Improvement in Information, Income Inequality, and Growth

by

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Abstract

We analyze the importance of information about individual skills for understanding economic growth and income inequality. The paper uses the framework of an OLG economy with endogenous investment in human capital. Agents in each generation differ by random individual ability, or talent, which realizes in the second period of life. The human capital of an agent depends on both his talent and his investment in education. The investment decision is based on a public signal (test outcome) which screens all agents for their talents. We analyze how a better information system, which allows more efficient screening, affects the co-movements of indicators for income inequality and human capital accumulation.

Keywords: Information system, human capital accumulation, income inequality.

JEL classification numbers: D80, J24, J30.

1 Introduction

In recent decades we have witnessed a growing body of research on the role of information in economic models. In particular, the welfare implications of information have been studied extensively. As a main result, these studies have revealed the ambiguous nature of information with regard to economic welfare when risk sharing arrangements are operative (Hirshleifer (1971), Green (1981), Campbell (2001), Schlee (2001), Eckwert and Zilcha (2003)). Surprisingly, the question how information interacts with economic growth and income inequality has received much less attention, even though this question is not unrelated to the welfare problem.

The idea that growth and income inequality are systematically related through equilibrium market mechanisms has stimulated a considerable amount of empirical and theoretical research. So far, these studies have produced mixed results. Based on empirical evidence, Persson and Tabellini (1994) concluded that higher growth induces less income inequality. Other papers (e.g., Forbes (2000), Quah (2002)) found a positive correlation between growth and inequality. In part, the inconclusiveness of the evidence is due to a lack of consensus with regard to the main factors by which inequality and growth are determined. Various studies focus on different factors and thereby produce conflicting results. Our paper also follows this route by singling out information about individual skills as an explanatory factor: we identify the effects of such information on indicators for economic growth and for income inequality; and we analyze the co-movements of both indicators due to changes in information.

Typically, the effects of information in general equilibrium models depend on the scope of risk sharing opportunities (Hirshleifer (1971,1975), Orosel (1996), Schlee (2001), Eckwert and Zilcha (2001)). In this paper we consider an economy where no explicit risk sharing arrangements are operative. Nevertheless, due to imperfect information, in equilibrium some individual risks will be shared across agents: the market treats all agents identically who, on the basis of the available information, cannot be distinguished according to their characteristics. Thus, even with risk sharing markets absent, better information goes hand in hand with less risk sharing. Therefore, information has important effects on the allocation of risks and, hence, on investment and economic growth. In addition, since better information allows more reliable identification of individual characteristics, it also affects in a very natural way the inequality of income distribution. This set up provides a theoretical platform for an analysis of the co-movements of economic growth and income inequality.

Our analytical framework is an OLG economy in which private investment in education (say, non-compulsory schooling), while young, affects an agent's human capital in his second period of life. Individual human capital depends also on random ability, or talent, which is still unknown when the agent decides how much 'effort' to invest in his/her education and training. The investment decision is made after observing a signal (test outcome) which screens agents for their abilities. Each individual signal contains imperfect public information about an agent's random talent. At the same time labor contracts are concluded. Since individual abilities are not yet known agents differ only by the signals they have received. As a consequence, all agents with the same signal are grouped together, and they are paid a wage equal to the mean marginal product of human capital in this group.

In this set up we analyze the effect of better information system, i.e., more efficient screening of individual skills, on the distribution of income within each generation and on the accumulation of the aggregate human capital stock. We find that income inequality always increases with better information. The effects on economic growth depend on the properties of the individual investment decisions which, in turn, are determined by the degree of intertemporal substitution in consumption: if individual preferences exhibit high elasticity of intertemporal substitution, agents with more favorable signals will choose higher investment levels. Under this constellation more efficient screening enhances growth and, hence, higher growth goes hand in hand with more income inequality. By contrast, growth and inequality are inversely related when the elasticity of intertemporal substitution is small.

The paper is organized as follows. In section 2 we describe the OLG economy and define our concept of informativeness. In section 3 we study the information-induced link between inequality and growth. All proofs are relegated to a separate Appendix.

2 The Model

Consider an overlapping generations economy with a single commodity and a continuum of individuals in each generation (but no population growth). The commodity can be either consumed or used as an input (physical capital) in a production process. Individuals live for three periods: 'youth' where they obtain education (while still supported by parents), 'middle-age' where they work and consume, and 'retirement' where they only consume. We denote generation t by $G_t, t = 0, 1, \cdots .G_t$ consists of all individuals born at date $t - 1$.

One of the main features of our economy is the heterogeneity of individuals with regard to their human capital generated by a random innate ability. When individual i is born his ability is yet unknown. The uncertainty about the agent's ability is described by some random variable $\tilde{A}^i$ which realizes at the beginning of the next period and takes values in some interval $\mathcal{A} \subset \mathbb{R}_+$. We assume that the random variables $\tilde{A}^i, i \in G_t, t = 0, 1, \cdots$, are i.i.d; thus, in particular, the ex ante distribution of ability is the same for all agents and does not depend on time or on the history of the economy.

Human capital of individual $i \in G_t$ depends on ability $\tilde{A}^i$ (which is random), effort $e^i \in \mathbb{R}_+$ invested in education by this individual, and the 'environment', represented here by the average human capital of agents in the previous generation (who are currently active economically). Thus we write,

$$\tilde{h}^i = \varphi(\tilde{A}^i) g(H_{t-1}, e^i) \tag{1}$$

where i belongs to generation t, and H_{t-1} is the average human capital of G_{t-1}, (see the role of H_{t-1} in generating human capital of G_t, for example, in Lucas (1988), Azariadis and Drazen (1990)).

