# Financial Frictions and Business Cycles in Developing Countries

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**Abstract**

Standard theory of small open economies predicts a smooth path for consumption and investment over time, and procyclical current account balances and employment. This contrasts with the data for emerging countries, where consumption, investment and employment are highly procyclical and volatile, and the current account balance is counter cyclical. Previous studies have reconciled theory and data using different households preferences for emerging countries vs. developed ones, either a lower intertemporal elasticity of substitution or the use of GHH type of preferences. This paper takes a different approach. It explores whether adding two financial frictions to an otherwise standard small open economy model, an external borrowing constraint and labor financing wedges, can reconcile theory and data without appealing to changes in preferences, but rather to market imperfections widely studied in the literature for the emerging world. I find that adding the external borrowing constraint makes consumption and investment more procyclical and volatile, and reproduces the counter cyclical path of the current account. However, it generates counter cyclical employment. Adding the labor financing wedges reproduces the cyclical path of this variable as well

# 1 Introduction

Developing countries' business cycles during the 1980s and 1990s were characterized by highly procyclical consumption and investment, counter cyclical current account balances and procyclical employment. This contrasts with the predictions of small open economy models when standard preferences from the real business cycles literature are considered. These models predict that consumption and investment should be smoothed through external borrowing, while the current account balance and employment should be procyclical.

Mendoza [25], [26] and [27], Correia et al. [14] and Schmitt-Grohe et al. [29] approached the issue assuming GHH type preferences or a lower intertemporal elasticity of substitution in developing countries than in developed ones. GHH type preferences makes the labor-leisure decision independent from consumption and wealth effect, setting it only as a function of the real wage. In this way, employment over react to shocks, which causes bigger fluctuations in output, consumption and investment, and a counter cyclical pattern for current account, everything in line with the data. A low intertemporal elasticity of substitution create the same cyclical fluctuations.

The approach in this paper is different. I will argue that is not necessary to assume different preferences for consumers in developing vs developed countries to reconcile data and theory, but rather private agents in the emerging economies face two types of financial frictions that limit their optimal response to shocks. First, they face a limited access to the external capital market, identified in the paper as an external borrowing constraint, which refrain them from smoothing consumption and investment over time, and second, they face an counter cyclical internal financial distortions that reduces the costs of credit to the firms with lack of collateral during booms, but increases them during recessions. This last distortion allows the model to capture the cyclical properties of employment observed in data.

The objective of this paper is to evaluate quantitatively, in a general equilibrium framework, whether adding these two financial frictions to a standard small open economy model, would help reconciling theory and data without appealing to non standard assumption of preferences. These two frictions have been studied in the literature as amplifiers or source of economic fluctuations in the developing economies. The quantitative analysis is made taken Chile as a case study, for which Caballero [8] and [9] argued that the interaction between both frictions played a major role amplifying its cycles during the 1990s.

The analysis is done in consecutive steps. First, a frictionless small open economy model is calibrated and simulated for exogenous sequences of shocks in terms of trade, external interest rate and total factor productivity between 1992 and 2000. Consecutively, each friction is added exogenously to the model to quantify their contribution to improve the matching of the model to the data.

The simulations show that the external borrowing constraint reduces consumption and investment smoothing over time, as well as replicates the counter cyclical path of current account. However, for standard specification of preferences, it produces counter cyclical hours worked and a smoother path of output of non tradable than in data. Adding counter cyclical sector specific labor financing wedges allows the model to solve this problem.

The average labor wedge that makes the model replicate the path of output in data is similar to the internal lending spread between 1993 and 1998, but higher in 1999 and 2000. This suggests that the lending spread in data is misleading as a measure of internal financing frictions during recessions. It does not account for the high spread faced by small and medium size firms, with lack of collateral, which are crowded out of the internal financial market in bad times.

The external constraint arises because contracts in external lending are not enforceable and there is a risk of default. Atkeson and Rios-Rull [4] and Caballero and Krishnamurthy [10] identified this friction as a collateral constraint, in which part of the exportable sector's profits or revenues could be seized by external lenders in case of default. Eaton and Gersovitz [15], Bulow and Rogoff [7], Atkeson [3], Kehoe and Levine [22], Kocherlacota [23], Alvarez and Jerman [2] and Jeske [21] considered the punishment by defaulting as the exclusion from the external capital market. Wright [29] argued that the country's concern for it's reputation can enforce repayment without placing any technological restriction on the ability of banks to offer contracts as long as there are incentives for banks to tacitly culled to punish a country in default.

This paper specifies the constraint as the external lenders' requirement to the internal households to finance at least a fraction of their expenditures with their current income, Mendoza [27]. This fraction is taken from data and introduced exogenously to the model assuming that such constraint was always binding. In this setup, a positive shock that makes the constraint less binding, will make households increase consumption of tradable and non tradable goods. The former can be obtained abroad, while the latter has to be produced domestically. Thus, there is a lower expansion in the tradable sector than in a frictionless model, which makes the current account counter cyclical. A negative shock triggers the opposite.

However, for standard specification of preferences, this setup produces counter cyclical employment and a smoother output of non tradable than in data. Mendoza [25], [26] and [27], and Neumeyer and Perry [28] used GHH preferences to attack this problem, for which the labor-leisure decision depends only on the wage and not on wealth effects. But, these preferences are incompatible with balanced growth path and differ from the standard specifications in the real business cycles literature. This paper shows that this problem can be solved by considering counter cyclical sector specific labor financing wedges, with no need of setting non standard preferences.

The financial friction for domestic producers arises from informational and moral hazard problems that create financial constraints among firms or with banks, which

limits a country's ability to reallocate resources in response to shocks. Holmstrom and Tirole [20] derive them from moral hazard problems, Bernanke and Gertler [6] from "costly state verification" problems, while Kiyotaki and Moore [24] and Caballero and Krishnamurthy [10] considered them as collateral constraints.

This paper sets this friction as sector specific labor financing wedges, in which the cost of paying wages in advance to production differs across sectors and fluctuates with the cycle. Chari at all [13] showed that a credit in advance friction can be represented in this way, methodology that is used in this paper to deduct the wedges that allow the model to replicate the path of output in data.

The paper is organized as follows: section 2 presents an empirical motivation, section 3 presents a literature review, section 4 discusses the objective and methodology, section 5 presents the model and simulations for the frictionless economy, sections 6 and 7 present the model and simulations when the external borrowing constraint and the labor financing wedges are added respectively, and section 9 concludes.

# 2 Empirical Motivation

This section presents empirical evidence of lack of consumption smoothing, counter cyclical current accounts and procyclical employment in developing countries. It then discusses why Chile is a good case study, and describes the cycles and shocks that this country faced during the 1990's as well as the methodology to construct them.

## 2.1 Business Cycles in Developing Countries

Standard real business cycles models predict a more procyclical consumption in developed countries than in developing ones. For the former group, the interest rate is endogenous and consumption varies in response to temporary shocks, while for the latter group, the interest rate is exogenously given by the external capital market and consumption is smooth and independent from these shocks.

Table 1 presents the data on standard deviations of detrended output and current account as percentage of GDP (columns 1 and 9), relative standard deviations to detrended GDP (columns 3, 5 and 7) and correlations to detrended GDP (columns 2, 4, 6 and 8) of detrended private consumption, gross fix capital formation, employment and current account balance as percentage of GDP for different groups of industrialized and emerging economies between 1980 and 2001. It shows that in data, emerging countries' consumption and investment were almost as procyclical, and more volatile relative to GDP, than in the industrialized countries, particularly in Latin America and South East Asia. Thus, developing countries did not smooth consumption and investment over time as the standard theories would predict. This fact motivates the introduction of a external borrowing constraint to the model, which refrains internal households from smoothing both variables over time.

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The correlation between consumption and GDP, as well as their volatility, has a direct correspondence to the correlation among GDP, employment and current account. Table 1 shows that in data, employment in developing countries is as procyclical, and more volatile relative to GDP, than in the industrialized countries, while the current account in Latin America and South East Asia is as counter cyclical, but more volatile, than for this group of countries.

## Table 1
## Business Cycles Indicators (1980-2001)[1]

|  | (1)<br>St.Dev.<br>Y | (2)<br>Correl.<br>C to Y | (3)<br>SD C/<br>SD Y | (4)<br>Correl.<br>I to Y | (5)<br>SD I/<br>SD Y | (6)<br>Correl.<br>L to Y | (7)<br>SD L/<br>SD Y | (8)<br>Correl.<br>CA to Y | (9)<br>St.Dev.<br>CA/Y |
|---|---|---|---|---|---|---|---|---|---|
| G7 | 0.019 | 0.90 | 1.14 | 0.89 | 3.11 | 0.47 | 0.49 | -0.40 | 0.016 |
| Others OECD | 0.024 | 0.82 | 1.10 | 0.86 | 3.65 | 0.53 | 0.40 | -0.30 | 0.030 |
| Developing Countries | 0.041 | 0.69 | 1.39 | 0.73 | 3.82 | 0.50 | 0.52 | -0.24 | 0.044 |
| Africa | 0.038 | 0.61 | 1.71 | 0.60 | 3.87 | na | na | -0.08 | 0.043 |
| Latin America | 0.043 | 0.76 | 1.20 | 0.83 | 3.85 | 0.44 | 0.61 | -0.29 | 0.034 |
| Chile | 0.053 | 0.94 | 1.41 | 0.96 | 3.10 | 0.20 | 0.75 | -0.29 | 0.037 |
| Middle East | 0.042 | 0.51 | 1.37 | 0.58 | 4.36 | 0.72 | 0.57 | -0.43 | 0.050 |
| South East Asia | 0.044 | 0.82 | 1.11 | 0.84 | 3.32 | 0.60 | 0.28 | -0.41 | 0.062 |

Therefore, business cycles in developing countries were characterized by a procyclical and non smooth path of consumption and investment over time, a procyclical and volatile employment, and a counter cyclical and volatile current account balance. This contrasts with standard theories for small open economies, in which consumption is smooth over time, and the current account and employment are procyclical. This motivates the introduction of financial frictions to the model to explore up what extent are able to reconcile theory and data.

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[1]GDP and Private Consumption and Investment correspond to log deviations from the trend.
Africa: Algeria, Burundi, Botswana, Cameroon, Ethiopia, Ghana, Kenya, Madagascar, Mauritius, Morocco, Nigeria, South Africa, Sudan, Tanzania, Zambia and Zimbawe.
Latin America: Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, El Salvador, Guatemala, Honduras, Mexico, Panama, Paraguay, Peru, Uruguay and Venezuela.
South East Asia: Hong Kong, Indonesia, South Korea, Malaysia, Philippines, Singapore and Thailand.
Middle East: Egypt, Israel, Jordan and Siryan Arab Republic.

## 2.2   Chile as a Case Study

Chile is taken as a case study for two reasons. First, it presents the same pattern in consumption, investment, employment and current account in data than the other emerging economies. Second, it is widely cited in the literature as one of the most successful and disciplined economies in Latin America since the mid 1980s, which makes reasonable to simplify the analysis to a simple exchange - production economy, for which fiscal or monetary policy shocks are not important as source or amplifiers of cycles.

This section discusses first Chile's economic policy and structural reforms to motivate the setting of the model as a simple exchange - production economy, abstracting from economic policy shocks. Second, it shows the lack of consumption smoothing as a motivation to consider an external borrowing constraint in the analysis. Third, it discusses the relevance of internal financial frictions as a motivation of sector specific labor financing frictions. Fourth, it presents this economy's business cycles, shocks and the methodology to construct them during the 1990s.

### 2.2.1   Economic Policy and Structural Reforms

Since the late 1970's, Chile undertook several structural reforms in trade policy, fiscal policy, privatization, banking and bankruptcy, to promote growth and reduce volatility. Bergoing et all [5] made the following review of them:

Trade policy: By 1979 Chile had eliminated all quantitative barriers on imports and imposed an uniform tariff of 10 percent. This tariff was increased to 20 percent in 1983 and to 35 percent in 1984, and then gradually lowered until it fell below its original level in 1991.

Fiscal policy: There was a major tax reform in 1975, a major social security reform in 1980, and another major tax reform in 1984.

Privatization: By 1973 a large fraction of the economy was under government control. From 1974 to 1979, the government reprivatized much of it. The number of state companies was reduced from 596 in 1973 to 48 in 1983, although the portion of GDP generated by them just fell from 39 percent to 24 percent.

Banking: In 1973 Chile had 18 national banks and 1 foreign bank. After several reforms that included deregulation, low reserve requirements, and opening to foreign competition, the number of financial institution increased by 1980 to 26 national banks, 19 foreign banks and 15 financieras (non-bank financial institutions subject to less stringent regulations). After the collapse in 1982, the government liquidated insolvent banks and financieras, quickly reprivatized solvent banks taken over because of liquidity problems, and set up a new regulatory framework to avoid mismanagement.

