02/04/04

Currency Substitution, Portfolio Diversification and Money Demand

Miguel Lebre de Freitas *
Universidade de Aveiro and NIPE

The paper explores the implications of means of payment substitutability and capital mobility on the properties of the money demand, using the Thomas (1985) stochastic dynamic optimising model, where the specific role of money is explicitly accounted for. Extending the model to a case in which the consumer has no access to bonds denominated in foreign currency, we are able to describe the double role (means of payment and store of value) that foreign bank notes may have in countries where asset markets are illiquid. We show that means of payment substitutability opens a channel through which portfolio decisions influence the demand for domestic money, even if the later is dominated as store of value. Contrary to what suggested by the Portfolio Balance Theory of Currency Substitution (Cuddington, 1983), the results obtained in this paper suggest that the significance of an expected exchange rate depreciation term in the demand for domestic money provides a valid test for the presence of CS.

JEL Classification: E41, F41, G11.
Keywords: Money Demand, Currency Substitution, Dollarisation, Portfolio Choice.

* Assistant Professor, DEGEI, Universidade de Aveiro. Campus Universitário, 3810-193 Aveiro, Portugal. Tel: 351-234-370200. Fax: 351-234-370215. E-mail: afreitas@egi.ua.pt. A previous version of this paper was presented in the 2003 meeting of LACEA - The Latin American and Caribbean Economic Association, in Puebla, Mexico.

1. Introduction

During periods of macroeconomic and political uncertainty, many developing countries experienced a partial replacement of the domestic currency by a foreign currency, either as store of value, unit of account or medium of exchange. This phenomenon is known as *currency substitution* (CS). CS results from the existence of substitutability between currencies (though it is not necessarily implied by it) and it may take place both at the domestic level and in the international arena. CS is a matter of concern for policymakers, as it rises the unpredictability of the money demand and reduces the effectiveness of monetary policy, even under flexible exchange rates (see Giovannini and Turtelboom, 1994, for a survey).

This paper explores the implications of imperfect means of payment substitutability on the properties of the money demand, using a stochastic dynamic optimising model in which the specific role of money is explicitly accounted for. In particular, it is assumed that money reduces the frictional losses from transacting in the goods market. This feature of the model is essential to distinguish the phenomenon of currency substitution from portfolio choice. The paper compares two extreme assumptions concerning capital mobility: the case in which the consumer has unrestricted access to bonds denominated in foreign currency and the case in which the consumer cannot hold such bonds. In both cases, the individual is allowed to hold an interest-bearing asset denominated in domestic currency, implying that domestic money is dominated as store of value.

The first case draws on Thomas (1985). This author demonstrated that borrowing and lending opportunities separate the ownership of currencies from portfolio decisions. That is, on one hand, a consumer selects his currency holding based on transaction services and user costs. On the other hand, she borrows or lends to achieve the desired overall portfolio currency composition. An

optimal currency hedge is created and the denomination structure of the individual portfolio is independent of the currency holdings.

The Thomas' separation result depends critically on the assumption of complete bond markets. As pointed out by Cuddington (1989), such assumption may not be suitable to describe the demand for money in developing countries where asset markets are illiquid. Our contribution is to extend the Thomas (1985) model to a case in which the consumer has no access to bonds denominated in foreign currency. This is the appropriate set-up to describe the demand for money in economies subject to capital controls or in economies where the openness of capital markets did not reach a significant part of the population.

An obvious implication of banishing foreign bonds from the individual portfolio is that foreign bank notes get a store of value role, in addition to the eventual means of payment role. The double role that foreign bank notes have under imperfect capital mobility and currency substitution is formally described in this paper. We show that, in case the domestic and foreign currencies are substitutes as means of payment, then the demand for domestic money will be influenced by portfolio decisions. This is not to say that there will be a portfolio demand for domestic money. Since domestic money is dominated by an interest-bearing asset, its demand will be driven by transactions purposes, only. Means of payment substitutability opens however a channel through which portfolio adjustments involving foreign money balances impact on the liquidity value of the domestic currency.

The money demand properties in this model are, thus, different from those postulated by the Portfolio Balance Model (PBM) of currency substitution (Cuddington, 1983). In light of that theory, money is a viewed as a simple asset, that is gross substitute of all other available assets. When foreign currency and foreign bonds are both available, this leads to a demand for domestic money that depends negatively on the expected exchange rate depreciation by two different channels:

currency substitution and capital mobility. For this reason, followers of the PBM have argued that currency substitution and capital mobility are statistically indistinguishable.

In contrast to that theory, the model explored in this paper assumes that domestic money provides liquidity services and is dominated as store of value. Comparing the results obtained under extreme assumptions concerning capital mobility and currency substitutability, we conclude that only in case the two currencies are substitutes as means of payment will the demand for domestic money differ from the closed economy specification. In order to qualify the results, we also investigate the odd case in which the domestic currency is not dominated by an interest-bearing asset. In general, the results suggest that the significance of an expected depreciation term in the demand for domestic money provides a valid test for the presence of Currency Substitution.