Assumption 1 *The function $g(H, e)$ is strictly increasing and $g_{12} \geq 0, g_{22} < 0$. $\varphi : \mathcal{A} \to \mathbb{R}_+$ is increasing and differentiable.*

A priori the distribution of random ability $\tilde{A}^i$ is the same for all agents i both within the same generation and across generations.[1] However, before choosing optimal

[1]In the sequel we will therefore suppress the index i and write $\tilde{A}$ instead of $\tilde{A}^i$. Note, however,

effort in the youth period each individual receives a signal (test outcome) which contains public information about his own ability. We model the informational structure of the economy as follows: let $\tilde{y}$ be a real-valued random variable which takes values in $Y \subset \mathbb{R}$ and is correlated to $\tilde{A}$. Each agent $i \in G_t, t = 0, 1, \cdots$, with ability A observes an individual signal y^i which is drawn randomly from the distribution of the random variable $(\tilde{y}|A)$.[2] By construction, this individual signal is correlated to i's ability. Therefore, when agent i makes his decision about how much effort e^i to invest in education, the relevant c.d.f. for random ability is the posterior distribution of $\tilde{A}$ given the individual signal y^i.

For convenience we normalize the measure of agents in each generation to 1:

$$\int_{\mathcal{A}} \nu(A)\, \mathrm{d}A = 1,$$

where $\nu(A)$ is the (Lebesgue)-densitiy of agents with ability A. Denote by $f(\cdot|A)$ the density of the random variable $(\tilde{y}|A)$, and by $\nu_y(\cdot)$ the density of the random variable $(\tilde{A}|y)$. Using this notation, the distribution of signals received by agents in the same generation has the density[3]

$$\mu(y) = \int_{\mathcal{A}} f(y|A)\nu(A)\, \mathrm{d}A. \tag{2}$$

And average ability of all agents who have received the signal y is

$$\bar{\varphi}(y) := E[\varphi(\tilde{A})|y] = \int_{\mathcal{A}} \varphi(A)\nu_y(A)\, \mathrm{d}A. \tag{3}$$

The agents are expected utility maximizers with von-Neumann Morgenstern lifetime utility function

$$U(e, c_1, c_2) = v(e) + u_1(c_1) + u_2(c_2). \tag{4}$$

Individuals derive negative utility from 'effort' while they are young and positive utility from consumption in the working period, c_1, and from consumption in the retirement period, c_2.

that in general the random variables $\tilde{A}^i$ and $\tilde{A}^j$ differ for $i \neq j$; only their distributions are the same.

[2]Throughout the paper we shall refer to the realizations of $\tilde{y}$ as signals, and to the realizations of the $\tilde{y}^i$'s as *individual* signals.

[3]Note that, by the law of large numbers, μ does not depend on t.

Assumption 2 *The utility functions v and u_j, $j = 1, 2$, have the following properties:*

(i) $v : \mathbb{R}_+ \to \mathbb{R}_-$ *is decreasing and strictly concave,*

(ii) $u_j : \mathbb{R}_+ \to \mathbb{R}$ *is increasing and strictly concave,* $j = 1, 2$.

In each period, production in our economy, is carried out by competitive firms who use two production factors: physical capital K and human capital H. The process is described by an aggregate production function $F(K, H)$, which exhibits constant returns to scale. If individual i supplies l^i units of labor in his 'working period', his supply of human capital equals $l^i h^i$. We assume inelastic labor supply, i.e., l^i is a constant and it is equal to 1 for all i.

Assumption 3 $F(K, H)$ *is concave, homogeneous of degree 1, and satisfies* $F_K > 0$, $F_H > 0, F_{KK} < 0, F_{HH} < 0$.

We assume throughout this paper full international capital mobility, while human capital is assumed to be immobile. Thus the interest rate $\bar{r}_t$ is exogenously given at each date t. This implies that marginal productivity of aggregate physical capital K_t must be equal to $1 + \bar{r}_t$ (assuming full depreciation of capital in each period). On the other hand, given the aggregate stock of human capital at date t, H_t, the stock K_t must adjust such that

$$1 + \bar{r}_t = F_K(K_t, H_t) \qquad t = 1, 2, 3, \cdots \tag{5}$$

holds. But this implies, by Assumption 3, that $\frac{K_t}{H_t}$ is determined by the international rate of interest $\bar{r}_t$. Hence the wage rate w_t (price of one unit of human capital), given in equilibrium by the marginal product of aggregate human capital, is also determined once $\bar{r}_t$ is given. Thus we may write

$$w_t = F_L\Big(\frac{K_t}{H_t}, 1\Big) =: \zeta(\bar{r}_t) \qquad t = 1, 2, 3, \cdots. \tag{6}$$

Labor contracts are concluded *after* agents have learned their signals but *before* their abilities become known.

Obviously, the wage income specified in a labor contract cannot be made contingent on individual human capital because individual ability is yet unknown. Therefore agents are unable to appropriate the full marginal product of their human capital. Instead, individuals are grouped according to the signals they have received. And, in the absence of any further information, the market treats all agents in the same group identically. Under these circumstances each individual will receive a wage equal to the mean marginal product of human capital of those with whom he is grouped. Within the group of all agents who have received the signal y, average ability is given by $\bar{\varphi}(y)$ in equation (3). Therefore, the wage income of agent $i \in G_t$ with signal y is $w_t \bar{h}^i$, where

$$\bar{h}^i = \bar{\varphi}(y) g(H_{t-1}, e^i). \tag{7}$$

In equilibrium, all agents with the same signal y choose the same effort level. As a consequence, aggregate wage income and aggregate human capital in this group are given by $\mu(y)(w_t \bar{h}^i)$ and $\mu(y)\bar{h}^i$, respectively. The firm therefore pays the competitive wage in (6), $\mu(y)(w_t \bar{h}^i)/\mu(y)\bar{h}^i = w_t$, for each unit of *aggregate* human capital supplied by agents with signal y.

