

Sticky prices and comovement of business cycle*

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Abstract

A defining characteristic of business cycle is comovements of economic variables across sectors. But it is not easy to replicate these comovements in standard real business cycle models. Traditionally, however, not only the productivity shocks emphasized in real business cycle models but also monetary shocks have been believed to be important in explaining business cycles. Following this tradition, a two sector sticky price model is constructed in this paper to examine the sectoral comovements of economic variables under nominal rigidities. It turns out that monetary shocks can generate comovements of sectoral variables.

Keywords: Comovement over business cycles, Sticky prices, Sticky wages

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1 Introduction

As noted in Huffman and Wynne (1998), a defining characteristic of business cycles, whether in the traditional sense of Burns and Mitchell (1946) or in the contemporary sense of Lucas (1977), is the comovement in the pace of economic activity in different sectors of the economy. Also according to Christiano and Fitzgerald (1998) and Huffman and Wynne (1998), levels of output, employment and investment in various sectors of the economy move in a procyclical manner although they do not move perfectly in tandem.

As an example, Huffman and Wynne (1998) divided the U.S. economy into consumption and investment sectors, and they report the correlations of output, capital, labor input and investment flow in consumption sector and investment sector with aggregate output as shown in table1. Using the 1987 input-output tables to determine how much of a sector's final output goes to consumption as opposed to investment or intermediate uses, they classified a sector as belonging to the consumption sector if the bulk of the sector's final output is allocated to final consumption demand and a sector as belonging to the investment sector otherwise. Using this criterion, they classified the finance, insurance and real estate(FIRE), retail trade and services sectors as the consumption sector and the mining, construction, manufacturing, transportation and public utilities and wholesale trade sectors as the investment sector. Table1 shows that the outputs in both sectors are procyclical and the output of the investment sector is more correlated with aggregate output than output of the consumption sector, and is also nearly three times more volatile. The labor inputs in both sectors are also strongly procyclical, and the labor input in the investment sector is nearly twice as volatile as the labor input in the consumption sector. Finally sectoral investment flows show procyclical movements. Huffinan and Wynne (1998) also reports that more detailed sectoral data show similar patterns.

However, it is not easy to replicate the sectoral comovement of economic variables in a business cycle model. In standard real business cycle models, as shown in Christiano and Fitzgerald (1998), a positive productivity shock induces labor hours and investment in the consumption sector to move negatively not positively, in contrast with data.¹ Christiano and Fitzgerald (1998) documents various approaches to solve this "comovement puzzle" in the real business cycle models. Such approaches include Benhabib et al. (1991) which incorporates household production as a third sector, Hornstein and Praschnik (1997) which stresses intermediate input channel, Huffman and Wynne (1998) which introduces intratemporal adjustment costs in producing investment goods, and Christiano and Fisher (1998) which modifies standard model by introducing labor immobility and habit persistence. But also limitations of these approaches

¹Christiano and Fitzgerald (1998) explains as follows. When a positive productivity shock hits the economy, the outputs of both consumption and investment goods sector increase. However, there is a relatively larger increase in the output of investment goods reflecting the rise of opportunity cost of applying resources to the consumption sector and the consumption smoothing motives of households. The increase in the demand for investment goods relative to consumption goods implies that capital and labor resources are shifted out of the production of consumption goods and into the production of investment goods.

are documented in Christiano and Fitzgerald (1998) and possible new lines of approaches to solve this "puzzle" such as incorporating strategic complementarity, information externalities, and efficiency wages are suggested in Christiano and Fitzgerald (1998).

On the other hand, economists have explained business cycle phenomena not only in terms of productivity shocks emphasized in real business cycle models, but also in terms of aggregate demand shocks. And traditionally monetary shocks have been believed to be important sources of business cycle fluctuations as in Friedman and Schwartz (1963). Along this line of thought, we can find sticky price and wage models such as Chari et al. (2000), Christiano et al. (2001) and Erceg et al. (2000) which try to explain fluctuations of economic variables in terms of monetary shocks. Thus it is very natural to examine the behavior of a sticky price and wage model in a two sector setting so that we can see whether monetary shocks can explain sectoral movements (particularly comovement) better than productivity shocks. Since monetary shocks, which are demand shocks by nature, can work differently from productivity shocks, which are supply shocks by nature, they may explain the sectoral comovement of business cycles better than productivity shocks. Simply put, when a monetary shock hits the economy, this can increase demand across all sectors, leading to the possible comovement of economic variables across the sectors.

But until now, there has been virtually no attempt to explain the comovement of sectoral variable in terms of aggregate demand shocks or more specifically monetary shocks. So in this paper, we construct a two sector sticky price and wage model to see whether monetary shocks can generate a realistic comovement of economic variables in the model economy.

The main findings from this attempt can be summarized as follows. First monetary shocks can generate comovement of sectoral variables in the model economy and volatility and correlation statistics in the model economy are similar to the actual data. And this result is obtained by a fairly standard two sector sticky price and wage model constructed below and thus we can say monetary shocks naturally and inherently generate the comovement of economic variables in sticky price and wage model without any major modifications. Second, productivity shocks do not generate comovement of economic variables in the model constructed below. We observe negative responses of aggregate inputs after a positive productivity shock due to the stickiness in price and wage as explained in Galí (2000). In addition, we observe that labor input in each sector moves in the opposite direction when there is a productivity shock in the investment sector, as can be seen in standard two sector real business cycle models.

Thus we can explain the comovement of economic variables very easily with monetary shocks in a sticky price and wage model. But we can not easily generate the comovement with productivity shocks in the model and at least some modifications, like those tried in real business cycle models, are needed to obtain the comovement when productivity shocks are used as sources of business cycles.

The rest of the paper is organized as follows. In section 2, we describe the model. In section 3, we characterize the equilibrium of the model and calibrate

parameters. In section 4, we summarize findings from our benchmark model and its variations. And in section 5, we conclude.

2 Model Economy

2.1 General Description

The model basically modifies Chari et al. (2000) which is one sector sticky price model and also incorporates several other minor modifications. In each period t , the model economy experiences an event s_t in S_t which is a set of all possible events at t .

We denote by $s^t = (s_0, \dots, s_t)$ the history of events up through and including period t . The probability as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given.

There are two sectors in this economy. One sector produces a consumption good and the other sector produces a durable investment good. This follows from standard two sector models such as Huffman and Wynne (1998) and Christiano and Fisher (1998). The consumption good is produced by aggregating a continuum of intermediate goods and is sold to the market competitively. Intermediate goods for the production of consumption good are produced using labor and capital and sold by imperfect competitors. And intermediate goods producers set prices in a staggered fashion as in Taylor (1980).

The investment sector works similar to the consumption sector except that there is an intratemporal adjustment costs in producing investment goods as in Huffman and Wynne (1998) which will be explained below.

In the labor market, wages are also determined in a staggered fashion. Namely we introduce sticky wages by letting labor be differentiated and introducing monopolistically competitive unions that set wages in a staggered way as in Chari et al. (2002). We also introduce intratemporal adjustment costs in the labor supply as in investment good production.

2.2 Agents' Problems

2.2.1 Consumption Good Sector

Final consumption good is produced by applying following technology:

$$y_c(s^t) = \left[\int y_c^d(i, s^t)^{\theta_c} di \right]^{\frac{1}{\theta_c}} \quad (1)$$

where $y_c(s^t)$ is the consumption good, $y_c^d(i, s^t)$ is an intermediate good of type $i \in [0, 1]$ used for the production of consumption good. And the elasticity of substitution between the intermediate goods is $1/(1 - \theta_c)$.

The technology for producing each intermediate good i is a standard Cobb-Douglas production function given as

$$y_c(i, s^t) = k_c^d(i, s^t)^{\alpha_1} (\lambda_c(s^t) l_c^d(i, s^t))^{1-\alpha_1} \quad (2)$$

where $k_c^d(i, s^t)$ and $l_c^d(i, s^t)$ are the capital and labor inputs used to produce the i th intermediate good. And $\lambda_c(s^t)$ is consumption sector productivity shock represented in labor augmenting form as in Huffman and Wynne (1998). Also α_1 is the parameter for the Cobb-Douglas production function.

Final consumption good producers behave competitively and in each period t , they choose intermediate inputs $y_c^d(i, s^t)$ for all $i \in [0, 1]$, and output $y_c(s^t)$ to maximize the profits given as

$$\max \bar{P}_c(s^t) y_c(s^t) - \int P_c(i, s^{t-1}) y_c^d(i, s^t) di \quad (3)$$

subject to (1), where $\bar{P}_c(s^t)$ is the price of the final consumption good in period t and $P_c(i, s^{t-1})$ is the price of intermediate good i used for the consumption good production in period t . We assume period t intermediate goods prices are set before the realization of the period t shocks, thus intermediate goods prices do not depend on s_t . Solving the problem in (3) gives the input demand functions:

$$y_c^d(i, s^t) = \left[\frac{\bar{P}_c(s^t)}{P_c(i, s^{t-1})} \right]^{\frac{1}{(1-\theta_c)}} y_c(s^t) \quad (4)$$

The zero-profit condition implies that

$$\bar{P}_c(s^t) = \left[\int P_c(i, s^{t-1})^{\frac{\theta_c}{\theta_c-1}} \right]^{\frac{\theta_c-1}{\theta_c}} \quad (5)$$

In equilibrium the consumption good price in period t depends only on s^{t-1} due to the price setting assumption of the intermediate goods producers.

Intermediate goods producers behave as imperfect competitors. They set prices for N periods and do so in a staggered fashion. In particular, in each period t , a fraction $1/N$ of these producers choose new prices $P_c(i, s^{t-1})$ before the realization of the event s_t . These prices are set for N periods, so for this group of intermediate goods producers, $P_c(i, s^{t+\tau-1}) = P_c(i, s^{t-1})$ for $\tau = 0, \dots, N-1$. The intermediate goods producers are indexed so that producers indexed $i \in [0, \frac{1}{N}]$ set new prices in $0, N, 2N$, and so on, while producers indexed $i \in [\frac{1}{N}, \frac{2}{N}]$ set new prices in $1, N+1, 2N+1$, and so on, for the N cohorts of intermediate goods producers. Intermediate goods producers whose price setting constraint is $P_c(i, s^{t+\tau-1}) = P_c(i, s^{t-1})$ for $\tau = 0, \dots, N-1$, maximize discounted profits from period t to period $t+N-1$ given as:

$$\max_{P_c(i, s^{t-1}), k_c^d(i, s^\tau), l_c^d(i, s^\tau)} \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) [P_c(i, s^{t-1}) y_c(i, s^\tau)]$$

$$-\bar{P}_c(s^\tau)r_c(s^\tau)k_c^d(i, s^\tau) - \bar{P}_c(s^\tau)w_c(s^\tau)l_c^d(i, s^\tau)] \quad (6)$$

subject to (2), (4), where $Q(s^\tau | s^{t-1})$ is the price of one dollar in s^τ in units of dollars at s^{t-1} , $r_c(s^\tau)$ is the rental rate on capital and $w_c(s^\tau)$ is the real wage rate evaluated in terms of final consumption good.

First order conditions imply²

$$P_c(i, s^{t-1}) = \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) \bar{P}_c(s^\tau)^{(2-\theta_c)/(1-\theta_c)} v_c(s^\tau) y_c(s^\tau)}{\theta_c \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) \bar{P}_c(s^\tau)^{1/(1-\theta_c)} y_c(s^\tau)} \quad (7)$$

where $v_c(s^\tau)$ is given as

$$v_c(s^\tau) = \frac{1}{(1-\alpha_1)\lambda_c(s^\tau)} w_c(s^\tau) \left(\frac{\lambda_c(s^\tau) l_c^d(i, s^\tau)}{k_c^d(i, s^\tau)} \right)^{\alpha_1} \quad (8)$$

And also from the first order conditions, we get

$$\left(\frac{1-\alpha_1}{\alpha_1} \right) \frac{k_c^d(i, s^t)}{l_c^d(i, s^t)} = \frac{w_c(s^t)}{r_c(s^t)} \quad (9)$$

Given the constant elasticity of substitution (CES) property of Cobb-Douglas function, this implies that capital-labor ratios are equated across the intermediate goods firms, so for all $i \in [0, 1]$,

$$\frac{k_c^d(i, s^t)}{l_c^d(i, s^t)} = \frac{k_c^d(0, s^t)}{l_c^d(0, s^t)} \quad (10)$$

2.2.2 Investment Good Sector

Durable investment goods are produced in the investment sector for the use of consumption sector and for its own use. The basic structure of the investment sector is similar to the consumption sector. But we will introduce intratemporal adjustment costs discussed in Huffman and Wynne (1998).³

Producers who produce investment goods for both sectors produce the required investment goods using a composite investment good $y_i(s^t)$. And the composite investment good $y_i(s^t)$ is produced in turn by aggregating its intermediate goods.

Production technology for the composite good $y_i(s^t)$ is given as follows, which is analogous to the final consumption good in the consumption sector.

$$y_i(s^t) = \left[\int y_i^d(j, s^t)^{\theta_i} dj \right]^{\frac{1}{\theta_i}} \quad (11)$$

²Appendix containing detailed derivations is available upon request.

³Huffman and Wynne (1998) generates sectoral comovement of investment by incorporating this type of intratemporal adjustment costs in a two sector real business cycle model.

where $y_i^d(j, s^t)$ is intermediate good of type $j \in [0, 1]$ used for the production of $y_i(s^t)$. The elasticity of substitution between the intermediate goods is $1/(1 - \theta_i)$.

