Discriminatory vs Uniform Price Auction: Auction Revenue Comparison in the Case of the Korean Treasury Auction Market

by

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Abstract: We compare auction revenues from discriminatory auctions and uniform price auctions in the case of the Korean treasury bonds auction market. For this purpose, we employ detailed bidder level data for each of 16 discriminatory auctions recently carried out in Korea. We first theoretically recover unobserved individual bidding functions under counter-factual uniform price auctions from the observed bidding functions under the actual discriminatory auctions, and then empirically estimate revenue differences. To test significance of the auction revenue differences, we use Bootstrap re-sampling methods where uncertainty in the cut-off yield spreads and uncertainty in the bidders are addressed individually as well as simultaneously. Our results indicate that uniform price auction increases the auction revenue relative to the discriminatory auction in most of the 16 cases, justifying the Korean government's decision to switch to the uniform price auction mechanism.

Keywords: Treasury bonds auction, discriminatory auction, uniform price auction, hazard rate, Bootstrap re-sampling, yield spread, bidding function, bid shading
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1. Introduction

We compare auction revenues from discriminatory auctions and uniform price auctions in the case of the Korean treasury bonds auction market. For this purpose, we use detailed bidder level data for each of 16 discriminatory auctions recently carried out in Korea. Using a theoretical model, we first recover unobserved individual bidding functions under counter-factual uniform price auctions from the observed bidding functions under actual discriminatory auctions, and then estimate the auction revenue differences. To test significance of the auction revenue differences, we use Bootstrapping re-sampling methods by which we address uncertainty in the cut-off yield spreads as well as uncertainty in the bidders. Our results indicate that the uniform price auction increases the auction revenue relative to the discriminatory auction in most of the 16 cases analyzed, justifying the Korean government's decision to switch to the uniform price auction in the year 2000.

Let us briefly overview the Korean treasury auction market. The Korean government has adopted a competitive bidding system in the treasury bonds auction market, September 1999. There exist 24 primary dealers and 6 candidate dealers for a total of 30 dealers who are exclusively entitled to submitting bids in the Korean treasury auction. The Korean treasury has nominated 6 candidate dealers for the purpose of "internship" before promoting them to primary dealers. Most of the 30 dealers are either banks or security companies. Their market powers are evenly distributed in terms of bidding and winning amounts. The Korean treasury auction market is competitive.

Under the discriminatory auction, "what you bid is what you pay." The bigger the uncertainty about the auction cut-off price, the less willing you become to bid, so called "bid shading." Bid shading is more severe as the uncertainty in the cut-off price increases. Lack of the so called "when-issued market" delays the price discovery process in the Korean treasury auction market on one hand, whereas it precludes the possibility "short squeeze" on the other.

Depending on whether the uncertainty is large or not, auction revenue criterion favors either the switch to the uniform price auction or the continuation of the discriminatory auction. We use historical auction spread distribution as a measure of the uncertainty in the treasury auction market. We only use those auction spreads that are observed under historical discriminatory auction cases. Using this measure of auction uncertainty, we identify the magnitude of bid shading and thus revenue gap between the actual discriminatory auction and the counter-factual uniform price auction for each of the 16 discriminatory auction cases actually implemented recently in Korea.

The rest of the paper is organized as follows. Section 2 briefly reviews the existing literature on the treasury bond auction. Section 3 introduces a standard auction model showing the differences in bidder behavior under the two different auction mechanisms. Section 4 explains the data and the econometric model. Results are reported in section 5. Section 6 concludes.

2. Literature Review

Auction revenue depends on the underlying auction mechanism. Under a set of conditions, Vickery (1961) shows that the auction revenue is the same whether one uses a first price auction or a second price auction. This famous “revenue equivalence theorem,” however, does not hold in divisible multiple unit auctions as in the case of the treasury bond auction. Revenue comparison becomes even more difficult as one considers the common value aspect of the treasury bond. Earlier, Friedman (1960) raises a possibility that the uniform price auction may result in more auction revenues than the multiple price auction.

Milgrom and Weber (1982), Bikchandani and Huang (1989, 1993), and Chari and Weber (1992) analyze the treasury auction market. Wilson (1979), Back and Zender (1993), and Wang and Zender (1996) introduces a divisible goods assumption to make their analyses compartible with the treasury auction.

Bikchandani and Huang (1989), and Chatterjea and Jarrow (1998) analyze the interaction between the primary and the seconary markets. According to Bikchandani and Huang (1989), bidders with maket power would like to bid more aggressively in the primary (auction) market to signal their strong valuation to the secondary market, and in fact they do so at a lower cost under the uniform price auction. Chatterjea and Jarrow (1998) argue that bidders with market power bid more aggressively in the primary market to squeeze out those auction participants with a short position in the "when issued markets," so called a "short squeeze" phenomenon.

Viswanathan and Wang (1998) view the primary dealers as market makers in the treasury auction market.

As theoretical approaches do not provide any conclusive results, there have arisen empirical approaches to compare auction revenues across the discriminatory and the uniform price auctions. Most empirical studies compare the observed auction spreads across those two auction mechanisms. Umlauf (1993a) reports that the Mexican government's auction revenue slightly increased as Mexico switched its treasury auction mechanism from the discriominatory one to the uniform one. Regarding the US, Simon (1994) reports that the auction revenue decreased as the US government switched its auction mechanism from the discriminatory to the unifrorm in the 70s, whereas Nyborg and Sundaresan (1996) and Malvey and Archibald (1998) report that the auction revenue increased under the uniform price auction in the 90s (statistically not significant, though).

Applicability of these empirical approaches is limited in the following two senses. First, they cannot be used unless a country has experimented both auction mechanisms. Second, they do not use detailed, micro-level bidding information. An alternative structural approach overcomes these drawbacks. The structural approaches recover "counter-factual" bidding fuctions under the uniform price auction from the observed bidding functions from the discriminatory auction. The approach can be applied even to a country which has only experienced the discrominatory auction mechanism. The structural approach relies on micro level bidding information to recover the counter-factual bidding function.

Nautz (1995) develops a theoretical model for the above structual approach. Heller and Lengwiller (1998) analyze the Swiss treasury auction using the structual approach. Hortacsu (2002a,b) adds strategic interactions among the bidders to the structural approach.

3. Theoretical Model

Treasury auction is a multiple unit, divisible goods auction. The treasury, as auctioneer, puts on the table a fixed amount of Treasury bonds. In the Korean treasury auction market, there are a total of 30 potential bidders who are more or less homogeneous. We assume that each bidder has a common belief on the distribution of the cut-off price in the auction, and that there are no strategic interactions either between auctioneers and the bidders or among the bidders. We consider a private value auction model where different bidders' private values are not affiliated (Milgrom and Weber 1982).

To be able to recover unobserved bidding behaviors under the counter-factual uniform price auction from the observed behaviors under the actually implemented discriminatory auction, we need to characterize theoretically bidders' bidding behaviors under each auction mechanism and to identify their relationship.

