Durable Goods Monopoly with Endogenous Quality

Jong-Hee Hahn
Keele University[*]
(*Preliminary*)

March 30, 2004

Abstract

This paper examines the dynamic pricing problem of a durable-good monopolist when product quality is endogenous. It is shown that the relationship between the firm's quality choice and the time-inconsistency problem crucially depends on how the unit production cost varies with quality. The monopolist may use quality as a strategic commitment device to eliminate the time-inconsistency problem. Also, it may have incentives to choose a quality higher or lower than the optimal commitment level. This contrasts with the planned obsolescence literature where durable goods monopolists reduces durability (often regarded as a measure of quality) to mitigate the time-inconsistency problem.

Keywords: Durable Goods, Quality, Time-inconsistency.

JEL classification: D42, L12.

1   Introduction

An important proposition in economic literature on durable-good monopolies is that a seller faces a problem of time inconsistency (Coase (1972), Bulow (1982), and others). The problem arises because durable goods sold in the future affect the future value of units sold today, and in the absence of the ability to commit to future prices the monopolist does not internalize this externality. There have been numerous investigations on the robustness of the result in the

---

[*]Department of Economics, Keele University, Keele, Staffs ST5 5BG, UK. Email: j.h.hahn@econ.keele.ac.uk

literature. Bond and Samuelson (1984) show that depreciation and replacement sales reduce the monopolist's tendency to cut price. Kahn (1986) shows a similar result when the monopolist faces an upward-sloping marginal cost schedule. Also, there have been many studies analyzing means and practices durable-good monopolists can employ to overcome or mitigate the time-inconsistency problem, notably leasing (Bulow, 1982), planned obsolescence (Bulow, 1986; Waldman, 1993), contractual provisions such as best-price provisions or most-favored-customer clauses (Butz, 1990), and more recently product-line extension using quality differentiation (Hahn, 2002; Inderst, 2002; Takeyama, 2002).

However, one of the aspects that received little attention in this literature is how the time-inconsistency result is affected when quality of product is chosen endogenously by the firm, as in real world durable goods markets. In the standard durable-goods monopoly model, quality of product is exogenous and therefore its effect on firm's strategic behavior is ignored. We analyze a durable-good monopoly model in which the firm is allowed to choose quality as well as prices. The main questions to be asked in this new setup are

- How does endogenizing quality affect the time-inconsistency problem?

- How does the time-inconsistency problem, if it is present, affect the monopolist's optimal choice of quality?

We consider a three-period dynamic game where a single-product monopolist first decides on the quality of good in period 0, and then chooses the price of the good sequentially over periods 1 and 2. The focus of our analysis is to investigate how the monopolist's decision on product quality is related to the time-inconsistency problem, and how the optimal quality differs from the one chosen under commitment to future prices. We show that the relationship critically depends on how the unit product cost varies with quality. The main findings of the analysis are as follows.

With endogenous quality the durable-goods monopolist may be naturally immune to the time-inconsistency problem. This happens when the average cost at the optimal commitment quality is high (i.e. the elasticity of scale of quality is small) enough for the firm to find selling the goods to low-valuation consumers in period 2 unprofitable. This result is quite different from the one observed in the standard durable-goods monopoly model with an exogenous quality and a constant unit cost. In the standard model the firm does not face time-inconsistency problems if the *absolute level* of the constant unit cost is sufficiently high (see Bulow, 1982 for example),

2

while here the relevance of the time-inconsistency problem depends on the *shape* of the unit cost function of quality. For instance, there is no time-inconsistency problem if the elasticity of scale of quality is sufficiently small even if the unit product cost itself is high.

As the average unit cost at the optimal commitment quality gets smaller (i.e. the elasticity of scale of quality increases) the time-inconsistency problem becomes relevant (i.e. the firm has the incentive to sell the good to low-valuation consumers in period 2). The firm's optimal reaction to the problem differs depending on the level of the elasticity of scale of quality. If it is relatively small (but still high enough for time consistency to be relevant), the firm finds it optimal to eliminate the time-inconsistency problem by choosing a quality higher than the optimal commitment level (therefore increasing the average unit cost of production). So, the monopolist uses quality as a strategic device of committing to no future sales, similar to the well-known burning-his-factory story in the standard durable-goods monopoly. Here, the firm also increases price up to a level where the equilibrium demand is smaller than the one in the commitment regime. As the elasticity of scale of quality increases further, increasing quality up to the point where the time-consistency problem disappears is too costly and the firm chooses to sell the goods in period 2. But, still the monopolist may have incentives to increase or decrease quality relative to the optimal commitment level. With second-period sales, the firm's decision on quality depends on the levels of the two marginal types and the relative importance (distribution) of the first and second-period demands. The second-period marginal type is more sensitive to the change of the average unit cost than the first-period marginal type, i.e. the second-period marginal type decreases more rapidly than the first-period marginal type as the average unit cost gets smaller. So, if the elasticity of scale of quality is very large (i.e. the average unit cost is sufficiently small at the optimal commitment quality), the equilibrium second-period marginal type is too low so that the firm chooses a quality lower than the optimal commitment level. If the elasticity of scale of quality is in an intermediate level, however, the firm increases quality, partially mitigating the time-inconsistency problem.

