Endogenous Business Cycles with Consumption Externalities

Xiaopeng Yin*

Zhi Li**

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* the correspondent address: Department of Economics, University of Windsor, 401 Sunset Ave., Windsor, Ontario, N9B 3P4, Canada; Tel. (519) 253-3000, Ext. 2374, Fax (519) 973-7096; E-mail: xyin@uwindsor.ca

** Department of Economics, University of Guelph, Guelph, Ontario, N1G 2W1

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Abstract

Empirical evidences tell us that in the recent years the expansion period is increased with reduction of the contraction period in the U.S. business cycles. Moreover, the business cycles in the United States also show the trend to be moderated with recent economic growth induced and supported by high technologies and their industries. We study endogenous business cycles by a modified synthesized endogenous business cycles model "in which expansions are neoclassical growth periods driven by productivity improvements and capital accumulation, while downturns are the results of Keynesian contractions in aggregate demand" (Francois and Lloyd-Ellis, 2002), with consumption externalities. By considering consumption externalities, the endogenized business cycles will be more likely to happen, the optimal consumption level will be higher, the technology growth rate will be bigger, the length of expansion will be longer and the length of contraction will be shorter. All of these results will lead to a faster and longer economic growth and smoother cycles. These theoretical results are significantly different from those in circumstances without the consumption externalities Francois and Lloyd-Ellis (2002) obtained, and are strongly supported by the data from the United States in the different periods.

Key Words: Endogenous Business Cycle, Consumption Externality, Endogenous Growth

JEL Classification: E3, D11, O3, O4

Canada; Tel. (519) 253-3000, Ext. 2374, Fax (519) 763-8497, E-mail: **zli01@uoguelph.ca**

1. Introduction

The economy in the United States has been achieved a great growth in the last half of the twentieth century. The business cycle, however, still exist as before. Many economists did plenty of research on it. For example, Christina Romer (1986) has done some researches about the change for the business cycle in the United States. She denied the claim that the business cycle moderated over time. Because of Romer's (1986) significant work, much new research has been done for different countries. In the recent years, especially from the end of 1980s, it was found there forms a new trend for business cycles in the United States. We found that with that the total length of the business cycle does not change much, the length of boom period becomes longer, and the length of the recession period becomes shorter. Up to now, as we know so far, no article can explain the new phenomena properly. In this paper, we try to interpret it by considering the consumption externalities into the endogenous growth model.

In the meantime, economic growth theory has been a very important topic in the research of economics in the last fifty years. Since technology has been improved at a more rapid rate than ever from the last half of the nineteenth century, the theories of the way of which technology affects the economic growth have been advanced substantially, which is considered as "the engine of economic growth". The impacts of factors on the economic growth are always at the heart of the study of modern economic growth theory, especially from the mid-1980s, in which the endogenous growth theory emerges and obtains the significant achievement to explain the relationship between technology innovation and the economic growth.

On the other hand, the real business cycle (RBC, hereafter) theory is unusually silent now. The reasons for this are discussed in many articles and even in university textbooks. They mentioned that one of the main problems is the extensions of the model included an exogenous growth trend which measures the level of technology, but this trend had no effect on the duration or size of the cycle. And the other shortcoming is the real business cycle theory can not explain the effect of the contractions on the employment. The RBC theory can not explain the volunteer unemployment in the recession but describe it as the access supply of the labor force. Such problems seem, however, to be solved with the rapid development of the economic growth theory.

Economists (Francois and Lloyd-Ellis (2003), Wälde (2003)) found that with the mechanism of "creative destruction" (Schumpeter, 1934), the business cycles can be endogenized and the reasons of cycles could have some relationship with the process of the technology growth or the activities in the R&D sector.

Precisely, according to Schumpeter (1934), productivity growth and the business cycles are closely interrelated. There are several possible reasons why business cycles or even recessions may exert positive effects on economic growth. One of them is that in the period of recessions, the efficient firms can survive out and the inefficient ones will stop their production because they can not make any profit by using the old technology while the market demand is decreasing. Another one, which is also the more important one, is that during the recession period, the more of the free capital or the skilled labor will be relocated in the R&D sector and their contribution to the new level of technology will add fresh blood to the economic growth in the next booming period.

Furthermore, on the consumer side, the consumption externality is also a very important factor of the optimal economic growth path. Generally, it discusses the effects of consumption externality on many aspects of the economic growth and finds the methods to control these kinds of effects. Consumption externalities have been extensively studied in the context of models of jealousy and “keeping up with the Joneses.” For example, Abel (1990) studies the effect of consumption externalities on asset pricing and the equity premium. Ljungqvist and Uhlig (2002) analyzed the impact of consumption externalities on the effect of short-run macroeconomic stabilization policy. Dupor and Liu (2002) defined different forms of consumption externalities and explored their relationship with equilibrium over-consumption. Ordinarily, there are two kinds of consumption externalities, positive ones and negative ones. In my paper, for the purpose of simplicity, I assume that there are only positive consumption externalities in my study. I do not want to consider the negative consumption externalities. This makes sense as one person benefits from the increased whole society’s consumption, with the higher utility and, perhaps, the higher consumption as well. And the positive consumption externalities are paid more attention than to the negative ones in the research by far.

In this area, Liu and Turnovsky (2002) made important contribution on the research of consumption externalities’ effects on the optimal economic growth path. They analyzed the effects of consumption and production externalities on the long-run rate of capital accumulation and, consequently, on the economic growth. They also discuss the different results with the inelastic and elastic labor supply.

There are several interesting papers discuss the RBC with forming a unified model to describe the engine of the economic growth and the size of the downturns, such as King and Rebelo (1986), Lucas (1988) and Aghion and Howitt (1998). In these papers, Francois and Lloyd-Ellis' (2003) paper is a significant example for such attempt.

Francois and Lloyd-Ellis (2003) developed a unified theory of endogenous business cycles in which expansions are neoclassical growth periods driven by productivity improvements and capital accumulation, while downturns are the results of Keynesian contractions in aggregate demand below potential output. Recessions allow skilled labor to be relocated to growth-promoting activities, which fuel subsequent expansions. However, rigidities in production and contractual limitations, which are inherent to the process of creative destruction, leave capital severely underutilized.

In their paper, they discuss the engine of economic growth and the interaction of the economic growth and the business cycles. But they do not mention the effects of the consumption externalities, which should be very important for determining the extent and length of business cycle. Especially, in the recent years, the high technology makes the effect of aggregate consumption of the society more important on the individual decision of the consumption. I follow Francois and Lloyd-Ellis' framework with considering the circumstances of having and not having consumption externalities. To make analyses simple and clear, I will compare the results of Francois and Lloyd-Ellis (2003) and mine with positive consumption externalities. In this way, readers can easily find out the differences and the importance of the role of consumption externalities. However, in my paper, rather than having variable labor supply, to simplify the model, I assume that the labor supply is fixed and the labor market is always clear.

I find that by considering consumption externalities, the business cycles are more likely to happen, i.e. the acyclical equilibrium is less likely to exist. The optimal consumption level will be higher due to a higher economic growth rate. In a general purpose technology model, comparing to that without the consumption externalities, the technology growth rate is higher and the length of recession is shorter, which means a smoother growth path. All of these results will lead to a faster and longer economic growth. The evidences in the resent years of business cycles and economic growth in the United States support these interesting results. I think the reason that the economy in the United States can keep the relatively low but sustained growth rate rather than having dramatically fluctuated along business cycles is able to explained at least partly by the effect or the support of the consumption externalities.