Section 2.1 introduces the basics. Sections 2.2 and 2.3 characterize the bidding behaviors under the discriminatory and the uniform price auctions. Section 2.4 recovers the bidding function under the counter-factual uniform price auction from the bidding function under the actual discriminatory auction.

3.1. Basics

Bidders determine their bidding strategy to maximize their expected profits. Each bidder is allowed to submit up to five (price, quantity) pairs. The bidding prices are denominated in terms of yield to maturity with a tick size of one basis point, that is, one hundredth of one percent point.

Let $P = \{p_1, \cdots, p_k\}$ be the support of the market clearing cut-off yield spread, say p. We arrange p_j's in an increasing order, $p_1 < \cdots < p_k$. As a result a more aggressive bid corresponds to a lower value of p_j.

The yield spread is the difference between the market clearing yield (so called the cut-off yield) and the secondary market yield of the same class of treasury bonds (at morning close on the same day). A higher yield spread means a higher yield to the bidders relative to the yield anticipated from the secondary market, and a lower bond price to the treasury.

Let the yield spread distribution (under the discriminatory auction) be represented by $f = (f_1, \cdots, f_{k-1})$, where $f_1 = \Pr(p = p_1)$, ···, $f_{k-1} = \Pr(p = p_{k-1})$. Of course, one has $\Pr(p = p_k) = 1 - (f_1 + \cdots + f_{k-1})$. For notational consistency, let us denote this value as f_k. We assume that $\Pr(p = p_j) > 0$ for each of $j = 1, \cdots, k$.* The yield spread distribution can be equivalently represented as a $(k-1) \times 1$ vector $h = (h_1, \cdots, h_{k-1})$ of the so called hazard rates, where $h_j = f_j / (f_j + \cdots + f_k)$, $j = 1, \cdots, k-1$ Let us further define $h_0 = 0$ and $h_k = 1$ for later use.

The hazard rate is the most important concept in life-time, mortality, and duration analyses. It is the conditional probability of "death" given survival up to now. To understand this concept, imagine

* In the empirical implementation, we only consider those support points p_j's where bids are ever made in the historical data, justifying the assumption $\Pr(p = p_j) > 0$ for each of $j = 1, \cdots, k$.

that you are conducting an "ascending yield" auction. Define death as the end of the auction process. The auction ends as soon as the total amount of bids exceeds the fixed total supply. Hazard rate at a given yield spread denotes the chance that the game ends exactly at that yield spread level conditional on that the auctioneer has just announced that level after passing all the previous levels. At a low enough yield spread level, say p_0, no bidder would bid. According to the common belief, the chance that the auction process ends at that low level is zero, $h_0 = 0$. On the other hand, suppose you have already reached a highest possible yield spread level, say p_k. At p_k, every bidder would bid so far as the bid is beneficial from her own perspective. According to the common belief, the chance that the auction process ends immediately at that high level is one, resulting in $h_k = 1$.

Let i index individual bidders in a given discriminatory auction case, $i = 1, \cdots, n$. Bidder i submits up to a maximum of 5 pairs of price and quantity, (p_{ij}, Δ_{ij}), $j = 1, \cdots, m_i$ where m_i is the number of bids submitted by bidder i. Of course, by regulation, $m_i \in \{1,2,3,4,5\}$. From $\{p_{ij}, \Delta_{ij}\}_{j=1,\cdots,m_i}$, one can construct bidder i's individual bidding function as a step function with m_i steps. Let $b_i(p_j)$ be the amount of bid submitted by bidder i at or below the yield spread p_j and denote it simply as b_{ij}. Do not forget that we are measuring the "price" using the yield spread which is inversely related with the bond price. We have $b_i(p_j) = \sum_{j':p_{ij'} \le p_j} \Delta_{ij'}$, Let $b_i(p) = b_i(p_j)$ for $p_j \le p < p_{j+1}$ ($j = 1, \cdots, k-1$), $b_i(p) = 0$ for $p < p_1$ and $b_i(p) = b_i(p_k)$ for $p \ge p_k$. It represents the demand function of bidder i. By summing the individual demand functions, we have the market demand function, $B(p) = \sum_{i=1}^{n} b_i(p)$.

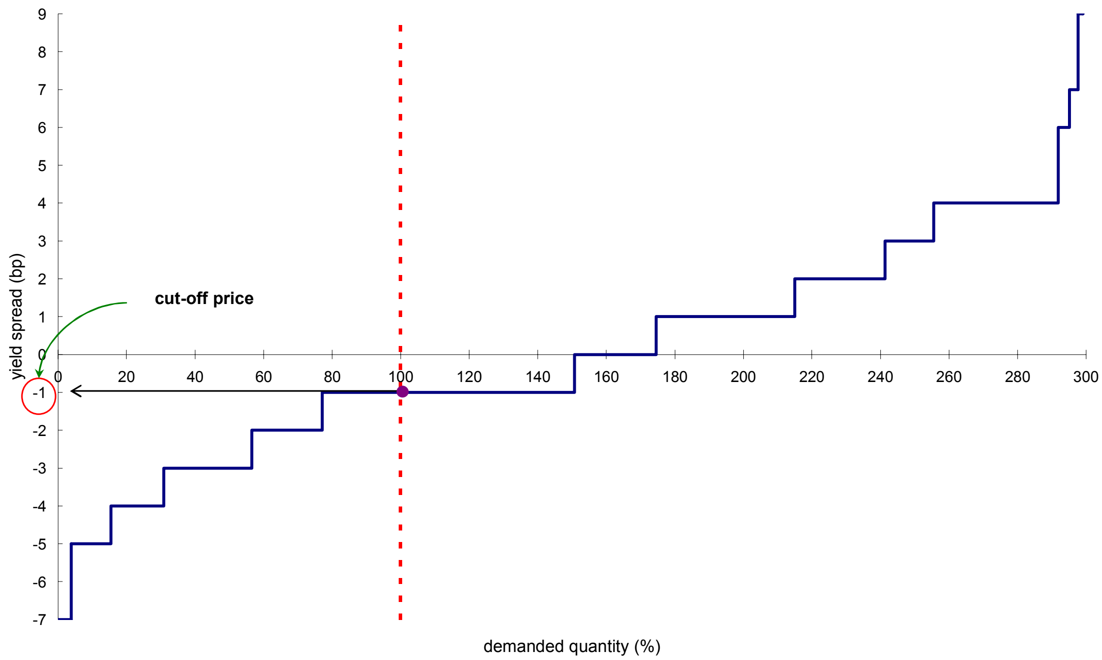
Let us normalize to one the total (fixed) supply of the treasury bonds in a given auction. Accordingly, we represent all the quantities as a fraction of the total supply. The cut-off yield spread is determined from solving $B(p) = 1$. In fact, due to discrete nature of the bidding, the market clearing yield spread is determined as the minimum price among the elements of p_j 's in the set $\{p_j \mid B(p_j) \geq 1\}$.

