

Male-Female Wage and Productivity  
Differentials:  
A Structural Approach Using Japanese  
Firm-Level Panel Data

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## **Abstract**

In an attempt to explain male-female wage differential, I estimated the relative marginal productivity and relative wage of female workers compared to those of male workers using panel data of Japanese firms. The relative wage of female workers is also estimated from the same data. Cross-sectional estimates that neglect firm-level, fixed effects indicate that the marginal productivity of female workers is 44 percent of that of male workers, while female wage is 31 percent of that of male workers. These estimates indicate that part of the wage differential cannot be explained by the productivity differential. However, the IV estimates that allow for firm-level, fixed effects indicate that both female marginal productivity and wage are about 50 percent of those of male workers. Thus we cannot reject the null hypothesis of no discrimination against female workers once the selection of workers into productive and high paying firms is accounted for. Evidence found in this study is consistent with the existence of employer sex discrimination at the point of job entry, but not afterward.

Keywords: Sex Discrimination, Wage, Productivity, Panel Data, Fixed Effects

JEL Classification Code: J31

# 1 Introduction

Wage differentials between the sexes are observed worldwide, and these differentials have persisted for a long time. Labor economists, as well as the general public, have argued about why these sex-wage differentials exist.

There are two major explanations for the sex wage differential (of course, this is not an exhaustive list of such explanations). First, the wage differential simply may reflect the productivity differential between two sexes. Second, the differential may be due to employer sex discrimination. If the marginal female worker is employed by a discriminatory employer, then a male-female wage differential emerges in the market equilibrium.

Labor economists have been using multiple regression to distinguish these two hypotheses. They regress the log of wage on independent variables that presumably capture workers' productivity and the residual male-female wage differential is attributed to sex discrimination. However, drawing a definitive conclusion from this regression is difficult because the productivity differential between the sexes that cannot be observed may be included in the residual, as discussed in Becker [1985] (See Altonji and Blank [1999] for this literature).

In an attempt to overcome the difficulties mentioned above, this paper directly estimates the relative productivity of male and female workers by estimating the production function using Japanese firm-level panel data. This estimated productivity differential is compared with the wage differential es-

estimated from individual firms' accounting data. This exercise reveals whether the wage differential is due to the productivity differential. This approach has been employed by Hellerstein and Neumark [1999] and Hellerstein et al. [1999] to analyze Israeli and US cross-sectional data respectively. They found supportive evidence for sex discrimination in the US but not in Israel; they found a larger sex wage gap than productivity gap in the US, but not in Israel.

It is worth mentioning a related approach for identifying sex discrimination using firm-level data. This approach examines the empirical implication of employers' discrimination theory. If the sex composition of workers in each firm is determined to maximize its profit, the sex composition of workers should not affect a firms' profits after conditioning on output and input prices because of the envelope theorem (Hotelling's lemma). This result no longer holds once the firms' objectives include satisfying the employers' preference for sex discrimination. If employers' objectives are heterogeneous and the equilibrium male-female wage differential reflects discrimination against women, then those employers without discriminatory preferences against women should hire more women and earn higher profits than discriminatory employers. This hypothesis was tested by Hellerstein et al. [2002] using US data and by Kawaguchi [2003] using the Japanese firm-level panel data that are used in this paper. Both papers found evidence that is consistent with the existence of sex discrimination. Kawaguchi [2003] also found that the firms that persistently earn high profits tend to hire fewer women; thus,

cross-sectional estimates of the effect of female proportion on profit are likely to be biased downward. However the estimates implied that most of the male-female wage differential is the product of the productivity differential.

This paper attempts to complement Kawaguchi [2003] by obtaining structural parameters, and consequently, it sheds light on the mechanism of the male-female wage differential more directly. Panel data adds a beneficial feature to this study that is preferable to previous structural studies by Hellerstein and Neumark [1999] and Hellerstein et al. [1999]. Using this panel data, I can allow for the heterogeneity in the individual firms' productivity that may be correlated with the sex composition of their workers. Controlling for unobserved technological heterogeneity across firms is important because those firms with productive technology may well accommodate female workers, while at the same time earning higher profits due to their productive technology.

The rest of this paper is organized as follows. Section 2 introduces the structural model of the production function and wage equation. This section also discusses the method of estimation. Section 3 explains the data used in this study. Section 4 reports the results, and Section 5 concludes.