And the technology for the production of intermediate good $y_i(j, s^t)$ is given as

$$y_i(j, s^t) = k_i^d(j, s^t)^{\alpha_2} (\lambda_i(s^t) l_i^d(j, s^t))^{1-\alpha_2} \quad (12)$$

where $k_i^d(j, s^t)$ and $l_i^d(j, s^t)$ are the capital and labor inputs used to produce j th intermediate good. $\lambda_i(s^t)$ is productivity shock in the investment sector, and α_2 is a parameter.

The production of investment goods is then allocated across the two sectors according to the relationship:

$$\Upsilon [\phi i_c(s^t)^{-\rho} + (1 - \phi) i_i(s^t)^{-\rho}]^{-1/\rho} = y_i^d(s^t) \quad (13)$$

where $i_c(s^t)$ is the investment good produced for consumption sector, and $i_i(s^t)$ is the investment good produced for investment sector. $y_i^d(s^t)$ is the composite good used for the production of investment goods. And ϕ , ρ and Υ are parameters.

We need some explanations concerning (13). With $\phi = 0.5$, $\rho = -1$ and $\Upsilon = 2$, (13) becomes standard resource constraint for the investment goods. That is, total investment ($y_i(s^t)$) is the sum of investment good produced for the consumption good sector and investment good sector. However, changing these parameters we can change the relative price of the two investment goods. Figure 1 illustrates some relevant facts where we change parameter ρ setting $\phi = 0.5$ and $\Upsilon = 2$. For the standard case when $\rho = -1$, there is an infinite elasticity of substitution between $i_c(s^t)$ and $i_i(s^t)$. This means that it is very easy to switch from the production of one type of investment good into that of another. Specifically, by cutting back the production of new investment good for one sector by one unit, it is possible to increase production of new investment good for the other sector by one unit without incurring further costs. It is plausible that an economy can alter its capacity for producing heavy equipment for industrial use on the one hand, and alternative equipment for services sector use on the other. However, in practice it can be costly to do so quickly. Now, as the absolute value of ρ gets bigger, it becomes more difficult to alter the composition of investment goods produced. The motivation of this specification is that it takes time and resources to change the composition of investment goods produced. This is referred to as intratemporal adjustment costs in Huffman and Wynne (1998), since we encounter decreasing marginal returns in producing more of one type of investment good while reducing the production of the alternative investment good at a particular moment in time.

Investment goods producers act competitively and their problem is specified as follows:

$$\max P_{ic} i_c(s^t) + P_{ii} i_i(s^t) - \bar{P}_i y_i^d(s^t) \quad (14)$$

subject to (13), where P_{ic} , P_{ii} , and \bar{P}_i are prices for $i_c(s^t)$, $i_i(s^t)$, and $y_i^d(s^t)$

respectively. The first order conditions for investment goods producers are:

$$\frac{P_{ic}(s^t)}{\bar{P}_i(s^t)} = \Upsilon[\phi i_c(s^t)^{-\rho} + (1 - \phi)i_i(s^t)^{-\rho}]^{-(1+\rho)/\rho} \phi i_c(s^t)^{-\rho-1} \quad (15)$$

$$\frac{P_{ii}(s^t)}{\bar{P}_i(s^t)} = \Upsilon[\phi i_c(s^t)^{-\rho} + (1 - \phi)i_i(s^t)^{-\rho}]^{-(1+\rho)/\rho} (1 - \phi)i_i(s^t)^{-\rho-1} \quad (16)$$

The composite good producers behave competitively and in each period t , they choose inputs $y_i(j, s^t)$ for all $j \in [0, 1]$, and output $y_i(s^t)$ to maximize profits:

$$\max \bar{P}_i(s^t) y_i(s^t) - \int P_i(j, s^{t-1}) y_i^d(j, s^t) dj \quad (17)$$

subject to (11). Solving the problem in (17) gives the input demand functions:

$$y_i^d(j, s^t) = \left[\frac{\bar{P}_i(s^t)}{P_i(j, s^{t-1})} \right]^{\frac{1}{(1-\theta_i)}} y_i(s^t) \quad (18)$$

The zero-profit condition implies that

$$\bar{P}_i(s^t) = \left[\int P_i(j, s^{t-1})^{\frac{\theta_i}{\theta_i-1}} \right]^{\frac{\theta_i-1}{\theta_i}} \quad (19)$$

The intermediate goods producers in this sector work analogous to those in the consumption sector. Namely, they set prices for M periods and do so in a staggered fashion. In particular, in each period t , a fraction $1/M$ of these producers choose new prices $P_i(j, s^{t-1})$ before the realization of the event s_t . These prices are set for M periods, so for this group of intermediate goods producers, $P_i(j, s^{t+\tau-1}) = P_i(j, s^{t-1})$ for $\tau = 0, \dots, M-1$. The intermediate goods producers are indexed so that producers indexed $j \in [0, \frac{1}{M}]$ set new prices in $0, M, 2M$, and so on, while producers indexed $j \in [\frac{1}{M}, \frac{2}{M}]$ set new prices in $1, M+1, 2M+1$, and so on, for the M cohorts of intermediate goods producers. Intermediate goods producers whose price setting constraint is $P_i(j, s^{t+\tau-1}) = P_i(j, s^{t-1})$ for $\tau = 0, \dots, M-1$, maximize discounted profits from period t , to period $t+M-1$. That is, each solves problem:

$$\begin{aligned} \max_{P_i(j, s^{t-1}), k_i^d(j, s^\tau), l_i^d(j, s^\tau)} & \sum_{\tau=t}^{t+M-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) [P_i(j, s^{\tau-1}) y_i(j, s^\tau) \\ & - \bar{P}_i(s^\tau) r_i(s^\tau) k_i^d(j, s^\tau) - \bar{P}_i(s^\tau) w_i(s^\tau) l_i^d(j, s^\tau)] \end{aligned} \quad (20)$$

subject to (12), (18), where $r_i(s^\tau)$, $w_i(s^\tau)$ are rental rate of capital and wage rate in the investment sector evaluated in terms of investment composite good, The first order conditions imply

$$P_i(j, s^{t-1}) = \frac{\sum_{\tau=t}^{t+M-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) \bar{P}_i(s^\tau)^{(2-\theta_i)/(1-\theta_i)} v_i(s^\tau) y_i(s^\tau)}{\theta_i \sum_{\tau=t}^{t+M-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) \bar{P}_i(s^\tau)^{1/(1-\theta_i)} y_i(s^\tau)} \quad (21)$$

where $v_i(s^\tau)$ is given as

$$v_i(s^\tau) = \frac{1}{(1 - \alpha_2)\lambda_i(s^\tau)} w_i(s^\tau) \left(\frac{\lambda_i(s^\tau) l_i^d(j, s^\tau)}{k_i^d(j, s^\tau)} \right)^{\alpha_2} \quad (22)$$

And also from the first order conditions

$$\left(\frac{1 - \alpha_2}{\alpha_2} \right) \frac{k_i^d(j, s^t)}{l_i^d(j, s^t)} = \frac{w_i(s^t)}{r_i(s^t)} \quad (23)$$

Given the CES property of Cobb-Douglas function, this implies that capital-labor ratios are equated across the intermediate goods firms, so for all $j \in [0, 1]$,

$$\frac{k_i^d(j, s^t)}{l_i^d(j, s^t)} = \frac{k_i^d(0, s^t)}{l_i^d(0, s^t)} \quad (24)$$

2.2.3 Labor and Capital Supplying Firms

We introduce labor and capital supplying firms for ease of analysis. Labor supplying firms will supply labor to both sectors in the presence of intratemporal adjustment costs analogous to the investment goods production. Namely, labor supplying firms will provide labor for both sectors using composite labor $l(s^t)$ and the constraint for the labor supply is given as:

$$\Phi[\varpi l_c(s^t)^{-\varkappa} + (1 - \varpi)l_i(s^t)^{-\varkappa}]^{-1/\varkappa} = l^d(s^t) \quad (25)$$

where $l_c(s^t)$ is the labor supply for consumption sector, and $l_i(s^t)$ is the labor supply for investment sector. $l^d(s^t)$ is the composite labor used for the provision of labor for each sector. And Φ , ϖ and \varkappa are parameters. We introduce intratemporal adjustment costs in labor supply because it is also costly to reallocate labor between sectors quickly.

Labor supplying firms act competitively and their problem is specified as follows:

$$\max \bar{P}_c(s^t)w_c(s^t)l_c(s^t) + \bar{P}_i(s^t)w_i(s^t)l_i(s^t) - \bar{W}(s^t)l^d(s^t) \quad (26)$$

subject to (25), where \bar{W} is price for labor $l(s^t)$. The first order conditions for labor supplying firms imply

$$\frac{\bar{P}_c(s^t)w_c(s^t)}{\bar{W}(s^t)} = \Phi[\varpi l_c(s^t)^{-\varkappa} + (1 - \varpi)l_i(s^t)^{-\varkappa}]^{-(1+\varkappa)/\varkappa} \varpi l_c(s^t)^{-\varkappa-1} \quad (27)$$

$$\frac{\bar{P}_i(s^t)w_i(s^t)}{\bar{W}(s^t)} = \Phi[\varpi l_c(s^t)^{-\varkappa} + (1 - \varpi)l_i(s^t)^{-\varkappa}]^{-(1+\varkappa)/\varkappa} (1 - \varpi)l_i(s^t)^{-\varkappa-1} \quad (28)$$

Composite labor is created by aggregating a continuum of differentiated labor inputs $l(q, s^t)$, provided by labor unions of type $q \in [0, 1]$. That is,

$$l(s^t) = \left[\int l^d(q, s^t)^\vartheta dq \right]^{1/\vartheta} \quad (29)$$

where $l(s^t)$ is composite labor, $l^d(q, s^t)$ is the amount of differentiated labor input of type q , and ϑ is a parameter. And the composite labor is provided competitively. Thus each composite labor providing firm solves following problem

$$\max \bar{W}(s^t) l(s^t) - W(q, s^{t-1}) l^d(q, s^t) \quad (30)$$

subject to (29). The first order conditions to this problem imply,

$$l^d(q, s^t) = \left(\frac{\bar{W}(s^t)}{W(q, s^{t-1})} \right)^{\frac{1}{1-\vartheta}} l(s^t) \quad (31)$$

where $\bar{W}(s^t) = \left[\int W(q, s^{t-1})^{\frac{\vartheta}{\vartheta-1}} dq \right]^{\frac{\vartheta-1}{\vartheta}}$.

Now let's turn to the capital supplying firms. Capital used for the consumption sector is provided by competitive capital leasing firms (below referred as the consumption sector capital leasing firm) which maximize the following problem

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau} Q(s^\tau | s^t) \{ \bar{P}_c(s^\tau) r_c(s^\tau) k_c(s^{\tau-1}) - P_{ic}(s^\tau) i_c^d(s^\tau) \} \quad (32)$$

subject to the law of motion for capital accumulation.⁴

$$k_c(s^t) = (1 - \delta_1) k_c(s^{t-1}) - \frac{b_c}{2} \left(\frac{i_c(s^t)}{k_c(s^{t-1})} - \delta_1 \right)^2 k_c(s^{t-1}) + i_c(s^t) \quad (33)$$

Here, δ_1 denotes the depreciation rate of capital.

The first order conditions are⁵

$$P_{ic}(s^t) = \xi(s^t) \left\{ 1 - b_c \left(\frac{i_c(s^t)}{k_c(s^{t-1})} - \delta_1 \right) \right\} \quad (34)$$

$$\begin{aligned} \xi(s^t) = & \sum_{s^{t+1}} Q(s^{t+1} | s^t) \left[\bar{P}_c(s^{t+1}) r_c(s^{t+1}) + \xi(s^{t+1}) \{ (1 - \delta_1) \right. \\ & \left. - \frac{b_c}{2} \left(\frac{i_c(s^{t+1})}{k_c(s^t)} - \delta_1 \right)^2 + b_c \left(\frac{i_c(s^{t+1})}{k_c(s^t)} - \delta_1 \right) \frac{i_c(s^{t+1})}{k_c(s^t)} \right] \end{aligned} \quad (35)$$

⁴This form of capital accumulation equation is from Chari et al. (2000). Also capital is immobile across sectors and thus we have a separate capital accumulation equation in each sector as in Huffman and Wynne (1998).

⁵Appendix containing detailed derivaitons is available upon request.

where $\xi(s^t)$ is Lagrangian multiplier associated with the law of motion for capital accumulation.

The capital used for the investment sector is provided similarly. Thus competitive capital leasing firms (below referred as the investment sector capital leasing firm) maximize the following objective function.

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau} Q(s^\tau | s^t) \{ \bar{P}_i(s^\tau) r_i(s^\tau) k_i(s^{\tau-1}) - P_{ii}(s^\tau) i_i^d(s^\tau) \} \quad (36)$$

subject to the law of motion for capital accumulation

$$k_i(s^t) = (1 - \delta_2) k_i(s^{t-1}) - \frac{b_i}{2} \left(\frac{i_i(s^t)}{k_i(s^{t-1})} - \delta_2 \right)^2 k_i(s^{t-1}) + i_i(s^t) \quad (37)$$

Here, δ_2 denotes the depreciation rate of capital

First order conditions imply

$$P_{ii}(s^t) = \kappa(s^t) \left\{ 1 - b_i \left(\frac{i_i(s^t)}{k_i(s^{t-1})} - \delta_2 \right) \right\} \quad (38)$$

$$\begin{aligned} \kappa(s^t) = & \sum_{s^{t+1}} Q(s^{t+1} | s^t) \left[\bar{P}_i(s^{t+1}) r_i(s^{t+1}) + \kappa(s^{t+1}) \{ (1 - \delta_2) \right. \\ & \left. - \frac{b_i}{2} \left(\frac{i_i(s^{t+1})}{k_i(s^t)} - \delta_2 \right)^2 + b_i \left(\frac{i_i(s^{t+1})}{k_i(s^t)} - \delta_2 \right) \frac{i_i(s^{t+1})}{k_i(s^t)} \right] \quad (39) \end{aligned}$$

where $\kappa(s^t)$ is Lagrangian multiplier associated with the law of motion for capital accumulation.