This paper will be consisted of the following sections: Section 2 reviews the instructive literature and sums up the relationship between the economic growth and business cycles. Section 3 sets up a model to analyze the different effects on economic growth with and without the consumption externalities within endogenous economic growth framework. The last section summarizes the conclusion and makes suggestions about the potential research. Some data and figures from the United States are attached at the end of this paper to verify my results.

2. REVIEW OF THE LITERATURE

2.1 Achievements of the theories of economic growth and business cycles

In the last two centuries, the whole world economy experienced an extremely rapid growth. And the world per capita GDP is almost more than two hundred times as that of two centuries ago. It is a valid question to ask what the mechanism of this kind of miracle is. In the first half of the twentieth century, British economist R. F. Harrod (1939) and American economist E. D. Domar (1946) (H-D, henceforth) pioneered the modern economic growth theory. They introduced an important exogenous variable, the rate of population growth, *n*, and this variable has been included routinely in the models of later research. Furthermore, Harrod initiated the research on the existence of a unique and stable "equilibrium output path", which became the orthodox theory that was extended by later modern growth models.

Following these two economists, Robert M. Solow (1956) founded a neoclassical model to improve the H-D model, which set fixed ratio between capital and labor. One assumption that both Solow and H-D have made was the full employment or market clear. Therefore, both Harold-Domar and Solow models are Walrasian models. Assuming Cobb-Douglas production function, Solow introduced Solow residue and balanced growth rate into the growth theory. Solow's residue was discovered by Solow when he put forward the total factor productivity method. He named the 87.5% of the output that can not be explained by the factors in the Solow's model as "Solow residue". Furthermore, as the residue stands for the technical progress which is exogenous, the research following Solow in the last half of the twentieth century is focused on making Solow's residue and the balanced growth rate endogenous.

To make Solow's residue endogenous, Kenneth J. Arrow and Paul M. Romer used different models to interpret the engine of economic growth. Arrow (1962) introduced his celebrated learning by doing model. In Arrow's Model, the factor of technical progress is an exponential function of the aggregate capital. Therefore, capital makes more contribution to the output in growth and in level, than that for the Solow model, and this kind of transformation made Solow's residue more interpretable than before. But Arrow can not totally endogenize the Solow's residue in his learing by doing model.

Paul Romer (1986,1990) pioneered the new growth theory. He rationalized the growth without increasing of population and endogenized the technology progress. This solves the shortcoming of Solow model. Unlike Arrow's model, which views technology progress as the side-effect of the physical capital accumulation; Romer introduced an R&D part and focused on the investment decisions of the firms. After Romer, the technology endogenous growth theory got more attention and incisive research.

The real business cycle theory is the center of the economic theory in the mid-twentieth century. According to this theory, business cycles are driven by the fundamental technological dynamic of the economy. With the emergence in the 1950s of the real business cycle literature, productivity shocks has been emphasized as a main driving force behind cyclical fluctuations. This theory gave people a good interpretation about the process of the higher level technology affecting on the output level. But it can not find proper relationship between the technology innovation and recessions.

Besides the important progress in the theory of the economic growth, current research on the real business cycles theory makes a link for these two theories. It explains the boom period by using the rising of the technology level. The research following it tries to make up its shortcomings and illustrates the reasons of the contractions.

In this spirit, Aghion and Howitt (1998) made remarkable success on modeling the endogenous growth, which also contributes to explain the business cycles. They also found some empirical evidence for such model. They not only use intermediate goods but also introduced a new term named templates to make the higher level of technology be used easier.

Because of the central role of the technology growth on the progress of economic growth and cycles, the endogenous growth model can help people understand the overall effects of the technology improvement on the growth process and business cycles.

Economic growth is driven by many factors: economic, political, and cultural. Economists have traditionally emphasized the accumulation of conventional inputs (e.g., labor and capital) as the primary forces behind output expansion. More recently, greater attention has been paid to technological factors. This paper concerns with the effect of technology innovation on the economic growth in a model with and without consumption externalities.

As the cycles and the growth theories are closely related in the modern economic growth theory, it will be useful to review some contributions of the research on cycles to the research on economic growth.

Firstly, some cycles come with the obsolescence of the existing capital. But this kind of obsolescence maybe caused by the decrease of aggregate demand or the increase of investment on R&D. And these two kinds of obsolescence will both stimulate growth in the following period with the increase of aggregate demand and the leap of the technology level.

Secondly, some cycles are coming from maximizing the dynamic profit. Because of the expectation of a big improvement of the new technology, firms should reduce their manufactory and wait for the adoption of the more productive machines. Such kind of cycles will also lead to a big boom.

Hence, some of the economic recessions may be the preparation for the rapid growth period that will follow, in Schumpeter's original word, these recessions are 'creative destruction'.

2.2 Recent progress on the theories of technology leading cycles

Economic historians have placed great weight on technology as the force of change to show the importance of the technological factors in explaining modern economic growth. See Rosenberg (1982) and Mokyr (1990), for example.

Because of the inability to analyze the forces that shape technological change, macroeconomists used to downplay its role. However, in recent years, this attitude has changed. Following the work of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), many macroeconomic studies now place technological progress at the center of the economic growth process. This kind of change has been triggered by theoretical developments that allow microeconomic aspects of the innovation process to be linked with macroeconomic outcomes.

To give technology an important role in economic growth, however, is not a new invention. We can find ideas like this among the classical authors. A famous example is Simon Kuznets, who wrote more than thirty years ago: '*. . . we may say that certainly since the second half of the nineteenth century, the major source of economic growth in* *the developed countries has been science-based technology—in the electrical, internal combustion, electronic, nuclear, and biological fields, among others'.* (Kuznets 1966, p. 10)

The incremental nature of technological progress has been well documented by economic historians, See Rosenberg (1982). But major inventions always had far-reaching and prolonged implications, such as the steam engine, electricity, and the computer, See Du Boff (1967) on electricity and David (1990) on the parallels between electricity and the computer. In this paper, I discuss a certain type of drastic innovations, termed General Purpose Technologies (GPTs) a la Bresnahan and Trajtenberg (1995). There is hardly a debate about incremental innovations. Small improvements take place in the regular course of business, both by chance and intentional. Many incremental innovations were following drastic innovations. For example, the introduction of the first steam engine triggered a sequence of secondary innovations that were designed to improve its operation. The same is true about electricity and the computer.

Economists have paid less attention to the role of drastic innovations when the modern growth theories had a rapid progress in the last half of the twentieth century. A drastic innovation introduces discontinuity, in the sense that it leads to the replacement of an old technology, which played a significant role in an industry. Or it replaces an old material that performed an array of designated functions (e.g., rubber) with a new one (e.g., plastics). But a discontinuity in this sense does not imply a necessary discontinuity in the observed pattern of resource allocation or the evolution of output, as the introduction of a superior technology can be gradual, starting with a negligible absorption of resources and followed by continuous expansion over time. Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) have done a great amount of research on the relationship between incremental and drastic innovations. They modeled the production and employment of intermediate goods to endogenize the impact of GPTs on the economic growth.

