

# Macroeconomic Forecasting with Independent Component Analysis

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## ABSTRACT

This paper considers a factor model in which independent component analysis (ICA) is employed to construct common factors out of a large number of macroeconomic time series. The ICA has been regarded as a better method to separate unobserved sources that are statistically independent to each other. Two algorithms are employed to compute the independent factors. The first algorithm takes into account the kurtosis feature contained in the sample. The second algorithm accommodates the time dependence structure in the time series data. A straightforward forecasting model using the independent factors is then compared with the forecasting models using the principal components in Stock and Watson (2002). The results of this research can help us to gain more knowledge about the underlying economic sources and their impacts on the aggregate variables. The empirical findings suggest that the independent component method is a powerful method of macroeconomic data compression. Whether the ICA method is superior over the principal component method in forecasting the U.S. real output and inflation variables is however inconclusive.

*JEL classification:* C32; C53; E60

*Keywords:* forecast, dynamic factors, independent component analysis, principal component analysis

# 1 Introduction

In the last two decades, enormous effort and progress have been made on the development of small-scale macroeconomic models. Both stationary univariate autoregressions and vector autoregressions (VARs) are standard benchmarks used to evaluate economic policies or forecasts. These models include only a small number of variables while economic theories usually suggest large categories of variables, such as output, money, interest rates, wages, stock prices, etc. The choice of a specific subset of variables then becomes a statistical problem (known as variable-selection problem).

As opposed to small-scale models, large-scale models explicitly incorporate information from a large number of macroeconomic variables, again suggested by economic theory, into a formal statistical framework. The large-model approach is more appealing in real world practice because practical forecasters and policymakers find it useful to extract information from many more series than are typically included in a VAR model. One advantage of the large-model approach over the small-model approach is argued by Leeper et al. (1996) that there may be substantial forecasting improvement as the number of variables increases from, say 18 to 50 or to 100. Therefore, a VAR model with variables arbitrarily chosen might have large biases associated with it. Furthermore, Watson (2000) shows empirical evidence in support of the large model approach that there are many non-zero regression coefficients in such models.<sup>1</sup>

One particular class of the large-scale model is known as factor models. Classical factor models were initiated by Sargent and Sims (1977) and Geweke (1977) and had been considered in Singleton (1980), Engle and Watson (1981), Chamberlain and Rothschild (1983), and Quah and Sargent (1983). More recent studies of this approach include Forni and Reichlin (1996, 1998), Forni et al. (2000), and Stock and Watson (1998, 2002).

The intuition behind macroeconomic factor models is that the comovement in economic time series is arising largely from a relatively few key economic factors, such as productivity, monetary policy, oil shock, and so forth. In the literature, various methods have been proposed to construct these common economic driving forces. Among them, the simplest

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<sup>1</sup>A detailed discussion on the relative merits of the small and large model approaches can be found in Watson (2000).

method of constructing latent factors is the principal component analysis (PCA) introduced by Stock and Watson (2002).<sup>2</sup>

Although economists have accumulated a vast amount of experiences and knowledge in the theories of estimation, inferences and identification in a stationary VAR framework, there are practical and theoretical questions applied to the large models need to be answered. In particular, the problem of efficient estimation in large models remains an open question. Naturally, the construction of the common economic factors would be crucial in determining the forecasting performance of a factor model. While Stock and Watson's method is readily applied, their results also show that model selection procedures can be improved.

As an alternative to PCA, independent component analysis (ICA) is proposed in this study to obtain economic factors. The reasons to advocate ICA are twofold. First, many macroeconomic time series have fat tails in distribution and thus are non-Gaussian distributed. According to the central limit theorem, this implies that the latent economic forces are farther away from being Gaussian distributed than the observed economic series. While the ICA could construct factors that are non-Gaussian distributed, the PCA would totally omit this possibility. Second, assumptions, such as orthogonality and stationarity, are often made to identify latent factors. However, a researcher might be more interested in factors that are statistically independent of each other. While the PCA is designed to separate factors that are uncorrelated, the ICA is able to separate factors that are statistically independent.

We experimented with two different ICA algorithms in this paper. The basic ICA algorithm is known as FastICA. The results suggest that the basic ICA is a powerful technique of economic data compression; however its advantage over the PCA in forecasting the real output and inflation of the United States is limited. One important issue had not been considered in the FastICA algorithm. The FastICA algorithm is ideally applied to data that have no particular order. It means that one could shuffle the sample in anyway and this would not affect the estimation of independent components (ICs). Nevertheless, macroeconomic time series data contain much more time structure that should not be ignored. It implies

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<sup>2</sup>Other methods include the Kalman filter approach (Engle and Watson, 1981) and the spectral density approach (Singleton, 1980; Forni and Reichlin, 1996; Forni et al., 2000)

that the basic ICA and PCA methods may then be far from optimal, since they do not use the whole structure of the data.<sup>3</sup> The second algorithm then is designed to accommodate the time dependence structure in macroeconomic time series data.