2.2.4 Consumer Problem

The consumer side of the market is organized into a continuum of unions indexed by $q \in [0, 1]$. Union q consists of all the consumers in the economy with type q labor. Each union realizes that it faces a downward-sloping demand for its own type of labor. Namely, the total demand for type q labor is given as (31). We assume that a fraction $1/G$ of unions set their wages in a given period and hold wages fixed for G subsequent periods. The unions are indexed so that those with $q \in [0, 1/G]$ set new wages in $0, G, 2G$, and so on, while those with $q \in [1/G, 2/G]$ set new wages in $1, G+1, 2G+1$, and so on, for the G cohorts of unions. In each period, these new wages are set before the realization of the event s_t . Notice that the wage-setting arrangement is analogous to the price-setting arrangement for intermediate goods producers in both consumption and investment sector.

Total discounted expected utility for the q th union is given as

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c(q, s^t), l(q, s^t), M(q, s^t)/\bar{P}_c(s^t)) \quad (40)$$

where $0 < \beta < 1$ is the discount factor and $c(q, s^t)$, $l(q, s^t)$, and $M(q, s^t)/\bar{P}_c(s^t)$ are consumption in period t , labor in period t , and real balances in period t respectively.

Temporal utility function is given as

$$\begin{aligned} & U(c(q, s^t), l(q, s^t), M(q, s^t)/\bar{P}_c(s^t)) \\ &= \frac{c(q, s^t)^{1-\sigma}}{1-\sigma} + \psi \frac{(1-l(q, s^t))^{1-\gamma}}{1-\gamma} + \omega \frac{(M(q, s^t)/\bar{P}_c(s^t))^{1-\eta}}{1-\eta} \end{aligned} \quad (41)$$

where ω and ψ are relative weight parameters, η is interest elasticity of real balance, σ is risk aversion, and γ is labor elasticity.

The budget constraints are given as

$$\begin{aligned} & \bar{P}_c(s^t)c(q, s^t) + M(q, s^t) + \sum_{s^{t+1}} Q(s^{t+1} | s^t) B(q, s^{t+1}) \\ & \leq W(q, s^{t-1})l(q, s^t) + M(q, s^{t-1}) + B(q, s^t) + \Pi(s^t) + T(s^t) \end{aligned} \quad (42)$$

where $\Pi(s^t)$ is the nominal profits of the intermediate goods producers, and $T(s^t)$ is nominal transfers. Each of the nominal bonds $B(q, s^{t+1})$ is a claim to one dollar in state s^{t+1} and costs $Q(s^{t+1} | s^t)$ dollars in state s^t . In terms of relating the prices in the intermediate goods producers' problem to these prices, note that $Q(s^\tau | s^t) = Q(s^{t+1} | s^t) Q(s^{t+2} | s^{t+1}) \dots Q(s^\tau | s^{\tau-1})$ for all $\tau > t$.

The problem of the q th union is thus to maximize (40) subject to the labor demand schedule (31), the budget constraints (42), and the wage setting constraints $W(q, s^{t+\tau-1}) = W(q, s^{t-1})$ for $\tau = 0, \dots, M-1$. The first order conditions are⁶

$$\zeta(q, s^t) = \frac{U_1(q, s^t)}{\bar{P}_c(s^t)} \quad (43)$$

$$W(q, s^{t-1}) = - \frac{\sum_{\tau=t}^{t+G-1} \sum_{s^\tau} \beta^{\tau-t+1} \pi(s^{t+1} | s^t) \bar{W}(s^\tau)^{\frac{1}{1-\vartheta}} l(s^\tau) U_2(q, s^\tau)}{\vartheta \sum_{\tau=t}^{t+G-1} \sum_{s^\tau} \beta^{\tau-t+1} \pi(s^{t+1} | s^t) \zeta(q, s^\tau) \bar{W}(s^\tau)^{\frac{1}{1-\vartheta}} l(s^\tau)} \quad (44)$$

$$\frac{U_3(q, s^t)}{\bar{P}_c(s^t)} - \zeta(q, s^t) + \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) \zeta(q, s^{t+1}) = 0 \quad (45)$$

$$Q(s^\tau | s^t) = \beta^{\tau-t} \pi(s^\tau | s^t) \frac{\zeta(q, s^{t+1})}{\zeta(q, s^t)} \quad (46)$$

for all $\tau > t$, where $\zeta(q, s^t)$ is Lagrangian multiplier associated with budget constraint in period t , $U_1(s^t)$, $U_2(s^t)$, and $U_3(s^t)$ denote the derivatives of the

⁶Appendix containing detailed derivaitons is available upon request.

utility function with respect to its arguments and $\pi(s^\tau | s^t) = \pi(s^\tau) / \pi(s^t)$ is the conditional probability of s^τ given s^t . Notice that (46) imply that

$$\frac{\zeta(q, s^{t+1})}{\zeta(q, s^t)} = \frac{\zeta(q', s^{t+1})}{\zeta(q', s^t)} \quad (47)$$

for all q , and q' . So, Lagrangian multipliers of different type of union are equated up to a factor of proportionality, namely the date 0 Lagrangian multiplier on their budget constraint. Here, we assume that initial debts and transfers among the G types in the economy are such that the multipliers are equated. In that case, we have

$$U_1(q, s^t) = U_1(q', s^t) \quad (48)$$

$$U_3(q, s^t) = U_3(q', s^t) \quad (49)$$

for all q , and q' from (43) and (45).

2.2.5 Money Supply

The nominal money supply process is given by

$$M(s^t) = \mu(s^t) M(s^{t-1}) \quad (50)$$

where $\mu(s^t)$ follows a first order autoregressive process

$$\log \mu(s^t) = \rho_\mu \log \mu(s^{t-1}) + \epsilon_\mu(s^t) \quad (51)$$

where $\epsilon_\mu(s^t)$ is independent and identically normally distributed mean zero shock with standard deviation σ_{ϵ_μ} . New money balances are distributed to consumers in a lump sum fashion by having nominal transfers satisfying $T(s^t) = M(s^t) - M(s^{t-1})$.

2.2.6 Productivity Shock

Productivity shocks $\lambda_c(s^t)$ and $\lambda_i(s^t)$ jointly obey the following law of motion:

$$\Lambda_t \equiv \begin{bmatrix} \log(\lambda_c(s^t)) \\ \log(\lambda_i(s^t)) \end{bmatrix} = \Gamma \Lambda_{t-1} + \epsilon_\lambda(s^t) \quad (52)$$

where Γ is autoregressive matrix and $\epsilon_\lambda(s^t)$ is independent, identically and normally distributed mean zero shock with covariance matrix $\Sigma_{\epsilon_\lambda}$ which is symmetric positive definite.

2.3 Market Equilibrium

In terms of market-clearing conditions, consider first the factor markets. The capital supply for the consumption sector is $k_c(s^{t-1})$.⁷ On the demand side, we need to aggregate demand for capital by each intermediate goods producer i in consumption sector $k_c^d(i, s^t)$, $i \in [0, 1]$. Analogous reasoning also holds for capital market for investment sector. Thus market clearing conditions for the capital used for the consumption sector and the investment sector are respectively,

$$k_c(s^{t-1}) = \int k_c^d(i, s^t) di \equiv k_c^d(s^t) \quad (53)$$

$$k_i(s^{t-1}) = \int k_i^d(j, s^t) dj \equiv k_i^d(s^t) \quad (54)$$

We denote aggregate demands for capital in consumption and investment sector as $k_c^d(s^t)$ and $k_i^d(s^t)$ respectively.

Similarly for labor market, labor demand for each sector equals labor supply for each sector.

$$l_c(s^t) = \int l_c^d(i, s^t) di \equiv l_c^d(s^t) \quad (55)$$

$$l_i(s^t) = \int l_i^d(j, s^t) dj \equiv l_i^d(s^t) \quad (56)$$

We denote aggregate demands for labor in each sector as $l_c^d(s^t)$ and $l_i^d(s^t)$.

Also demand for composite labor equals supply of composite labor.

$$l(s^t) = l^d(s^t) \quad (57)$$

And market for the labor input of the q th union clears such that

$$l(q, s^t) = l^d(q, s^t) \quad (58)$$

Bond market clearing requires that

$$B(s^{t+1}) = 0 \quad (59)$$

The consumption goods market clears such that

$$y_c(s^t) = c(s^t) \quad (60)$$

The investment goods market also clears such that

$$i_c(s^t) = i_c^d(s^t) \quad (61)$$

$$i_i(s^t) = i_i^d(s^t) \quad (62)$$

⁷And note that capital provided to each sector is determined one period ahead.

The market clearing condition for the investment composite good holds,

$$y_i(s^t) = y_i^d(s^t) \quad (63)$$

Finally, intermediate goods markets for the consumption and investment sectors clear trivially such that

$$y_c(i, s^t) = y_c^d(i, s^t) \quad (64)$$

$$y_i(i, s^t) = y_i^d(i, s^t) \quad (65)$$

An *equilibrium* for this economy is, then, a collection of allocations for the q th union $c(q, s^t)$, $l(q, s^t)$, $M(q, s^t)$, $B(q, s^{t+1})$ for all $q \in [0, 1]$; allocations for the labor providing firms $l_c(s^t)$, $l_i(s^t)$, $l(s^t)$; allocations for the composite labor providing firms $l(s^t)$, $l(q, s^t)$ for all $q \in [0, 1]$; allocations for the consumption sector capital leasing firms $k_c(s^{t-1})$, $k_c(s^t)$, $i_c(s^t)$; allocations for the investment sector capital leasing firms $k_i(s^{t-1})$, $k_i(s^t)$, $i_i(s^t)$; allocations for the consumption goods producers $y_c(s^t)$, $y_c(i, s^t)$ for all $i \in [0, 1]$; allocations for the intermediate goods producers in consumption sector $y_c(i, s^t)$, $k_c(i, s^t)$, $l_c(i, s^t)$ for all $i \in [0, 1]$; allocations for the investment goods producers $i_c(s^t)$, $i_i(s^t)$, $y_i(s^t)$; allocations for the investment composite goods producers $y_i(s^t)$, $y_i(j, s^t)$ for all $j \in [0, 1]$; allocations for the intermediate goods producers in investment sector $y_i(j, s^t)$, $k_i(j, s^t)$, $l_i(j, s^t)$ all for $j \in [0, 1]$; together with prices $\overline{W}(s^t)$, $W(q, s^t)$ for all $q \in [0, 1]$, $w_c(s^t)$, $w_i(s^t)$, $r_c(s^t)$, $r_i(s^t)$, $Q(s^\tau | s^t)$ for $\tau = t, \dots$, $\overline{P}_c(s^t)$, $P_c(i, s^{t-1})$ for all $i \in [0, 1]$, $P_{ic}(s^t)$, $P_{ii}(s^t)$, $\overline{P}_i(s^t)$ and $P_i(j, s^{t-1})$ for all $j \in [0, 1]$ that satisfy the following conditions: (a) taking all prices but its own wage as given, each union's wage and allocations solve the union's problem; (b) taking all prices as given, the labor providing firm's allocations solve the labor providing firm's problem; (c) taking all prices as given, the composite labor providing firm's allocations solve the composite labor providing firm's problem; (d) taking all prices as given, the consumption sector capital leasing firm's allocations solve the consumption sector capital leasing firm's problem; (e) taking all prices as given, the investment sector capital leasing firm's allocations solve the investment sector capital leasing firm's problem; (f) taking all prices as given, the final consumption goods producer's allocations solve the final consumption goods producer's problem; (g) taking all prices but his own as given, allocations of each intermediate goods producer in the consumption sector solve problem (6); (h) taking all prices as given, the investment goods producer's allocations solve the investment goods producer's problem; (i) taking all prices as given, the investment composite goods producer's allocations solve the composite goods producer's problem; (j) taking all prices but his own as given, each intermediate goods producer's price and allocations in the investment sector solve problem (20); and (k) the market clearing conditions (53) – (65) hold.

3 Computation of Equilibrium and Parametrization

3.1 Computing the equilibrium

Here computation of the equilibrium in the model economy is described.⁸ We begin by substituting out a number of variables and reducing the equilibrium to several equations. The reduction of the number of equations characterizing the model economy is not absolutely necessary but it helps to represent the model more compactly and it also helps to find analytical expression for the nonstochastic values of the steady-state variables. Once we have these reduced equations, we compute Markov equilibria.