As we observe $\{p_{ij}, \Delta_{ij}\}$ for $j = 1, \cdots, m_i$, $i = 1, \cdots, n$, we can compute the market demand function, the market clearing price, and the auction revenue under discriminatory auction. Remember that p_{ij} 's are spreads (in terms of 1 basis point) between the bidding yield and the market yield in the secondary market. That is, $p_{ij} = -2$ denotes that the bidding yield is 2 basis point lower than the market yield for the same class of bonds (or close substitutes). The Δ_{ij} is the amount of bid, as a percentage fraction of the total supply, that bidder i bids at the bidding yield spread p_{ij} . For example, $\Delta_{ij} = 10$ (%) denotes that the quantity bid by bidder i at the bidding yield spread p_{ij} is 10% of the total supply.

Figure 1 illustrates how the equilibrium cut-off yield spread is determined using the micro-level bidding data on the discriminatory auction which occurred on July 18, 2000. (A full set of micro level bidding data is available upon request.)

Figure 1: Market demand function and cut-off price

(discriminatory price auction on July 18, 2000)



Now, suppose that you have derived individual demand functions under the counter-factual uniform price auction. Let $\{s_i(p)\}_{i=1,\cdots,n}$ be a collection of such counter-factual demand functions constructed for each individual who has participated in the discriminatory auction. (Here we are implicitly assuming that bidder composition does not change as the auction mechanism changes, which assumption sounds a bit strong. We will revisit this issue later.) By summing the individual demand functions, we have the market demand function, say $S(p)=\sum_{i=1}^{n} s_i(p)$. Under the hypothetical uniform price auction, the market clearing price would have been determined as the minimum price among the elements of p_j's in the set $\{p_j \mid S(p_j) \geq 1\}$.

In sections 2.2 through 2.4, we would like to characterize $b_i(p)$ and $s_i(p)$, and their relationship. This relationship will turn out most important when comparing auction revenues later on.

3.2. Discriminatory price auction

Assume that individual bidders enjoy private values from the treasury bonds, that the private values are not affiliated (Milgrom and Weber 1982), and that individual bidders do not strategically interact. Let $d_i^{-1}(q)$ be the marginal value to bid i arising from securing one additional share of the Treasury bond when she has already secured q. Under the assumptions, we neither put any functional form restrictions across different $d_i^{-1}(q)$'s nor solve the individual bidding behaviors considering strategic interactions (the same setting as in Lengwiler 1998, for example).

Consider the "price" of a treasury bond (with face value of one) which pays coupons at the prevailing yield. Let d be the duration of the bond and p the yield using one basis point (=10^{-4}) as the measurement unit. (The same p now denotes yields, not the yield spreads. Here we are obviously abusing notations for the purpose of notational simplicity.) Using a linear approximation, we can approximate the bond's price as "$\text{price} = 1 - d \times p_j \times 10^{-4}$."

Bidder i determines the optimal bidding strategy $\{b_{ij} = b_i(p_j)\}_{j=1,\cdots,k}$ to maximize the expected profit

$$\max_{b_{i1},\cdots,b_{ik}} \sum_{j=1}^{k} f_j \left[\int_0^{b_{ij}} d_i^{-1}(q)\,dq - \sum_{j'=1}^{j} (1 - 10^{-4}\, d\, p_{j'})(b_{ij'} - b_{ij'-1}) \right],$$

where $\text{price}_{j'} = 1 - 10^{-4}\, d\, p_{j'}$, $\Delta_{ij'} = b_{ij'} - b_{ij'-1}$, $b_{i0} = 0$ and "no rationing at the margin" are used.

In fact, this optimization is a bit different from the treasury auction practices in Korea where participants are allowed only up to a maximum of 5 bids which are in general smaller than k. We do not think ignoring this difference would bias our empirical results. See Appendix Table A1 for the number of bids per bidder.

By solving the k first-order conditions, we have

$$f_j[d_i^{-1}(b_{ij}) - p_j^*] = (p_j^* - p_{j+1}^*)\sum_{j'=j+1}^{k} f_{j'} \iff b_{ij} = d_i[p_j^* + (p_j^* - p_{j+1}^*)(1/h_j - 1)],$$

where we have used (i) $h_j = f_j/(f_j + \cdots + f_k)$, (ii) $f_j \neq 0$, and (iii) the definition of $p_j^* = 1 - 10^{-4} d\, p_j$. Note that the general solution takes the form $b_{ij} = b_i(p_j) = d_i(p_j^* + \delta_j^*)$ with $\delta_j^* = (p_j^* - p_{j+1}^*)(1/h_j - 1) \geq 0$.

Not to cause confusion to the readers, let us comment on the usage of notations in the rest of the paper. We are moving back and forth between the unit bond prices and the bond yields. Measurements in terms of unit bond prices are denoted with an asterisk (*), and measurements in terms of bond yields are denoted without an asterisk. For example, p_j^* and δ_j^* are measured in bond prices, whereas p_j and δ_j (as to be defined shortly) in bond yields.

The solutions allow nice economic interpretations. First, we observe that individual bidders shade their bids in the sense that their actual bids are smaller than the "truth-revealing" bids, $b_i(p_j) = d_i(p_j^* + \delta_j^*) \leq d_i(p_j^*)$. As is well known, the reason for bid shading here under the discriminatory price auction is essentially the same as bid shading in the first price sealed bid auction.

What is less well known is that it is also similar to "shirking" in a typical principal-agent model. The solution has exactly the same form if you map bidding to effort level, and cut-off yield distribution to agent type distribution. "What you pay" is the bidding yield in the case of the discriminatory auction and the effort level in the case of agent models. "What you get," however, is not exactly one for one. Rather a fraction of what you pay, resulting in bid shading and effort shirking. Under the discriminatory auction, if a bidder believes that she still has a chance to secure an additional share of the bond, she faces an incentive to under-bid, that is, to shade. To save on payment, she bids less so far as chances are there, resulting in bid shading.

Second, only at p_k shading does not occur, $b_i(p_k) = d_i(p_k^* + \delta_k^*) = d_i(p_k^*)$ since $h_k = 1$ and thus $\delta_k^* = 0$. According to the belief about the cut-off yield, you have already reached the maximum possible yield level at p_k. There is absolutely no chance that the market clearing yield further goes up passing p_k Knowing this, bidders face no incentive to shade. Bidder i submits a bid if she wants an additional share in the sense that $d_i^{-1}(q) \geq p_k^*$, and not otherwise, leading her to bid $d_i(p_k^*)$ at p_k. At each "candidate" yield level, the bidder ask herself, "Would the ascending auction further go up?" If the answer were positive, she would shade. Otherwise, she would reveal the truth.

Third, shading depends on your belief about the market clearing yield level. As well known in the literature, there exists one-to-one relationship between $(h_1, \cdots, h_k)$ and $(f_1, \cdots, f_k)$. Let me explain this relationship using the previous example of ascending auction. For the market clearing yield to be determined at level p_j, the ascending auction process should not have stopped at each of the previous yield levels p_1 through p_{j-1}, and then it should stop immediately at the current yield level p_j. Thus, the probability that the market clearing yield will be p_j is the product of the initial marginal probability and the subsequent conditional probabilities, leading to $f_j = (1-h_1) \cdot \cdots \cdot (1-h_{j-1}) \cdot h_j$.