Now let us consider the optimization problem that each $i \in G_t$ faces, given $\bar{r}_t, w_t$, and H_{t-1}. At date $t-1$, when 'young', this individual chooses an optimal level of savings, s^i, and an optimal level of effort employed in obtaining education. These decisions are made under random ability $\tilde{A}$, but after the individual signal y^i has been observed.

For given levels of H_{t-1}, w_t and $\bar{r}_t$, the optimal saving and effort decisions of individual $i \in G_t$ are determined by

$$\begin{aligned} \max_{s^i, e^i} \; & E[v(e^i) + u_1(c_1^i) + u_2(c_2^i) | y^i] \\ \text{s.t. } & c_1^i = w_t \bar{h}^i - s^i \\ & c_2^i = (1 + \bar{r}_t) s^i. \end{aligned} \tag{8}$$

Since income is determined by *average ability*, given the signal y^i, saving s^i is based on average human capital $\bar{h}^i$ (and not on h^i); as a consequence, period 2 consumption c_2^i is non-random when e^i is chosen.

The necessary and sufficient first order conditions are

$$-u_1'(w_t\bar{h}^i - s^i) + (1+\bar{r}_t)u_2'((1+\bar{r}_t)s^i) = 0 \tag{9}$$

$$v'(e^i) + w_t g_2(H_{t-1}, e^i)\bar{\varphi}(y)u_1'(w_t\bar{h}^i - s^i) = 0, \tag{10}$$

where $\bar{h}^i$ is given by equation (7).

Observe that the signal y enters the first order conditions only via the term $\bar{\varphi}(y)$. Thus we may express the optimal decisions as functions of $\bar{\varphi}(y)$ rather than as functions of the signal itself, i.e., $s^i = s_t(\bar{\varphi}(y))$, $e^i = e_t(\bar{\varphi}(y))$. Similarly, $\bar{h}^i = \bar{h}_t(\bar{\varphi}(y))$.

Using (2) and (3) the aggregate stock of human capital at date t can be expressed as

$$H_t = E_y[\bar{h}_t(\bar{\varphi}(y))] = \int\limits_Y \bar{h}_t(\bar{\varphi}(y))\mu(y)\mathrm{d}y, \tag{11}$$

where

$$\bar{h}_t(\bar{\varphi}(y)) := \bar{\varphi}(y)g(H_{t-1}, e_t(\bar{\varphi}(y))) \tag{12}$$

is the average human capital of agents in G_t who have received the signal y.

Definition 1 *Given the international interest rates $(\bar{r}_t)$ and the initial stock of human capital H_0, a competitive equilibrium consists of a sequence $\{(e^i, s^i)_{i\in G_t}\}_{t=1}^{\infty}$, and a sequence of wages $(w_t)_{t=1}^{\infty}$, such that:*

(i) *At each date t, given $\bar{r}_t$, H_{t-1}, and w_t, the optimum for each $i \in G_t$ in problems (9) and (7) is given by (e^i, s^i).*

(ii) *The aggregate stocks of human capital, $H_t, t = 1, 2, \cdots$, satisfy (11).*

(iii) *Wage rates $w_t, t = 1, 2, \cdots$, are determined by (6).*

2.1 Information Systems

The ability of each individual i is a random variable $\tilde{A}^i$. We assume that the random variables $\tilde{A}^i$ are i.i.d. across individuals in G_t, $t = 0, 1, 2...$, and that they all have the same distribution as $\tilde{A}$. We shall refer to the realizations of $\tilde{A}$ as the states of nature. Before a young agent with ability A chooses an optimal effort level he observes an individual signal which is drawn randomly from the distribution of the random variable $(\tilde{y}|\tilde{A}^i = A) = (\tilde{y}|\tilde{A} = A) =: (\tilde{y}|A)$. Thus, ex ante the conditional distributions of the individual signals are identical. For convenience, we shall refer to the realizations of $\tilde{y}$ simply as signals.

An information system, which will be represented by $f : Y \times \mathcal{A} \to \mathbb{R}_+$ throughout the paper, specifies for each state of nature A a conditional probability function over the set of signals. The positive real number $f(y|A)$ defines the conditional probability (density) that if the state of nature is A, then the signal y will be sent. $F(y|A)$ denotes the c.d.f. for the density $f(y|A)$. We assume throughout the paper that the densities $\{f(\cdot|A), A \in \mathcal{A}\}$ have the strict monotone likelihood ratio property (MLRP): $y' > y$ implies that for any given (nondegenerate) prior distribution for $\tilde{A}$, the posterior distribution conditional on y' dominates the posterior distribution conditional on y in the first-order stochastic dominance. This implies that higher signal is 'good news' (see Milgrom (1981)). As a consequence, $\int_{\mathcal{A}} \vartheta(A)\nu_{y'}(A)\,\mathrm{d}A > \int_{\mathcal{A}} \vartheta(A)\nu_y(A)\,\mathrm{d}A$ holds for any strictly increasing function ϑ.