Bankruptcy: By the end of the 1970's Chile had an obsolete and unwieldy bankruptcy law, which relied on poorly paid public officials and highly bureaucratic proce-

dures, which made bankruptcy proceedings languish for years in court. Following an administrative reform of the bankruptcy management service in 1978, the 1982 bankruptcy reformed law clearly defined the rights of each creditor and replaced public officials by private officials, making the bankruptcy proceedings much quicker.

Besides these reforms, Chile maintained fiscal and monetary discipline since the mid 1980s. On the fiscal side, Figure 1 shows that the fiscal balance as percentage of GDP was stable during all 1990's, which contrasts with its behavior during the 1970's and first half of the 1980's. Based on this, fiscal shocks were excluded from the analysis of Chile's cycles during the 1990's, arguing that they did not play an important role in the origin or amplification of them.

**Figure 1: Fiscal Balance (% GDP)**
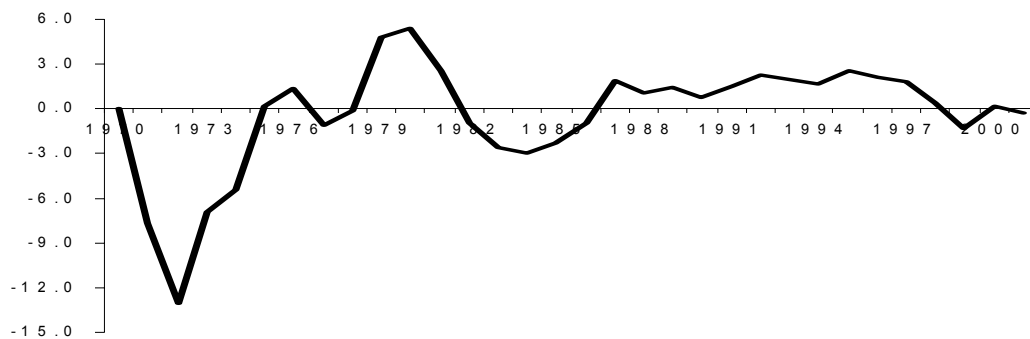


In the monetary side, Central Bank's independence was imposed in 1990, and since then inflation fell from 26 percent in 1989 to 3 percent in 2000, see Figure 2. The price stability achieved during the 1990s together with the impediment by law for the central bank to buy fiscal debt, suggests that monetary imbalances did not play an important role in the origin or amplification of cycles. For that reason, the monetary side of the economy is excluded from the analysis, leaving a model focussed only on real variables.

**Figure 2: Inflation (yoy %)**



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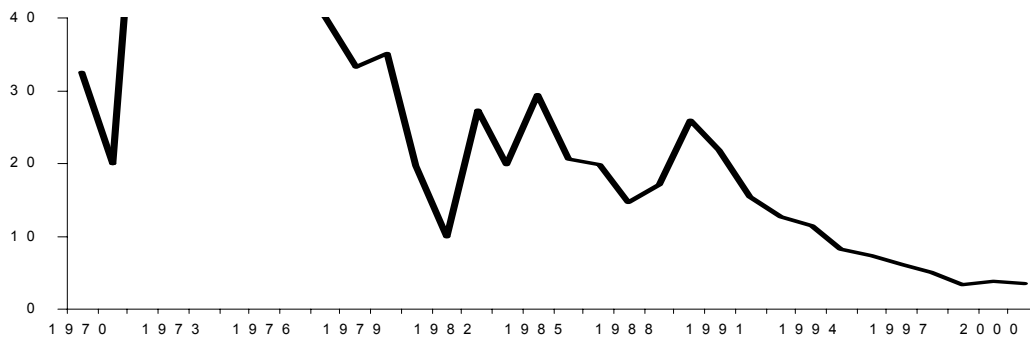
### 2.2.2 Absence of Consumption Smoothing

Figure 3 presents the rates of growth of GDP, private consumption and investment in Chile during the 1990's. It shows a high correlation between consumption and GDP, a symphom of lack of consumption smoothing. Investment is also highly correlated to GDP, although more volatile. This contrasts with the implications of the standard theory, in which consumption is smoother than output. This motivates the introduction of a external borrowing constraint into the analysis, which could have been always binding given the high correlation between consumption and GDP.

**Figure 3: GDP and Spending (yoy %)**
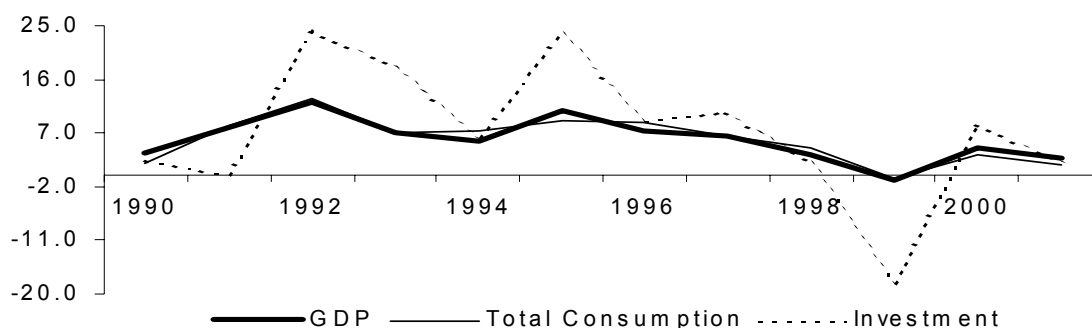


### 2.2.3 Internal Financial Frictions:

This section is based on Caballero [9]. He argued that during the recession in 1999, domestic banks in Chile reacted slowing down private loans even though domestic deposit continue growing fast. They substituted private domestic loans by public debt and external assets. At the same time, there was a "flight to quality" phenomenon, in which domestic banks allocated a higher fraction of their credit to big size firms, reducing considerably the access to credit to small and medium size firms. Big firms could substitute their financial needs in the domestic financial market, while many small and medium size firms were crowded out from it.

This analysis suggests that small and medium size firms have a better access to internal financing in booms than in recessions. Thus, the lending spread in data is misleading as a measure of this distortion, because it does not account for the high spread that firms crowded out of the financial market face in bad times.

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### 2.2.4   Shocks and the Internal Cycle

Figure 4 presents Chile's real GDP index and its trend for the 1990's. Chile experienced a boom between 1995 and 1998, and a recession in 1999, from which it had not recovered. The boom coincided with a fall in unemployment, a deterioration in the current account balance and big capital inflows. The recession coincided with a sharp improvement in the current account balance, a sudden reversal in capital inflows and an abrupt increase in unemployment.

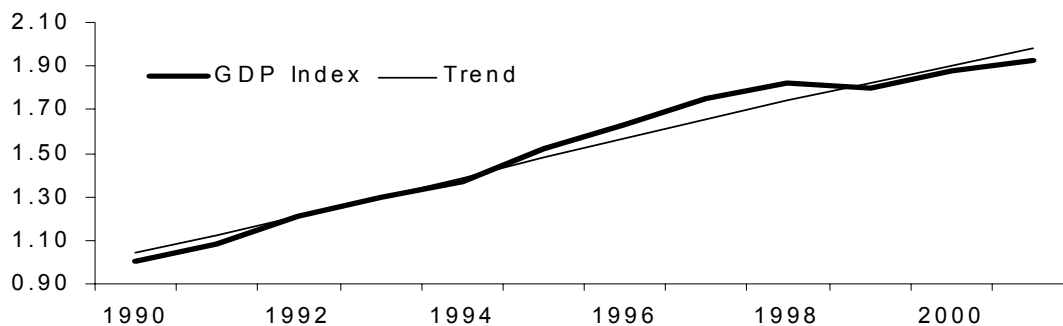### Figure 4:  Real GDP index and Trend



Figure 5 presents the sequences of external interest rate, terms of trade and total factor productivity, in log-deviations from 1992. The external interest rate corresponds to the federal funds rate minus ex-post inflation in the US and the terms of trade to the price exports over the price of imports. To estimate total factor productivity was necessary to build the sectorial series output, capital stock and hours worked.

The exportable sector's GDP corresponds to Mining, Agriculture and Forestry, Fishery and Manufacturing, equivalent to 36 percent of total GDP. The Non Tradable sector's output corresponds to Wholesale and Retail Trade, Construction, Electricity, Gas and Water, Financial Services, Housing, Personal Services, Public Administration and Transport, Storage and Communication, equivalent to 64 percent of total GDP.

The following law of motion for capital was used to construct the series of aggregate capital stock in stationary terms:

$$\gamma k_{t+1} = (1 - \delta) k_t + i_t - \left(\frac{i_t}{k_t}\right)^v k_t \tag{1}$$

The ratio capital stock to GDP was taken from Bergoing et all [5]. The depreciation rate was set in 8 percent according the United Nations estimations, and $v$ was set in 2.1. This parameters, together with investment from data, yield a capital stock consistent with balanced growth path and with Aguilar and Collinao [1] estimations.

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Total hours worked were constructed using the series of total employment from the National Institute of Statistics and average hours worked per employee reported by the ILO. The normalized value of hours worked corresponds to the average hours worked times the number of employed people, divided by the potential working time of the total working age population.

Aggregate total factor productivity was estimated using aggregate GDP, capital stock and hours worked, and assuming a Cobb-Douglas production function with a capital share fraction of 0.47. To estimate the relative allocation of hours worked and capital stock in each sector, it was assumed that capital is sector specific and labor is freely mobile across sectors. Assuming that both sectors present Cobb-Douglas production functions, the relative allocation of labor corresponds to:

$$\frac{l_t^N}{l_t^X} = \frac{\left(1 - \alpha_N\right) P_t^N y_t^N}{\left(1 - \alpha_X\right) P_t^X y_t^X} \tag{2}$$

Having the sectorial output and hours worked, the sectorial allocation of capital and total factor productivity were estimated using both sectors production functions, the market equilibrium condition for capital, and the condition that equates the average total factor productivity to the aggregate estimation of it.

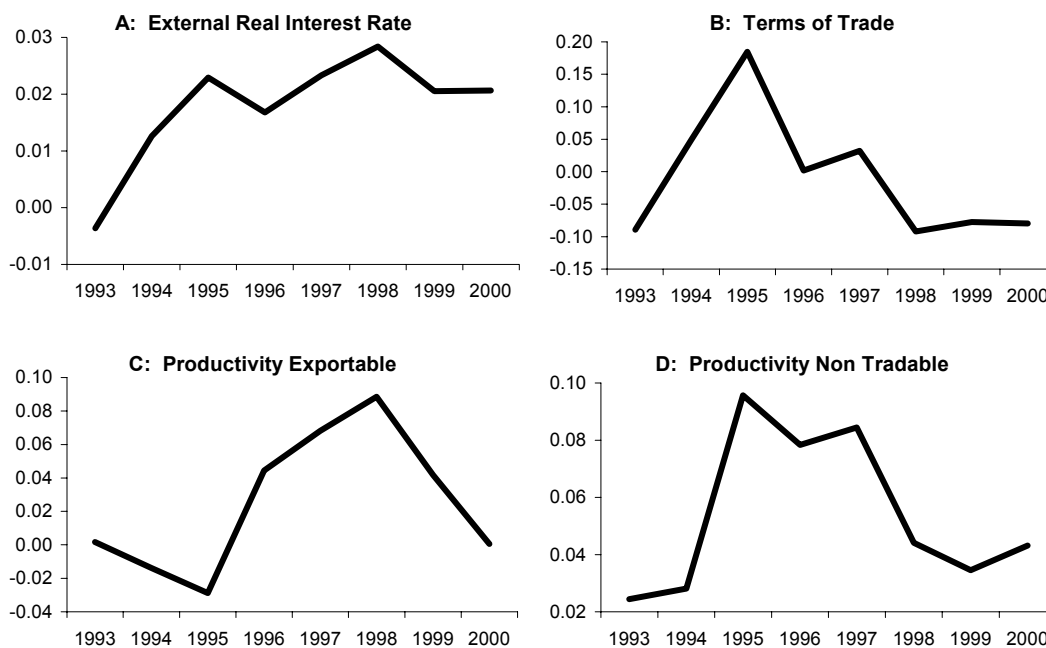**Figure 5: Internal and External Shocks (log-deviations from 1992)**



Figure 5 suggests that the internal cycles were induced by the terms of trade shocks rather than by internal shocks or by shocks to the external interest rate. The terms

of trade present large deviations, a maximum of 20 percent in 1995, and a minimum of -10 percent in 1999, which coincides with the beginning of each cycle. Total factor productivity and external interest rate shocks presented smaller deviations.

# 3   Literature Review

The literature has studied the idiosyncratic properties of business cycles in developing countries in deep. Mendoza [25] highlighted two main regularities in data for the developing countries business cycles: The positive correlation between domestic savings and domestic investment, and the countercyclical patter of the current account and the balance of trade. On the other hand, Correia et al. [14] defined the following regularities for the Portuguese economy: Consumption, output, investment and employment are positively correlated, while the trade balance is counter cyclical.