Models of currency substitution in which money is viewed as providing liquidity services include Agénor and Khan (1996), Ratti and Jeong (1994), Rogers (1990) and Végh (1989). Since these models assume away uncertainty, however, they cannot describe the portfolio role of foreign currency. Imrohoroglu (1994) uses a stochastic version of the liquidity services model to obtain testable first order conditions, but does not explore the implications of exchange rate uncertainty on the properties of money demand. Sahay and Végh (1996) refer to the Thomas (1985) model to describe a case of asymmetric CS in which individuals have no access to bonds denominated in foreign currency. However, in their framework, consumers are allowed to hold interest bearing foreign currency deposits, which play in the model the role of the missing bond. Hence, their analysis does not depart from the original Thomas (1985) model.

This paper focuses on a particular case of "asymmetric" CS, in which a *local* currency is replaced by an *international* currency in a function performed by the former in the domestic

economy[1]. This case shall be distinguished from "international CS", which refers to the displacement of an international currency by another international currency in functions performed by the former in the international economy (for the roles of international currencies, see Krugman, 1984). The model shares with Thomas (1985) the fact that only imperfect means of payment substitutability is allowed for. The implications of perfect means of payment substitutability are analysed in Kareken and Wallace (1981), for the symmetrical case, and in Freitas (2004), for the asymmetric case.

The paper proceeds as follows. In Section 2, we present the basic model. In Section 3, we discuss the money demand properties under imperfect means of payment substitutability and complete bond markets. In Section 4, we examine the implications means of payment substitutability on the properties of the money demand when the consumer cannot hold foreign bonds. In Section 5 we investigate the case in which domestic money is not dominated by an interest-bearing asset. In Section 6 we discuss the optimal choices under alternative portfolio restrictions. Section 7 addresses the empirical implications of the results obtained. Section 8 concludes.

[1] Ramirez-Rojas (1985) suggested that currency substitution should be classified as "symmetrical" when residents and non-residents simultaneously hold domestic and foreign currency, and as "asymmetrical", when there is no demand for domestic currency by non-residents.

2. The model

Consider a one-good small open economy populated by a large number of identical, infinitely lived consumers. There is only one (non-storable) consumption good, which domestic price is equal to *P*. The representative consumer is endowed with a constant flow of the good, denoted by *y*. The consumer maximises the expected value of a discounted sum of instantaneous utility functions of the form:

$$\mathrm{E}\int_o^{\infty} e^{-\beta t}\,\frac{c_t^{1-\phi}}{1-\phi}dt\,, \tag{1}$$

where c_t denotes for real consumption at time *t*, β is a positive and constant subjective discount rate and $\phi > 0$ is the Arrow-Pratt measure of relative risk aversion.

The representative individual has unrestricted access to domestic currency (called peso, M), a foreign currency (dollar, F) and bonds denominated in domestic currency (*A*). It may, however, face a zero restriction on the holdings of bonds denominated in foreign currency (*B*). In this section, we formulate the unrestricted case. The case in which foreign bonds are not available is analysed in Section 4.

The individual' real wealth is defined as:

$$w = m + f + a + b\,, \tag{2}$$

where $m = M/P$, $f = EF/P$, $a = A/P$, $b = EB/P$, *P* is the domestic price level and *E* is the price of the dollar in peso-units.

Domestic and foreign securities have certain nominal returns, represented by *i* and *j*, respectively. Currency holdings earn zero nominal returns. There is uncertainty concerning real returns, because the domestic price level and the exchange rate evolve stochastically, according to[2]:

$$\frac{dP}{P} = \pi dt + \sigma dZ \tag{3}$$

$$\frac{dE}{E} = \varepsilon dt + \gamma dX\,, \tag{4}$$

where dZ and dX are standard Wiener processes. Denoting by ρ the covariance between the stochastic processes (3) and (4) and using the Ito's lemma, we obtain the real returns to domestic bonds, domestic money, foreign bonds and foreign money:

$$\frac{da}{a} = \left(i + \sigma^2 - \pi\right)dt - \sigma dZ \tag{5}$$

$$\frac{dm}{m} = \left(\sigma^2 - \pi\right)dt - \sigma dZ \tag{6}$$

$$\frac{db}{b} = \left(j + \varepsilon + \sigma^2 - \pi - \rho\right)dt - \sigma dZ + \gamma dX \tag{7}$$

$$\frac{df}{f} = \left(\varepsilon + \sigma^2 - \pi - \rho\right)dt - \sigma dZ + \gamma dX\,. \tag{8}$$

