Some Methods for Assessing the Need for Non-linear Models in Business Cycle Analysis

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1. Setting the Scene

Writers on the business cycle and forecasting often emphasize that non-linear models are needed to account for the former phenomenon and to improve the latter. On the business cycle front it is often said that either the asymmetry of the duration of business cycle expansions and contractions or the variability of these quantities demand a non-linear model. Such comments are rarely made precise however and mostly consist of references to such assertions from the past. Thus the asymmetry in the cycle is mostly accompanied by references to Keynes (1936) and Burns and Mitchell (1946). But these authors were looking at what we call today the "classical "cycle i.e. movements in the level of GDP, and so the fact that there are long expansions and short contractions can arise simply due to the presence of long-run growth in the economy. It seems possible that there is little asymmetry left over once one has accounted for the consequences of long-run growth. If so there would be little contribution to the explanation of this phenomenon from using a non-linear statistical model of the cycle.

In this paper we subject this view that non-linear models are important to an explanation of business cycles to some critical analysis. In section 2 we discuss ways of measuring the characteristics of the business cycle and discuss which features of the U.S. cycle cannot be reproduced by a linear model. Section 3 then utilizes data on US GDP to consider the characteristics of cycles generated by three non-linear models that have recently been proposed.

1. The SETAR model of van Dijk and Franses(2003)
2. The bounceback model of Kim, Morley and Piger (2002)
3. The tension index model of De Jong, Liesenfeld and Richard (2003).

In each case we measure how much each of these models contributes to the explanation of the primary features of the business cycle. We regard the latter as being

1. Asymmetries of phases.
2. The shape of phases.
3. The variability of phases.
4. The nature of the transition between phases.

We find that the non-linear models add little to the explanation for the asymmetry of phases, provide a little of the explanation of the shape of phases, have the potential to explain why expansions are *actually less variable* in the data than predicted by linear models, and, finally, that the use of these non-linear models does not provide a better explanation of the transition between phases than that available from linear models. Indeed, one might well argue that it is worse. In the latter context we introduce the idea of separating out the influence on the business cycle of contemporaneous shocks from lagged ones. Effectively this gives us a measure of how much of the cycle is due to unpredictable events from those that are predictable owing to their dependence on the past. Technically this decomposition might be regarded as isolating the role of impulses and the propogation mechanism in producing a cycle. By eliminating the influence of an unknown exogenous event, the current shock, it may also provide some insight into the ability to forecast cyclical developments.

2. Measuring the Cycle

2.1. The Average Cycle

Much of our discussion of business cycle features will be assisted by Figure 1 which shows a stylized recession. There is an equivalent one for expansions. Here the y axis is y_t (the log of the level of economic activity, Y_t) and the x axis is time. Thus the graph shows a peak at A and a trough at C. The length of AB is the duration of the recession and the vertical distance between A and C is the amplitude of the cycle, effectively expressed as a fraction of the peak, since it is the difference $logY_C - \log Y_A$. Once we know where the turning points are located in time, we can compute these measures. The method we will use is the BBQ algorithm of Harding and Pagan (2002). It was shown in Harding and Pagan (2003) that this rule provides a good reproduction of the turning points in the US cycle selected by the NBER, as one might expect from the comments by the NBER Business Cycle Dating Committee in dating the turning points of the 2001 recession -see “The NBER’s Recession Dating Procedure” at http://www.nber.org/cycles/recessions.html.

In the BBQ algorithm turning points in the *business cycle* at time t are defined in the following way.

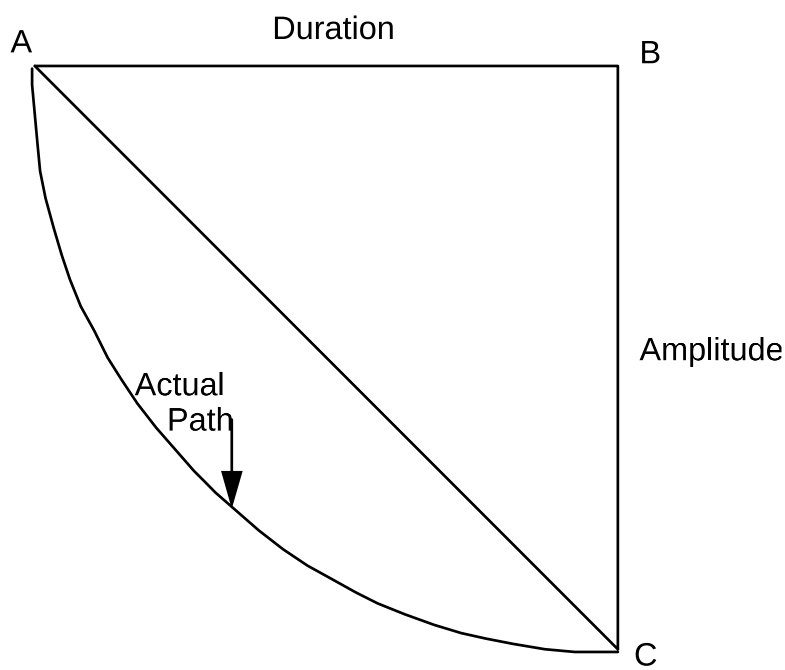

Figure 2.1:

$$\begin{aligned} peak\ at\ t &= \{(y_{t-2}, y_{t-1}) < y_t > (y_{t+1}, y_{t+2})\} \\ trough\ \ at\ t &= \{(y_{t-2}, y_{t-1}) > y_t < (y_{t+1}, y_{t+2})\}. \end{aligned} \tag{2.1}$$

These definitions could be re-expressed as

$$\begin{aligned} peak\ at\ t &= \{(\Delta_2 y_t, \Delta y_t) > 0, (\Delta y_{t+1}, \Delta_2 y_{t+2}) < 0\} \\ trough\ \ at\ t &= \{(\Delta_2 y_t, \Delta y_t) < 0, (\Delta y_{t+1}, \Delta_2 y_{t+2}) > 0\} \end{aligned} \tag{2.2}$$

where $\Delta_2 y_t = y_t - y_{t-2}$. In words, a recession occurs if the level of economic activity declines for over a single quarter as well as over a six monthly period, while an expansion needs increases over the same intervals. In practice, the Bry and Boschan (1971) algorithm from which BBQ was derived also applied some extra *censoring* procedures to the dates that emerged from using the above rule. In particular, the contraction and expansion phases must have a minimum duration of six months and a completed cycle must have a minimum duration of fifteen months. We emulate this by imposing two quarter and five quarter minima on the phase lengths and complete cycle duration respectively. Another important operation in BBQ is to ensure that peaks and troughs alternate. When this does not happen the excess ones must be eliminated by choosing between them with rules such as selecting the date with the lower (higher) value of y_t for troughs (peaks). This form of censoring is extremely important in Monte Carlo simulations and great care has to be taken to ensure that it is properly applied when dating cycles with simulated data. In this paper we have used an extensively re-written version of the program employed in Harding and Pagan (2002) in order to ensure that the alternation is performed accurately. Some differences have emerged to those reported previously.

