

Risk Aversion and Income Tax Enforcement

Parkash Chander¹

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Abstract

This paper characterizes optimal income tax and audit schemes in the presence of costly enforcement when the agent is risk averse and not necessarily risk neutral. It is shown that the results under risk-neutrality (Chander and Wilde (1998)) largely hold under risk aversion. We first show that in an optimal scheme the tax evasion decision of the agent is equivalent to risking his entire income against a possible gain in terms of lower tax payment. We then introduce a measure of aversion to such large risks. In contrast, the Arrow-Pratt coefficients of risk aversion measure aversion to small risks only. We show that the optimal tax function is non-decreasing and concave if the agent's aversion to large risks, as defined in terms of our measure, is decreasing with income. The optimal audit function is non-increasing and the audits may be random or deterministic.

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¹ Indian Statistical Institute and National University of Singapore. E-mail: ecsparka@nus.edu.sg. I am thankful to Jacques Drezè, Edi Karni, Basant Kapur, Ali Khan, Maurice Marchand, Rajiv Vohra and Peyton Young for comments and discussion. I have also benefited from seminar presentations at Brown, CORE, Hopkins, NUS, and Yale.

I. Introduction

Unlike the optimal income tax model of Mirrlees (1971),² the income tax enforcement model³ rules out supply side effects of income taxation or the problem of moral hazard and treats the agent's income as exogenous. However, it assumes that the agent's income can be observed only by performing an audit which is costly. In this setting the equity vs. efficiency issue arises in a form which is different from the one in the optimal income tax model. A progressive tax function generates stronger incentives for the agent to underreport income and thereby necessitates more auditing. Since audit expenditure is a direct resource cost, an optimal policy must weigh the welfare gains from progressive taxes against the concomitant rise in audit expenditure. Indeed, Chander and Wilde (1998) show that the optimal tax function must be generally increasing and concave,⁴ but under the rather strong assumption that the agent is risk neutral. The same result need not obtain when the agent is risk averse. First, the incentive to underreport income, which is equivalent to taking a risk, is weaker if the agent is risk averse, thus making it easier or less costly for the principal to enforce any given tax function. Second, the incentive to underreport income is even weaker if the income of the agent is higher and aversion to risk is not decreasing with income, thus making it easier for the principal to charge proportionately higher taxes on higher incomes. Both these effects can tilt the

² For recent exposition, see Myles (1995).

³ See e.g. Reinganum and Wilde (1985), Border and Sobel (1987), Melumad and Mookherjee (1989), Cremer, Marchand and Pestieau (1990), Sanchez and Sobel (1993), and Chander and Wilde (1998).

⁴ This means that the inability of the tax authority to costlessly observe true income restricts its ability to redistribute income in the same manner as does its inability to observe individual effort and skill in the original optimal income tax model.

balance in favor of a progressive tax function.

We characterize optimal income tax and audit schemes when the agent is risk averse and not necessarily risk neutral. We show that the results under risk-neutrality (Border and Sobel (1987) and Chander and Wilde (1998)) largely hold under risk aversion. In contrast to Winton (1995), who restricts attention to deterministic audits, we allow for random audits and show that the tax schedule is non-decreasing and the audit schedule is non-increasing.⁵ We proceed by showing that in an optimal scheme the agent's decision to underreport income is equivalent to risking his entire income against a possible gain in terms of lower tax payment. We then introduce a measure of aversion to such large risks. We show that the optimal tax function is increasing and concave if the agent's aversion to such large risks, as defined in terms of this measure, is decreasing with income. We call this the "strong decreasing risk aversion" or SDRA in short. This is briefly contrasted with the Arrow-Pratt (Arrow (1971) and Pratt (1964)) measures of risk aversion, which measure aversion to small risks only. We show that SDRA implies decreasing absolute risk aversion, but the converse is not true. Furthermore, SDRA does not imply decreasing or increasing relative risk aversion and the two are not generally comparable.

The paper is organized as follows. The next section states a general model of income tax enforcement and introduces the definition of an optimal tax and audit scheme. Section 3

⁵ Mookherjee and Png (1989) also study the problem under risk aversion and under random auditing but leave open the question of monotonicity of the optimal tax and audit schemes.

introduces the notion of SDRA and contrasts it with the Arrow-Pratt measures. Section 4 characterizes the optimal tax and audit schemes. Section 5 draws the conclusion.