Money is distinguished from bonds because it provides liquidity services. In particular, we assume that money reduces frictional losses from transacting in the good markets[3]. Purchases of the

[2] With such specification, asset demands will be neutral in respect to the domestic inflation rate. Thomas (1985) deflated domestic assets by the domestic price level and foreign assets by the foreign price level and introduced uncertainty in the foreign inflation rate, instead as on the exchange rate. Although the two approaches are equivalent for the issues being discussed here, the specification above looks more appealing to describe the case of dollarisation, in which a foreign currency can be used along with the domestic currency as vehicle for transactions that take place in the *domestic* economy.

consumption good are subject to a transaction cost, τ, that depends positively on the real consumption level (c) and negatively on the amount of real money balances. To allow for currency substitution, it is assumed that both the domestic currency and the foreign currency serve as a media of change. To keep in shape with simplicity, we use a particular transactions technology, introduced by Végh (1989):

Assumption 1. *(The transactions technology):* $\tau(.)$ *is a non-negative, twice continuously differentiable and convex function of the form:*

$$\tau = cv\left[\frac{m}{c}, \frac{f}{c}\right], \tag{9}$$

with $v(.) > 0$, $v_1 < 0$, $v_2 < 0$, $v_{11} > 0$, $v_{22} > 0$, $v_{12} \geq 0$ *and* $\Delta = v_{11}v_{22} - v_{12}^2 > 0$.

In (9), τ refers to the amount of real resources spent in transacting, m and f denote domestic and foreign real money balances, respectively and a subscript k *(k=1,2)* to the function *v(.)* denotes partial differentiation with respect to the k argument. Linear homogeneity and the assumption that additional real money balances (either domestic or foreign) bring about diminishing reductions in transaction costs are not necessary for the main propositions to hold, but they help, respectively, to simplify the algebra and to obtain well behaved money demand functions[4].

---

[3] An alternative specification would assume that money enters in the utility function. The two approaches become functionally equivalent when the utility function is weakly separable, as happens to be the case in most of the literature. For a stochastic model with money in utility, currency substitution and complete bond markets, see Smith (1995).

[4] As shown by Sahay and Végh (1996) and briefly reviewed in Section 3, in the case with complete bond markets, these conditions are sufficient to obtain sensible money demand functions. In Section 4, we show that, when domestic agents have no access to bonds denominated in foreign currency, further assumptions are needed so as to obtain unambiguous interest-rate elasticities.

The fact that foreign currency provides liquidity services does not necessarily imply means of payment substitutability. Suppose, for example, that some fraction of the consumption bundle is purchased using pesos only and that the remaining fraction is purchased using dollars only. In that case, there is no substitutability. Means of payment substitutability occurs when some fraction of the consumption bundle can be purchased with either currency. Formally, this can be stated in the following way:

Definition 1. *(Means of payment substitutability): the domestic and foreign currency are said to be substitutes as means of payment if the cross derivative* v_{12} *in equation (9) is strictly positive.*

The flow budget constraint depends on the amount of saved wealth allocated to the available assets and on real returns:

$$dw = dm + df + da + db + [y - c - \tau(.)]dt\,. \tag{10}$$

Using (9) and (5)-(8), the flow budget constraint of the representative consumer becomes:

$$dw = \Phi dt + (w - a - m)\gamma dX - w\sigma dZ\,, \tag{11}$$

with

$$\Phi = (\sigma^2 - \pi)m + (\varepsilon + \sigma^2 - \pi - \rho)f + (i + \sigma^2 - \pi)a + (j + \varepsilon + \sigma^2 - \pi - \rho)b + y - c[1 + v(.)].$$

In the following, two cases will be analysed: the case in which the individual has unrestricted access to the foreign bond (Section 3) and the case in which foreign bonds are not available (Section 4).

3. The case with complete bond markets (Thomas, 1985)

The consumer problem is to maximise (1), subject to (11). The Hamilton-Jacobi-Bellman equation of the quasi-stationary problem is:

$$rV(w) = \max_{c,m,f,a}\left\{\frac{c^{1-\phi}}{1-\phi} + V'(w)\Phi + \frac{1}{2}V''(w)\left[\gamma^2(w-m-a)^2 + \sigma^2 w^2 - 2\sigma w(w-m-a)\rho\right]\right\} \qquad (12)$$

where Φ is defined as in (11). Using (2) to eliminate b from Φ and using the first order conditions with respect, respectively, to a, m and f, one obtains:

$$\frac{b+f}{w} = \left(\frac{1}{\phi}\right)\left(\frac{j+\varepsilon-i}{\gamma^2}\right) + \left(1-\frac{1}{\phi}\right)\left(\frac{\rho}{\gamma^2}\right) \qquad (13)$$

$$i + v_1\left(\frac{m}{c}, \frac{f}{c}\right) = 0 \qquad (14)$$

$$j + v_2\left(\frac{m}{c}, \frac{f}{c}\right) = 0\,. \qquad (15)$$

Equation (13) is the well known optimal portfolio rule in a world with two assets (see, for example, Branson and Henderson, 1985). It states that the optimal share of assets denominated in foreign currency is a weighted average of two terms, the weights depending on the coefficient of relative risk aversion, ϕ. The first term is the speculative component. The second term is the hedging component. The term ρ/σ^2 gives the proportion of assets denominated in dollars that minimises the portfolio's purchasing power risk. According to (13), the consumer is induced to move away from the minimum risk portfolio by the expected return differential and the extend to which it moves depend on its risk aversion.