Mendoza [25], [26] and [27], Correia et al. [14] and Schmitt-Grohe et al. [29] approached the issue assuming GHH type preferences or a low intertemporal elasticity of substitution for developing countries. Correia et al. [14] showed that assuming GHH type preferences makes the labor-leisure decision independent from consumption and wealth, setting it only as a function of the real wage. In this way, employment over react to shocks, creating bigger fluctuations in output, consumption and investment, and a counter cyclical current account. Using the standard preferences used in close economy real business cycles models does not capture important features of small open economies business cycles. The volatility of consumption is much lower that suggested by the data, and the trade balance is procyclical instead of counter cyclical.

Other studies of idiosyncratic properties of business cycles in developing countries have emphasized the relevance of market incompleteness and credit constraints in these economies when compared to the industrialized ones. For example, Caballero [8] stated: "*Latin American economies are weak in two dimensions: links with the international financial markets and development of domestic financial markets. Those two ingredients, either directly or by leveraging a variety of standard shocks, probably account for much of fluctuations and crises in modern Latin America*".

"*Underdeveloped financial markets ultimately limit the prompt reallocation of resources, creating wasteful contractions in those markets most affected by shocks or less plugged into the financial system*". "*Weak links to international financial markets are simply financial constraints, possible time varying, that limit the public and private international borrowing of emerging countries. These constraints limit the smoothing of shocks over time and are themselves a source of shocks*".

Caballero and Krishnamurthy [10] analyzed the interaction of this two frictions in a model that features two types of collateral constraints. First, firms in a domestic economy have limited borrowing capacity from international investors, and second, they also have limited borrowing capacity with respect to each other. They found that

these two constraints can interact in important ways. The first is disintermediation: a fire sale of domestic assets causes banks to fail reallocating resources across firms leading to wasted international collateral. The second is a dynamic effect, in which firms with limited domestic collateral and a binding international collateral constraint will not adequately precaution against adverse shocks, increasing the severity of them.

Weak links to the external markets has been identified in previous studies as collateral external borrowing constraints, in which part of the exportable sector's profits or revenues can be seized by the external lenders in case of default. See for example Atkeson and Rios-Rull [4] and Caballero and Krishnamurthy [10]. Another approach used to identify this constraint is to take the punishment by defaulting as the exclusion from the external capital market after this event. See for example Eaton and Gersovitz [15], Bulow and Rogoff [7], Atkeson [3], Kehoe and Levine [22], Kocherlacota [23], Alvarez and Jerman [2] and Jeske [21]. Wright [30] argued that the country's concern for it's reputation can enforce repayment without placing any technological restriction on the ability of banks to offer contracts, and without appealing to any mechanisms outside of the credit market, as long as there are incentives for banks to tacitly culled in punishing a country in default.

Mendoza [27] set this constraint as the external lender's requirement to the households to finance at least a fraction of their expenditures with current income, arguing that it could result from an optimal contract model with traditional financial-market frictions, and that it is consistent with the lending criteria used in mortgage and consumers loans. In his setup, a sudden stop in capital inflows is generated by an abrupt switch on the constraint from slack to binding in response to negative shocks.

Domestic financial frictions are microeconomics financial constraints, either among firms or banks, limit a country's ability to reallocate resources in response to shocks. Holmstrom and Tirole [20] derive them from moral hazard problems, Bernanke and Gertler [6] from "costly state verification" problems, while Kiyotaki and Moore [24] and Caballero and Krishnamurthy [10] assumed internal collateral constraints.

Other authors have identified them assuming that firms pay the rent of capital and wages in advance to production, facing a credit in advance constraint. See for example. Edwards and Vegh [17] and Neumeyer and Perry [28]. Chari at all [13] showed that an input financing friction of this type can be represented as a time varying productivity wedge, under which a firm specific input financing tax would capture the magnitude of the distortion for each firm.

# 4  Objective and Methodology

The goal of this paper is to evaluate quantitatively, in a general equilibrium framework, the role played by two financial frictions, an external borrowing constraint and labor wedges, in the origin and amplification of the Chilean economy's cycles during

the 1990s. First, a friction-less small open economy model is calibrated and simulated for exogenous sequences of shocks in terms of trade, external interest rate and total factor productivity. Consecutively each friction is added to the model exogenously, quantifying their role in the amplification and origin of the internal cycles.

Capital is sector specific, while labor is freely mobile across sectors. The external borrowing constraint is set as the external lenders' requirement to the households to finance at least a fraction $\Psi$ of their expenditure with their current income. The labor financing friction is identified as a sector specific labor wedge. The simulations are presented as log-deviations from the steady state.

# 5 Friction-Less Small Open Economy

The standard setup for a friction less small open economy assumes that the home economy has free and costless access to the external market. However, in this setup consumption and external assets have random walk processes when the model is log-linearized. Both variables reach new steady state values in response to shocks.

To have a unique steady state, it is necessary to introduce a device that anchors the level of external debt in this equilibrium. Previous studies have considered either an upward sloping supply of external funds, a cost function of adjusting the external asset portfolio or an endogenous discount factor. Schmitt-Grohe and Uribe [28] showed that any of this formulations yield the same dynamics in terms of first and second moments. To be consistent with the following sections, an exogenous upward sloping supply of external funds was used to set up the frictionless version of the model.

## 5.1 Model

Consider a small open economy perfectly integrated with the rest of the world in goods, but faces an upward sloping supply of external funds at the aggregate level:

$$R_t = R_t^* + \eta \left( \overline{b} - b_t \right) \tag{3}$$

Where $\eta$ and $\overline{b}$ are parameters, $R_t$ is the internal rate of return, $b_t$ is the net external assets position and $R_t^*$ the external rate of return, which is stochastic:

$$R_t^* = \exp(\varepsilon_t^R) R^* \tag{4}$$

Where $R^*$ is its unconditional mean and $\varepsilon_t^R$ its stochastic shock, which follows a first order auto regressive process:

$$\varepsilon_{t+1}^R = \rho^R \varepsilon_t^R + v_{t+1}^R \qquad Where \ E\left[v_{t+1}^R\right] = 0 \ \ and \ \ V\left[v_{t+1}^R\right] = \sigma_R^2. \tag{5}$$

There are three goods, an exportable $(X)$, an importable $(M)$ and a non tradable $(N)$, and two production factors, labor $(L)$ and capital $(K)$. Capital is sector specific,

while labor is freely mobile across sectors. The home economy produces $X$ and $N$, using $L$ and $K$ inputs, but consumes $M$ and $N$. The law of one price holds for both tradable goods. The external price of $M$ was normalized to one and assumed to be constant, while the external price of $X$ is stochastic with the following process:

$$P_t^X = \exp(\varepsilon_t^{P^X}) P^{X*} \tag{6}$$

Where $P^{X*}$ is its unconditional mean and $\varepsilon_t^{P^X}$ its stochastic shock in the external market, which is the source of the terms of trade shocks. This shock follows a first order auto regressive process:

$$\varepsilon_{t+1}^{P^X} = \rho^{P^X} \varepsilon_t^{P^X} + v_{t+1}^{P^X}, \qquad Where \ E\left[v_{t+1}^{P^X}\right] = 0 \ \ and \ \ V\left[v_{t+1}^{P^X}\right] = \sigma_{P^X}^2 \tag{7}$$

There are two types of internal agents: households and firms. Households own firms, consume $M$ and $N$ goods, and supply $L$ and $K$ to the firms. They are the only ones with access to external lending. There are two firms, the $X$ producer and the $N$ producer. Both demand $L$ and $K$ to produce their goods, being the former more capital intensive. The economy follows a balanced growth path and population is assumed to be constant. In the following, the model is set in stationary terms.

### 5.1.1 Households

The households' problem is to maximize their lifetime utility:

$$U = E_0\left[\sum_{t=0}^{\infty} \beta^t \left\{\frac{1}{\rho}\log(\varpi c_t^{M\rho} + (1-\varpi)\, c_t^{N\rho}) + \theta_l \log(1-l_t)\right\}\right] \tag{8}$$

Where $c_t^M$ is consumption of importable, $c_t^N$ consumption of non tradable and $l_t$ total hours worked. Having a external lending spread increasing on external debt not only reproduce a unique steady state, but also allow the model to have a rate of time preference different from the external interest rate.

Since the international traded bond and capital are the only assets in this economy, markets of contingent claims are incomplete and the economy's wealth varies with the state of nature. The households flow budget constraint is:

$$w_t l_t + q_t^X k_t^X + q_t^N k_t^N + R_t b_t + \pi_t^X + \pi_t^N = c_t^M + P_t^N c_t^N + i_t^X + i_t^N + \gamma b_{t+1} \tag{9}$$

Where $w_t$ is the wage, $q_t^j$ the rental rate of capital in sector $j = X, N$, $\pi_t^j$ profits in firm $j = X, N$, $P_t^N$ the relative price of non tradable to importable and $i_t^j$ investment in capital $j = X, N$. Investment and capital are expressed in units of the importable good, which is reasonable if we consider that the non tradable good is composed mainly by services. The law of motion for both types of capital are:

$$\gamma k_{t+1}^{j} = (1 - \delta) k_{t}^{j} + i_{t}^{j} - \left(\frac{i_{t}^{j}}{k_{t}^{j}}\right)^{\upsilon} k_{t}^{j} \qquad For \ \ j = X, N \qquad (10)$$

Households choose the sequence $\left\{c_{t}^{M}, c_{t}^{N}, l_{t}, i_{t}^{X}, i_{t+1}^{N}, b_{t+1}\right\}_{t=0}^{\infty}$ to maximize (8), subject to (9) and (10). Their first order conditions are:

$$\frac{\varpi c_{t}^{M(\rho-1)}}{\varpi c_{t}^{M\rho} + (1 - \varpi) c_{t}^{N\rho}} = \lambda_{t} \qquad (11)$$

$$\frac{(1 - \varpi) c_{t}^{N(\rho-1)}}{\varpi c_{t}^{M\rho} + (1 - \varpi) c_{t}^{N\rho}} = P_{t}^{N} \lambda_{t} \qquad (12)$$

$$\frac{\theta_{l}}{(1 - l_{t})} = \lambda_{t} w_{t} \qquad (13)$$

$$\phi_{t}^{X} = \lambda_{t} + \upsilon \phi_{t}^{X} \left(\frac{i_{t}^{X}}{k_{t}^{X}}\right)^{(\upsilon-1)} \qquad (14)$$

$$\phi_{t}^{N} = \lambda_{t} + \upsilon \phi_{t}^{N} \left(\frac{i_{t}^{N}}{k_{t}^{N}}\right)^{(\upsilon-1)} \qquad (15)$$

$$\gamma \phi_{t}^{X} = \beta E_{t} \left\{\lambda_{t+1} q_{t+1}^{X} + \phi_{t+1}^{X} \left((1 - \delta) - (1 - \upsilon) \left(\frac{i_{t+1}^{X}}{k_{t+1}^{X}}\right)^{\upsilon}\right)\right\} \qquad (16)$$

$$\gamma \phi_{t}^{N} = \beta E_{t} \left\{\lambda_{t+1} q_{t+1}^{N} + \phi_{t+1}^{N} \left((1 - \delta) - (1 - \upsilon) \left(\frac{i_{t+1}^{N}}{k_{t+1}^{N}}\right)^{\upsilon}\right)\right\} \qquad (17)$$

$$\gamma \lambda_{t} = \beta E_{t} \left\{\lambda_{t+1} R_{t+1}\right\} \qquad (18)$$

$$E_{t} \left[\lim_{t \to \infty} \beta^{t} \lambda_{t} \left(k_{t+1}^{X} + k_{t+1}^{N} + b_{t+1}\right)\right] = 0 \qquad (19)$$

Where $\lambda_{t}$, $\phi_{t}^{X}$ and $\phi_{t}^{N}$ are the lagrange multipliers on (9) and (10), respectively.

