Multi-Agent Bilateral Bargaining with Endogenous Protocol*

Sang-Chul Suh†
University of Windsor

Quan Wen‡
Vanderbilt University

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Abstract

Consider a multilateral bargaining problem where negotiation is conducted by a sequence of bilateral bargaining sessions. We are interested in an environment where bargaining protocols are determined endogenously. During each bilateral bargaining session of Rubinstein (1982), two players negotiate to determine who leaves the bargaining and with how much. A player may either make an offer to his opponent who would then leave the game or demand to leave the game himself. Players' final distribution of the pie and a bargaining protocol constitute an equilibrium outcome. When discounting is not too high, we find multiple subgame perfect equilibrium outcomes, including inefficient ones. As the number of players increases, both the set of discount factors that support multiple equilibrium outcomes and the set of the first proposing player's equilibrium shares are enlarged. The inefficiency in equilibrium remains even as the discount factor goes to one.

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†Department of Economics, University of Windsor, Windsor, Ontario, N9B 3P4, Canada. E-mail: scsuh@uwindsor.ca

‡Department of Economics, Vanderbilt University, VU Station B #351819, 2301 Vanderbilt Place, Nashville, TN 37235, U.S.A. E-mail: quan.wen@vanderbilt.edu

1 Introduction

Bargaining problems deal with situations where a number of players negotiate how to share their gains obtained through trade. In a seminal paper, Rubinstein (1982) studied a highly stylized non-cooperative bilateral bargaining model with discounting, and showed that the subgame perfect equilibrium is unique and its outcome is efficient. Multilateral bargaining problems are generally more complicated. When it is infeasible or too costly for all players to negotiate at the same time and the same place, the bilateral bargaining framework provides a simple and attractive alternative.

Here we analyze a multi-agent bilateral bargaining model where players negotiate via a sequence of bilateral bargaining sessions. In each bilateral bargaining session, two players negotiate via Rubinstein's (1982) bilateral bargaining framework. After a partial agreement, one player effectively leaves the game and the other player moves to the following bilateral bargaining sessions. The proposing player may either make an offer to his opponent to leave or demand to leave the bargaining himself. The bargaining protocol is hence determined endogenously by the proposing player's choice of the type of proposals.

Consider a situation where, for simplicity, three players negotiate to share a pie of size one, and every player has a simple linear utility function and a common discount factor $\delta \in (0, 1)$. The negotiation is conducted by two separate bilateral bargaining sessions. Without loss of generality, assume that players 1 and 2 bargain during the first session. Who bargains with player 3 in the second session is crucial in finding equilibrium outcomes. If player 2 always bargains with player 3 in the second session, by backward induction, it is not hard to see that the players' shares in the equilibrium are

$$\left(\frac{1}{1+\delta}, \frac{\delta}{(1+\delta)^2}, \frac{\delta^2}{(1+\delta)^2}\right). \tag{1}$$

Suppose the negotiation is conducted by the "demand protocol" where during the first session between players 1 and 2, the proposing player always demands a certain amount of the pie to leave the bargaining, which leaves the other player to bargain with player 3 in the second session. Now which player, either player 1 or 2, will bargain with player 3 in the second bargaining session depends on who is the proposing player when a partial agreement is reached between players 1 and 2. It is involved but nevertheless straightforward to show that such a protocol leads to a unique equilibrium outcome where player 2 accepts player 1's demand immediately, then agrees with player 3 to split the remainder of the pie according to Rubinstein's shares. Players' shares in the equilibrium are given by

$$\left(\frac{1}{1+\delta+\delta^2}, \frac{\delta}{1+\delta+\delta^2}, \frac{\delta^2}{1+\delta+\delta^2}\right). \tag{2}$$

Alternatively, we may consider the "offer protocol" where the proposing player always offers certain amount of the pie to his opponent to leave the bargaining in the first session. Then the proposing player of a partial agreement between players 1 and 2 will bargain with player 3 in the second session. This offer protocol also predicts a unique and efficient equilibrium outcome, where players' shares are given by

$$\left(\frac{1}{1+2\delta}, \frac{\delta}{1+2\delta}, \frac{\delta}{1+2\delta}\right). \tag{3}$$

It is straightforward to show that both (2) and (3) converge to the corresponding Nash bargaining solution (1/3,1/3,1/3) as δ goes to one, but solution (1) does not.[1]

In this paper, we consider this kind of multi-agent bilateral bargaining model where the bargaining protocol is determined endogenously by proposing strategies. Except in the last bargaining session, the proposing player may choose between making an offer to his opponent to leave and demanding to leave the bargaining himself. Solutions (2) and (3) suggest that a proposing player would prefer demanding to making an offer. When the proposing player may choose between an offer and a demand, there is indeed an equilibrium where the proposing player always demands to leave the bargaining in every period, with (2) as the equilibrium outcome. When the discount factor is not too small, however, there are other equilibria as well in games with more than two players. But the offer protocol never

[1] Suh and Wen (2003) show that, in a general setup, the unique equilibrium outcome from either the demand protocol or the offer protocol converges to the Nash (1950) cooperative bargaining solution in the corresponding bargaining problem.

arises in equilibrium for any discounting factor. The issue of inefficient equilibria appears due to the multiplicity of equilibrium outcomes. As the number of players increases, both the set of discount factors that supports multiple equilibrium outcomes, and the set of the first proposing player's equilibrium shares are enlarged. The maximum loss of efficiency hence increases with respect to the number of players and the discount factor. For example in games with four players, as players become sufficiently patient, the first player's highest equilibrium share could be sufficiently close to one and his lowest equilibrium share could be sufficiently close to zero. The game with four players has multiple equilibrium outcomes (including inefficient ones) as long as the discount factor is not less than 0.544.

This paper follows the line of research that extends Rubinstein's (1982) bilateral model to the multilateral case. Because of its simplicity and strong predictability, Rubinstein's model has been widely adopted as a basic bargaining framework in the literature.[2] Generalizing Rubinstein's (1982) result to multilateral bargaining models has been less successful. For example, bargaining models of Haller (1986), Herrero (1985), and Sutton (1986) predict that any partition can be supported as an equilibrium outcome when discounting is not too high, while all these models reduce to Rubinstein's (1982) model in the bilateral case. One key factor for the existence of multiple equilibrium outcomes in these models is the unanimity of agreement. Although our model also has multiple equilibrium outcomes, it is more closely related to the multilateral bargaining models of Chae and Yang (1988, 1990, 1994), Huang (2002), Jun (1987), Krishna and Serrano (1996), Suh and Wen (2003), and Yang (1992) where partial agreements are allowed. In these bargaining models, one player makes a proposal and his opponents may either accept or reject, either sequentially or simultaneously. A player effectively leaves the bargaining after accepting an offer. The ability to accept an offer and leave is not affected by other players' rejections of the proposal. Consequently, these models restore the uniqueness of equilibrium with outcome akin to either (2) or (3). Since there is

[2]For recent developments in the multilateral bargaining literature, refer to Asheim (1992), Baliga and Serrano (1995), Cai (2003), Chatterjee and Sabourian (2000), Muthoo (1999), Osborne and Rubinstein (1990), Serrano (1993), and Vanntelbosch (1999).

always a unique equilibrium that is also efficient, these results are not affected by when a partial agreement is honored, either immediately or until a full agreement is reached. When there are multiple and inefficient outcomes, such as in the model studied in this paper, the time to honor a partial agreement becomes important.[3]

The rest of this paper is organized as follows. In Section 2, we set up the model where the proposing player can choose between two types of proposals, demand and offer. Section 3 is devoted to the analysis of the model with three players, in order to demonstrate some of the key arguments. In Section 4, we investigate the general case and analyze its equilibrium outcomes. Section 5 provides some concluding remarks. The proofs of propositions are given in the Appendix.