Our result is in contrast with the planned obsolescence literature (Bulow, 1986; Waldman, 1993) in which durable goods monopolists tends to *reduce* durability (often regarded as a measure of product quality) to mitigate the time-inconsistency problem. This result is also reminiscent of the reverse Averch-Johnson effect established by Bulow (1982), i.e. durable-good monopolists may invest less in fixed costs in order to keep their marginal costs high (a signal of lower future output and thus high future prices).

2 The model

We incorporate a quality dimension into the classical durable-goods monopoly model of Bulow (1982). There is a monopolist selling durable goods over two periods. The good is characterized by a one-dimensional quality index $q \geq 0$. The unit cost of production is constant and is given by $c(q)$, where $c(0) = 0, c'(\cdot) > 0$, and $c''(\cdot) > 0$. There is no economies of scale or learning-by-doing over time. Before production begins (say, in period 0) the firm chooses quality of the good (q) it will sell in periods 1 and 2. We assume constant fixed costs of production independent of quality, and also assume that the fixed cost corresponding to producing goods of a single quality (such as costs of setting up production lines) is too large so that the firm has no incentives to produce multiple qualities or alter the quality of the good later. So, we confine ourselves to a single-quality monopolist, i.e. the firm is constrained to produce goods of the single quality determined in period 0 and the quality choice is virtually irreversible.

On the demand side, there are a large number of consumers who live two periods. The total number of consumers is normalized to 1. Consumers have unit demands, and each consumer is indexed by a type parameter θ. The per-period (non-discounted) gross surplus a type-θ consumer derives from the consumption of a good of quality q is given by

$$v(\theta) = \theta q.$$

So, the marginal utility of quality is linear and increasing in type parameter θ. Consumer type is private information. The good purchased and used during period 1 can be used again in period 2 without depreciation. After period 2 the good becomes obsolete or is replaced by a new product. Consumers purchase if they are indifferent between buying and not buying. We assume that there is no upgrade or technological innovation during the time horizon considered. Consumers and the firm have a common discount factor denoted $\delta \in [0, 1]$. All agents have complete and perfect information (except consumer type), and have perfect foresight on future outcomes. The solution concept we are using is subgame perfection.

3 Two types of buyers

Consumers come in two types: $\theta \in \{\theta_l, \theta_h\}$ where $\theta_h > \theta_l > c'(0)$. There is a continuum of consumers for each type with mass μ for type θ_h and $1 - \mu$ for type θ_l. So, consumers act as a

price taker and all the bargaining power is given to the firm.[1]

First, consider the benchmark case where the firm can commit to future prices. The firm has two options. If it sells to only type-h consumers at a high price the total profit is

$$\pi_h \equiv \max_q : \mu[(1+\delta)\theta_h q - c(q)],$$

and the optimal quality q_h is given by the first order condition,

$$(1+\delta)\theta_h = c'(q_h). \tag{1}$$

If it sells to both types of consumers at a low price the total profit is

$$\pi_l \equiv \max_q : [(1+\delta)\theta_l q - c(q)],$$

and the optimal quality q_l is given by

$$(1+\delta)\theta_l = c'(q_l). \tag{2}$$

Note that both profit functions are concave, and therefore q_h and q_l given above are the true optimal quality in each case. The firm will choose to serve type-h consumers only if $\pi_h \geq \pi_l$, i.e.

$$\mu[(1+\delta)\theta_h q_h - c(q_h)] \geq (1+\delta)\theta_l q_l - c(q_l), \tag{3}$$

and choose to serve both types of consumers otherwise.

It is easily observed that $q_h > q_l$ from the convexity of $c(\cdot)$. In order to highlight the time-consistency issue in the classical durable-goods monopoly pricing, we will focus on the cases where condition (3) holds, i.e. the firm optimally chooses the intertemporal sales rather than the immediate market clearing and achieves the optimal commitment profit π_h with the optimal quality q_h. Condition (3) simply says that the marginal utility of quality is sufficiently differentiated between the two types of consumers and or the proportion of type-h consumers is sufficiently large relative to type-l consumers.