By the law of large numbers, the prior distribution over $\mathcal{A}$ coincides with the ex post distribution of ability across agents. Also the prior distribution over Y coincides with the ex post distribution of individual signals across agents and, hence, is given by equation (2). Finally, the density function for the updated posterior distribution over $\mathcal{A}$ is

$$\nu_y(A) = f(y|A)\nu(A)/\mu(y). \tag{13}$$

Next we define our criterion of informativeness. Let $G(A|y)$ be the c.d.f. for the conditional density $\nu(A|y)$.

<u>Remark 1</u>: $G(A|y)$ is a decreasing function of y. This follows from MLRP. Choose $\hat{A} \in \mathcal{A}$ arbitrarily but fixed and define

$$U(A) = \begin{cases} 0 & ; \quad A \le \hat{A} \\ 1 & ; \quad A > \hat{A} \end{cases} .$$

Since $EU(\tilde{A}|y) = 1 - G(\hat{A}|y)$ is increasing in y by virtue of MLRP, $G(A|y)$ is decreasing in y for all $A \in \mathcal{A}$.

Consider the transformation $\tilde{\pi} := F \circ \tilde{y}$, where F is the c.d.f. for the probability density μ defined in (2) under information system f,

$$\mu(y) = \int_{\mathcal{A}} f(y|A)\nu(A)\,\mathrm{d}A.$$

Thus for any $y \in Y$, the transformed signal $\pi = F(y)$ represents the probability that under the information system f an agent receives a signal less than y. Obviously, $\tilde{\pi}$ is uniformly distributed over $[0,1]$, i.e., the distribution of the transformed signal across agents does not depend on the information system f. We will exploit this fact later when we define our concept of income inequality.

An information system will be regarded as more informative if the observable signal realizations have a uniformly stronger impact on the posterior distribution of states:

Definition 2 (informativeness) *Let $\bar{f}$ and $\hat{f}$ be two information systems with corresponding c.d.f's $\bar{G}(A|y)$, $\hat{G}(A|y)$ for the densities $\bar{\nu}(A|y)$, $\hat{\nu}(A|y)$. $\bar{f}$ is more informative than $\hat{f}$ (expressed by $\bar{f} \succ_{\text{inf}} \hat{f}$), if*

$$\bar{G}_{\pi}\big(A|\bar{F}^{-1}(\pi)\big) \leq \hat{G}_{\pi}\big(A|\hat{F}^{-1}(\pi)\big) \tag{14}$$

holds for all $A \in \mathcal{A}$ and $\pi \in (0,1)$.

According to Remark 1, $G\big(A|F^{-1}(\pi)\big) = \text{prob}\ \big(\tilde{A} \leq A|F^{-1}(\pi)\big)$ is decreasing in the (transformed) signal π. Inequality (14) says that under a more informative system the posterior distribution over states is more sensitive with respect to changes in the signal.

In the economics literature various concepts of informativeness have been used, dating back to the seminal work by Blackwell (1951,1953) where the ordering of information has been linked to a statistical sufficiency criterion for signals. More recently, concepts have been developed which represent informativeness as a stochastic dominance order over posterior distributions (Kim (1995), Athey and Levin (1988), Demougin and Fluet (2001)). Some of these partial orderings contain the Blackwell ordering as a subset.[4] Our concept of information in (14) also imposes a restriction

[4] E.g. Kim's criterion can be shown to be strictly weaker than Blackwell's criterion.

on the sensitivities of the posterior state distributions. It has an advantage in terms of tractability over the above mentioned criteria as it involves only signal derivatives of the posteriors rather than more complex measures of stochastic dominance.

3 Income Inequality and Growth: The Role of Information

Our analysis of income inequality focuses on the distribution of labor income within a given generation G_t. Labor income depends both on the information system and on the (transformed) signal received by an agent,

$$I_t^f(\pi) = w_t \bar{\varphi}^f(\pi) g\big(H_{t-1}, e_t\big(\bar{\varphi}^f(\pi)\big)\big), \tag{15}$$

where

$$\bar{\varphi}^f(\pi) := E^f\big[\varphi(\tilde{A})|F^{-1}(\pi)\big]. \tag{16}$$

To study the impact of information on income inequality we use a concept which is based on the following comparison of distributions:

Definition 3 *Let Y and X be real-valued random variables with zero-mean normalizations $\check{Y} = Y - EY$ and $\check{X} = X - EX$. The distribution of Y is 'more unequal' than the distribution of X if $\check{Y}$ differs from $\check{X}$ by a MPS.*

The following lemma facilitates the application of our inequality concept in Definition 8:

Lemma 1 *Let $\tilde{\pi}$ be a random variable which is distributed over $[0,1]$. Let $y : [0,1] \to \mathbb{R}$, $x : [0,1] \to \mathbb{R}$ be continuous increasing functions such that*

(i) $\tilde{y} := y \circ \tilde{\pi}$ differs from $\tilde{x} := x \circ \tilde{\pi}$ by a MPS,

(ii) $y(\pi) - x(\pi)$ is monotone in π.

For any continuous strictly increasing function $\vartheta : \mathbb{R} \to \mathbb{R}$ the distribution of $\vartheta \circ \tilde{y}$ is more unequal than the distribution of $\vartheta \circ \tilde{x}$.

Our concept of income inequality is based on the dominance criterion for normalized distributions in Definition 3.