2 The Model

A finite number of players, $1, 2, \ldots, n$, negotiate how to split a pie of size 1. The negotiation is conducted through $(n-1)$ bilateral bargaining sessions where players 1 and 2 negotiate in the first session. In each bilateral bargaining session, two players bargain for a partial agreement. A partial agreement specifies a share to the player who agrees to leave the negotiation. The other player moves to the following bargaining sessions to split the remainder of the pie with the rest of the players in a similar fashion. In the last session, the two players (one of them must be player n) simply negotiate how to split the remainder of the pie between them.

Each bilateral bargaining session follows the bilateral bargaining framework of Rubinstein (1982). In any period, one player, called the proposing player, makes a proposal and the other player, called the responding player, may either accept or reject the standing proposal. If the responding player rejects the proposal then the game proceeds to the next period where the current responding player will be the proposing player, and so on. If the responding player accepts the standing proposal then the current bargaining session ends and the accepted proposal becomes a partial agreement. Which player moves to the following sessions depends

[3] Refer to Cai (2000) where the cause of the inefficiency or delay is not from the existence of multiple equilbria but from the advantage of holding up the bargaining process.

on the nature of the partial agreement. The proposing player i can make two types of proposals: either demand x_i for himself to agree to leave the game or offer y_i to the responding player to agree to leave the game. Since the proposing player can always make one type of proposal unacceptable, for example the proposing player could demand all the remainder of the pie or offer nothing to the responding player, it is equivalent whether the proposing player is allowed to make both types of proposals or to make only one type proposal. If proposing player i's demand x_i is accepted then player i will receive x_i and leave the game and the responding player will negotiate with the rest of the players. If the responding player accepts offer y_i then the responding player will receive y_i and leave the game, and player i will negotiate with the rest of the players. Assume that the player with a lower index initiates a proposal in any bargaining session. Denote the bilateral bargaining session between players i and j as $BG(i, j)$, that has the following structure:

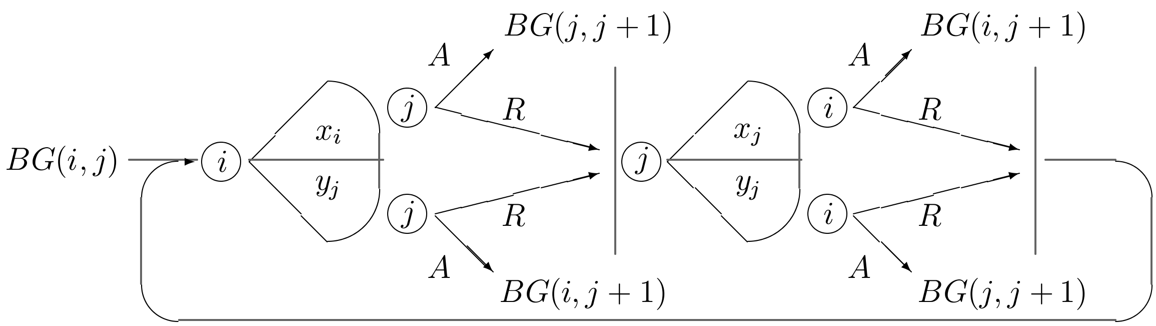

Figure 1. The Bilateral Session Between Players i and j.

By allowing the proposing player to choose between two types of proposals, the bargaining protocol is determined endogenously in equilibrium by the proposing player's strategy choices. Since this model has multiple equilibrium outcomes, including inefficient ones, assume that the player who accepts an offer or whose demand is accepted will consume his share immediately for simplicity. Otherwise, we would need to trace when the last session ends.

A typical outcome is as follows. Players 1 and 2 negotiate during the first session. After reaching a partial agreement, one of them (either player 1 or player 2, depending on the type of the proposal accepted) negotiates with player 3, and so on. All players have a common discount factor $\delta \in (0, 1)$ per bargaining period. It is assumed that there is no discounting between consecutive bargaining sessions. The existence of discounting between consecutive sessions will not affect the qualitative aspect of our results. According to those specifications, player i's payoff is $\delta^{t_i} \cdot s_i$ where $s_i \in [0, 1]$ is the share of the pie player i receives in period t_i. In the case where player i never reaches any partial agreement, his payoff is zero and t_i is set to be infinity. Accordingly, there are certain restrictions on the specification of outcomes. For example, the feasibility implies that $s_1 + s_2 + \cdots + s_n = 1$. If $t_i = \infty$ then $t_j = \infty$ for all $j > i$, which means if player i does not have an agreement neither does any player after player i. For any player i, t_i is either the first or the second largest element of $\{t_1, t_2, \cdots, t_i\}$, which means that there is at most one player before player i will have an agreement after player i by the sequential nature of bilateral bargaining sessions.

The model has perfect information and so histories and strategies are defined in the usual way. A history summarizes all the actions played in the past and a strategy profile specifies an appropriate action for the acting player after every history. Any strategy profile induces a unique (either finite or infinite) outcome path and the players evaluate their strategies based on their payoffs from the induced outcome path. In the rest of this paper, we will study subgame perfect equilibrium outcomes.

3 The Three-Player Game

In this section, we consider the model with 3 players. Since the model has finite bargaining sessions, the model is analyzed by backward induction. Therefore, studying the model with 3 players is not only the starting point of our analysis, but also provides us with some basic intuition and methodology used in the general analysis.

Note that there are only two bargaining sessions. Recall the last session is a standard Rubinstein game. Even if the proposing player proposes either an offer or a demand, such a modification will not change players' equilibrium strategies in this last session. Therefore, there is a unique equilibrium that is also efficient. The equilibrium outcome depends on the partial agreement in the first session. In particular, if we denote the share to the player who left the game after the first session as X then the equilibrium outcome in the second session is that two players agree on the following shares immediately,

$$\left(\frac{1-X}{1+\delta}, \frac{(1-X)\delta}{1+\delta}\right), \tag{4}$$

where $(1-X)/(1+\delta)$ is the share to the player from the first session and $(1-X)\delta/(1+\delta)$ is player 3's share. We now analyze the players' strategic interaction during the first session. Because of the proposing player's option to choose between two types of proposals, the analysis is made significantly different by the new elements in the model. In what follows, we will first establish the existence of a perfect equilibrium for any discount factor, and then characterize all perfect equilibrium outcomes.

