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Three-Candidate Competition when Candidates Have Valence: The Base Case

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Abstract: We study the Nash Equilibrium of three-candidate unidimensional spatial competition when candidates differ in their non-policy characteristics (valence). If the voters' policy preferences are represented by a strictly convex loss function, and if the voter density is unimodal and symmetric, then a unique, modulo symmetry, local Nash Equilibrium exists under fairly plausible conditions. The global Nash Equilibrium, however, exists when only one candidate has a valence advantage (or disadvantage) while the other two candidates have the same valence.

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1 Introduction[1]

The median voter theorem (Hotelling 1929; Downs 1957; and Black 1958) is one of the most widely used results in analytical political economy. In a setup also known as the Hotelling-Downs model, the theorem states that two-candidate unidimensional spatial competition has a unique pure strategy Nash Equilibrium (PSNE) in which both candidates adopt the policy most preferred by the median voter. In the model, there is a continuum of voters. The voting is sincere, non-stochastic, and without abstention. The candidates are vote-maximizing, and they may differ only in their policies; otherwise they are identical. Although the model is commonly used to study two-candidate competition when candidates are homogeneous, it cannot be easily extended to other cases: More specifically, the PSNE does not exists when the candidates have different non-policy characteristics (also known as valence) in two-candidate competition[2], nor when three homogeneous candidates compete (Eaton and Lipsey 1975). In this paper, however, we show that a unique (modulo symmetry) PSNE exists when one considers competition among three candidates with valence differences.

In our base model (presented in Section 2) we impose two main restrictions on the standard Hotelling-Downs model: Voters' policy preferences are represented by a strictly convex loss function; and the density of voters is both unimodal and symmetric. We then show that a (modulo symmetry) unique Local Nash Equilibrium (LNE) exists for a large set of valence parameters. The set of parameters for which the PSNE exists is smaller, yet it is non-empty. For a symmetric density of voters, the PSNE exists only when two of the three competing candidates have the same valence, *and* the valence difference between these candidates and the third one exceeds a certain threshold. That is, for PSNE to exists, two of the three candidates must have the

[1]I would like to thank Şule Atahan-Evrenk, Darlene Chisholm, Tim Groseclose, Hsueh-Ling Hyunh, Dilip Mookherjee, and Jörgen Weibull for comments. All errors are mine

[2]For a partial list of people who established this claim, see (Groseclose 2001, footnote 10).

same appeal, and the appeal of the third candidate must be sufficiently different from the appeal of the other two. Further, under an alternative set of assumptions, such as asymmetric voter density or plurality-maximizing candidates, the PSNE exists for a much larger set of parameters including cases where each candidate has different valence.

We find that the location of candidates in equilibrium depends on the number of the candidate(s) with the highest valence. When only one candidate, say Candidate 2, has higher valence than the rest, in any LNE, Candidate 2 locates between the two lower-valence candidates (Candidates 1 and 3). Candidates 1 and 3 locate *equidistantly* from the center, and receive the same vote share. Candidate 2 is located closer to the candidate with the second highest valence. A PSNE for this case does not exist unless Candidates 1 and 3 have the same valence. In the PSNE, Candidate 2 is located at the center.

When two of the candidates have the same valence and the third one has a lower valence, the former candidates choose the same policy platform on one side of the center, receiving the same vote share. The lower valence candidate locates at the other side of (and further away from) the center. His vote share is lower than that of the other two candidates.[3]

We characterize both the LNE and the PSNE of the base model for a general strictly convex loss function (that is, for a general strictly concave utility function). We show that the quadratic loss function, commonly used in models of two-candidate competition, restricts the set of candidate valences under which an LNE exists; it rules out many plausible equilibria. This does not happen under many other loss functions.

When we consider some modifications of the base model, such as plurality maximizing candidates or asymmetric voter density, we find that the LNE of the base model is also an LNE in these models. More important, in these models PSNE exists

[3]When all three candidates have the same valence, we return to competition between three identical candidates, and neither a PSNE nor an LNE exists.

for a much larger set of parameters including the case in which none of the candidates have the same valence. Although both the model and its extensions mentioned above are quite stylized, they have two advantages. Both the LNE and the PSNE (i) are unique (modulo symmetry) when they exist, and (ii) can be characterized analytically. Thus, for instance, the model could be used for comparative statics concerning both marginal and large changes in candidate valences.[4] Further, given the existence of a unique equilibrium with an analytical solution, the model could also serve as a benchmark case for understanding the effects of additional modelling assumptions, such as probabilistic voting, on the equilibrium outcome.

Lin, Enelow and Dorussen (1999) considers spatial competition among homogenous candidates under probabilistic voting. They prove that when the variance of the uncertainty about the voting decision is large enough, there exists a PSNE. However, in this PSNE *all* candidates locate at the mean of the voter density. Two recent book-length treatments of multi-party competition, Adams, Merrill and Grofman (2005), and Schofield and Sened (2006), study detailed models of probabilistic voting. These models predict party divergence in equilibrium. Adams, Merrill and Grofman (2005) considers features such as party loyalty, policy discounting, and abstention in a unidimensional setup. Schofield and Sened (2006) has an integrated and more detailed approach. They study models of policy making that include many factors such as post election bargaining, activist valence, party principals, and a multi-dimensional policy space. Characterizing PSNE of these models in analytical form is not possible in general; the authors estimate the equilibrium using simulations based on country specific estimates of several parameters in their models. Adams (1999) runs simulations to calculate the PSNE of a more stylized model: three candidate competition

[4] For instance, Evrenk (2004) studies the support for anti-corruption reforms in a setup with three candidates who differ both in the level of their honesty and ability. The reform will raise the minimum level of honesty among the candidates, and thus will change the valence vector significantly. A candidate's support for the reform depends on the change in his vote shares before and after the reform. When we have an equilibrium in analytical form both before and after the reform, one can determine the sign of the change in vote share.

with valence differences under probabilistic voting and uniform voter density. The structure of the equilibrium in his Figures 2B and 2C look similar to the equilibrium locations that we find in Propositions 1 and 2 *without* probabilistic voting.[5]

Chisik and Lemke (2006) studies a model of simultaneous move unidimensional spatial competition among three homogeneous candidates without probabilistic voting. They prove that under a uniform density a *continuum* of PSNE exists when one assumes that candidates care only about winning a majority of votes. This, however, is a strong assumption. It implies that a candidate is indifferent between receiving a 49 percent vote share and no vote at all, if another candidate receives 50 percent of votes in both situations. Hug (1995) studies PSNE of three-party competition when there is uncertainty about the policy that a party will implement, –extending Enelow and Hinich (1984, section 7.4) to three-party competition. This model is isomorphic to a special case of the model we study, as we discuss in greater detail at the end of Section 2.