Definition 4 *Let $\bar{f}$ and $\hat{f}$ be two information systems. Income inequality under $\bar{f}$ is higher than under $\hat{f}$, if the distribution of $I_t^{\bar{f}}(\pi)$ is more unequal (in the sense of Definition 3) than the distribution of $I_t^{\hat{f}}(\pi)$ for all $t \geq 1$.*

Under a better information system individual ability can be assessed more accurately at the time when labor contracts are concluded. We may conjecture, therefore, that firstly the income distribution will be more discriminating with respect to differences in abilities, and secondly that it will be better in line with the distribution of human capital across agents. The following proposition confirms our first conjecture, i.e., the informational mechanism results in higher income inequality.

Proposition 1 *Let $\bar{f}$ and $\hat{f}$ be two information systems such that $\bar{f} \succ_{\text{inf}} \hat{f}$. Income inequality is higher under $\bar{f}$ than under $\hat{f}$.*

Next we look into the effects of better information on the aggregate stock of human capital and, hence, on economic growth. Aggregate human capital of generation t is

$$H_t^f = \int_0^1 \bar{h}_t\big(\bar{\varphi}^f(\pi)\big)\,\mathrm{d}\pi, \tag{17}$$

where

$$\bar{h}_t\big(\bar{\varphi}^f(\pi)\big) := \bar{\varphi}^f(\pi)g\big(H_{t-1}, e_t\big(\bar{\varphi}^f(\pi)\big)\big). \tag{18}$$

Since $\bar{h}_t(0) = 0$, the function $\bar{h}_t(\cdot)$ is convex (concave) in $\bar{\varphi}^f$ if $e_t(\cdot)$ is increasing (decreasing) in $\bar{\varphi}^f$.

Depending on the well-known interaction between an income effect and a substitution effect, effort $e_t(\cdot)$ may be increasing or decreasing in $\bar{\varphi}^f$. If the elasticity of intertemporal substitution is sufficiently small, the income effect will be dominant and, hence, a better signal results in lower effort. By contrast, the substitution effect will be dominant, if preferences exhibit sufficiently high elasticity of intertemporal substitution. In that case agents step up their efforts when they receive more favorable signals. In Section 3.1 we will characterize the monotonicity properties of the effort decision for the special case of constant elasticity of substitution.

The expected marginal product of investment in education, $\bar{\varphi}^f(\pi)g_2(H_{t-1}, e_t)$, is higher for agents with better signals. Thus, from the viewpoint of economic growth an increasing effort function would be more efficient (i.e., more conducive to the growth of the human capital stock) than a decreasing effort function. We shall therefore call individual behavior *efficiency-inducing* if good news (higher signal) induces higher investment in education; and individual investment behavior will be called *inefficiency-inducing* if good news results in lower effort.

With this terminology, the function $\bar{h}_t(\cdot)$ in (12) is convex under efficiency-inducing investment behavior, and concave under inefficiency-inducing investment behavior.

Proposition 2 *Let $\bar{f}$ and $\hat{f}$ be two information systems such that $\bar{f} \succ_{\inf} \hat{f}$. Consider the competitive equilibrium for a given initial H_0.*

(i) *Under efficiency-inducing behavior better information (weakly) enhances growth, i.e., $H_t^{\bar{f}} \geq H_t^{\hat{f}}$ for all $t \geq 1$.*

(ii) *Under inefficiency-inducing behavior better information (weakly) reduces growth, i.e., $H_t^{\bar{f}} \leq H_t^{\hat{f}}$ for all $t \geq 1$.*

From propositions 1 and 2 we obtain, as a corollary, a characterization of the information-induced link between growth in income inequality:

Corollary 1 *As the result of an improvement of the economy's information system,*

(i) *under efficiency-inducing investment behavior, higher growth goes hand in hand with more income inequality;*

(ii) *under inefficiency-inducing investment behavior, lower growth goes hand in hand with more income inequality.*

The characterization in Proposition 2 can be interpreted in terms of a simple economic mechanism. Consider part (i), i.e., assume that investment behavior is growth-efficient. The implementation of a better information system enhances the reliability of the individual signals. As a consequence, high signals become even better news and induce higher investment in education. Similarly, under a better information system the bad news conveyed by a low signal becomes even worse (because now the news is more reliable). As a result, investment in education declines. Thus, under growth-efficient investment behavior, better information tends to increase the efforts of agents with high signals and decrease the efforts of agents with low signals. Since the expected marginal product of effort (in terms of human capital) is higher for agents with higher signals, aggregate human capital increases when the information system becomes more informative. If investment behavior is growth-inefficient, the same mechanism results in lower aggregate human capital under a more informative system.

The relationship between economic growth and income inequality has been widely debated in the literature in the last decade. Based on empirical evidence, Persson and Tabellini (1994) show that higher growth results in less income inequality – a finding that was challenged by other authors, e.g., Forbes (2000) and Quah (2002). Our study contributes to this controversy with a narrow, information-based focus: we identify the effects of information on indicators for economic growth and income inequality; and we analyze the co-movements of both indicators due to changes in information. In this sense, growth and income inequality are positively related if agents respond to better signals with higher investment in education. Yet, the model is also consistent with an inverse relationship between growth and income inequality. Such a pattern arises when better signals induce agents to reduce investment in education.

3.1 An Example: CEIS Preferences

To illustrate the critical role of the elasticity of intertemporal substitution for the information-induced link between income inequality and growth we restrict the utility functions $u_1(\cdot)$, $u_2(\cdot)$, and $v(\cdot)$ to be in the family of CEIS (**C**onstant **E**lasticity of **I**ntertemporal **S**ubstitution) :

$$u_1(c_1) = \frac{c_1^{1-\gamma_u}}{1-\gamma_u}; \quad u_2(c_2) = \beta \frac{c_2^{1-\gamma_u}}{1-\gamma_u}; \quad v(e) = -\frac{e^{\gamma_v+1}}{\gamma_v+1}. \tag{19}$$

γ_u and γ_v are strictly positive constants. $1/\gamma_u$ parametrizes the elasticity of intertemporal substitution in consumption.