Next, consider the case where the firm cannot commit to future prices. Let us first define $\widetilde{q}$ such that $\theta_l \widetilde{q} = c(\widetilde{q})$, which denotes the level of quality where the price the firm can charge for a type-l consumer is just equal to the unit cost of production. Note that if the firm chooses $q \geq \widetilde{q}$

---

[1]The assumption of a continuum of buyers rules out the perfectly discriminating equilibria proposed by Bagnoli, Salant, and Swierzbinski (1989, 1995). See von der Fehr and Kühn (1995) for more details on how the relative commitment power between the seller and buyers affects the equilibrium outcome.

it does not have incentives to sell in period 2 and therefore does not face the time-inconsistency problem. But, for $q < \widetilde{q}$ it is optimal for the firm to sell to type-l consumers in period 2, which constrains the first-period price the firm can charge according to the type-h consumers' intertemporal incentive constraint,

$$(1 + \delta)\theta_h q - p_1 \geq \delta(\theta_h - \theta_l)q.$$

Given q, the firm's profit is given by

$$\pi(q) = \begin{cases} \mu[\theta_h q + \delta\theta_l q - c(q)] + \delta(1 - \mu)[\theta_l q - c(q)] & \text{if } q < \widetilde{q} \\ \mu[\theta_h q + \delta\theta_h q - c(q)] & \text{if } q \geq \widetilde{q} \end{cases}.$$

The profit function is discontinuous and non-differentiable at $q = \widetilde{q}$, but it is differentiable and concave elsewhere.[2] In fact, it jumps up at $q = \widetilde{q}$.[3]

Now we examine the firm's optimal choice of product quality without commitment. First, suppose that $q_h \geq \widetilde{q}$, i.e. θ_l is small relative to θ_h and or the unit cost of production is sufficiently large at the optimal commitment quality (subject to condition (3)). Then, from the concavity of the profit function (except for $q = \widetilde{q}$) and the fact that $\lim_{q \to \widetilde{q}-} \pi(q) < \pi(\widetilde{q})$ and $\lim_{q \to \widetilde{q}-} \pi'(q) < \pi'(\widetilde{q})$, it is optimal for the firm to choose the optimal commitment quality q_h. In this case, there is no time-inconsistency problem since it is optimal for the firm not to serve type-l consumers in period 2 ($\theta_l q_h < c(q_h)$), and therefore the firm can achieve the optimal commitment outcome even without commitment power.

Next, suppose that $q_h < \widetilde{q}$, i.e. θ_l is close to θ_h (subject to condition (3)) and or the unit cost of production is sufficiently small at the optimal commitment quality. Then, the concavity of the profit function in $q \in [\widetilde{q}, \infty]$ implies that $\lim_{q \to \widetilde{q}+} \pi'(\widetilde{q}) < 0$. Also, from condition (1), the definition of $\widetilde{q}$, the convexity of $c(\cdot)$, and the fact that $\theta_h > \theta_l$ we have $\lim_{q \to \widetilde{q}-} \pi'(q) = \mu[\theta_h + \delta\theta_l - c'(\widetilde{q})] + \delta(1 - \mu)[\theta_l - c'(\widetilde{q})] < 0$. So, we need to compare $\pi(\widetilde{q})$ and $\pi(\overline{q})$ where $\overline{q}$ is such that

$$\mu[\theta_h + \delta\theta_l - c'(\overline{q})] + \delta(1 - \mu)[\theta_l - c'(\overline{q})] = 0,$$

---

[2] $\pi'(q) = \begin{cases} \mu[\theta_h + \delta\theta_l - c'(q)] + \delta(1 - \mu)[\theta_l - c'(q)] & \text{if } q < \widetilde{q} \\ \mu[\theta_h + \delta\theta_h - c'(q)] & \text{if } q \geq \widetilde{q} \end{cases}$ and

$\pi''(q) = \begin{cases} -\mu c''(q) - \delta(1 - \mu)c''(q) & \text{if } q < \widetilde{q} \\ -\mu c''(q) & \text{if } q \geq \widetilde{q} \end{cases}.$

[3] $\lim_{q \to \widetilde{q}-} \pi(q) = \mu[\theta_h \widetilde{q} + \delta\theta_l \widetilde{q} - c(\widetilde{q})] < \pi(\widetilde{q}) = \mu[\theta_h \widetilde{q} + \delta\theta_h \widetilde{q} - c(\widetilde{q})].$

6

and the optimal quality is given by

$$q^* = \arg\max\{\pi(\widetilde{q}), \pi(\overline{q})\}.$$

Here, two qualitatively different equilibria can emerge. If the optimal quality is determined at $q^* = \widetilde{q} > q_h$, the demand is exactly the same as in the commitment case, but the firm, by increasing quality, commits itself to not serving type-l consumers in period 2 and effectively avoids the time-inconsistency problem.[4] This strategic commitment, however, comes with costs: the optimal profit $\pi(\widetilde{q})$ is smaller than the commitment profit π_h. If the optimal quality is determined at $q^* = \overline{q}$, on the other hand, the firm chooses to serve type-l consumers in period 2 and therefore the total demand increases. Let us compare $\overline{q}$ with q_h in this case. Recall that q_h satisfies condition (1). Evaluating the derivative of the profit function $\pi(\cdot)$ at $q = q_h$, we have

$$\pi'(q_h) = \mu[\theta_h + \delta\theta_l - c'(q_h)] + \delta(1 - \mu)[\theta_l - c'(q_h)] < 0.$$