In what follows we will focus on the symmetric equilibrium in which all the intermediate goods producers of the same cohort make identical decisions. Thus, $P_c(i, s^t) = P_c(i', s^t)$, $k_c(i, s^t) = k_c(i', s^t)$, $l_c(i, s^t) = l_c(i', s^t)$, $y_c(i, s^t) = y_c(i', s^t)$, for all $i, i' \in [0, 1/N]$, and so on, for the N cohorts of intermediate goods producers in consumption sector. And $P_i(j, s^t) = P_i(j', s^t)$, $k_i(j, s^t) = k_i(j', s^t)$, $l_i(j, s^t) = l_i(j', s^t)$, $y_i(j, s^t) = y_i(j', s^t)$, for all $j, j' \in [0, 1/M]$, and so on, for the M cohorts of intermediate goods producers in investment sector. Similarly labor unions of the same cohort make identical decisions. Thus $W(q, s^t) = W(q', s^t)$, $l(q, s^t) = l(q', s^t)$, $B(q, s^t) = B(q', s^t)$ for all $q, q' \in [0, 1/G]$, and so on, for the G cohorts of labor unions.⁹

We begin with the intermediate goods equilibrium in the consumption sector. Equating supplies of and demands for each intermediate good using equations (2) and (4), then integrating gives

$$\bar{P}_c(s^t)^{1/(1-\theta_c)} \left(\int P_c(i, s^{t-1})^{1/(\theta_c-1)} di \right) y_c(s^t) = k_c(s^{t-1})^{\alpha_1} (\lambda_c(s^t) l_c(s^t))^{1-\alpha_1}$$

where we have exploited the fact that the production function has a constant elasticity of substitution so that the capital-labor ratios are equated across producers, as seen in (10) and we also used the definition $l_c(s^t) \equiv \int l_c(i, s^t) di$ in (55). Rearranging above equation gives

$$y_c(s^t) = k_c(s^{t-1})^{\alpha_1} (\lambda_c(s^t) l_c(s^t))^{1-\alpha_1} \left(\frac{\bar{P}_c(s^t)^{1/(\theta_c-1)}}{\int P_c(i, s^{t-1})^{1/(\theta_c-1)} di} \right) \quad (66)$$

With symmetric equilibrium assumptions, all the intermediate goods prices are equal within each cohort. And we need only to record one intermediate good price per cohort and not the index identifying the intermediate goods. Thus, from now on, we drop the index i , and we let $P(s^{t-1})$ denote the wages set at

⁸ Appendix containing details on the computations is available upon request.

⁹ Note that $c(q, s^t)$ and $M(q, s^t)$ is same regardless of the type of union due (48) and (49).

the beginning of period t , $P(s^{t-2})$ denote the wages set at the beginning of $t-1$, etc. Thus using (5) and symmetric equilibrium assumption, we can rewrite the final consumption good price as

$$\bar{P}_c(s^t) = \left[\frac{1}{N} P_c(s^{t-1})^{\theta_c/(\theta_c-1)} + \dots + \frac{1}{N} P_c(s^{t-N})^{\theta_c/(\theta_c-1)} \right]^{(\theta_c-1)/\theta_c} \quad (67)$$

Substituting equation (67) in (66), we get our first equation to be used for computation.

Similarly we obtain an equation derived from the intermediate goods equilibrium in the investment sector.

$$y_i(s^t) = k_i(s^{t-1})^{\alpha_2} (\lambda_i(s^t) l_i(s^t))^{1-\alpha_2} \left(\frac{\bar{P}_i(s^t)^{1/(\theta_i-1)}}{\int P_i(j, s^{t-1})^{1/(\theta_i-1)} dj} \right) \quad (68)$$

And using (13), we get

$$\Upsilon[\phi i_c(s^t)^{-\rho} + (1-\phi) i_i(s^t)^{-\rho}]^{-1/\rho} = k_i(s^{t-1})^{\alpha_2} (\lambda_i(s^t) l_i(s^t))^{1-\alpha_2} \left(\frac{\bar{P}_i(s^t)^{1/(\theta_i-1)}}{\int P_i(j, s^{t-1})^{1/(\theta_i-1)} dj} \right) \quad (69)$$

Using (19) and the symmetric equilibrium assumption, we can write the price index for investment composite good as

$$\bar{P}_i(s^t) = \left[\frac{1}{M} P_i(s^{t-1})^{\theta_i/(\theta_i-1)} + \dots + \frac{1}{M} P_i(s^{t-M})^{\theta_i/(\theta_i-1)} \right]^{(\theta_i-1)/\theta_i} \quad (70)$$

Substituting equation (70) in (69), we get our second equation.

Next we transform the wage equation (44). We use (46) to substitute for $Q(s^t | s^t)$, (43) to substitute for $\zeta(q, s^t)$, and (25) to substitute for $l(s^t)$. Also, we rewrite the wage index $\bar{W}(s^t)$ as

$$\bar{W}(s^t) = \left[\frac{1}{G} W(s^{t-1})^{\vartheta/(\vartheta-1)} + \dots + \frac{1}{G} W(s^{t-G})^{\vartheta/(\vartheta-1)} \right]^{(\vartheta-1)/\vartheta} \quad (71)$$

using $\bar{W}(s^t) = \left[\int W(q, s^{t-1})^{\frac{\vartheta}{\vartheta-1}} dq \right]^{\frac{\vartheta-1}{\vartheta}}$ and the symmetric equilibrium assumption. Then we get our third equation.

Now we can develop another equation using (7). To do so, we rewrite $v_c(s^t)$ in (8) as

$$v_c(s^t) = \frac{1}{(1-\alpha_1)\lambda_c(s^t)} \left(\frac{\lambda_c(s^t) l_c(s^t)}{k_c(s^t)} \right)^{\alpha_1} \times \Phi[\varpi l_c(s^t)^{-\varkappa} + (1-\varpi) l_i(s^t)^{-\varkappa}]^{-(1+\varkappa)/\varkappa} \varpi l_c(s^t)^{-\varkappa-1} \frac{\bar{W}(s^t)}{\bar{P}_c(s^t)} \quad (72)$$

using (10) and (27). Using (46), (66), (72), (67) and (71) in (7), we obtain the pricing equation for consumption sector, which is our fourth equation.

We can do the same work for the investment sector. First, we express $v_i(s^t)$ in (22) as

$$v_i(s^t) = \frac{1}{(1 - \alpha_2)\lambda_i(s^t)} \left(\frac{\lambda_i(s^t)l_i(s^t)}{k_i(s^t)} \right)^{\alpha_2} \times \Phi[\varpi l_c(s^t)^{-\varkappa} + (1 - \varpi)l_i(s^t)^{-\varkappa}]^{-(1+\varkappa)/\varkappa} (1 - \varpi)l_i(s^t)^{-\varkappa-1} \frac{\overline{W}(s^t)}{\overline{P}_i(s^t)} \quad (73)$$

using (24) and (28). Using (46), (69), (73) (70) and (71) in (21), we obtain the pricing equation for investment sector, which is our fifth equation.

And we rewrite the Euler condition for consumption (43) substituting $\overline{P}_c(s^t)$ using (67). This is our sixth equation. Also we rewrite the Euler condition for money (45), substituting $\overline{P}_c(s^t)$ using (67), and then we get our seventh equation.

And, we can rewrite the first order conditions for consumption sector capital (34) and (35) substituting $r_c(s^{t+1})$ using (9), (27) and (71), and substituting $P_{ic}(s^t)$ using (15). These are our eighth and ninth equations.

Similarly we can rewrite the first order conditions for investment sector capital (38) and (39) substituting $r_i(s^{t+1})$ using (23), (27) and (71), and substituting $P_{ii}(s^t)$ using (16). These are our tenth and eleventh equations.

In addition to these, we use the law of motion for the money supply, the laws of motion for the productivity shocks and the capital accumulation equations for each sector as our twelfth to sixteenth equations.

After these successive substitutions, we get 16 equations and 16 variables c , P_c/M , P_i/M , W/M , l_c , l_i , μ , λ_c , λ_i , k_c , k_i , i_c , i_i , ξ/M , κ/M , ζ/M their past variables, and their future variables in expectation. Note that since we are interested in a stationary equilibrium, we have normalized prices(P_c , P_i), wage rate(W) and Lagrange multipliers(ξ , κ , ζ) by either dividing or multiplying them by the money stock(M) as in Chari et al. (2000) and Cho and Cooley (1995).

And then we log-linearize the resulting equations around the nonstochastic steady-state of the model. After the log-linearization, we can cast the resulting 16 equations characterizing the model economy, equations defining lagged variables and equations defining lagged expectations in the following form

$$\Pi_0 \hat{x}_t = \Pi_1 \hat{x}_{t-1} + \Pi_2 \varepsilon_t + \Pi_3 (\hat{x}_t - E_{t-1}[\hat{x}_t]) \quad (74)$$

where \hat{x}_t is the vector of log differences from the steady state of the 16 variables as well as their lagged variables and their lagged expectations. And ε_t is a vector of the exogenous error terms, namely the monetary and productivity shocks.

Then this system of linear stochastic difference equations can be solved using the QZ decomposition method by Sims (2001).¹⁰ The solution, which is unique and bounded in the model, takes the following form:

$$\hat{x}_t = \Psi_1 \hat{x}_{t-1} + \Psi_2 \varepsilon_t \quad (75)$$

¹⁰Sims (2001) contains details on the solution methods. The *Matlab* code is available at <http://www.priceton.edu/~sims/>

3.2 Parameterization

The time period in the model is assumed to be a quarter. The parameter values for the benchmark model economy are summarized in Table 2.

The production parameters, depreciation rates, disturbance parameters for the productivity shocks, and intratemporal adjustment parameters are from Huffman and Wynne (1998). The market demand parameters are from Chari et al. (2002), the monetary shock parameters are from Cho and Cooley (1995), and the preference parameters are basically from Chari et al. (2000).

Huffman and Wynne (1998) calculate the elasticities of output with respect to the labor inputs in the two sectors (i.e., $1 - \alpha_1$ and $1 - \alpha_2$) as the average values over the post-war period of the ratio of the sum of compensation of employees plus proprietor's income to output in each sector. Also δ_1 and δ_2 are calibrated using annual depreciation to the net capital stock in the fixed reproducible tangible wealth data, and Γ and $\Sigma_{\epsilon_\lambda}$ are calibrated using the same sectoral input and output data. Note that $\lambda_c^{1-\alpha_1}$ and $\lambda_i^{1-\alpha_2}$ can be interpreted as Solow residuals in each sector given our labor augmenting form of productivity shock in the production function.

ρ is calibrated using nominal and real investment flows. That is from (15) and (16), we have

$$\frac{P_{ic}i_c}{P_{ii}i_i} \equiv \frac{I_c}{I_i} = \frac{\phi}{1-\phi} \left(\frac{i_i}{i_c} \right)^\rho \quad (76)$$

Using this relationship and Hodrick-Prescott filtered nominal investment flows and real investment flows we can calibrate ρ . This calibrated value ranges from -1.3 to -1.1. And Huffman and Wynne (1998) picked -1.1 to be conservative. And ϕ is chosen so that the price of each type of capital in each sector is equal in the nonstochastic steady state. \varkappa was picked to be -1.1 so that intratemporal adjustment costs in investment and labor is roughly same. And ϖ is calibrated in a similar way as ϕ .

Chari et al. (2002) chose market parameters based on the work of Basu (1996), Basu and Fernald (1994), Basu and Fernald (1995), and Basu et al. (1997). And in this paper, we set $\theta_c = \theta_i$ assuming same market demand parameters in consumption and investment sectors¹¹. Cho and Cooley (1995) calibrated monetary shocks fitting first auto-regressive process to the M1 stock.

We calibrate the preference parameters as in Chari et al. (2000). They set β assuming a 4% annual discount rate, and they set $\sigma = \gamma = \eta$ based on the balanced growth requirement. Also since the model can be used to price a nominal bond that costs one dollar at s^t and pays a gross interest rate of $R(s^t)$ dollars in all states s^{t+1} , we can get a first order condition for this asset, which is $\frac{U_3(s^t)}{P_c(s^t)} = \zeta(s^t) (R(s^t) - 1) / R(s^t)$ where $\zeta(s^t)$ is Lagrangian multiplier

¹¹ And from simulation results, setting reasonable different values of θ in each sector (e.g. 15% difference in the elasticity of substitution between sectors) does not change the features of sectoral comovement.

in (43)¹². This can be rewritten as¹³

$$\log \frac{M}{P_c} = \frac{1}{\eta} \log \omega + \log C - \frac{1}{\eta} \log (R(s^t) - 1) / R(s^t) \quad (77)$$

And Chari et al. (2002)'s calibration gives $\eta = 2.56$ and $\omega = 0.66$. ψ is calibrated so that a share of time allocated to labor is around 1/3.

Finally the capital accumulation parameters b_c and b_i will be adjusted so that the relative standard deviation of total investment to that of consumption and the relative standard deviation of consumption sector investment to that of investment sector investment are similar to the corresponding statistics for the U.S. economy in line with Chari et al. (2000).¹⁴ In the simulation of the model, we will set $N = M = Q = 4$ as in Chari et al. (2002) so that prices and wages are set for four quarters.

4 Findings

Before examining the behavior of the model, it needs to be noted that the presence of multiple sectors gives rise to a measurement issue of aggregates. In this paper, a fixed-weight price deflator is employed to measure the aggregates as in Huffman and Wynne (1998). Namely, for example, to measure aggregate output, we add up the amount of investment to that of consumption by using steady state price level.¹⁵ The basic comovement behavior of the model is summarized below.

4.1 Benchmark Model

In the benchmark model, we can generate the comovement of economic variables including sectoral variables when we perturb the model economy with monetary

¹²This can be shown as follows. From (45)

$$\frac{U_3(s^t)}{P_c(s^t)} - \zeta(s^t) \left[1 - \beta \sum_{s_{t+1}} \pi(s^{t+1} | s^t) \frac{\zeta(s^{t+1})}{\zeta(s^t)} \right] = 0$$

From (46) and by definition

$$\beta \sum_{s_{t+1}} \pi(s^{t+1} | s^t) \frac{\zeta(s^{t+1})}{\zeta(s^t)} = \sum_{s_{t+1}} Q(s^t | s^t) = \frac{1}{R(s^t)}$$

¹³ $\zeta(s^t) = \frac{U_1(s^t)}{P_c(s^t)}$ from (43).