2 The model

Consider an Hotelling-Downs model of political competition where each candidate $j \in \{1, 2, 3\}$ chooses a policy platform, p_j, from $\mathbb{R}$. Unlike the standard model, we assume that each candidate j has exogenous non-policy characteristics (known as valence and denoted by $v_j \in \mathbb{R}$) favored by voters, such as competency, honesty, and charisma.[6] There is a continuum of voters (of measure one), and i denotes the voter whose most preferred policy platform is $i \in \mathbb{R}$. Voting is sincere: i votes for candidate j who provides the highest $U_i^j(p_j, v_j)$, and randomizes when there are more than one such j's, where

[5] However, using these simulations and our results, one cannot identify how probabilistic voting would affect the equilibrium. Because, under the uniform voter density used in these simulations, no equilibrium exists without probabilistic voting.

[6] For more on valence, see Stokes (1963).

$$U_i^j(p_j, v_j) = -L(|i - p_j|) + v_j. \tag{1}$$

In (1), $L(x) : \mathbb{R}_+ \to \mathbb{R}_+$ is the "loss function" representing the voter's policy preferences, and $|i - p_j|$ is the distance between i and the policy platform of candidate j. Let $f(i) : \mathcal{I} \to \mathbb{R}_+$ denote the the density of i. We assume that: (*A1*) The loss function, $L(.)$, is twice continuously differentiable and strictly convex with $L'(0) = 0$ and $\lim_{x\to\infty} L'(x) = \infty$; (*A2*) The domain of the density is (i) closed, convex and symmetric around zero, and the density is (ii) continuous, (iii) symmetric with $f(i) = f(-i)$, (iv) differentiable on all its domain with the possible exceptions at zero and $\sup \mathcal{I}$, (at these points we assume that at least the directional derivatives exist), and (v) unimodal; (*A3*) Each candidate j simultaneously chooses his policy platform to maximize his vote share, $V_j(p_j, p_{-j})$.

We study the Nash Equilibria of the above model in which each candidate receives a non-zero vote share, –if a candidate receives no votes in equilibrium, his policy platform does not matter for any practical purposes. The existence of a global equilibrium is hard to verify, so, we follow a method proposed in Schofield (2005). We first identify the set of Local Nash Equilibria, LNE, i.e. any strategy profile (p_1^*, p_2^*, p_3^*) where p_j^* maximizes $V_j(p_j, p_{-j}^*)$ over a (small) ε neighborhood of p_j^*. We, then, use simulations to check if a given LNE is a PSNE.[7]

We normalize candidate valences as $v_2 \geq v_1 \geq v_3$. Then, the following cases are collectively exhaustive and mutually exclusive; (a) $v_1 = v_2 = v_3$, (b) $v_2 > v_1 > v_3$, (c) $v_2 > v_1 = v_3$, and (d) $v_2 = v_1 > v_3$. Let δ_j denote the valence difference between Candidate 2, and Candidate j, i.e., $v_2 - v_j$. Case (a) with no valence difference is the same as the competition between three identical candidates, and therefore has no LNE. For the other cases, we find that the number of candidate(s) with the highest valence determines the structure of the equilibrium policy platforms. When

[7] Schofield (2005) works from Local Strict Nash Equilibrium, LSNE, to PSNE. In our model, however, LNE and LSNE coincide generically.

one candidate has higher valence than the rest, i.e., cases (b) and (c), we have the following result.

Proposition 1 *When $v_2 > v_1$, if the LNE exists, then it is unique modulo symmetry, and it is given by* $[\frac{-L^{-1}(\delta_3)-L^{-1}(\delta_1)}{2}, \frac{L^{-1}(\delta_1)-L^{-1}(\delta_3)}{2}, \frac{L^{-1}(\delta_3)+L^{-1}(\delta_1)}{2}]$. *A necessary (sufficient) condition for the existence is* $\frac{-2f'(p_2^*+L^{-1}(\delta_3))}{f(p_2^*+L^{-1}(\delta_3))} \geq (>) \frac{L''(0)}{[L'(L^{-1}(\delta_3))]^2} + \frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}$. *For non-zero vote shares we need* $p_3^* < \sup \mathcal{I}$.

Remark 1 *When $v_2 > v_1 = v_3$, the LNE becomes $[-L^{-1}(\delta_1), 0, L^{-1}(\delta_3)]$, and the sufficient condition becomes* $\frac{f'(-L^{-1}(\delta_1))}{f(L^{-1}(\delta_1))} > \frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}$.

Next, we consider case (d). Let $F(i)$ denote the cumulative density of i, and let z be defined by[8] $F(z) = \frac{F(z+L^{-1}(\delta_3))}{2}$.

Proposition 2 *When $v_2 = v_1 > v_3$, if the LNE exists, then it is unique modulo symmetry, and is given by $[z, z, z + L^{-1}(\delta_3)]$. A necessary (sufficient) condition for the existence is $f(z) \geq (>) 2f(z + L^{-1}(\delta_3))$.*

We prove Propositions 1 and 2 in the Appendix. Here we first discuss the equilibrium locations and the vote shares. When one candidate has higher valence than the others, the lower-valence candidates are located symmetrically around the mean, $p_1^* = -p_3^*$. Note that this is true even when the candidates 1 and 3 have *unidentical* valence, i.e., in case (b). The high-valence candidate always locates in between the lower-valence candidates. The higher-valence candidate locates exactly at the center only when both of the lower-valence candidates have the same valence. Otherwise, he locates at the same side of the center as the second highest valence candidate does.

When two of the candidates have exactly the same valence, and the third one has a lower valence, case (d), the policy platforms of high-valence candidates converge,

[8] To ensure the existence and uniqueness of z, a sufficient condition is that the valence difference is small, more precisely, $F(-L^{-1}(\delta_3)) < \frac{1}{4}$. To see that under this assumption there exists a unique $z \in (-L^{-1}(\delta_3), 0]$, note that the function $D(z) = 2F(z) - F(z + L^{-1}(\delta_3))$ is continuous with $D(-L^{-1}(\delta_3)) < 0$, $D(0) \geq 0$, and $D'(z) = 2f(z) - f(z + L^{-1}(\delta_3)) > 0$.