Again, imagine that you are attending an ascending yield auction and that you are now at "node" p_j. Based on your belief, if you are sure that the yield will be determined at the current level and thus you are in the terminal node $(j = k)$, then you face no incentive to shade. On the other hand, if you believe that the current node may not be the terminal node with positive probability $(j < k)$, you face an incentive to save money by under-bidding, that is, by bid-shading.

To shade or not, and how much to shade if you do, really depends on the relative strength of these two opposing forces. At p_j, you believe that this yield is the market clearing level with strength proportional to f_j, and you believe that this yield will further go up with strength proportional to $f_{j+1} + \cdots + f_k$. The relative strength of these two forces is nothing but $1/h_j - 1 = (f_{j+1} + \cdots + f_k)/f_j$. As you see from the equilibrium bidding function $b_i(p_j) = d_i(p_j^* + \delta_j^*)$, the amount of shading $d_i(p_j^*) - d_i(p_j^* + \delta_j^*)$ is increasing in δ_j^*, which is equal to $(p_j^* - p_{j+1}^*)(1/h_j - 1)$, and thus increasing in $1/h_j - 1$ and decreasing in h_j.

At each yield level, say p_j, the degree of shading really depends on h_j, which is the hazard rate at that yield level. This hazard measures the strength with which you believe that the market clearing yield will be determined at the current level without going up any further (conditional on that the auction process has already reached that level). Shading would be depressed as you believe more strongly that the current level is the final node, and it would be encouraged as you believe more strongly that the market clearing yield would further go up.

3.3 Uniform price auction

Unlike in the discriminatory auction, you do not pay what you bid. Rather, you pay what other participants bid “at the margin.” This aspect of the uniform price auction is quite similar to the second price sealed bid auction, resulting in “truth revealing” in both cases. The Korean treasury auction market is highly competitive as there are many market participants and as none of them has dominant market power. A bidder believes that she can influence neither other bidders' bidding behaviors nor the market clearing price, a reasonable description of the Korean treasury auction market.

At all yield levels, bidders now do not face any incentive to shade. Bidder i bids if she wants an additional share in the sense that $d_i^{-1}(q) \geq p_j^*$, and not otherwise, leading her to bid truthfully $d_i(p_j^*)$ at p_j. Recall that in the case of the discriminatory auction, bidders only reveal the truth when they are 100% sure that there is absolutely no chance that the market clearing yield will further increase (that is, only at level p_k). Now, in the case of the uniform price auction, bidders reveal truth at all price levels $p_1, \cdots, p_k$.

Of course, we can verify the above heuristics by formally solving the bidders' optimization problem. Let p^s be the market clearing yield under the uniform price auction. Bidder i determines the optimal bidding strategy, say $\{s_{ij} = s_i(p_j)\}_{j=1,\cdots,k}$ to maximize the expected profit.

$$\max_{s_{i1},\cdots,s_{ik}} \sum_{j=1}^{k} \Pr(p^s = p_j)\left[\int_0^{s_{ij}} d_i^{-1}(q)\,dq - (1-10^{-4}d\ p_j)s_{ij}\right],$$

where "$1-10^{-4}d\ \ p_j$," say p_j^*, is again the unit bond price corresponding to the yield level of p_j.

By solving the first-order conditions, we have

$$\Pr(p^s = p_j)\left[d_i^{-1}(s_{ij}) - p_j^*\right] = 0 \iff s_{ij} = d_i(p_j^*) \text{ for all } j = 1,\cdots,k.$$

Note that the solution takes the form $s_i(p_j) = d_i(p_j^*)$, $j = 1,\cdots,k$, that beliefs about the market clearing yield do not play any role, that bidders always reveal truth, and that there arises no shading at any yield level.

3.4 Recovering uniform price bidding from discriminatory price bidding

Through the above two sub-sections, we have characterized the equilibrium bidding strategies under each of the discriminatory auction and the uniform price auction. Bidders shade their bids under the discriminatory auction, whereas they do not under the uniform price auction. The degree of shading is determined by the bidders' belief about the market clearing yield under the discriminatory auction.

Once we identify the bidders' common belief about the market clearing yield under the discriminatory auction, then we can recover their counter-factual bidding functions under the uniform price auction.

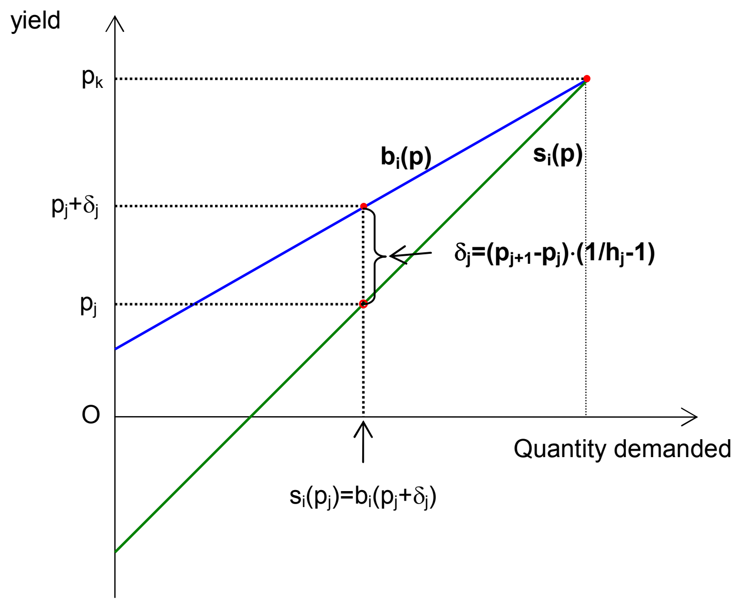
In the foregoing sub-sections, by implicitly assuming that bidder composition does not change as the auction mechanism changes, we have not addressed an important issue whether the set of auction participants grows or shrinks. It is arguably agreed that auction participants increase under the uniform price auction. It is because the uniform price auction is more friendly to individual participants in that individuls when making bids do not have to worry about the uncertainty in the cut-off yield level. Individuals are more likely to join the treasury auction (of course, through their primary dealers) under the uniform price auction mechanism. If that is the case, our analysis, by assuming that auction participants do not change, would underestimate the market demand under the uniform price auction, and thus underestimate the auction revenue increase resulting from switching to the uniform price auction.

Using the results in sections 2.2 and 2.3, we derive the relationship.

$$b_i(p_j) = d_i(p_j^* + \delta_j^*)\,,\ \ s_i(p_j) = d_i(p_j^*) \Rightarrow s_i(p_j) = b_i(p_j + \delta_j)\,,$$

where $\delta_j = 10^4(\delta_j^*/d) = (p_{j+1} - p_j)(1/h_j - 1)$. Figure 2 shows this relationship.

