Optimal Devaluations

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Abstract

We analyze optimal fiscal, monetary and exchange rate policy in a simple small open econonomy model with price setting frictions. We perform our analysis in the tradition of optimal dynamic Ramsey problems. We characterize optimal allocations and the government policies that implement the optimal allocation.

1 Introduction

The purpose of this paper is to provide a theoretical framework to characterize optimal fiscal, monetary and exchange rate policy in a small open economy model with varying degrees of price setting restrictions. The contribution of this paper is to carry on the analysis following the dynamic Ramsey literature. Thus, the mapping from policies to allocations is derived from a fully articulated dynamic general equilibrium monetary model with taxes. An important consequence of this approach is that we can jointly study optimal fiscal and monetary policy. In addition, the explicit introduction of preferences provides a natural welfare criteria to evaluate policies.

We consider a model in which a fraction of firms is restricted to set prices one period in advance and characterize the optimal cyclical properties of the Ramsey solution. For this economy, we first extend results derived in Correia, Nicolini and Teles (2001a) and show that the set of implementable allocations is independent of the price setting restrictions. Thus, the optimal allocation under sticky prices is the same as the optimal allocation under flexible prices. We then show that the cyclical properties of optimal short run monetary policy depend on the nature of the shock driving the cycle. Except in strange cases, if the boom is caused by a shock to the technology of final goods (non-tradables), optimal monetary policy must be procyclical and a devaluation must follow; if it is driven by a technology shock to intermediate goods (tradeable), optimal monetary policy is countercyclical and the exchange rate must decrease. Finally, if the boom is induced by an international terms of trade shock, optimal monetary policy is procyclical and the exchange rate depends on the source of the term of trade shock: if it is driven by a decrease in the price of importable, the exchange rate must be revalued, while if it is driven by an increase in the price of exportables, the exchange rate must be devalued. Another remarkable result is that neither optimal allocations nor the policy instruments that implement it depends on the degree of price stickiness.

There is an extensive literature that studies optimal monetary and exchange rate policies and characterizes it in terms of its cyclical properties. Obviously, this properties do depend on the mapping from policies to allocations that is derived form the particular model used and on the welfare criterion used. Most of the literature has used reduced form models not explicitly derived from preferences and technologies. Our results will differ form most of the literature, sometimes because of the particular model we use, sometimes because of the welfare criteria used.

The model we analyze is very simple. As such, it has at least two weaknesses we want to discuss. First, as most of the modern literature, we impose ad-hoc restrictions on the price setting process, instead of modelling the price setting decision and deriving the optimal price setting rules. Thus, we take as a fundamental parameter the fraction of firms that can adjust prices within the period. Thus, the model is subject to the Lucas critique, and this raises doubts of its usefulness for policy analysis. We do not view this as a significant problem, since we show that both the optimal allocation and the optimal policy are independent of that assumed fundamental parameter. Thus, potential changes in the parameter due to changes in policy will not alter our conclusions regarding optimal policy.

Second, a model as simple as this one is not able to replicate the evidence of open economies, particularly at the business cycle frequency we will be focusing on. Why performing optimal policy exercises in models that are not able to match the data?. This is indeed a serious shortcoming, but there does not seem to be obviously better choices available. We went ahead with the analysis, despite this issue, for two reasons: first, we hope that the intuitions we unravel here will prove useful to understand the workings of monetary and fiscal policy in models that can replicate observed patterns for aggregate variables at business cycle frequencies, if these do exhibit price stickiness and second, we want to explore the implications of price setting restriction for the conduct of optimal fiscal and monetary policy in open economies, above and beyond the empirical relevance of this restrictions in explaining real time dynamics. Since many times policy advice is offered based on the alleged workings of models with sticky prices, clarifying the ways these models work was, for us, a natural question to raise.

The characterization of optimal monetary and exchange rate policies is an old time question. There also seems to be a certain consensus with respect to the way the nominal exchange rate ought to be managed given shocks such as government spending, real exchange rate or productivity shocks. On the other hand, these questions have only very recently started being addressed in general equilibrium dynamic models. The policy implications derived form the models we analyze are at odds with the conventional wisdom many times. In addition, some of those policy implications do not appear robust to small changes in the environment. Our first, very simple approximation suggest that general equilibrium economics does not seem to support the conventional wisdom in all dimensions.

2 The economy

Our model economy follows closely the structure in Correia, Nicolini and Teles (2001a) modified to allow for international trade along the lines suggested by Rebelo (XXX).