$p_1^* = p_2^*$. However, they do not converge on the center. These two candidates locate on one side of the center and the third (lower-valence) candidate locates on the other side. The unimodal and symmetric voter density implies that the higher-valence candidates' platform is closer to the center, i.e., $|z| < |z + L^{-1}(\delta_3)|$.

Given the equilibrium locations, we can calculate equilibrium vote shares. As Lemma 2 in Appendix shows, in any LNE, all voters located between a low valence candidate and a high valence candidate vote for the latter. Thus, in (b) and (c), low valence candidates receive $V_1^* = V_3^* = F(\frac{-L^{-1}(\delta_3)-L^{-1}(\delta_1)}{2})$, where Candidate 2 receives the rest of the votes, $V_2^* = 1 - 2F(\frac{-L^{-1}(\delta_3)-L^{-1}(\delta_1)}{2})$. Similarly, in (d), $V_3^* = 1 - F(z + L^{-1}(\delta_3))$, and each of the high valence candidates receives $\frac{1}{2}F(z + L^{-1}(\delta_3))$, or, equivalently, $F(z)$, fraction of votes. The LNE vote shares in cases (b) and (c) can not be further ordered. For instance, depending on δ_1 in some equilibria V_1^* is less than V_2^* and in some other equilibria V_1^* is larger than V_2^*.[9] In case (d), we can rank the equilibrium vote shares of all candidates: since $|z| < |z + L^{-1}(\delta_3)|$, each high-valence candidate receives a larger vote share than the candidate with the lower valence, $V_1^* = V_2^* > V_3^*$. It is also worth noting that in cases (b) and (c), the low valence candidates are not locally competing with each other in the equilibrium. That is to say, when one of them locally deviates from his equilibrium platform, then the vote share of the other low valence candidate remains unchanged. In contrast, in case (d) when the low valence candidate locally deviates from his LNE location, the vote share of all candidates change.

Before we study when LNE is PSNE, let us note an implication of the quadratic loss function specification on LNE. Quadratic loss function has been used extensively in the literature on two-candidate competition with valence differences, (Groseclose (2001) is a notable exception); when the loss function is quadratic, it is quite easy to calculate the location of the indifferent voter analytically. Yet, using this specification in *three*-candidate competition restricts the set of valence differences under which an

[9] As we discuss below, in PSNE we can always order the vote shares.

LNE exists, ruling out many plausible LNE. To see why, let us compare the set of LNE under $L(x) = x^2$ with the same set under $L(x) = x^4$, –or, for that matter, under any $L(x) = x^{2n}$ where n is an integer larger than one. Both loss functions satisfy *A1*, but $L''(0) = 2$ in the former and $L''(0) = 0$ in the latter. To see how the value of $L''(0)$ imposes a lower bound on the set of valence differences, δ_j's, under which an equilibrium exists, consider the following voter densities that differ from each other in terms of support and curvature: (i) Gaussian with zero mean, $f_1(x;\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2}$ for $x \in \mathbb{R}$, (ii) Symmetric Exponential, $f_2(x;\lambda) = \frac{\lambda}{2}e^{-|x|\lambda}$ for $x \in \mathbb{R}$, and (iii) Symmetric Zero-mode Triangular, $f_3(x;b)$ defined as $\frac{b+x}{b^2}$ for $x \in [-b, 0]$, and as $\frac{b-x}{b^2}$ for $x \in [0, b]$. The necessary condition in *Remark 1* becomes $\frac{L^{-1}(\delta_1)}{\sigma^2} \geq \frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}$ for (i), $\lambda \geq \frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}$ for (ii), and $\frac{1}{b-L^{-1}(\delta_1)} \geq \frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}$ for (iii). Note that the larger the $L''(0)$, the larger is the minimum valence difference, δ_j, that satisfy these inequalities: When $L''(0) = 0$, there exists an LNE for any $0 < \delta < L(\sup \mathcal{I})$. Yet, when $L''(0) = 2$, the LNE does not exists under small valence differences. We need $\frac{1}{2^{2/3}} < \delta < L(\sup \mathcal{I})$ for $f_1(x;1)$, $\frac{1}{2} < \delta < L(\sup \mathcal{I})$ for $f_2(x;1)$, and $\frac{1}{4} < \delta < L(\sup \mathcal{I})$ for $f_3(x;1)$. Thus,

Remark 2 *Although under any strictly convex loss function with* $L''(0) = 0$ *the LNE exists for any* $\delta > 0$*, when* $L''(0) = 2$*, one needs the variance of the voter density to converge to infinity for this to happen.*

Since the variance of the voter density is a parameter over which the modeler has the least control, the constraint pointed out in Remark 2 is especially important when one considers actual voter data. Even more important, the set of valence parameters (and thus the LNE) ruled out by the quadratic specification contain many plausible equilibria. For instance, when voter density is $f_3(x;1)$ we are immediately ruling out any LNE where the high valence candidate receives a vote share less than $\frac{3}{4}$.

In case (d), to calculate the equilibrium locations in analytical form, we need a cumulative density in analytical form.[10] Then, one can identify the set of valence

[10] Thus, under the Gaussian density one cannot calculate the equilibrium locations analytically,

differences that give rise to a non-zero vote share LNE as well: $\delta_3 > L(\frac{Log[25/8]}{\lambda})$ for $f_2(x;\lambda)$, and $L((2-\sqrt{2})b) < \delta_3 < L(b)$ for $f_3(x;b)$. Unlike (b) and (c), in case (d) the boundaries of the LNE locations does not depend on the curvature of the loss function and are solely determined by the valence differences. In other words, independent of $L(.)$, $p_1^* = p_2^*$ is in $(\frac{1}{\lambda}(Log[\frac{1}{2}(1-\frac{8}{25}\sqrt{(\frac{25}{8})^2-\frac{50}{8}})]), 0)$ for $f_2(x;\lambda)$, and in $(\frac{b(2\sqrt{2}-3)}{3}, 0)$ for $f_3(x;b)$.

After identifying the LNE, now we can study the PSNE.