### 5.1.2  Firms

Both firms have Cobb-Douglas constant returns to scale technologies. Their static problem to is choose the allocation $\left\{l_{t}^{j}, k_{t}^{j}\right\}$ to maximize profits, for $j = X, N$. Their first order conditions are:

**Non Tradable Firm**

$$w_{t} = (1 - \alpha_{N}) P_{t}^{N} \exp\left(\varepsilon_{t}^{N}\right) \left(\frac{k_{t}^{N}}{l_{t}^{N}}\right)^{\alpha_{N}} \qquad (20)$$

$$q_t^N = \alpha_N P_t^N \exp\left(\varepsilon_t^N\right) \left(\frac{l_t^N}{k_t^N}\right)^{(1-\alpha_N)} \tag{21}$$

**Exportable Firm**

$$w_t = (1 - \alpha_X) P_t^X \exp\left(\varepsilon_t^X\right) \left(\frac{k_t^X}{l_t^X}\right)^{\alpha_X} \tag{22}$$

$$q_t^X = \alpha_X P_t^X \exp\left(\varepsilon_t^X\right) \left(\frac{l_t^X}{k_t^X}\right)^{(1-\alpha_X)} \tag{23}$$

Where $\varepsilon_t^j$ is the productivity shock in each sector $j = X, N$ respectively, which are assumed to follow a first order auto regressive process:

$$\varepsilon_{t+1}^j = \rho^j \varepsilon_t^j + v_{t+1}^j, \qquad Where\ E\left[v_{t+1}^j\right] = 0\ \ and\ \ V\left[v_{t+1}^j\right] = \sigma_j^2 \tag{24}$$

### 5.1.3 Definition of a Competitive Equilibrium

Given $b_0$, $k_0^X$ and $k_0^N$, and shocks' processes $\left(\varepsilon_t^R, \varepsilon_t^{P^X}, \varepsilon_t^X, \varepsilon_t^N\right)$, a competitive equilibrium is a sequence of allocations $\left\{c_t^M, c_t^N, l_t, l_t^X, l_t^N, i_t^X, i_t^N, k_{t+1}^X, k_{t+1}^N, b_{t+1}\right\}_{t=0}^{\infty}$ and prices $\left\{P_t^X, P_t^N, q_t^X, q_t^N, w_t, R_t\right\}_{t=0}^{\infty}$, such that:

Given prices, $b_0$, $k_0^X$, $k_0^N$ and shocks' processes, $\left\{c_t^M, c_t^N, l_t, i_t^X, i_t^N, b_{t+1}\right\}_{t=0}^{\infty}$ solve household's problem.

Given prices and shocks' processes, $\left\{l_t^X, k_t^X\right\}_{t=0}^{\infty}$ solve firm X's problem.

Given prices and shocks processes, $\left\{l_t^N, k_t^N\right\}_{t=0}^{\infty}$ solve firm N's problem.

Market clearing conditions are satisfied: $c_t^N = y_t^N$ and $l_t = l_t^X + l_t^N$

The resource constraint is satisfied: $R_t b_t + P_t^X Y_t^X - c_t^M - i_t^X - i_t^N - \gamma b_{t+1} = 0$

## 5.2 Steady State and Calibration

The calibrated values of parameters are pinned down by introducing the average ratios of the main macroeconomic aggregates in Chile during the 1990s into the model's set of equilibrium conditions. Table 2 presents them, while Table 3 shows the macroeconomic ratios in data and implied by the model.

The value of $\gamma$ corresponds to the average rate of growth of GDP. Consumption of $N$ is equal to the non tradable output, while consumption of $M$ is the rest of total consumption. Note that in steady state the current account balance has to be equal to zero. However, it was in deficit for most of the 1990s. Thus, some adjustments to the ratios in the model were made to calibrate a consistent steady state. The fraction of exportable GDP on total GDP was increased from 0.36 in data to 0.40 in

the model, and the fraction of investment on total GDP was reduced from 0.28 in data to 0.27 in the model. The relative prices of $X$ and $N$ to $M$ were both set equal to 2 in the steady state.

The value of $\rho$ was chose to match the value of elasticity of substitution between $c^M$ and $c^N$ estimated by Ostry and Reinhart [31] for developing countries. The values of $\varpi$, $\lambda$, $\phi^X$, $\phi^N$ and $\beta$ were pinned down from the households first order conditions. The external interest rate was set in 5 percent, while the internal one in 5.06 percent. The value of $\eta$ was taken from Schmitt-Grohe and Uribe [29]. The coefficients for the production functions in each sector, $\alpha_X$ and $\alpha_N$, were set such that the sectorial allocation of labor were consistent with average in data, 30 percent in the exportable sector and 70 percent in the non tradable sector, and with an average share of capital income of 0.46 in the whole economy.

<div align="center">

**Table 2**
**Calibrated parameters values for Model 1**

| Parameters | $\beta$ | $\gamma$ | $\rho$ | $\varpi$ | $\theta_l$ | $\alpha_X$ | $\alpha_N$ |
|---|---|---|---|---|---|---|---|
| Values | 0.95 | 1.05 | -0.32 | 0.14 | 2.05 | 0.60 | 0.37 |
| Parameters | $P$ | $\lambda$ | $\phi$ | $\delta$ | $v$ | $A_X$ | $A_N$ |
| Values | 2.0 | 0.014 | 0.019 | 0.08 | 2.10 | 3.68 | 16.4 |

</div>

Table 3 shows that the calibration is consistent with the macroeconomic ratios in data, except by the adjustments made to calibrate a consistent steady state. The current account and trade balance deficit as percentage of GDP were reduced from 3.4 and 0 in data to zero and -2.4 in the model.

<div align="center">

**Table 3**
**Main aggregate ratios**

| Ratios | $y^N/y$ | $y^X/y$ | $i/y$ | $k/y$ | $k^N/k$ | $k^X/k$ | $h$ |
|---|---|---|---|---|---|---|---|
| Data | 0.64 | 0.36 | 0.28 | n.a. | n.a. | n.a. | 0.27 |
| Model | 0.60 | 0.40 | 0.27 | 1.85 | 0.48 | 0.52 | 0.27 |
| Ratios | $h^N/h$ | $h^X/h$ | $c^N/y$ | $c^M/y$ | $b/y$ | $TB/y$ | $CA/y$ |
| Data | 0.72 | 0.28 | 0.64 | 0.13 | 0.19 | 0.000 | -0.034 |
| Model | 0.70 | 0.30 | 0.60 | 0.13 | 0.19 | 0.024 | 0.000 |

</div>

## 5.3   Simulations

### 5.3.1   Shocks processes

The model is simulated for exogenous and independent trajectories of shocks in external interest rate, terms of trade and total factor productivity. Table 4 presents the estimated first order autocorrelation coefficients.

These coefficients are consistent with previous studies. The coefficient for the external interest rate is close to the 0.77 estimated by Mendoza [25] between 1988 and 1998, while the coefficient for the terms of trade is close the value of 0.35 estimated by Mendoza [26] between 1960 and 1990. The coefficients for the productivity shocks are in line with the standard values considered in the real business cycles literature.

**Table 4**
**First order Autoregressive Coefficients**

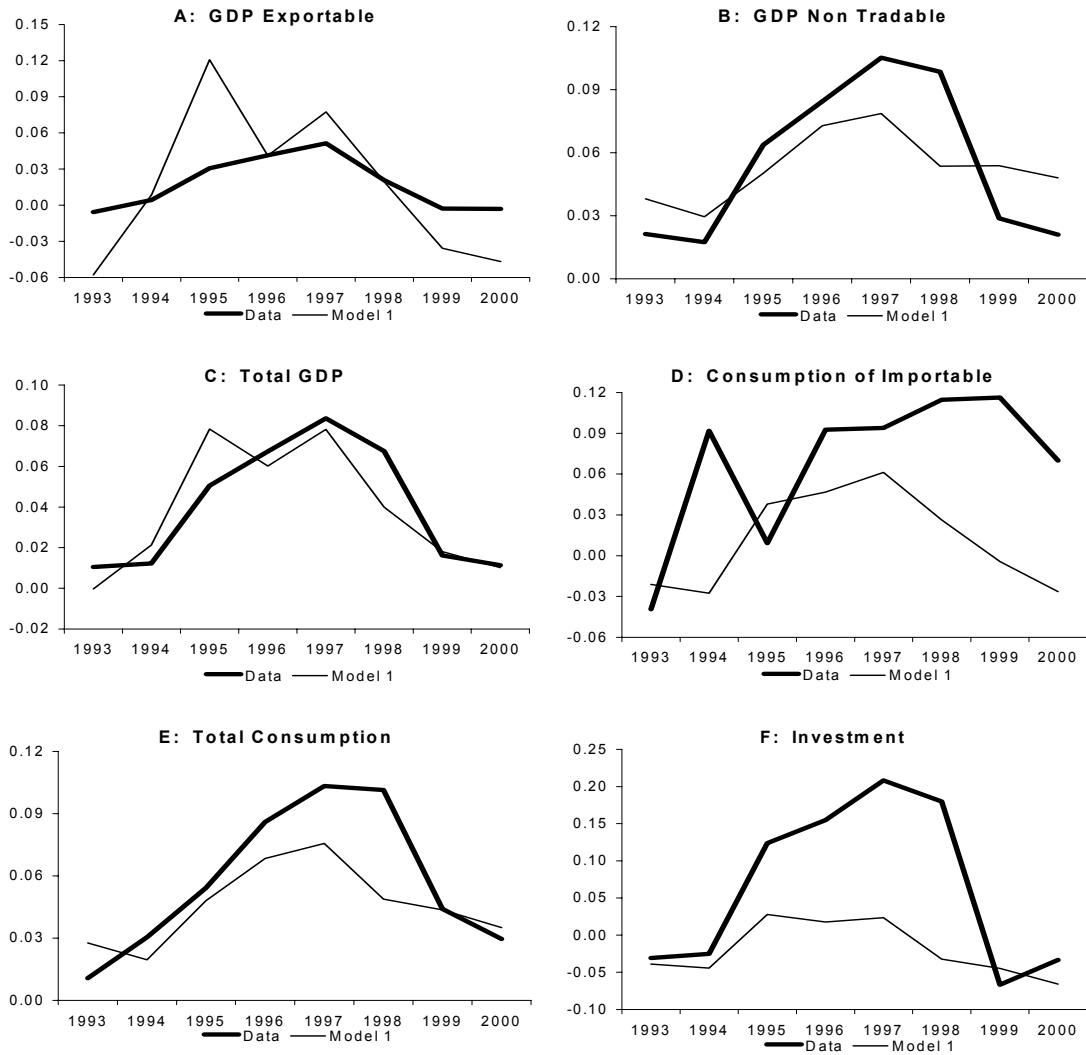|  | $r_t^*$ | $TOT$ | Prod. X | Prod. N |
|---|---|---|---|---|
| $Coefficient$ | 0.68 | 0.36 | 0.88 | 0.88 |

### 5.3.2   Results

Table 5 presents the average log deviation, and volatility, of the main variables in data and different models. Comparing the data to model 1 (columns 1 and 2), it shows that the model predicts a smoother path of consumption of both goods, investment, capital in both sectors and hours worked in the no tradable sector, but a more volatile path of output and hours worked in the exportable sector than in data.