So, it is clear that $\overline{q} < q_h$, i.e. the firm decreases quality in this case. This is because the marginal revenue of quality is smaller than in the commitment regime due to the expanded demand and the time-consistency constraint. With the general cost function $c(\cdot)$ it is a bit cumbersome to characterize the exact situations under which the firm chooses a particular level of quality. But, from the following numerical example we can clearly see that it depends on the structure of the unit cost function of quality, more specifically the elasticity of scale of the technology with respect to quality, which measures how the unit cost of production varies with quality.[5]

Example 1: Suppose that $\theta_h = 1, \theta_l = \frac{1}{2}, \delta = 1, \mu = \frac{1}{2}$, and $c(q) = q^\alpha$ ($\alpha > 1$). Note that the parameter α is the elasticity of scale of quality of the cost function.[6] From the first-order conditions (1) and (2), we find that $q_h = (\frac{2}{\alpha})^{\frac{1}{\alpha-1}}$ and $q_l = (\frac{1}{\alpha})^{\frac{1}{\alpha-1}}$. The average cost of quality is $k(q_h) = \frac{2}{\alpha}$ at q_h and $k(q_l) = \frac{1}{\alpha}$ at q_l, which are decreasing in α. So, we can regard α as a (indirect) measure of the average cost of quality at the commitment equilibrium. Given that $\alpha > 1$, condition (3) is always satisfied, i.e. $(\frac{1}{2})^{\frac{\alpha}{\alpha-1}} \leq \frac{1}{2}$ for all $\alpha > 1$. Also, we have $\widetilde{q} = (\frac{1}{2})^{\frac{1}{\alpha-1}}$. The condition for no time-inconsistency problem ($q_h \geq \widetilde{q}$) reduces to

---

[4]This demand rigidity is in fact an artifact of the two type model and is not generally true. In the next section, using a continuous type model we characterize how the (first-period) demand is affected by the firm's attempt of changing quality in order to avoid the time-consistency problem.

[5]In the present context, the elasticity of scale of the technology measures the percent increase in cost due to a one percent increase in quality.

[6]Note that α also represents the degree of homogeneity of the cost function.

$(\frac{2}{\alpha})^{\frac{1}{\alpha-1}} \geq (\frac{1}{2})^{\frac{1}{\alpha-1}}$, which holds for $\alpha \leq 4$. If $\alpha > 4$, then $q_h < \widetilde{q}$ and we need to compare $\pi(\widetilde{q})$ and $\pi(\overline{q})$ where $\overline{q} = (\frac{1}{\alpha})^{\frac{1}{\alpha-1}} < q_h = (\frac{2}{\alpha})^{\frac{1}{\alpha-1}} < \widetilde{q}$, i.e.

$$\pi(\widetilde{q}) = \frac{3}{2}(\frac{1}{2})^{\frac{\alpha}{\alpha-1}} \text{ and } \pi(\overline{q}) = (\alpha-1)(\frac{1}{\alpha})^{\frac{\alpha}{\alpha-1}},$$

and it turns out that the optimal quality is $\widetilde{q}$ for $\alpha \leq 9.8265$ and $\overline{q}$ for $\alpha \geq 9.8265$.

Hence, we have three different equilibria depending upon the parameter α. For $\alpha \leq 4$, the cost function increases more steadily as quality increases, leading to a relatively high level of unit production cost at the optimal commitment quality. In this case, there is no time-inconsistency problem and the firm can achieve exactly the same outcome as in the commitment regime. For $4 \leq \alpha \leq 9.8265$, the time-inconsistency problem becomes relevant but the firm increases quality relative to the optimal commitment level in order to avoid the time-inconsistency problem. For $\alpha \geq 9.8265$, the firm finds it more profitable to accept the time-inconsistency problem and serve type-l consumers in period 2, but decrease quality as the marginal revenue of quality is smaller than the commitment regime.

4   A continuum-of-types case

In this section we generalize our analysis to a situation with a continuum of consumer types. For simplicity, we assume that there is a continuum of consumers with unit mass, and the type parameter θ is uniformly distributed on the unit interval $[0, 1]$.