A straightforward forecasting model using the independent factors is compared with the forecasting models using the principal components in Stock and Watson (2002). The forecasting results comparing the ICA technique and the PCA technique however are mixing. In this paper, the same data set studied in Stock and Watson (2002), which contains 146 monthly time series over 1959:1 - 1999:12, is used to demonstrate the estimation of independent common factors. The remainder of this paper is organized as follows. Section 2 presents a dynamic factor model and the basic concept of constructing independent factors using the ICA technique. In Section 3, the forecasting results of using these two different methods of common source separation are compared. Section 4 concludes.

## 2 Dynamic Factor Models

The notion that the comovement in economic time series is arising largely from a relative few key economic factors can be represented in a statistical factor model as,

$$\mathbf{X}_t = \mathbf{A}\mathbf{F}_t + \mathbf{e}_t, \quad (1)$$

where  $\mathbf{X}_t = (x_{1t}, x_{2t}, \dots, x_{Nt})'$  is a  $N \times 1$  vector of economic variables,  $\mathbf{F}_t = (f_{1t}, \dots, f_{kt})'$ , contains  $k$  latent factors, and  $\mathbf{e}_t = (e_{1t}, e_{2t}, \dots, e_{Nt})'$  is a  $N \times 1$  vector of idiosyncratic disturbances. The elements of  $\mathbf{e}_t$  are assumed to be cross sectionally and temporally uncorrelated. We assume that a predictive relationship between  $\mathbf{X}_t$  and  $y_{t+h}$  (an individual series to be forecasted) exists as:

$$y_{t+h} = \alpha(L)\mathbf{F}_t + \beta(L)y_t + \varepsilon_{t+h}. \quad (2)$$

Namely, to evaluate the  $h$ -step-ahead forecast performance of using common factors, one can apply an  $h$ -step-ahead projection to construct the forecast directly. This is the same forecast approach adopted in Stock and Watson (2002).

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<sup>3</sup>The principal component method used in Stock and Watson shares this same problem.

Because  $\mathbf{F}_t$ ,  $\alpha(L)$ , and  $\beta(L)$  in the large-scale factor model of (1) and (2) are unknown, Stock and Watson (2002) suggest three steps to construct the forecast. First, a consistent estimate of  $\mathbf{F}_t$ , denoted by  $\hat{\mathbf{F}}_t$ , is constructed from the in-sample data,  $t = 1, \dots, T$ . Second, the coefficients in (2) are estimated by regressing  $y_{t+h}$  onto  $\hat{\mathbf{F}}_t$ ,  $y_t$ , and possibly their lags, for  $t = 1, \dots, T - h$ . Finally, the forecast is formed as  $\hat{y}_{T+h} = \hat{\alpha}(L) + \hat{\beta}(L)y_T$ .

Stock and Watson (2002) estimate the factors by principal components because the PCA technique is feasible even for very large  $N$ . Obviously, the method of how to construct common economic driving forces would be crucial in determining the forecasting performance of a dynamic factor model. It is worthy further investigating such models with alternative method of factor construction. A relatively newly-developed statistical technique that can also be applied to a very large panel dataset is known as independent component analysis. In this paper, the forecast models that employ factors estimated by ICA are compared with those employ principal components.

## 2.1 Diffusion Indexes: PCA method versus ICA method

The diffusion indexes from a large panel dataset is constructed by decomposing  $\mathbf{X}$  into a loading matrix  $\mathbf{A}$  ( $N \times k$ ) and a factor matrix  $\mathbf{F}$  according to the following equation:

$$\mathbf{X} = \mathbf{A}\mathbf{F},$$

where  $\mathbf{X}(N \times T)$  and  $\mathbf{F}(k \times T)$  are observation stacked signals and diffusion indexes, respectively.

In either PCA or ICA, the factor matrix  $\hat{\mathbf{F}}_t$  is estimated by determining a weighting matrix  $\mathbf{W}$  with orthogonal rows so that the estimated components are

$$\hat{\mathbf{F}}_t = \mathbf{W}\mathbf{X}_t.$$

By constraining the rows of  $\mathbf{W}$  to be orthogonal vectors, the elements in  $\hat{\mathbf{F}}_t$  are uncorrelated factors. However, there is an infinity of different matrices that give orthogonal vectors. Once  $\mathbf{W}$  is determined,  $\mathbf{X}$  can be obtained by the pseudo-inverse of  $\mathbf{W}$ . So one would need further identification criterion. In PCA, principal components are obtained by maximizing variances of the estimated factors. In contrast, basic ICA obtains independent components by maximizing some other criterion, such as higher moments of the factors.