**Table 5**
**Average Log-Deviations from Steady State and Volatility**

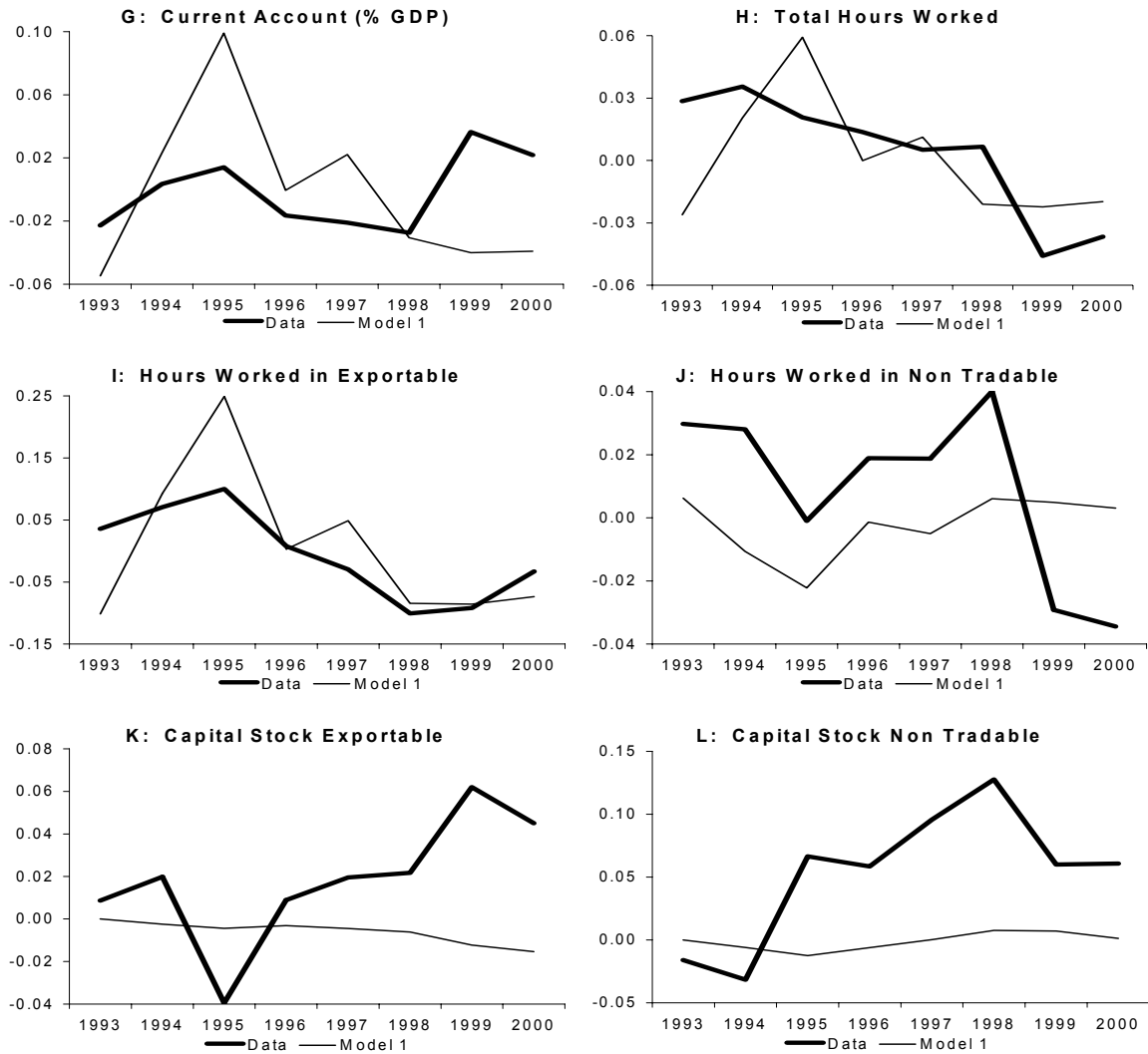|  |  | Data | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|---|
| GDP | Mean | 0.040 | 0.038 | 0.037 | 0.040 |
|  | Std. Dev. | 0.031 | 0.031 | 0.016 | 0.031 |
| Exportables | Mean | 0.017 | 0.016 | 0.015 | 0.017 |
|  | Std. Dev. | 0.022 | 0.063 | 0.032 | 0.022 |
| Non Tradables | Mean | 0.055 | 0.053 | 0.052 | 0.055 |
|  | Std. Dev. | 0.037 | 0.016 | 0.014 | 0.037 |
| Consumption | Mean | 0.057 | 0.046 | 0.047 | 0.051 |
|  | Std. Dev. | 0.035 | 0.019 | 0.018 | 0.042 |
| Importable | Mean | 0.069 | 0.012 | 0.027 | 0.030 |
|  | Std. Dev. | 0.055 | 0.036 | 0.060 | 0.064 |
| Investment | Mean | 0.064 | -0.020 | 0.025 | 0.028 |
|  | Std. Dev. | 0.113 | 0.037 | 0.129 | 0.123 |
| Hours Worked | Mean | 0.003 | 0.000 | -0.010 | -0.005 |
|  | Std. Dev. | 0.030 | 0.029 | 0.026 | 0.030 |
| Exportables | Mean | -0.005 | 0.006 | -0.015 | -0.010 |
|  | Std. Dev. | 0.072 | 0.121 | 0.097 | 0.071 |
| Non Tradables | Mean | 0.009 | -0.002 | -0.008 | -0.002 |
|  | Std. Dev. | 0.028 | 0.010 | 0.017 | 0.043 |
| Capital | Mean | 0.011 | -0.004 | 0.006 | 0.006 |
|  | Std. Dev. | 0.030 | 0.003 | 0.020 | 0.020 |
| Exportables | Mean | 0.018 | -0.006 | 0.007 | 0.007 |
|  | Std. Dev. | 0.030 | 0.005 | 0.021 | 0.021 |
| Non Tradables | Mean | 0.053 | -0.001 | 0.005 | 0.004 |
|  | Std. Dev. | 0.053 | 0.007 | 0.019 | 0.018 |

Figures 6 presents the trajectories of the macroeconomic aggregates in data and Model 1. The model predicts a bigger expansion in the exportable sector, and a bigger contraction in the non tradable sector, than in data in 1995, year in which the terms of trade showed a positive deviation of 20 percent. At the same time, it predicts a bigger contraction in the exportable sector, and a bigger expansion in the non tradable sector, than in data between 1998 and 2000, period in which the average deviation on terms of trade was of -8 percent. These differences in the productive structure in the model when compared to data are explained by a bigger reallocation of hours worked from the non tradable sector to the exportable one when facing positive terms of trade shocks, and the opposite when facing negative shocks.

**Figure 6**
**Data and Simulation of Model 1**

**A: GDP Exportable**

**B: GDP Non Tradable**

**C: Total GDP**

**D: Consumption of Importable**

**E: Total Consumption**

**F: Investment**

At the same time, the model predict smoother paths of consumption and investment, which is consistent with the hypothesis of consumption smoothing. Individuals isolate their expenditure decision from output volatility through external borrowing, which causes an improvement in the current account balance when facing positive terms of trade shocks, and a worsening in it when facing the opposite shocks.

**Figure 6 (Continuation)**
**Data and Simulation of Model 1**

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**G: Current Account (% GDP)**

**H: Total Hours Worked**

**I: Hours Worked in Exportable**

**J: Hours Worked in Non Tradable**

**K: Capital Stock Exportable**

**L: Capital Stock Non Tradable**

In conclusion, the friction-less model not only fails to predict volatility of aggregate output, consumption, investment and capital, but also fails to predict the relative reallocation of hours worked across sectors in response to the shocks. These results show that considering a time invariant upward sloping supply of funds, dependent only on the current level of external debt, is not appropriate to characterize the Chilean economy's business cycles during the 1990s.

In the next section we will explore wether an endogenous time varying upward sloping supply of funds, dependent not only on the level of external debt, but also on the current levels of income and expenditures, would improve the matching of the model to the data for the period of analysis.

# 6   Introducing a External Borrowing Constraint

The functional form for the external borrowing constraint is taken from Mendoza [26], and is set as the external lenders' requirement to the internal households to finance at least a fraction $\Psi_t$ of their expenditures with their current income at date $t$. This fraction is taken from data, and introduced exogenously into the model, assuming that such constraint was always binding. This is:

$$w_t l_t + q_t^X k_t^X + q_t^N k_t^N + \pi_t^X + \pi_t^N = \Psi_t \left( c_t^M + P_t^N c_t^N + i_t^X + i_t^N - R_t b_t \right) \tag{25}$$

Where the left hand side is the households' current income and the right hand side the self financed fraction of expenditures. Combining $(9)$ with $(25)$, and using equilibrium conditions, this yields the following lower bound for external debt:

$$b_{t+1} = -\frac{1 - \Psi_t}{\gamma \Psi_t} \left( P_t^X Y_t^X + P_t^N Y_t^N \right) \tag{26}$$

Equation (26) relates the amount of new debt contracted today to current output, and not to next period output as a standard collateral constraint. This condition can be interpreted as a collateral constraint in which external lenders are myopic. This is, the best forecast they have about next period output is today's output. This formulation is in line with the argument that current fundamentals matter for external lenders to decide how much to lend to a country in a given period.

External lenders maximize profits facing a complete asset market. The tacitly collude to impose this borrowing constraint over the households, which yields an endogenous upward sloping supply of funds, where the spread between the internal and external interest rates depends on the borrowing constraint multiplier.

## 6.1   Model

Consider a small open economy perfectly integrated with the rest of the world in goods, but faces individual specific external borrowing constraints identified as the requirement to the internal households to finance at least a fraction $\Psi_t$ of their expenditures with their current income at date $t$. This new setup only modifies the households problem, while the rest of the model is the same. Thus, only the modified households' problem is presented in this section.

There are three types of agents in this economy: Internal households and firms, and foreign lenders. Internal households own firms, consume $M$ and $N$, and supply $L$ and $K$ to the firms. They face an external borrowing constraint. There are two firms, firm $X$ and firm $N$, both demand $L$ and $K$ to produce their goods, but the former is more capital intensive. External lenders set the borrowing constraint on the internal households. The economy follows a balanced growth path and population is assumed constant. In the following, the model is set in stationary terms.

### 6.1.1 Households

Households choose the sequence $\left\{ c_t^M, c_t^N, l_t, i_t, k_{t+1}, b_{t+1} \right\}_{t=0}^{\infty}$ to maximize $(8)$, subject to $(9)$, $(10)$ and $(25)$. Their first order conditions are:

$$\frac{\varpi c_t^{M(\rho-1)}}{\varpi c_t^{M\rho} + (1-\varpi) c_t^{N\rho}} = (\lambda_t + \mu_t \Psi_t) \tag{27}$$

$$\frac{(1-\varpi) c_t^{N(\rho-1)}}{\varpi c_t^{M\rho} + (1-\varpi) c_t^{N\rho}} = P_t^N (\lambda_t + \mu_t \Psi_t) \tag{28}$$

$$\frac{\theta_l}{(1-l_t)} = (\lambda_t + \mu_t) w_t \tag{29}$$

$$\phi_t^X = (\lambda_t + \mu_t \Psi_t) + v \phi_t^X \left( \frac{i_t^X}{k_t^X} \right)^{(v-1)} \tag{30}$$

$$\phi_t^N = (\lambda_t + \mu_t \Psi_t) + v \phi_t^N \left( \frac{i_t^N}{k_t^N} \right)^{(v-1)} \tag{31}$$

$$\gamma \phi_t^X = \beta E_t \left\{ (\lambda_{t+1} + \mu_{t+1}) q_{t+1}^X + \phi_{t+1}^X \left( (1-\delta) - (1-v) \left( \frac{i_{t+1}^X}{k_{t+1}^X} \right)^v \right) \right\} \tag{32}$$

$$\gamma \phi_t^N = \beta E_t \left\{ (\lambda_{t+1} + \mu_{t+1}) q_{t+1}^N + \phi_{t+1}^N \left( (1-\delta) - (1-v) \left( \frac{i_{t+1}^N}{k_{t+1}^N} \right)^v \right) \right\} \tag{33}$$

$$\gamma \lambda_t = \beta E_t \left\{ (\lambda_{t+1} + \mu_{t+1} \Psi_{t+1}) R_{t+1} \right\} \tag{34}$$

$$E_t \left[ \lim_{t \to \infty} \beta^t \lambda_t \left( k_{t+1}^X + k_{t+1}^N + b_{t+1} \right) \right] = 0 \tag{35}$$

Where $\lambda_t$, $\phi_t^X$, $\phi_t^N$ and $\mu_t$ are the lagrange multipliers on $(9)$, $(10)$ and $(25)$.

### 6.1.2 Definition of a Competitive Equilibrium

Given $b_0$, $k_0^X$ and $k_0^N$, and shocks' processes $\left( \varepsilon_t^R, \varepsilon_t^{PX}, \varepsilon_t^X, \varepsilon_t^N, \Psi_t \right)$, a competitive equilibrium is a sequence of allocations $\left\{ c_t^M, c_t^N, l_t, l_t^X, l_t^N, i_t^X, i_t^N, k_{t+1}^X, k_{t+1}^N, b_{t+1} \right\}_{t=0}^{\infty}$ and prices $\left\{ P_t^X, P_t^N, q_t^X, q_t^N, w_t, r_t \right\}_{t=0}^{\infty}$ such:

Given prices, $b_0$, $k_0$ and shocks' processes, $\left\{ c_t^M, c_t^N, l_t, k_{t+1}, b_{t+1} \right\}_{t=0}^{\infty}$ solve the household's problem.

Given prices and shocks' processes, $\left\{ l_t^X, k_t^X \right\}_{t=0}^{\infty}$ solve firm X's problem.

Given prices and shocks processes, $\left\{l_t^N, k_t^N\right\}_{t=0}^{\infty}$ solve firm N's problem.

Market clearing conditions are satisfied: $c_t^N = y_t^N$ and $l_t = l_t^X + l_t^N$

The resource constraint is satisfied: $R_t b_t + P_t^X Y_t^X - c_t^M - i_t^X - i_t^N - \gamma b_{t+1} = 0$

### 6.1.3 External Lenders

External lenders are risk neutral and face a complete asset market. Their problem is to maximize the present value of their profits subject to the borrowing constraint they impose over the internal households. Their first order conditions are:

$$Q_t = Q_{t+1} R_{t+1} \left(1 - \mu_{t+1} \Psi_{t+1}\right) \qquad With \quad Q_t = \frac{1}{\prod_{s=0}^{t} R_s^*} \tag{36}$$

This yields the following endogenous upward sloping supply of funds:

$$R_t - R_t^* = R_t \mu_t \Psi_t \tag{37}$$

This is an upward sloping on external debt and expenditures and downward sloping on income, given by the changes in the multiplier $\mu_t$ in response to these variables. This functional form allows the model to have a unique steady state.

## 6.2 Steady State and Calibration

Table 6 presents the calibrated parameters for this new model. The ones in common with Model 1 are the same, while $\Psi$ and $\mu$ were pinned down from the households FOCs. The macroeconomic ratios in the model are the same than in Model 1.

**Table 6**
**Calibrated parameters values for Model 2**

| Parameters | $\beta$ | $\gamma$ | $\rho$ | $\varpi$ | $\theta_l$ | $\Psi$ | $\alpha_X$ | $\alpha_N$ |
|---|---|---|---|---|---|---|---|---|
| Values | 0.95 | 1.05 | -0.32 | 0.14 | 2.06 | 0.834 | 0.60 | 0.37 |
| Parameters | $P$ | $\lambda$ | $\phi$ | $\delta$ | $v$ | $\mu$ | $A_X$ | $A_N$ |
| Values | 2.0 | 0.013 | 0.019 | 0.08 | 2.10 | 0.00069 | 3.68 | 16.4 |

## 6.3 Simulations

### 6.3.1 Shocks Processes

The model is simulated for the exogenous trajectories of the shocks on external interest rate, terms of trade, total factor productivity and $\Psi_t$. All shocks are assumed to follow a first order autoregressive process and to be independent among them. The

estimated first order autocorrelation coefficients are presented in table 7. The first four of them are the same than in Model 1.