2. The Model and Some Results

Consider an agent with random income y which can take values over an interval $[0, \bar{y}]$, $\bar{y} > 0$, according to a probability density function g with $g(y) > 0$ for all $y \in (0, \bar{y})$. The income of the agent is private information. The principal knows the distribution of income g but does not know the actual income of the agent. The agent has a von Neumann-Morgenstern (vN-M) utility function $u(y)$ which is continuously differentiable and satisfies $u(0) = 0, u'(y) > 0, u''(y) \leq 0$ for all $y \geq 0$.⁶

The principal may set up a mechanism for extracting the income of the agent. The *mechanism* consists of a set M of messages; a *tax function* $t : M \rightarrow \mathfrak{R}_+$; an *audit function* $p : M \rightarrow [0,1]$; and a *penalty function* $f : M \times [0, \bar{y}] \rightarrow \mathfrak{R}_+$.⁷ The agent who reports message m to the principal is audited with probability $p(m)$. If no audit occurs, his payment to the principal is $t(m)$. If an audit occurs, his true income is discovered

⁶ Assuming $u(0) = 0$ is a legitimate normalization (as we know from the expected utility theory), though not always possible. For instance, if $u(y) = (1/\alpha) y^\alpha$ with $\alpha < 0$. Nevertheless, it can be approximated by the utility function $(1/\alpha)(y + \varepsilon)^\alpha - (1/\alpha)\varepsilon^\alpha$ where $\varepsilon > 0$ is arbitrarily small. The assumption rules out schemes that may attain approximate first- best optima by imposing arbitrarily large penalties on the agent for underreporting.

⁷ We require both the tax and penalty functions to be nonnegative. However, none of our results below is affected if we allow them to take negative values so long as they are bounded below.

without error, and his payment to the principal is $f(m, y) \geq t(m)$ if his true income is y , i.e., the payment after an audit is not lower than the payment without an audit.⁸ We assume that the agent cannot submit a report that requires a payment which is larger than his true income. Therefore, for the mechanism to be feasible, it must satisfy certain requirements. Define $M(y) = \{m \in M : t(m) \leq y\}$ to be the set of feasible messages. Then the first feasibility requirement on the mechanism is that for all $M(y) \neq \phi$ for each y and $f(m, y) \leq y$ for all $m \in M(y)$. Thus, the agent with income y would submit a report m so as to maximize $[(1 - p(m))u(y - t(m)) + p(m)u(y - f(m, y))]$, subject to $m \in M(y)$. The second feasibility requirement is that this maximization problem has a solution for each $y \in [0, \bar{y}]$.

Most analyses of the principal-agent problem rely on the revelation principle. This states that without loss of generality we can confine our attention to mechanisms in which the agent is asked to report his income and is provided incentives to report truthfully. We shall refer to such mechanisms as *incentive compatible direct revelation mechanisms*. Since the revelation principle can indeed be shown to apply to our setting, we shall consider only such mechanisms. In view of the above mentioned feasibility requirements on the mechanism, the relevant class of such mechanisms consists of schemes (t, p, f) that satisfy for each y the following conditions:

⁸ This restriction rules out rewards for truth-telling. As noted by Melumad and Mookherjee (1989), such rewards violate the principle of horizontal equity (see e.g. Stiglitz (1982)).

$$0 \leq p(y) \leq 1;$$

$$0 \leq t(y) \leq f(y, y) \leq y; \text{ and}$$

$$(1 - p(y))u(y - t(y)) + p(y)u(y - f(y, y)) \geq (1 - p(x))u(y - t(x)) + p(x)u(y - f(x, y))$$

for all x with $t(x) \leq y$.

The last inequality says that the agent's expected utility is maximized if he reports his income truthfully. Let $f(y) \equiv f(y, y)$. Then the *revenue function* is defined as $r(y) \equiv (1 - p(y))t(y) + p(y)f(y)$. Since u is concave, if $f(y) > t(y)$, we can increase $t(y)$ and lower $f(y)$ such that the agent's incentive to report income truthfully is not affected and $r(y)$ is not lower. Note that raising the tax corresponding to income level y does not reverse the incentive constraints corresponding to income levels other than y . This means that $f(y)$ should be as small as possible. Therefore, in view of the above inequalities, $f(y) = t(y) = r(y)$.⁹ Similarly, the incentive constraints are as weak as possible if $f(x, y)$, $x \neq y$ and $t(x) \leq y$, is made as large as possible which is accomplished by setting $f(x, y) = y$ for $x \neq y$. This means that the expected utility of the agent with income y who reports instead x is equal to $(1 - p(x))u(y - t(x))$ or

$$(1 - p(x))u(y - t(y) + (t(y) - t(x))) + p(x)u(y - y), \text{ since by assumption } u(0) = 0.$$

Whereas the expected utility of the agent is equal to $u(y - t(y))$ if he reports his income truthfully. Comparison of the two utility levels implies that the agent's tax evasion decision is equivalent to risking his entire income y against a possible gain of $t(y) - t(x)$.

⁹ The argument here is different from that in the risk neutral case. However, risk aversion only reinforces the case for the equality of $f(y)$ and $t(y)$ as it reduces risk.

In view of the above observations, we need to consider only those tax and audit schemes that satisfy for each y ,

$$0 \leq p(y) \leq 1;$$

$$0 \leq t(y) \leq y; \text{ and}$$

$$u(y - t(y)) \geq (1 - p(x))u(y - t(x)) \text{ for all } x \text{ with } t(x) \leq y, \text{ since } u(0) = 0.$$

As in most analysis of the principal-agent problem we assume henceforth that an agent reports truthfully whenever reporting true income is optimal.