Proposition 3 *The PSNE exists only in cases (c) and (d).*

Proof. To see why there is no PSNE in (b), assume that an LNE with $p_2^* < 0$ is PSNE (one can use symmetric arguments to show that an LNE with $p_2^* > 0$ can not be a PSNE either). Then, by Proposition 1, we have $|p_1^* - p_2^*| < |p_3^* - p_2^*|$ with $V_1^* = V_3^* = F(p_1^*)$. But, if Candidate 1 deviates to $p_1' = p_3^*$, then all the voters located at the right side of p_3^* vote for him, since $v_1 > v_3$. In addition to these voters, some of the voters located at the immediate left of p_3^* also vote for him. That is because now there is a candidate with higher valence located at p_3^*, thus the voter who is indifferent between Candidate 2 and the candidate at p_3^* is located somewhere at the left of p_3^*. Then the symmetry of density implies $V_1(p_3^*, p_2^*, p_3^*) > V_1(p_1^*, p_2^*, p_3^*) = F(p_1^*)$, contradicting that the LNE was a PSNE. To show PSNE exists in cases (c) and (d), we provide examples: Consider $f_3(x;1)$. Using simulations one can show that in case (c), when $L(x) = x^4$, for any valence difference, δ_1, in $[0.012, 1]$ the PSNE exists, and when $L(x) = x^2$ all LNE is PSNE, i.e., there is a PSNE under any $\frac{1}{4} < \delta_1 < 1$. In case (d), under both quartic and quadratic loss functions, any LNE is PSNE. ■

Although in two-candidate competition the PSNE does not exist when one candidate has valence advantage, in three-candidate competition, the PSNE exist when only *one* candidate has a valence advantage (or, a valence "disadvantage"), i.e., cases

but $z = \frac{1}{\lambda}(Log[\frac{1}{2}(1-e^{-\lambda L^{-1}(\delta_3)}\sqrt{e^{2\lambda L^{-1}(\delta_3)} - 2e^{\lambda L^{-1}(\delta_3)}})])$ when the voter density is $f_2(x;\lambda)$, and $z = \frac{1}{3}\left(-b - L^{-1}(\delta_3) + \sqrt{-2b^2 + 8bL^{-1}(\delta_3) - 2(L^{-1}(\delta_3))^2}\right)$ for $f_3(x;b)$.

(c) and (d). Furthermore, the size of the valence difference between this candidate and the other two candidates must be larger than a threshold that depends on, among other things, the density of voters and the curvature of the loss function. Since the PSNE is a subset of the LNE, PSNE is unique modulo symmetry, and can be characterized analytically.

The method by which we calculate the equilibrium has restrictions on the analysis. We can characterize equilibrium locations analytically, but we cannot analytically check which ones are PSNE. For this, we use numerical simulations.[11] With this method we can provide examples of PSNE under many specific densities, however, unlike the LNE, we are unable to provide sufficient conditions for the existence of PSNE that would apply to *any* density that satisfies *A2* (i)-(v). Neither can we show that the PSNE exists for *any* such density. Such conditions are provided in Hug (1995) for a special case of this model, however, there are problems with these conditions.

Hug (1995) does not address competition with valence differences, however, the mathematical structure of the model studied there is isomorphic to a special case of the base model in this paper. He examines the PSNE of unidimensional political competition among three parties when there is uncertainty about the policy platforms of parties, –an extension of Enelow and Hinich (1984, section 7.4) to three parties. In that model, the platform chosen by party k is perceived with the same noise by all the voters. More formally, when party k locates on $\mu_k \in R$, each voter expects party k to implement the lottery $\mu_k + \varepsilon_k$, where ε_k is an independent random variable with zero mean and standard deviation σ_k. As Enelow and Hinich (1984), Hug (1995) also assumes that the voter whose most preferred policy platform, or bliss point, is $i \in R$ has a quadratic loss utility function, $U_i(\mu_k) = -(i - \mu_k)^2$. Then, the expected utility of voter i from party k located at μ_k is $-(x_i - \mu_k)^2 - \sigma_k^2$: when the loss function is quadratic, the effect of uncertainty is reduced to one parameter, the variance. In

[11] For all simulations, *Mathematica* notebooks with calculations are available from the author.

Hug (1995), the density of voters, $f(x_i)$, is twice differentiable, continuous, symmetric around zero, strictly increasing on $(-\infty, 0)$. Thus, his model of policy uncertainty is isomorphic to a model of political competition with valence differences, where $L(x) = x^2$, $\mathcal{I} = \mathbb{R}$, and $f(.)$ twice differentiable everywhere.

In Propositions 1 and 2, Hug (1995) claims to find sufficient conditions for PSNE in all cases of present interest: (b), (c), and (d). As the proof of Proposition 2 should make clear, the claim about case (b) is incorrect: the author does not check for the global deviation by the candidate with the second highest valence, that is, $p_1' = p_3^*$. The claim about PSNE in case (c) must also be corrected: as we show the PSNE exists, but the conditions that the author claims as sufficient for PSNE are not sufficient even for an LNE. This is because, the second-order condition (the last equation on page 179) is incorrect. For the correct sufficient conditions for LNE, note that the condition in Remark 1 would translate as $\frac{f'(-\sqrt{\sigma_2^2-\sigma_1^2})}{f'(-\sqrt{\sigma_2^2-\sigma_1^2})} > \frac{2}{4(\sigma_2^2-\sigma_1^2)}$ into the setup the author studies. Similar to case (b), the author claims that when other candidates are located at their equilibrium platforms, for any $f(i)$, one can analytically show that the vote share of the high-valence candidate is maximized in his policy platform at the relevant region, i.e., $p_2' \in (p_2^*, p_2^* + L^{-1}(\delta_3))$. However, for neither (c) nor (d), could we prove this.[12]

The model we use is a variant of the Hotelling-Downs model and it, too, is stylized. In the unique equilibrium of the Hotelling-Downs model, both candidates choose the same policy and each receives exactly the same vote share. The PSNE of the model we study has two non-plausible features as well. First, the PSNE does not exist for the most likely case, (b). Second, in the PSNE of case (c), the highest valence candidate always receives a majority of votes ($V_2^* > \frac{1}{2}$).[13] To see why, note that in any LNE in

[12] For instance, in the proof of Proposition 2, Hug (1995, p. 177), the argument for the existence of equilibrium in (d) is that, using our notation, for any $p_2' \in (p_2^*, p_2^* + L^{-1}(\delta_3))$, the function $V_2(p_2, p_{-2}^*)$ has at most one critical value. This, we could not prove or disprove for the case he studies. We can show, on the other hand, that when the loss function is quadratic as in Hug (1995), the density is given by $f_2(x, 1)$, and the valence/variance difference is equal to $Log[\frac{25}{8}]^2$ (or slightly larger than this), the vote share function has *two* critical points.