### Table 7
### First order Autoregressive Coefficients

|             | $r_t^*$ | $TOT$ | $Prod.X$ | $Prod.N$ | $\Psi_t$ |
|-------------|---------|-------|----------|----------|----------|
| $Coefficient$ | 0.68  | 0.36  | 0.88     | 0.88     | 0.90     |

Figure 7 presents the trajectories of $\Psi_t$ and $\mu_t$. Panel (A) shows that households were required to self finance an increasing fraction of their expenditures between 1993 and 1995, but a decreasing one after. This suggests that this constraint could have contributed to the boom between 1995 and 1998 and could have allowed the country to smooth partially the recession in 1999 and 2000.

### Figure 7:
### Self Financing Requirement and Ext. Borrowing Const.'s Multiplier
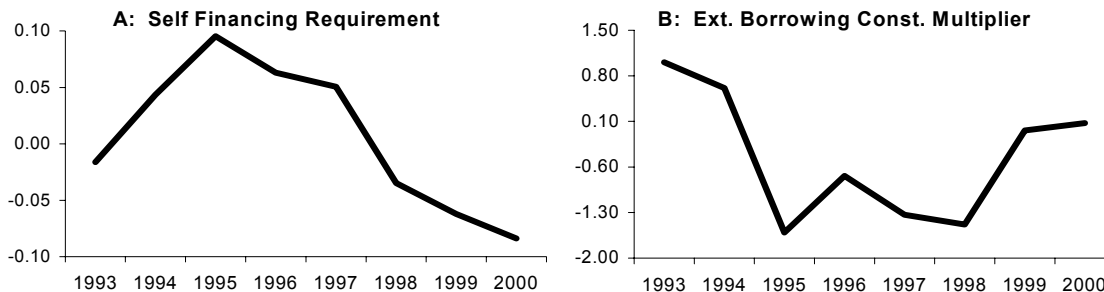


Panel (B) shows a negative correlation between $\mu_t$ and terms of trade shocks. The external borrowing constraint was tighter between 1999 and 2000 than in 1995, despite that $\Psi_t$ reached its maximum in 1995 and its minimum between 1998 and 2000. This figure that the fall in $\Psi_t$ during the recession was not enough to isolate consumption and investment completely from it.

### 6.3.2   Results

Column 3 in table 5 presents the average log deviations and volatility of Model 2's simulations. It shows, with respect to model 1, an improvement in the predicted volatility of consumption of importable and investment, as well as in output, hours worked and capital in the exportable sector. However, it predicts a smoother path of non tradable output, induced by the counter cyclical path of total hours worked.

Figure 8 presents the time series in data and simulations of Models 1 and 2. It shows that introducing the external borrowing constraint reduces the smoothing of consumption of importable and investment, as well as it reduces the reallocation of

labor across sectors. The slackening in the constraint between 1995 and 1998 allows the model to reproduce the boom in investment and consumption of importable, and its tightening in 1999 and 2000 allows the model to capture the fall in both.

**Figure 8**
**Data and Simulation of Models 1 and 2**
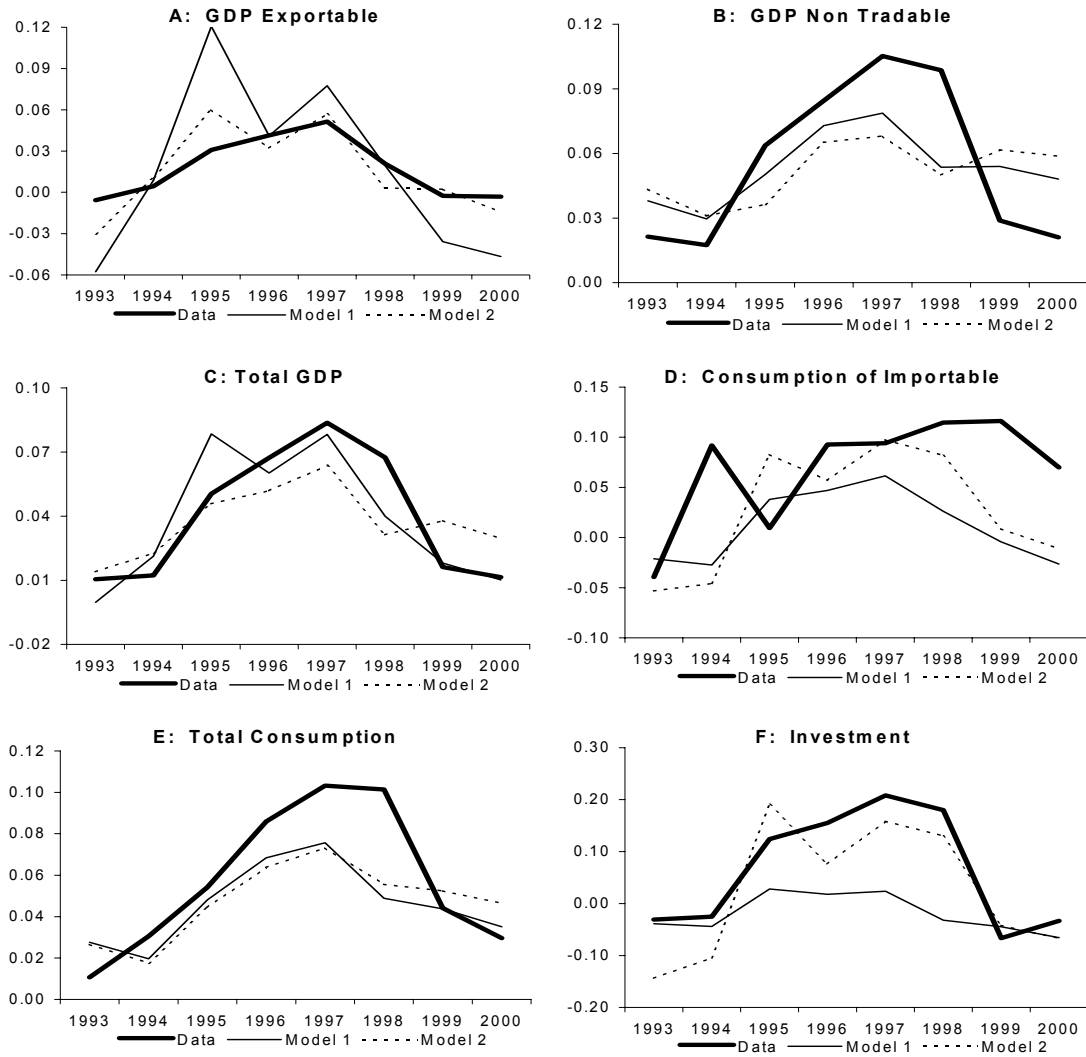


At the same time, the slackening in the constraint during the boom reduces the reallocation of labor from the non tradable sector to the exportable one, as well as its tightening during the recession reduces the reallocation of labor in the opposite direction. When the economy receives a positive shock in terms of trade, which makes the constraint slacker, individuals want to consume more of both goods. The importable one can be obtained in the external market, while the non tradable has to

be produced domestically. This effect prevents the economy from reallocating labor to the exportable sector from the non tradable one in response to the shock. A negative shock in terms of trade, which makes the constraint tighter causes the opposite.

## Figure 8 (Continuation)
## Data and Simulation of Models 1 and 2
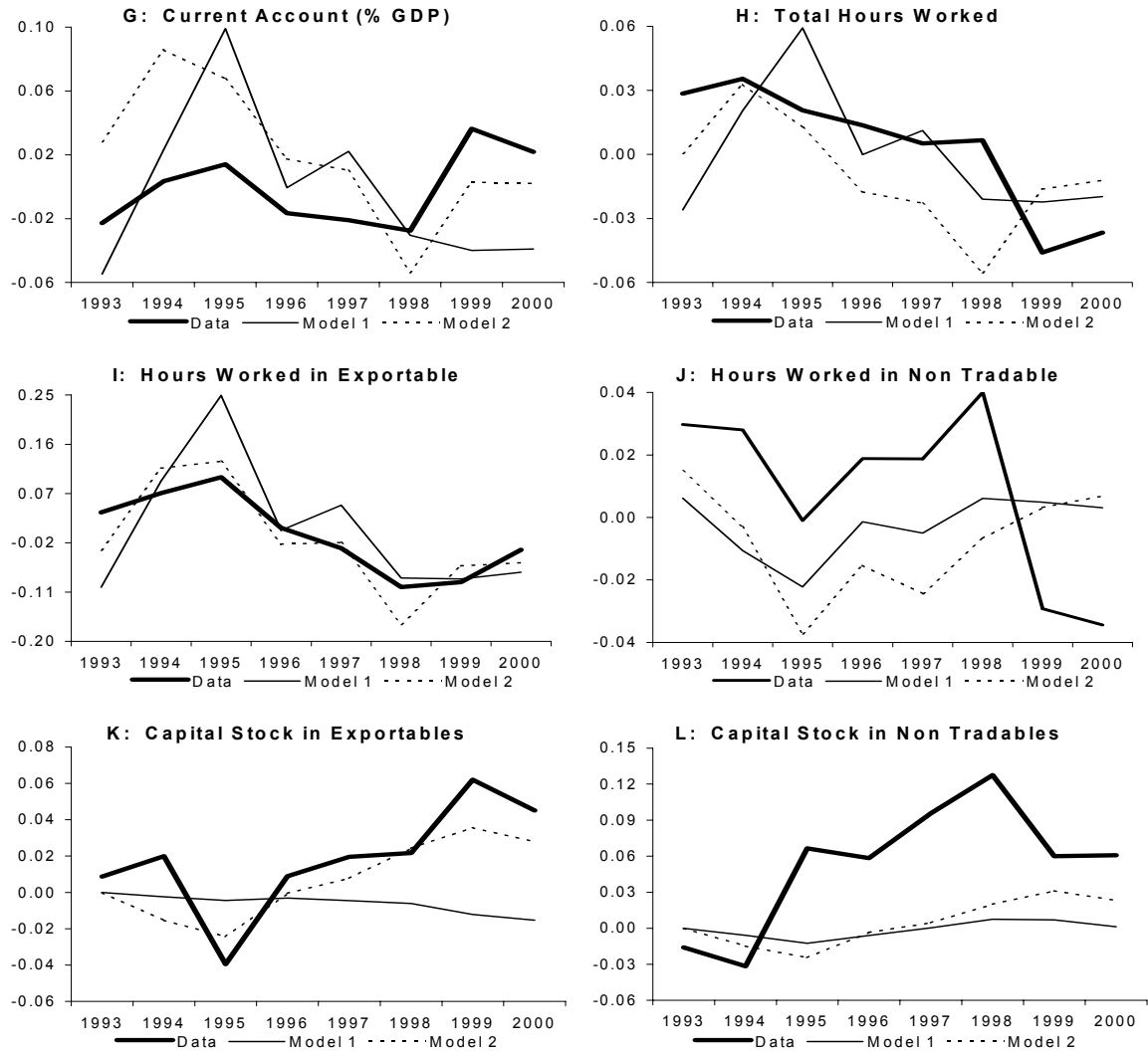


However, introducing the constraint makes total hours to be counter cyclical. When the constraint gets slacker, individuals also want to have more leisure, while when the constraint gets tighter makes them to reduce leisure. The final outcome is that individuals work less during the boom and more during the recession, which is opposite to what is observed in data. This ends up affecting output in the non tradable sector. The economy produce and consume less non tradable goods in good

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times and more of it in bad times.

This problem has been attacked in the literature assuming that households have GHH type preferences, where the labor-leisure decision depends only on the real wage and not on wealth effects. However this specification it is not compatible with balanced growth path, and implicitly assumes that agents in developing countries have different preferences than the standard ones used for developed ones. This next section shows that it is not necessary to make this extreme assumption to solve the problem. A sector specific labor financing friction that drops during booms, and increases during recessions, would make the model match both series.

# 7    Introducing Labor Wedges

This section introduces sector specific labor wedges to the model as a measure of differentiated labor market distortions in each sector. Appendix 1 shows that such specification has an equivalent reduced form than a model with labor financing frictions. The objective is to evaluate wether distortions in the labor market could help the model predicting better hours worked and output in the non tradable sector.

The exercise is deductive. It consists on finding the sector specific labor wedges such that the model replicates the trajectories of output in both sectors. Its equivalence with a model with labor financing frictions will allow us to compare the average wedge implied by the model with the average lending spread in data, as a measure of relevance of this friction to characterize Chile's business cycles during the 1990s.