Let Q denote the set of all schemes (t, p) that satisfy the inequalities above. Then Q is the *set of all feasible schemes*.

Let S be some subset of Q , i.e. $S \subset Q$. A scheme $(t, p) \in S$ is *efficient* in S if there is no other scheme $(t', p') \in S$ such that $p' \leq p$, $t' \geq t$, and $p' \neq p$ or $t' \neq t$. That is, it is not possible to not lower the taxes and decrease an audit probability without increasing any other audit probability, and it is not possible to not increase the audit probabilities and raise the tax at some income level without lowering it at any other level.

Note that this notion of efficiency is independent of the probability density function g , that is, if a scheme is efficient, it is efficient with respect to every g . An optimal scheme must

clearly be efficient if the objective of the principal is to maximize revenue net of audit cost, that is

$$\underset{t,p}{\text{maximize}} \left[\int_0^{\bar{y}} t(y) g(y) dy - c \int_0^{\bar{y}} p(y) g(y) dy \right],$$

where $c > 0$ is the cost per audit. It is possible to show that for a variety of other objectives also an optimal scheme must be efficient including when the objective of the principal is purely redistributive. We thus characterize efficient schemes. Note that efficiency is a general requirement for optimality. Additional properties of optimal schemes depend on the particular objective function and the probability density function and are usually obtained by means of numerical analysis, as in Chander et al. (2003).

We characterize the efficient schemes in two steps. In the first step we show that in an efficient scheme the tax and audit functions must be monotonic. In the second step we show that if the agent's utility function satisfies SDRA, then the tax function must be concave.

Proposition 1: A scheme $(t, p) \in Q$ is efficient in Q only if t is non-decreasing and p is non-increasing.

Proposition 1 distinguishes itself from other results on this subject (Border and Sobel (1987) and Chander and Wilde (1998)) by allowing the agent to be risk averse. This

proposition is consistent with certain stylized facts in insurance markets (see e.g. Krasa and Villamil (1994)). We also need it for the proof of Theorem 3 below.

Proof of Proposition 1: We first show that if $(t, p) \in Q$ is efficient in Q , then the incentive constraints for each income level y must be binding at some x , that is, for each y there exists an x such that $u(y - t(y)) = (1 - p(x))u(y - t(x))$.

Suppose not, i.e., for some y , $u(y - t(y)) > (1 - p(x))u(y - t(x))$ for all x with $t(x) \leq y$. This implies $t(y) < y$ and we can find $t'(y)$ satisfying $y > t'(y) > t(y)$ such that $u(y - t'(y)) > (1 - p(x))u(y - t(x))$ for all x with $t(x) \leq y$. Since raising $t(y)$ to $t'(y)$ will also not reverse the incentive constraints for income levels other than y , we have been able to find a scheme $(t', p') \in Q$ such that $t' \geq t, t' \neq t$, and $p' = p$ which contradicts that (t, p) is efficient in Q .

We now show that t must be non-decreasing. Suppose not, that is, $y' > y$ and $t(y') < t(y)$. Then, $y \geq t(y) > t(y') \geq t(x')$ where x' is the point at which the incentive constraints are binding for the agent with income y' . It follows from the incentive constraints that $u(y - t(y)) \geq (1 - p(x'))u(y - t(x'))$. Since u is concave and $t(y) > t(x')$, $u(y' - t(y)) \geq (1 - p(x'))u(y' - t(x'))$. Since $t(y) > t(y')$ by supposition, $u(y' - t(y)) > (1 - p(x'))u(y' - t(x'))$. But this contradicts that the incentive constraints for the agent with income y' are binding at x' . Hence our supposition is wrong and $t(y') \geq t(y)$ for $y' > y$.

Finally, we prove that p is non-increasing. The incentive constraints imply that for each x , $p(x)$ must satisfy

$$p(x) \geq 1 - \frac{u(y - t(y))}{u(y - t(x))} \text{ for all } y \geq t(x).$$

Given that (t, p) is efficient, we must have

$$p(x) = 1 - \inf_{y > t(x)} \frac{u(y - t(y))}{u(y - t(x))}.$$

Note that if there is no y such that $t(x) < y$, then $p(x) = 0$. Since $t(x)$ is non-decreasing with x (as shown above), it follows from the above equality that $p(x)$ is non-increasing. This completes the proof.

The above equality defining the audit probabilities is at the heart of the result that taxes and audit probabilities move in opposite directions: the higher the tax $t(x)$, the weaker (and fewer) the incentive constraints to be satisfied at x and therefore the lower the audit probability $p(x)$.

The result that the tax is non-decreasing with income is along the expected lines. The result that the audit probability is non-increasing with income may be, however, explained intuitively as follows: since the low income reports are the most attractive for any taxpayer, the optimal auditing policy must devote more resources to the audit of low

reports. Since this monotonicity result applies to a population of taxpayers who are otherwise indistinguishable by occupation, residential location and source of income, a more careful interpretation of this result is that within each category of taxpayers (defined according to some exogenously available information on other characteristics) a taxpayer is more likely to be audited if he claims to have a lower income in that category.