Let the turning point dates produce K expansions and contractions with the duration of the $i'th$ $(i = 1, ..., K)$ expansion being D_i^E and that of the contractions being D_i^C. Then, dropping the i subscript, the quantities summarizing durations of the average phases will be

$$\overline{D}^E = \frac{1}{K} \sum_{i=1}^{K} D_i^E, \overline{D}^C = \frac{1}{K} \sum_{i=1}^{K} D_i^C.$$

Generally these refer to completed durations i.e. if the expansion (contraction) is still on-going at the end (beginning) of the sample period it will not be counted when computing the average. It is also possible to produce the same measures $\overline{A}^E$ and $\overline{A}^C$ for the amplitudes of the phases of the average cycle.

But the graph shows that there are other things we might like to measure. One of these is the cumulated gain or loss during a phase i.e. the area under the curve that describes the actual path for the log of GDP. This is measured by (see Harding and Pagan (1992))

$$F = \sum_{j=1}^{D}(y_j - y_0) - \frac{A}{2}.$$

A useful comparison can be made of F with the area of the triangle ABC ($AR = \frac{D \times A}{2}$) and the quantity we use for this is the *excess area*

$$E = \frac{F - AR}{D}.$$

This shows how much extra output per quarter is gained or lost during an expansion or contraction as compared to the situation if the economy had expanded or contracted at a constant growth rate. It is computed for both phases, although it is unlikely to be very reliable for contractions, as these are very short.

2.2. The Diversity of Cycles

It is also possible to compute all the characteristics mentioned above for the average cycle for each individual cycle. But this produces a large amount of information that is hard to readily compare with the equivalent quantites from simulated data. Consequently we seek some summary characteristics of the diversity of cycle outcomes. Simple indices of this type come from coefficients of variation, in particular the ratio of the standard deviation of the durations and amplitudes to their means. Thus, for durations we have

$$CV_D^E = \frac{\sqrt{(\frac{1}{K}\sum_{i=1}^{K}(D_i^E - \overline{D}^E)^2)}}{\frac{1}{K}\sum_{i=1}^{K} D_i^E}$$

$$CV_D^C = \frac{\sqrt{(\frac{1}{K}\sum_{i=1}^{K}(D_i^C - \overline{D}^C)^2)}}{\frac{1}{K}\sum_{i=1}^{K} D_i^C}$$

The same quantities can be computed to measure the variability of amplitudes.

2.3. The Transition Between Phases

It will prove useful to think about the rules above as marking a move from one phase (state) to another. Thus a peak at time t demarcates a state of expansion at time t, $S_t = 1$, from a state of contraction at time $t+1$, $S_{t+1} = 0$. The states S_t are then a binary Markov process which can be summarized with the transition probabilities $\Pr(S_{t+1}|S_t)$. As we will see later, examination of these quantities will prove to be fruitful.

In fact it is hard to analytically derive the transition probabilities for the BBQ rule. A rule that is close to it and which is amenable to analysis is that two periods of negative (positive) growth after a point in time t, when an expansion (contraction) held at t, initiates a contraction (expansion). When y_t follows a random walk with drift, the transition probabilities coming from application of this modified rule can be shown to depend only upon S_{t-1}, with no further dependence on Δy_{t-1}. But most GDP series have serial correlation in their growth rates. To gain some appreciation of how this is likely to affect transition probabilites we use a simple dating rule that has been called the calculus rule in which the phases are defined as $S_t = 1(\Delta y_t > 0)$ i.e when there is a positive growth rate at time t, the economy is an expansion (E) state while a negative one signifies a contraction (C). Now, if $\Delta y_t = \mu + \sigma e_t$, where e_t is $i.i.d.(0,1)$, then the probabilities of a change in phase at time t would be

$$
\begin{aligned}
\Pr(EC) &: \Pr(S_{t+1} = 0|S_t = 1) = \Pr(\Delta y_{t+1} < 0|\Delta y_t > 0) \\
\Pr(CE) &: \Pr(S_{t+1} = 1|S_t = 0) = \Pr(\Delta y_{t+1} > 0|\Delta y_t < 0),
\end{aligned}
$$

where $P(EC)$ is the probability of a switch from an expansion to a contraction state etc. Let us look at $\Pr(EC)$ which, in this context, has the form

$$
\begin{aligned}
\Pr(\Delta y_{t+1} &< 0|\Delta y_t > 0) = \Pr(\mu + \sigma e_{t+1} < 0|\Delta y_t > 0) \\
&= \Pr(e_{t+1} < -\frac{\mu}{\sigma})
\end{aligned}
$$

due to the independence of Δy_t. If e_t was $N(0,1)$ then we would have $\Pr(EC) = \Phi(-\frac{\mu}{\sigma}) = \psi$ and $\Pr(CE) = 1 - \Phi(-\frac{\mu}{\sigma})$, where Φ is the distribution function of an $N(0,1)$ random variable. So the probability of a turning point depends solely upon $\frac{\mu}{\sigma}$ i.e. the ratio of long-run growth to the volatility of shocks. The modified rule discussed above is also found to produce transition probabilities that depend solely upon $\frac{\mu}{\sigma}$.