¹⁴We set $b_c = b_i = 0$ when the relative volatility of total investment is too small compared with data.

¹⁵As explained in Huffman and Wynne (1998), this method of combining consumption and investment in an aggregate is close to the actual national income data generating method.

shocks. But we do not generate the comovement when we perturb the model economy with productivity shocks.

4.1.1 Monetary Shock

Figure 2 plots the responses of economic variables to a one standard deviation monetary shock in the benchmark model. Total output, total labor, consumption and total investment all increase due to the monetary shock. And labor in both sectors move very similarly although the amplitude of response is bigger in the investment sector than in the consumption sector. The investment in both sectors also move similarly but the response of investment in the consumption sector is bigger in amplitude than the response of investment in the investment sector. The prices in both sectors $\left(\log \frac{\bar{P}_c}{M}, \log \frac{\bar{P}_i}{M}\right)$ and wage $\left(\log \frac{\bar{W}}{M}\right)$ in this economy decrease.

When the monetary shock hits the economy, the prices of the consumption and investment goods and the wage all become relatively lower than before due to the stickiness of prices and wage. Then output, consumption, investment, and labor in the economy all increase due to high demands following the relatively lower prices and wages. But this economic boom induced by monetary shock does not last long since prices and wage will adjust after a while as can be seen in the diagram.

In Table 3, we also report the relative standard deviation of each economic variables to the consumption as well as the correlation coefficients of each economic variables with the output when the economy is perturbed by monetary shocks. Consistent with the impulse response diagrams in Figure 2, all the correlation coefficients of important variables in the model with the total output are positive, showing contemporaneous comovement of those variables with output. And the correlation coefficients of consumption and investment with output are slightly higher than the correlation coefficient of labor with output. In terms of volatility, investment is more volatile than output, and thus output is more volatile than consumption. And labor in the investment sector is more volatile than labor in the consumption sector and investment in the consumption sector is more volatile than investment in the investment sector, somewhat consistent with the actual data in table 1.

In sum, monetary shocks generate comovement of economic variables in this model.¹⁶ And this can be considered a major improvement compared with negative correlations of sectoral variables in a standard two sector real business cycle models.

4.1.2 Consumption sector productivity shock

¹⁶Even when we do not impose intratemporal adjustment costs in labor and investment so that $\rho = \chi = -1$, we have the comovement of variables when perturbed by a monetary shock.

Figure3 plots the responses of the economic variables to a standard deviation productivity shock in the consumption sector. Total output decreases slightly in the beginning and overall it increases. Consumption increases but total labor and investment decrease. And labor in both sectors decrease. Investment in both sectors decrease in the beginning but investment in investment sector rebounds above steady state thereafter. The prices in both sectors and wage in this economy decrease.

When there is a positive productivity shock in the consumption sector, consumption good production and thus consumption increases. And due to this increased production of the consumption good, total output also increases.

If the prices and wages were all flexible, the increase of marginal product of labor and capital in consumption sector due to the positive productivity shock would raise the labor and capital inputs in the consumption sector and also it would raise the labor and capital inputs in the investment sector due to the equalization of marginal product across sectors. But when price and wages are sticky, a positive technology shock can have a negative effect on the inputs as explained in Gali (2000). That is, the combination of a constant money supply and predetermined prices implies that real balance thus aggregate demands for consumption goods remain unchanged in the period when the productivity shock occurs in the consumption sector. Producing same amount of consumption goods given the positive productivity shock will require less inputs thus lowering labor and capital inputs in consumption sector. Lower amount of required capital in consumption sector induces lower amount of output in the investment sector. And due to the large decrease in the production of investment goods in the beginning, total output also decreases slightly in the beginning. And labor and investment in the consumption sector decrease more sharply than those in the investment sector reflecting the fact that the productivity shock hits the consumption sector. Prices and wages fall due to the positive productivity shock given constant aggregate demands and decrease in consumption good price $\left(\log \frac{P_c}{M}\right)$ is sharper than investment good price $\left(\log \frac{P_i}{M}\right)$ or wages $\left(\log \frac{W}{M}\right)$ since the positive productivity shock occurred in the consumption sector.

Productivity shocks in the consumption sector overall do not generate the comovement of economic variables observed in the data. Particularly the correlation between total output and total investment and the correlation between total output and total labor show negative sign.

4.1.3 Investment sector productivity shock

Figure4 plots the responses of economic variables initiated by a standard deviation productivity shock in the investment sector. Total output, total investment, and total labor decrease in the beginning and then increase. Consumption increases but the response of consumption is at least 10^{-1} smaller than the response of other aggregate variables. Labor in both sectors move in the opposite

direction and the magnitude of response in consumption sector is 10^{-1} smaller than the magnitude in the investment sector. Investment in both sectors move together showing similar movement as total investment. The price in the investment sector decreases, but wage increase. The price in the consumption sector increases for relatively short periods in the beginning and then decreases thereafter.

When a positive productivity shock hits the investment sector, investment good production thus total investment increases. But we see small decrease of total output and total investment in the beginning due to the sticky price and wage. If the prices and wages were fully flexible, investment good production would increase from the beginning but when prices and wages are sticky, we do not need to produce more investment good initially to meet relatively unchanged demands. And actually we demand and produce less investment good in the beginning expecting a decrease in the relative price of investment good in the future.¹⁷ And thus total investment decreases slightly in the beginning and total output and investment in each sector reflect this movement of total investment.

Labor input in the investment sector decreases in the beginning due to the stickiness of prices and wages following the positive productivity shock. But labor input in the consumption sector increases in the beginning to produce more consumption goods. The consumption increases from the beginning reflecting the positive productivity shock and consumption smoothing motive. In a relatively longer time horizon, however, labor in the investment sector increases while labor in the consumption sector decreases following the positive productivity shock in the investment sector. But this is a standard result from two sector real business cycle models as discussed in Christiano and Fitzgerald (1998).

Concerning the prices and wages, the investment good price decreases $\left(\log \frac{\bar{P}_i}{M}\right)$ due to the positive productivity shock given constant aggregate demands. But wages $\left(\log \frac{\bar{W}}{M}\right)$ increase due to the rise of marginal productivity of labor following the productivity shock in the investment sector. Consumption sector price $\left(\log \frac{\bar{P}_c}{M}\right)$ increases initially due to the rise of wages but decreases thereafter due to the reduced capital costs following positive productivity shock in the investment sector.

In sum, productivity shocks in investment sector do not generate comovement of economic variables observed in the data. Particularly correlation coefficient between labor in both sectors show negative sign as in standard real business cycle models.

4.1.4 Aggregate productivity shock

We can consider an aggregate productivity shock that affects all the sectors equally. The result is that an aggregate productivity shock also does not gener-

¹⁷Investment good is transformed into capital which depreciates gradually over time. Thus it is important to consider future price of investment good.

ate comovement of the variables as in either the consumption sector productivity shock and investment sector productivity shock. This is very natural since we can think of an aggregate productivity shock as a combination of consumption sector productivity shock and investment sector productivity shock. Particularly total labor and output shows negative correlation coefficients due to the reason explained by Gali (2000).

4.2 Variations

4.2.1 Sticky prices or Sticky Wage

We can consider the case when there is stickiness only in either prices or the wage. But in those cases, the basic movement patterns of the variables in the model economy are not much changed from the benchmark model.

For instance, the responses of economic variables corresponding to a monetary shock when only prices are sticky are depicted in Figure 5 and the responses when only the wage is sticky are depicted in Figure 6. Total output, total labor, consumption, total investment, and sectoral labor all show similar pattern to the benchmark model. But investment in investment sector is more volatile than investment in consumption sector when there is stickiness only in the wage contrary to the benchmark model. And the responses of economic variables are more persistent when there is stickiness only in the wage compared with benchmark model¹⁸. Prices and wage movements in these are different from the benchmark case but it is very natural since we assumed stickiness either in prices or wage instead of stickiness in both.

Productivity shocks also induce similar movements patterns in the variables as in the benchmark economy and they do not generate comovement of economic variables as in benchmark model.

4.2.2 Persistent and Hump Shaped Responses

Christiano et al. (2001) shows that habit formation and variable capacity utilization are helpful in matching the persistence and hump shape in the impulse responses of model economies to the US economy's.

To examine this in our model, we can introduce habit formation in the utility function, and different law of motions for the accumulation of capital in each sectors as well as variable utilization of capital in the benchmark model following Christiano et al. (2001).

¹⁸See Huang and Liu (2002).

The results of these modifications can be seen in Figure 7. The responses of economic variables to a monetary shocks are now more persistent and they are generally hump shaped. But the volatility statistics and correlation coefficients are basically unchanged compared with benchmark model. Thus these modification generate persistence and hump shapes in the responses of economic variables without altering basic features of movement patterns of economic variables.

5 Conclusion

The comovement of sectoral economic variables such as sectoral labor inputs and sectoral investment is one of the defining characteristics of business cycle fluctuations.

But recent real business cycle models have not been able to successfully generate the comovement of sectoral variables. In this paper, we considered the possibility of comovement of sectoral economic variables during a business cycle induced by monetary shocks. It is very natural in that monetary shock has been traditionally believed to be one of most important candidates among the sources of business cycle. Following this tradition, we constructed a sticky prices and sticky wage model to see the monetary effects on the economy.

The main results from this attempt can be summarized as follow. Unlike productivity shocks, monetary shocks can generate the comovement of economic variables across sectors in the model economy, and volatility statistics and correlations among economic variables in the model economy are similar to the real world counterparts when the economy are continuously perturbed by monetary shocks.

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Table1¹⁹

Correlations with aggregate output(annual data)

	%Std.	Corr(x_{t-j}, y_t)				
		-2	-1	0	1	2
Output						
Consumption sector	1.24	-0.110	0.317	0.856	0.272	-0.297
Investment sector	3.69	-0.153	0.376	0.991	0.393	-0.097
Capital						
Consumption sector	1.60	0.269	0.293	-0.023	-0.215	-0.076
Investment sector	1.22	0.369	-0.004	-0.408	-0.324	-0.213
Labor Input(Household data)						
Consumption sector	1.66	0.054	0.638	0.931	0.453	-0.031
Investment sector	3.15	0.215	0.673	0.864	0.148	-0.444
Investment flows						
Consumption sector	8.59	-0.06	0.10	0.54	0.16	-0.11
Investment sector	7.24	0.07	0.63	0.67	-0.12	-0.34

Table 2

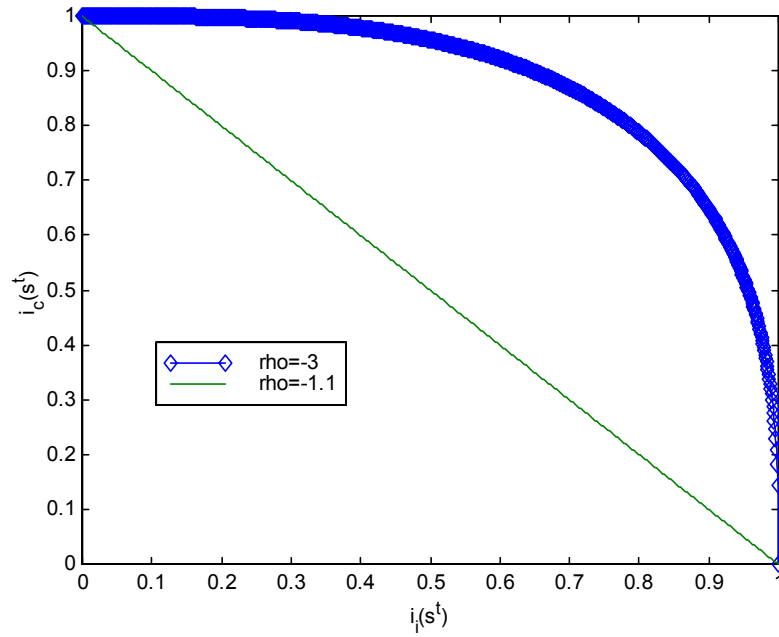
Benchmark Model Parameters

	parameter values
Preferences	$\beta = 0.97^{1/N}$, $\omega = 0.66$, $\psi = \text{adjusted}$, $\sigma = \gamma = \eta = 2.56$
Production	$\alpha_1 = 0.41$, $\alpha_2 = 0.34$,
Market demand	$\theta_c = \theta_i = 0.9$, $\vartheta = 0.87$
Intrateporal Adjustment cost	$\rho = -1.1$, $\phi = \text{adjusted}$, $\Upsilon = 2$, $\varkappa = -1.1$, $\varpi = \text{adjusted}$, $\Phi = 2$,
Capital accumulation	$1 - \delta_1 = 0.982$, $1 - \delta_2 = 0.98$ $b_c = \text{adjusted}$, $b_i = \text{adjusted}$
Monetary Shock	$\rho_\mu = 0.48$, $\sigma_\varepsilon = 0.00985$
Productivity Shock	$\Gamma = \begin{bmatrix} 0.928 & 0.000 \\ 0.000 & 0.786 \end{bmatrix}$, $\Sigma_{\varepsilon_\lambda} = \begin{bmatrix} 0.000179 & 0.000332 \\ 0.000332 & 0.000873 \end{bmatrix}$

¹⁹Correlations are between HP filtered series with smoothing parameter set equal to 100.