Comparing outcomes from (2) and (3), it is obvious that player 1 prefers outcome (2) to outcome (3) for any $\delta \in (0,1)$. This observation suggests that the proposing player prefers making an acceptable demand rather than an acceptable offer. Proposition 1 asserts that for any discount factor, there is a perfect equilibrium where the proposing player always makes an acceptable demand in any period, which leads to (2) as the equilibrium outcome.

Proposition 1 *For any $\delta \in (0,1)$, the model has an efficient equilibrium where the proposing player always demands $1/(1+\delta+\delta^2)$, and the responding player accepts any demand no more than $1/(1+\delta+\delta^2)$ or any offer no less than $\delta/(1+\delta+\delta^2)$ during the first session. The equilibrium outcome is given by (2).*

The equilibrium of Proposition 1 is stationary and symmetric between players 1 and 2. It is the standard argument that the proposing player should not make any unacceptable proposal. When comparing between an acceptable demand and an acceptable offer, the acceptable demand dominates the acceptable offer given that the proposing player always makes an acceptable demand in the future.

On the other hand, it is not an equilibrium where both players always make acceptable offers during the first session. Given players' proposing strategies, solution (3) suggests that a responding player in the first period would accept any proposal that leads to $\delta/(1+2\delta)$ and the proposing player would have $1/(1+2\delta)$ from making the acceptable offer. If the proposing player demands x then the responding player will accept as long as

$$\frac{1-x}{1+\delta} \geq \frac{\delta}{1+2\delta} \;\Rightarrow\; x \leq \frac{1+\delta-\delta^2}{1+2\delta},$$

which is higher than $1/(1+2\delta)$. Therefore, the proposing player would prefer to make the acceptable demand $(1+\delta-\delta^2)/(1+2\delta)$ rather than the acceptable offer which leaves him $1/(1+2\delta)$.[4]

It is not always the case that the acceptable demand dominates the acceptable offer, however. The following Proposition 2 demonstrates that when the discount factor is not too small, the model has a stationary but asymmetric perfect equilibrium where player 1 always makes an acceptable demand but player 2 always makes an acceptable offer during the first session.

Proposition 2 *If $\delta \in (0,1)$ satisfies the following inequality*

$$\delta \geq \frac{1}{1+\delta}, \tag{5}$$

then there is a perfect equilibrium where, during the first session, player 1 always demands $1/(1+\delta)$ and accepts any proposal which gives him no less than $\delta/(1+\delta)$, and player 2 always offers $\delta/(1+\delta)$ and accepts any proposal which gives him no less than $\delta/(1+\delta)^2$. The equilibrium outcome is given by (1).

Condition (5) requires that $\delta \geq (\sqrt{5}-1)/2 \simeq 0.618$. Notice that player 1 always makes an acceptable demand and player 2 always makes an acceptable offer so that the equilibrium strategies specify the bargaining protocol where players 1 and 2 bargain in the first session

[4]Outcome (3) can, nevertheless, be supported by a non-stationary perfect equilibrium when there are multiple equilibrium outcomes. Refer to Proposition 5 below.

and players 2 and 3 in the second session. Player 2's strategy of making an acceptable offer may seem counter-intuitive. Given the switching in proposing strategies, player 1 certainly benefits since player 1's payoff when proposing is $1/(1+\delta)$. Therefore, player 2's acceptable demand cannot be too high given that player 1's final payoff from such a proposal has to be at least $\delta/(1+\delta)$, which implies that player 2 cannot demand more than $(1-\delta)$. On the other hand, player 2 can guarantee himself $1/(1+\delta)^2$ by offering $\delta/(1+\delta)$ to player 1, which is not less than $(1-\delta)$ under condition (5):

$$\delta \geq \frac{1}{1+\delta} \iff \frac{1}{(1+\delta)^2} \geq 1-\delta.$$

Analogic to Proposition 2, the model has another stationary but asymmetric equilibrium where player 1 always makes an acceptable offer and player 2 always makes an acceptable demand. The proof of Proposition 3 is similar to that of Proposition 2.

Proposition 3 *Under condition (5), there is a perfect equilibrium where during the first session, player 1 always offers $\delta/(1+\delta)$ and accepts any proposal which gives him no less than $\delta/(1+\delta)^2$, and player 2 always demands $1/(1+\delta)$ and accepts any proposal which gives him no less than $\delta/(1+\delta)$. The equilibrium outcome is*

$$\left(\frac{1}{(1+\delta)^2}, \frac{\delta}{1+\delta}, \frac{\delta}{(1+\delta)^2}\right).$$

Propositions 2 and 3 indicate that the model has multiple perfect equilibrium outcomes when δ is not too small. Indeed Propositions 1, 2 and 3 give all the extreme equilibrium outcomes for different values of δ. In what follows, we derive the supremum and infimum of a player's equilibrium payoffs by adopting Shaked and Sutton's (1984) method.

By Proposition 1, we know that the set of equilibrium payoffs to any player is not empty. Also by symmetry between player 1 and player 2 during the first session, the supremum and infimum of player 1's equilibrium payoffs when player 1 proposes are the same as those of player 2's when player 2 proposes. Denote the supremum and infimum of the proposing player's equilibrium payoffs during the first session in the three-player model as M_3 and m_3, respectively. Both M_3 and m_3 depend on the discount factor δ.

Note that the proposing player will never make a proposal where the responding player's payoff is more than $\delta \cdot M_3$. On the other hand, the responding player will never accept any proposal where his payoff is less than $\delta \cdot m_3$. Denote the proposing player's demand by x and offer by y. The conditions for demand x and offer y to be acceptable are, respectively,

$$\delta \cdot m_3 \;\leq\; \frac{1-x}{1+\delta} \;\leq\; \delta \cdot M_3, \quad \text{and} \quad \delta \cdot m_3 \;\leq\; \frac{y}{1+\delta} \;\leq\; \delta \cdot M_3. \tag{6}$$

Then m_3 and M_3 are the infimum and supremum of the maximum of x and $y/(1-\delta)$, since the proposing player chooses between making a demand and an offer, subject to the acceptability conditions (6). More specifically, we have the following two conditions for m_3 and M_3:

$$\begin{aligned} m_3 \;&=\; \max \left\{ \begin{array}{ll} x & \text{subject to } (1-x)/(1+\delta) \geq \delta M_3 \\ (1-y)/(1+\delta) & \text{subject to } y \geq \delta M_3 \end{array} \right. \\ &=\; \max \left\{ 1-\delta(1+\delta)M_3, \; \frac{1-\delta M_3}{1+\delta} \right\}. \end{aligned} \tag{7}$$

Notice that in (7), the responding player's continuation payoff after rejection is δM_3 which is the best situation for the responding player. On the other hand, in the worst situation to the responding player where his continuation payoff is δm_3, the proposing player will obtain M_3, so we have

$$M_3 \;=\; \max \left\{ 1-\delta(1+\delta)m_3, \; \frac{1-\delta m_3}{1+\delta} \right\}. \tag{8}$$

Solving (7) and (8), we have the following proposition:

Proposition 4 *Conditions (7) and (8) yield*

$$m_3 = M_3 = \frac{1}{1+\delta+\delta^2} \qquad \textit{when} \quad \delta < \frac{1}{1+\delta}, \tag{9}$$

$$m_3 = \frac{1}{(1+\delta)^2}, \quad M_3 = \frac{1}{1+\delta} \quad \textit{when} \quad \delta \geq \frac{1}{1+\delta}. \tag{10}$$

Proposition 4 asserts that the perfect equilibrium outcomes of Propositions 1—3 are indeed extreme equilibrium outcomes. Even in the case of the three-player case where the second session has a unique equilibrium outcome, allowing players to choose proposing strategies leads to multiple equilibrium outcomes. Figure 2 below illustrates player 1's equilibrium shares in the model with three players.