[13] The PSNE of other three-candidate competition models with non-stochastic voting has similar

(c), the high valence candidate is located at the origin and the two other low valence candidates are located equidistantly on both sides. Suppose that we have a PSNE where the high valence candidate does not receive a majority of votes, then he could simply deviate to the location of one of the low valence candidates and receive a vote share that is strictly larger than one half. As we discuss in the next section, under alternative assumptions these features disappear.

3 Discussion and Conclusion

In this section, we briefly discuss three related issues. First, we examine the role of the assumptions *A1*, *A2* and *A3* in establishing the equilibrium. Second, we consider if (and how) these assumptions could be relaxed or altered without eliminating the equilibrium. And, third, we identify how the PSNE of the modified model differs from the PSNE of the base model. Here, we consider only the alternative assumptions under which the LNE of the model is a superset of the LNE of the base model. Then, we conclude.

We assume that the voter preferences are strictly concave (the loss function is strictly convex), *A1.* With a linear loss function, even an LNE does not exist in any of the cases.[14] Under a strictly concave loss function, even if it exists, the equilibrium is supported by an unusual pattern of voting: both the center voters and the voters located around the supremum and infimum of $\mathcal{I}$, i.e., the far-right and the far-left voters, always vote for the center candidate.

We impose several conditions on the voter density. These assumptions are sufficient, not necessary, and sometimes are just convenient. Consider the assumption

strong conclusions. For instance, in Chisik and Lemke (2006) in every equilibrium, one candidate receives more than a majority of votes. For other examples see Adams (2001, p.39-40)

[14] The reason is as follows. With a linear loss function, the low-valence candidates still choose the locations of indifference, i.e., Lemma 2 applies. However, if a high valence candidate moves slightly towards the low-valence candidate, then all the voters who voted for the latter will switch to the former. Since by moving slightly towards a low-valence candidate, the high-valence candidate's loss of vote share from the voters on the other side is infinitesimal, such a deviation always increases his vote share, and thus an LNE does not exist.

that the voter density is unimodal, *A2* (v). While this assumption is not necessary, it is not as strong as what one needs for PSNE in standard Downsian spatial multi-candidate competition: "..the number of firms does not exceed twice the number of nodes," (Eaton and Lipsey 1974, p.35). To see how unimodality simplifies the condition for the existence of LNE, note that using (3), the sufficient condition in Proposition 1 can be written as $\frac{2f'(p_2^*-L^{-1}(\delta_1))}{f(p_2^*-L^{-1}(\delta_1))} > \frac{L''(0)}{[L'(L^{-1}(\delta_3))]^2} + \frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}$. The right-hand side cannot be negative. Thus, if we want to construct a density where these sufficient conditions hold generically, then $f'(x)$ should decrease around p_3^*, and should increase around p_1^*. The density need not to be strictly decreasing on each side of the mean. For example, both LNE and PSNE would exist under a density first decreasing on each side of the mean, then having a sufficiently small local mode sufficiently far from the mean. However, with such densities, there will be "holes" in the set of valence differences that gives rise to the LNE, making the characterization of LNE (and, for instance, the discussion of the impact of a quadratic loss function on LNE) especially cumbersome.

We also assume that voters are distributed symmetrically around the mean, *A2* (iii). The symmetric density, however, has strong implications on both the existence of PSNE and the structure of equilibrium platforms: Under symmetry, a PSNE exists only when the vector of candidate valences has a certain kind of symmetry and a certain degree of asymmetry. Unlike cases (c) and (d) where two of the three candidates have the same valence, there is no symmetry in candidate valences (and thus no PSNE) in case (b).[15] Since (b) is the most likely case, the PSNE under asymmetric densities must be considered. The analysis of equilibrium under a general asymmetric density is difficult: to study PSNE under an asymmetric density, one has to impose more structure on the density.[16] Through examples we are able to show

[15] Note that the important step in the proof of Proposition 3 uses the fact that both lower-valence candidates receive the same vote share in equilibrium, an implication of symmetric density, (see (3)). However, also note that the symmetry of voter density does not require full symmetry in candidate valences, as in case (a), either. Furthermore, under any valence distribution that is close to full symmetry there is no PSNE either, cf. the examples in the proof of Proposition 3.

[16] The symmetric structure is analytically convenient because it imposes a structure. Asymmetry,

that the PSNE in case (b) exists under asymmetric voter densities: assume that (i) $f(x) = 1+2x$ for $-\frac{1}{2} \leq x \leq 0$ and $f(x) = 1-\frac{2}{3}x$ for $-0 \leq x \leq \frac{3}{2}$, (ii) the loss function is quadratic, (iii) $\sqrt{\delta_3} = \frac{4}{3}\sqrt{\delta_1}$. Then one can show that for any $\sqrt{\delta_1} \in [\frac{4}{10}, \frac{48}{100}]$ a unique, *not* modulo symmetry, PSNE exists at $(\frac{7}{4}\sqrt{\delta_1}, \frac{3}{4}\sqrt{\delta_1}, \frac{-7}{12}\sqrt{\delta_1})$. Asymmetric density is not the only solution; another way to restore the equilibrium in case (b) is to consider alternative candidate motivations.

We assume that candidates are vote-maximizing, *A3*. Although *A3* is the most commonly used objective function in models of multi-candidate competition, it may imply paradoxical behavior: "when there are more *[than two]* candidates: a candidate who wins outright may, if she moves her position closer to that of a neighbor, increase the number of votes that she receives but at the same time increase the number of voters received by her other neighbor enough that she is no longer the outright winner", (Osborne 1995, p. 278). To check if the equilibrium we find is supported by this kind of behavior, we calculate both the LNE and PSNE of the model under the assumption that each candidate j maximizes his plurality,[17] $PLU_j(p_1, p_2, p_3) = V_j(p_1, p_2, p_3) - \max_{k \neq j}\{V_k(p_1, p_2, p_3)\}$. We find that such a paradoxical behavior does not occur in the LNE we find. Further, we find that

Proposition 4 *Under plurality maximization, (i) the policy platforms in Propositions 1 and 2 are still LNE, and (ii) now PSNE exists in every case except (a).*

Proposition 4 is proved in the Appendix. We do not provide a full characterization of Nash equilibrium under plurality maximization here, as our purpose is to note that the equilibria we find in Section 2 are robust. It is worth noting that when each candidate maximizes his plurality, there exists a PSNE in the most likely case, (b). This is because, a high(er)-valence candidate, such as Candidate 1 in case (b), could

however, simply implies a lack of structure. Then, to study the model with an asymmetric density, the researcher needs to impose some structure. For example, studying the equilibrium under a certain asymmetric Triangular distribution is relatively straightforward, but studying it under *any* asymmetric distribution is difficult.