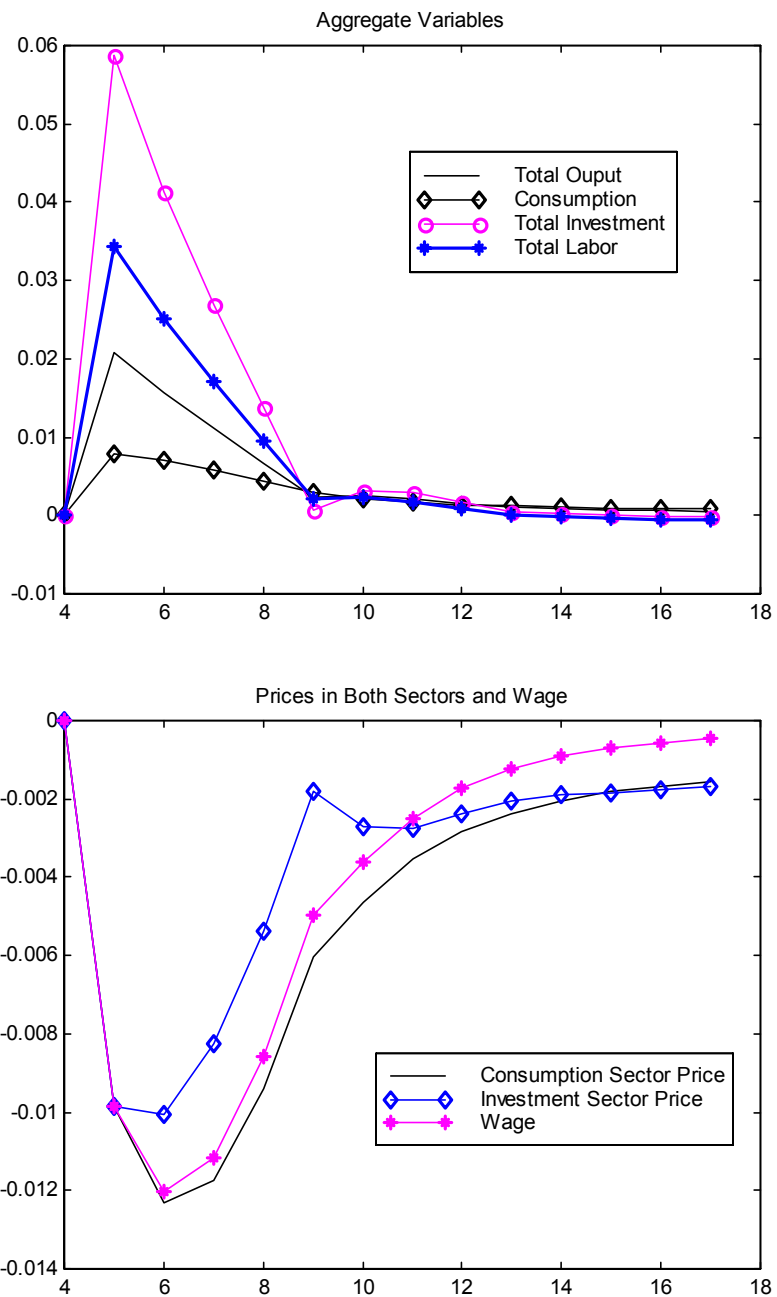
	STD. Relative to Consumption	Corr. with Output
c	1.00 (1.00)	0.99 (0.86)
i	2.95 (2.98)	0.99 (0.99)
k_c	0.80 (1.29)	0.51 (-0.02)
k_i	0.67 (0.98)	0.55 (-0.41)
l_c	1.50 (1.34)	0.98 (0.93)
l_i	4.40 (2.54)	0.97 (0.86)
i_c	3.05 (6.93)	0.99 (0.54)
i_i	2.59 (5.84)	0.98 (0.67)

Figure1



²⁰Numbers in parentheses are corresponding statistics from Table1. The statistics for the model economy are obtained by simulating the model for 5,000 annual periods.

Figure 2²¹
 Monetary Shock in Benchmark Model



²¹All variables are in log-deviation form. The shock hits the economy at 5th period.

Figure2 continued
 Monetary Shock in Benchmark Model

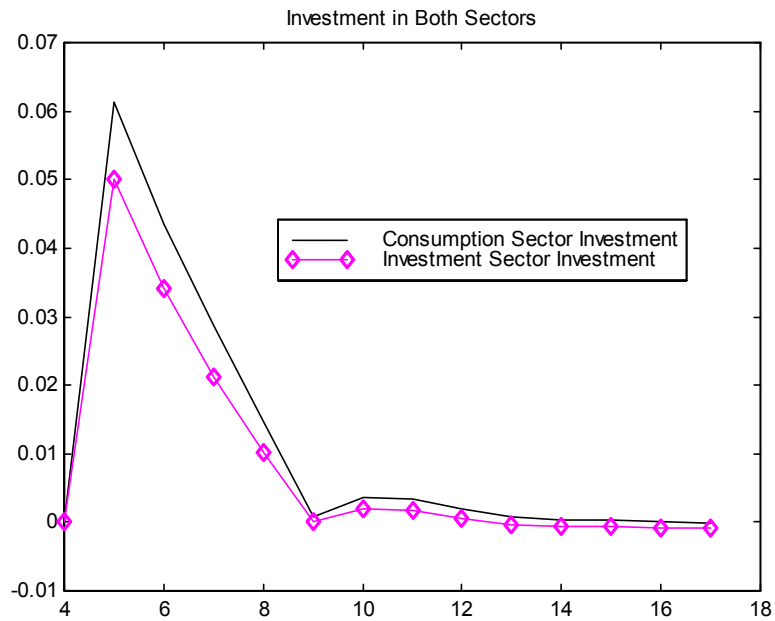
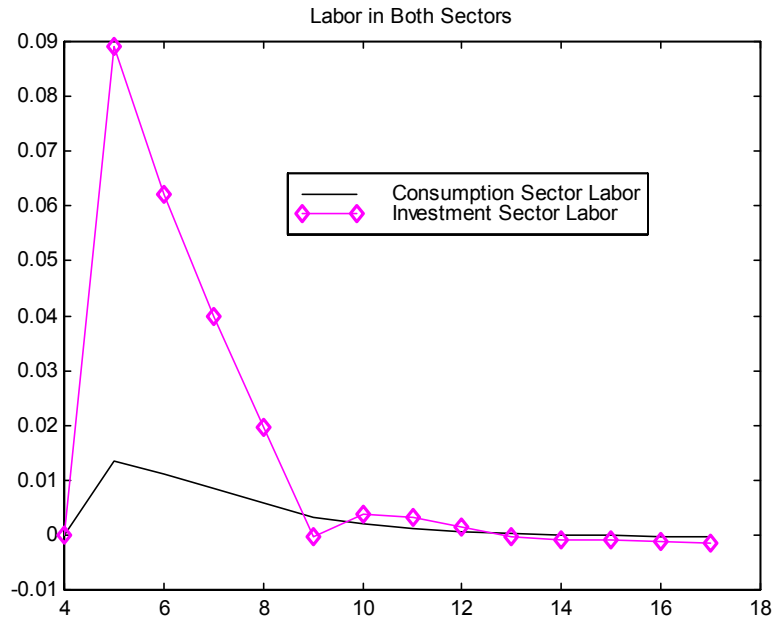
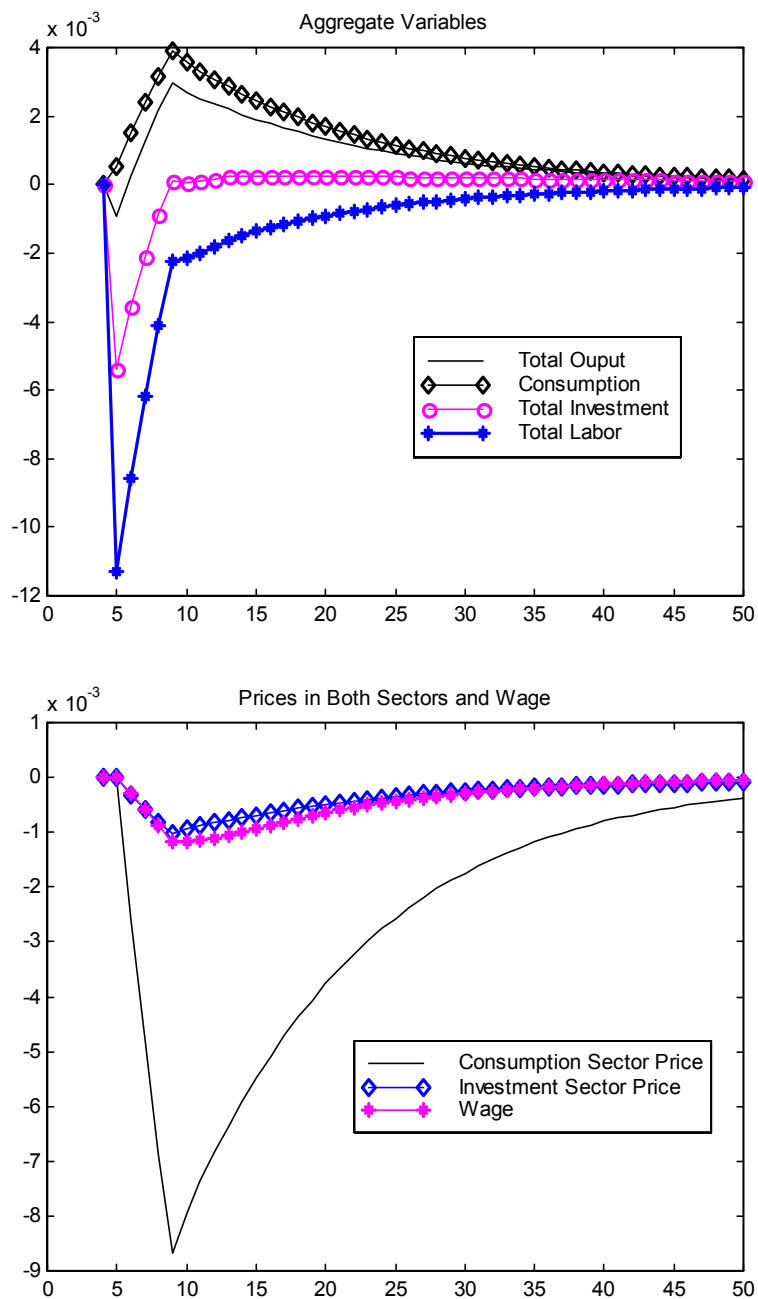


Figure 3²²
 Consumption Sector Productivity Shock in Benchmark Model



²²See footnote for Figure 2.