We prove an additional monotonicity property that motivates further characterization of efficient schemes.

Proposition 2: If the incentive constraints for income levels y and y' are binding at x and x' , respectively, and $y' > y$, then $t(x') \geq t(x)$. That is, if $y' > y$, $u(y - t(y)) = (1 - p(x))u(y - t(x))$ for some x with $t(x) \leq y$ and $u(y' - t(y')) = (1 - p(x'))u(y' - t(x'))$ for some x' with $t(x') \leq y'$, then $t(x') \geq t(x)$.

Proof of Proposition 2: Suppose not, that is, suppose $y' > y$, but $t(x') < t(x)$. Then from the incentive constraints $u(y' - t(y')) = (1 - p(x'))u(y' - t(x')) \geq (1 - p(z))u(y' - t(z))$ for all z with $t(z) \leq y'$ and $u(y - t(y)) = (1 - p(x))u(y - t(x)) \geq (1 - p(z))u(y - t(z))$ for all z with $t(z) \leq y$. In particular, since $y \geq t(x) > t(x')$,

$$u(y' - t(y')) = (1 - p(x'))u(y' - t(x')) \geq (1 - p(x))u(y' - t(x))$$

and

$$u(y - t(y)) = (1 - p(x))u(y - t(x)) \geq (1 - p(x'))u(y - t(x')).$$

Thus,

$$\frac{u(y'-t(x'))}{u(y'-t(x))} \geq \frac{1-p(x)}{1-p(x')} \geq \frac{u(y-t(x'))}{u(y-t(x))},$$

which means

$$\frac{u(y'-t(x'))}{u(y-t(x'))} \geq \frac{u(y'-t(x))}{u(y-t(x))}.$$

This inequality can be rewritten as

$$\frac{\int_0^{y-t(x')} u'(s) ds + \int_{y-t(x')}^{y'-t(x')} u'(s) ds}{\int_0^{y-t(x')} u'(s) ds} \geq \frac{\int_0^{y-t(x)} u'(s) ds + \int_{y-t(x)}^{y'-t(x)} u'(s) ds}{\int_0^{y-t(x)} u'(s) ds}.$$

Thus,

$$\frac{\int_0^{y'-t(x')} u'(s) ds}{\int_0^{y-t(x')} u'(s) ds} \geq \frac{\int_0^{y'-t(x)} u'(s) ds}{\int_0^{y-t(x)} u'(s) ds}.$$

Since u is concave, that is, u' is decreasing, the above inequality cannot be true if $t(x') < t(x)$. Hence our supposition is wrong. This proves Proposition 2.

Examples can easily be constructed in which the inequality is strict, i.e., $t(x') > t(x)$ and

therefore $x' > x$, since as shown t is non-decreasing.

Proposition 2 shows that the audit probabilities in an efficient scheme may not be determined by the incentives of the highest income taxpayers. The audit probability $p(x)$ which is just sufficient to deter the taxpayer with income y from underreporting may be more than sufficient to deter the taxpayer with income higher than y . Intuitively, this indicates that the taxes may not be rising proportionately with income in an efficient scheme.

3. The Strong Decreasing Risk Aversion

We had noted above that the tax evasion decision of the agent is equivalent to risking his entire income y against a possible gain of $t(y) - t(x)$. More generally, consider a risk averse agent with vN-M utility function u who is considering a bet in which he risks his entire wealth w against a possible gain of x . This gain would have to be sufficiently large in order for him to be indifferent between such a bet and retaining his current wealth. Moreover, the more unwilling he is to risk his entire wealth, the larger x will be. Thus, x is a direct measure of the agent's aversion to risking his entire wealth. Furthermore, the higher the agent's current wealth w or the higher the amount risked, the higher must be the potential gain x . More formally, let

$$u(w) = (1 - p)u(w + x) + pu(0) = (1 - p)u(w + x), \quad (1)$$

since by assumption $u(0) = 0$. Then

$$\frac{dx}{dw} = \frac{1}{1-p} \frac{u'(w)}{u'(w+x)} - 1 \quad (2)$$

for a given w . Risk aversion or concavity of u implies that for any $p \in (0,1)$, x is strictly increasing with w , i.e. $dx/dw > 0$. Note that unlike the Arrow-Pratt model, the size of the risk or bet itself is increasing with wealth. Therefore, the risk premium must be increasing in any case and the only question that remains is whether it is increasing at a non-increasing rate. This motivates us to say that u satisfies *strong decreasing risk aversion (SDRA)* at w if x is increasing with w at a non-increasing rate, that is, if $d^2x/dw^2 \leq 0$ for all $p \in (0,1)$. We call it ‘strong’ for the following reason. In view of (2),

$$\begin{aligned} \frac{d^2x}{dw^2} &= \frac{1}{1-p} \left[\frac{u''(w)}{u'(w+x)} - \frac{u'(w)u''(w+x)}{(u'(w+x))^2} \left(1 + \frac{dx}{dw} \right) \right] \\ &= \frac{1}{1-p} \frac{u'(w)}{u'(w+x)} \left[\frac{-u''(w+x)}{u'(w+x)} \left(1 + \frac{dx}{dw} \right) - \frac{-u''(w)}{u'(w)} \right]. \end{aligned} \quad (3)$$

Since $x > 0$ and $dx/dw > 0$, $d^2x/dw^2 \leq 0$ only if $-u''(w)/u'(w)$ i.e. absolute risk aversion is decreasing at w .