## 7.1    Model

Consider a small open economy perfectly integrated with the world in goods, but faces individual specific external borrowing constraints. The structure of the model is the same than in section 6, but now both firms face sector specific labor wedges that increase the labor cost differently across sectors. The revenues from the labor wedges are rebated to the households such that the resource constraint remain unchanged.

External lenders set the external borrowing constraint on the internal households. The economy follows a balanced growth and population is assumed to be constant. In the following the model is set in stationary terms. This setup will only change the firms' problem, the rest of the model is the same. Thus, only the modified firms' problem is presented in this version of the model.

### 7.1.1    Firms

The total cost of production for each firm is:

$$q_t^j k_t^j + w_t l_t^j (1 + \tau^j) \qquad For \;\; j = X, N \tag{38}$$

Where $\tau^j$ is the labor wedge in sector $j$. The firms' static problem is to choose the allocation $\left\{l_t^j, k_t^j\right\}$ to maximize profits. Their first order conditions are:

**Non Tradable Firm**

$$w_t(1 + \tau^N) = (1 - \alpha_N)P_t^N \exp\left(\varepsilon_t^N\right) \left(\frac{k_t^N}{l_t^N}\right)^{\alpha_N} \tag{39}$$

$$q_t^N = \alpha_N P_t^N \exp\left(\varepsilon_t^N\right) \left(\frac{l_t^N}{k_t^N}\right)^{(1-\alpha_N)} \tag{40}$$

**Exportable Firm**

$$w_t(1 + \tau^X) = (1 - \alpha_X)P_t^X \exp\left(\varepsilon_t^X\right) \left(\frac{k_t^X}{l_t^X}\right)^{\alpha_X} \tag{41}$$

$$q_t^X = \alpha_X P_t^X \exp\left(\varepsilon_t^X\right) \left(\frac{l_t^X}{k_t^X}\right)^{(1-\alpha_X)} \tag{42}$$

### 7.1.2 Competitive Equilibrium

Given $b_0$, $k_0^X$ and $k_0^N$ and shocks' processes $\left(\varepsilon_t^R, \varepsilon_t^{P^X}, \varepsilon_t^X, \varepsilon_t^N, \Psi_t, \tau^X, \tau^N\right)$, a competitive equilibrium is a sequence of allocations $\left\{c_t^M, c_t^N, l_t, l_t^X, l_t^N, i_t^X, i_t^N, k_{t+1}^X, k_{t+1}^N, b_{t+1}\right\}_{t=0}^{\infty}$ and prices $\left\{P_t^X, P_t^N, q_t^X, q_t^N, w_t, R_t, \right\}_{t=0}^{\infty}$, such that:

Given prices, $b_0$, $k_0^X$, $k_0^X$, and shocks' processes, $\left\{c_t^M, c_t^N, l_t, i_t^X, i_t^N, k_{t+1}^X, k_{t+1}^N, b_{t+1}\right\}_{t=0}^{\infty}$ solve the household's problem.

Given prices and shocks' processes, $\left\{l_t^X, k_t^X\right\}_{t=0}^{\infty}$ solve firm X's problem.

Given prices and shocks processes, $\left\{l_t^N, k_t^N\right\}_{t=0}^{\infty}$ solve firm N's problem.

Market clearing conditions are satisfied: $c_t^N = y_t^N$ and $l_t = l_t^X + l_t^N$

The resource constraint is satisfied: $R_t b_t + P_t^X Y_t^X - c_t^M - i_t^X - i_t^N - \gamma b_{t+1} = 0$

## 7.2 Steady State and Calibration

The labor wedges were set equal for both sectors in steady state, such that they do not distort the relative allocation of labor and capital with respect to Model 2. Therefore, the calibrated parameters as well as the implied macroeconomic ratios by the model are the same than in Model 2.

## 7.3    Simulations

### 7.3.1    Shocks processes

The model is simulated for the same exogenous trajectories of shocks than in section 6. The labor wedges are built such that the model replicates the path output in each sector. All shocks follow a first order autoregressive process, which estimated coefficients are presented in table 9. It is assumed that the labor wedges are persistent.

### Table 8
### First order Autoregressive Coefficients

|  | $r_t^*$ | $TOT$ | $Prod.X$ | $Prod.N$ | $\Psi_t$ | $\tau^X$ | $\tau^N$ |
|---|---|---|---|---|---|---|---|
| $Coefficient$ | 0.68 | 0.36 | 0.88 | 0.88 | 0.90 | 0.90 | 0.90 |

Figure 9 presents the sector specific labor wedges, the lending spread in data and the borrowing constraint's multiplier. Panels (C) and (D) show that, to match output in both sectors, it is necessary to have decreasing wedges from 1993 to 1998, and higher ones in 1999 and 2000, although more volatile for the exportable sector.

### Figure 9:
### Average Lending Spread, and Sector Specific Labor Wedges
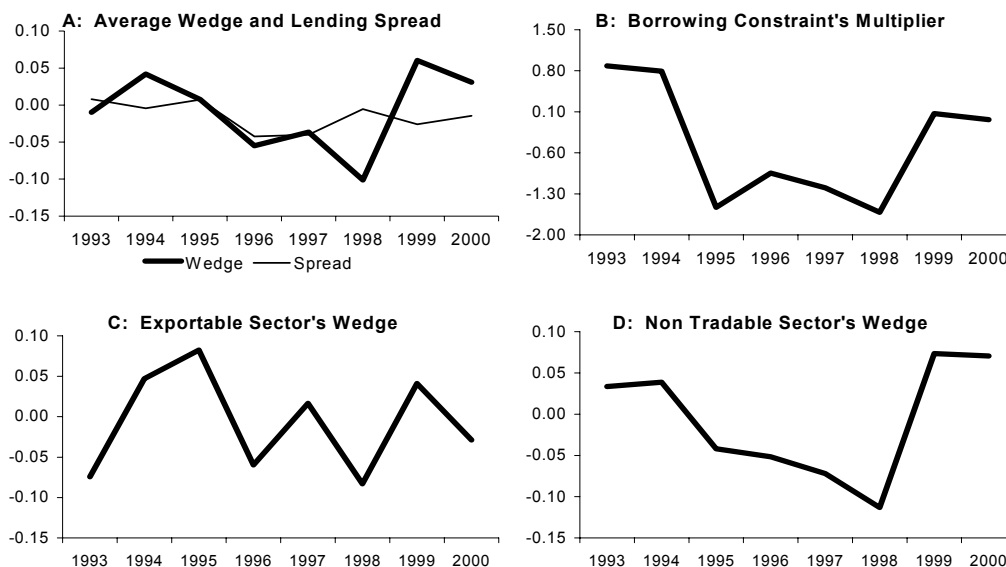


Panel (A) shows that the average wedge in the model is similar to the lending spread in data between 1993 and 1998, but higher in 1999 and 2000. This suggests that the labor distortion during the 1990s could be due to the labor financing friction. The gap in the recession could be attributed to the high spread faced by small and medium size firms crowded out of the internal financial market in bad times, which

is not reflected in the lending spread. Finally, panel (B) shows a similar path for the borrowing constraint multiplier than in Model 2.

### 7.3.2   Results

Column 4 in table 5 presents model 3's average log deviations and volatility. It shows an improvement not only on sectorial output and total consumption, but also in hours worked and its sectorial allocation. It does not change much the prediction of investment, consumption of importable and current account.
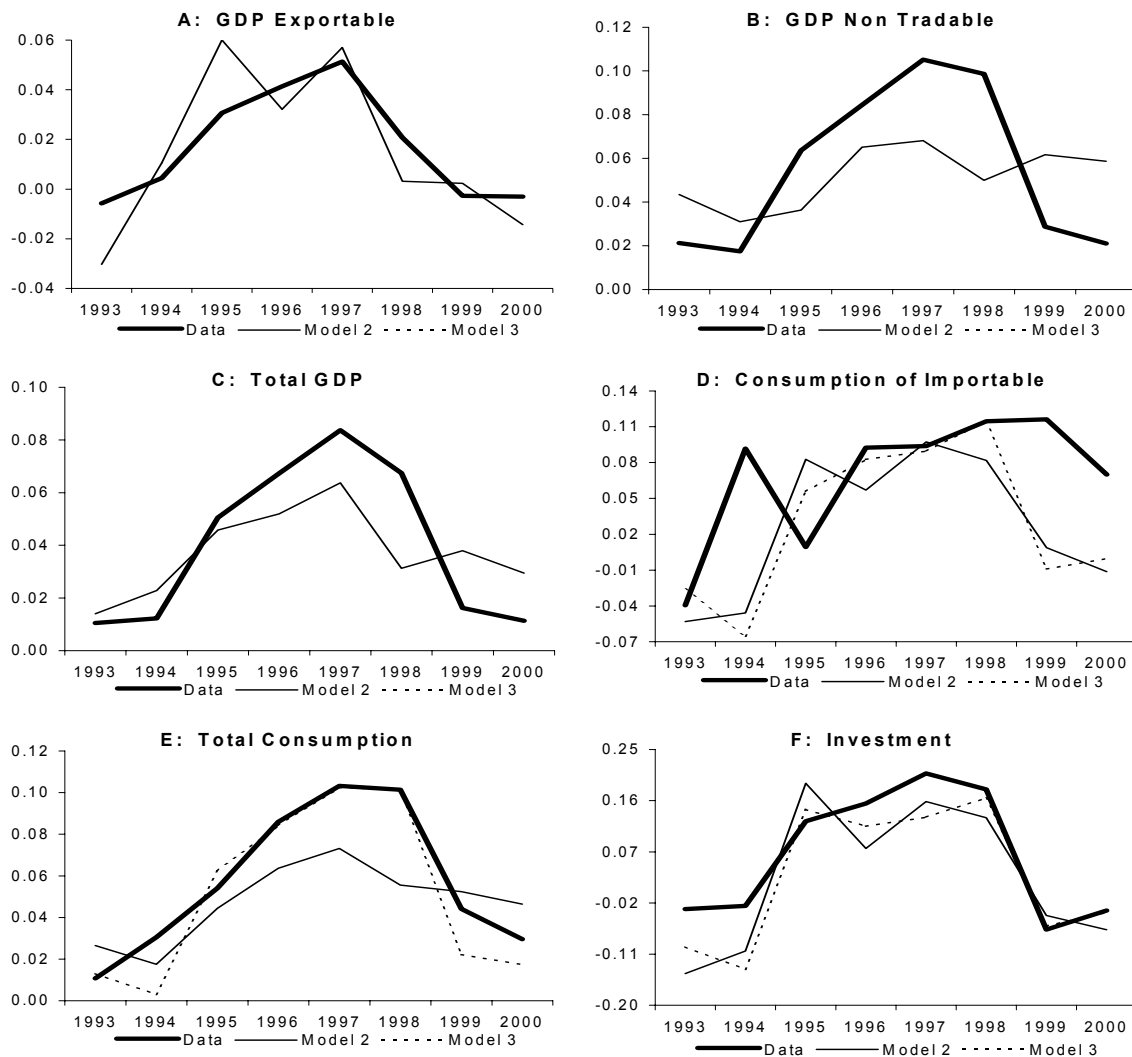
**Figure 10**
**Data and Simulation of Models 2 and 3**



Figures 10 presents the time series in data and simulations of models 2 and 3. Model 3 match better the data in aggregate consumption, hours worked and its relative

allocation. In particular, the spike in the non tradable sector's wedge in 1999 and 2000 replicates the fall in hours in this sector and the whole economy.