The PCA method is to get linear combinations of the observed signals such that the factors  $\hat{f}_{it}$  and  $\hat{f}_{jt}$  are uncorrelated. Differently, the ICA method is to get the linear combinations such that the transformed factors, say  $g(\hat{f}_{it})$  and  $h(\hat{f}_{jt})$ , with  $g$  and  $h$  some suitable nonlinear functions, are uncorrelated. In other words, the ICA technique is proposed to separate observed signals into statistically independent source components. Note that independence is a much stronger property than uncorrelatedness. If we are interested in separating independent sources of economic variations, uncorrelatedness is not enough to achieve this task.

The PCA method requires the rows of  $\mathbf{W}$  (or the columns of  $\mathbf{A}$ ) to be orthogonal. Given this requirement, the PCA method ranks factors by maximizing their variances. We think the orthogonal requirement on  $\mathbf{W}$  is unrealistic or too restrictive. While the ICA techniques makes a more stringent statistical-independent requirement on the estimated factors, it imposes no restriction on the matrix  $\mathbf{W}$ .

The principle of ICA estimation is to find the linear combination of observed signals that are maximally nongaussian. The idea is that, according to the central limit theorem, the sums of nongaussian random variable are closer to gaussian than the original ones (i.e. the latent factors). Given the fact that many macroeconomic time series have fat tails in distribution, it is likely that latent factors are nongaussian distributed.<sup>4</sup> In this paper, we first adopt a fixed-point algorithm that uses Newton iteration to maximize negentropy, which is a measure of nongaussianity. This iteration algorithm is called FastICA.<sup>5</sup> We summarize the FastICA algorithm in the next section. Further details should be referred to Hyvarinen and Oja (1997) and Hyvarinen, Karhunen and Oja (2001).<sup>6</sup>

## 2.2 FastICA Algorithm

Although we can measure nongaussianity by kurtosis, that measure is however very sensitive to outliers. An alternative measure of nongaussianity is negentropy, which is based on the

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<sup>4</sup>The kurtosis is often used as a first practical measure of nongaussianity.

<sup>5</sup>The estimated ICA factors are calculated using Matlab software version 6.1.

<sup>6</sup>There are various information theoretic contrast functions available for solving  $\mathbf{W}$ . In addition to maximum negentropy, mutual information, maximum entropy, informax, and the maximum likelihood approach are discussed in Hyvarinen, Karhunen and Oja (2001).

information-theoretic quantity of entropy. The entropy of a random vector  $\mathbf{z}$  with density  $p_z(\mu)$  is:

$$H(\mathbf{z}) = - \int p_z(\mu) \log p_z(\mu) d\mu.$$

Then negentropy is defined as follow:

$$J(\mathbf{z}) = H(\mathbf{z}_{gauss}) - H(\mathbf{z}),$$

where  $\mathbf{z}_{gauss}$  is a Gaussian random variable of the same correlation matrix as  $\mathbf{z}$ . A fundamental result of information theory is that a gaussian variable has the largest entropy among all random variables of equal variance. Therefore, the negentropy above is always nonnegative, and it is zero if  $\mathbf{z}$  has a gaussian distribution. In practice, an approximation version of the negentropy is used.

The FastICA algorithm for estimating the independent components consists the following steps:

1. Remove non-zero sample means from the data.
2. Whiten the data to yield variables  $\mathbf{x}^*$  that are zero-mean uncorrelated and have unit variances. This transformation can be accomplished by the PCA method.
3. Randomly choose an initial vector  $\mathbf{w}$  of unit norm.
4. Use a fixed point iteration to estimate the weighting vector  $\mathbf{w}$  such that the negentropy of  $\mathbf{w}'\mathbf{x}^*$ , i.e.  $J(\mathbf{w}'\mathbf{x}^*)$ , is maximized.

A remarkable property of the FastICA algorithm is the high speed of convergence in the iterations.

### 2.3 TSICA Algorithm

Since both the PCA method and FastICA method ignore the time structure in macroeconomic data, they may not be optimal. In this section, we consider an alternative independent component technique that takes the time dependence feature of our sample into account. We name such algorithm TSICA.

When data have time dependence, their autocovariances can be used to estimate the common components. Denote the  $l$ -th order autocovariance matrix of  $\mathbf{X}_t$  as

$$\mathbf{C}_X(l) = E[\mathbf{X}_t \mathbf{X}'_{t-l}]. \quad (3)$$

Given (3), we have

$$\mathbf{C}_{\hat{\mathbf{F}}}(l) = \mathbf{W} E[\mathbf{X}_t \mathbf{X}'_{t-l}] \mathbf{W}' = \mathbf{W} \mathbf{C}_X(l) \mathbf{W}'.$$

For independent components, the lagged covariances are all zero. That is,  $E[\hat{f}_{i,t}, \hat{f}_{j,t-l}] = 0$  for all  $l$ , and  $i \neq j$ . This implies that these  $\mathbf{C}_{\hat{\mathbf{F}}}(l)$ 's are diagonal matrices. Unless the data are truly generated by the ICA model, exact diagonal matrices are unlikely to obtain. The intuition then guides us to minimize the sum of the off-diagonal elements of several lagged covariances of  $\hat{\mathbf{F}}_t$ , under the constraint that the rows of  $\mathbf{W}$  are orthogonal. Namely, the estimation method is to minimize the objective function:

$$J(\mathbf{W}) = \sum_{l=1}^p \text{offdiag}(\mathbf{W} \mathbf{C}_X(l) \mathbf{W}'),$$

subject to the constraint that the rows of  $\mathbf{W}$  are orthogonal. Here, 'offdiag' indicates the sum of squares of the off-diagonal elements of a matrix.

