11), (15), and (23) continue to prevail if the disturbance variances are interpreted as those applicable to the unpredictable component of behavior in period t, such as ξ_t instead of η_t.

There are, however, other modest changes in the model's specification which will not leave our results unaffected. One that is of considerable importance involves the dating of the expectation operator in the IS equation. According to specification (1), agents do not have knowledge of period-t magnitudes when forming expectations of inflation between periods t and t+1, expectations used in converting nominal into real interest rates. An alternative specification that gives agents knowledge of period-t values is as follows:

$$y_t = b_o + b_1[r_t - (E_t p_{t+1} - p_t)] + v_t \tag{27}$$

Here $E_t p_{t+1}$ is the expectation of p_{t+1} within the model conditional upon values of y, m, p, r, and h in period t, as well as periods prior to t. Given this change, the innovation version of the IS function, equation (7), no longer obtains. In its place we have instead

$$(28)\quad y_t - E_{t-1}y_t = b_t(r_t - E_{t-1}r_t) - b_1(E_tp_{t+1} - E_{t-1}p_{t+1}) + b_1(p_t - E_{t-1}p_t) + v_t,$$

in which the change in the expectation (between t-1 and t) of p_{t+1} appears. With this change, the model becomes dynamic in a more thoroughgoing sense. As a result, it ceases to be one in which $E(m_t - m_t^*)^2$ is computable without reference to the behavioral rules specifying m_t^*.

There is one special case of some interest in which the results presented in previous sections continue to hold precisely even if (27) is used instead of (1). That case is the one in which no lagged terms appear in the IS, LM, or aggregate supply equations -- i.e., in which equations (27), (2), and (3) prevail with $a_2 = 0$ -- and the policy rule specifying m_t^* makes these target values exogenous and known in advance (as, for example, a constant money-growth rule). Under these conditions, the usual undetermined-coefficient solution equations for the endogenous variables will include only m_t^* and current disturbances on the right-hand side. In particular, p_t will obey

$$(29)\quad p_t = \pi_{10} + \pi_{11}m_t^* + \pi_{12}u_t + \pi_{13}v_t + \pi_{14}\eta_t + \pi_{15}\zeta_t$$

where the π_{ij}'s are constant coefficients related to the parameters of (27), (2), (3) and the policy rule. Consequently, we have

$$(30)\quad E_tp_{t+1} - E_{t-1}p_{t+1} = \pi_{10} + \pi_{11}E_tm_{t+1}^* - (\pi_{10} + \pi_{11}E_{t-1}m_{t+1}^*) = 0$$

and the change in the expectation of p_{t+1} disappears from (28), leaving (7) to be used as before in the computation of $m_t - m_t^*$.

In general, however, the results will differ from those based on the IS specification (1) and will depend upon the policy rule that governs m_t^*. For any given rule the model is solvable in principle, but the calculations may be much more complex than those presented above. Furthermore, the results for $E(m_t - m_t^*)^2$ will depend upon the precise distributed-lag specifications in the IS, LM, aggregate supply, and reserve demand equations. This sensitivity makes it unlikely that any conclusions of wide applicability would be forthcoming from an analysis that uses specification (27).

VI. Conclusions

The two main substantive results of the paper are easily summarized, as follows. First, from the standpoint of monetary control, a reserve instrument will[19] perform less well with lagged reserve accounting than with contemporaneous reserve accounting. Second, it seems likely that a reserve instrument will, with comtemporaneous reserve accounting, permit tighter monetary control than will an interest rate instrument (which is equally effective under lagged and contemporaneous accounting).

In terms of analytical interest, one conclusion is that the messages concerning monetary control are not drastically different in a dynamic, rational expectations model than in a static model provided that current information is unavailable to agents in forming expectations about the inflation rate relevant to the real-nominal interest rate distinction. If this current information is available, however, the substantive results described above will hold only for highly special cases of the model and it seems unlikely that other general results are obtainable.

APPENDIX

The purpose here to derive the expression for the mean-squared money control error, $E(m_t - m_t^*)^2$, for the case in which a reserve instrument is used under LRA and the reserve demand equation is of the form mentioned in footnote 11, namely,

$$\text{(A-1)} \quad \delta m_t + (1 - \delta)m_{t-1} = \nu_0 + \nu_1 r_t + \nu_2 h_t + \zeta_t$$

with $0 \leq \delta \leq 1$. Here it is presumed that the right-hand side parameters (including σ_ζ^2) have the same values as in equations (4) and (4').

Before beginning the analysis, let us briefly motivate specification (A-1). Suppose that the reserve instrument used is the monetary base, $H_t = R_t + C_t$, where R_t and C_t denote total reserves and currency in circulation, respectively. Also let the money stock be $M_t = C_t + D_t$, where D_t = demand deposits. (These are raw values, not logarithms.) Then assume that money holders keep a constant fraction, k, of their money balances in the form of currency; thus $C_t = kM_t$ and $D_t = (1-k)M_t$. Next assume that banks' desired reserves can be expressed as

$$\text{(A-2)} \quad R_t = \rho(D_{t-1}, r_t)$$

with $\rho_1 > 0$ and $\rho_2 < 0$. That desired reserves in period t must depend positively upon D_{t-1} is clear, given LRA. The lower is r_t, in addition, the more excess reserves will be held for possible use in period t + 1.