Figure 2: Bidding functions: discriminatory vs uniform



From the relationship, we observe that the degree of shading can be represented in two alternative terms: either by δ_j^* using bond prices as the measurement unit or by δ_j using bond yields as the measurement unit. Let me explain bid shading using the latter term. It is a product of the following two terms. The first term, $p_{j+1} - p_j$, measures how much you gain in terms of the bond yield when you wait for another "node," and the second term, $1/h_j - 1$, measures the relative strength of your belief with which you believe you would reach the next node rather than "burst."

Once we secure information on $(h_1, \cdots, h_{k-1})$ representing the bidders' common belief about the market clearing yield under the discriminatory auction, we can recover the counter-factual bidding function $s_i(p)$ from the observed bidding function $b_i(p)$.

Let $m_i (m_i \leq 5)$ be the number of the bids submitted by bidder i in the discriminatory price auction, and $\{p_{ij}, \Delta_{ij}\}$ be those pairs, $j = 1, \cdots, m_i$, $i = 1, \cdots, n$. Of course, $p_{ij} \in \{p_1, \cdots, p_k\}$ by construction. We have

$$s_i(p_j) = b_i(p_j + \delta_j) = \sum_{j': p_{ij'} \le p_j + \delta_j} \Delta_{ij'}$$

It is the "derived" bidding function of bidder i under the counter-factual uniform price auction. By summing the individual demand functions, we can also derive the hypothetical market demand function, $S(p_j) = \sum_{j=1}^{n} s_i(p_j)$.

The market clearing price will be determined as the minimum p_j among the elements of the set $\{p_j : S(p_j) = \sum_{i=1}^{n} s_i(p_j) \ge 1\}$. Once we compute the market clearing price, we can compute the auction revenue in the hypothetical uniform price auction. Thus, we can compute the percentage auction revenue increase which one enjoys by switching the auction mechanism from the discriminatory auction to the uniform price one.

Depending on the amount of uncertainty regarding the market clearing yield spread, it may or may not pay to switch to the uniform price auction from the discriminatory one. It is an empirical issue after all.

4. Data, Econometric Model

The data are micro-level, individual bidding data for the recent discriminatory auction cases held in the Korean treasury auction market. Table 1 shows basic characteristics of each auction analyzed in this paper. Out of a total of 16 cases in the sample, 10 are taken consecutively from September 6, 1999 to January 10, 2000, and the remaining 6 from May 15, 2000 to July 18, 2000. During the in-between period, discriminatory auctions were used with variable supply. Variable supply itself would make bidders shade. Not to confound the effect of discriminatory/uniform price auctions with the effect of fixed/variable supplies, we exclude these interim auction cases with variable supply.

Table 1: Background information on each of 16 discriminatory auctions

auction date	cut-off yield (bp)	market yield (bp)	maturity (year)	duration (year)	total supply (billion won)
19990906	865	850	1.00	0.97	1195.16
19990913	944	930	2.65	2.65	1196.19
19990928	989	975	4.01	4.01	790.40
19991004	839	836	1.00	0.97	1164.90
19991011	841	837	2.68	2.68	1357.40
19991018	938	939	4.05	4.05	798.30
19991108	811	807	1.00	0.97	776.30
19991115	838	832	2.68	2.68	1184.90
19991206	870	869	2.67	2.67	349.80
20000110	906	902	1.00	0.97	738.67
20000515	929	929	3.79	4.05	279.60
20000605	823	828	1.00	0.97	287.20
20000612	865	863	2.51	2.67	578.70
20000619	902	901	3.79	4.05	767.90
20000710	795	790	2.45	2.67	586.00
20000718	815	816	4.07	4.16	778.00

From the discussions in section 2, we readily notice that the whole revenue comparison boils down to an issue of estimating uncertainty surrounding the market clearing yield spread. One may think of several approaches. First, use an empirical distribution of the historically observed yield spreads under the 16 discriminatory auction cases. Yield spreads are defined as the differences between the observed cut-off yields and the yields in the secondary market at morning of the auction date.

The first approach is simple, but naive. Market clearing yield spreads may depend on a number of variables. Here comes the second approach. Second, use a more sophisticated method. For this purpose, run a multiple regression of the yield spread, defined as the cut-off yield minus the secondary market yield, on a constant, maturity dummies, year dummies, number of bidders, and the auction size (face value).

Once you estimate the regression coefficients (reported in Appendix Table A2), compute the 16 residual terms (graphed in Appendix Figure A1). Then, we approximate the yield spread distribution as the empirical distribution function of these 16 residuals, shifted to the right by a relevant regression function. The regression function is obtained by combining the coefficient estimates with the current auction characteristics. We use this sophisticated approach in this paper. (The results were basically the same when we alternatively used the empirical distribution of the yield spreads historically observed, shifted to the right by the current secondary market yield.)

Given the historically observed yield spreads, we would like to estimate $h=(h_1,\cdots,h_{k-1})$. For this purpose, let $p_{(1)}\le\cdots\le p_{(16)}$ be those 16 realizations of the yield spread, that is, the 16 residuals shifted to the right by the regression function. We measure them using 1 basis point as the measurement unit after approximating them upto the nearest integer values. Let k be the number of the distinctive elements among the $p_{(j)}$'s. Let us take these distinctive elements as $\{p_1,\cdots,p_k\}$, the support set of the market clearing yield spread. Let $f_1,\cdots,f_{k-1}$ be the empirical frequencies of $p_{(j)}$'s which are equal to $p_1,\cdots,p_{k-1}$. We estimate $h=(h_1,\cdots,h_{k-1})$ using $h_j=f_j/(f_j+\cdots+f_k)$, $j=1,\cdots,k-1$.

Often, it is tempting to impose monotonicity on $h=(h_1,\cdots,h_{k-1})$ that $h_1\le\cdots\le h_{k-1}$ holds. Imposing this monotonicity assumption is useful for the following two reasons. First, it will smooth out the empirical hazard estimates. Second, it is a priori reasonable to assume that the hazard rate of the "ascending yield" auction increases as the yield spread level further goes up.

Estimating the empirical hazard rates under this assumption is easy using the so called "moving to the right" idea. Let us explain this idea. You first estimate h_j's as above, $h_j = f_j /(f_j + \cdots + f_k)$. If the estimated hazard rates satisfy the monotone hazard property, stop. If not, search for the yield spread level sequentially from the lowest where the monotonicity breaks down for the first time. Let p_j be such level, that is, $h_1 \leq \cdots \leq h_{j-1} > h_j$. You move one observation observed at p_{j-1} to the right such that it behaves as if it were observed at p_j. As a result of this "moving to the right," f_{j-1} decreases by one whereas f_j increases by one. Accordingly, h_{j-1} decreases whereas h_j increases. Repeat this moving to the right procedure until you have $h_{j-1} < h_j$.