Now consider what happens when there is serial correlation in the growth rates of output i.e.

$$\Delta y_t = \mu + \rho \Delta y_{t-1} + \sigma e_t$$

where e_t is $n.i.d.(0,1)$. Then we can write the transition probability $P(EC)$ as

$$\begin{aligned}
\Pr(\Delta y_t &< 0|\Delta y_{t-1} > 0) = \Pr(\mu + \rho\Delta y_{t-1} + \sigma e_t < 0|\Delta y_{t-1} > 0) \\
&= \Pr(e_t < -a - \phi\Delta y_{t-1}|\Delta y_{t-1} > 0) \\
&= \int_0^{\infty} [\int_{-\infty}^{-\alpha-\phi z} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}e^2)de)]dz \qquad (2.3) \\
&= \int_0^{\infty} \Phi(-\alpha - \phi z)dz,
\end{aligned}$$

where $\Phi(\cdot)$ is the cumulative standard normal, $\alpha = \frac{\mu}{\sigma}, \phi = \frac{\rho}{\sigma}$ and $z = \Delta y_{t-1}$. Now consider the derivative of $\Phi(-\alpha - \phi z)$ with respect to ϕ. From Liebniz' rule this is

$$-\frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}(-\alpha - \phi z)^2)z$$

and so a rise in ϕ (given $\phi > 0$) will cause a decline in the probability of moving from an expansion to a contraction. This makes sense given that positive serial correlation in the growth rate (Δy_{t-1}) makes it more likely that Δy_t will be of the same sign, and so the expansion state is likely to be preserved. To map this into the length of an expansion is more complex however. As Harding and Pagan (2002, p373) show

$$\Pr(S_t = 1) = \frac{\Pr(CE)}{\Pr(EC) + \Pr(CE)}$$

and, from the arguments above, $P(CE)$ would also decline with rises in ϕ, since negative growth rates would likely be perpetuated. The upshot of such consideras-tions will be that the effect of serial correlation in growth rates upon the length of expansions is likely to be indeterminate.

From the analysis above we would expect that the probability of switching from an expansion to a contraction state depends upon the magnitude of Δy_{t-1} for any dating rule. When there is a small positive growth rate observed during $t-1$ there is a bigger chance of a switch to a recession phase than if the Δy_{t-1} had been larger. This is sensible due to the positive serial correlation and it points to the fact that it will be useful to look at how $P(EC)$ (or its equivalent $P(EE) = 1 - P(EC)$)

will vary with Δy_{t-1} in the data and with different models. In passing, it should be noted that it has often been said that a turning point rule does not enable one to predict the change in state, and the preceding analysis shows that to be a fallacy. Of course it may not be easy to determine the mapping between the conditional probability and Δy_{t-1} when there are more realistic DGP's and dating rules. Later we will use non-parametric estimation methods to compute the transition probabilities for many models for which analytic solutions are not readily available.

One also has to give some thought as to what conditional probability should be analysed. It is natural to think of $\Pr(S_t|S_{t-1}, \Delta y_{t-1})$, but this may be less useful than it would appear to be when the dating rule is the BBQ one. To see why, suppose we try to find how $\Pr(S_t|S_{t-1} = 1, \Delta y_{t-1})$ varies with Δy_{t-1} when the BBQ rule is used. The fact that $S_{t-1} = 1$ means that the expansion did not terminate in $t-2$. This puts constraints on the relation between y_t, y_{t-1} and y_{t-2}. There are a number of these but one that is particularly relevant occurs when $y_{t-1} < y_{t-2}$. When such growth eventuates, in order to ensure that a peak does not occur at $t-2$ it will be necessary that $y_t > y_{t-2}$ i.e. we must have

$$\{\Delta y_{t-1} \quad < \quad 0, \Delta y_t > 0 > |\Delta y_{t-1}|\}$$

Now consider what happens if we experiment with a negative value for Δy_{t-1} but $S_{t-1} = 1$. As just seen this means that a positive value for Δy_t is necessary. But, if that happens, it is impossible for $S_t = 0$, since the positive value for Δy_t means that $y_t > y_{t-1}$ and so one cannot satisfy the criterion for a peak at $t-1$ ie $S_t \neq 0$. Such an outcome would not arise with the calculus rule. It occurs with the BBQ rule since knowledge about S_{t-1} and Δy_{t-1} implies some knowledge of future growth outcomes. In a prediction environment the simplest way to avoid the constraints which arise from the use of the BBQ determined dates is to work with $\Pr(S_t|S_{t-2}, \Delta y_{t-1})$, since then we are not using any information about Δy_t. Consequently, in later work we use $\Pr(S_t|S_{t-2} = 1, \Delta y_{t-1})$ to compare models of the cycle.

3. Accounting for the Cycle with Statistical Models

3.1. The Value of Linear Models

We can fit a number of statistical models to US GDP over a given data period. Unfortunately, the three non-linear models that we consider have all been estimated over different data periods. We will act as if they actually apply to 1947/1-2002/2, as we believe that any biases would be such as to reinforce our conclusions.

We use two linear models. With y_t being the log of US GDP the models fitted to data over 1947/1-2002/2 are a random walk with drift for y_t and an AR(1) in growth rates Δy_t.

$$\begin{aligned} RW &: \Delta y_t = .00836 + .0102 e_t \\ AR(1) &: \Delta y_t = .0055 + .3438 \Delta y_{t-1} + .0096 e_t \end{aligned}$$

Data was simulated from each of these models assuming that e_t is $n.i.d.(0,1)$ and then passed through the BBQ program to produce the range of measures summarizing the business cycle discussed above. One thousand simulated series were generated. Table 1 compares the actual and simulated characteristics. Quantities that relate to amplitudes, cumulated movements and excess area are all multiplied by 100 to make them percentage changes. Focussing on the duration and amplitude measures, it is clear that an AR(1) in growth rates can replicate the US cycle very well, but the random walk with drift model produces expansions that are much too strong. This role of positive serial correlation in growth rates as being needed to produce realistic expansions was noted in Harding and Pagan (2002). One feature both models fall down badly on is the shape of the expansions, as was also noted in Harding and Pagan (2002). Basically these linear models always predict that expansion phases look like triangles.

As we noted above it is sometimes said (without any evidence being given) that the variability of U.S. cycle durations and amplitudes is such that a linear model would not explain them. In fact it is the other way around; these linear models produce *too much variability* in the duration and amplitude of expansions i.e. the CV ratio for expansions is much higher than the data, and there is some evidence of this for contractions as well. So if one is looking for a non-linear effect to improve on the explanation of business cycle phenomena it would need to produce *less* rather than more variability in phases.