From (A-2) and the identities we have

$$\begin{aligned} \text{(A-3)} \quad H_t &= C_t + \rho(D_{t-1}, r_t) \\ &= kM_t + \rho[(1-k)M_{t-1}, r_t] \end{aligned}$$

which relates H_t positively to both M_t and M_{t-1}, and negatively to r_t. Equation (A-1) is simply a rearranged version of the best approximation to (A-3) that is linear in $h_t = \log H_t$, $m_t = \log M_t$, $m_{t-1} = \log M_{t-1}$, and r_t.

To determine the value of $E(m_t - m_t^*)^2$ in the case at hand we use equations (7), (8), and (9) plus the innovation version of (A-1), which is

$$\text{(A-4)} \quad \delta(m_t - m_t^*) = \nu_1(r_t - E_{t-1}r_t) + \zeta_t.$$

Elimination of the innovations in y_t, p_t, and r_t from these four equations then yields

$$\text{(A-5)} \quad m_t - m_t^* = \frac{\eta_t + \phi_1 v_t + (c_2 - \phi_1)u_t - \psi_1\zeta_t}{1 - \delta\psi_1}$$

where ϕ_1 and ψ_1 are defined above. From (A-5), then we immediately obtain

$$\text{(A-6)} \quad E(m_t - m_t^*)^2 = \frac{\sigma_\eta^2 + \phi_1^2\sigma_v^2 + (c_2 - \phi_1)^2\sigma_u^2 + \psi_1^2\sigma_\zeta^2}{(1 - \delta\psi_1)^2}$$

which is the desired expression.

To what extent do the conclusions of Section IV survive the change from (4') to (A-1)? First, since $\psi_1 < 0$, the value of $(1 - \psi_1)^2$ in (11) is greater than the value of $(1 - \delta\psi_1)^2$ in (A-6). Thus the mean squared control error with the h_t instrument continues to be unambiguously larger under LRA than under CRA. And with small values of δ the quantitative magnitude of the difference may again be large.

Second, since $(-\psi_1)^2 > 0$, the numerator on the right-hand side of (A-6) is greater than the expression in (15). Thus, for small values of δ, the mean squared error in (A-6) will exceed that in (15). For a substantial range of parameter values, then, the h_t- LRA combination continues to provide the poorest monetary control, even with the modified reserve demand equation.

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FOOTNOTES

1. These studies employ an analytical approach developed in the well-known and justly influential paper by Poole (1970).

2. The source of this belief is the famous paper by Sargent and Wallace (1975). The result developed in that paper (and discussed further by Sargent (1979)) obtains when the interest rate policy rule is not designed to have a desired effect on the money stock or the price level.

3. In this case, the system includes one economic actor -- the monetary authority -- who is concerned with nominal magnitudes. Price level indeterminacy occurs, as Patinkin (1965, pp. 303-309) clearly describes, when there is <u>no one</u> whose real supply or demand behavior depends upon nominal magnitudes.

4. "Poorer" in terms of our criterion, described above, and in our model.

5. Here we are accepting at face value the Fed's claim to have altered procedures to more nearly reflect a reserve instrument procedure. It would probably not be accurate, however, to describe this recent period as one in which a reserve instrument is used in any very pure sense.

6. Severe criticism of IS-LM relationships, even when used with classical supply functions, has been expressed by Kareken and Wallace (1980).

7. Of course the same is true of the studies of Pierce and Thomson (1972), Friedman (1975) (1977), Le Roy (1979), and virtually all writers on the subject of monetary control. A partial exception is provided by Goodfriend (1981), who develops a dynamic optimizing analysis of banks' behavior with respect to borrowed reserves.

8. The possibility that agents have knowledge of period t magnitudes when forming expectations regarding p_{t+1} is considered below, in Section V.

9. See, for example, Pierce and Thomson (1972, p. 118), Friedman (1975, p. 445) (1977, p. 305), LeRoy (1979, p. 461), Sivesind and Hurley (1980, p. 200), and -- implicitly -- Axilrod and Lindsey (1981, p. 247).

10. We recognize, nevertheless, that in some cases equations analogous to (4) may reflect not only the portfolio behavior of banks but also the desires of the non-bank public regarding the composition of its money holdings.

11. More generally, reserve demand might be dependent upon both current and lagged values of the money stock, in which case the relationship comparable to (4) would be $\delta m_t + (1-\delta)m_{t-1} = \nu_0 + \nu_1 r_t + \nu_2 h_t + \zeta_t$ with $0 \leq \delta \leq 1$. This sort of relationship would be implied if h_t were interpreted as the monetary base and the non-bank public maintained a constant ratio of currency to demand deposits. For more discussion, and an analysis of this case, see the Appendix.

12. The practice of counting lagged vault cash toward current reserves, for example, tends to make total reserves more nearly controllable than the base.

13. Previous arguments along lines similar to ours have been made by Burger (1972) and Meltzer (1969).

14. We do, however, presume that fractional reserve banking is maintained.

15. We feel compelled, however, to dispute the contention that total reserves could not possibly be used as an instrument under LRA. Clearly, such a regime could be effected if the discount window were closed before the cessation of open-market activities on the last day of the statement period. Banks would then simply maintain a large enough stock of excess reserves that defaults would occur infrequently. How infrequently would of course depend upon the severity of default penalties (which at present are formally surprisingly mild).

16. The same result was obtained, within a static model, by Axilrod and Lindsey (1981). Also see LeRoy and Lindsey (1978).

17. This and subsequent statements in this paragraph are based on the reserve demand specification (4') and are not strictly applicable to the more general version used in the Appendix.

18. This conclusion does not obtain for all parameter values when (4') is modified as in the Appendix. It still holds, however, for small values of the parameter δ.

19. In the model considered.