## **2 Structural Model**

To estimate the marginal product of the labor of male and female workers, I need to specify the functional form of the production function. I assume the Cobb-Douglas production function, as in Hellerstein and Neumark [1999]

and Hellerstein et al. [1999]. The production function is specified as:

$$y_{it} = (\exp a_i)(ql_{it})^\alpha k_{it}^\beta m_{it}^\gamma (\exp u_{it}), \quad (1)$$

where  $i$  and  $t$  are the subscripts for firm and time respectively,  $a_i$  is the firm-specific, time-constant technology,  $ql$  is the labor input that is measured in efficiency units,  $k$  is capital input,  $m$  is the intermediate input, and  $u$  is the unobserved idiosyncratic shock to production. The labor input in the efficiency unit is the weighted sum of the numbers of male employees and female employees as follows:

$$ql_{it} = l_{m,it} + \psi l_{f,it} = l_{it}(1 + (\psi - 1)(\frac{l_f}{l})_{it}), \quad (2)$$

where  $l_m$  stands for the number of male employees,  $l_f$  stands for the number of female employees, and  $l$  stands for the total number of employees. The parameter  $\psi$  indicates the relative productivity of female workers to male workers. By taking a logarithm of (1) and substituting (2) into  $ql$ , we obtain:

$$\ln y_{it} = a_i + \alpha \ln(l_{it}(1 + (\psi - 1)(\frac{l_f}{l})_{it})) + \beta \ln k_{it} + \gamma \ln m_{it} + \text{ind}\delta + \text{year}\tau + u_{it}. \quad (3)$$

I included one-digit industry dummies (in nine categories) to allow for the differences in  $a$  across industries. Time dummies presumably capture the effect of macro-economic shocks and inflation. Parameters in this equation can be consistently estimated by a pooled, nonlinear, least-squares estimation under the strict exogeneity assumption of idiosyncratic error:

$$E(u_{it}|a_i, l_i, f_i, k_i, m_i, \text{ind}, \text{year}) = 0 \quad (4)$$

and the strict exogeneity of firm-specific technology:

$$E(a_i | l_i, f_i, k_i, m_i, ind, year) = 0 \quad (5)$$

where  $x_i \equiv [x_{i1}, x_{i2}, \dots, x_{iT}]$ . The second assumption is violated when an individual firm has its own technology, which affects the optimal input mix. This assumption is relaxed by estimating (3) by a nonlinear instrument variable estimation, using the following mean deviation variables as instruments:

$$(l_{it} - \bar{l}_i), \left( \left( \frac{f}{l} \right)_{it} - \left( \frac{f}{l} \right)_i \right), (\ln k_{it} - \ln \bar{k}_i), (\ln m_{it} - \ln \bar{m}_i), \quad (6)$$

where  $\bar{x}_i = (1/T) \sum_{t=1}^T x_{it}$ . These instrumental variables  $x_{it} - \bar{x}_i$  are correlated with  $x_{it}$ , if  $x_{it}$  is time variant, and not correlated with  $a_i$  because  $E[(x_{it} - \bar{x}_i)a_i] = E[(x_{it} - (1/T) \sum_{t=1}^T x_{it})a_i] = E[x_{it}a_i] - (1/T) \sum_{t=1}^T E[x_{it}a_i] = 0$ .

The estimated relative productivity of women is compared to the relative wage of women. The data used in this study contain the total wage bill, but do not contain its sex breakdown. Thus, I estimate the relative wage of women under the assumption that all firms behave as price takers. The total wage bill is defined as:

$$wb_{it} = w_{m,t}l_{m,it} + w_{f,t}l_{f,it} = w_{m,t}(l_{it} - l_{f,it} + \frac{w_f}{w_m}l_{f,it}) = w_m l_{it} (1 + (\lambda - 1) \frac{l_{f,it}}{l_{it}}), \quad (7)$$

where  $wb_{it}$  is the wage bill,  $w_m$  is male wage,  $w_f$  is female wage, and  $\lambda$  is the relative female wage to male wage. This equation is a definitional equation, rather than a behavioral equation. I estimate this equation by taking a natural logarithm and allowing for unobserved factors. The estimation

equation is:

$$\ln(wb_{it}/l_{it}) = \ln w_m + \ln(1 + (\lambda - 1)(\frac{l_f}{l})_{it}) + ind\delta + year\tau + e_{it}, \quad (8)$$

where  $e_{it}$  is the error term that satisfies  $E(e_{it}|l_{m,it}, l_{f,it}, ind, year) = 0$ . I included industry dummies, assuming that the inter-industry wage differential could persist because of friction in the labor movement across industries. Year dummies capture the effect of inflation or macro-economic shock. This estimated relative wage of women  $\lambda$  is compared to the relative female productivity  $\psi$ . A consistent estimation of parameters is possible via NL2SLS when independent variables are exogenous.