Table 1 US Business Cycle Characteristics,
Data and Linear Models: 1947/1-2002/2

	Data	RW	AR(1)
Dur Con	2.9	2.6	3.1
Dur Expan	20.2	30.0	21.5
Amp Con	-2.1	-1.3	-1.7
Amp Expan	22.0	28.6	22.3
Cum Con	-4.3	-2.5	-3.9
Cum Expan	310.9	834.0	474.0
Excess Con	-.056	.02	.02
Excess Expan	.755	.02	-.00
CV Dur Con	.343	.332	.447
CV Dur Expan	.601	.857	.869
CV Amp Con	-.521	-.558	-.653
CV Amp Expan	.554	.838	.894

We investigate the transition between phases by simulating data from the AR(1) model over a 750 year period. This produces at least 2800 eligible values for S_t and Δy_t (owing to the use of completed phases the number is never 3000). We then used these to estimate

$$\Pr(S_t = 1|S_{t-2} = 1, \Delta y_{t-1}) = E(S_t|S_{t-2} = 1, \Delta y_{t-1})$$

non-parametrically with a Gaussian kernel and a window width $\hat{\sigma}_{\Delta y_{t-1}} T^{-1/5}$. One can compute the same quantity with the data, Figure 2 shows a plot of these estimated transition probabilities for the AR(1) model and data conditioned upon the values of Δy_{t-1} found in the data. Although the fit is good for positive growth rates in period $t-1$, there is an over-statement of the probability that an expansion will continue in the face of a large negative growth rate in output. It should be noted that there are not many observations in the data at the left hand end of the graph. Only twice in the sample was growth as small as -2% for the quarter.

As mentioned earlier we also conduct an experiment that attempts to isolate the impact of current versus past shocks upon the nature of the business cycle.

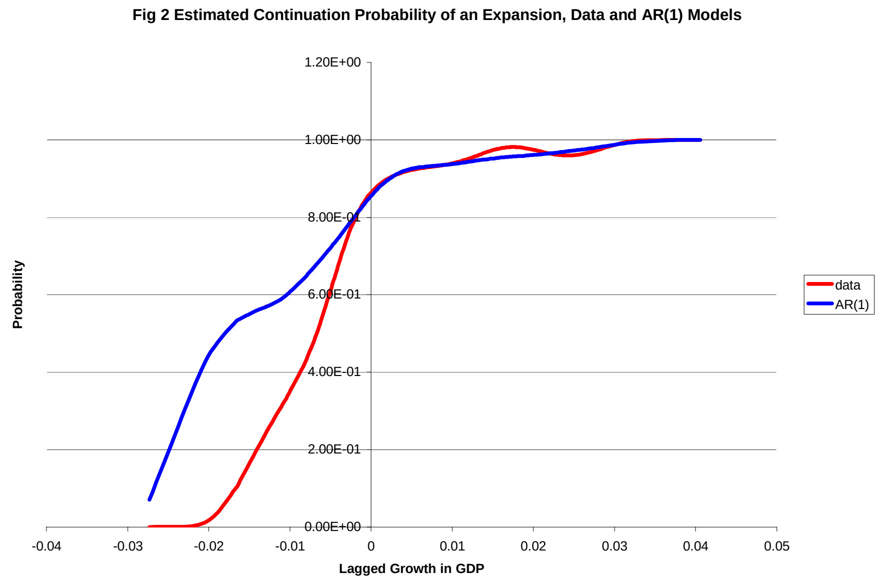

Figure 3.1:

Take the random walk model. We define $y_t^* = y_t - e_t = \mu + y_{t-1}$ and then pass y_t^* through BBQ rather than y_t. If the calculus rule was being used for dating purposes i.e. $S_t = 1(\Delta y_t^* > 0)$, we see that

$$Pr(S_t = 1) = \Pr(\Delta y_t^* > 0) = \Pr(e_{t-1} > -\frac{\mu}{\sigma})$$

and this would be the same as $\Pr(\Delta y_t > 0)$ i.e. we would find that there would be no difference in the average duration of expansions. The situation is more complicated when BBQ is used but it is probably not too surprising to find that the average durations of expansions for the random walk model using y_t^* are unchanged from that with y_t. This is not true for the AR(1) where the durations of 3.1/21.5 become 3.1/18.5. Nevertheless it is clear that the main features of the cycle generated by an AR(1) model come from the propogation mechanism.

3.2. The Value of a SETAR Model

Various types of SETAR models exist in the literature e.g. Pesaran and Potter (1997), but the version we use here is that set out in van Dijk and Franses (2003).

It has a number of extra features compared to other SETAR models in the literature, notably an ARCH effect in the errors. The model consists of the following set of equations defining $\Delta y_t = \frac{1}{4}\psi_t$

$$\begin{aligned} \psi_t &= \phi_1\psi_{t-1} + \phi_2\psi_{t-2} + \phi_0 + \theta_1 CDR_{t-1} + \theta_2 OH_{t-1} + v_t \\ & \quad v_t\tilde{\ } N(0, H_t) \end{aligned}$$

where

$$\begin{aligned} F_t &= 1(\psi_t < r_F) \text{ if } F_{t-1} = 0 \\ &= 1(CDR_{t-1} + \psi_t < 0) \text{ if } F_{t-1} = 0 \\ C_t &= 1(F_t = 1)1(\psi_t > r_C)1(\psi_{t-1} > r_C) \\ CDR_t &= (\psi_t - r_F)F_t \text{ if } F_{t-1} = 0 \\ &= (CDR_{t-1} + \psi_t)F_t \text{ if } F_{t-1} = 1 \\ OH_t &= C_t(OH_{t-1} + \psi_t - r_t) \\ H_t &= \sigma_F^2 F_{t-1} + \sigma_{COR}^2 COR_{t-1} + \sigma^2 C_{t-1} \\ COR_t &= 1(F_t + C_t = 0) \\ \phi_0 &= 1.52, \phi_1 = .35, \phi_2 = .21, \theta_1 = -.45, \theta_2 = -.041 \\ \sigma_F &= 5.03, \sigma_{COR} = 3.64, \sigma_C = 2.81, r_F = -3.51, r_C = 2.04 \end{aligned}$$

Table 2 presents business cycle characteristics of the SETAR model as well as other non-linear models. It is clear that the SETAR model is of little value. It makes expansions too strong and produces much the same variability of durations and amplitudes as the linear AR(1) model. The only way in which it improves on the linear models is in terms of the fact that it predicts that the cumulated growth from expansions will be around 3% higher than what would happen if growth was constant i.e. it is closer to the shape of actual expansions, although falling well short of the 11% registered in the data.