When the above step is over, you might have disturbed the previously holding inequality. If you happen to see $h_{j-2} > h_{j-1}$, you move one observation observed at p_{j-2} two steps to the right such that it behaves as if it were observed at p_j. Moving one step to the right to p_{j-1}, would cause $h_{j-1} > h_j$, so this move is ruled out. As a result of this "moving to the right," f_{j-2} decreases by one whereas f_{j-1} stays the same and f_j increases by one. Repeat this "moving to the right" until you have $h_{j-2} \leq h_{j-1} \leq h_j$.

When the above second step is over, you might have disturbed the previously holding inequality. If you happen to see $h_{j-3} > h_{j-2}$, you move one observation observed at p_{j-3} to the right such that it behaves as if it were observed either at p_{j-1} or at p_j. Think again why moving to p_{j-2}, one-step to the right, is ruled out. (It is because it would have caused $h_{j-2} > h_{j-1}$.) Repeat this procedure until $h_{j-3} \leq h_{j-2} \leq h_{j-1} \leq h_j$ holds. In this process, note the following two points. First, in a given move, you do not want to move further to the right unless necessary. In the previous step, you would stop moving at p_{j-1} rather than further advancing to p_j unless needed. Second, you stop the process of moving to the right immediately when you have $h_1 \leq \cdots \leq h_j$.

Then, you search for a new yield spread at which the monotonicity breaks down (again for the first time), and repeat the whole procedures as explained above. Finally, you have $h_1 \leq \cdots \leq h_{k-1}$. As we have mentioned above, these hazard estimates are advantageous in that they are smoother, and that they satisfy an a priori appealing monotonicity property.

So far we have explained an approach to deriving the common belief about the cut-off yield level. Other approaches include (i) introducing GARCH type models into the error terms in the above regression approach, and (ii) estimating the yield spread distribution using information contained in the yields themselves and/or interest derivative products. Adding GARCH idea to the above procedure should be easy. Extracting additional information from the interest derivative products, should be a bit involved.

For a given discriminatory auction case, once you estimate the percentage revenue difference, you want to compute its sampling error. There are two sources of the sampling error. One is the uncertainty arising from estimation of the yield spread distribution. The other is the uncertainty arising from who participate in a given auction.

First, recalling that we have estimated this distribution using the 16 cases, we would like to measure this uncertainty by applying Bootstrap re-sampling techniques to those 16 cases. More concretely, out of 16 integers, 1 through 16, you select a set of 16 numbers through random sampling with replacement. Then, by using the auction cases corresponding to these 16 selected numbers, you re-estimate the yield spread distribution. Using the re-estimated spread distribution, re-estimate the percentage revenue difference. Then, repeat the whole procedures. This way, you can generate as many percentage revenue differences as you wish.

Second, given an estimate of the yield spread distribution, it is the set of the participating individual bidders who determine the percentage revenue difference. We also would like to address this second source of uncertainty by using the Bootstrap re-sampling techniques. We will re-sample the same size of the bidders from the original set of the bidders through sampling with replacement. More concretely, out of integers, 1 through n, you select a set of n numbers by sampling randomly with replacement. Then, by using the bidders corresponding to these selected numbers, you re-estimate the percentage revenue difference. Then, repeat the whole procedures. This way, you can generate as many percentage revenue differences as you wish.

To sum, we can measure the sampling uncertainty using the Bootstrap re-sampling techniques. There are two sources of sampling uncertainty. One type of uncertainty lies in estimating the yield spread distribution. The other, in sampling (drawing) individual bidders. Of course, we want to consider both sources of uncertainty jointly as well as separately.

To address both sources of uncertainty, you apply the re-sampling scheme at both stages. In a single run, you first re-sample the 16 auction cases. Estimate the yield-spread distribution. Re-sample the set of bidders. Then, finally by combining the estimated yield distribution with the set of re-sampled bidders, you come up with an estimate of the percentage revenue difference. Repeat the whole procedure as many times as you wish.

To address only the uncertainty in the yield spread estimates, you only apply the re-sampling scheme at the first stage. In a single run, you first re-sample the 16 auction cases. Estimate the yield-spread distribution. (Use the original set of bidders. Do not apply re-sampling at the second stage.) Then, by combining the estimated yield distribution with the set of the original bidders, you come up with an estimate of the percentage revenue difference. Repeat the whole procedure as many times as you wish.

Finally, to address only the uncertainty in the set of the participants in the auction, you only apply the re-sampling scheme at the second stage. (Use the 16 historically observed yield spreads to estimate the yield spread distribution. That is, do not apply re-sampling at the first stage.) In a single run, you use the yield-spread distribution estimated from the original 16 auction cases. You stick to this estimate throughout the replications. Only re-sample the set of bidders. Then, by combining the original yield distribution with the set of re-sampled bidders, you come up with an estimate of the percentage revenue difference. Repeat the whole procedure as many times as you wish.

5. Results

The Korean treasury auction market is highly competitive as there are 30 potential participants none of whom possesses dominant market power. Short squeeze does not arise as there is no "when-issued market" in the Korean treasury auction market. Lack of when-issued markets, though, increases uncertainty facing the cut-off price.

Tables 2 to 4 show the estimation results obtained under monotonicity assumption imposed on the hazard etimates. Results obtained without imposing the monotonicity assumption, are basically the same (available upon request). The auction carried out on Dec. 6, 1999 is much smaller in size. The results are basically the same whether we include or exclude this case. Also, regarding those 6 discriminatory auction cases carried out between May 15, 2000 and July 18, 2000, our results are robust to inclusion/exclusion of these cases.

Our empirical results show that uniform price auctions, had they been implemented, would have increased auction revenues during the sample period when the Korean government in fact used discriminatory auctions. Judging from Bootstrap re-sampling standard errors, we have established evidences that the uniform price auction, had it been adopted in Korea back in the years 1999 and

2000, would have increased auction revenue for 11 cases out of a total of 16 cases with an average of 0.13 % revenue increase.

To sum, the uniform price auction is revenue enhancing in the Korean treasury auction market, and thus the Korean government's switch to the uniform price auction in August 2000 was a right policy choice.

Table 2: Percentage revenue difference between discriminatory and uniform auctions (both yield-spreads and bidders re-sampled; monotone hazard imposed)

Auction Date	Mean(%)	Median(%)	SE(%)	maximum (%)	minimum (%)
19990906	0.05	0.05	0.02	0.10	-0.10
19990913	0.15	0.15	0.06	0.31	-0.25
19990928	0.36	0.38	0.09	0.54	-0.23
19991004	0.00	0.00	0.00	0.01	0.00
19991011	0.00	0.00	0.00	0.01	0.00
19991018	0.00	0.00	0.01	0.12	0.00
19991108	0.03	0.03	0.01	0.05	-0.01
19991115	0.05	0.05	0.01	0.09	0.00
19991206	0.05	0.02	0.08	0.33	-0.32
20000110	0.06	0.06	0.01	0.07	-0.05
20000515	0.31	0.32	0.04	0.41	0.11
20000605	0.00	0.00	0.00	0.02	0.00
20000612	0.09	0.09	0.03	0.17	-0.06
20000619	0.13	0.14	0.02	0.19	0.00
20000710	0.18	0.18	0.03	0.24	-0.07
20000718	0.07	0.07	0.03	0.17	0.00