But even if a drastic innovation penetrates an economy gradually and the pattern of resource allocation and the level of output change only slowly, it does not mean that it is not helpful to think about drastic innovations on its own right. Because if we only think about the incremental innovations, we will miss the true cause of incremental innovations that are triggered by a drastic innovation, and we may misleadingly attribute all changes in resource allocation and output to the induced secondary innovations. In order to properly understand the relationship between cause and consequence; it is useful to distinguish between drastic and incremental innovations. A drastic (or major) innovation often sets the stage for a series of incremental innovations. Secondly, it is possible that forces that drive incremental innovations are different from those that drive drastic innovations. For example, incremental innovations are more susceptible to standard profitability calculations. Although they involve externalities and are subject to risk, markets can evaluate their profitability. In comparison, drastic innovations face much larger uncertainties, producing risks that are much harder to evaluate and therefore harder to insure, See for example, Rosenberg (1996). As a result, drastic innovators can engage in little risk-sharing and have to bear most of the risk themselves.

Furthermore some drastic innovations may produce discontinuities at the industry level, if not at the economy-wide level. When we are looking at the data, we will not search for such discontinuities if we are preconditioned to think only about incremental innovations. As a result we may misinterpret the evidence. This is another reason that we can not value the technology contribution to the economic growth precisely.

A drastic innovation qualifies as a GPT if it has the potential for pervasive use in a wide range of sectors in ways that drastically change their modes of operation. To quote from Bresnahan and Trajtenberg (1995), who coined the term GPT and provided a highly original discussion of its usefulness,

'*Most GPTs play the role of "enabling technologies," opening up new opportunities rather than offering complete, final solutions. For example, the productivity gains associated with the introduction of electric motors in manufacturing were not limited to a reduction in energy costs. The new energy sources fostered the more efficient design of factories, taking advantage of the newfound flexibility of electric power. Similarly, the users of micro-electronics benefit from the surging power of silicon by wrapping around the integrated circuits their own technical advances. This phenomenon involves what we call "innovational complementarities" (IC), that is, the productivity of R&D in a downstream sector increases as a consequence of innovation in the GPT technology. These complementarities magnify the effects of innovation in the GPT, and help propagate them throughout the economy*.'

This description makes clear two important features of drastic technological innovations that qualify as GPTs: generality of purpose and innovational complementaries. When these effects are particularly strong, as, for example, in the case of electricity, they lead to historic changes in economic organizations. Sometimes they also affect the organization of society through working hours, constraints on family life, social stratification, and so on.

GPTs bring two types of externalities: one between the GPT and the application sectors; another across the application sectors. The former comes from the difficulties that a GPT inventor may have in appropriating the fruits of her invention. When institutional conditions can not make full appropriation, the GPT is effectively underpriced and therefore undersupplied. The latter comes from the fact that, since the application sectors are not coordinated, each one conditions its expansion on the available general purpose technology. But if they coordinated a joint expansion, they would raise the profitability of the GPT and encourage its improvement.

The research on GPTs is desirable and feasible in the new century, since the GPT-driven economic growth has shown very active role in the engine of modern economic growth theory.

In this paper, General Purpose Technologies is the engine of booming. I will choose parameters to make the technology innovation activities become more certain to happen and more possible to success, and this makes the two externalities mentioned above not affect the economy too much.

There is another important group of literature I must discuss here, which is about consumption externality. On the basis of the endogenous business cycles, the consumption externality is a very important factor that can help to find the process of the economic growth. From the fifties in the last century, the consumption externalities began to interest some economists such as Duesenberry (1952). In his paper, he first introduced the term of the consumption externality, which is also referred to as interdependent preference. The main point of this introduction is to reconcile the cross-sectional and time series evidence on consumption. Duesenberry pointed out that the consumers try to *"keep* *up with the Joneses"*, which means the consumers care about not only their absolute amount of consumption but also the relative standard of living. After Duesenberry, economists began to pay more attention to the consumption externalities and find out the effects of it in many aspects in the economics. In fact, by considering the consumption externalities, the realization of the economy of society has got some different results. First of all, the resource allocation may not be the Pareto ones and economists try to find some methods to improve it from the second best to the first best situation. For example, the optimal taxation and social welfare is a hot topic in the field of consumption externalities. In Sadka (1978), he found out some way to settle the optimal taxation. In Diamond (1973), he considered another effect, imperfectly correcting price, which is brought to the economy by the consumption externalities.

In Liu and Turnovsky (2002), they consider the consumption externalities in the analysis of the long-run economic growth. They checked some result of the optimal growth path with the consumption externalities.

In my paper, I think the positive externalities are becoming reasonable to be introduced in the model of endogenous economic growth model. The economy in the whole world gets a very rapid rate of growth. Keeping up with the living standard of rich men is popular in the modern society. So the effects of the consumption externalities on the economy have become a considerable one. On the other hand, by comparing the results of the balanced growth path with and without considering the consumption externalities, I find some interesting differences and these are useful to interpret the business cycle with new economy emerges in the recent decades in the United States.

From all of the above, the combination of consumption externalities and the technology engined growth is a good implement to the economic growth theory.

3. THE MODEL

3.1 Assumptions and Notations

To compare the endogenous business cycles with and without the consumption externalities using the endogenous growth theory, I adopt the framework of Francois and Lloyd-Ellis (2003). Follows are the basic assumptions and notations in Francois and Lloyd-Ellis (2003) which will be used after introducing the consumption externalities into this model.

3.1.1 Households

There are a discrete finite number of households in this economy. Each household is sufficiently small to neglect the effects of individual behavior on aggregate variables. Households maximize expected utility *U (t)* given by the sum of instantaneous utility *u (.)* resulting from a household's consumption flows $c(\tau)$ and the aggregate consumption flows $C(\tau)$, discounted at the time preference rate ρ,

$$U(t) = E_t \int_{t=0}^{\infty} e^{-\rho\tau} u(c(\tau), C(\tau)) d\tau \qquad (1)$$

The household maximizes (1) subject to the intertemporal budget constraint,

$$\int_{t=0}^{\infty} e^{-R(\tau)} c(\tau) d\tau \leq S(t) + \int_{t=0}^{\infty} e^{-R(\tau)} w(\tau) d\tau \qquad (2)$$

where *w(t)* denotes wage income, *S(t)* denotes the household's stock of assets (firm shares and capital) at time t and *R(t)* denotes the discount factor from time zero to *t*.

In equation (1), the instantaneous utility function *u(.)* is characterized by considering the consumption externalities, which is modified from Liu and Turnovsky (2002),

$$u\,(c,\, C) = \frac{1}{1-\sigma}(cC^{\beta})^{1-\sigma} \qquad (3)$$

where σ is a parameter in the utility function, I assume that $0 < \sigma < 1$. β is the power of the aggregate consumption contributes to the household's utility, because the effect of the aggregate consumption on individual's utility can not greater than the individual's consumption, here $0 < \beta < 1$.

3.1.2 Production

Every household has one unit of labor to supply and the population is normalized to unity so that the amount of labor supply is fixed and there is a full employment, which is consistent with my implicit assumption in the utility: labor supply is fixed.

In the final output sector, it is produced by competitive firms according to a Cobb-Douglas production function utilizing a continuum of intermediates, x_i, indexed by $i \in [0, 1]$. For simplicity, Francois and Lloyd-Ellis (2003) assume there is no physical depreciation and no adjustment cost, final output can be used for the production of consumption, *C(t)*, investment, $\dot{K}(t)$, or can be stored at a small flow cost of $\nu > 0$ per unit time.

$$C(t) + \dot{K}(t) \leq Y(t) = \exp(\int_0^1 \ln x_i(t) di) \tag{4}$$

As the production of the intermediate goods is the second last process of the production of the final goods, the utilization of the input of intermediate goods is also very important for the output of final goods.