Table 2 US Business Cycle Characteristics,
Data and Non-Linear Models: 1947/1-2002/2

	Data	SETAR	Bounce	Tension
Dur Con	2.9	3.3	3.1	3.2
Dur Expan	20.2	25.3	27.7	26.3
Amp Con	-2.1	-1.8	-2.2	-1.6
Amp Expan	22.0	24.7	27.1	27.4
Cum Con	-4.4	-4.1	-5.1	-3.1
Cum Expan	310.8	611.1	701.3	657.1
Excess Con	-.056	-.177	-.078	.21
Excess Expan	.755	.296	.549	.66
CV Dur Con	.343	.449	.395	.398
CV Dur Expan	.601	.830	.807	.758
CV Amp Con	-.521	-.653	-.614	-.576
CV Amp Expan	.554	.842	.745	.728

Figure 3 looks at the transition probability computation. For comparison we include the AR(1) model as well as the value found from the data. Although there is little difference between the estimated probabilities from the SETAR and AR(1) models for most values of Δy_{t-1}, the striking feature is that, for large negative growth rates, the SETAR model actually predicts that there will be a high probability of a continuation of an expansion. This is because, in simulation, it produces only a few observations that have negative growth of this magnitude, and they coincide with S_t mainly being unity i.e. an expansion. As we have said previously there are only two observations in the data with Δy_{t-1} being less than -.02 and both of these are associated with $S_t = 0$. A priori one might have expected that this would happen so that the opposite prediction by the SETAR model suggests that it has a bias towards expansions (indeed we have already seen that in the duration statistics)

In order to understand the results on the transition probabilities it is worth computing $E(\psi_t|\psi_{t-1})$ for the SETAR model and the linear model. These are presented in Figure 4 as a cross plot of $E(\psi_t|\psi_{t-1})$ against the values of ψ_{t-1} in

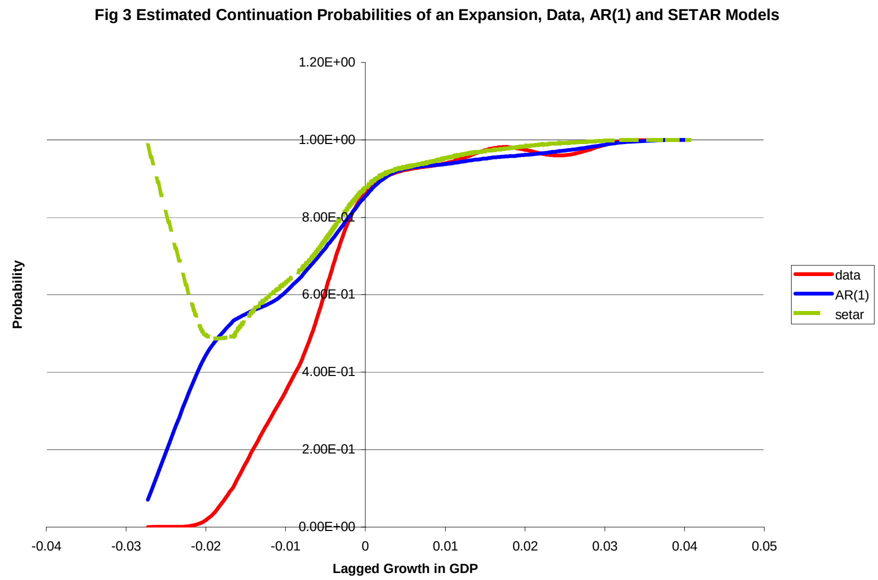

Figure 3.2:

the data set. Also on the same graph are the matched data points. We computed $E(\psi_t|\psi_{t-1})$ using a non-parametric estimator from 30000 simulated data points and a Gaussian kernel (note that the data has been annualized and made into percentage changes in this graph) It is apparent that a few extreme observations have had a large effect upon the estimates of the SETAR model. The largest negative growth rate in the sample was in 1958/1 at an annualized rate of around -11%, but it was followed in 1958/2 with a positive growth rate of around 2.4%. This somewhat rare occurrence becomes a population characteristic of the calibrated SETAR model and, whenever a very large negative growth rate occurs in the simulations, it induces a very rapid "bounce-back" effect.

Roughly speaking we might think that recessions are those points that have two successive periods of negative growth and these are in the lower left hand quadrant. Notice that the SETAR model never predicts any events in that quadrant; the linear model does much better on that score. The upshot of the effect seen in these graphs is to make recessions terminate too readily and that is what the transition probabilities tell us. It is also interesting to observe that the conditional mean from the SETAR model is virtually identical to that from the linear model, except

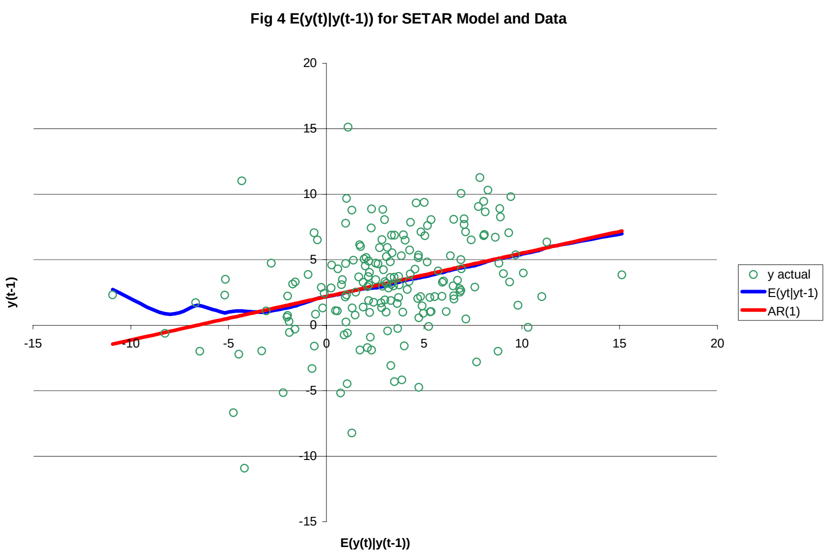

Figure 3.3:

when growth rates become very negative. This will mean that the forecasts are unlikely to differ much unless one has encountered a steep drop in output. At that point it is not clear which of the forecasts one would prefer. In any case one should be aware that the differences are associated with extreme realizations.