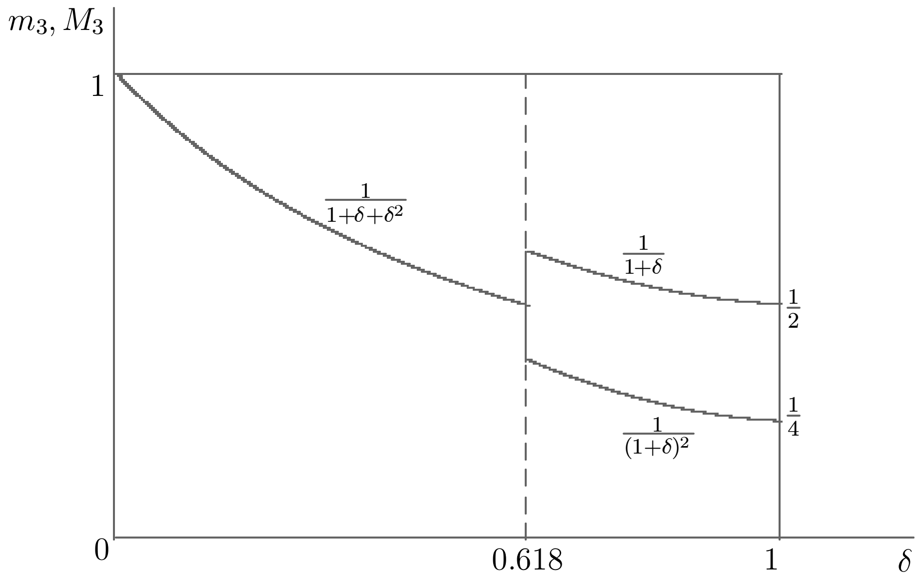

FIGURE 2. Player 1's equilibrium shares when $n = 3$.

In addition to those efficient equilibrium outcomes, there are other efficient and inefficient equilibrium outcomes. Since either player 1 or player 2 bargains with player 3 in the second session where there is a unique equilibrium outcome, the restriction on the outcome that can be supported by equilibrium is that player 3's payoff has to be δ fraction of the payoff to either player 1 or 2. Proposition 5 completely characterizes the equilibrium outcomes in the three-player case.

Proposition 5 *Under condition (5), vector (v_1, v_2, v_3) can be supported by a subgame perfect equilibrium if and only if $\exists\ T \geq 1$ and $x \in (0,1)$ such that for $i, j = 1, 2$ and $i \neq j$*

$$v_i = \delta^{T-1} \cdot x, \quad \textit{and} \quad v_j = \delta^{T-1} \cdot \frac{1-x}{1+\delta}, \tag{11}$$

and players' payoffs satisfy

$$v_1 \geq \frac{1}{(1+\delta)^2}, \quad v_2 \geq \frac{\delta}{(1+\delta)^2}, \quad v_3 = \delta v_j. \tag{12}$$

Proposition 5 provides not only efficient equilibrium outcomes ($T = 1$) but also inefficient equilibrium outcomes ($T > 1$) as well. The set of efficient equilibrium payoffs shapes like an "X" on the unit simplex. Any inefficient equilibrium payoff is a convex combination of 0 and a point in such an "X" where player 1's payoff is bounded below by $1/(1+\delta)^2$ and player 2's payoff is bounded below by $\delta/(1+\delta)^2$. It is interesting to observe that player 3's equilibrium payoffs are bounded from below by $\delta^2/(1+\delta)^2$, but bounded from above by

$$\frac{\delta}{1+\delta}\left[1-\frac{\delta}{(1+\delta)^2}\right]=\frac{\delta(1+\delta)^2-\delta^2}{(1+\delta)^3}\to\frac{3}{8}\ \text{ as } \delta\to 1.$$

The inefficiency of equilibrium outcomes is "persistent" in the sense that the inefficiency does not disappear as δ goes to one. This can be seen from conditions (11) and (12) since only δ^T is restricted by these two conditions. We can use $1-\delta^T$ to measure the inefficiency. As δ approaches to one, T can be chosen to be sufficiently large as long as δ^T satisfies (11) and (12).

From the case of three players, we learned that the set of equilibrium payoffs can be quite irregular and multiple equilibrium outcomes emerge even the subgame after the first session has a unique equilibrium outcome. In analyzing the general case, we will concentrate on equilibrium shares to the proposing player in the first session.

4 The General Analysis

In this section, we consider the general case with n-players. From the last section, we know the set of equilibrium outcomes could be very complicated. Therefore, we will concentrate our analysis on the equilibrium shares to the proposing player in the first session. Note that the continuation game after the first session is the game with $(n-1)$ players. The general case is analyzed by mathematical induction. First, we establish the existence of a perfect equilibrium in the general n-player case.

Proposition 6 *For any $\delta \in (0, 1)$, the model with n players has an efficient perfect equilibrium where the proposing player always makes an acceptable demand in any period in any* *session. The equilibrium payoff vector is* $(\alpha_n, \delta\alpha_n, \cdots, \delta^{n-1}\alpha_n)$ *where*

$$\alpha_n = \frac{1}{1+\delta+\delta^2+\cdots+\delta^{n-1}}.$$

Denote m_n and M_n to be the infimum and supremum of equilibrium shares to the proposing player in the first session. Proposition 6 implies that both m_n and M_n are well defined and $m_n \leq \alpha_n \leq M_n$. Rubinstein (1982) implies that $m_2 = M_2 = 1/(1+\delta)$, and our Proposition 4 asserts that

$$(m_3, M_3) = \begin{cases} (\alpha_3, \alpha_3) & \text{if } \delta < \alpha_2; \\ (\alpha_2^2, \alpha_2) & \text{if } \delta \geq \alpha_2. \end{cases}$$

Now we derive the dynamics that determine the values of m_n and M_n. First consider m_n, the infimum of the proposing player's equilibrium payoffs during the first session in the case of n players. Note that if the responding player rejects the standing proposal, his highest possible payoff will be δM_n, resulting from a one period delay and the best possible payoff available in the following period. If the proposing player demands x such that $m_{n-1}(1-x) \geq \delta M_n$ then the responding player will certainly accept since his worst possible payoff from accepting x is not less than his best possible payoff from rejecting x. On the other hand, if the proposing player offers $y \geq \delta M_n$ then the responding player will also accept. Then the lowest possible payoff to the proposing player cannot be less than $m_{n-1}(1-y)$ from the remaining sessions. Therefore, m_n cannot be less than the highest possible payoff to the proposing player from either the highest acceptable offer or the lowest acceptable demand. That is,

$$\begin{aligned} m_n &= \max \begin{cases} x & \text{subject to} \quad m_{n-1}(1-x) \geq \delta M_n \\ m_{n-1}(1-y) & \text{subject to} \quad y \geq \delta M_n \end{cases} \\ &= \max \left\{ 1 - \frac{\delta}{m_{n-1}} M_n,\ m_{n-1}(1-\delta M_n) \right\} \end{aligned} \tag{13}$$