The economy consists of a representative household, firms that produce an exportable intermediate input, a continuum of producers of final goods indexed by $i \in [0,1]$, and a government. Each firm in the final goods sector produces a distinct, perishable consumption good, indexed by i.

The state of the economy will be represented by the realization of a random process $\sigma_t \in \Sigma$ that follows a first order Markov process with stationary transition function. For simplicity, we will assume that there exist a density function $\mu(\sigma_{t+1} | \sigma_t)$ describing the law of motion of σ. The shocks to the economy will be time invariant functions of the state. The economy faces the following shocks: government expenditure shocks, $G_t = G(\sigma_t)$, two types of productivity shocks, to the final consumption good $s_t^y = s^y(\sigma_t)$ and to the intermediate (exportable) good $s_t^x = s^x(\sigma_t)$. In addition, we will let policy instruments to be functions of the state. That is, labor income taxes $\tau_t^n = \tau^n(\sigma_t)$, dividend taxes $\tau_t^d = \tau^d(\sigma_t)$, consumption taxes $\tau_t^c = \tau^c(\sigma_t)$, taxes to foreign contingent claims $\tau^*(\sigma_t)$, non-negative lump-sum transfers $T(\sigma_t)$ (e.g. social security), money supplies $M_t = M(\sigma_t)$ and exchange rate $\varepsilon_t = \varepsilon(\sigma_t)$. These are all the natural policy instruments to consider in this environment.

We model monetary policy as a mapping from the state of the economy σ_t to the positive real numbers, $M(\sigma_t)$ for $t \geq 0$. As in any rational expectations version of sticky price models, the effects of anticipated or unanticipated monetary policy are very different. Therefore, it is convenient to introduce the following notation. Let

$$\begin{aligned} M(\sigma_t) &= M(\sigma_{t-1})\mu(\sigma_{t-1})\delta(\sigma_t), \text{ where} \\ E_{t-1}\left[\delta(\sigma_t)\right] &= 1 \end{aligned}$$

which implies

$$\overline{M}_t = E_{t-1}\left[M_t\right] = M(\sigma_{t-1})\mu(\sigma_{t-1})$$

Thus, we identify the expected rate of money change between time $t-1$ and t with $\mu(\sigma_{t-1})$, while $\delta(\sigma_t)$ represents the state contingent deviation on the rate of money growth. In the same way we decompose the exchange rate in an expected element $\overline{\varepsilon}(\sigma_{t-1})$ and an unexpected shock $\zeta(\sigma_t)$. That is,

$$\begin{aligned} \varepsilon\left(\sigma_t\right) &= \overline{\varepsilon}(\sigma_{t-1})\zeta(\sigma_t) \\ \text{with } E_{t-1}\zeta(\sigma_t) &= 1, \text{ so } E_{t-1}\left(\varepsilon_t\right) = \overline{\varepsilon}(\sigma_{t-1}) \end{aligned}$$

We assume that the economy is open to international trade, as well as to the international credit markets. The intermediate good is traded internationally and is produced with labor, the sole input, with a constant returns to scale technology. The consumption goods are assumed to be non traded in international markets and are produced using labor and two intermediate inputs, one home produced and the other produced in the rest of the world. The international price of both intermediate inputs are, respectively, P_t^{x*} and P_t^{m*}. The law of one price holds for both tradable goods. The economy is fully integrated in world capital markets and the price of foreign assets will also be allowed to depend on the state, $z^*\left(\sigma_{t+1}, \sigma_t\right)$ so we can analyze the optimal response of policy to these shocks.

3 Households

Preferences are described by the expected utility function:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t U\left(C_t, 1-N_t\right)\right\} \tag{1}$$

where N_t is labor effort, $\beta \in (0,1)$ is a discount factor and the composite C_t is

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1. \tag{2}$$

The timing of events is taken from Lucas and Stokey (1987): assets markets open at the beginning of the period. After all trading takes place, assets markets close until the beginning of the next period and the rest of the markets open. And at the end of the period households receives their labor income and dividends that can be used to buy assets or consumption goods the next period.

Households start period t with outstanding nominal wealth, $\mathbb{W}_t$, and decide to buy money balances M_t and B_{t+1}^h units of money in nominal bonds that pay $R_{t+1}B_{t+1}^h$ units of money one period later. They also buy $Z_{t+1}^h(\sigma_{t+1})$ units of one-period state contingent nominal securities promising one unit of money in the next period conditional on the state σ_{t+1} being realized, whose nominal price, normalized by the probability of the state σ_t at t, is given by $z(\sigma_{t+1} | \sigma_t)$. They can also buy $A_{t+1}(i)$ units of stocks of firm i, that cost $a_t(i)$ in units of currency. Households have to pay flat labor income, dividend and consumption taxes at the rates τ_t^n, τ_t^d and τ_t^c respectively. Notice that since there are complete markets, risk free bonds are redundant, however we will introduce them to be able to talk about the Friedman rule and other monetary policies more clearly. To avoid cumbersome notation, we will keep the argument σ_t implicit in most of the paper, although the reader should keep in mind that everything is function of the state.