Fig. 1 illustrates the relationship between w and $z = w + x$ as implied by SDRA of u . The “level curve” as defined by (1) is parameterized by the probability p of the unfavorable outcome. The lower the p , the higher the level curve.

Substituting from (1) and (2) and rearranging the terms in (3), we obtain

$$\frac{d^2x}{dw^2} = \frac{1}{1-p} \left(\frac{u'(w)}{u(w)} \frac{u'(w)}{u'(w+x)} \right) \left[\frac{-u''(w+x)}{u'(w+x)} \frac{u(w+x)}{u'(w+x)} - \frac{-u''(w)}{u'(w)} \frac{u(w)}{u'(w)} \right]. \quad (4)$$

Since, as seen from equation (1), x can be made as close to zero as desired by choosing the probability p of the unfavorable outcome to be sufficiently small, $d^2x/dw^2 \leq 0$ for all p if and only if $(-u''(w)/u'(w))(u(w)/u'(w))$ is non-increasing.

Let

$$R(y) = \frac{-u''(y)}{u'(y)} \frac{u(y)}{u'(y)}. \quad (5)$$

Then strong decreasing risk aversion of u at income level y is equivalent to $R'(y) \leq 0$, that is,

$$R'(y) = \frac{u''(y)}{u'(y)} \left[\left(\frac{u(y)}{u'(y)} \right) \left(\frac{-u'''(y)}{u''(y)} - 2 \frac{-u''(y)}{u'(y)} \right) - 1 \right] \leq 0. \quad (6)$$

This inequality reconfirms that decreasing absolute risk aversion (DARA) of u , that is, $-u'''(y)/u'(y) \geq -u''(y)/u'(y)$ is a necessary condition for SDRA of u . The ratio $-u'''(y)/u''(y)$ is the index of absolute prudence. The relative magnitudes of $-u''(y)/u'(y)$ and $-u'''(y)/u''(y)$ are known to play an important part in many applications¹⁰. Next we compare SDRA with non-increasing relative risk aversion, that is,

$$rr'(y) = \frac{u''(y)}{u'(y)} \left[y \left(\frac{-u'''(y)}{u''(y)} - \frac{-u''(y)}{u'(y)} \right) - 1 \right] \leq 0. \quad (7)$$

Since u is concave, that is, $u'(x)$ is non-increasing in x and $u(0) = 0$, $u(y)/u'(y) =$

$(1/u'(y)) \int_0^y u'(x) dx \geq y$. Therefore, the inequalities (6) and (7) are generally not comparable.

However, when (6) and (7) hold with equality they become identical. In fact, by equating the expression on the right in (6) or (7) to zero and integrating it twice and using $u(0) = 0$, we obtain $u(x) = ax^\alpha$ with $0 < \alpha < 1$ and $a > 0$, that is, the relative risk aversion is constant.¹¹ Our argument is completed if we can exhibit a utility function which satisfies (6) with strict inequality. It is easily seen that one such utility function is $u(y) = y + y^\alpha$ with $0 < \alpha < 1$. We also provide an example of a utility function such that

¹⁰ Sinclair-Desgagné and Gabel (1997), who study the problem of environmental auditing, show that the condition $-u'''(y)/u''(y) \geq -2u''(y)/u'(y)$ (which is necessary but not sufficient for SDRA) is sufficient for the condition under which an audit has to be performed. Drezè and Modigliani (1972) implicitly use the same condition to sign a precautionary saving effect. Aumann and Kurz (1977) use it to characterize the outcome of their income redistribution game.

¹¹ This might explain why the utility function $u(y) = ay^\alpha$, $a > 0, 0 < \alpha < 1$, has been so successful in many applications, as it implies constant aversion to not only small risks but also to some large risks.

$rr'(y) < 0$ but $R'(y) > 0$. Let $u(y) = y/(y^\alpha + 1)$, $\frac{1}{2} < \alpha < 1$. It is seen that $rr'(y) < 0$ for y sufficiently large but $R'(y) > 0$ for all y . A convenient utility function for which $R'(y) < 0$ but $rr'(y) > 0$ is not easy to find. Inequalities (6) and (7) suggest that such a utility function must be such that $u(y)/u'(y)$ is very high compared to y but the absolute risk aversion $-u''(y)/u'(y)$ is very low compared to the absolute prudence $-u'''(y)/u''(y)$.