Equations (14) and (15) define implicitly the money demand functions. They state that the consumer should hold each currency until the marginal peso (dollar) produces additional transaction

services equal in value to its user cost[5]. To investigate the money demand properties, we take differences in (14)-(15):

$$di = -\frac{v_{11}}{c}dm - \frac{v_{12}}{c}df + \left(\frac{mv_{11}}{c^2} + \frac{fv_{12}}{c^2}\right)dc,$$

$$dj = -\frac{v_{21}}{c}dm - \frac{v_{22}}{c}df + \left(\frac{mv_{21}}{c^2} + \frac{fv_{22}}{c^2}\right)dc.$$

Solving for *dm* and *df*, and computing the partial derivatives, we obtain[6]:

$$m = cL^m(i,j), \text{ with } L_i^m = -\frac{cv_{22}}{\Delta} < 0 \text{ and } L_j^m = \frac{cv_{12}}{\Delta} \geq 0, \tag{16}$$

$$f = cL^f(i,j), \text{ with } L_i^f = \frac{cv_{21}}{\Delta} \geq 0 \text{ and } L_j^f = -\frac{cv_{11}}{\Delta} < 0. \tag{17}$$

A particular case occurs when there is no means of payment substitution ($v_{12} = 0$). In that case, the system simplifies to:

$$m = cL^m(i), \qquad \text{with } L_i^m = -\frac{c}{v_{11}} < 0 \tag{16a}$$

$$f = cL^f(j), \qquad \text{with } L_j^f = -\frac{c}{v_{22}} < 0. \tag{17a}$$

To interpret, consider a rise in the expected exchange rate depreciation not imbedded in the interest rates *(dε>0, di=dj=0)*. According to (13), this causes a portfolio adjustment from domestic

---

[5] Conditions (14) and (15) were first obtained by Miles (1978), in the context of the *two-step* liquidity services model of currency substitution. In that approach, however, separability between portfolio decisions and currency substitution decisions was postulated. The proof that separability holds in the dynamic optimising model with complete bond markets is from Thomas (1985).

[6] Equations (16) and (17) are not in the reduced form because changes in the interest rates also impact on money demands through wealth effects. However, the aim of the exercise is to learn about money velocity, so as to obtain testable money demand functions.

assets to foreign assets, for speculative reasons. As long as the user costs of holding money remain constant, however, such adjustment does not impact on currency holdings.

Now assume that the rise in expected exchange rate depreciation is accompanied by a rise in the domestic interest rate, so that the expected return differential remains unchanged ($d\varepsilon = di > 0$, $dj = 0$). From equation (13), we know that the optimal proportion of assets (money and bonds) denominated in each currency remains unchanged. However, since the user cost of the domestic currency rises, the consumer will optimally reduce the amount of domestic currency held for transaction purposes. The remaining effects depend on whether the two currencies are substitutes or not as means of payment.

If the currencies are not substitutes ($v_{12} = 0$), the adjustment involves only domestic assets. The consumer keeps the currency composition of the portfolio unchanged swapping peso-currency with peso-bonds. The demand for domestic money (16a) is the same as in a closed economy[7].

If the two currencies compete ($v_{12} > 0$), the fall in peso-currency holdings rises the liquidity value of the dollar-currency. Thus, the demand for dollar-currency rises, for transaction purposes. In order to keep the currency composition of the overall portfolio unchanged, the consumer offsets such move buying peso-denominated bonds and selling dollar-denominated bonds.

These examples illustrate the Thomas (1985) separation result: on one hand, a consumer selects his currency holdings, based on each money's transaction services and its associated user cost. On the other hand, she borrows or lends to achieve his desired overall portfolio currency composition. An optimal currency hedge is created and the denomination structure of the individual

---

[7] In the extreme case in which the dollar currency provided no liquidity services at all (that is, if $v_2 = v_{22} = 0$), then condition (15) would not hold in equality and the optimal demand for dollars would be zero. Still, the demand for pesos would be as described by (16a).

portfolio is independent of the currency holdings. Changes in the expected exchange rate depreciation not embedded in the nominal interest rates affect the overall portfolio denomination but fail to influence the individual money demands.