We also assume that the function g in (1) has the form

$$g(H, e) = \hat{g}(H)e^{\alpha}, \tag{20}$$

where $\hat{g}$ is strictly increasing in H, and $\alpha \in (0, 1)$.

Using the functional forms of u_j, $j = 1, 2$, in (19), it follows from equation (9) that, given $\bar{r}_t$ and w_t, the saving s^i is proportional to the human capital level h^i. In other words, for each t there is a constant m_t such that for all $i \in G_t$ we have:

$$s^i = m_t h^i, \quad 0 < m_t < w_t, \quad t = 1, 2, \cdots \tag{21}$$

Setting $s = F(y)$, the specifications in (19), (20) and (21) allow us to solve equation (10) for the optimal effort level as a function of average ability $\bar{\varphi}^f(s)$:

$$e_t\big(\bar{\varphi}^f(s)\big) = \delta_t\big(\bar{\varphi}^f(s)\big)^{\rho(1-\gamma_u)/\alpha} \tag{22}$$

where

$$\delta_t := \left[\frac{\alpha w_t(\hat{g}(H_{t-1}))^{1-\gamma_u}}{(w_t - m_t)^{\gamma_u}}\right]^{\rho/\alpha}; \quad \rho = \frac{\alpha}{\gamma_v + \alpha(\gamma_u - 1) + 1}.$$

The income of an agent with signal $y = F^{-1}(s)$ is

$$I_t^f(s) = w_t \delta_t^{\alpha} \hat{g}(H_{t-1})\big(\bar{\varphi}^f(s)\big)^{\tau}, \tag{23}$$

and aggregate human capital of generation t is

$$H_t^f = \delta_t^{\alpha} \hat{g}(H_{t-1}) \int_0^1 \Big(E^f\left[\varphi(A)|F^{-1}(s)\right]\Big)^{\tau} \mathrm{d}s, \tag{24}$$

where

$$\tau := 1 + \rho(1 - \gamma_u) = \frac{1 + \gamma_v}{\gamma_v + \alpha\gamma_u + (1 - \alpha)} > 0. \tag{25}$$

Corollary 2 *Let $\bar{f}$ and $\hat{f}$ be two information systems such that $\bar{f} \succ_{\text{inf}} \hat{f}$, and assume that the specifications in (19) and (20) are valid.*

(i) *<u>High EIS:</u> For $1/\gamma_u \geq 1$ better information (weakly) enhances growth, i.e., $H_t^{\bar{f}} \geq H_t^{\hat{f}}$ for all $t \geq 1$.*

(ii) <u>Moderate EIS:</u> *For* $1/\gamma_u \leq 1$ *better information (weakly) reduces growth, i.e.,* $H_t^{\bar{f}} \leq H_t^{\hat{f}}$ *for all* $t \geq 1$.

<u>Proof:</u> Since $e_t(\bar{\varphi}^f)$ in (22) is increasing for $1/\gamma_u \geq 1$ and decreasing for $1/\gamma_u \leq 1$ the claim is implied by Proposition 2. □

From Proposition 1 and Corollary 2 we obtain the following characterization of the information-induced link between growth in income inequality:

Corollary 3 *Assume that the specifications in (19) and (20) are valid. As the result of an improvement of the economy's information system,*

(i) *higher growth goes hand in hand with more income inequality, if the elasticity of intertemporal substitution in consumption is high, i.e.,* $1/\gamma_u \geq 1$,

(ii) *lower growth goes hand in hand with more income inequality, if the elasticity of intertemporal substitution in consumption is small, i.e.,* $1/\gamma_u \leq 1$.

4 Conclusion

The conjecture that inequality of income distribution is systematically related to economic growth is an idea which has triggered many debates and controversies in economics. Our paper analyzes the inequality-growth link from the narrow perspective of information: what are the joint effects on income inequality and economic growth if decisions with respect to investment in education can be based on more reliable information about the agents' abilities? It turns out that there is no unambiguous answer to this question, a fact which reflects the inconclusiveness of the various empirical studies in the field (Persson and Tabellini (1994), Forbes (2000), Quah (2003)).

In our framework the inequality-growth link depends on a monotonicity property of individual investment in education. If consumer preferences exhibit high intertemporal substitution in consumption, agents with better test results and, hence, better ability prospects, choose higher investments in education. Under this constellation both income inequality and growth increase when the information system is improved. Similarly, if consumer preferences exhibit low intertemporal substitution in consumption, agents with more favorable signals invest less and, hence, higher inequality goes hand in hand with lower growth.

In the literature various formalizations of what constitutes more information in an economic model have been suggested. Blackwell's prominent sufficiency criterion has been widely used, but this concept is understood to be quite demanding and, in fact, stronger than needed for many economic applications. In recent years other concepts based on the sensitivity of the posterior state distribution with regard to signals have been developed and successfully applied to economic problems (e.g., Kim (1995), Athey and Levin (1998)). Our notion of informativeness belongs to this class of extensions, i.e., the informativeness order emerges from a restriction on the distribution of state posteriors.