By a similar argument, M_n should satisfy the following condition:

$$M_n = \max \left\{ 1 - \frac{\delta}{M_{n-1}} m_n,\ M_{n-1}(1-\delta m_n) \right\} \tag{14}$$

We summarize these results in the following proposition:

Proposition 7 *For all $n \geq 3$, m_n and M_n satisfy conditions (13) and (14).*

It is not difficult to show that both m_n and M_n determined by (13) and (14) can be supported as equilibrium shares of the proposing player for the corresponding discount factor δ. A more interesting question is then what value of δ will actually support multiple equilibrium outcomes, i.e., $m_n < M_n$. The analysis of the case with three players suggests that there are multiple equilibrium outcomes as long as δ is not too "small." In the following proposition, we are able to establish this result formally in the general case.

Proposition 8 *For $n \geq 3$, the model has a unique equilibrium outcome if and only if $\delta < \alpha_{n-1}$. The unique equilibrium is the efficient equilibrium stated in Proposition 6.*

Proposition 8 implies that as the number of players increases, the set of discount factors that supports multiple equilibrium outcomes, hence inefficient outcomes, increases as well. Generally speaking, as n increases, it is more likely that the model will have multiple equilibrium outcomes. Our next proposition, which is even more striking and seems counter intuitive, is that when the model has multiple equilibrium outcomes, the set of the proposing player's equilibrium shares is widened as the number of players increases. In other words,

Proposition 9 *For $n \geq 3$ and $\delta \in [\alpha_n, 1)$, we have $m_{n+1} < m_n < M_n < M_{n+1}$.*

When there are multiple equilibrium outcomes, player 1's best equilibrium share increases and his worst equilibrium share decreases with respect to the number of players. This polarized effect is due to the fact that in player 1's best equilibrium, player 1 will obtain the best equilibrium during or after the first session from either accepting an offer or making an acceptable demand, while player 2 will always obtain the worst equilibrium during or after the first session from either making an acceptable offer or accepting a demand. In order to support M_n to be player 1's equilibrium share, player 1 should always make the highest acceptable demand and player 2 should always make the highest acceptable offer during the first session, after which player 2 will receive the worst equilibrium outcome in the continuation game. If the model with $(n-1)$ players continues to have multiple equilibrium outcomes, then player 3 will always make the highest acceptable demand and player 2 will continue to make the highest acceptable offer in the second session, from which player 3's payoff is $\delta M_{n-1}(1-M_n)$. Otherwise, players 2 and 3 will make acceptable demands on the remainder of the pie, from which player 2's and 3's payoffs are $\alpha_{n-1}(1-M_n)$ and $\delta\alpha_{n-1}(1-M_n)$, respectively. This means that when player 1 obtains the highest equilibrium payoff, payoffs to all the other players are uniquely determined. For example, when $\delta < \alpha_k$, Proposition 8 asserts that the model with no more than $(k+1)$ players will have a unique equilibrium outcome. This means that when there are n players, each of the last k sessions has a unique equilibrium outcome, and each of the fist $(n-k-1)$ sessions has multiple equilibrium outcomes. When player 1 receives M_n, player 2 will have to make acceptable offers in each of the first $(n-k-1)$ session and will make the acceptable demand in session $(n-k)$, while the first other $(n-k-2)$ players (except player 2) will always make the highest acceptable demands in the first $(n-k-1)$ sessions. Likewise, in the equilibrium where player 1 receives m_n, player 1 will always make acceptable offers during the first $(n-k-1)$ sessions and will make the acceptable demand in session $(n-k)$, while the other players in the first $(n-k-1)$ sessions will always make acceptable demands. Proposition 10 provides the dynamics for M_n and m_n when δ is not too small. Proposition 10 asserts that, for example, in the equilibrium where player 1 receives the highest payoff M_n, player 1 should always make the highest acceptable demand and player 2 should always make the lowest acceptable offer during the first session. The continuation equilibrium after the first session depends on how the first session ends. After player 2 accepts player 1's demand, player 2 will receive the highest share M_{n-1} on the remaining pie. After player 1 accepts player 2's offer, however, player 2 will receive the lowest share m_{n-1} on the remaining pie.

Proposition 10 *Under (5), $\forall\ n \geq 3$, we have*

$$m_n = \frac{m_{n-1}(1-\delta)}{1-\delta^2 m_{n-1}/M_{n-1}}, \quad \textit{and} \quad M_n = \frac{1-\delta m_{n-1}/M_{n-1}}{1-\delta^2 m_{n-1}/M_{n-1}}. \tag{15}$$

Propositions 8 and 9 characterize two key properties of the set of proposing player's equilibrium shares. It is important to emphasize that as the number of players increases, not only the set of discount factors that supports multiple equilibrium outcomes but also the set of the proposing player's equilibrium shares will be widened when there are multiple perfect equilibrium outcomes.

5 Concluding Remarks

We studied a non-cooperative multilateral bargaining model where the negotiation is conducted by a sequence of bilateral bargaining sessions. Bargaining protocol is determined by the players' equilibrium strategies. In contrast to the model where the bargaining protocol is exogenously given, the model studied here, in which the proposing player chooses between making an offer and a demand, has multiple equilibrium outcomes in general. The model always has a perfect equilibrium where the proposing player always makes an acceptable demand in every period. In the case in which the bargaining protocol is exogenous where the proposing player always makes an acceptable demand, the corresponding equilibrium outcome converges to the Nash (1950) bargaining solution as players become sufficiently patient even under a more general setup. We have shown that as the number of players increases, not only the set of discount factors that support multiple equilibrium outcomes, but also the set of proposing player's equilibrium shares increases. The threshold of discount factor that supports multiple equilibria is α_{n-1} where there are n players. When $n > 3$ and the model has multiple equilibrium outcomes, the first proposing player's best equilibrium share could be much higher than α_2 (the case when the other player always bargains with the rest of the players), but his worst equilibrium share could be much lower then α_2^{n-1} (the case when the first proposing player always bargains with the rest of the players). In order to demonstrate this result, consider the case of $n = 4$. Solving (13) and (14), the set of the first proposing player's equilibrium shares is

$$[m_4,\ M_4] \ = \ \begin{cases} \left\{\dfrac{1}{1+\delta+\delta^2+\delta^3}\right\} & \text{when } \delta \in (0, \alpha_3) \\ \left[\dfrac{1}{(1+\delta)(1+\delta+\delta^2)},\ \dfrac{1}{1+\delta}\right] & \text{when } \delta \in [\alpha_3, \alpha_2) \\ \left[\dfrac{1-\delta}{(1+\delta)(1+\delta-\delta^2)},\ \dfrac{1}{1+\delta-\delta^2}\right] & \text{when } \delta \in [\alpha_2, 1) \end{cases}$$

When $\delta < \alpha_3$, the model has a unique perfect equilibrium where player 1's share is equal to α_4. When $\delta \geq \alpha_3$, the model has multiple perfect equilibrium outcomes. It is interesting to observe that as δ goes to one, player 1's highest equilibrium share M_4 converges to 1, and player 1's lowest equilibrium share m_4 converges to zero. These properties can be easily demonstrated in Figure 3 where both M_4 and m_4 are depicted with respect to δ.