Table 3: Percentage revenue difference between discriminatory and uniform auctions (only yield-spreads are re-sampled; monotone hazard imposed)

auction date	Mean(%)	Median(%)	SE(%)	maximum (%)	minimum (%)

19990906	0.04	0.05	0.02	0.06	-0.10
19990913	0.15	0.16	0.03	0.17	-0.20
19990928	0.37	0.39	0.08	0.45	-0.22
19991004	0.00	0.00	0.00	0.00	0.00
19991011	0.00	0.00	0.00	0.00	0.00
19991018	0.00	0.00	0.00	0.00	0.00
19991108	0.03	0.03	0.01	0.03	0.00
19991115	0.05	0.04	0.01	0.06	0.00
19991206	0.02	0.02	0.00	0.03	0.00
20000110	0.06	0.06	0.01	0.06	-0.05
20000515	0.30	0.32	0.04	0.41	0.12
20000605	0.00	0.01	0.00	0.01	0.00
20000612	0.08	0.09	0.02	0.11	-0.01
20000619	0.13	0.14	0.02	0.20	0.00
20000710	0.18	0.19	0.02	0.22	-0.01
20000718	0.07	0.08	0.02	0.10	0.00

Table 4: Percentage revenue difference between discriminatory and uniform auctions

(only bidders are re-sampled; monotone hazard imposed)

auction date	Mean(%)	Median(%)	SE(%)	maximum (%)	minimum (%)
19990906	0.03	0.04	0.01	0.06	0.00
19990913	0.17	0.16	0.04	0.32	0.08
19990928	0.40	0.40	0.04	0.51	0.29
19991004	0.00	0.00	0.00	0.02	0.00
19991011	0.00	0.00	0.00	0.01	0.00
19991018	0.00	0.00	0.01	0.10	0.00
19991108	0.03	0.03	0.01	0.05	0.02
19991115	0.06	0.06	0.01	0.09	0.03
19991206	0.06	0.02	0.08	0.40	0.00
20000110	0.06	0.06	0.00	0.08	0.04
20000515	0.33	0.32	0.01	0.37	0.32
20000605	0.01	0.01	0.00	0.02	0.00

20000612	0.10	0.10	0.02	0.17	0.02
20000619	0.15	0.14	0.01	0.18	0.12
20000710	0.19	0.19	0.01	0.24	0.16
20000718	0.09	0.09	0.03	0.19	0.02

6. Concluding Remarks

In this paper, we compare auction revenues across the two different auction mechanisms, discriminatory vs uniform price auctions, using a structural approach. The auction revenue difference critically depends on the hazard function estimates of the market clearing yield. We have estimated the hazard rates using the historically observed auction yield data, adjusted for several factors such as secondary market rates, maturities, years, number of participants, and the auction size. Using these historical data set, we estimate the hazard rates with and without imposing the monotone hazard assumptions. We believe that monotone hazard property makes senses, and that imposing it reduces the sampling uncertainty.

We measure sampling uncertainty using the Bootstrap re-sampling methods. We address the sampling uncertainty arising from the hazard estimates as well as the sampling uncertainty arising from who joins the auction. We address these two types of uncertainty separately as well as jointly.

This paper theoretically has clarified the role of the hazard rates in the discriminatory auction by comparing it to a typical principal agent model, and empirically has offered new ways of estimating the hazard rates. This paper has also suggested the use of Bootstrap re-sampling techniques to address sampling uncertainty from two different sources. Using the re-sampling method, we can identify these two types of uncertainty separately.

This research leaves room for improvements, though. As the auction scheme changes, participants may change. For example, as the auction mechanism switches from the discriminatory auction to the uniform price one, it is expected that more would participate in the auction. If so, our analyses based on "no change" assumption of the participants, would underestimate the revenue increase resulting from the switch. In this paper, as we have obtained such results that the uniform price auction increases the auction revenue under the "no participant change" assumption, our results would only have been strengthened if we had considered auction participation decision as well.

This paper has not formally considered the auction participation decision. Theoretical as well as empirical analyses of the auction participation decision, would be interesting, and are left for future research.

In this paper, we have assumed that all the auction participants have the same belief about the market clearing cut-off yields, and additionally that this belief is well approximated by the distribution of the historically observed cut-off yields once adjusted for factors like secondary market rates, maturities, years, number of participants, and the auction size. It would be interesting to see how the results change as one uses different distributions.

In this paper, we have explained how to recover bidding functions under the uniform price auction from those under the discriminatory price auction. One can apply the similar techniques to solve the reverse problem, that is, to derive bidding functions under the discriminatory auction from those under the uniform price auction. As the Korean government has switched to the uniform price auction from August 2000, we can also compare auction revenues across different auction mechanisms using the observed uniform price auction data. These results will be reported in a separate paper.

Using micro-level auction data, one can potentially think of three different approaches to measuring the percentage revenue difference between the uniform price and the discriminatory auctions.

First, "discriminatory to uniform." This is an approach we have adopted in this paper. Given individual bids under the discriminatory auction, we recover individual bids under the counter-factual uniform price auction.

Second, "uniform to discriminatory." Given individual bids under the uniform price auction, we recover individual bids under the counter-factual discriminatory auction.

Third, we can compare historically observed auction revenues (or auction cut-off yields) from two different auction mechanisms. For this purpose, one may simply run a multiple regression of (auction revenue/face value) (or auction cut-off yield) on a constant, uniform price auction dummy, maturity dummies, year dummies, number of bidders, auction size, and the secondary market yield. Then, test the statistical significance of the coefficient of the uniform price auction dummy. If it turns out positive and statistically significant, then the uniform price auction is more revenue enhancing relative to the discriminatory auction and vice versa. (The opposite is true if one uses the cut-off yield as the dependent variable in the regression).

Among the three approaches, the first two are structural in nature in the sense that one has to use a theoretical model to derive the counter-factual individual bids from the observed ones. The third is purely statistical and reduced-form in nature in the sense that one does not need any theoretical model.

In terms of data requirement, the first two are less demanding as they only require data from one auction mechanism, discriminatory or uniform price. The third requires data from both auction mechanisms. In terms of "statistical control," the first two approaches are advantageous. It is because, in the first two approaches, an auction case is compared to itself, eliminating the need for statistical control. However, the first two approaches critically depends on the theoretical model used and also on the empirical estimates of the cut-off yield distribution.

In the third approach, one discriminatory auction is compared with another uniform price auction. These two auctions are different not only in terms of the auction mechanism but also in many other aspects such as maturities, years, number of bidders, auction sizes, secondary market rates at morning of the auction day, interest rate expectations, yield uncertainties, financial and macro shocks, and many other factors. Observable differences are controlled to a certain extent by including them as regressors in the multiple regression equation. However, it is simply impossible to control even the major differences in an adequate way, let alone all the differences.

We leave it as a future research to compare revenue differences across these three different approaches. Specifically we would like to address issues like (i) whether results from the structural approaches and results from the reduced-form approach would agree, and (ii) whether the two structural approaches would yield mutually consistent results.