### 3 Data

I used the basic survey of firms' activity collected by the Ministry of Economy, Trade, and Industry (METI) of the Japanese government to implement the test. The survey is a firm-level census survey that covers all firms that hire more than 50 employees and hold more than 30 million yen in capital. The available data cover 7 years, 1992 and every year between 1995 and 2000; and the sample size is about 25,000 firms for each year. From the data sets, I extracted each firm's total sales, sales cost, or overhead cost, data on the firm's employees, such as the number of employees with sex breakdown, the book value of its fixed assets, the year of the firm's origin, and the three-digit code indicating the industry in which the firm operates. There were originally 180,838 firm-year observations in the 7 years of data, but



after excluding observations with missing sales information or inconsistent employee records, there remained 177,868 firm-year observations. The survey record unfortunately does not distinguish missing values and zeros, except when a firm did not answer the entire survey. Since replying to the survey is compulsory due to the Statistics Law and because the METI exerts its best effort to fill in the missing values with a follow-up phone survey, missing values are presumably rare. Thus, all values of zero in the record are treated as actual zeros.

The descriptive statistics of the analysis sample are reported in Table 1.

## 4 Results

### 4.1 Nonlinear, Least-Squares Estimates

The results of separate estimations of the production function (3) and the wage equation (8) appear in Table 2. The estimate of  $\psi$  indicates that the marginal product of female workers is 44 percent of that of male workers. On the other hand, the estimate of  $\lambda$  indicates that female workers earn 31 percent of what male workers earn.

Table 3 reports the results of the joint estimation of the production function and wage equation, allowing for the correlation of error terms across equations. A generalized, non-linear, least-squares estimation was applied for the system, using the estimated variance matrix under the homoscedasticity assumption as the weight. The point estimates virtually did not change from the results of the separate estimations, but standard errors grew by about ten

times. This joint estimation allows us to estimate  $\psi - \lambda$  and the associated standard error. The estimate of the difference is 0.14, with a standard error of 0.08. Thus, the null hypothesis of no discrimination against women, which is  $H_0 : \psi - \lambda = 0$ , is marginally rejected ( $t = 1.75$ ) at a 10% significance level.

If we take the point estimates seriously, of the 69 percentage points of wage differentials observed in data, 14 percentage points cannot be explained in the productivity difference between men and women. Thus 20 percent of male-female wage differential ( $=0.14/0.69$ ) arguably can be attributed to employers' discrimination.

## 4.2 Nonlinear IV Estimates (Fixed Effects Estimates)

The pooled, non-linear, least-squares estimator discussed in the previous section is a consistent estimator when each firm's time-constant, unobserved heterogeneity is not correlated with the firm's input mix. This is a rather restrictive assumption because if there is firm-specific heterogeneity in production technology, then the optimal input mix is likely to be heterogenous. If each firm's technological heterogeneity is correlated with inputs, the pooled, non-linear, least-squares estimator is an inconsistent estimator. To work around this potential endogeneity issue, the production function and wage equations are estimated via a non-linear, instrumental variable estimation using the mean deviation of explanatory variables from each firm's mean. These mean deviation variables are not correlated with firms' time-constant,

unobserved heterogeneity by its construction. In this estimation, only within-firm variations of input mix over time are used for the identification. To assure that the idiosyncratic error term of (3) is exogenous from each firm's mean of independent variables, the strict exogeneity of the error term stated as (4) is required.

I also allow for the firm's time-constant heterogeneity in the wage equation (8) and estimate the following wage equation:

$$\ln((wb/l)_{it}) = \ln w_m + \ln(1 + (\lambda - 1)\frac{l_{fit}}{l_{it}}) + ind\delta + year\tau + d_i + e_{it}, \quad (9)$$

where  $d_i$  is time-constant, firm-level, unobserved heterogeneity that affects the per capita labor cost. This firm-level heterogeneity could arise due to the heterogeneity of workers' quality across firms. Even if firms operate in a perfectly competitive labor market and pay the same wage for an efficiency unit of labor, those firms that hire eligible workers pay a higher wage per capita. If the quality of workers in a specific firm is time-constant, then the effect of heterogeneity of workers' quality is captured by  $d_i$ . If male workers are more skilled on average,  $d_i$  and female worker proportion are negatively correlated. Accordingly, the pooled, non-linear, least-squares estimator of  $\lambda$  is downward inconsistent. On the other hand, a non-linear IV estimation that uses the mean deviation of independent variables from each firm's mean renders a consistent estimator.