In the intermediate inputs sector, intermediate *i* depends upon the state of technology in sector *i*, $A_i(t)$, utilized capital, $K_i^u(t)$ which is no more than the actual stock of installed capital in sector *i*, $K_i(t)$, and labor, $L_i(t)$, according to the following putty-clay technology:

$$x_i^s(t) = \begin{cases} [K_i^u(t)]^\alpha [A_i(t) L_i(t)]^{1-\alpha}, K_i^u(t) = K_i(t) \\ \kappa_i(z)^\alpha A_i(t)^{1-\alpha} L_i(t), K_i^u(t) = \kappa_i(z) L_i(t) < K_i(t) \end{cases} \tag{5}$$

As I understand, if the manufacture is putty-clay, once capitalists invest their capital in the production sector, they can not withdraw or transfer their capital in a certain period. This also means that the utilized capital should be proportional to the amount of the labor after the economy reaches the equilibrium point.

The present value of the capitalist's net income in sector i under the price, $q_i(\tau)$, and utilization sequence $\{K_i^u(\tau), q_i(\tau)\}_{\tau=0}^{\infty}$ is:

$$V_i^k(t) = \int_{t=0}^{\infty} e^{-R(\tau)}[q_i(\tau)K_i^u(\tau) - \dot{K}_i(\tau)]d\tau \qquad (6)$$

In this paper, the growth of technology is the main reason that leads to the endogenous business cycle. Because the R&D sector attracts some skilled labor from the production sector, the capital in the production sector can not be full utilized and the output growth rate should be negative and that means the growth rate is less than before.

In the R&D sector, the probability of an entrepreneurial success in instant t is $\delta H_i(t)$, where δ is the easy index of the R&D and H_i is the labor effort allocated to the R&D sector i. The entrepreneurs should decide whether to hire labor to conduct innovation and how much labor they should hire. The aggregate labor effort allocated to entrepreneurship is given by

$$H(t) = \int_0^1 H_i(t)dt \qquad (7)$$

The entrepreneurs also should decide whether to implement their innovations at every time, the way they obey is compare the value of delay and implementation. In Francois and Lloyd-Ellis' (2003), $V_i^I(t)$ denotes the expected present value of profits from implementing an innovation at time t, and $V_i^D(t)$ denotes that of delaying implement from time *t* until the most profitable time in future. When $V_i^I(t) > V_i^D(t)$, the entrepreneurs will implement their new innovation.

3.2 Equilibrium

I know that at the beginning of the boom or the implementation of the new technology, the economy will experience a great rate of growth and the investment will increase, too. But with the capital in the production sector increasing, the marginal productivity of labour will decrease and the profit of capital investment will decrease together. After this kind of growth, the whole economy would reach an equilibrium point that is defined to satisfy the following conditions:

1. Households allocate consumption over time to maximize (1) subject to (2):

In fact, the equilibrium is defined by a series of contracts signed between the intermediate goods suppliers and the firms and between the capitalists and the firms.

2. Labor market clearing:

$$\int_0^1 \hat{L}_i(t)di + \hat{H}(t) = 1 \qquad (8)$$

The whole labor force in the production sector and R&D sector is hold to be unity and this guarantees the full employment.

3. Market free entry:

Both for the R&D sector and for the production sector, the market entry is always free and that also means the complete competition and zero profit.

To discuss the balanced growth path of the whole economy, I consider the acyclical and cyclical ones respectively.

3.2.1 The acyclical balanced growth path

The acyclical balanced growth path means that the final output won't fall down and never go through a negative growth. To approach this kind of economy growth, we need that the utilized capital of firms grows with the investment of capital grows, R&D sector's activities are continuous and implementation is never delayed. Although the acyclical balanced growth path is not the main point that we are interested in, this kind of consideration is useful to our following work.

By using the same method as that in the Ramsey-Cass-Koopmans model, one can get that the consumption satisfies the differential equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma - \beta + \sigma\beta} \qquad (9)$$

In this equation, comparing with the similar utility function that one used to use without the consumption externalities, the growth rate of consumption is bigger than that without the consumption externalities. Here, I suppose $0 < \sigma < 1$.

Furthermore, since the capital is always full utilized in the production sector before the recession and the condition of free-entry into the capital markets implies that the capital price is equal to the interest rate. By assuming the prices of intermediate goods as given and the free-entry into the final goods markets, one can get the firms' cost for intermediate goods is equal to the profit of the final output.

From equation (9) and the balanced growth path, it follows that,

$$r(t) = g(\sigma - \beta + \sigma\beta) + \rho \qquad \textit{(10)}$$

Following the way in appendix, one can get,

$$g(\sigma - \beta + \sigma\beta) + \rho + \frac{g}{\gamma} = \frac{\left(1 - e^{-(1-\alpha)\gamma}\right)\delta}{(1-\alpha)e^{-(1-\alpha)\gamma}} + g \qquad \textit{(11)}$$

Solving for *g* yields

$$g^{a} = \max\left[\frac{\gamma\delta(1 - e^{-(1-\alpha)\gamma}) - \gamma\rho(1-\alpha)e^{-(1-\alpha)\gamma}}{[\gamma(\sigma-1)(1+\beta)+1](1-\alpha)e^{-(1-\alpha)\gamma}}, 0\right] \qquad \textit{(12)}$$

To ensure an acyclical balanced growth path, I need $r(t) > g^{a}(t)$, which means delay the implementation of a successful innovation is unprofitable, and this means all successful innovations are implemented immediately rather than delay. Referring the method in appendix, one can derive,

$$\rho > (1-\omega)(1-\theta)\delta\gamma H(t) \tag{13}$$

$$\frac{\left(1-e^{-(1-\alpha)\gamma}\right)\gamma(1-\sigma)(1+\beta)}{(1-\alpha)e^{-(1-\alpha)\gamma}} < \frac{\rho}{\delta} \tag{14}$$

Then I can have the following proposition:

Proposition 1: *If*

$$\frac{\left(1-e^{-(1-\alpha)\gamma}\right)\gamma(1-\sigma)(1+\beta)}{(1-\alpha)e^{-(1-\alpha)\gamma}} < \frac{\rho}{\delta}$$

then there exist an acyclical equilibrium with a constant growth rate given by

$$g^a = \max\left[\frac{\gamma\delta(1-e^{-(1-\alpha)\gamma}) - \gamma\rho(1-\alpha)e^{-(1-\alpha)\gamma}}{[\gamma(\sigma-1)(1+\beta)+1](1-\alpha)e^{-(1-\alpha)\gamma}}, 0\right]$$

Remark: this result is based on the consideration of the consumption externalities. This is different from that of Francois and Lloyd-Ellis (2003). To compare this condition and growth rate with those without the consumption externalities, I find that this condition will hold when there are bigger value of ρ and smaller value of δ. These indicate that an acyclical balanced growth path could exist only when people prefer to consume now rather than in future and the possibility to get a successful innovation is very small to the R&D sector. But in reality, consumer will make some saving for future and the technology innovation always can get some success. So the acyclical equilibrium is more unlike to happen considering the consumption externalities.

3.2.2 The cyclical growth path

Since the acyclical balanced growth path is not so possible to exist in this paper, I will check the propositions of the cyclical growth path with the consumption externalities compared with those without consumption externalities.