Note that unlike the Arrow-Pratt coefficients of risk aversion $R(y)$ is independent of the units in which income y is measured. It is of course also invariant with respect to the units in which utility $u(y)$ is measured.¹² Additional properties of $R(y)$ as a measure of risk aversion can be found in Chander (2000).

4. The Optimal Tax Function

We are now well prepared to show that the optimal tax function is concave even when the agent is risk averse. As argued in the introduction to this paper, such a generalization is not straightforward. However, we are able to show that the result holds at least when the agent's utility function satisfies SDRA, that is, when absolute risk aversion is decreasing at a sufficiently high rate with income. This means that under SDRA the agent's incentive to

¹² More explicitly, $R(y) = (-u''(y)/u'(y))(u(y) - u(0))/u'(y)$. It is worth noting that the coefficient $R(y)$ also depends on the value of the utility function at zero and is not defined entirely by the local characteristics of the utility function at point y .

underreport income remains strong enough so as to render a progressive tax function less cost effective than a concave (or regressive) tax function.

Theorem 3: If the vN-M utility function u satisfies SDRA, then a scheme (t, p) is efficient in Q only if t is concave.

The proof involves showing that for each $w \in [0, \bar{y}]$ there exists a probability $p \in [0, 1]$ and a level curve passing through $(w, z(w))$ as in Fig. 1 such that the function $y - t(y)$ lies entirely above it.

Proof of Theorem 3: Since (t, p) is efficient, given any $\hat{y} \in [0, \bar{y}]$ (as shown in the proof of Theorem 1) there exists an \hat{x} such that $\hat{y} \geq t(\hat{x})$ and $u(\hat{y} - t(\hat{y})) = (1 - p(\hat{x}))u(\hat{y} - t(\hat{x}))$.

Three cases arise: $p(\hat{x}) = 0$, $p(\hat{x}) = 1$, and $0 < p(\hat{x}) < 1$. We consider these in that order.

If $p(\hat{x}) = 0$, $u(\hat{y} - t(\hat{y})) = u(\hat{y} - t(\hat{x}))$ and therefore $t(\hat{y}) = t(\hat{x})$. Since $u(y - t(y)) \geq u(y - t(\hat{x}))$ for all $y \geq t(\hat{x})$ (from the incentive constraints and that $p(\hat{x}) = 0$) and t is non-decreasing (by Theorem 1), it follows that $t(y) = t(\hat{x})$ for all $y \geq \hat{x} \geq t(\hat{x})$. On the

other hand, since t is non-decreasing, $t(y) \leq t(\hat{x})$ for $y \leq \hat{x}$. Thus, all of t lies below the line $l(y) = t(\hat{x}) = t(\hat{y})$ for all $y \in [0, \bar{y}]$.

If $p(\hat{x}) = 1$, $u(\hat{y} - t(\hat{y})) = 0$, i.e., $t(\hat{y}) = \hat{y}$. Since $y \geq t(y)$ for all $y \geq 0$, it follows that all of $t(y)$ lies below the 45⁰ line $y = l(y)$ and $l(\hat{y}) = t(\hat{y})$.

For the remaining case, that is, $0 < p(\hat{x}) < 1$, define $z = y - t(\hat{x})$, $\hat{w} = \hat{y} - t(\hat{y})$, and $\hat{z} = \hat{y} - t(\hat{x})$. Consider the level curve corresponding to $p(\hat{x})$, that is, the set of w and z satisfying $u(w) = (1 - p(\hat{x})) u(z)$ for all $z \geq 0$. As seen from Figure 1, there exists a line $k(z) = a(z - \tilde{z})$, $a > 0$, $\tilde{z} \geq 0$, such that $\hat{w} = k(\hat{z})$ and $w \geq k(z)$ for all w satisfying $u(w) \geq (1 - p(\hat{x})) u(z)$, $z \geq \tilde{z} \geq 0$.

Since in view of the incentive constraints $u(y - t(y)) \geq (1 - p(\hat{x})) u(y - t(\hat{x}))$ for all $y \geq t(\hat{x})$, $z = y - t(\hat{x})$ and $\tilde{z} = \tilde{y} - t(\hat{x})$ for some $\tilde{y} \geq t(\hat{x})$,

$$\begin{aligned} y - t(y) &\geq a[(y - t(\hat{x})) - (\tilde{y} - t(\hat{x}))] \text{ for all } y \geq \tilde{y} \geq t(\hat{x}) \\ &= a(y - \tilde{y}) \text{ for all } y \geq \tilde{y} \geq t(\hat{x}), \end{aligned}$$

and

$$\hat{y} - t(\hat{y}) = a(\hat{y} - \tilde{y}).$$

Since $y - t(y) \geq 0$ for all $y \geq 0$, the above inequality is true for $y \leq \tilde{y}$ as well. This means that there exists α and β such that

$$t(y) \leq \alpha y + \beta \text{ for all } y \geq 0$$

and

$$t(\hat{y}) = \alpha \hat{y} + \beta.$$

We have thus shown that in all the three cases, given any $\hat{y} \in [0, \bar{y}]$, there exists an affine function $l(y)$ such that $t(\hat{y}) = l(\hat{y})$ and $t(y) \leq l(y)$ for all $y \geq 0$. This proves that t is concave.