Table 4 reports the results of the non-linear, least-squares estimation applied to each equation separately. The result in Column (1) shows that

female workers' productivity relative to male workers' is 55 percent. Compared with the cross-sectional estimate reported in Column (1) of Table 2, this number is 11 percentage points higher, which implies that  $a_i$  and  $\frac{l_{fit}}{l_{it}}$  are negatively correlated. We can roughly test whether these two estimates are significantly different in a statistical sense by using Hausman statistics. Under the homoscedasticity assumption for the idiosyncratic error term, the non-linear, least-squares estimator is an efficient estimator under the null of no correlation between  $a_i$  and  $\frac{l_{fit}}{l_{it}}$ . Accordingly, Hausman statistics can be constructed for the difference of these two estimators as

$$H = (\hat{\psi}_{NLIV} - \hat{\psi}_{NL})^2 / (\text{Var}(\hat{\psi}_{NLIV}) - \text{Var}(\hat{\psi}_{NL})) \sim \chi(1). \quad (10)$$

for the two estimates,  $\hat{H} = 4.89$  ( $p = 0.03$ ), and thus we can conclude that the two estimates are different in a statistical sense. The implied positive correlation between  $a_i$  and  $\frac{l_{fit}}{l_{it}}$  is consistent with the hypothesis that firms with a technological advantage hire fewer women because employers face less pressure of market competition and have room to indulge their preference for discrimination against females. This finding is consistent with the finding in Kawaguchi [2003]. This earlier study found that firms with a persistent, high-profit factor tend to hire fewer women.

As for the relative payment to female workers, Column (2) of Table 4 shows that they receive 52 percent of male workers' wage. The difference between this estimate and the cross-sectional estimate is 0.21 ( $=0.52-0.31$ ), and the Hausman statistics for this difference are  $\hat{H} = 459.38$  ( $p < 0.000$ ). This

difference of estimates implies that high-paying firms tend to hire fewer female workers. Different from the results of the pooled, non-linear estimation, this relative payment is very comparable to relative productivity. Once conditioned on the firms where they work, female workers seem to receive their wage according to their productivity. To make this point more rigorously, I estimated the production function and wage equation jointly by using a non-linear 3-step, least-squares technique and estimating  $\phi - \lambda$  and the associated standard error. The estimation result appears in Table 5. Due to the correlation of the idiosyncratic error terms of the two equations, the estimates for the production function differ from the estimates from the previous, separate estimation. Now the relative productivity of female workers to male workers is estimated to be 50 percent, while the relative payment is 52 percent. The parameter  $\phi - \lambda$  is precisely estimated to be  $-0.02$  ( $s.e = 0.0006$ ). This difference is economically negligible. From this result, I conclude that relative wage of female workers compared to that of male workers reflects their relative productivity if they work for the same company.

### 4.3 Discussion

A comparison of the pooled, non-linear estimates and the IV non-linear estimates reveals that women are less likely to work in higher productivity and higher wage firms (i.e.  $Cov(d_i, (l_f/l)_{it}) < 0$  in (3) and  $Cov(a_i, (l_f/l)_{it}) < 0$  in (9)). Female workers' selection into low productivity firms may be due to their low productivity because high technology and high skills can be com-

plemented in the production process, or simply a part of  $a_i$  may reflect the workers' skill level.

The fact that I did not find a larger male-female wage differential than productivity differential does not imply the non-existence of discrimination against women because I cannot pinpoint the reason why women are less productive. This may be simply because women are less productive or because employers with productive technology discriminate against women at the time of hiring. Taking an example from anti-age-discrimination legislation, Posner [1999] pointed out that many legal disputes are observed among workers and firms that are already have contracts, for example when there are issues related to dismissal, promotion, or wage payment. Job applicants who are rejected for a reason that may be due to discrimination have only a weak incentive to sue the firms because they can try other employers who may be non-discriminatory. Thus, discrimination is likely to occur at the entry point of jobs. This discussion also may apply to the Japanese case. Further investigation is needed to determine whether there is discrimination against women. In particular, why women are less likely to work in less productive and low wage firms should be investigated, perhaps using employer-employee matched data.