I will discuss the cycle from two parts, the expansion and the downturn. First, the expansion is following Solow's neoclassical growth theory. As Francois and Lloyd-Ellis (2003) denote, the improvement in aggregate productivity during implementation period T_v and the growth in the average unit cost is $e^{(1-\alpha)\Gamma_v}$, where

$$\Gamma_v = \ln[\overline{A}_v / \overline{A}_{v-1}] \ \textit{and} \ \overline{A}_v = \exp(\int_0^1 \ln A_i(T_v) di) \qquad (15)$$

Referring to the appendix, one can derive that:

$$\frac{\dot{C}(t)}{C(t)} = \frac{\alpha e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(t)^{\alpha-1} - \rho}{\sigma - \beta + \sigma\beta} \qquad (16)$$

Then I can have the following proposition:

Proposition 2: *During the expansion, consumption evolves according to:*

$$\frac{\dot{C}(t)}{C(t)} = \frac{\alpha e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(t)^{\alpha-1} - \rho}{\sigma - \beta + \sigma\beta}$$

Remark: this result is based on the consideration of the consumption externalities. This is different from that of Francois and Lloyd-Ellis (2003). To compare the growth rate of consumption with those without the consumption externalities, I find that the consumption growth rate considering the consumption externalities is bigger than that without the consumption externalities. It implies that, people will increase their consumption more than the level without the consumption externalities. This means the aggregate consumption gives some kind of support to people's confidence to the whole economy. On the other hand, people's increasing consumption also gives much support to the whole economic growth. In fact, this is the way that the economy in the United States can hold on from falling into the recession for quite a long period.

To be convenient for the analysis below, Francois and Lloyd-Ellis (2003) define the discount factor that will be used to discount from some time t during the cycle to the beginning of the next cycle. It is given by

$$\beta(t) = R(T_v) - R(t) = R(T_v) - R(T_{v-1}) - \int_{T_{v-1}}^{t} r(s)ds \qquad (17)$$

Since the capital investment after the equilibrium point falls to zero, the interest rate should change to zero, too. And the growth rate of the economy changes to $\frac{-\rho}{\sigma - \beta + \beta\sigma}$.

So the labor located in R&D sector at time *t* is given by

$$H(t) = 1 - (1 - H_v)e^{\frac{-\rho[t - T_v^E]}{\sigma - \beta + \sigma\beta}} \qquad (18)$$

Think about the putty-clay nature of capital investment, it implies that the capital utilization rate in the downturn period is given by

$$\lambda(t) = (1 - H_0(T_v^E))e^{\frac{-\rho[t - T_v^E]}{\sigma - \beta + \sigma\beta}} \qquad (19)$$

It follows that the capital amount in the equilibrium point should satisfy:

$$\int_{T_v^E}^{T_v} q(\tau)(1 - H_v)e^{\frac{-\rho[t - T_v^E]}{\sigma - \beta + \sigma\beta}} d\tau = 1\text{-}\ e^{-\beta(T_v)} \qquad (20)$$

Since the average price level of capital in the downturn should be unique, the average in the *vth* cycle should be

$$\bar{q}_v \equiv \frac{\int_{T_v^E}^{T_v} q(\tau)(1 - H_v)e^{\frac{-\rho(\tau - T_v^E)}{\sigma - \beta + \sigma\beta}} d\tau}{\int_{T_v^E}^{T_v} (1 - H_v)e^{\frac{-\rho(\tau - T_v^E)}{\sigma - \beta + \sigma\beta}} d\tau} \qquad (21)$$

Using (20) and integrating the denominator through the downturn, this implies:

$$\overline{q}_v = \frac{1 - e^{-\beta(T_v)}}{(1 - H_v)\left(\frac{1 - e^{\frac{-\rho\Delta_v^E}{\sigma - \beta + \sigma\beta}}}{\rho / (\sigma - \beta + \sigma\beta)}\right)} \quad (22)$$

Then I can have the following proposition:

Proposition 3: *For a capital-supply contract to be efficient through the downturn it is necessary that capital is installed only up to the point at which the marginal return to capital is equal to its average rental price:*

$$\overline{q}_v = \frac{1 - e^{-\beta(T_v)}}{(1 - H_v)\left(\frac{1 - e^{\frac{-\rho\Delta_v^E}{\sigma - \beta + \sigma\beta}}}{\rho / (\sigma - \beta + \sigma\beta)}\right)}$$

Remark: this result is based on the consideration of the consumption externalities. This is different from that of Francois and Lloyd-Ellis (2003). Here I suppose Δ_v^E, the length of the downturn, is fixed, then I find that the average price of capital in the downturn considering the consumption externalities is bigger. This implies that the capital utilization level will be lower than that without the consumption externalities, so that the average price should be higher.

In the downturn the growth in productivity is also an important variable that we care for with the consumption externalities. In Francois and Lloyd-Ellis (2003), the growth in productivity during a successful innovation is given by

$$\Gamma_v = (1 - P(T_v))\gamma \tag{23}$$

where $P(T_v)$ is the probability of not being displaced at the next implementation. And

$$1 - p(T_v) = \delta \int_{T_v^E}^{T_v} \left(1 - (1 - H_v) e^{\frac{-\rho\left[t - T_v^E\right]}{\sigma - \beta + \sigma\beta}} \right) d\tau \tag{24}$$

Substitution into (23) and integrating gives:

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma(1 - H_v) \frac{1 - e^{-\frac{\rho\Delta_v^E}{\sigma - \beta + \sigma\beta}}}{\rho / (\sigma - \beta + \sigma\beta)} \tag{25}$$

Then I can have the following proposition:

Proposition 4: *In an equilibrium where there is positive entrepreneurship only over the interval* $(T_v^E, T_v]$*, the growth in productivity during the succeeding boom is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma(1 - H_v) \frac{1 - e^{-\frac{\rho\Delta_v^E}{\sigma - \beta + \sigma\beta}}}{\rho / (\sigma - \beta + \sigma\beta)}$$

Remark: this result is based on the consideration of the consumption externalities. This is different from that of Francois and Lloyd-Ellis (2003). Similar with the result of proposition 3, the growth in productivity is bigger with the consumption externalities given the length of the recession period. This is because the lower level of the capital utilization during the downturn makes more skilled labor flow to the R&D sector and the possibility to success should be higher than that without the consumption externalities.

Since the discount factor is $\beta(T_v)$ and the technology growth is powered by $(1-\alpha)\Gamma_v$, the innovation will be implemented when the discount factor and the growth in productivity satisfy

$$\beta(T_v) = (1-\alpha)\Gamma_v \tag{26}$$

If this condition is not satisfied there should be no free entry or still delay the implementation.

Then I can have the following proposition:

Proposition 5: *Assets market clearing at the boom requires that*

$$\beta(T_v) = (1-\alpha)\Gamma_v$$

Remark: this result is based on the consideration of the consumption externalities. I find that, as the growth in productivity is bigger considering the consumption externalities and to satisfy the condition of equation (17), I have to make the length of the downturn shorter than that without the consumption externalities. This will show people good news about the economic growth. And this is also observed by most people in resent years.

Here the growth rate of final output is given by:

$$\frac{\dot{Y}}{Y} = (1-\alpha)\Gamma_v + (1-\alpha)\frac{\dot{L}}{L} + \alpha\frac{\dot{K}}{K}$$

$$= (1-\alpha)\Gamma_v + \frac{\rho\Delta_v^E}{\sigma - \beta + \sigma\beta} - \ln(1 - H_v) \qquad (27)$$

Suppose the whole length of the cycle, which means the sum of that of the boom and recession, is fixed. One can find that, as the length of the recession is shorter, the length of the boom should become longer than that without thinking out the consumption externalities.