By definition, the net revenue maximizing scheme must be such that raising an additional dollar of revenue requires exactly an additional dollar of audit expenditure. Raising the taxes corresponding to the low-income levels not only increases the tax revenue collected but also deters the higher income agent from underreporting and thus saves audit expenditure. It is therefore optimal to impose proportionately higher taxes on low-income levels. Theorem 3 basically confirms this intuition.

Though in our formulation the audit probabilities are determined endogenously and allowed to be random, our characterization does not rule out the possibility of optimal audit probabilities being deterministic. In particular, it does not preclude the optimality of the following type of tax and audit schemes: $t(y) = y$ for $0 \leq y \leq y^*$ and $t(y) = y^*$ for $y \geq y^*$; $p(y) = 1$ for $0 \leq y \leq y^*$ and $p(y) = 0$ for $y \geq y^*$. The tax function is clearly concave and non-decreasing and the audit function is non-increasing as required by our characterization. Such deterministic audit schemes are of particular interest as they

resemble debt contracts, provided the act of auditing is identified with bankruptcy, and have been shown to be optimal in a variety of contexts (see e.g. Dye (1986)). Examples are however easily constructed in which deterministic audit probabilities are not optimal.

5. Conclusions

The contribution of this paper is two fold. First, it characterizes the optimal income tax and audit schemes when the agent is risk averse and not necessarily risk neutral by applying tools and concepts from the theory of risk aversion that have not been used before in this literature. Second, the paper advances the theory of risk aversion itself by not only discovering a new application, but also by suitably extending/generalizing it. This enriches both the areas. Alternatively, we could have adopted a convenient utility function, namely, $u(y) = ay^\alpha$ and obtained all our results. However, we have avoided this approach not merely for the sake of greater generality, but also for understanding better the role of the agent's risk aversion in the determination of the optimal policy. The concept of SDRA complements that of DARA as it implies that absolute risk aversion is decreasing with income at a higher rate.

We assumed that penalties like taxes and audit probabilities are determined endogenously subject only to an upper and lower bound. We know from the literature on economics of crime prevention that endogenous determination of penalties gives rise to what is often called the “Becker conundrum” – it is optimal to increase penalties as far as possible and

minimize the probability of costly auditing. Accordingly, in the present context the penalty for misreporting, no matter how insignificant, is extreme. This is clearly at odds with the actual practice. Doing away with this optimal penalty will however only reinforce our results. This is so because milder penalties only strengthen the incentives for misreporting and thus make it even harder to enforce a progressive tax function.

It may appear that the application of the revelation principle simplifies the analysis and tells us a lot about the effective taxes but little about the nominal ones. This is however not entirely so and by a suitable interpretation of our results we can also learn a few things about the nature of the nominal taxes and the pattern of underreporting. Let (t, p, f) be a tax and audit scheme that does not induce truthful reporting but that maximizes the net tax revenue. Suppose that the tax function t is increasing and let $\alpha(y) \leq y$ be the optimal report of the agent with income y . Let (t', p', f') be the equivalent scheme defined as follows: for each y , $t'(y) \equiv t(\alpha(y))$, $p'(y) \equiv p(\alpha(y))$, and $f'(x, y) = f(\alpha(x), y)$. Since the original scheme (t, p, f) is net revenue maximizing, so must be the equivalent scheme (t', p', f') - besides inducing truthful reporting. Thus, (t', p', f') must be efficient and therefore, as shown, t' must be non-decreasing. Since both t and t' are monotonic and $t(\alpha(y)) = t'(y) \geq t'(y') = t(\alpha(y'))$ for $y > y'$, it follows that $\alpha(y) \geq \alpha(y')$ for $y > y'$, i.e., the higher income agents report higher incomes. Furthermore, if the nominal tax function t is progressive, then since the effective tax function t' is regressive, tax evasion must be increasing with income

References:

1. Arrow, K. J. (1971), *Essays in the Theory of Risk-Bearing*, Chicago: Markham.
2. Aumann, R. J. and Kurz, M. (1977), "Power and Taxes", *Econometrica*, 45, 1137-1161.
3. Border, K. and Sobel, J. (1987), "Samurai Accountant: A Theory of Auditing and Plunder ", *Review of Economic Studies*, 54, 525-540.
4. Chander, P. and Wilde, L. (1998), "A General Characterization of Optimal Income Tax Enforcement", *Review of Economic Studies*, 165-183.
5. Chander, P. (2000), "A Simple Measure of Risk Aversion in the Large and an Application", CORE Discussion Paper No. 2000/41, in the process of publication.
6. Chander, P., Chakraborty, B. and Banerjee, D. (2003), "Optimal Income Tax Enforcement and Redistribution", mimeo, Indian Statistical Institute.
7. Drèze, J. H. and Modigliani, F. (1972), "Consumption Decisions Under Uncertainty", *Journal of Economic Theory*, 5, 308-335.
8. Dye, R. A. (1986), "Optimal Monitoring Policies in Agencies", *Rand Journal of*

Economics, 17, 339-350.