Some iterative algorithms that solve the above minimization problem are discussed in Hyvarinen et al. (2001). One particular computation difficulty we would encounter is the determination of the number of factors to use in the factor model. Two different approaches can be applied here. Stock and Watson (1998) propose a modified information criterion in the context of the forecasting problem. Alternatively, Bai and Ng (2001) propose estimators that are based on the fit of (2). Both methods yield estimators that are consistent and asymptotically efficient under some assumptions.

## 2.4 A Dynamic Factor Model with Independent Components

With fixed number of factors, the weighting matrix in the ICA is obtained by rotating the weighting matrix in the PCA. Therefore, the forecasting model of (1) and (2) with PC and that with IC produce identical results. An alternative forecasting model with IC is proposed as the following.

$$\hat{\mathbf{F}}_t = \mathbf{W} \mathbf{X}_t. \quad (4)$$



$$\hat{f}_{j,t+h} = \gamma_j(L)\hat{f}_{j,t}, \quad j = 1, \dots, k. \quad (5)$$

$$\hat{\mathbf{X}}_{t+h} = \mathbf{A}\hat{\mathbf{F}}_{t+h}, \quad (6)$$

where  $\mathbf{A}$  is computed as the pseudo-inverse of  $\mathbf{W}$ . Namely, the IC's are estimated in the first step using either FastICA or TSICA algorithm. In the second step, coefficients in (5) are estimated in each individual factor regression. The final step is to construct the forecast as formulated in (6).

### 3 Empirical Results

#### 3.1 The Data

The data studied here were taken from the website of the authors of Stock and Watson (2002). The full dataset used in Stock and Watson (2002) contains 215 monthly series for the U.S. from 1959:1 to 1998:12. The series can be grouped into 14 main categories: real output and income; employment and hours; real retail, manufacturing, and trade sales; consumption; housing starts and sales; real inventories and inventory-sales ratios; orders and unfilled orders; stock prices; exchange rates; money and credit quantity aggregates; price indexes; average hourly earnings; and miscellaneous.<sup>7</sup>

Among these 215 series, 146 variables are available for the full sample period and form the balanced panel.<sup>8</sup> The remaining 69 series in the full dataset contain missing observations or are available over a diminished time span. In this paper, the IC factors are estimated based on the balanced panel dataset.

We have carried out normality test on each individual series. The normality assumption was tested using the Jarque-Bera skewness-kurtosis statistic.<sup>9</sup> The test results show that 120 out of the 146 time series can have the null hypothesis of normality rejected at the 1% significance level. This strong evidence motivates our application of the ICA method to estimate non-Gaussian distributed factors.

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<sup>7</sup>The detailed list of series is given in Stock and Watson (2002).

<sup>8</sup>Stock and Watson (2002) miscalculate the number of the balanced panel as 149.

<sup>9</sup>The Jarque-Bera test statistic,  $TS^2/6 + T(K - 3)^3/24$ , is asymptotically  $\chi^2(2)$  distributed, where  $T$  is the number of observations,  $S$  is sample skewness, and  $K$  is sample kurtosis. The 10% critical value is 4.61.

### 3.2 Estimated Independent Components

We first apply the FastICA algorithm to estimate the independent components. A different perspective about the PCA and ICA methods deserves some attention. The PCA factors are ordered in the magnitude of variances. The resulting largest  $k$  factors are invariant to the total number of principal components calculated. In contrast, the ICA computation is sensitive to the selection of the total number of independent components. Due to convergence failure in the iteration process of FastICA, the resulting estimated number of independent components could be less than how many that has been asked to compute. We have experimented with various number (6, 10, 15, and 20) of the total independent components. The resulting forecast outcomes are not too different. In the following, we report results where 10 independent factors are requested to be estimated.<sup>10</sup>

Figure 1 plots the squared correlation coefficient ( $R^2$ ) between the 215 individual time series and each of the first six independent factors (ranked according to excess-kurtosis). The 215 time series are grouped by category with detailed list stated in Stock and Watson (2002). The figure shows a more impressive clear-cut in identifying the sources of economic variations than the principal component factors plot in the paper by Stock and Watson. The first independent factor loads primarily on prices; the second independent factor on output and employment; the third, on interest rates and interest rate spreads; the fourth, on stock prices, exchange rates, and interest rates; the fifth, on retail sales and inventory-sale ratios; and the sixth, on orders.