[17]The following discussion would apply under the assumption of complete plurality maximization, CPM, as well. For a definition of CPM, see Cox (1987).

increase his vote share by deviating to the position of a low-valence candidate, however such a move would reduce his plurality. For this reason, the LNE for smaller valence differences in cases (c) and (d), along with many of the LNE for case (b), survives the test for PSNE.

In this paper, we study both the local and the global Nash Equilibrium of three-candidate competition in standard Downsian model with exogenous candidate valence. We show that, for a large set of valence differences, a modulo symmetry unique LNE with an analytical characterization exists. We also show that the commonly used quadratic loss specification significantly restricts the set of parameters under which an LNE exists. For a PSNE to exist, the valence differences need more structure. The PSNE of the base model does not exists in case (b), however, this feature disappears when one considers either asymmetric voter density, or plurality maximizing candidates. In PSNE of case (c), Candidate 2 receives a majority of votes. Under plurality maximization, this feature also disappears; when candidates are plurality maximizing in (still modulo symmetry unique) PSNE, the center candidate does not necessarily receive a majority of the votes. Interestingly, this PSNE in which $V_2^* < \frac{1}{2}$ is supported by another type of paradoxical behavior: in such an equilibrium, Candidate 2 does not move towards a low valence candidate as this would reduce his plurality, even though the move would secure him a majority. It is possible to impose other assumptions such as probabilistic voting where both features disappears. A detailed analysis of these cases is left for future research.

4 Appendix

Proof of Proposition 1. To prove Proposition 1, we need the following three Lemmas. Consider competition between two candidates, j and k, where $v_j > v_k$. Let $I(p_j, p_k) \in \mathbb{R}$ denote a location of indifference, i.e., $L(|I(p_j, p_k) - p_j|) - L(|I(p_j, p_k) - p_k|) = v_j - v_k$. Obviously when $p_j = p_k$ all voters prefer candidate j, thus $I(p_j, p_k)$ does not exist. However,

Lemma 1 *Whenever candidates choose non-identical policy platforms, (i) there always exists a unique location of indifference (Groseclose 2005, Appendix III), and (ii)* $I(p_j, p_k)$ *is closer to the candidate with lower valence.*

Proof. For (ii) note that $v_j - v_k > 0$ implies that $L(|I(p_j, p_k) - p_j|) >$
$L(|I(p_j, p_k) - p_k|)$. Since $L'(x) > 0$, we have $|I(p_j, p_k) - p_j| > |I(p_j, p_k) - p_k|$. ■

The location of indifference determines the equilibrium locations of candidates with relatively lower valences. Let $P_k(p_j)$ denote the best response correspondence for candidate k. Then, if k receives a non-zero vote share in his best response, then we have $I(p_j, p_k) \in P_k(p_j)$ (with equality if $p_j \neq 0$). To see this, note that,

$$\frac{\partial I(p_j, p_k)}{\partial p_k} = \frac{\mathrm{sgn}(\Delta_k)L'(|\Delta_k|)}{\mathrm{sgn}(\Delta_j)L'(|\Delta_j|) - \mathrm{sgn}(\Delta_k)L'(|\Delta_k|)}, \tag{2}$$

where $\Delta_j = I(p_j, p_k) - p_j$, $\Delta_k = I(p_j, p_k) - p_k$, and sgn(.) denotes the sign function.

Lemma 2 *Candidate k's best response correspondence always includes a location of indifference. Furthermore, if Candidate k receives any votes in that location, then* $|p_j - P_k(p_j)| = L^{-1}(\delta_k)$*, and, unless* $p_j = 0$*, the best response correspondence is single valued (where* $P_k(0) = \{L^{-1}(\delta_k), -L^{-1}(\delta_k)\}$*).*

Proof. Let us fix $p_j \neq 0$. Now consider Candidate k's "conditional" best response, $P_k^+(p_j) = \{p_k > p_j : V_k(p_j, p_k) \geq V_k(p_j, p'_k)\ \forall p'_k > p_j\}$. For any $p_k > p_j$, Lemma 1.(ii) implies $I(p_j, p_k) > p_j$, and $|I(p_j, p_k) - p_j| > |I(p_j, p_k) - p_k|$. Thus, the denominator of $\frac{\partial I(p_j,p_k)}{\partial p_k}$ is positive, and the sign of $\frac{\partial I(p_j,p_k)}{\partial p_k}$ is determined by the sign of $I(p_j, p_k) - p_k$. Note that for $p_k > p_j$, we have $V_k(p_j, p_k) = 1 - F(I(p_j, p_k))$, so $\frac{\partial V_k(p_j,p_k)}{\partial p_k} = -f(I(p_j, p_k))\frac{\partial I(p_j,p_k)}{\partial p_k}$. Now, if $\mathcal{I} = \mathbb{R}$, then for $p_k > I(p_j, p_k)$, we have $\frac{dV_k}{dp_k} < 0$, and for $p_k < I(p_j, p_k)$, we have $\frac{dV_k}{dp_k} > 0$. But, by *A1*(ii), the only stable location for Candidate k is that of the indifferent voter, i.e., for $p_k = I(p_j, p_k)$ we have $\frac{dV_k}{dp_k} = 0$. Then $I(p_j, p_k)$ is located $L^{-1}(\delta_k)$ off of the high-valence candidate. Thus, $f(p_j + L^{-1}(\delta_k)) > 0$ implies

$P_k^+(p_j) = p_j + L^{-1}(\delta_k)$. If $\mathcal{I} \neq \mathbb{R}$, and $f(p_j + L^{-1}(\delta_k)) = 0$, then we still have $p_j + L^{-1}(\delta_k) \in P_k^+(p_j)$, as in this case any $p_k > p_j$ results in $V_k(p_j, p_k) = 0$. Similarly, maximizing $V_k(p_j, p_k)$ conditional on $p_k < p_j$, implies that $p_j - L^{-1}(\delta_k) \in P_k^-(p_j)$ (again with equality if $f(p_j - L^{-1}(\delta_k)) > 0$). Now, let us assume that Candidate k receives a non-zero vote share, $\max\{f(p_j - L^{-1}(\delta_k)), f(p_j + L^{-1}(\delta_k))\} > 0$. By *A2*(iii), we have $V_k(p_j, P_k^-(p_j)) <(>) V_k(p_j, P_k^+(p_j))$ if and only if $p_j <(>) 0$. The same assumption also implies that $V_k(0, L^{-1}(\delta_k)) = V_k(0, -L^{-1}(\delta_k))$. ■

Hug (1995) derives the following result for $L(x) = x^2$.