Now, let us find out the condition that the R&D sector hold their innovation and delay the implement. In this paper I suppose the innovator can implement his innovation without delay if he will pay the buy-out price for using the whole capital that is utilized and unutilized by the firms. So it is not reasonable to implement the innovation under the following condition:

$$\int_t^{T_v} \pi(\tau)d\tau + \int_t^{T_v} (1 - H_v) e^{-\frac{\rho(\tau - T_v^E)}{\sigma - \beta + \sigma\beta}} \overline{q} K(T_v^E) d\tau \geq \int_t^{T_v} \overline{q} K(T_v^E) d\tau \qquad (28)$$

Rearrange it one can get:

$$\int_t^{T_v} \pi(\tau) d\tau \geq \int_t^{T_v} \overline{q} K\left(T_v^E\right)\left(1-\left(1-H_v\right) e^{-\frac{\rho\left(\tau-T_v^E\right)}{\sigma-\beta+\sigma\beta}}\right) d\tau \tag{29}$$

$$\left(1-e^{-(1-\alpha)\gamma}\right) Y\left(T_v^E\right) \int_t^{T_v}\left(1-H_v\right) e^{-\frac{\rho\left(\tau-T_v^E\right)}{\sigma-\beta+\sigma\beta}} d\tau \geq \overline{q} K\left(T_v^E\right) \int_t^{T_v}\left(1-\left(1-H_v\right) e^{-\frac{\rho\left(\tau-T_v^E\right)}{\sigma-\beta+\sigma\beta}}\right) d\tau \tag{30}$$

Since $\overline{q} K\left(T_v^E\right)=\alpha e^{-(1-\alpha)\gamma} Y\left(T_v^E\right)$, this can be expressed as

$$\left(1-e^{-(1-\alpha)\gamma}\right) \int_t^{T_v}\left(1-H_v\right) e^{-\frac{\rho\left(\tau-T_v^E\right)}{\sigma-\beta+\sigma\beta}} d\tau \geq \alpha e^{-(1-\alpha)\gamma} \int_t^{T_v}\left(1-\left(1-H_v\right) e^{-\frac{\rho\left(\tau-T_v^E\right)}{\sigma-\beta+\sigma\beta}}\right) d\tau \tag{31}$$

$$\left(1-(1-\alpha) e^{-(1-\alpha)\gamma}\right)\left(1-H_v\right) \int_t^{T_v} e^{-\frac{\rho\left(\tau-T_v^E\right)}{\sigma-\beta+\sigma\beta}} d\tau \geq \alpha e^{-(1-\alpha)\gamma}\left(T_v-t\right) \tag{32}$$

Since this holds with equality at $t = T_v$, a sufficient condition is that the left-hand side declines more rapidly with t than the right-hand side. That is

$$\left(1-(1-\alpha) e^{-(1-\alpha)\gamma}\right)\left(1-H_v\right) e^{-\frac{\rho\Delta_v^E}{\sigma-\beta+\sigma\beta}} \geq \alpha e^{-(1-\alpha)\gamma} \tag{33}$$

Then I can have the following proposition:

Proposition 6: *If*

$$\left(1-(1-\alpha)e^{-(1-\alpha)\gamma}\right)(1-H_v)e^{-\frac{\rho\Delta_v^E}{\sigma-\beta+\sigma\beta}} \geq \alpha e^{-(1-\alpha)\gamma}$$

then entrepreneurs who successfully innovate during the downturn prefer to wait until the beginning of the next cycle rather than displace the incumbent, produce now and store until the boom.

Remark: this result is based on the consideration of the consumption externalities. This is different from that of Francois and Lloyd-Ellis (2003). I find that considering the consumption externalities, the length of recession will be shorter than that without the consumption externalities. This result is similar with that from the proposition 5.

4. CONCLUSIONS AND FURTHER DISCUSSION

This paper examines the effects of the consumption externalities on the endogenized business cycle in endogenous economic growth framework. The main conclusions are follows. Firstly, by considering consumption externalities, the endogenized business cycles will be more likely to happen. Compare to the result in Francois and Lloyd-Ellis' (2003), the condition to satisfy an acyclical equilibrium is harder to be met. This is the reason that an economy can not experience an endless extension. Secondly, the growth rate of the consumption is greater than that in Francois and Lloyd-Ellis' (2003), this is one of reasons that the economy can have a relative longer boom period. The higher level of aggregate consumption supports a higher level of final output, which means the economy experiences an extension. Thirdly, assuming the length of the business cycles is unchanged in the circumstances of having and not having the consumption externalities, the capital price will become higher because the recession period is shorter. The difference of the cycle size between that in my paper and that in Francois and Lloyd-Ellis' (2003) is one of the main results I obtain. The trend that economy extensions become longer and contractions become shorter is observed in recent years, which is explained reasonably in this paper while few papers provide good explanations. Fourthly, the growth rate of productivity is higher than that in Francois and Lloyd-Ellis' (2003), this is another one of reasons that the economy can have a relative longer boom period because the technology growth is the engine of the economic growth. This result is also a positive one that can ensure a faster and longer economic growth. All of these results will lead to a faster and longer economic growth and can be supported by the empirical evidences as figures in the appendix.

There is some potential research to do further. In this paper the Keynesian contraction has a main character that using contracts to hold the capital in the production sector and to show the putty-clay symptom. But the way to determine the prices and quantities in contracts is also a little unclear and confused. What the real way to get a stationary cycle is also a puzzle. At the moment, however, an agreed-upon theoretical framework for dealing with the complex dynamics that are associated with endogenous cycles should be done in the further study.

To make some further discussion of potential research, one can try to take the elasticity of the labor supply into consideration. I think there should be some other important conclusion to be followed by that.

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Figures

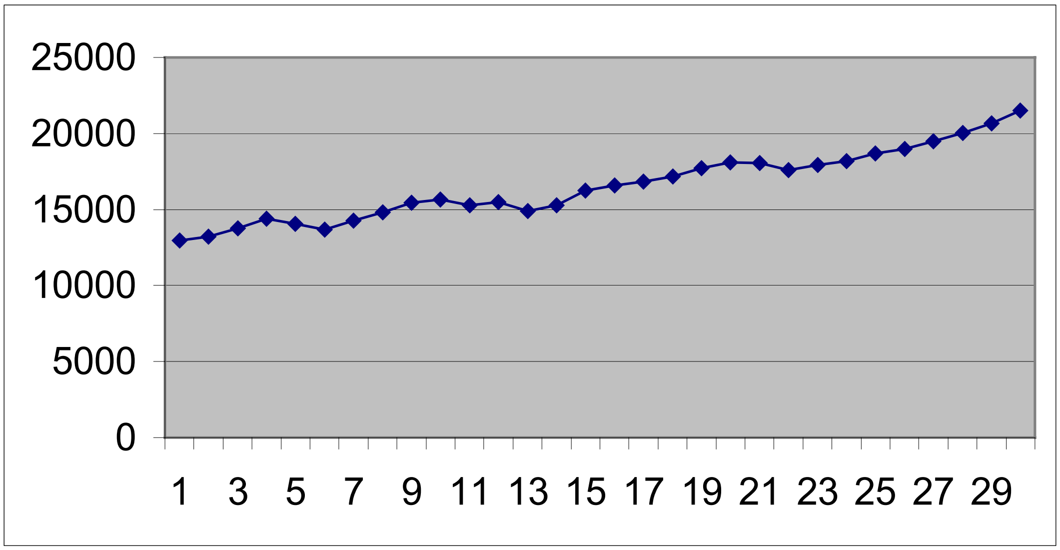

FIGURE 1 **The real GDP per capita in the U.S. between 1970 and 1999**

Figure 1 is the real GDP per capita from the year 1970 to year 1999 as the chart shows above. The vertical axis is the Real GDP Per Capita in constant dollars (international prices, base year 1985) and the horizontal axis is the year from 1970 to 1999.