Figure3 continued
 Consumption Sector Productivity Shock in Benchmark Model

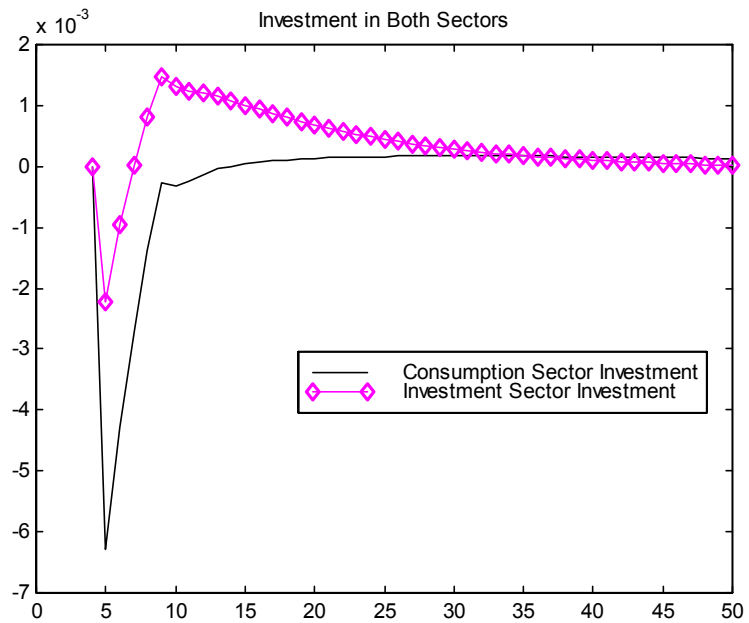
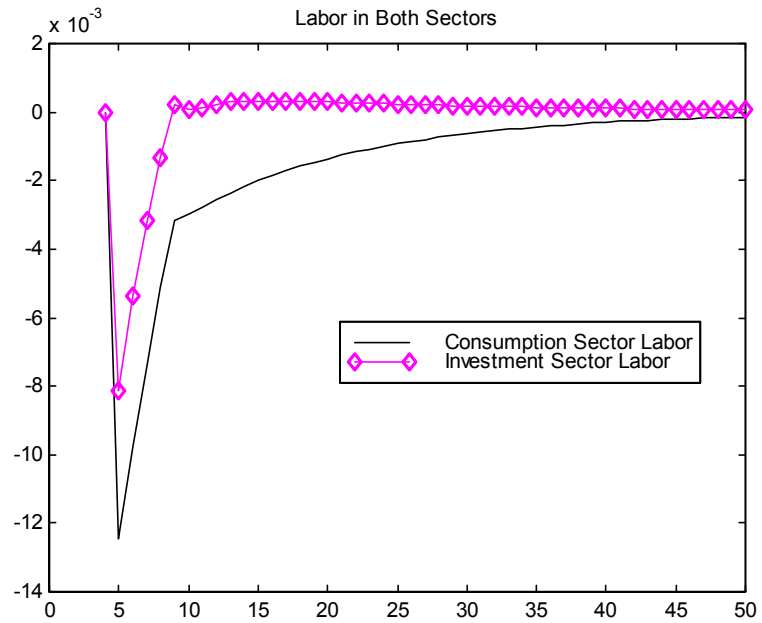
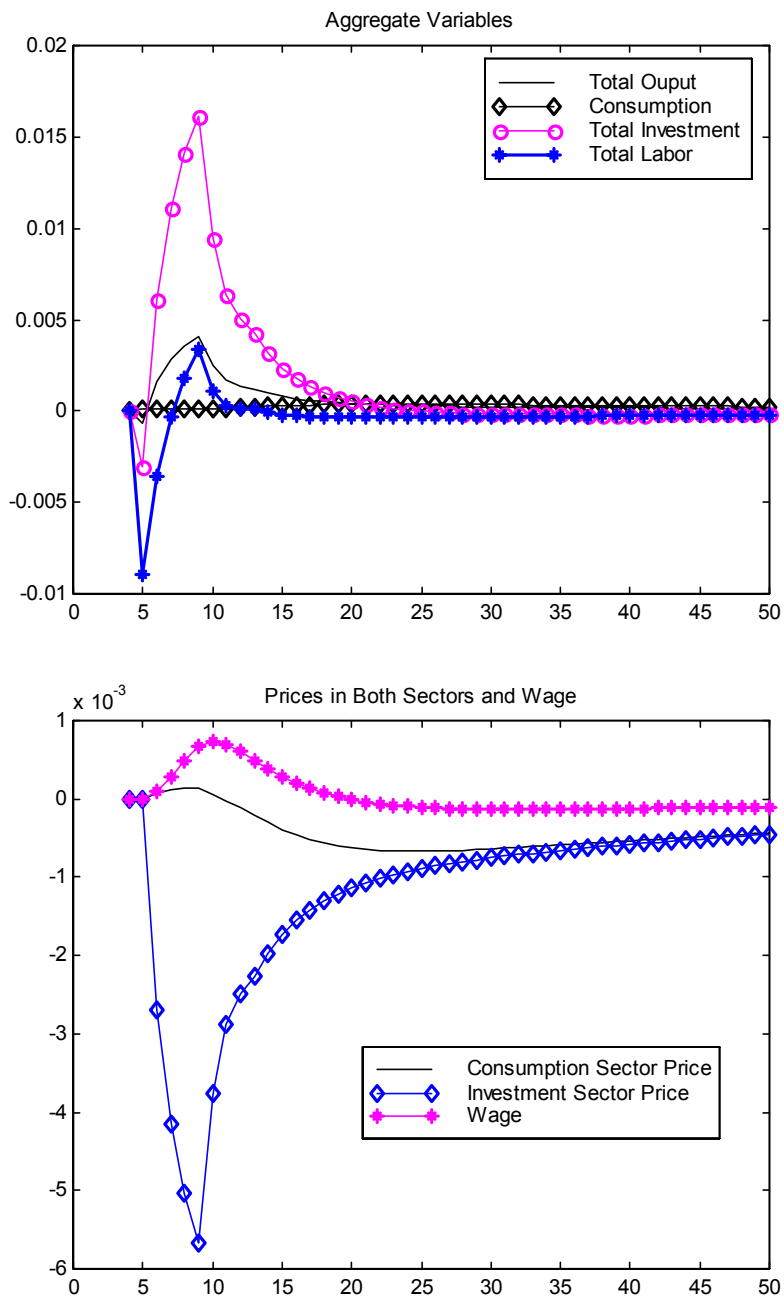


Figure4²³
 Investment Sector Productivity Shock in Benchmark Model



²³See footnote for Figure2.

Figure4 continued
 Investment Sector Productivity Shock in Benchmark Model

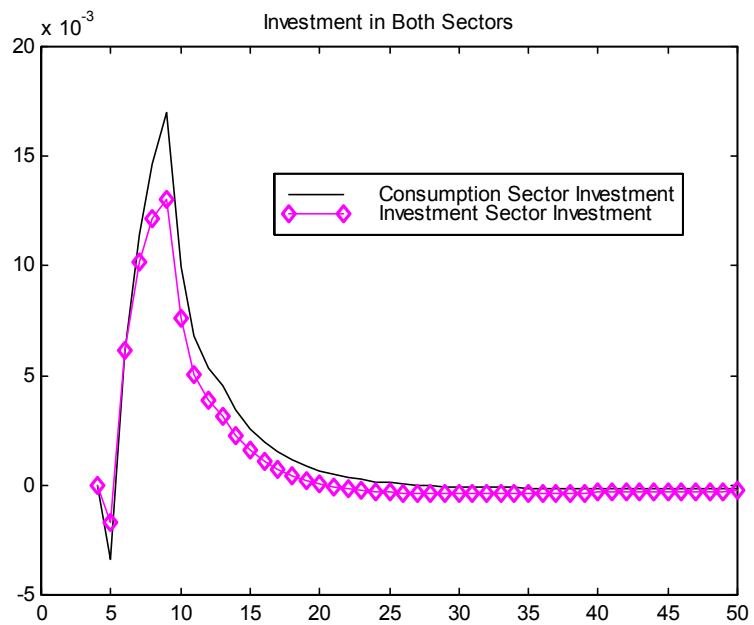
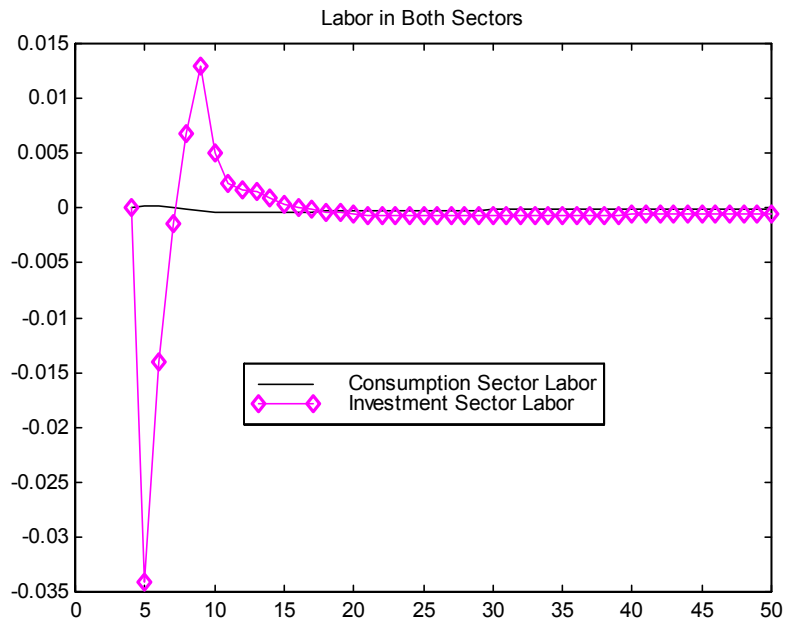


Figure4 continued
Investment Sector Productivity Shock in Benchmark Model

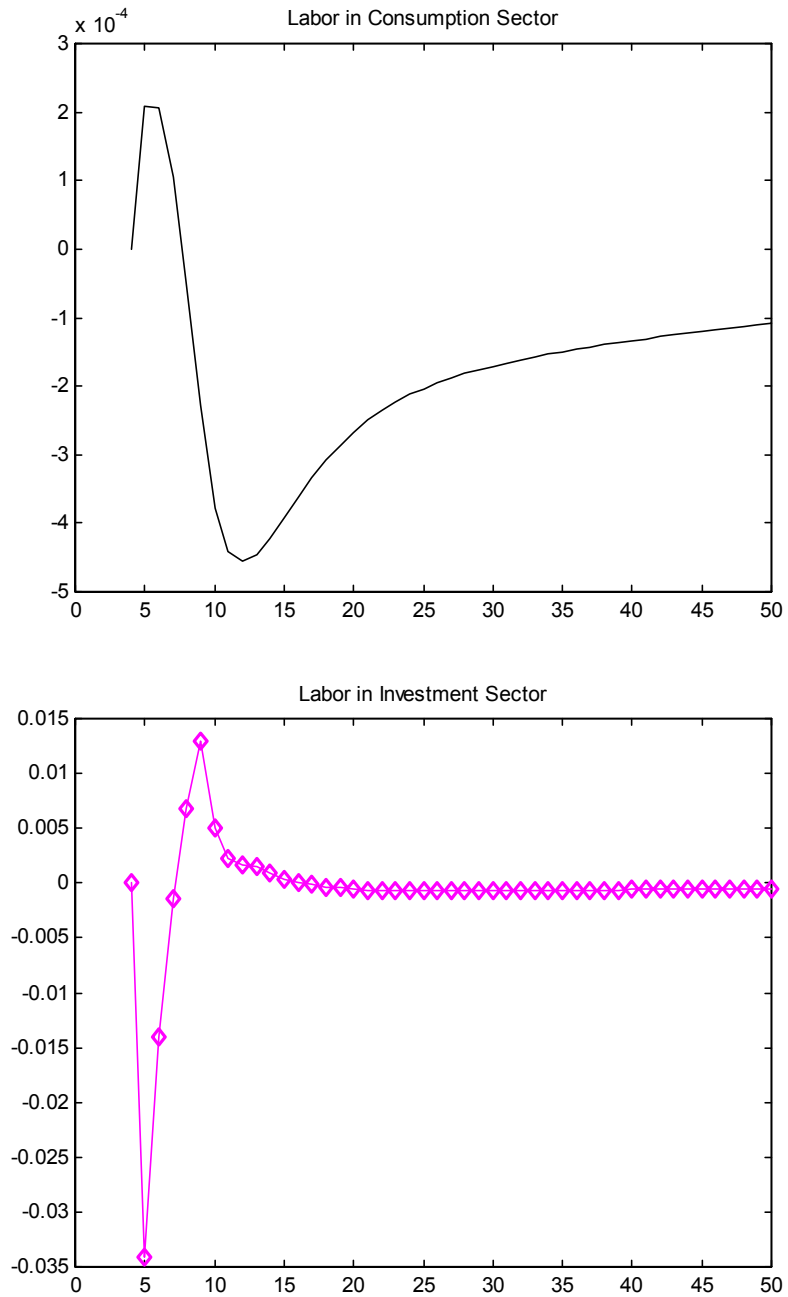
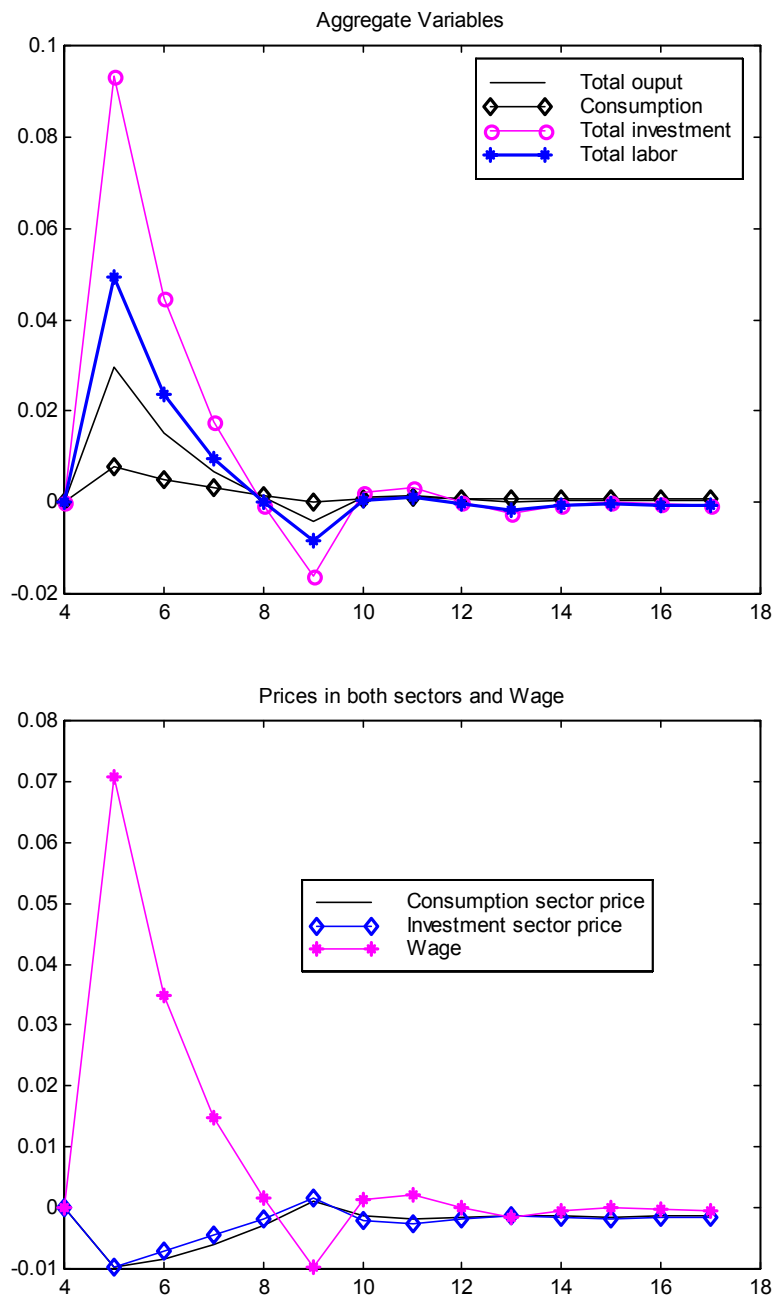


Figure 5²⁴
 Monetary Shock When Prices are Sticky



²⁴See footnote for Figure 2.