4. Currency substitution when foreign bonds are not available

We now assume that individuals cannot hold bonds denominated in foreign currency. In this case, the following proposition holds:

Proposition 1 *(non-separation). If the consumer is constrained to have zero holdings of the foreign bond, then the optimal holdings of foreign bank notes obey to:*

$$\frac{f}{w} = \left(1 - \frac{1}{\phi}\right)\left(\frac{\rho}{\gamma^2}\right) + \left(\frac{1}{\phi}\right)\left(\frac{-v_2 + \varepsilon - i}{\gamma^2}\right) \tag{13a}$$

Proof: Maximise (1) subject to the stochastic differential (11) under the restriction b=0. Use equation (2) to eliminate f in Φ *in (11) and substitute in the Hamilton-Jacobi-Bellman equation. The first order condition in respect to m leads to equation (14). The first order condition with respect to a simplifies to (13a).*

Equation (13a) is similar to (13) with the difference that foreign currency plays the role of the missing bond[8]. As in (13), the consumer is induced to move away from the minimum risk portfolio by the expected return differential and the extend to which it moves depend on its degree of risk aversion, ϕ. The novelty here is that, to the extent that foreign money holdings help reducing transaction costs (that is, $v_2 < 0$), this will be accounted for in the assessment of the expected return differential. Equation (13a) captures the double role that foreign bank notes have in high inflation countries where asset markets are illiquid.

The following implication is straightforward:

---

[8] Note that nothing in this models prevents the individual to be short of foreign currency. For *f* to become negative, one needs either a low coefficient of relative risk aversion or a negative return differential. Our aim, however, is to discuss the case of high inflation countries, where the optimal demand for foreign currency is positive.

Corollary 1 *(domestic money influenced by portfolio decisions). In the conditions of Proposition 1, if the domestic currency and the foreign currency are substitutes as means of payment, then the demand for domestic currency will be influenced by portfolio considerations.*

The intuition underlying Corollary 1 is quite simple. If the amount of dollar-currency holdings affects the moneyness of the peso-currency, then any variable influencing the demand for dollar-currency will influence the demand for peso-currency, even if the later is dominated as store of value.

To illustrate, we solve the system (13a)-(14) for the money demands, using the transactions technology (9). Proceeding as before, the following properties are obtained:

$$m = cL^m(i,\varepsilon,\gamma),$$

$$\text{with } L_i^m = \frac{w(v_{22}-v_{12})+c\phi\gamma^2}{\Omega},\ L_\varepsilon^m = \frac{wv_{12}}{\Omega} < 0 \text{ and } L_\gamma^m = -\frac{2f\gamma\phi v_{12}}{\Omega} > 0, \tag{16b}$$

$$f = cL^f(i,\varepsilon,\gamma),$$

$$\text{with } L_i^f = \frac{w(v_{11}-v_{21})}{\Omega}, L_\varepsilon^f = \frac{-wv_{11}}{\Omega} > 0 \text{ and } L_\gamma^f = \frac{2f\gamma\phi v_{11}}{\Omega} < 0, \tag{17b}$$

where $\Omega = -\Delta w/c - v_{11}\phi\gamma^2 < 0$.

In the particular case in which the two currencies are not substitutes as means of payment ($v_{12}=0$) the demand for domestic money is the same as in a closed economy, (16a). The demand for foreign money is such that $L_i^f = -L_\varepsilon^f < 0$.

To interpret, consider first a rise in the expected exchange rate depreciation not imbedded in the domestic interest rate *(d*ε>0, *di=0).* From (13a), this induces a portfolio shift from peso-bonds to dollar-currency. In the same manner, a decline in the exchange rate volatility (*d*γ<0) induces

agents to move away from peso bonds to buy dollar-currency. When the moneyness of the peso does not depend on the amount of dollar holdings, these developments do not impact on the domestic money demand. In the presence of means of payment substitutability, however, the demand for peso-currency will fall in response to these portfolio changes.

When the rise in the expected exchange rate depreciation is embedded in the domestic interest rate ($di=d\varepsilon>0$), the demand for dollar-currency does not change for portfolio reasons. However, the rise in the user cost of the peso-currency leads agents to reduce the amount of money balances in domestic currency. If the two currencies are substitutes, this will induce a rise in the demand for dollars as vehicle for consumption. This is a pure currency substitution effect: the demand for pesos declines and the demand for dollars rises[9].