Lemma 3 *When $v_2 > v_1$, in any LNE Candidate 2 should locate strictly between the other two candidates.*

Proof. Assume otherwise, i.e., that we have an LNE with non-zero vote shares where both low valence candidates are located, say, at the right side of Candidate 2. By Lemma 1.(ii), $p_2 < I(p_2, p_j)$ for $j \in \{1, 3\}$. Then by (2), $\frac{\partial I(p_2, p_j)}{\partial p_2} > 0$ for $j \in \{1, 3\}$, i.e., by converging towards the others, Candidate 2 can increase his vote share. Contradicting this was an LNE. Also note that if Candidate 2 shares his location with the other candidate(s), then these candidate(s) would get zero votes, again contradicting that these candidate(s) were receiving non-zero vote shares. ■

Now, without loss of generality, assume that in the equilibrium, Candidate 3 is on the right side of Candidate 2. Then, if an LNE exists, we have $p_2^* \in (p_1^*, p_3^*)$. Note that at the policy platforms in Proposition 1, the low-valence candidates are not competing with each other. That is because for any voter i, high-valence Candidate 2's policy platform is closer than at least one low-valence candidate, so Candidate 2, when located between the other two candidates, ranks at worst as the second candidate in any voter's preference ordering. This observation implies that as long as it is on the other side of Candidate 2, the location of the third candidate does not have any effect on Candidate 1's vote share, and thus Lemma 2 applies, i.e., the location of indifference between a low valence and a high valence candidate is still the best response for the former. Thus we have $p_1^* = p_2^* - L^{-1}(\delta_1)$, and $p_3^* = p_2^* + L^{-1}(\delta_3)$. We also need to show that Candidate 2 does not have any incentive to deviate from p_2^*. His vote share is $F(I(p_2^*, p_3^*)) - F(I(p_2^*, p_1^*))$, and the f.o.c. is $f(I(p_2^*, p_3^*))\frac{\partial I(p_2^*,p_3^*)}{\partial p_2} - f(I(p_2^*, p_1^*))\frac{\partial I(p_2^*,p_1^*)}{\partial p_2} = 0$. Using (2) and *A1*, we have $\frac{\partial I(p_2^*,p_3^*)}{\partial p_2} = \frac{\partial I(p_2^*,p_1^*)}{\partial p_2} = 1$. Then, the symmetry of $f(.)$ implies that the f.o.c. will hold if and only if,

$$I(p_2^*, p_1^*) = -I(p_2^*, p_3^*). \tag{3}$$

Equation 3 implies that $p_1^* = -p_3^*$. Using this with Lemma 2, we have the LNE platform in Proposition 1 as the only candidate for LNE. A sufficient condition for (3) to characterize a local maximum is the s.o.c., $\frac{d^2 V_2(p_1^*,p_2^*,p_3^*)}{(dp_2)^2} < 0$, or

$$f'(I_3^*)(\frac{\partial I_3^*}{\partial p_2})^2 + f(I_3^*)\frac{\partial^2 I_3^*}{(\partial p_2)^2} < f'(I_1^*)(\frac{\partial I_1^*}{\partial p_2})^2 + f(I_1^*)\frac{\partial^2 I_1^*}{(\partial p_2)^2}, \tag{4}$$

where $I_3^* = I(p_2^*, p_3^*)$, and $I_1^* = I(p_2^*, p_1^*)$. We assumed that $L(.) \in \mathcal{C}^2$, then by implicit function theorem, whenever it exists, $I(p_2, p_j)$ is also locally $\mathcal{C}^2$. One can show that, when $p_1 < p_2 < p_3$, we have

$$\frac{\partial^2 I(p_2, p_1)}{(\partial p_2)^2} = \frac{L''(p_2 - I(p_2, p_1))(\frac{\partial I(p_2,p_1))}{\partial p_1})^2 - L''(I(p_2, p_1) - p_1)(\frac{\partial I(p_2,p_1))}{\partial p_2})^2}{[L'(p_2 - I(p_2, p_1)) + L'(I(p_2, p_1) - p_1)]^2},$$

$$\frac{\partial^2 I(p_2, p_3)}{(\partial p_2)^2} = \frac{L''(I(p_2, p_3) - p_3)(\frac{\partial I(p_2,p_3))}{\partial p_2})^2 - L''(-p_2 + I(p_2, p_3))(\frac{\partial I(p_2,p_3))}{\partial p_3})^2}{[L'(-p_2 + I(p_2, p_3)) - L'(I(p_2, p_3) - p_3)]^2}.$$

Hence, evaluating the second derivatives at $p_1^* = p_2^* - L^{-1}(\delta_1)$, and $p_3^* = p_2^* + L^{-1}(\delta_3)$, we find $\frac{\partial^2 I(p_2^*,p_1^*)}{(\partial p_2)^2} = -\frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}$, and $\frac{\partial^2 I(p_2^*,p_3^*)}{(\partial p_2)^2} = \frac{L''(0)}{[L'(L^{-1}(\delta_3))]^2}$. Then, (4) becomes

$$f'(I_3^*) + f(I_3^*)\frac{L''(0)}{[L'(L^{-1}(\delta_3))]^2} < f'(I_1^*) - f(I_1^*)\frac{L''(0)}{[L'(L^{-1}(\delta_1))]^2}, \tag{5}$$

establishing the sufficient condition in Proposition 1, and completing the Proof of Proposition 1.