**Figure 10 (Continuation)**
**Data and Simulation of Models 2 and 3**
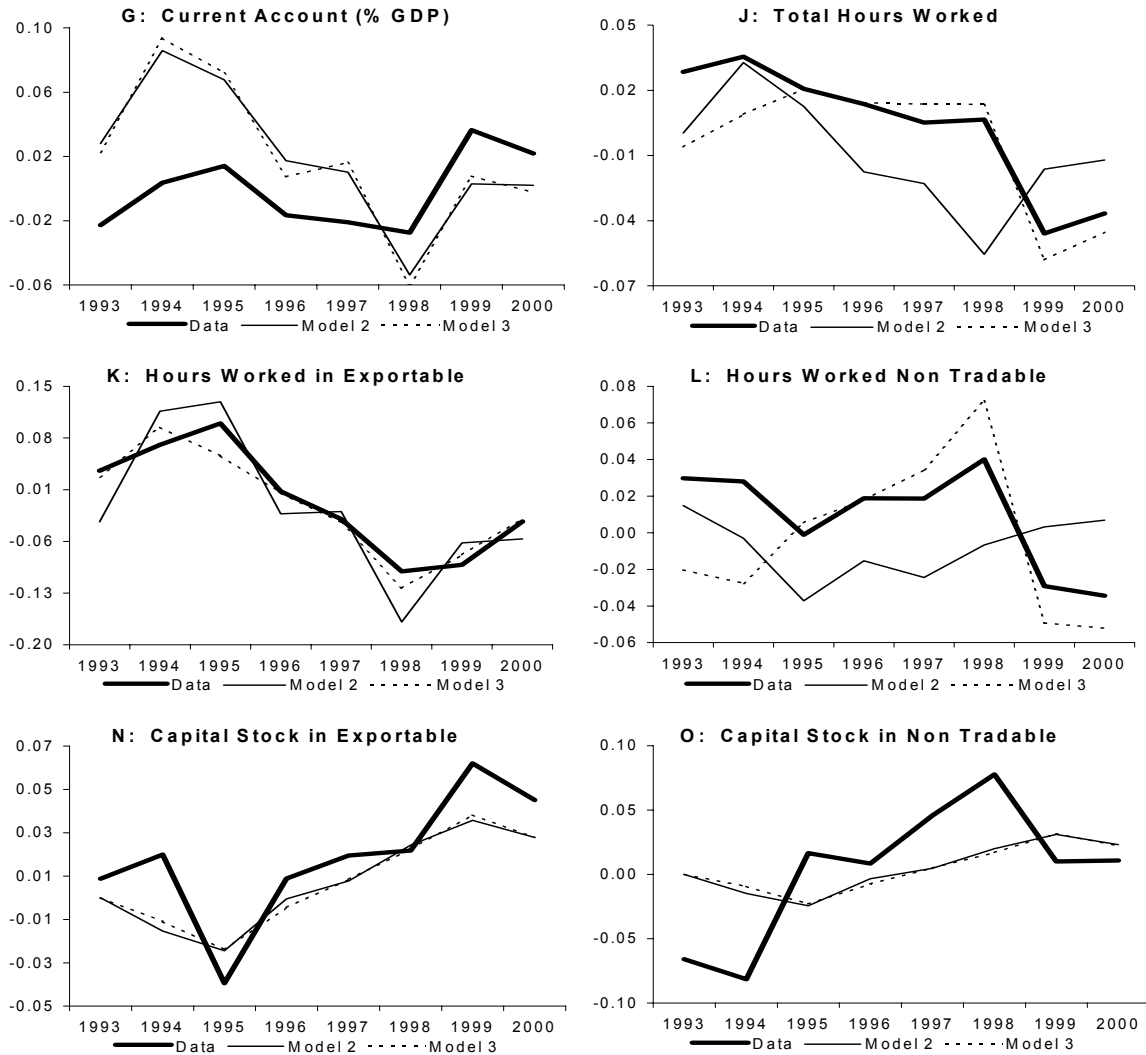


The exercise suggests that an adequate characterization of Chile's cycles during the 1990's should consider sector specific labor distortions, which could be attributed to labor financing frictions for the whole period. The average wedge and lending spread in data are similar between 1993 and 1998, while their difference in 1999 and 2000 could be explained by the increase in the spread charged to small and medium size firms, with lack of collateral, crowded out of the financial market in bad times.

# 8    Conclusions

The differences on the cyclical pattern of consumption, investment, current account and employment between standard models of small open economies and data can be explained by the interaction of two financial frictions, an external borrowing constraint and a sector specific labor financing wedges, without recurring to non standard assumption of preferences. Both frictions modify the optimal reaction of internal agents to shocks.

A standard frictionless model predicts smooth consumption and investment over time, and procyclical current account and employment. It predicts a bigger expansion in exportable output, and a lower one in non tradable output, than in data for positive terms of trade shock, and the opposite for negative terms of trade shock. Thus, it predicts an improvement in current account in good times and a worsening on it in bad times. This contrasts with data, in which consumption is highly procyclical, current account is counter cyclical and employment is procyclical.

Introducing an external borrowing constraint, to the otherwise frictionless model, reduces the smoothing over time of consumption of tradables and investment. It also reduces the volatility of exportable output and replicates the path of current account in data. However, for standard specification of preferences, it predicts a counter cyclical employment and an even smoother path of output of non tradable.

A positive terms of trade shock, which makes the constraint less binding, causes the households demand more importable and non tradable goods. This reduces the predicted expansion in the exportable sector in response to the shock, while a negative shock that tights the constraint, reduces the predicted fall in it. At the same time, a less binding constraint, induces the individuals to work less and to produce less non tradable goods. A more binding one, makes the individuals work more and produce more non tradable goods. The final outcome is a better forecast of exportable output, but a counter cyclical employment and a too smooth path of non tradable output.

Introducing counter cyclical sector specific labor wedges to the model, as a measure of labor market distortions, allows the model to replicate hours worked and non tradable output in data. The wedge implied by the model suggest that the internal lending spread is not a good measure of labor financing distortions during recessions. It does not account for the high spread faced by small and medium size firm crowded out of the internal financial market in bad times.

In conclusion, both financial frictions seem to be relevant to characterize developing countries business cycles. The external borrowing constraint reduces the smoothing of consumption of importable and investment over time, improves the relative reallocation of factors and captures the cyclical pattern of current account. However, it produces counter cyclical employment and a excessive smoothing of output of non tradable. Counter cyclical sector specific labor financing wedges would capture the

cyclical pattern of employment and non tradable output without the need of setting non standard preferences.

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## 8.1 Appendix 1: Introducing Labor Financing Frictions

Consider a small open economy perfectly integrated with the rest of the world in goods, but faces individual specific external borrowing constraints identified as the external lenders requirement to the internal households to finance at least a fraction $\Psi_t$ of their expenditures with their current income at date $t$.

There are four types of agents: Internal households, firms and banks, and external lenders. Households own firms and banks, consume $M$ and $N$, and supply $L$ and $K$ to the firms. They face the external borrowing constraint, and supply funds infinitely elastically to internal banks within the period at the internal rate of return $R_t$, and demand funds infinitely elastically from the firms within the period at the same rate.

There are two firms, $X$ and $N$. Both have constant return to scale technologies, and use $L$ and $K$ inputs. They pay their wages before production is realized, facing a credit in advance constraint in the internal financial market. Banks face an infinitely elastic supply of deposits from households within the period at the rate of return $R_t$, and lend to each firm subject to sector specific collateral constraints. They can only be sure that each firm produces a fraction of its sector average output, which is the collateral. All costs caused by this distortion are rebated to the households such that the resource constraint does not change. The economy follows a balanced growth path and population is constant. In the following the model is set in stationary terms.

### 8.1.1 Households

Households choose the sequence $\left\{ c_t^M, c_t^N, l_t, i_t, k_{t+1}, b_{t+1} \right\}_{t=0}^{\infty}$ to maximize (8) subject to (9), (10) and (25). Their first order conditions are:

$$\frac{\varpi c_t^{M(\rho-1)}}{\varpi c_t^{M\rho} + (1-\varpi) c_t^{N\rho}} = (\lambda_t + \mu_t \Psi_t) \tag{43}$$

$$\frac{(1-\varpi) c_t^{N(\rho-1)}}{\varpi c_t^{M\rho} + (1-\varpi) c_t^{N\rho}} = P_t^N (\lambda_t + \mu_t \Psi_t) \tag{44}$$

$$\frac{\theta_l}{(1-l_t)} = (\lambda_t + \mu_t) w_t \tag{45}$$

$$\phi_t^X = (\lambda_t + \mu_t \Psi_t) + v \phi_t^X \left( \frac{i_t^X}{k_t^X} \right)^{(v-1)} \tag{46}$$

$$\phi_t^N = (\lambda_t + \mu_t \Psi_t) + v \phi_t^N \left( \frac{i_t^N}{k_t^N} \right)^{(v-1)} \tag{47}$$

$$\gamma \phi_t^X = \beta E_t \left\{ (\lambda_{t+1} + \mu_{t+1}) q_{t+1}^X + \phi_{t+1}^X \left( (1-\delta) - (1-v) \left( \frac{i_{t+1}^X}{k_{t+1}^X} \right)^v \right) \right\} \tag{48}$$

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$$\gamma\phi_t^N = \beta E_t \left\{ \left(\lambda_{t+1} + \mu_{t+1}\right) q_{t+1}^N + \phi_{t+1}^N \left((1-\delta) - (1-v)\left(\frac{i_{t+1}^N}{k_{t+1}^N}\right)^v\right) \right\} \tag{49}$$

$$\gamma\lambda_t = \beta E_t \left\{ \left(\lambda_{t+1} + \mu_{t+1}\Psi_{t+1}\right) R_{t+1} \right\} \tag{50}$$

$$E_t \left[\lim_{t\to\infty} \beta^t \lambda_t \left(k_{t+1}^X + k_{t+1}^N + b_{t+1}\right)\right] = 0 \tag{51}$$

Where $\lambda_t$, $\phi_t^X$, $\phi_t^N$ and $\mu_t$ are the lagrange multipliers on $(9)$, $(10)$ and $(25)$.

### 8.1.2 Firms

Both firms get the credit at the beginning of each period, after the shocks are realized, and repay it at the end of the same period once production is realized. They lend this credit within the period to the households at the rate of return $R_t$. Their optimal decision is to hold just the necessary credit to pay wages in each period. Thus, the credit in advance constraint is satisfied in equality:

$$z_t^j = w_t l_t^j \qquad For \;\; j = X, N \tag{52}$$

Where $z_t^j$ is the credit received by firm $j$. The total cost of production for each firm is:

$$w_t l_t^j (1 + R_t^{lj} - R_t) + q_t^j k_t^j \qquad For \;\; j = X, N \tag{53}$$

The static problem for each firm is to choose the allocation $\{l_t^j, k_t^j\}$ in each period to maximize profits. Their first order conditions are:

**Non Tradable Firm**

$$w_t(1 + R_t^{lN} - R_t) = (1 - \alpha_N)P_t^N \exp\left(\varepsilon_t^N\right) \left(\frac{k_t^N}{l_t^N}\right)^{\alpha_N} \tag{54}$$

$$q_t^N = \alpha_N P_t^N \exp\left(\varepsilon_t^N\right) \left(\frac{l_t^N}{k_t^N}\right)^{(1-\alpha_N)} \tag{55}$$

**Exportable Firm**

$$w_t(1 + R_t^{lX} - R_t) = (1 - \alpha_X)P_t^X \exp\left(\varepsilon_t^X\right) \left(\frac{k_t^X}{l_t^X}\right)^{\alpha_X} \tag{56}$$

$$q_t^X = \alpha_X P_t^X \exp\left(\varepsilon_t^X\right) \left(\frac{l_t^X}{k_t^X}\right)^{(1-\alpha_X)} \tag{57}$$

### 8.1.3 Banks

The banking industry is perfectly competitive. The role of banks is to take deposits from the households and lend them to the firms. To produce credit they need some collateral to back up credit from firms.

$$\Omega_t^j Y_t^j \geq z_t^j \qquad For \;\; j = X, N \tag{58}$$

Where $z_t^j$ is the credit given to firm $j$. The banks static problem is to choose the allocation $\left\{ z_t^X, z_t^N \right\}$ in each period to maximize their profits. Their first order conditions are:

$$R_t^{lX} - R_t = \eta_t^X \tag{59}$$

$$R_t^{lN} - R_t = \eta_t^N \tag{60}$$

Where $\eta_t^X$ and $\eta_t^N$ are the lagrange multipliers on $(9)$, for X and N respectively.

### 8.1.4 Definition of a Competitive Equilibrium

Given initial values of foreign assets $b_0$, capital $k_0^X$ and $k_0^X$ and shocks' processes $\left( \varepsilon_t^R, \varepsilon_t^{P^X}, \varepsilon_t^X, \varepsilon_t^N, \Psi_t, \Omega_t^X, \Omega_t^N \right)$, a competitive equilibrium is a sequence of allocations $\left\{ c_t^M, c_t^N, l_t, l_t^X, l_t^N, i_t^X, i_t^N, k_{t+1}^X, k_{t+1}^N, b_{t+1}, z_t^X, z_t^N \right\}_{t=0}^{\infty}$ and prices $\left\{ P_t^X, P_t^N, q_t^X, q_t^N, w_t, R_t, R_t^{lX}, R_t^{lN} \right\}_{t=0}^{\infty}$, such that:

Given prices, $b_0$, $k_0^X$, $k_0^X$ and shocks' processes, $\left\{ c_t^M, c_t^N, l_t, i_t^X, i_t^N, k_{t+1}^X, k_{t+1}^N, b_{t+1} \right\}_{t=0}^{\infty}$ solve the household's problem.

Given prices and shocks' processes, $\left\{ l_t^X, k_t^X, z_t^X \right\}_{t=0}^{\infty}$ solve firm X's problem.

Given prices and shocks processes, $\left\{ l_t^N, k_t^N, z_t^N \right\}_{t=0}^{\infty}$ solve firm N's problem.

Given prices and shocks processes, $\left\{ z_t^X, z_t^N \right\}_{t=0}^{\infty}$ solve bank's problem.

Market clearing conditions are satisfied: $c_t^N = y_t^N$, and $l_t = l_t^X + l_t^N$.

The resource constraint is satisfied: $R_t b_t + P_t^X Y_t^X - c_t^M - i_t^X - i_t^N - \gamma b_{t+1} = 0$