It is important to observe that the signs of the partial derivatives in respect to the domestic interest rate in (16b) and (17b) are uncertain. To understand this, assume that the domestic interest rate raises alone ($di>0$, $d\varepsilon=0$). If there was no currency substitution, this would lead to decline in the demand for peso-currency (as in a closed economy) accompanied by a portfolio shift from dollar currency to peso-bonds (with $v_{12}=0$, L_i^m and L_i^f are unequivocally negative). Under currency substitutability, however, less money holdings in one currency denomination lead to a rise in the liquidity value of the other currency. To obtain negative elasticities ($L_i^m<0$ and $L_i^f<0$), it is sufficient to assume that own effects dominate over currency substitution effects (that is $v_{kk}>v_{12}$, with k=1,2). Other results are however consistent with $\Delta>0$, in equation (9). For example, with $v_{22}>v_{12}>v_{11}$, one would obtain $L_i^m>0$ and $L_i^f<0$.

---

[9] Note that, since the marginal contribution of dollar-holdings to the reduction of transaction costs ($-v_2$) is decreasing, the expected return differential in (13a) is affected. This gives rise to a second round of effects, via portfolio adjustment, that partially offsets the initial currency substitution effect.

5. Currency substitution when domestic money has a portfolio role

In this section we discuss briefly the cases in which the consumer has no access to bonds denominated in domestic currency[10]. In this case, the domestic currency plays both a store of value and a means of payment role.

Two sub-cases shall be distinguished: the case in which no bonds (either domestic or foreign) are available and the case in which the individual has access to bonds denominated in foreign currency.

Maximisation of (1) subject to (11) under the restriction $a=b=0$ leads to the following modified optimal portfolio rule:

$$\frac{f}{w} = \left(1 - \frac{1}{\phi}\right)\left(\frac{\rho}{\gamma^2}\right) + \left(\frac{1}{\phi}\right)\left(\frac{v_1 - v_2 + \varepsilon}{\gamma^2}\right) \qquad (13c)$$

In this case, the demand for domestic currency will be influenced by the expected exchange rate depreciation by two different channels: store of value substitution and means of payment substitution (if any). The influence of an expected exchange rate depreciation term in the demand for domestic money will still capture currency substitution, though in a broad sense, not distinguishing the two channels. Currency substitution in a world without bonds was first discussed in the context of the monetary model by Calvo and Rodrígues (1977) and is analysed in the context of the liquidity services model by Rojas-Suarez (1992).

The only case in which domestic money would substitute for foreign bonds as store of value is the when the foreign currency is dominated and the domestic currency is not. To see this, maximise (1) subject to (11) under the restriction $a=0$, obtaining (15) and:

$$\frac{b}{w} = \left(1 - \frac{1}{\phi}\right)\left(\frac{\rho}{\gamma^2}\right) + \left(\frac{1}{\phi}\right)\left(\frac{v_1 + j + \varepsilon}{\gamma^2}\right) \qquad (13b)$$

Of course, in that case, the influence of an expected exchange rate depreciation term in the demand for domestic currency could wither capture currency substitution and portfolio choices. The assumption that foreign currency is dominated while the domestic currency is not is however unrealistic.

6. Implications for empirical work

In the earlier empirical literature on currency substitution (Miles, 1978, Ramirez-Rojas, 1985), the currency substitution hypothesis was tested evaluating the significance of a term capturing the expected exchange rate depreciation in the demand for domestic money. This procedure - which has also been adopted in recent estimates for the euro-area money demand (references in Artis, 1996) - was criticised by Cuddington (1983), in the context of the Portfolio Balance Model (PBM) of currency substitution.

The PBM postulates gross substitutability between money and all other assets, leading to money demand functions that depend positively on income and wealth and negatively on the return of each alternative asset. When the available assets are domestic money, foreign money, domestic bonds and foreign bonds, the proposed functional form is (see Branson and Henderson, 1985):

$$\frac{M}{P} = m\left(\overset{-}{i}, \overset{-}{j + \varepsilon}, \overset{-}{\varepsilon}, \overset{+}{y}, \overset{+}{w}\right) \qquad (18)$$

---

[10] In Appendix 1 we argue that, extending the analysis so as to include a real asset on the consumer portfolio does not change qualitatively the results.

In (18), the second term in the right hand side captures substitutability between the domestic currency and the foreign bond and the third term captures substitutability between the domestic currency and the foreign currency. Since the demand for domestic currency depends negatively on the expected exchange rate depreciation both through substitutability vis-à-vis the foreign currency and substitutability vis-à-vis the foreign bond, followers of the PBM have claimed that currency substitution and capital mobility are statistically indistinguishable. Moreover, in light of that approach, it has been argued that CS does not constitute a qualitative difference relative to capital flight (Cuddington, 1983). Empirical exercises based on the PBM include Cuddington (1983), Rogers (1992), Mizen and Pentecost (1994), Akçay et. al (1997).

The PBM has two main shortcomings. First, as noted by Branson and Henderson (1985), gross substitutability is not always consistent with individual optimisation. Second, the model does not explain why money is held, despite being dominated by interest-bearing assets. A closer scrutiny of the properties of the money demand in light of firmer microeconomic foundations was made by Thomas (1985), for the case with complete bond markets. As shown in Section 3, in this case, there is no portfolio demand for money.