Figure5 continued
 Monetary Shock When Prices are Sticky

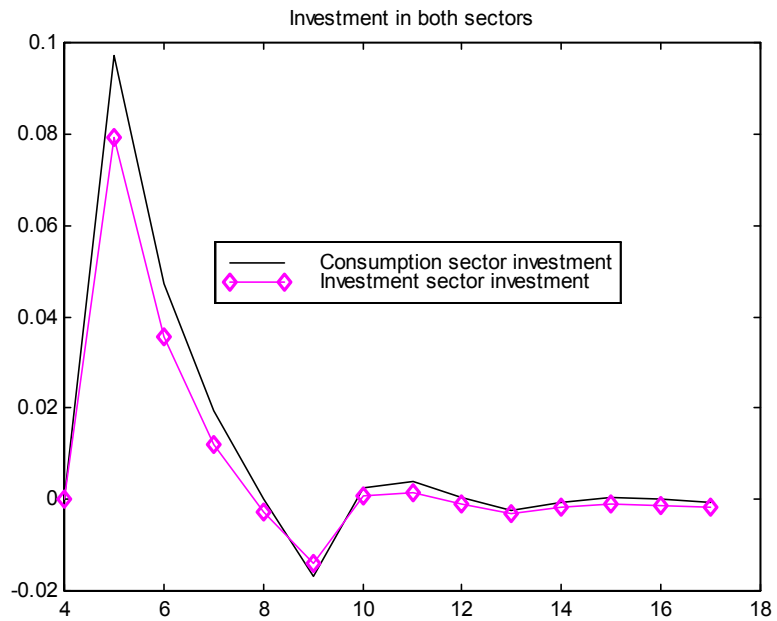
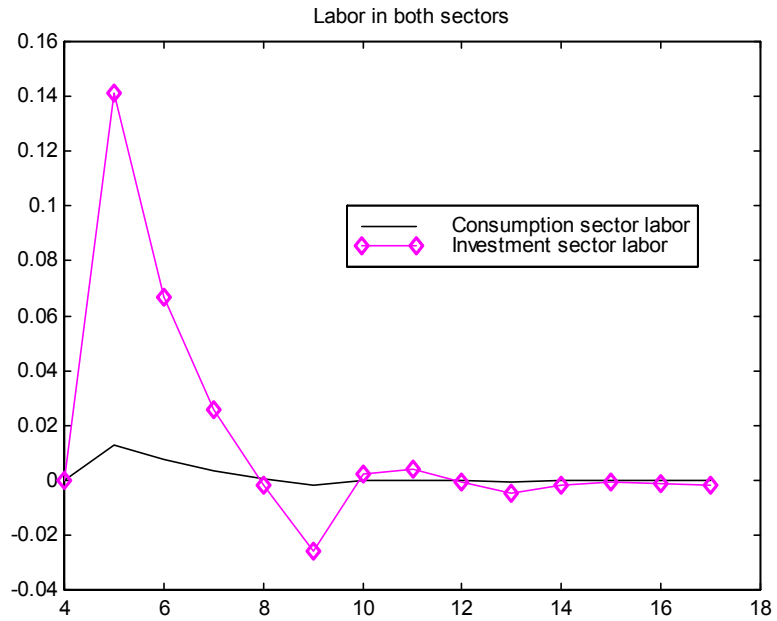
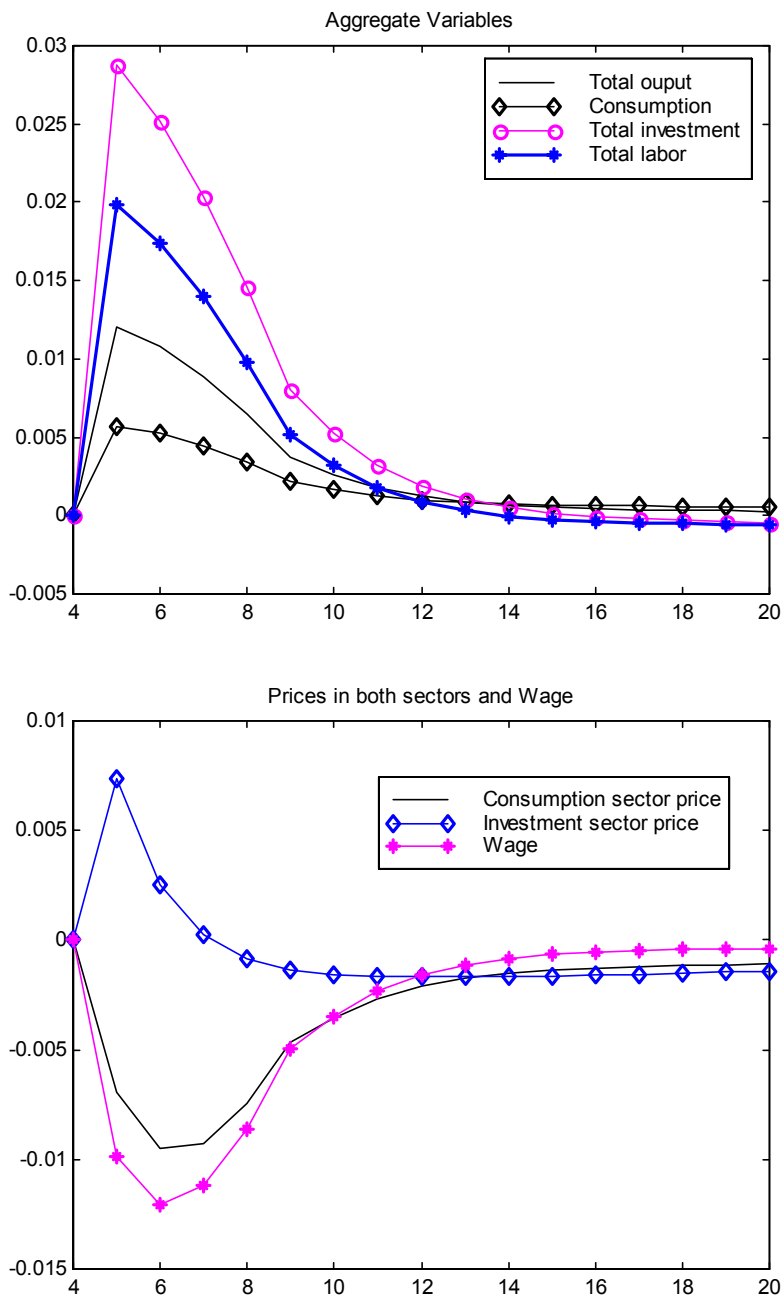


Figure6²⁵
 Monetary Shock When Wages are Sticky



²⁵See footnote for Figure2.

Figure 6 continued
 Monetary Shock When Wages are Sticky

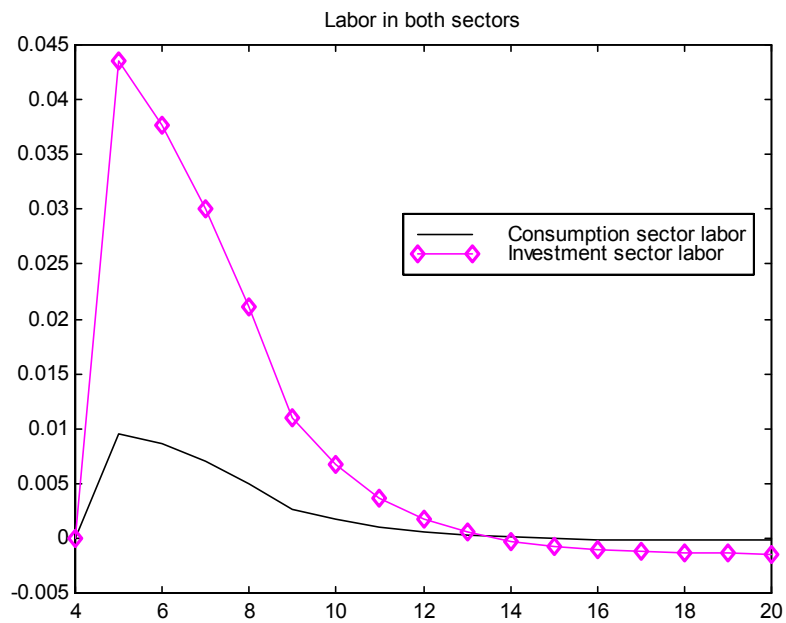
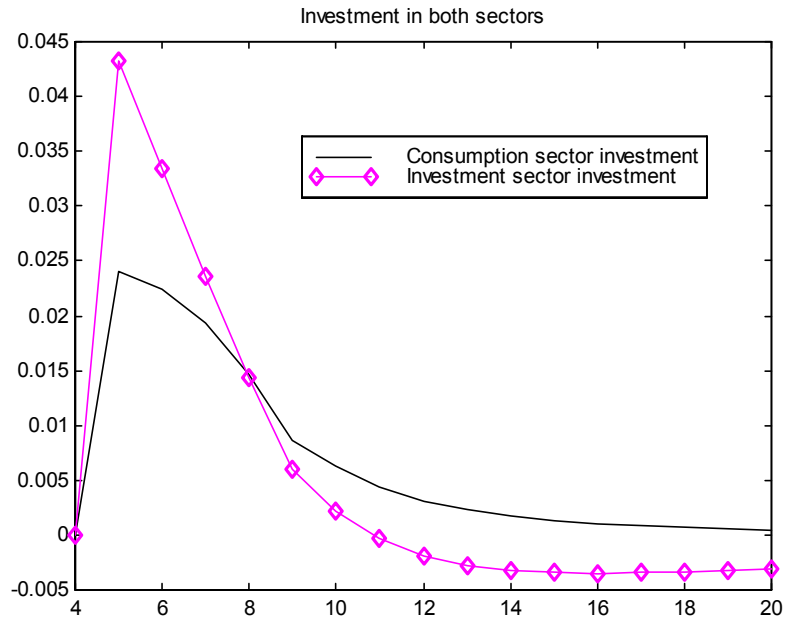
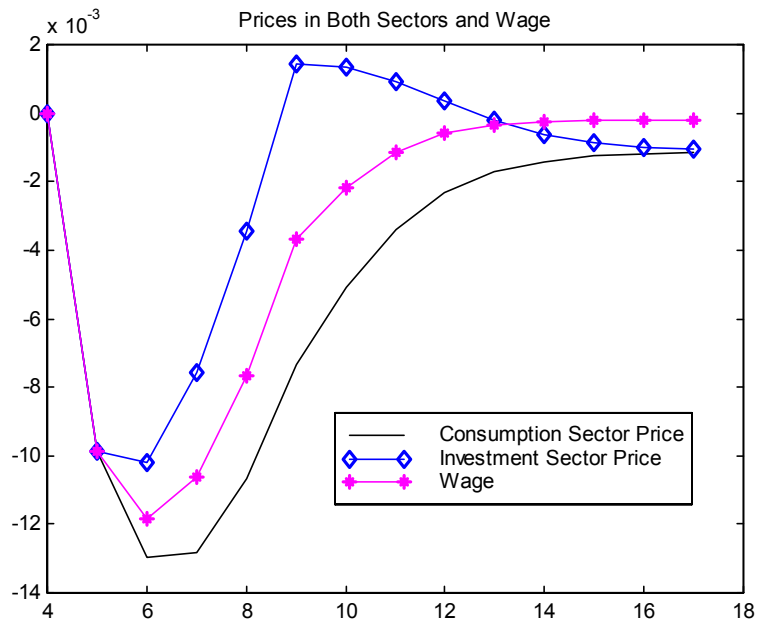
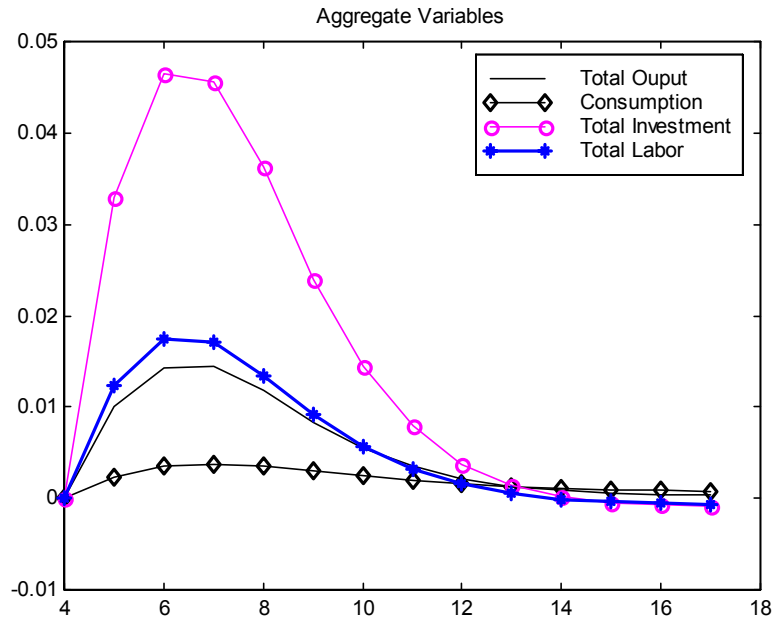


Figure 7²⁶
 Monetary Shock with Persistent and Hump Shaped Responses



²⁶See footnote for Figure 2.

Figure7 continued
 Monetary Shock with Persistent and Hump Shaped Responses