Our final exercise with this model was to construct y_t^* from $\psi_t^* = \psi_t - v_t$. The durations of contractions and expansions in y_t^* become 3.2/20.1, and it would therefore seem that the current shock v_t is a major determinant of the characteristics of the business cycle predicted by this model. Such dependence on an unpredictable shock may not make it a good vehicle for forecasting business cycle developments.

3.3. The Value of a Bounce-back Model

3.3.1. The Cycle Characteristics of the Model

The SETAR model has the property that there are forces leading to recovery that derive from the current depth of the recession, captured by the CDR_t variable. So there is a type of error correction type mechanism at work. An alternative approach that is motivated by observation of rapid growth coming out of a recession is to provide a "bounceback" mechanism. A model that was suggested to endogenize such a bounceback response was set out in Kim, Morley and Piger (2002) and is recorded below. Here $\Delta y_t = \frac{\psi_t}{100}$ and

$$
\begin{aligned}
\psi_t &= \mu_0 + \mu_1 z_t + \lambda(\sum_{j=1}^{6} \xi_{t-j}) + \varepsilon_t \\
& \quad \varepsilon_t \tilde{} N(0, \sigma^2) \\
\Pr(\xi_t &= 0|\xi_{t-1} = 0) = p; \Pr(\xi_t = 1|\xi_{t-1} = 1) = q \\
\mu_0 &= .868, \mu_1 = -1.877 \\
\lambda &= .246, q = .952, p = .693 \\
\sigma &= .745
\end{aligned}
$$

Table 2 shows that, just like the SETAR model, expansions are very strong and inconsistent with the data. It improves on the SETAR model in having variability in the amplitudes of expansions being closer to that of the data and it comes quite close to replicating the shape of expansions as summarized by the "excess area" statistic.

Figure 5 presents the transition probability computations. The results certainly look a little peculiar. Nevertheless, the model clearly shares the difficulty of the SETAR model that there is a tendency for expansions to continue in the face of a large negative growth rate and this does not seem desirable. Figure 6 is the equivalent of figure 4 for this model. The influence of the observation from1958/1 is still apparent but is not as extreme as before.

Finally, unlike the SETAR model, the cycle in the y_t^* after the elimination of ε_t features contractions of 3.0 quarters and expansions of 27.7 quarters, showing that the overall cycle is largely due to the past history of shocks rather than the contemporaneous one.

3.3.2. A Look at MS Models with the Bounceback Model

The history of recent business cycle analysis is replete with statements about asymmetry in the business cycle and the need for non-linear models to produce this feature. After the obligatory reference to Keynes (1936) and Burns and Mitchell (1946) as mentioning such a characteristic e.g. in Psaradakis and Sola (2002), such authors either proceed to design tests of symmetry of the "cyclical component" or to estimate non-linear models which are said to account for the

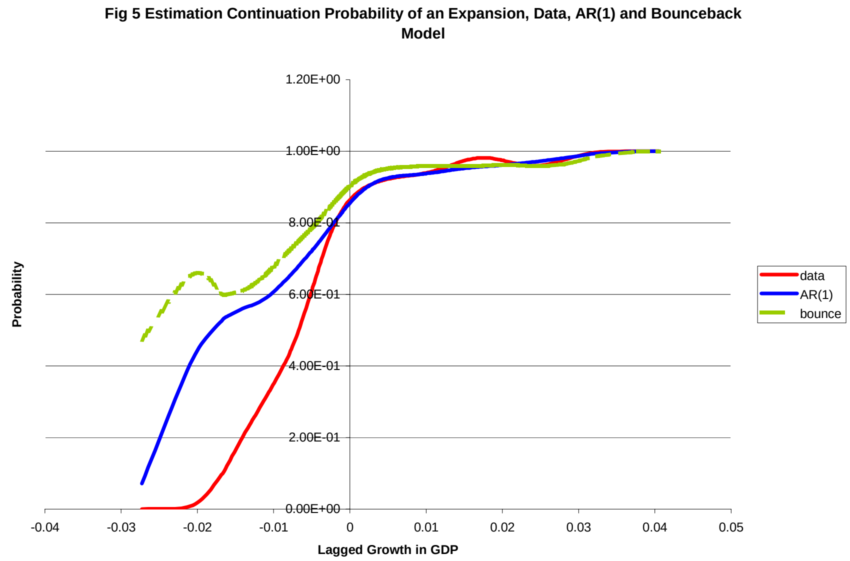

Figure 3.4:

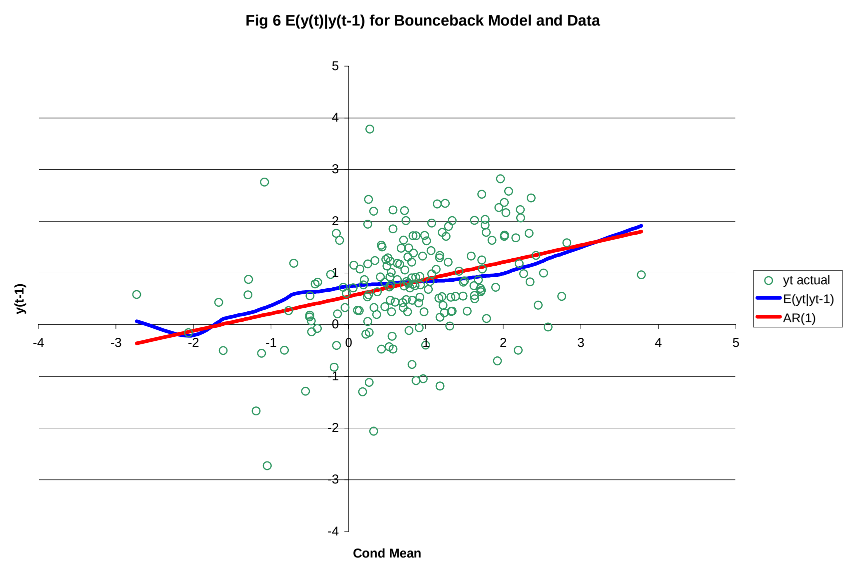

Figure 3.5:

asymmetry. Nowhere is it mentioned that Keynes and Burns and Mitchell were referring to the asymmetry in the y cycle, which is clearly evident in Table 1 where expansions are four to five time more durable than contractions. If Keynes and Burns and Mitchell had been looking at the z cycle, which the authors mentioned above are mostly doing, then they would have seen such mild asymmetry as to probably not evince any comment.