Figure 2 to Figure 5: indicate the business cycle expansions and contractions of the economy in the United States from the year 1854 to the year 2001.

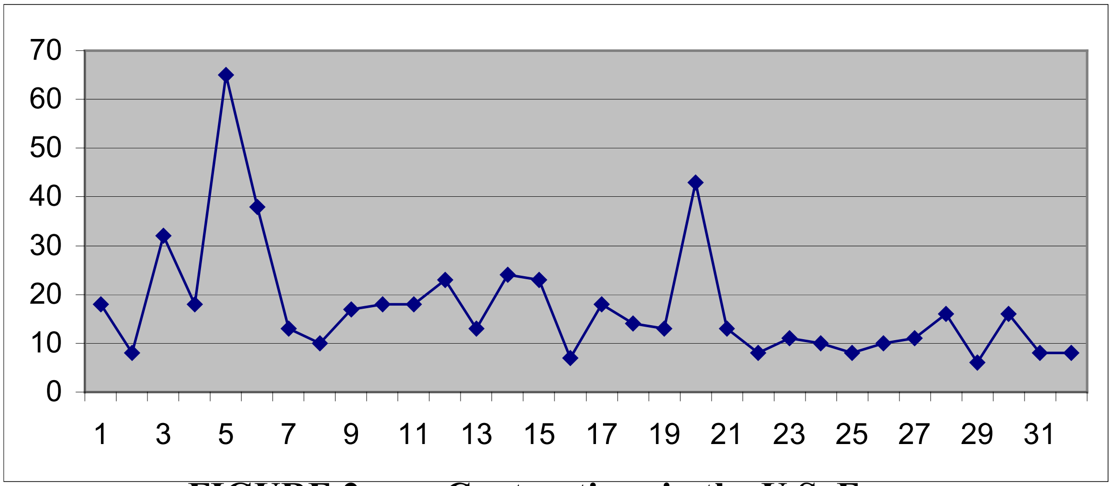

FIGURE 2 Contractions in the U.S. Economy

Figure 2 is the duration in months of the contractions (Peak to Trough). The vertical axis measures the duration in months of the contractions (Peak to Trough) and the horizontal axis means the number of the contractions (Peak to Trough).

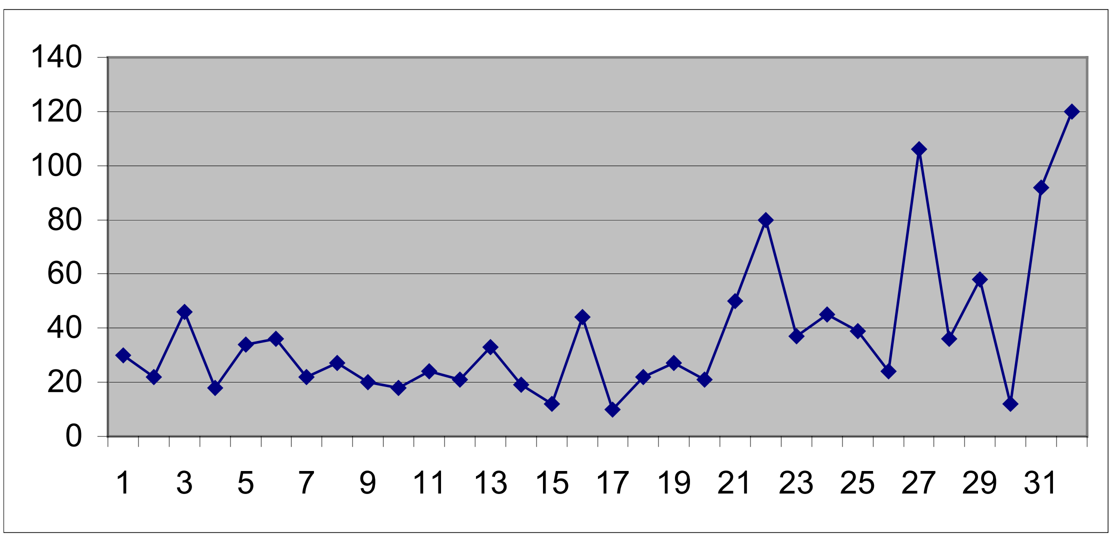

FIGURE 3 The duration of the expansions: 1

Figure 3 is the duration in months of the expansions (Previous Trough to This Peak). The vertical axis is the duration in months of the expansions (Trough to Peak) and the horizontal axis is the number of the expansions (Trough to Peak).

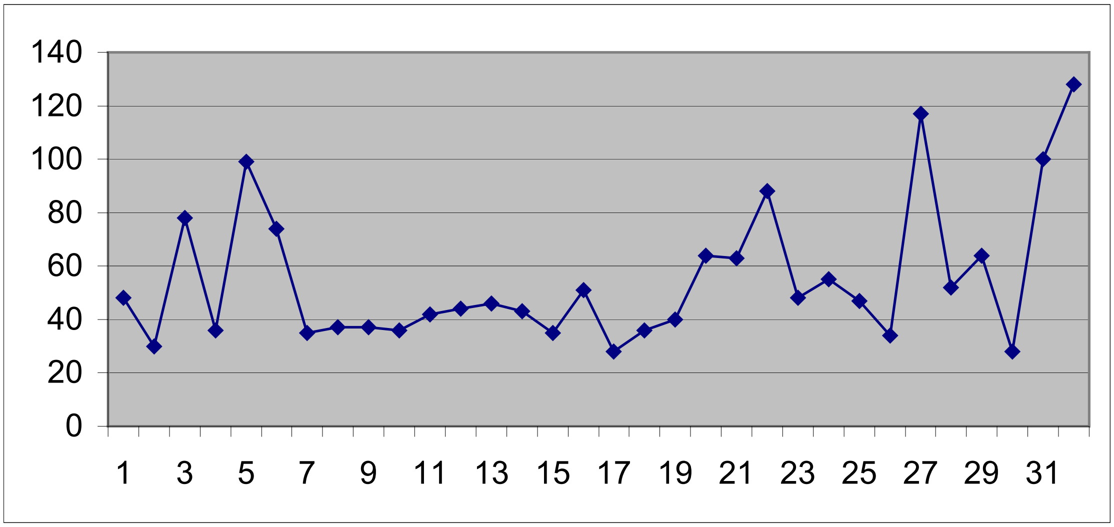

FIGURE 4 The duration of the cycles: 2

Figure 4 is the duration in months of the cycles (Peak to Peak). The vertical axis is the duration in months of the cycles (Peak to Peak) and the horizontal axis means the number of the cycles (Peak to Peak).