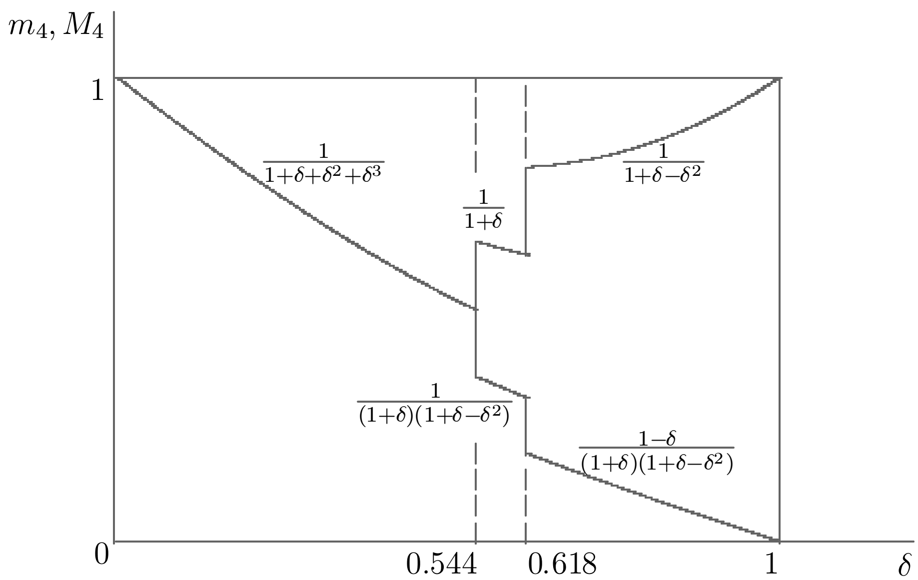

FIGURE 3. Player 1's equilibrium shares when $n = 4$.

Comparing Figures 2 and 3, in the region of discount factor where there are multiple equilibrium outcomes, the set of player 1's equilibrium shares becomes larger by adding one more player in the bargaining. In the case of $n = 4$, $\alpha_3 = \delta$ yields $\delta = 0.544$.

Our studies suggest that the most robust perfect equilibrium in this type of multi-agent bilateral bargaining model is the symmetric and stationary equilibrium where the proposing players always make acceptable demands, not only to the discount factor, but also with respect to many other aspects of the model specifications, such as players' utility functions, disagreement payoffs and discount factors. Because of the possibility of delay in reaching an agreement, the assumption that the player who effectively exits the bargaining will consume the share immediately significantly simplifies the general analysis. Otherwise, one needs to trace how the whole game concludes in order to determine the equilibrium conditions in the early sessions. Nevertheless, the general characteristics of our prediction will not change and delay will occur in the early bargaining sessions as long as the discount factor is not too low.

Appendix

Proof of Proposition 1: As we argued, the continuation equilibrium in the second session is unique so we will concentrate on the players' strategies during the first session. By symmetry, we will simply examine strategies of the proposing player and the responding player in every period.

First consider the responding player's strategies. If the responding player rejects the standing proposal (either an offer or a demand), he will be the proposing player in the next period with an acceptable demand of $x = 1/(1+\delta+\delta^2)$. Therefore, if the responding player rejects the standing proposal, he can secure himself a payoff of δx. This implies that the responding player will not accept any proposal that gives himself less than δx.

Given the responding player's strategies, the proposing player has three alternatives; making an acceptable offer δx, making an acceptable demand, or making an unacceptable demand or offer. From the last alternative, the proposing player will receive a payoff of $\delta^2 x$. From making an acceptable offer δx, the proposing player's payoff is

$$\frac{1}{1+\delta}\left[1-\delta x\right] = \frac{1+\delta^2}{1+\delta}x. \tag{16}$$

On the other hand, the proposing player will have a payoff x' by making the acceptable demand x'. The condition for demand x' to be acceptable is that the responding player's payoff from accepting demand x' is not less than δx:

$$\frac{1-x'}{1+\delta} \geq \delta x \quad \Rightarrow \quad x' \leq 1-(1+\delta)\delta x = x. \tag{17}$$

(17) implies that x is the best acceptable demand for the proposing player. Comparing the proposing player's payoffs from the three alternatives, x, (16) and $\delta^2 x$, it is easy to conclude that the proposing player will make the acceptable demand x during the first session, which leaves $1-x$ for players 2 and 3. Then in the second session, players 2 and 3 split $1-x$ according to Rubinstein's shares, which give δx and $\delta^2 x$. Note that the equilibrium outcome is efficient. **Q.E.D.**

Proof of Proposition 2: Note that player 1's payoff from rejecting any standing proposal is $\delta/(1+\delta)$ and player 2's payoff from rejecting any standing proposal is $\delta/(1+\delta)^2$. Therefore both players' responding strategies are subgame perfect.

Now consider player 1's proposing strategies in the first session. If player 1 does not make any acceptable proposal, his payoff will be $\delta^2/(1+\delta)$ since player 2 will offer $\delta/(1+\delta)$ in the following period. If player 1 makes a demand $x_1 \in [0,1]$, player 2 will accept demand x_1 if

$$\frac{1-x_1}{1+\delta} \geq \frac{\delta}{(1+\delta)^2} \quad \Rightarrow \quad x_1 \leq \frac{1}{1+\delta},$$

which means that player 1's demand $1/(1+\delta)$ is acceptable, so making any unacceptable proposal is dominated by demanding $1/(1+\delta)$. If player 1 makes an acceptable offer $y_1 \geq \delta/(1+\delta)^2$, player 1's payoff will be

$$\frac{1-y_1}{1+\delta} \leq \frac{(1+\delta)^2-\delta}{(1+\delta)^3} = \frac{1}{1+\delta} \cdot \frac{1+\delta+\delta^2}{(1+\delta)^2} < \frac{1}{1+\delta}.$$

Therefore, for player 1 during the first session, making any acceptable offer is strictly dominated by making the acceptable demand $1/(1+\delta)$.

Next we examine player 2's proposing strategies during the first session. If player 2 does not make any acceptable proposal, his payoff will be $\delta^2/(1+\delta)^2$. If player 2 makes an acceptable offer $y_2 = \delta/(1+\delta)$, player 2's payoff will be

$$\frac{1-y_2}{1+\delta} = \frac{1}{(1+\delta)^2}, \tag{18}$$

which implies that making any unacceptable proposal is dominated by making the acceptable offer. If player 2 makes an acceptable demand x_2 such that

$$\frac{1-x_2}{1+\delta} \geq \frac{\delta}{1+\delta} \quad \Rightarrow \quad x_2 \leq 1-\delta, \tag{19}$$

then player 2 could not demand more than $1-\delta$. Under condition (5), we have that (19) is no more than (18) since

$$(1-\delta) \leq \frac{1}{(1+\delta)^2} \quad \Leftrightarrow \quad 1+\delta-\delta^2-\delta^3 \leq 1 \quad \Leftrightarrow \quad (5).$$

Therefore, during the first session, player 2 should always make the acceptable offer rather than making any acceptable demand.