Proof of Proposition 2 In case (d), an LNE exists only when Candidates 1 and 2 share the same location, otherwise at least one of them has the incentive to move towards the other one. Also note that the location Candidates 1 and 2 share should be the mean point of the voters who vote for one of these strong candidates, otherwise one of the high-valence candidates would move slightly to the side with more voters and increase his vote share. By Lemma 2, Candidate 3's platform (the location of indifference) is a best response to the other candidates' platform; having two candidates with the same valence at the same location does not change the result of Lemma 2. Thus, we have (i) $p_1^* = p_2^*$, and, under the assumption that[18] $p_3^* > p_2^*$, (ii) $F(p_1^*) = \frac{F(p_3^*)}{2}$. None of the high-valence candidates has any incentive to move to the left of z, as this would reduce that candidate's vote share below $F(z)$. The sufficient condition guarantees that moving infinitesimally to the right does not increase a high-valence candidate's vote share either, i.e., $\lim_{dp \to 0^+} V_2(z, z + dp, z + L^{-1}(\delta_3)) < 0$. To obtain the sufficient condition from this inequality, note that evaluating (2) at equilibrium locations in Proposition 2, we have $\frac{\partial I(p_2^*, p_1^*))}{\partial p_2} = \frac{1}{2}$, and $\frac{\partial I(p_2^*, p_3^*))}{\partial p_2} = 1$.

Proof of Proposition 4 Note that by Propositions 1, 2, and 3 we know p_j^* maximizes $V_j(p_j, p_{-j}^*)$ locally for LNE and globally for PSNE. Here we need to show that it maximizes $V_j(p_j, p_{-j}^*) - \max_{k \neq j}\{V_k(p_j, p_{-j}^*)\}$ as well.

For part (i), consider the cases of (b) and (c) first. We need to show that a local deviation by a candidate does not increase his plurality when the rest are located at their LNE policy platforms. Let us consider a low valence candidate j who is maximizing his plurality $V_j(p_j, p_{-j}^*) - V_k(p_j, p_{-j}^*)$. In his LNE location, his plurality is either negative, i.e., $k = 2$, or equal to zero. Note, once more, that at his current location this low-valence candidate is competing only against Candidate 2. Therefore by a local deviation his plurality cannot increase. To see why, note that with any such deviation $V_j(p_j, p_{-j}^*)$ decreases (by the argument in the proof of Lemma 2, any local deviations would decrease his vote share) and $V_k(p_j, p_{-j}^*)$ either does not change,

[18] Of course, by symmetry, one could assume $p_3^* \leq p_2^*$, then (ii) would read $1 - F(p_1^*) = \frac{1-F(p_3^*)}{2}$.

i.e., $k \neq 2$, or increases, i.e., $k = 2$. So, a plurality-maximizing low-valence candidate has no incentive to locally deviate from his LNE location in Proposition 1. Now let us check if the high-valence candidate has any incentive to locally deviate from his LNE platform when he maximizes his plurality. Indeed, he has no incentive to deviate either. To see why, assume without loss of generality that he deviates to the right. Then his plurality is equal to $V_2(p_2, p^*_{-2}) - V_1(p_2, p^*_{-2})$. We know that $V_2(p^*_2, p^*_{-2}) \geq V_2(p_2, p^*_{-2})$ for any $p_2 \geq p^*_2$. By (2), we also know that $\frac{\partial I(p_1,p_2)}{\partial p_2} > 0$, thus $\frac{\partial V_1(p^*_1,p_2,p^*_3)}{\partial p_2} > 0$.

Now, let us prove (i) for case (d). Again, consider the low-valence candidate first. Both high-valence candidates locate at the same platform and thus they share the voters who do not vote for the low valence candidate. So, it is as if the low-valence candidate is competing against one high valence candidate. Thus, when the formers vote share decreases, so does his plurality. Then his LNE policy platform locally maximizes his plurality. When we consider either of the high-valence candidates, each has a plurality of zero in the equilibrium. If one of them moves to the left, his vote share will decrease and the vote share of the other high-valence candidate increases. But, the vote share of low-valence candidate will remain the same. Thus, a local deviation to the left by a high-valence candidate, say, Candidate 2, decreases his plurality. To see that a local deviation to the right by Candidate 2 is also plurality-decreasing, note that, again, as we are considering an LNE platform, by deviating, Candidate 2 cannot increase his vote share. Further, when Candidate 2 deviates to the right, the vote share of Candidate 1 increases. Thus, the plurality of Candidate 2 decreases. Hence, for all possible valences, if (p^*_1, p^*_2, p^*_3) is an LNE under vote-maximization, then it is an LNE under plurality-maximization.

For part (ii), we again use numerical simulations to show the existence of PSNE.

References

[1] Adams J (1999) Policy divergence in multi-candidate probabilistic spatial voting. Public Choice 100: 103-122.

[2] Adams J (2001) Party Competition and responsible party government. The University of Michigan Press.

[3] Adams J, Merrill S III, and Grofman B (2005) A unified theory of party competition: A cross-national analysis integrating spatial and behavioral factors. Cambridge University Press, New York

[4] Black D (1958) The theory of committees and elections. Cambridge University Press, London

[5] Chisik RA, and Lemke RJ (2006) When winning is the only thing: pure strategy Nash Equilibria in a three-candidate spatial competition model, Soc Choice Welf 26: 209-215.

[6] Cox GW (1987) Electoral equilibrium under alternative voting institutions, Am J of Pol Sci. 31: 82-108.

[7] Downs A (1957) An Economic Theory of Democracy. (Harper & Row, New York)

[8] Eaton, BC, and Lipsey RG (1974) The principle of minimum differentiation reconsidered: Some new developments in the theory of spatial competition, Rev of Econ Stud 42: 27-49.

[9] Evrenk H (2004) Political Economy of Anti-Corruption Reforms, PhD. Dissertation, Boston University.

[10] Groseclose T (2001) A model of candidate location when one candidate has a valence advantage. Am J of Pol Sci 45: 862-886.

[11] Hotelling H (1929) Stability in competition. Econ J 39: 41-57.

[12] Hug S (1995) Third parties in equilibrium. Public Choice, 82: 159-180.

[13] Lin T, Enelow JM, and Dorussen H (1999) Equilibrium in multicandidate probabilistic spatial voting. Public Choice 98: 50-82.

[14] Osborne M (1995) Spatial models of political competition under plurality rule. Can J Econ 27: 261-301.

[15] Schofield N (2005) Local Political Equilibria. In: Austen-Smith J, and Duggan J (eds.) Social choice and strategic decisions: Essays in honor of Jeffrey S. Banks. Springer, Berlin

[16] Schofield N, and Sened I (2006) Multiparty democracy. Cambridge University Press, New York

[17] Stokes D (1963) Spatial models of party competition. Am Pol Sci Rev 57:368-377