### 3.3 Forecasting Results

In exercising real-time forecasting simulation, the factor estimation, parameter estimation, and forecast forming are conducted recursively. The first out-of-sample forecast is made in 1970:1 for forecasting  $y_{1970:1+h}^h$ . The factors are estimated using observations over 1959:1 - 1970:1.<sup>11</sup> The regression model of (5) is then run for  $t = 1960:1, \dots, 1970:1-h$  and the forecast value for  $y_{1970:1+h}^h$  is computed according to (6). The final simulated out-of-sample forecast is made in 1980:12- $h$  for  $y_{1980:12}^h$ .

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<sup>10</sup>In our forecast experiment, we find that at least 6 factors are yielded.

<sup>11</sup>All data are standardized to have zero mean and unit standard deviation before factors are estimated.

Following Stock and Watson (2002), the forecasting experiment simulates real-time forecasting for four real economic activity measures and four price inflations measures. The four real activity variables are: industrial production, real personal income, real manufacturing and trade sales, and number of employees on non-agricultural payrolls. The four price variables are: the consumer price index, the personal consumption price deflator, the consumer price index excluding food and energy, and the producer price index.

In order to fulfill the assumption that  $\mathbf{X}_t$  is  $I(0)$ , we apply the same transformation listed in Stock and Watson (2002) to each series.<sup>12</sup> The real variables are all modeled as being  $I(1)$  in logarithms and the price indexes are all modeled as being  $I(2)$  in logarithms.

According to Stock and Watson (2002), the best-performing forecast models in their work are models with small number of PC factors. Some models conform with Stock and Watson's (2002) paper are listed below.

**Benchmark models:**

AR: Univariate autoregression with lag order chosen by BIC (Bayesian information criterion).

LI: The model contains a vector of leading indicators (and their lags) and lags of  $y_t$  with lag orders determined by BIC recursively.

**Index models (1)-(2) with PCA factors:**

PC,  $k$  fixed: The model includes  $k$  contemporaneous PCA factors. This seems to be the best-performing model when the variable to be forecasted is real variable.

PC-AR,  $k$  fixed: The model includes  $k$  contemporaneous PCA factors and lags of  $y_t$  with the lag order chosen by BIC, with  $0 \leq p \leq 6$ . This seems to be the best-performing model when the variable to be forecasted is the price variable (or inflation rate).

**Index models (4)-(6) with ICA factors:**

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<sup>12</sup>The possible transformation includes taking logarithms, first differencing, second differencing, and screening for outliers.

IC computed with FastICA algorithm (all factors are included).

IC computed with TSICA algorithm (all factors are included).

To avoid ambiguity, one should note that the model label ‘PC’ is used here to replace the ‘DI’ presented in Stock and Watson (2002). The benchmark models and the models using PC components are completely identical to what have been reported in Stock and Watson (2002). In the presentation of empirical results below, figures associated with these models are copied from their work directly.

All ICA-based forecasts include all available factors in the model. This is because that the number of the independent factors estimated is varying over sub-sample periods. And the estimated factors are quite sensitive to the number of factors chosen. Ignoring any subset of the factors might have deteriorated the complete information that might would have been able to convey in the factors. In addition, matrix  $\mathbf{A}$  should be obtained using all factors in the system for the decomposition to be accurate.

Following Stock and Watson (2002), the 6-month-ahead, 12-month-ahead, and 24-month-ahead forecasts are experimented. Relative-MSE of a forecasting model is computed as the ratio of its MSE to the MSE of the univariate AR model. Hence, the AR model has a relative-MSE of 1.00. Tables 1(A), 1(B), and 1(C) are forecast results for real activity variables at forecasting horizon of  $h=6, 12, \text{ and } 24$ , respectively.

The main findings of Stock and Watson (2002) is that in most cases the simple PC model can account for all the predictable dynamics of the forecasted series and outperform the benchmark autoregression model.

Comparing PC-based forecasts with IC-based forecast, we find the results disappointing. Opposite to our intuition stated in the introduction, the forecast performance of IC-based forecasts are doing worse than that of the PC-based forecasts. We notice that models based on TSICA algorithm often deliver forecasting improvement over models based on FastICA algorithm. It somehow supports the idea of capturing time dependent structure in the macroeconomic data.

Tables 2(A), 2(B), and 2(C) report forecast results for price inflation variables. Stock and Watson (2002) find that including autoregressive components in addition to PC can dramatically improves the forecasts. That is, using ‘PC-AR’ with fixed single PC factor can

yield the best performance. Using ICA technique somewhat has its payoff shown in these tables. Model with TSICA algorithm perform a fine job in forecasting consumption deflator ( $h=6$  and  $12$ ), in forecasting no food and energy CPI ( $h=12$  and  $24$ ) and in forecasting PPI ( $h=6$ ).