The time structure of our model implies that agents receive wage payments which are conditional on the agents' signals rather than on their true (ex post) abilities. Thus individual wage incomes are based on assessments of each agent's 'potential' rather than on the human capital the agent actually contributes in the production process. We believe that this feature of our model is reasonably well in line with various remuneration schemes that can be observed in labor markets when individual human capital is not verifiable. Even so, the case where wage contracts can be made contingent on ex post individual human capital might be of some interest as well. In such a setting each agent i is characterized by a pair (y^i, A^i), but his economic decisions are based solely on the signal y^i (while A^i is still random). Under this specification information no longer plays a role in some process of risk sharing across agents. Therefore, the impact of information on income inequality and growth will presumably be weaker than in our model. This conjecture requires further analysis and we intend to examine it in some future work.

Appendix

In this appendix we prove Lemma 1 and the two propositions.

<u>Proof of Lemma 1:</u> (i) and (ii) imply that $\vartheta_\circ y(\pi) - \vartheta_\circ x(\pi)$ is monotone increasing.

Thus there exists $\pi^* \in (0,1)$ such that

$$\vartheta{\circ}y(\pi) - E[\vartheta(\tilde{y})] \overset{(\geq)}{\leq} \vartheta{\circ}x(\pi) - E[\vartheta(\tilde{x})] \quad \text{for} \quad \pi \overset{(\geq)}{\leq} \pi^*.$$

This inequality implies that $\check{y} := \vartheta{\circ}\tilde{y} - E[\vartheta(\tilde{y})]$ differs from $\check{x} := \vartheta{\circ}\tilde{x} - E[\vartheta(\tilde{x})]$ by a MPS and, hence, the distribution of $\vartheta{\circ}\tilde{y}$ is more unequal than the distribution of $\vartheta{\circ}\tilde{x}$.

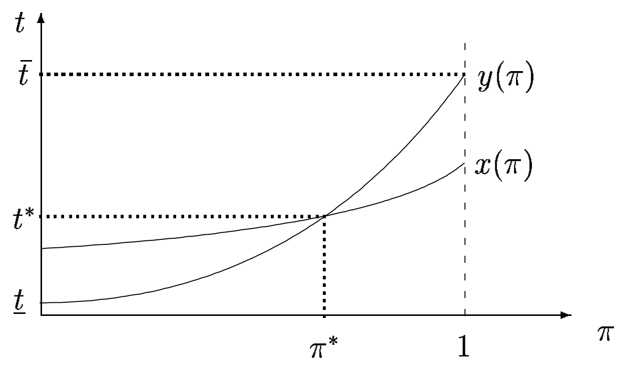
□

We prove two preliminary results before we proceed with the proofs of the propositions.

Lemma 2 (MPS) *Let $\tilde{\pi}$ be a random variable which is distributed over $[0,1]$ according to the Lebesgue density ϕ. Let $y : [0,1] \to [\underline{t}, \bar{t}]$ and $x : [0,1] \to [\underline{t}, \bar{t}]$ be differentiable strictly increasing functions such that $E[y{\circ}\tilde{\pi}] = E[x{\circ}\tilde{\pi}]$, i.e.,*

$$\int_0^1 y(\pi)\phi(\pi)\,\mathrm{d}\pi = \int_0^1 x(\pi)\phi(\pi)\,\mathrm{d}\pi. \tag{26}$$

Assume further that $y(\pi)$ and $x(\pi)$ have the single crossing property with $y(\pi^*) = x(\pi^*) =: t^*$ and $y(\pi) \overset{(\geq)}{\leq} x(\pi)$ for $\pi \overset{(\geq)}{\leq} \pi^*$. Then $Y := y{\circ}\tilde{\pi}$ differs from $X = x{\circ}\tilde{\pi}$ by a MPS.



Remark 4: If $y(\pi)$ and $x(\pi)$ are strictly decreasing and the other conditions in Lemma 1 are satisfied, then $X = x \circ \tilde{\pi}$ differs from $Y = y \circ \tilde{\pi}$ by a MPS.

Proof of Lemma 2: Let G and F be the c.d.f.'s for Y and X. Denote by g and f the (Lebesgue) densities of G and F, and define $S := G - F$. From $y(\pi) \overset{(\geq)}{\leq} x(\pi)$ for $\pi \overset{(\geq)}{\leq} \pi^*$ we conclude $S(t) \overset{(\leq)}{\geq} 0$ for $t \overset{(\geq)}{\leq} t^*$ and, hence,[5]

$$\begin{aligned}\int_{\underline{t}}^{\bar{t}} S(t)\,\mathrm{d}t &= \underbrace{tS(t)\Big|_{\underline{t}}^{\bar{t}}}_{=0} - \int_{\underline{t}}^{\bar{t}} t[g(t) - f(t)]\,\mathrm{d}t \\ &= \int_0^1 [y(\pi) - x(\pi)]\phi(\pi)\,\mathrm{d}\pi = 0. \end{aligned} \tag{27}$$

$$\int_{\underline{t}}^{\hat{t}} S(t)\,\mathrm{d}t = \int_{\underline{t}}^{t^*} S(t)\,\mathrm{d}t + \int_{t^*}^{\hat{t}} S(t)\,\mathrm{d}t \geq 0. \tag{28}$$

The inequality in (28) follows from (27) and the fact that $S(t) \overset{(\leq)}{\geq} 0$ for $t \overset{(\geq)}{\leq} t^*$. (27) and (28) together imply that Y differs from X by a MPS. □

Lemma 3 *Let $\bar{f}$ and $\hat{f}$ be two information systems with $\bar{f} \succ_{\inf} \hat{f}$. For any increasing differentiable function $\vartheta : \mathcal{A} \to \mathbb{R}_+$ the random variable $\bar{\theta}(\pi) := E^{\bar{f}}[\vartheta(A)|\bar{F}^{-1}(\pi)]$ differs from $\hat{\theta}(\pi) := E^{\hat{f}}[\vartheta(A)|\hat{F}^{-1}(\pi)]$ by a MPS. Also, $\bar{\theta}(\pi) - \hat{\theta}(\pi)$ is monotone in π.*

Remark 5: If ϑ is a decreasing function, $\hat{\theta}(\pi)$ differs from $\bar{\theta}(\pi)$ by a MPS.