## 5 Conclusion

In an attempt to explain the male-female wage differential, I estimated the relative marginal productivity and relative wage of female workers to male

workers using a panel data of Japanese firms. Estimates that were obtained by neglecting individual firm's heterogeneity indicated that the wage differential is larger than the productivity differential between female and male workers. Preferable estimates indicate that the marginal productivity of female workers is 44 percent of that of male workers while the female wage is 31 percent of that of male workers. These estimates are consistent with employers' discrimination against women.

However, the IV estimates, which allow for correlated firm-level Heterogeneity, indicated that both female workers' marginal productivity and wage are around 50 percent of male workers'. Thus we cannot reject the null hypothesis of no discrimination against female workers once the selection of workers into firms is controlled for.

We cannot draw a definitive conclusion regarding why female workers are likely to select into low productivity and low paying firms from this data. It may be either because of their lower skill or because of the discrimination against them. This evidence could be consistent with the employers' discrimination against women at the entry point to the higher paying jobs.

More detailed study on the selection of workers into firms is very important to identify the reason for the male-female wage differential.

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Table 1: Descriptive Statistics

Sample Period: 1992, 1995-2000

Number of Observations: 177868 (Number of Firms: 37131)

Variable Name	Mean	Standard Deviation
Log (Total Sales)	8.68	1.29
Log (Wage Bill/Total Employment)	1.48	0.40
Log (Total Employment)	5.17	0.97
Female Proportion	0.32	0.20
Log (Fixed Assets)	6.99	1.62
Log (Cost of Materials)	8.16	1.48
Industry		
Agriculture, Forestry, and Fishery	0.0004	
Mining and Construction	0.02	
Light Manufacturing	0.16	
Material Manufacturing	0.17	
Electronics and Machinery	0.22	
Public Utilities and Transportation	0.003	
Wholesale and Retail	0.41	
Finance, Insurance, and Real Estate	0.001	
Service	0.02	

Table 2: Non-Linear Separate Estimation of the Production Function and Wage Equation

	(1)	(2)
	Production Function	Wage Equation
$\varphi, \lambda$	0.44 (0.005)	0.31 (0.002)
Log (Employment)	0.35 (0.001)	-
Log (Asset Fix)	0.04 (0.0006)	-
Log (Cost Material)	0.64 (0.0007)	-
Constant	1.42 (0.03)	1.55 (0.04)
$R^2$	0.96	0.29

Note: Standard errors are in parentheses. Year and industry dummies are included. N=177868.

Table 3: Non-Linear Joint GLS Estimation of the Production Function and Wage Equation

	(1)	(2)	(3)
	Production Function	Wage Equation	$\varphi - \lambda$
$\varphi, \lambda$	0.44 (0.07)	0.31 (0.02)	0.14 (0.08)
Log (Employment)	0.35 (0.01)	-	
Log (Asset Fix)	0.04 (0.004)	-	
Log (Cost Material)	0.64 (0.004)	-	
Constant	1.42 (0.50)	1.55 (0.30)	

Note: Standard errors are in parentheses. Year and industry dummies are included. N=177868.

Table 4: Non-Linear IV (NL2SLS) Estimation of the Production Function and Wage Equation

	(1)	(2)
	Production Function	Wage Equation
$\varphi, \lambda$	0.55 (0.05)	0.52 (0.01)
Log (Employment)	0.30 (0.02)	-
Log (Asset Fix)	0.02 (0.002)	-
Log (Cost Material)	0.51 (0.05)	-
Constant	2.70 (0.07)	1.44 (0.04)
$R^2$	0.93	0.25

Note: Standard errors are in parentheses. Year and industry dummies are included.  
N=177868.

Table 5: Non-Linear IV Joint (NL3SLS) Estimation of the Production Function and Wage Equation

	(1)	(2)	(3)
	Production Function	Wage Equation	$\varphi - \lambda$
$\varphi, \lambda$	0.50 (0.003)	0.52 (0.01)	-0.02 (0.006)
Log (Employment)	0.41 (0.002)	-	
Log (Asset Fix)	0.00008 (0.0003)	-	
Log (Cost Material)	0.52 (0.0004)	-	
Constant	2.25 (0.007)	1.44 (0.004)	

Note: Standard errors are in parentheses. Year and industry dummies are included.  
N=177868.