## 4 Conclusion

This paper has explored the possibility of using ICA technique to yield economic common components from a large number of macroeconomic variables. The ICA method has been regarded as a better method to separate sources that are non-Gaussian distributed and statistically independent to each other. However, in terms of forecasting the real activity, the independent component method does not seem to demonstrate its advantage over the principal component method. In forecasting inflation variable, in some cases, the ICA technique shows its forecasting strength. Overall, the forecasting results comparing the ICA technique with the PCA technique are mixing.

Possible extensions to this line of research are raised here. First, the dynamic factor model considered in this paper is restricted to be linear. This leads to the possibility that a nonlinear ICA-based model can generate forecasting gains. Second, the results reported here are based on 146 series of the balanced panel. Will the ICA method produce better empirical results when it is applied to a larger panel, such as the 215 full dataset? Particularly, when the full dataset contains more outliers and data irregularity. Third, the transformation applied to the raw data before estimating the IC factors may have produced too much noise in the time series for the estimated factors to be accurate. One might want to estimate the independent factors from the raw data directly and apply suitable filtering to smooth the factors before he proceeds to forecast macroeconomic variables.

## References

- Bai, J. and S. Ng (2002). "Determining the number of factors in approximate factor models." *Econometrica* 70, NO. 1, 191-221.

- Chamberlain, G. and M. Rothschild (1983). "Arbitrage factor structure and mean-variance analysis of large asset markets." *Econometrica* 51, No. 5, 1305-1324.
- Engle, R.F., and M.W. Watson (1981). "A one-factor multivariate time series model of metropolitan wage rates." *Journal of the American Statistical Association*, 76, 774-781.
- Forni, M. and L. Reichlin (1996). "Dynamic common factors in large cross-sections." *Empirical Economics*, 21, 27-42.
- Forni, M. and L. Reichlin (1998). "Lets get real: A dynamic factor analytical approach to disaggregated business cycle." *Review of Economic Studies* 65, 453-474
- Forni, M., M. Hallin, M. Lippi, and L. Reichlin (2000). "The generalized dynamic factor model: Identification and estimation." *The Review of Economics and Statistics* 82(4), 540-552.
- Geweke, J. (1977). "The dynamic factor analysis of economic time series." in D.J. Aigner and A.S. Goldberger (eds.), *Latent Variables in Socio-Economic Models*, North-Holland, Amsterdam, Ch. 19.
- Geweke, J. and K.J. Singleton (1981). "Maximum likelihood 'confirmatory' factor analysis of economic time series." *International Economic review* 22, 37-54.
- Hyvarinen, A., J. Karhunen, and E. Oja (2001). *Independent Component Analysis*, John Wiley & Sons, Inc., New York.
- Hyvarinen, A. and E. Oja (1997). "A fast fixed-point algorithm for independent component analysis." *Neural Computation* 9(7), 1483-1492.
- Leeper, E.M., C.A. Sims and T. Zha (1996). "What does monetary policy do?" *Brookings Papers on Economic Activity*.
- Quah, D. and T.J. Sargent (1983). "A dynamic index model for large cross sections." In *Business Cycles, Indicators, and Forecasting*, eds. J.H. Stock and M.W. Watson, Chicago: University of Chicago Press, 285-306.
- Sargent and Sims (1977). "Business cycle modeling without pretending to have too much a-priori economic theory." In *New Methods in Business Cycle Research*, ed. C. Sims et al., Minneapolis: Federal Reserve Bank of Minneapolis.
- Singleton, K.J. (1980). "A latent time series model of the cyclical behavior of interest rates." *International Economic Review* 21, 559-575.
- Stock, James H. and Mark W. Watson (2002). "Macroeconomic forecasting using diffusion indexes." *Journal of Business and Economic Statistics* 20(2), 147-162.

- Stock, James H. and Mark W. Watson (1989). "New indexes of coincident and leading economic indicators." *NBER Macroeconomics Annual*, 351-393.
- Stock, J.H. and M.W. Watson (1998). "Diffusion indexes." NBER Working Paper No. W6702.
- Stock, James H. and Mark W. Watson (1999). "Forecasting inflation." *Journal of Monetary Economics* 44, 293-335.
- Watson, M.W. (2000). "Macroeconomic forecasting using many predictors." Manuscript, Department of Economics and Woodrow Wilson School, Princeton University.  
*Econometrica* 64, 1067-1084.