9. Krasa, S. and Villamil, A. (1994), "Optimal Contracts with Costly State Verification: The Multilateral Case", *Economic Theory*, 4, 167-187.
10. Mirrlees, J. (1971), "An Exploration in the Theory of Optimal Income Taxation", *Review of Economic Studies*, 38, 175-208.
11. Melumad, N.D. and Mookherjee, D. (1989), "Delegation as Commitment: the case of income tax audits", *Rand Journal of Economics*, 20, 139-163.
12. Mookherjee, D. and P'ng, I. (1989), "Optimal Auditing, Insurance, and Redistribution", *Quarterly Journal of Economics*, 103, 399-415.
13. Myles, G.D. (1995), *Public Economics*, Cambridge: Cambridge University Press, Chapter 5.
14. Pratt J. W. (1964), "Risk Aversion in the Large and in the Small", *Econometrica*, 32, 122-136.
15. Reinganum, J. F. and Wilde, L. L. (1985), "Income Tax Compliance in a Principal-Agent Framework", *Journal of Public Economics*, 26, 1-18.

16. Sanchez, I. and Sobel, J. (1993), "Hierarchical Design and Enforcement of Income Tax Policies", *Journal of Public Economics*, 50, 345-369.

17. Sinclair-Desgagné, B. and Gabel, H. L. (1997), "Environmental Auditing in Management Systems and Public Policy", *Journal of Environmental Economics, and Management*, 33, 331-346.

18. Winton, A. (1995), "Costly State Verification and Multiple Investors: The Role of Seniority", *Review of Financial Studies*, 8, 91-123.

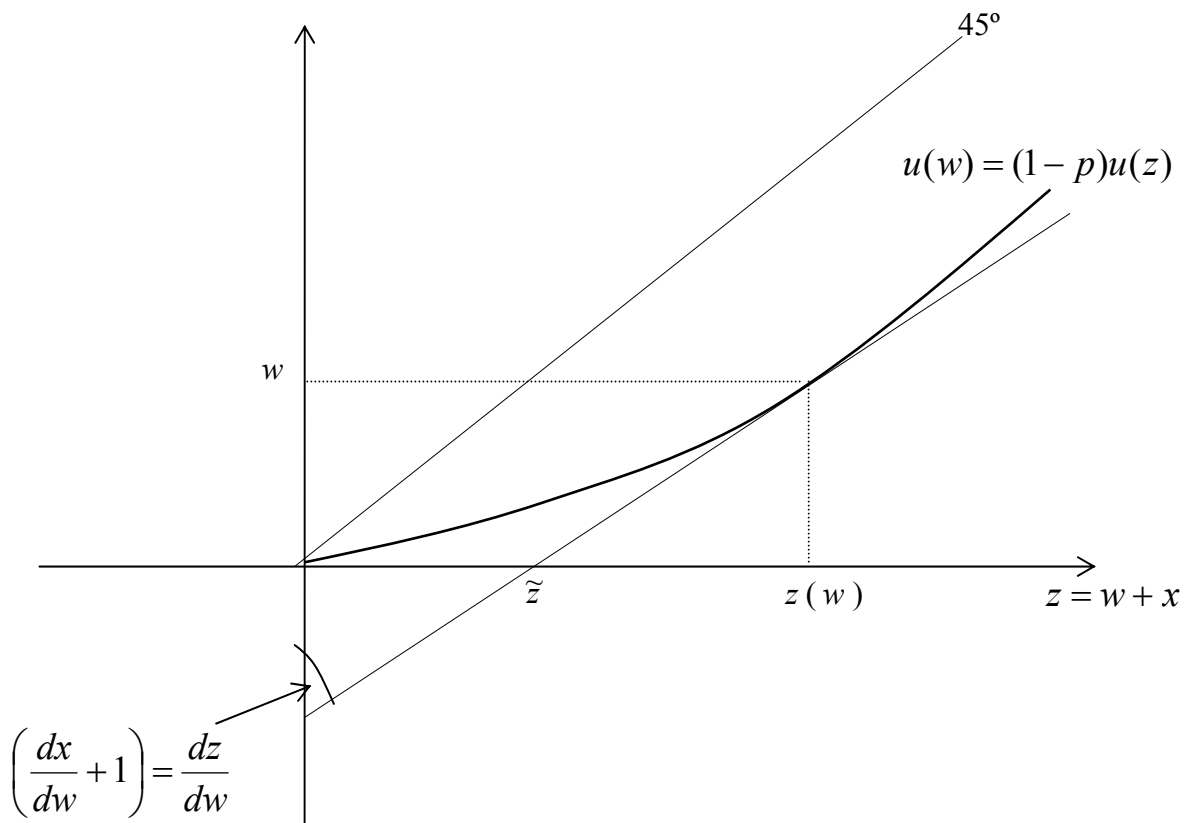


Figure 1