The bounceback model is a Markov Switching (MS) model and so is a useful vehicle for examining a number of issues that arise over the utility of MS models in business cycle analyis. Foremost among these is the question of how much of the asymmetry in the y cycle comes from the fact that there is long-run growth and how much comes from the non-linearity induced by an MS model. It was argued in Pagan (1997) and Harding and Pagan (2002) that it was quite likely to be due to the first of these features. Neverthless there is a substantial literature asserting the latter e.g. Psaradakis and Sola (2002), who actually misinterpret the contention in Harding and Pagan (2002), claiming that the latter's investigation is into symmetry in cycles in Δy_t. Whilst one *uses* the information in Δy_t to determine the nature of the cycle in y_t, it is *not* the cycle in Δy_t that is being investigated.

To determine the relative contributions of both of the effects we simply mean correct the simulated Δy_t. This eliminates the long-run growth but preserves any non-linearity. When we do this the duration of contractions become 5.5 quarters and that of expansions 6.5 quarters, versus the 3.1 and 27.8 of the model with the long-run growth included. This illustrates the fact that little of the asymmetry in the business cycle is accounted for by MS models.[1]

So why is it that MS models are often advocated as a tool for business cycle analysis? One reason is that, despite what we have shown above, it is often claimed that MS models produce the asymmetry that is seen in the y cycle. Such a claim is based upon identification of $\frac{1}{1-\Pr(\xi_t=1|\xi_{t-1}=1)}$ with the average duration of expansions. Thus, based upon the numbers for those probabilities found above, the bounceback model would imply that contractions were 3.3 quarters long and expansions are 20 quarters long. Now, as we have also seen above, the actual expansion lengths coming from the S_t generated by the bounceback model is very much higher at 27.7 quarters. How does this discrepancy come about? To answer that it needs to be recognized that quantities such as $\frac{1}{1-[\Pr(\xi_t=1|\xi_{t-1}=1)]}$ measure the expected time spent in the state $\xi_t = 1$ *not that spent in* $S_t = 1$. One might

[1] In fact this is true of all the non-linear models discussed in this paper.

roughly summarize the differences by saying that the transition probabilities for the ξ_t states are concerned with the time spent in *low* and *high* growth states, whereas those associated with S_t involve the time spent in sequences of *negative* and *positive* growth rates. These are clearly very different events. It is also the case that, even within the MS modelling tradition, ξ_t does not measure the states of expansion and contraction. Instead, that is done with the index $\zeta_t = 1([\Pr(\xi_t = 1)|\Delta y_{t-1}, ...] - .5)$ if one restricts attention to the predicted state (rather than the smoothed one based on all information). In most cases it is a series on ζ_t that is compared to the S_t from BBQ type dating operations when authors comment that the MS model produces turning point dates that match with those made with NBER type methods. So one really needs to compute $\frac{1}{1-\Pr(\zeta_t=1|\zeta_{t-1}=1)}$ and not $\frac{1}{1-\Pr(\xi_t=1|\xi_{t-1}=1)}$ if one wants to find an estimate of the expected expansion phase length implied by the MS model. The alternative approach is to do what was done above - pass the simulated data from an estimated MS model through BBQ and then compare that to the data. Then one is performing the same operation and the sets of computed durations are fully comparable.

The failure to carefully distinguish between the states ξ_t, ζ_t and S_t (where the latter designate the binary indicator established by applying some turning point rule) leads to other confusions. Thus there have been many studies of whether there is duration dependence within business cycles i.e. does the conditional probability (say) $P(S_t|S_{t-1} = 0)$ depend upon the period of time that one has been in the state $S = 0$? This hypothesis has been tested in a number of ways, mostly with duration data derived from the S_t e.g. Diebold and Rudebusch (1990). More recent studies however e.g. Durland and McCurdy (1994), ask whether $\Pr(\xi_t|\xi_{t-1} = 0)$ depends on the time spent in the state $\xi = 0$. Now it is highly likely that there will be duration dependence in the S_t states since Kedem (1980) showed that the S_t would have infinite order serial correlation when S_t was found using the calculus rule and Δy_t was finitely serially correlated i.e. the transition probability from S_{t-1} to S_t depends upon $S_{t-j}(j > 1)$.[2] Thus either an AR(1) model or an MS model with constant $\Pr(\xi_t|\xi_{t-1})$ will certainly produce duration dependence in the S_t with any dating rule, given what we know about Δy_t. Accordingly, findings about the presence/absence and nature of duration dependence in the ξ_t have few implications for the same features in the S_t. Models in which $\Pr(\xi_t|\xi_{t-1})$ depend on the past history of ξ_t are interesting non-linear models but they should be looked

[2] Harding and Pagan (2001) find that the serial correlation in the S_t is of higher than first order when the "modified" rule given earlier is used and the Δy_t have no serial correlation. This outcome is likely to be true of the BBQ statistics as well

at in this vein and not because they tell us anything about duration dependence in the cycle.

The other reason often given for favouring MS models is that the estimated model can be used to compute $P(\xi_t = 1|\Delta y_{t-1}...)$, and this might be used as an input into a forecast of what ζ_t would be. It is generally said that the turning point approach cannot do this but, since our phase transition measure is exactly about computing such quantities with S_t, this contention is incorrect.