In this equilibrium, player 1 demands $1/(1+\delta)$ and player 2 accepts, which leaves $\delta/(1+\delta)$ to players 2 and 3 to share according to Rubinstein's solution. So player 2's share is $\delta/(1+\delta)^2$ and player 3's share is $\delta^2/(1+\delta)^2$. This equilibrium outcome is efficient since there is no delay involved. **Q.E.D.**

Proof of Proposition 4: Recall (7) and (8), there are four cases to consider. Since two of the four cases are symmetric, we only have to deal with three cases.

Case 1: Assume that $1-\delta(1+\delta)M_3 \leq (1-\delta M_3)/(1+\delta)$ and $1-\delta(1+\delta)m_3 \leq (1-\delta m_3)/(1+\delta)$. This is the situation where both players prefer to make acceptable offers. Consequently (7) and (8) become

$$m_3 = \frac{1-\delta M_3}{1+\delta}, \; M_3 = \frac{1-\delta m_3}{1+\delta} \;\Rightarrow\; m_3 = M_3 = \frac{1}{1+2\delta}.$$

However, for $m_3 = M_3 = 1/(1+2\delta)$ and for all $\delta \in (0,1)$, we have

$$1-\delta(1+\delta)M_3 = \frac{1+\delta-\delta^2}{1+2\delta} > \frac{1}{1+2\delta} = \frac{1-\delta M_3}{1+\delta},$$

which contradicts the inequalities that define Case 1. Therefore, Case 1 is impossible.

Case 2: Assume that $1-\delta(1+\delta)M_3 \geq (1-\delta M_3)/(1+\delta)$ and $1-\delta(1+\delta)m_3 \geq (1-\delta m_3)/(1+\delta)$. This is the situation where both players prefer to make acceptable demands. Then (7) and

(8) become

$$m_3 = 1 - \delta(1+\delta)M_3,\ M_3 = 1 - \delta(1+\delta)m_3 \ \Rightarrow\ m_3 = M_3 = \frac{1}{1+\delta+\delta^2}.$$

It is straightforward to verify that the two inequality conditions hold for $m_3 = M_3 = 1/(1+\delta+\delta^2)$ and for all $\delta \in (0,1)$. This means Case 2 is possible.

Case 3: Assume that $1-\delta(1+\delta)M_3 \leq (1-\delta M_3)/(1+\delta)$ and $1-\delta(1+\delta)m_3 \geq (1-\delta m_3)/(1+\delta)$. This is the situation where the player who receives M_3 prefers to make acceptable demands and the player who receives m_3 prefers to make acceptable offers. Then (7) and (8) become

$$m_3 = \frac{1-\delta M_3}{1+\delta},\ M_3 = 1 - \delta(1+\delta)m_3 \ \Rightarrow\ m_3 = \frac{1}{(1+\delta)^2},\ M_3 = \frac{1}{1+\delta}.$$

With these values of m_3 and M_3, the second inequality in Case 3 holds for all $\delta \in (0,1)$ as

$$1 - \delta(1+\delta)m_3 = \frac{1}{1+\delta} > \frac{1+2\delta}{(1+\delta)^3} = \frac{1-\delta m_3}{1+\delta}.$$

The first inequality in Case 3 holds if and only if (5) holds,

$$1 - \delta(1+\delta)M_3 = 1 - \delta \geq \frac{1}{(1+\delta)^2} = \frac{1-\delta M_3}{1+\delta} \ \Leftrightarrow \delta \geq \frac{1}{1+\delta}.$$

Case 4: Assume that $1-\delta(1+\delta)M_3 \geq (1-\delta M_3)/(1+\delta)$ and $1-\delta(1+\delta)m_3 \leq (1-\delta m_3)/(1+\delta)$. As in Case 3, one would find that $m_3 = 1/(1+\delta)$ and $M_3 = 1/(1+\delta)^2$ when $\delta \geq 1/(1+\delta)$. However, $M_3 < m_3$ is contradictory. Therefore, Case 4 is also impossible.

Summarizing Cases 2 and 3, m_3 and M_3 are given by (9) and (10). **Q.E.D.**

Proof of Proposition 5: Under (5), Propositions 2 and 3 hold. Consider the following strategy profile where players 1 and 2 disagree in the first $(T-1)$ periods of the first session and, depending which player proposes in period T, either player j agrees to player i's demand x or player i agrees to player j's offer x. Note that if $T = 1$ then $i = 1$ and $j = 2$ by construction. During the first $(T-1)$ periods, the proposing player either offers 0 or demands 1, and the responding player will reject. If player 1 ever deviates from the outcome described above, the continuation equilibrium will be the equilibrium from Proposition 3, and if player 2 deviates then the continuation equilibrium will be the equilibrium from Proposition 2.

It is obvious that player i's payoff is v_i and player j's payoff is v_j in the strategy profile described above. The strategies after any deviation of either player 1 or 2 are subgame perfect due to either Proposition 2 or 3. It remains to be shown that neither player 1 nor player 2 has any incentive to deviate during the first $(T-1)$ periods of disagreement. Based on the strategy profile, the proposing player will have at least $1/(1+\delta)^2$ and the responding player will have at least $\delta/(1+\delta)^2$ due to (11) and (12)during the first $(T-1)$ periods of disagreement. On the other hand, the proposing player will obtain at most $1/(1+\delta)^2$ and the responding player will obtain at most $\delta/(1+\delta)^2$ from deviation. Therefore, neither player 1 nor 2 has any incentive to deviate in the first session with T periods, which concludes the proof of Proposition 5. **Q.E.D.**

Proof of Proposition 6: The proof is inductive. Proposition 1 is the special case of $n = 3$. Suppose that Proposition 6 holds with n players. Now we prove Proposition 6 for $(n+1)$.

During the first session of bargaining between players 1 and 2, assume that the player who bargains with the remaining $(n-1)$ players will receive α_n of the remaining share after the first session. Consider a symmetric and stationary strategy profile between players 1 and 2 where the proposing player always makes the highest acceptable demand x. The responding player's payoff from accepting x should not less be than $\delta \cdot x$. That is,

$$\alpha_n(1-x) = \delta x \ \Rightarrow \ x = \frac{\alpha_n}{\delta + \alpha_n} = \alpha_{n+1}.$$

If the proposing player decides to make the lowest acceptable offer y, then $y = \delta x = \delta\alpha_{n+1}$, which leaves the proposing player a payoff of

$$\alpha_n(1-\delta\alpha_{n+1}) = \alpha_{n+1} \cdot \frac{1+\delta^2+\delta^3+\cdots+\delta^n}{1+\delta+\delta^2+\cdots+\delta^{n-1}},$$

which is less than the acceptable demand α_{n+1}. Therefore, the proposing player will always make the acceptable demand α_{n+1}. **Q.E.D.**

Proof of Proposition 8: The proof is inductive. Proposition 8 reduces to Proposition 2 when $n = 3$. Suppose that Proposition 8 holds for n. In what follows, we establish Proposition 8 for $(n+1)$.