4.1   Equilibrium with commitment

Suppose the firm can commit to prices it will charge in the future. Then, the firm's optimal pricing policy is to sell the goods in period 1 at its (two-period) monopoly price and sell nothing in period 2. Given quality q and price p, consumers of type θ such that $(1 + \delta)\theta q \geq p$ will buy the good in period 1 and others will not. It is convenient to solve the firm's profit-maximization problem in two stages. We first find the optimal marginal type (which is equivalent to choosing the optimal price) for a given quality, and the optimal quality is determined later. Given q, the firm chooses θ in order to maximize profits:

$$\max_{\theta} : q(1 - \theta)[(1 + \delta)\theta - k(q)]$$

subject to $0 \leq \theta \leq 1$, where $k(q) = \frac{c(q)}{q}$ is the average cost of quality at the given q. The corresponding optimal price is then determined by the equation $(1 + \delta)\theta q = p$. The profit

8

function is concave in θ, and therefore from the first-order condition the optimal marginal type is given as

$$\theta^*(q) = \tfrac{1}{2} + \tfrac{k(q)}{2(1+\delta)} \tag{4}$$

for $0 < k(q) \leq 1 + \delta$.[7]

Plugging in the optimal marginal type in (4), the firm's quality choice problem is as follows:

$$\max_q : \tfrac{q}{4(1+\delta)}[(1+\delta) - k(q)]^2.$$

Note that the profit function is quasi-concave (single-peaked) for the relevant region where $0 < k(q) \leq 1 + \delta$. Then, from the first-order condition the optimal quality q^* is given by

$$k(q^*) + 2q^* k'(q^*) = 1 + \delta \tag{5}$$
$$\implies 2c'(q^*) - \tfrac{c(q^*)}{q^*} = 1 + \delta.$$

Robustness of the time-consistency problem: In order to examine the relevance of the time-inconsistency problem when the firm can not commit to future prices, we consider the firm's incentive to sell the goods in period 2 after serving consumers of type greater than or equal to θ^* (the optimal marginal type at the commitment equilibrium) in period 1. The marginal consumers' willingness-to-pay for the good for the second-period consumption is

$$v(\theta^*) = \theta^* q^* = \left[\tfrac{1}{2} + \tfrac{k(q^*)}{2(1+\delta)}\right] q^*.$$

The firm would not face the time-inconsistency problem even without commitment if the equilibrium quality under commitment is chosen so that serving consumers of type $\theta \leq \theta^*$ in period 2 is not profitable, i.e.

$$v(\theta^*) \leq c(q^*) \implies k(q^*) \geq \tfrac{1+\delta}{1+2\delta}, \tag{6}$$

which simply says that the average cost of quality is sufficiently large at the commitment equilibrium level of quality.

This clearly shows that when quality is endogenous the relevance of the time-inconsistency problem in durable goods monopoly crucially depends on how the unit production cost varies with quality, and in some cases a durable-good monopolist is naturally immune to the time-inconsistency problem. For example, for $c(q) = \beta q^\alpha$ ($\alpha > 1, \beta > 0$) condition (6) holds if

---

[7]We ignore the cases of $k(q) \geq 1 + \delta$ in which the firm gets zero profits.

$\alpha \le 1 + \delta$, which does not depend upon the scaling index β. This means that the condition for no time-inconsistency problem hinges on the *shape* of the unit cost function of quality (i.e. the elasticity of scale of quality) rather than its *absolute level*. It should be noted that this result is quite different from the one observed in the standard (exogenous-quality) durable-goods monopoly model with a constant unit cost where the firm does not face time-inconsistency problems if the constant unit cost is sufficiently large. For example, in the two-period durable-goods model of Bulow (1986) there is no time-inconsistency problems if the constant marginal cost c is greater than $\frac{1+\delta}{1+2\delta}$. But, here with endogenous quality the firm is immune to the time-inconsistency problem when the average cost of quality is larger than $\frac{1+\delta}{1+2\delta}$ at the optimal commitment quality, i.e. the elasticity of scale of quality is sufficiently large, even if the unit cost of production is very low (a low β).

4.2 Equilibrium without commitment

We now consider the monopolist's choice of quality when the firm cannot commit to future prices. The firm problem is to choose quality q in period 0 and a sequence of the first-period and second-period prices p_1 and p_2 in periods 1 and 2 to maximize total profits, given consumers' rational expectation about second-period outcomes. Given q and (p_1, p_2), $\theta_1 \in [0,1]$ denotes the type of consumers who are indifferent between buying in the first period and waiting to buy in the second period, i.e.

$$(1 + \delta)\theta_1 q - p_1 = \delta(\theta_1 q - p_2),$$

and similarly $\theta_2 \in [0, \theta_1]$ denotes the type of consumers who are indifferent between buying in the second period and buying nothing, i.e.

$$\theta_2 q - p_2 = 0.$$

We find a subgame perfect Nash equilibrium by solving the problem backward. It is useful to express the monopolist's profits in terms of θ_1 and θ_2 rather than p_1 and p_2. First, given q and θ_1 the firm's second-period problem is

$$\max_{\theta_2} : q(\theta_1 - \theta_2)[\theta_2 - k(q)]$$

subject to $\theta_2 \le \theta_1$. Since $k(\cdot)$ is monotone increasing (from the convexity of $c(\cdot)$), for a given q we can treat $k(q)$ as a constant unit cost in solving the problem at this stage. The solution is

$$\theta_2^*(q, \theta_1) = \begin{cases} \theta_1 & \text{for } \theta_1 \le k(q) \\ \frac{1}{2}[\theta_1 + k(q)] & \text{for } \theta_1 > k(q) \end{cases}.$$