The Thomas' model shall be seen as the centrepiece to test the CS hypothesis in countries with developed financial markets. Not surprisingly, this model has been used to test the presence of currency substitutability among major currencies (see Joines, 1985, Bergstrand and Bundt, 1990, Mizen and Pentecost, 1994). A common procedure - remember equations 14 and 14a - has been to investigate the significance of the foreign interest rate, j, in:

$$\frac{M}{P} = m\left(\overset{-}{i}, \overset{+}{j}, \overset{+}{y}\right). \tag{19}$$

The Thomas model is less suitable, however, to describe the phenomenon of CS in countries where asset markets are illiquid. As pointed out by Cuddington (1989), in that case, one expects the demand for foreign currency to have both a means of payment and a store of value role.

The results obtained in Section 4 give support to the Cuddington (1989) claim that, in the presence of capital controls, there will be a portfolio demand for foreign currency. They also suggest that, in case the two currencies are substitutes as means of payment, the demand for domestic currency will be influenced by portfolio considerations. However, our findings do not give support to an empirical test based on the PBM (18). In alternative, equations (16a) and (16b) suggest that a valid test for the presence of currency substitution in countries facing capital controls is:

$$\frac{M}{P} = m\left(\overset{-}{i}, \overset{-}{\varepsilon}, \overset{+}{\gamma}, \overset{+}{y}\right). \tag{20}$$

This corresponds to the traditional test (Ramirez-Rojas, 1985), except for the inclusion of a term capturing exchange rate volatility.

Of course, equations (19) and (20) are based on extreme assumptions. In (19), individual are allowed to borrow or lend any amount of foreign money, at a given interest rate. In (20), individuals are not allowed to hold bonds denominated in foreign currency. One may think, however, in economies composed by individuals of the two types. If a positive fraction of the population has access to bonds denominated in foreign currency while another positive fraction has not, probably a functional form combining elements of (19) and (20) would provide a suitable specification to start with when testing for CS in developing countries.

It may be argued that, under uncovered interest rate parity, the choice of the particular model to be estimated is less relevant. This does not change, however, the main message of the paper: irrespectively of the degree of capital mobility, only in case of currency substitution will the

demand for domestic money depart from the closed economy specification (16a). Thus, the CS hypothesis may be investigated, without ambiguity concerning the identification of the relevant effect.

7. Conclusions

We extend the Thomas (1985) dynamic optimising model of money demand and currency substitution to a case in which the consumer has no access to bonds denominated in foreign currency. In this case, foreign bank notes may have a portfolio role in addition to a means of payment role. We show that the presence of means of payment substitutability opens a channel through which portfolio decisions influence the demand for domestic money.

A particular result obtained is that, if domestic money is dominated as store of value, its demand will depend on open economy variables only in case the currency substitution hypothesis holds. This result contradicts the Cuddington (1983, 1989) very influential claim that the significance of an expected depreciation term in the demand for domestic money does not provide a valid test for the presence of currency substitution. Extending the analysis to other combinations of available assets, we find that only in the odd case in which the consumer has access to foreign bonds but not to domestic bond would the Cuddington claim make sense. This is not, however, a realistic scenario.

Our results are rather convenient for empirical purposes. A well known limitation in the empirical analysis of currency substitution is that of measurability. Since data on foreign bank notes circulating in an economy are not easily available, many empirical studies have measured the extent of currency substitution by the proportion of foreign currency denominated deposits (FCD) in M2[11]. Such proxy, has been, however, under criticism: on one hand, it is sensitive to swaps between

[11] In some developing countries, residents' bank deposits denominated in foreign currency were legalised, with the aim to raise credibility on the domestic monetary policy and to stop capital flight. Where this was done, monetary statistics are providing figures on a regular basis. Since these deposits are likely to grow fast in periods of erosion of macroeconomic conditions, thus accompanying the general shift towards foreign monetary assets, they are not surprisingly being used as a proxy for the extent of CS in many empirical studies.

foreign bank notes and FCD, which do not necessarily reflect currency substitution[12]; on the other hand, in countries with underdeveloped capital markets, interest bearing FCD may have a role more comparable to that of foreign bonds than that of foreign currency (Sahay and Végh, 1996). If, according to our results, the currency substitution hypothesis can be assessed estimating directly the demand for domestic currency, then these data limitations are circumvented.

[12] This problem is likely to occur when the institutional framework of FCD is not stable. For example, in Mexico, Peru and Argentina, FCD declined in periods of rising inflation, either because of changes in their legal status or due to the erosion of the public confidence in their legal status (Rogers, 1992, Kamin and Ericsson, 1993, Savastano, 1996).