3.4. The Value of a Tension Indicator Model

The model is developed in De Jong et al(2003). It consists of the equations

$$\begin{aligned}
\Delta y_t &= g^* + [m_t - vG_{t-2} + \lambda(\Delta y_{t-2} - g^*)] + \sigma_t \varepsilon_t \\
&\quad \varepsilon_t \tilde{\ } n.i.d.(0,1) \\
G_t &= \sum_{i=0}^{\infty} \delta^i (\Delta y_{t-i} - g^*) \\
m_t &= a_j + b_j \xi_t \sum_{\tau=1}^{t-t(j-1)-1} d^{\tau-1} \\
&\quad a_j \tilde{\ } N(\alpha_0 + \alpha_1 z_j, \sigma_a^2) \\
z_j &= 1 \text{ if expansion;} \\
&= 0 \text{ otherwise} \\
&\quad b_j \tilde{\ } \exp\text{onential}(\delta_0) \\
\xi_t &= 1 \text{ if expansion} \\
&= -1 \text{ otherwise} \\
\Pr(\xi_{t+1} &= -\xi_t | \xi_t; G_t) = \frac{1}{1 + \exp(\beta_0 - \beta_1 \xi_t G_t)} \\
\sigma_t^2 &= \omega_0 + \omega_1 \sigma_{t-1}^2 + \omega_2 y_{t-1}^2
\end{aligned}$$

In their model g^* is allowed to vary over time according to a third degree polynomial but this creates problems for a long simulation so we have set it to a constant value, the average growth rate in GDP over the period 1947:2-2002:3. The primary feature of the model is the tension indicator G_t, but the growth rate also changes according to regimes determined by ξ_t, just as in a Markov switching model. The values of the mean growth rate in each regime are also realizations of a binary random variable. Finally there is some GARCH type behaviour in the growth rate. Parameter values for $v, \lambda, \delta, d, a_j.b_j, \omega_j, \beta_j, \alpha_j$ and σ_a^2 are taken from their Table 2.

The results for the business cycle characteristics are in Table 2. The model improves on the bounce-back model by reducing the length and strength of expansions, although these are still much larger than in the data. It has good success in replicating the variability and shapes of cycles, however, which points to the need for statistical models of the business cycle to incorporate the type of effects it contains.

Given that it seems to be a reasonable match to many of the characteristics it is somewhat disappointing to compute the transition probability in Figure 7 and to observe that the model has a very strong tendency to continue in the expansion phase in the face of negative growth. In this respect it is inferior to the other non-linear models. It is possible that this outcome is partly due to the fact that g^* is a constant in the simulation but we cannot allow it to change as a polynomial in time as in their empirical work. In any case it is clear that one could not continue with a polynomial term even with empirical work. Figure 8 suggests that the parameter estimates have been greatly influenced by a small number of data points and this seems the most likely explanation for the business cycle behaviour it predicts. Overall the model looks promising in that it does produce phases with less variability than the linear models.

4. Conclusion

The non-linear cycle models studied in this paper manage to get an improved fit to the business cycle features of shape and variability over what linear models can produce. But they do so by having a very strong re-bound effect. This tends to make expansions continue much longer than they do in reality. Interestingly, the fitted models seem to be very influenced by a single point in 1958 when a large negative growth rate in GDP was followed by good positive growth in the next quarter.This seems to have become embedded as a population characteristic and results in overly long and strong expansions. That feature is likely to be a problem for forecasting if another large negative growth rate was observed.

We need a model that can explain a large number of cycle features simultaneously. When univariate models of GDP have been built it seems as if one manages to explain one feature only at the expense of another. The current generation of

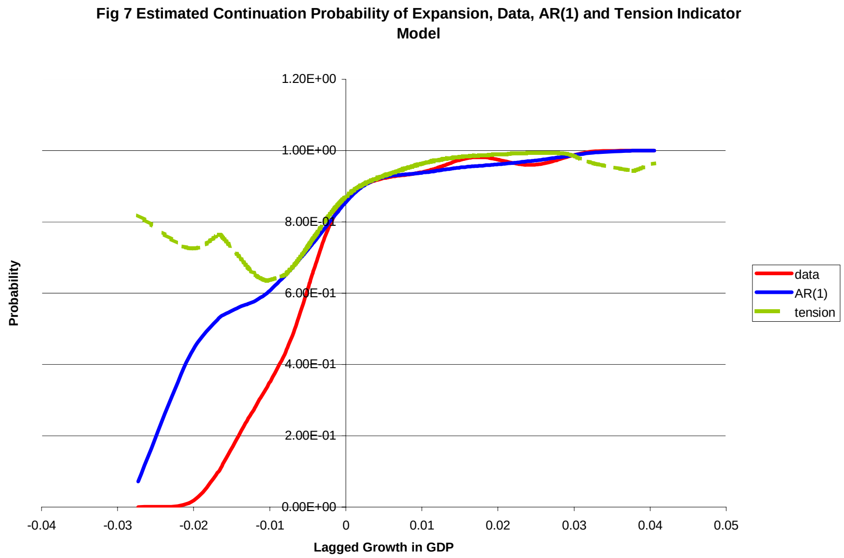

Figure 3.6:

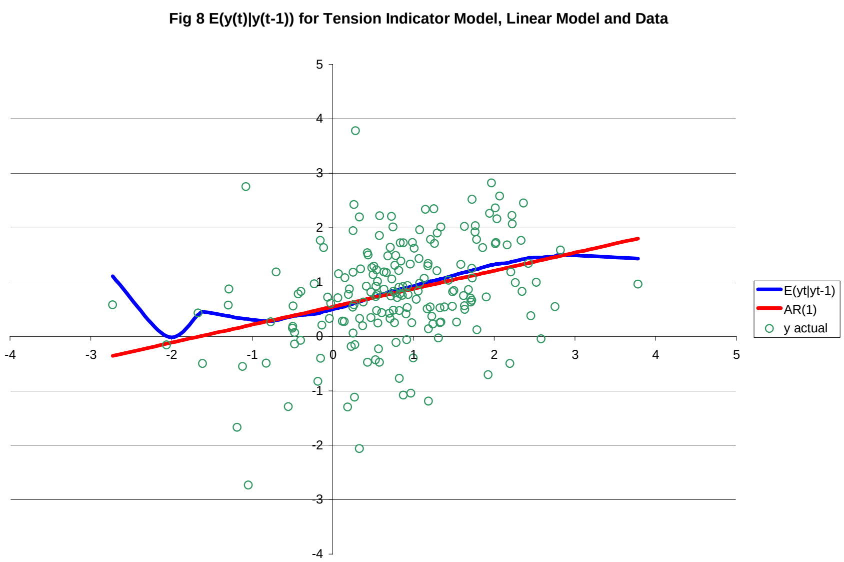

Figure 3.7:

non-linear univariate models have become extremely complicated and one suspects that we have reached the limit concerning the ability of these models to generate realistic business cycle and that the introduction of multi-variate models would be beneficial.

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