10

Given the second-period equilibrium outcome, in period 1 the firm chooses θ_1 maximize total profits:

$$\pi(q) \equiv \max_{\theta_1} : q\left\{(1-\theta_1)[\theta_1 + \delta\theta_2^* - k(q)] + \delta(\theta_1 - \theta_2^*)[\theta_2^* - k(q)]\right\}$$

subject to $0 \le \theta_1 \le 1$. Again we focus on the cases where $0 < k(q) \le 1 + \delta$. The solution is given by

$$\theta_1^*(q) = \begin{cases} \frac{2+\delta}{4+\delta} + \frac{2(1-\delta)}{4+\delta}k(q) & \text{for } 0 < k(q) \le \frac{2+\delta}{2+3\delta} \\ k(q) & \text{for } \frac{2+\delta}{2+3\delta} \le k(q) \le \frac{1+\delta}{1+2\delta} \\ \frac{1}{2} + \frac{1}{2(1+\delta)}k(q) & \text{for } \frac{1+\delta}{1+2\delta} \le k(q) \le 1 + \delta \end{cases} \ .$$

Plugging in the optimal marginal types, the total profit is then rewritten as

$$\pi(q) = \begin{cases} \frac{q}{4(4+\delta)^2}\{4[1-(1-\delta)k(q)][(2+\delta)^2 - (4+\delta^2)k(q)] \\ \qquad\qquad + \delta[2+\delta-(2+3\delta)k(q)]^2\} & \text{for } 0 < k(q) \le \frac{2+\delta}{2+3\delta} \\[2mm] \delta q[1-k(q)]k(q) & \text{for } \frac{2+\delta}{2+3\delta} \le k(q) \le \frac{1+\delta}{1+2\delta} \\[2mm] \frac{q}{4(1+\delta)}[(1+\delta)-k(q)]^2 & \text{for } \frac{1+\delta}{1+2\delta} \le k(q) \le 1+\delta \end{cases} \ .$$

The profit function $\pi(q)$ is continuous and differentiable except at $q = k^{-1}(\frac{1+\delta}{1+2\delta})$ and $q = k^{-1}(\frac{2+\delta}{2+3\delta})$ ($k^{-1}(\cdot)$ is the inverse function of $k(\cdot)$) where it is continuous but non-differentiable. Also, note that it is quasi-concave in each of the three regions. Differentiating the profit function where it is possible, we have

$$\pi'(q) = \begin{cases} \frac{1}{4(4+\delta)^2}\{4[1-(1-\delta)k(q)][(2+\delta)^2 - (4+\delta^2)k(q)] \\ \quad + \delta[2+\delta-(2+3\delta)k(q)]^2 \\ \quad + qk'(q)[(32-24\delta+32\delta^2+10\delta^3)k(q) - 2(4+\delta)(4+\delta^2)]\} & \text{for } 0 < k(q) < \frac{2+\delta}{2+3\delta} \\[2mm] \delta\{[1-k(q)]k(q) + qk'(q)[1-2k(q)]\} & \text{for } \frac{2+\delta}{2+3\delta} < k(q) < \frac{1+\delta}{1+2\delta} \\[2mm] \frac{1}{4(1+\delta)}[(1+\delta)-k(q)][(1+\delta)-k(q)-2qk'(q)] & \text{for } \frac{1+\delta}{1+2\delta} < k(q) \le 1+\delta \end{cases} \ .$$

We now analyze how the monopolist' optimal choice of quality is affected by the lack of commitment power, and how it is related to the structure of the unit cost function.

First, if $\frac{1+\delta}{1+2\delta} \le k(q^*) < 1 + \delta$, i.e. the average cost of quality is sufficiently high at the optimal commitment quality, the monopolist does not face the time-consistency problem and

can achieve exactly the same outcome as in the commitment regime. Note that this condition for no time-inconsistency problem is identical to condition (6) derived in the previous subsection.

Second, if $\frac{2+\delta}{2+3\delta} \leq k(q^*) < \frac{1+\delta}{1+2\delta}$, i.e. the average cost of quality at the optimal commitment quality is in an intermediate range, the derivative of the profit function evaluated at $q = q^*$ is always positive in the region, i.e.

$$\pi'(q^*) = \frac{\delta}{2}[(\delta+1) - (1+2\delta)k(q^*)] > 0,$$

where the use was made of condition (5). This means that it is optimal for the monopolist to commit to not selling in period 2 by increasing quality from the optimal commitment level. With a continuum of consumer types we can now see how this strategic commitment using quality affects the equilibrium demand, an aspect that has not been analyzed in the previous discrete type case. Since the convexity of $c(\cdot)$ implies that $\theta_1^*(q) = k(q)$ is increasing in q, the first-period demand is smaller than the equilibrium demand in the commitment regime. Increasing both quality and the first-period marginal type immediately implies an increase of the first-period price relative to the optimal price in the commitment regime. So, in this case the monopolist finds it profitable to increase quality as well as price up to a level where the first-period demand is smaller than the equilibrium demand in the commitment regime in order to avoid the time-inconsistency problem.