References

Agenor, P., Khan, M. , 1996. "Foreign Currency Deposits and the demand for money in developing countries". *Journal of Development Economics* 50, 101-118.

Akçay, O. C., Alper, C. and Karasulu, M., 1997. "Currency Substitution and Exchange Rate Instability: The Turkish Case", *European Economic Review,* 41 (3-5), 827-835.

Artis, M. , 1996. "Currency Substitution in European Financial Markets", in P. Mizen and E. Pentecost (eds), *The Macroeconomics of International Currencies: Theory, policy and Evidence*, Cheltenham: Edward Elgar.

Bergstrand, J. and Bundt, T. , 1990. "Currency substitution and monetary autonomy: the foreign demand for US deposits", *Journal of International Money and Finance* 9, 325-34.

Branson, W. and Henderson, D., 1985. The specification and influence of asset markets, in: R. W. Jones and P. B. Kenen, eds., *Handbook of International Economics, Vol. II* (North Holland, Amsterdam).

Calvo, G. and Rodríguez, C., 1977. "A model of exchange rate determination under currency substitution and rational expectations". *Journal of Political Economy* 85, 617-24.

Cuddington, J., 1983. "Currency Substitutability, Capital Mobility and Money Demand", *Journal of International Money and Finance,* 2, 111-133.

Cuddington, J., 1989. "Review of 'Currency Substitution: Theory and Evidence from Latin America" by V.A. Canto and G. Nickelburg. *Journal of Money, Credit and Banking* 21, 267-271.

Freitas, M., 2004. "The dynamics of Inflation and Currency Substitution in a Small Open Economy". *Journal of International Money and Finance* 23 (1), 133-142.

Giovannini, A. and Turtelboom, B., 1994. "Currency Substitution." in *The Handbook of International Macroeconomics*, edited by Frederick Van Der Ploeg, pp. 390-436. Oxford UK: Blackwell, 1994.

Imrohoroglu, S. , 1994. "GMM Estimates of Currency Substitution between the Canadian Dollar and the US Dollar", *Journal of Money, Credit and Banking,* 26 (3) , 792-807.

Kamin, S. and Ericsson, N., 2003. "Dollarisation in Post-hyperinflationary Argentina." *Journal of International Money and Finance* 22, 185-211.

Kareken, J. and Wallace, N., 1981. “On the indeterminacy of equilibrium exchange rates”, *Quarterly Journal of Economics* 96, 207-22.

Krugman, P., 1984. "The international role of the dollar: theory and prospect". In *Exchange Rate in Theory and Practice* (ed. J. F. Bilson and R.C. Martson). Chicago: University of Chicago Press.

Miles, M, 1978. "Currency Substitution, Flexible Exchange Rates, and Monetary Independence.” *American Economic Review* 68, 428-436.

Mizen, P. and Pentecost, E., 1994. "Evaluating the Empirical Evidence for Currency Substitution: A Case Study of the Demand for Sterling in Europe", *The Economic Journal*, 104 (426), 1057-1069.

Ramirez-Rojas, C., 1985. "Currency substitution in Argentina, Mexico and Uruguay", *Staff Papers,* International Monetary Fund, 32 (3), 629-667.

Ratti, R. and Jeong, B. 1994. "Variation in the Real Exchange Rate as a Source of Currency Substitution", *Journal of International Money and Finance,* 13 (4), 537-550.

Rojas-Suárez, L., 1992. "Currency substitution and inflation in Peru". *Revista de Análisis Económico* 7(1), 151-176.

Rogers, J.., 1990. "Foreign Inflation Transmission under Flexible Exchange Rates and Currency Substitution", *Journal of Money, Credit and banking,* 22 (9), 195-208

Rogers, J. , 1992. "Convertibility Risk and Dollarisation in Mexico: a Vector-Autorregressive Analysis", *Journal of International Money and Finance*, 11, 188-207.

Sahay, R. and Végh, C., 1996. “Dollarisation in Transition Economies, Evidence and Policy Implications”, in *The Macroeconomics of International Currencies: Theory, Policy and Evidence*, edited by P. Mizen and E. Pentecost, pp. 193-224. Cheltenham: Edward Elgar.

Savastano, M., 1996. "Dollarisation in Latin America: recent evidence and policy issues", in *The Macroeconomics of International Currencies: Theory, Policy and Evidence*, edited by P. Mizen and E. Pentecost, pp. 193-224. Cheltenham: Edward Elgar, 1996.

Smith, C., 1995. "Substitution , Income and Intertemporal Effects in Currency-Substitution Models", *Review of International Economics,* 3 (1), 53-59.

Thomas, L. , 1985. "Portfolio Theory and Currency Substitution", *Journal of Money, Credit and Banking*, 17 (2), 347-357.

Végh, C., 1989. " The optimal inflation tax in the presence of currency substitution", *Journal of Monetary Economics* 24, 139-146.