When $\delta < \alpha_{n-1} < \alpha_{n-2}$, the supposition implies that the model with $(n-1)$ players has a unique equilibrium outcome and $m_{n-1} = M_{n-1} = \alpha_{n-1}$ by Proposition 6. From conditions (13) and (14), as in the proof of Proposition 4, the only case with multiple equilibrium outcomes occurs when

$$1 - \frac{\delta}{\alpha_{n-1}} M_n \leq \alpha_{n-1}(1 - \delta M_n), \text{ and } 1 - \frac{\delta}{\alpha_{n-1}} m_n \geq \alpha_{n-1}(1 - \delta m_n) \tag{20}$$

Then conditions (13) and (14) become

$$\begin{aligned} & m_n = \alpha_{n-1}(1 - \delta M_n), \quad \text{and} \quad M_n = 1 - \frac{\delta}{\alpha_{n-1}} m_n, \\ \Rightarrow \quad & m_n = \frac{\alpha_{n-1}}{1+\delta}, \quad \text{and} \quad M_n = \frac{1}{1+\delta}. \end{aligned} \tag{21}$$

The first inequality condition of (20), together with solutions obtained from (21), yields

$$1 - \frac{\delta}{1+\delta} \cdot \frac{1}{\alpha_{n-1}} \leq \frac{\alpha_{n-1}}{1+\delta} \Leftrightarrow (1 - \alpha_{n-1})(\delta - \alpha_{n-1}) \geq 0,$$

which is false as when $\delta \in (0, \alpha_{n-1})$. Therefore, Proposition 6 predicts the unique equilibrium outcome in the case of n players when $\delta < \alpha_{n-1}$. By induction, the first part of Proposition 8 holds for all finite $n \geq 3$.

The second part of Proposition 8 asserts multiple equilibrium outcomes when $\delta \geq \alpha_{n-1}$. Note that Proposition 6 implies that α_{n-1} can always be supported as the first player's equilibrium share in the model with $(n-1)$ players. In the first part of this proof, we derived (21) and showed that the first inequality of (20) holds when $\delta \geq \alpha_{n-1}$. It remains to be shown that the second inequality of (20) holds when $\delta \geq \alpha_{n-1}$. With (21), the second inequality of (20) reduces to

$$\frac{1}{1+\delta} \geq \alpha_{n-1}\left(1 - \frac{\delta\alpha_{n-1}}{1+\delta}\right) \quad \Leftrightarrow \quad \alpha_{n-1}(1 + \delta - \delta\alpha_{n-1}) \leq 1,$$

which is trivial. Therefore, there are multiple equilibrium outcomes when $\delta \geq \alpha_{n-1}$. **Q.E.D.**

Proof of Proposition 9: For $\delta \in [\alpha_n, \alpha_{n-1})$, Proposition 8 and (21) assert that

$$M_{n+1} = \alpha_2 > \alpha_n = M_n, \quad \text{and} \quad m_{n+1} = \frac{\alpha_n}{1+\delta} < \alpha_n = m_n,$$

which establishes Proposition 9 for $\delta \in [\alpha_n, \alpha_{n-1})$.

For $\delta \in [\alpha_{n-1}, 1)$, we will prove the proposition by induction. From Proposition 4, we have the following inequalities:

$$m_3 = \alpha_2^2 < m_2 = \alpha_2 = M_2 = M_3. \tag{22}$$

Now suppose we have the following inequalities:

$$m_n < m_{n-1} \leq M_{n-1} \leq M_n. \tag{23}$$

Note that the two weak inequalities in (23) are needed to accommodate (22) for the case of $n = 3$. Also $m_n < M_n$ so the model with n players has multiple equilibrium outcomes. In the remainder of this proof, we will prove (23) for $(n+1)$ where the last inequality holds strictly as stated in Proposition 9. By (23), we can support both M_n and m_n as the proposing player's equilibrium shares in the case of n players in the same way as in the case of $(n-1)$ players since both m_{n-1} and M_{n-1} can be supported as the proposing player's equilibrium shares in the model with n players. This implies that $m_{n+1} \leq m_n < M_n \leq M_{n+1}$. Since $m_n < m_{n-1}$ and $M_n \leq M_{n+1}$, we have

$$1 - \frac{\delta}{m_n} \cdot M_{n+1} < 1 - \frac{\delta}{m_{n-1}} \cdot M_n, \quad m_n(1 - \delta M_{n+1}) < m_{n-1}(1 - \delta M_n).$$

Therefore, (13) implies that $m_{n+1} < m_n$. Next, since $m_{n+1} < m_n$ and $M_{n-1} \leq M_n$, we have

$$1 - \frac{\delta}{M_{n-1}} \cdot m_n < 1 - \frac{\delta}{M_n} \cdot m_{n+1}, \quad M_{n-1}(1 - \delta m_n) < M_n(1 - \delta m_{n+1}).$$

Hence (14) implies that $M_n < M_{n+1}$. Therefore (23) holds for $(n+1)$ with all strict inequalities. **Q.E.D.**

Proof of Proposition 10: Proposition 6 implies that $m_n \leq \alpha_n$ for all $n \geq 2$ and $\delta \in (0, 1)$. Propositions 4 and 9, on the other hand, imply that $M_n \geq \alpha_2$ for $n \geq 3$ and $\delta \geq \alpha_2$ by (5). First notice that

$$1 - \frac{\delta}{m_{n-1}} M_n \;\leq\; m_{n-1}(1 - \delta M_n) \;\Leftrightarrow\; \delta M_n \geq \frac{m_{n-1}}{1 + m_{n-1}}, \tag{24}$$

which is trivial under $M_n \geq \alpha_2$ and $\delta \geq \alpha_2 \geq \alpha_{n-1} \geq m_{n-1}$ due to the fact that

$$\delta M_n \geq \delta\alpha_2 = \frac{\delta}{1+\delta} \geq \frac{m_{n-1}}{1+m_{n-1}}.$$

Next, notice that

$$1 - \frac{\delta}{M_{n-1}} m_n \;\geq\; M_{n-1}(1-\delta m_n) \;\Leftrightarrow\; \delta m_n \leq \frac{M_{n-1}}{1+M_{n-1}}, \tag{25}$$

which is also trivial under $M_n \geq \alpha_2$ and $m_n \leq \alpha_n \leq \alpha_3$ due to the fact that

$$\delta m_n \leq \delta\alpha_3 = \frac{\delta}{1+\delta+\delta^2} \leq \frac{1}{2+\delta} \leq \frac{M_{n-1}}{1+M_{n-1}}.$$

With (24) and (25), conditions (13) and (14) can be simplified as

$$m_n = m_{n-1}(1-\delta M_n), \quad \text{and} \quad M_n = 1 - \frac{\delta}{M_{n-1}} m_n,$$

which yield the dynamics of m_n and M_n by (15) in the proposition. **Q.E.D.**

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