Last, if $0 < k(q^*) < \frac{2+\delta}{2+3\delta}$, i.e. the average cost of quality is sufficiently low at the optimal commitment quality, increasing quality to commit to not selling in period 2 is too costly and therefore the firm finds it more profitable to accept the time-inconsistency problem and sell to some low types of consumers in period 2. But, the monopolist may wish to increase or decrease quality relative to the optimal commitment level. Let $\overline{q}$ denote the optimal quality in this case, where $\overline{q}$ is given by the first-order condition,

$$4[1 - (1-\delta)k(\overline{q})][(2+\delta)^2 - (4+\delta^2)k(\overline{q})] \;+ \delta[2 + \delta - (2+3\delta)k(\overline{q})]^2$$
$$+ \overline{q}k'(\overline{q})[(32 - 24\delta + 32\delta^2 + 10\delta^3)k(\overline{q}) - 2(4+\delta)(4+\delta^2)] = 0.$$

Evaluating the first derivative of profit function at $q = q^*$ we have

$$\pi'(q^*) = \tfrac{5\delta^3}{4(4+\delta)}[k(q^*) - \tfrac{1}{5}].$$

So, given the quasi-concavity of the profit function the monopolist will decrease quality if $0 < k(q^*) < \frac{1}{5}$ and increase quality if $\frac{1}{5} \leq k(q^*) < \frac{2+\delta}{2+3\delta}$. The intuition for this result is as follows.

12

The firm's decision on quality basically depends on the levels of the two marginal types and the relative importance (distribution) of the first and second-period demands. In this case of $0 < k(q^*) < \frac{2+\delta}{2+3\delta}$, the two marginal types are given as

$$\theta_1^* = \frac{2+\delta}{4+\delta} + \frac{2(1-\delta)}{4+\delta}k(\overline{q})$$

and

$$\theta_2^* = \frac{1}{2}[\theta_1 + k(\overline{q})] = \frac{2+\delta}{2(4+\delta)} + \frac{6-\delta}{2(4+\delta)}k(\overline{q}).$$

Note that the second-period marginal type is more sensitive to the level of the average unit cost than the first-period marginal type.[8] This implies that the second-period marginal type decreases more rapidly than the first-period marginal type as the average unit cost gets smaller. In the former case, with a relatively small average unit cost the equilibrium second-period marginal type is too low so that the optimal quality is determined lower than the optimal commitment level. In the latter case, however, the average unit cost is relatively high and therefore the firm increases quality, partially mitigating the time-inconsistency problem.

Combining the above results leads to the following proposition.

Proposition 1 *Compared with the commitment equilibrium, the monopolist i) decreases quality if $0 < k(q^*) < \frac{1}{5}$, ii) increases quality if $\frac{1}{5} \leq k(q^*) < \frac{2+\delta}{2+3\delta}$ with second-period sales (still subject to the time-inconsistency problem), iii) increases quality if $\frac{2+\delta}{2+3\delta} \leq k(q^*) < \frac{1+\delta}{1+2\delta}$ without second-period sales (avoiding the time-inconsistency problem), and iv) offers the same quality as in the commitment case if $\frac{1+\delta}{1+2\delta} \leq k(q) < 1 + \delta$.*

Example 2: Suppose $c(q) = q^\alpha$ ($\alpha > 1$). From condition (5), the optimal commitment quality is given by $q^* = (\frac{1+\delta}{2\alpha-1})^{\frac{1}{\alpha-1}}$, which initially decreases and then increases after some critical value of α (e.g. $\alpha \cong 2.6555$ for $\delta = 1$). The average unit cost at the optimal commitment quality is then given by $k(q^*) = \frac{1+\delta}{2\alpha-1}$, which is monotone decreasing for $\alpha > 1$. Recall that α is the elasticity of scale of quality. So, the average unit cost of production at the optimal commitment quality is higher when the cost function is more elastic to quality change. If $0 < \frac{1+\delta}{2\alpha-1} < \frac{1}{5} \Rightarrow \alpha > \frac{6+5\delta}{2}$, the monopolist decreases quality. If $\frac{1}{5} \leq \frac{1+\delta}{2\alpha-1} < \frac{2+\delta}{2+3\delta} \Rightarrow \frac{4+6\delta+3\delta^2}{2(2+\delta)} < \alpha \leq \frac{6+5\delta}{2}$, the firm increases quality but chooses to sell the good in period 2. If $\frac{2+\delta}{2+3\delta} \leq \frac{1+\delta}{2\alpha-1} <$

---

[8] For instance, for $\delta = 1$ (no time discounting) the first-period marginal type (θ_2^*) is constant independent of the average unit cost.