References

[1] Back, K and J. Zender (1993), "Auctions of divisible goods : On the rationale for the Treasury experiment," *Review of Financial Studies*, vol. 6, 733-764.

[2] Bank of Korea (2000), “The Korean Treasury Bonds Market: Future Prospects and Current Tasks” (in Korean).

[3] Bikchandani, S., and C. Huang, (1989), “Auctions with Resale Markets: An Exploratory Model of Treasury Bill Markets,” *Review of Financial Studies*, vol. 2, 311-339.

[4] Bikchandani, S. and C. Huang (1993), “The Economics of Treasury Securities Markets,” *Journal of Economic Perspectives*, vol. 7, 117-134.

[5] Cammack, E.B. (1991), “Evidence on bidding strategies and the information in Treasury bill auctions,” *Journal of Political Economy*, vol. 99, 100-130.

[6] Chari, V. and R.J. Weber (1992), "How the U.S. Treasury Should Auction Its Debt," *Federal Bank of Minneapolis Quarterly Review*, vol. 16, no.4.

[7] Chatterjea, A. and R. J. Jarrow (1998), "Market Manipulation, Price Bubbles, and a Model of the U.S. Treasury Securities Auction Market," *Journal of Financial and Quantitative Analysis*, vol. 33(2), 255-289.

[8] Cottarelli, C. (1997), “Treasury Bill Auctions: Issues in Design,” in *Coordinating Public Debt and Monetary Management*(IMF), 190-208.

[9] Dattels, P. (1997), “Microstructure of Government Securities Markets”, in *Coordinating Public Debt and Monetary Management*(IMF), 209-282.

[10] Feldman, R.A. and R. Mehra (1993), “Auctions: Theory and Applications,” *IMF Staff Papers*, No.40, 485-511.

[11] Friedman, M. (1960), *A Program for Monetary Stability*, 1959, NY : Fordham University Press.

[12] Goldreich, D. (1998), “Underpricing in Treasury Auctions,” *IFA Working Paper 263*, London Business School.

[13] Gray, S. (1997), “Government Securities : Primary Issuance,” in *Handbooks in Central Banking*, Bank of England.

[14] Heller, D. and Y. Lengwiler(1998b),"The Auctions of Swiss Government Bonds : Should the Treasury Price Discriminate or Not ?", FRB.

[15] Hortacsu, A. (2002a), “Mechanism Choice and Strategic Bidding in Divisible Good Auctions : An Empirical Analysis of the Turkish Auction Market.”

[16] Hortacsu, A. (2002b), “Bidding Behavior in Divisible Good Auctions: Theory and Evidence from the Turkish Treasury Auction Market.”

[17] Kim, Gyung-Rok (2001), “The Impact of the Treasury Bonds Auction Techniques on the Auction Revenue: In the Case of the Korean Treasury Bonds Auction Market,” Ph.D. dissertation, Dept of Economics, Seoul National University (in Korean).

[18] Jegadeesh, N. and Y. Hamao (1997), “An analysis of Bidding in the Japanese Government Bond Auctions,” Working Paper.

[19] Kroszner, R. (1993), *An International Comparison of Primary Government Securities Markets*, Working paper, University of Chicago, Jan. 1993.

[20] Lee, S., Y. Ghaugh, K. Oh, A. Fayolle (1999), “Issuing and Managing the Treasury Bond,” World Bank FCRA Project (in Korean).

[21] Lengwiler, Y. (1998), “The Multiple Unit Auction with Variable Supply”, FRB.

[22] Malvey, P.F. and C.M. Archibald (1998), Uniform-Price Auctions: Update of the Treasury Experience. U.S. Treasury.

[23] Milgrom, P. and R.J. Weber (1982), “A Theory of Auctions and Competitive Bidding,” *Econometrica* 50, 1089-1122.

[24] Ministry of Finance and Economy (1999), “Rules Governing Primary Dealers in the Korean Treasury Bonds Auction Market,” The Korean Government (in Korean).

[25] Nautz, D. (1995), "Optimal bidding in multi-unit auctions with many bidders," *Economics Letters*, vol. 48 301-306.

[26] Nyborg, K. G. and Sundaresan, S. (1996), "Discriminatory versus Uniform Treasury Auctions : Evidence from When-Issued Transactions," *Journal of Financial Economics*, vol. 42(1), 63-104.

[27] Simon, D.P. (1994), "Markups, quantity risk, and bidding strategies at Treasury coupon auctions," *Journal of Financial Economics*, vol. 35, 43-62.

[28] Umlauf, S.R. (1993a), “An Empirical study of the Mexican Treasury bill auction,” *Journal of Financial Economics*, vol. 33, 313-340.

[29] Umlauf, S.R. (1993b), “Information Asymmetries and Security Market Design: An Empirical Study of the Secondary Market for U.S. Government Securities,” *Journal of Finance*, vol. XLVI, 929-953.

[30] Vickery, W. (1961), "Counter-speculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16, 8-37.

[31] Viswanathan, S. and J.D. Wang (1998), "Auctions with When-Issued trading: A Model of the U.S. Treasury Markets," U.S. Treasury.

[32] Wang, J. J. D. and J. F. Zender (1996), "Auctioning Divisible Goods," Working Paper, University of Utah.

[33] Wilson, R. (1979), "Auctions of Shares," *Quarterly Journal of Economics*, 93, 675-689.

Appendix

Table A1 shows the average and the standard deviation of the number of bids per bidder for each of the 16 discriminatory auction cases analyzed in this paper.

Table A1: Summary statistics on the number of bids per bidder

auction date (yyyymmdd)	average	SD
19990906	3.16	1.37
19990913	3.89	1.12
19990928	3.30	1.44
19991004	3.58	1.24
19991011	3.83	1.36
19991018	3.96	1.25
19991108	3.60	1.39
19991115	3.85	1.23
19991206	1.73	0.79
20000110	3.75	1.27
20000515	2.00	1.13
20000605	1.96	1.11
20000612	2.95	1.32
20000619	2.13	1.26
20000710	2.60	1.29
20000718	3.58	1.47
16 auctions pooled	3.16	1.45

Table A2 shows the results of regressing the cut-off yield spreads on several covariates using the 16 discriminatory auction data. Figure A1 is a histogram of the resulting residuals.

Table A2: Regression of the cut-off yield spread

explanatory variable	coefficient estimate	t-value
$d_{mat=1}$	-1.32	-0.18
$d_{mat=3}$	-0.56	-0.08
$d_{mat=5}$	-1.67	-0.20
D_{2000}	-3.46	-0.73
# of bidders	0.09	0.19
Auction size	0.00	0.65

※ regression equation: (cut-off yield spread) = α_1(maturity 1 year dummy)+α_2(maturity 3 year dummy)+α_3(maturity 5 year dummy)+α_4(auction year 2000 dummy)+α_5(number of bidders)+α_6(auction size)+(error)

Figure A1: Residuals from the cut-off yield spread regression