Proof of Lemma 3: By the law of iterated expectations, $\int_0^1 \bar{\theta}(\pi)\,\mathrm{d}\pi = \int_0^1 \hat{\theta}(\pi)\,\mathrm{d}\pi$. Therefore, in view of Lemma 1, it suffices to show that $\bar{\theta}(\pi) - \hat{\theta}(\pi)$ is increasing in

[5] The first equality in the second line of (27) follows from

$$\int_{\underline{t}}^{\bar{t}} tg(t)\,\mathrm{d}t = \int_{y^{-1}(\underline{t})}^{y^{-1}(\bar{t})} y(\pi)\underbrace{g(y(\pi))y'(\pi)}_{=\phi(\pi)}\,\mathrm{d}\pi = \int_0^1 y(\pi)\phi(\pi)\,\mathrm{d}\pi.$$

π.

$$\begin{aligned}\bar\theta'(\pi)-\hat\theta'(\pi) &= \int_{\mathcal{A}} \vartheta(A)\frac{\partial}{\partial\pi}\Big[\bar\nu\big(A|\bar F^{-1}(\pi)\big)-\hat\nu\big(A|\hat F^{-1}(\pi)\big)\Big]\,\mathrm{d}A \\ &= -\int_{\mathcal{A}} \vartheta'(A)\frac{\partial}{\partial\pi}\Big[\int_{\underline{A}}^{A} \bar\nu\big(A'|\bar F^{-1}(\pi)\big)-\hat\nu\big(A'|\hat F^{-1}(\pi)\big)\,\mathrm{d}A'\Big]\,\mathrm{d}A \\ &= -\int_{\mathcal{A}} \vartheta'(A)\Big[\bar G_\pi\big(A|\bar F^{-1}(\pi)-\hat G_\pi\big(A|\hat F^{-1}(\pi)\big)\Big]\,\mathrm{d}A \geq 0.\end{aligned}$$

The last inequality follows from (14), since $\vartheta' \geq 0$ has been assumed. □

Proof of Proposition 1: Incomes under the two information systems are given by

$$I_t^{\bar f}(\pi)=w_t\bar\varphi^{\bar f}(\pi)g\big(H_{t-1},e_t\big(\bar\varphi^{\bar f}(\pi)\big)\big),\qquad I_t^{\hat f}(\pi)=w_t\bar\varphi^{\hat f}(\pi)g\big(H_{t-1},e_t\big(\bar\varphi^{\hat f}(\pi)\big)\big),$$

where

$$\bar\varphi^{\bar f}(\pi):=E^{\bar f}\big[\varphi(\tilde A)|\bar F^{-1}(\pi)\big],\qquad \bar\varphi^{\hat f}(\pi):=E^{\hat f}\big[\varphi(\tilde A)|\hat F^{-1}(\pi)\big].$$

According to Lemma 3, $\bar\varphi^{\bar f}(\pi)$ and $\bar\varphi^{\hat f}(\pi)$ differ by a MPS and $\bar\varphi^{\bar f}(\pi)-\bar\varphi^{\hat f}(\pi)$ is monotone in π. Below we show that $\bar h_t(\bar\varphi)=\bar\varphi g\big(H_{t-1},e_t(\bar\varphi)\big)$ is monotone increasing in $\bar\varphi$. Lemma 1 then implies that the income distribution is more unequal under $\bar f$ than under $\hat f$.

First observe that $s_t\big(\bar\varphi(y)\big)$ and $w_t\bar h_t\big(\bar\varphi(y)\big)-s_t\big(\bar\varphi(y)\big)$ are both co-monotone with $\bar h_t\big(\bar\varphi(y)\big)$. This observation is immediate from (9) since u_1' is a decreasing function. Now assume, by contradiction, that as $\bar\varphi$ increases $\bar h_t(\cdot)$ declines. By co-monotonicity, $w_t\bar h_t(\cdot)-s_t(\cdot)$ declines as well. As a consequence, $\bar\varphi u_1'\big(w_t\bar h_t(\cdot)-s_t(\cdot)\big)$ increases and, according to (10), $e_t(\cdot)$ increases. However, in view of (7), an increase in $e_t(\cdot)$ contradicts our assumption that $\bar h_t(\cdot)$ declines as $\bar\varphi$ increases. □

Proof of Proposition 2: According to Lemma 2, $\bar\varphi^{\bar f}(\pi)$ differs from $\bar\varphi^{\hat f}(\pi)$ by a MPS. In addition, if the investment behavior is growth-efficient (growth-inefficient), $\bar h_t(\cdot)$ in (18) is a convex (concave) function. Therefore,

$$\int_0^1 \bar h_t\big(\bar\varphi^{\bar f}(\pi)\big)\,\mathrm{d}\pi \overset{(\leq)}{\geq} \int_0^1 \bar h_t\big(\bar\varphi^{\hat f}(\pi)\big)\,\mathrm{d}\pi$$

holds (see Rothschild/Stiglitz, 1970) and, hence, $H_t^{\bar f}$ in (17) is larger (smaller) than $H_t^{\hat f}$. □

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