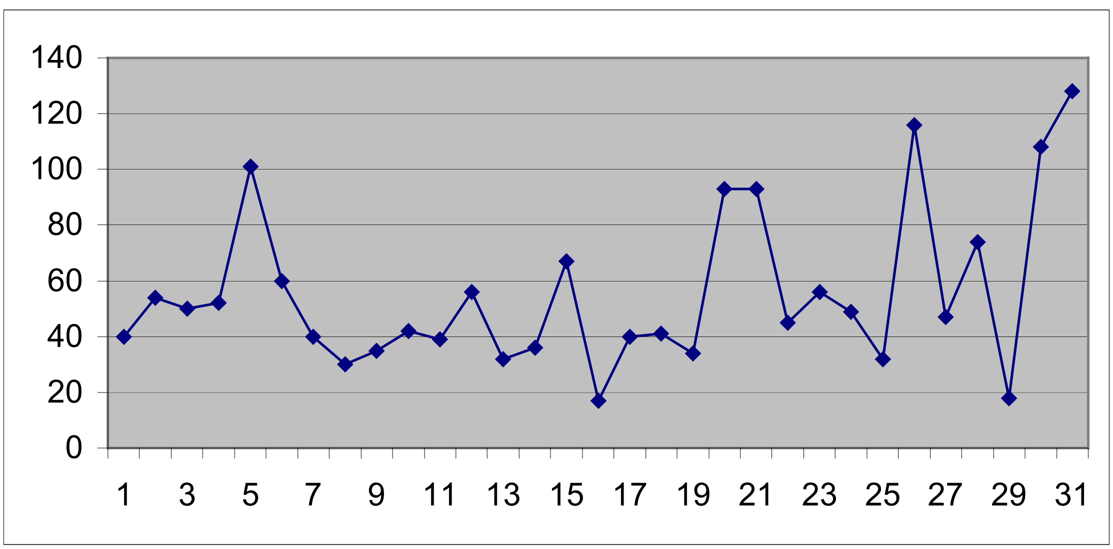

FIGURE 5 The duration of the cycles:3

Figure 5 is the duration in months of the cycles (Trough to Trough). The vertical axis measures the duration in months of the cycles (Trough to Trough) and the horizontal axis means the number of the cycles (Trough to Trough).

Data source:

Figure 1 :(http://www.worldbank.org/research/growth/GDNdata.htm)

Figure 2 to Figure 5: (http://www.nber.org/cycles.html)

APPENDIX

Proof of Proposition 1:

The capital is always full utilized in the production sector before the recession means:

$$K_i^u(t) = K_i(t) \tag{A1}$$

The condition of free-entry into the capital markets implies that:

$$q_i(t) = q(t) = r(t) \forall i \tag{A2}$$

By assuming the prices of intermediate goods as given and the free-entry into the final goods markets, one can get the firms' demand for intermediate *i* is:

$$x_i^d(t) = \frac{Y(t)}{p_i(t)} \tag{A3}$$

Through the cost minimization of the producer of the intermediate goods, one can derive out the unit price of the intermediate *i*. Given factor prices *q (t)* and *w (t)*, entrepreneurs choose the combination of capital and labour that minimizes the cost of producing $x_i(t)$:

$$K_i(t) = \frac{x_i(t)}{A_i^{1-\alpha}(t)}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{w(t)}{q(t)}\right]^{1-\alpha} \text{ and } L_i(t) = \frac{x_i(t)}{A_i(t)^{1-\alpha}}\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{q(t)}{w(t)}\right]^{\alpha} \qquad (A4)$$

The unit cost is:

$$\frac{w(t)}{A_i(t)^{1-\alpha}}\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{q(t)}{w(t)}\right]^{\alpha} + \frac{q(t)}{A_i^{1-\alpha}(t)}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{w(t)}{q(t)}\right]^{1-\alpha} = \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu A_i^{1-\alpha}(t)} \qquad (A5)$$

Since the productivity of the next lower level of technology is $e^{-\gamma} A_i(t)$, where in Francois and Lloyd-Ellis' (2003) the e^{γ} is the technology level jump after a successful innovation and implementation, the limit price is given by:

$$p_i(t) = \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(t)} \qquad (A6)$$

where $u = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

Then I can calculate the instantaneous profit earned in each sector is given by:

$$\pi(t) = (1 - e^{-(1-\alpha)\gamma})Y(t) \qquad (A7)$$

Aggregate final output can be expressed as

$$Y(t) = [K(t)]^{\alpha}[\overline{A}(t)L(t)]^{1-\alpha} \tag{A8}$$

where $\overline{A}(t) = \exp(\int_0^1 \ln A_i(t)di)$.

When I consider the acyclical balanced growth path, assuming that there is a constant amount of skilled labour located in the R&D sector and by using the above equations, one can derive the acyclical balanced growth path, what I need in this part:

Using (A3) and (A6) to substitute for p_i and x_i yields $K_i = K and L_i = L = 1$ for all i with q and w given by:

$$q(t) = \frac{\alpha e^{-(1-\alpha)\gamma} Y(t)}{K(t)} \tag{A9}$$

$$w(t) = (1-\alpha)e^{-(1-\alpha)\gamma} Y(t) \tag{A10}$$

Since $q(t) = r(t) > 0$ and economy goes through an acyclical balanced growth path, capitalist will investment and firms will not make storage, so that:

$$\dot{K}(t) = Y(t) - C(t) \tag{A11}$$

Since the acyclical balanced growth path means all successful innovations are implemented immediately rather than delay, the aggregate rate of productivity growth is

$$g(t) = \delta r H(t) \quad \text{(A12)}$$

No-arbitrage implies that

$$r(t) + \delta H(t) = \frac{\pi(t)}{V^I(t)} + \frac{\dot{V}^I(t)}{V^I(t)} \quad \text{(A13)}$$

Since the R&D sector is also free-entry, so that

$$\delta V^I(t) = w(t) \quad \text{(A14)}$$

From equation (9) and the balanced growth path, it follows that,

$$r(t) = g(\sigma - \beta + \sigma\beta) + \rho \quad \text{(A15)}$$

Differentiating (A10) and (A14) write respect to time, using these to substitute for $\frac{\dot{V}^I(t)}{V^I(t)}$ in (A13), and using (A15) to substitute for *r(t)* and (A7) to substitute for $\pi(t)$, one can get

$$g(\sigma - \beta + \sigma\beta) + \rho + \frac{g}{\gamma} = \frac{\left(1 - e^{-(1-\alpha)\gamma}\right)\delta}{(1-\alpha)e^{-(1-\alpha)\gamma}} + g \quad \text{(A16)}$$

Solving for *g* yields

$$g^{a} = \max\left[\frac{\gamma\delta(1-e^{-(1-\alpha)\gamma}) - \gamma\rho(1-\alpha)e^{-(1-\alpha)\gamma}}{[\gamma(\sigma-1)(1+\beta)+1](1-\alpha)e^{-(1-\alpha)\gamma}}, 0\right] \qquad (A17)$$

To ensure an acyclical balanced growth path, I need $r(t) > g^{a}(t)$, which means delay the implementation of a successful innovation is unprofitable, and this means all successful innovations are implemented immediately rather than delay. Using (A12) and (A15) one can get

$$\rho > (1-\omega)(1-\theta)\delta\gamma H(t) \qquad (A18)$$

Differenting (A10) and (A14) write respect to time, using these to substitute for $\frac{\dot{V}^{I}(t)}{V^{I}(t)}$ in (A13) and $\delta H(t)$ in (A18), the condition to ensure an acyclical balanced growth path is that:

$$\frac{\left(1-e^{-(1-\alpha)\gamma}\right)\gamma(1-\sigma)(1+\beta)}{(1-\alpha)e^{-(1-\alpha)\gamma}} < \frac{\rho}{\delta} \qquad (A19)$$

Proof of Proposition 2:

From the equation (9), (A2), (A8), (A9) and set $L_{i} = L = 1$, one can derive that:

$$\frac{\dot{C}(t)}{C(t)} = \frac{\alpha e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(t)^{\alpha-1} - \rho}{\sigma - \beta + \sigma\beta} \qquad (A20)$$