$\frac{1+\delta}{1+2\delta} \Rightarrow 1+\delta < \alpha \leq \frac{4+6\delta+3\delta^2}{2(2+\delta)}$, the firm increases quality in order to commit to no sales in period 2 (avoiding the time-inconsistency problem). Finally, if $\frac{1+\delta}{1+2\delta} \leq \frac{1+\delta}{2\alpha-1} < 1+\delta \Rightarrow 1 \leq \alpha < 1+\delta$, the firm is free from the time-inconsistency problem and offers the optimal commitment quality, achieving the same outcome as in the commitment case. The following figure exhibits the range of parameters corresponding to each of the above four cases.
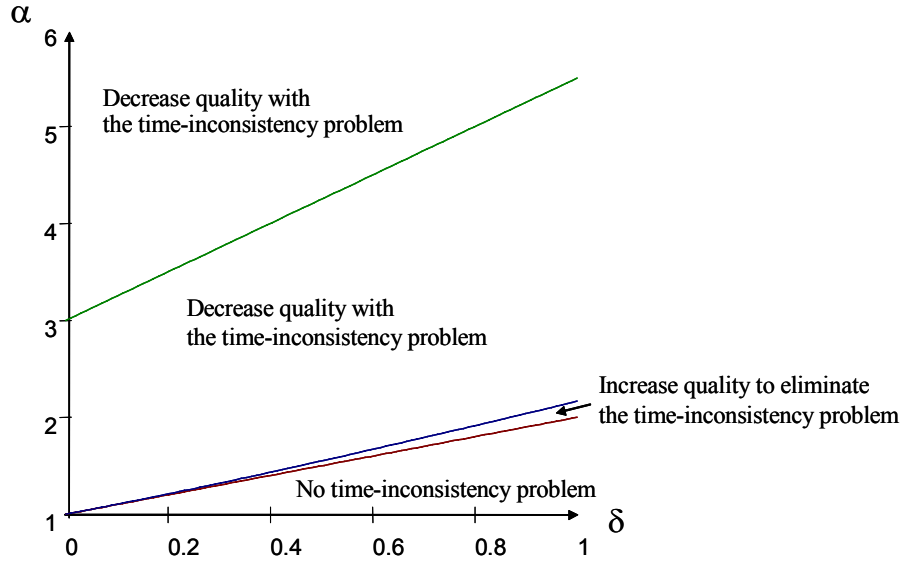

Fig 1: The optimal quality without commitment.

5 Concluding remarks

This paper has examined how a firm's choice of quality interacts with the time-inconsistency problem in a durable-good monopoly framework. It has been shown that the relationship depends on the structure of unit production function of quality. Durable-goods monopolists may have incentives to raise quality in order to eliminate or ameliorate the time-consistency problem, and the equilibrium level of quality can be higher or lower than the one in the commitment regime.

The basic model can be extended in several lines. The most demanding is probably is to extend the model to a finite (but more than two period) or infinite horizon setup. Also interesting is to examine the robustness of the result to allowing the firm to produce multiple qualities or alter the quality of the good later at some fixed costs.

References

[1] Bagnoli, M., S. Salant, and J. Swierzbinski (1989), Durable-Goods Monopoly with Discrete Demand, *Journal of Political Economy* 97, 1459-1478.

[2] Bagnoli, M., S. Salant, and J. Swierzbinski (1995), Intertemporal Self-Selection with Multiple Buyers, *Economic Theory* 5, 513-526.

[3] Bond, E. and L. Samuelson (1984), Durable Good Monopolies with rational Expectations and Replacement Sales, *RAND Journal of Economics* 15, 336-345.

[4] Bulow, J. (1982), Durable Goods Monopolists, *Journal of Political Economy* 90, 314-332.

[5] Bulow, J. (1986), An Economic Theory of Planned Obsolescence, *Quarterly Journal of Economics* 51, 729-749.

[6] Butz, D. (1990), Durable Good Monopoly and Best Price Provisions*, American Economic Review* 80*,* 1062-1075.

[7] Coase, R. (1972), Durability and Monopoly, *Journal of Law and Economics* 15, 143-149.

[8] Hahn, J. (2002), damaged durable Goods, Keele University, mimeo.

[9] Inderst, R. (2002), Why Durable Goods May Be Sold Below Costs, University College London, mimeo.

[10] Kahn, C. (1986), The Durable Goods Monopolist and Consistency with Increasing Costs, *Econometrica* 54, 275-294.

[11] Takeyama, L. (2002), Strategic Vertical Differentiation and Durable Goods Monopoly, *Journal of Industrial Economics* 50(1), 43-56.

[12] von der Fehr, N.-H. and K.-U. Kühn (1995), Coase versus Pacman: Who Eats Whom in the Durable-Goods Monopoly?, *Journal of Political Economy* 103, 785-812.

[13] Waldman, M. (1993), A New Perspective on Planned Obsolescence, *Quarterly Journal of Economics* 58, 272-283.