**Figure 1:  $R^2$  Between Factors and Individual Time Series, Grouped by Category.**

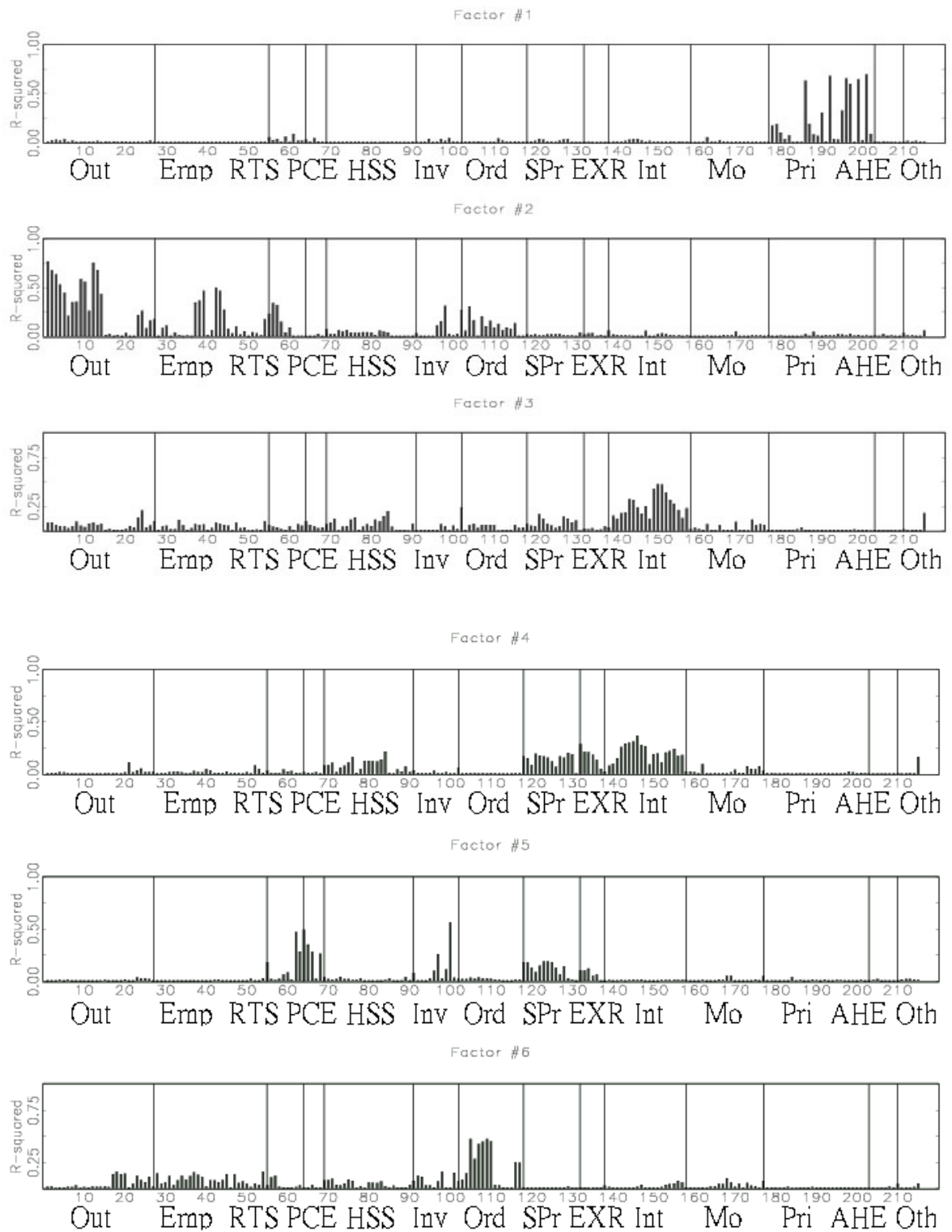




TABLE 1(A): Out-of-Sample Forecasting Results : Real Variables 6-month Horizon

<i>Forecast</i>	<u>Industrial Production</u>	<u>Personal Income</u>	<u>Mfg &amp; Trade sale</u>	<u>Nonag. Employment</u>
<i>Method</i>	<i>Rel. MSE</i>	<i>Rel. MSE</i>	<i>Rel.MSE</i>	<i>Rel. MSE</i>
<i>Horizon h = 6 months</i>				
<i>Benchmark models</i>				
AR	1.00	1.00	1.00	1.00
LI	.70	.83	.77	.75
<i>Models with PCA factors, k fixed</i>				
PC, k=1	.88	.83	.93	.88
PC, k=2	.67	.77	.74	.91
PC, k=3	.66	.77	.69	.91
PC, k=4	.66	.77	.69	.91
<i>Models with ICA factors (maximum k is 10)</i>				
FastICA algorithm	1.12	1.03	.83	.97
TSICA algorithm	.92	.84	.75	.89
<i>RMSE, AR Model</i>	.030	.016	.028	.008

TABLE 1(B): Out-of-Sample Forecasting Results : Real Variables 12-month Horizon

<i>Forecast</i>	<u>Industrial Production</u>	<u>Personal Income</u>	<u>Mfg &amp; Trade sale</u>	<u>Nonag. Employment</u>
<i>Method</i>	<i>Rel. MSE</i>	<i>Rel. MSE</i>	<i>Rel.MSE</i>	<i>Rel. MSE</i>
<i>Horizon h = 12 months</i>				
<i>Benchmark models</i>				
AR	1.00	1.00	1.00	1.00
LI	.86	.97	.82	.89
<i>Models with PCA factors, k fixed</i>				
PC, k=1	.94	.91	.94	.90
PC, k=2	.62	.81	.64	.83
PC, k=3	.55	.78	.59	.81
PC, k=4	.56	.81	.59	.84
<i>Models with ICA factors (maximum k is 10)</i>				
FastICA algorithm	1.23	1.17	.78	1.03
TSICA algorithm	1.06	.89	.61	.94
<i>RMSE, AR Model</i>	.049	.027	.045	.017

TABLE 1(C): Out-of-Sample Forecasting Results : Real Variables 24-month Horizon

<i>Forecast</i>	<u>Industrial Production</u>	<u>Personal Income</u>	<u>Mfg &amp; Trade sale</u>	<u>Nonag. Employment</u>
<i>Method</i>	<i>Rel. MSE</i>	<i>Rel. MSE</i>	<i>Rel.MSE</i>	<i>Rel. MSE</i>
<i>Horizon h = 24 months</i>				
<i>Benchmark models</i>				
AR	1.00	1.00	1.00	1.00
LI	1.09	1.29	1.08	1.07
<i>Models with PCA factors, k fixed</i>				
PC, k=1	1.00	.99	.99	.92
PC, k=2	.77	.89	.71	.74
PC, k=3	.55	.71	.65	.67
PC, k=4	.56	.75	.66	.75
<i>Models with ICA factors (maximum k is 10)</i>				
FastICA algorithm	1.40	1.11	.83	1.08
TSICA algorithm	1.19	.94	.70	.99
<i>RMSE, AR Model</i>	.075	.046	.070	.031

TABLE 2(A): Out-of-Sample Forecasting Results : Inflation Variables 6-month  
Horizon

<i>Forecast</i>	<u>CPI</u>	<u>Consumption deflator</u>	<u>CPI exc. food &amp; energy</u>	<u>PPI.</u>
<i>Method</i>	<i>Rel. MSE</i>	<i>Rel. MSE</i>	<i>Rel.MSE</i>	<i>Rel. MSE</i>
<i>Horizon h = 6 months</i>				
<i>Benchmark models</i>				
AR	1.00	1.00	1.00	1.00
LI	.82	1.04	1.10	1.00
<i>Models with PCA factors, k fixed</i>				
PC-AR, k=1	.77	.89	.90	.88
PC-AR, k=2	.76	.89	.80	.89
PC-AR, k=3	.78	.93	.85	.91
PC-AR, k=4	.77	.93	.85	.90
<i>Models with ICA factors (maximum k is 10)</i>				
FastICA algorithm	.80	.88	.91	.86
TSICA algorithm	.82	.84	.85	.89
<i>RMSE, AR Model</i>	.010	.007	.009	.017

TABLE 2(B): Out-of-Sample Forecasting Results : Inflation Variables 12-month Horizon

<i>Forecast</i>	<u>CPI</u>	<u>Consumption deflator</u>	<u>CPI exc. food &amp; energy</u>	<u>PPI.</u>
<i>Method</i>	<i>Rel. MSE</i>	<i>Rel. MSE</i>	<i>Rel.MSE</i>	<i>Rel. MSE</i>
<i>Horizon h = 12 months</i>				
<i>Benchmark models</i>				
AR	1.00	1.00	1.00	1.00
LI	.79	.95	1.00	.82
<i>Models with PCA factors, k fixed</i>				
PC-AR, k=1	.70	.83	.79	.80
PC-AR, k=2	.71	.83	.73	.82
PC-AR, k=3	.77	.93	.87	.87
PC-AR, k=4	.73	.89	.87	.82
<i>Models with ICA factors (maximum k is 10)</i>				
FastICA algorithm	.74	.86	.89	.87
TSICA algorithm	.66	.83	.71	.84
<i>RMSE, AR Model</i>	.021	.015	.019	.033

TABLE 2(C): Out-of-Sample Forecasting Results : Inflation Variables 24-month Horizon

<i>Forecast</i>	<u>CPI</u>	<u>Consumption deflator</u>	<u>CPI exc. food &amp; energy</u>	<u>PPI.</u>
<i>Method</i>	<i>Rel. MSE</i>	<i>Rel. MSE</i>	<i>Rel.MSE</i>	<i>Rel. MSE</i>
<i>Horizon h = 24 months</i>				
<i>Benchmark models</i>				
AR	1.00	1.00	1.00	1.00
LI	.70	.70	.99	.65
<i>Models with PCA factors, k fixed</i>				
PC-AR, k=1	.66	.72	.66	.76
PC-AR, k=2	.66	.74	.66	.73
PC-AR, k=3	.74	.77	.87	.80
PC-AR, k=4	.70	.70	.87	.75
<i>Models with ICA factors (maximum k is 10)</i>				
FastICA algorithm	.71	.90	.74	.79
TSICA algorithm	.72	.81	.64	.75
<i>RMSE, AR Model</i>	.052	.038	.046	.077