Denote by P_t^{x*} and P_t^{m*} the international price of the home produced and imported intermediate inputs in units of foreign currency, respectively. The law of one price holds for both goods, thus, if the nominal exchange rate is ε_t, we have that $P_t^x = \varepsilon_t P_t^{x*}$ and $P_t^m = \varepsilon_t P_t^{m*}$. We assume also that there exist (redundant) state contingent claims to deliver one unit of the foreign currency next period, Z_{t+1}^*, at the price (in units of foreign currency) $z_{t+1,t}^*$.

We also assume that the international credit market is inhabited by risk neutral investors that price the foreign currency contingent claims as

$$z_{t+1,t}^* = \beta^* \frac{P_t^{x*}}{P_{t+1}^{x*}}$$

where, to avoid non-stationary paths of the trade balance, we assume that $\beta^* = \beta$.

The purchases of consumption goods have to be made with cash, so we introduce the following cash in advance constraint

$$\int_0^1 P_t(i)c_t(i)(1+\tau_t^c)di \leq M_t v_t \tag{3}$$

where $P_t(i)$ is the money price of final good i and v_t is a velocity shock. In a cash-in-advance framework the velocity shock can be given the interpretation that in each period (and at a particular state σ_t), a fraction $\frac{1}{v_t}$ of the final goods must be paid with cash.

At the end of the period, the households receive labor income, $W_t N_t$ where W_t is the nominal wage rate, and collect dividends, given by current period profits $D_t(i)$ that can be used to purchase consumption and trade in assets in the following period. Therefore, the households face the budget constraints

$$M_t + B_{t+1}^h + E_t Z_{t+1}^h z_{t+1,t} + \int_0^1 A_{t+1}(i)a_t(i)di \leq \mathbb{W}_t \tag{4}$$

$$\begin{aligned} \mathbb{W}_{t+1} \quad = \quad & M_t + R_{t+1}B_{t+1}^h + Z_{t+1}^h - (1+\tau_t^c)\int_0^1 P_t(i)c_t(i)di + \\ & \int_0^1 A_{t+1}(i)a_{t+1}(i)di + T_t + W_t N_t\left(1-\tau_t^n\right) + \int_0^1 A_{t+1}(i)D_t(i)di\left(1-\tau_t^d\right) \end{aligned}$$

The Bellman equation describing the households' problem is

$$J\left(\mathbb{W}_t\right) = \max\left\{U\left(C_t, 1-N_t\right) + \beta E_t J\left(\mathbb{W}_{t+1}\right)\right\}$$

subject to (2), the cash in advance constraint (3) and the budget constraints (4). As is customary in this literature, $P_t \equiv \left(\int_0^1 P_t\left(i\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}$ defines the aggregate price index. After rearranging the first order conditions, the following equations describe the consumer's problem

$$\frac{c_t\left(i\right)}{C_t} = \left(\frac{P_t\left(i\right)}{P_t}\right)^{-\theta}$$

$$\frac{U_C\left(t\right)}{U_{1-N}\left(t\right)} = \frac{P_t\left(1+\tau_t^c\right)}{W_t\left(1-\tau_t^n\right)}\frac{\left(R_{t+1}+v_t-1\right)}{v_t}$$

$$\frac{U_{1-N}\left(t\right)}{W_t\left(1-\tau_t^n\right)} = \beta E_t\left[\frac{U_{1-N}\left(t+1\right)R_{t+2}}{W_{t+1}\left(1-\tau_{t+1}^n\right)}\right]$$

$$z_{t+1,t} = \beta\frac{U_{1-N}\left(t+1\right)R_{t+2}}{U_{1-N}\left(t\right)R_{t+1}}\frac{W_t\left(1-\tau_t^n\right)}{W_{t+1}\left(1-\tau_{t+1}^n\right)}$$

$$E_t\left[z_{t+1,t}\right] = \frac{1}{R_{t+1}}$$

$$\frac{U_{1-N}\left(t\right)R_{t+1}}{W_t\left(1-\tau_t^n\right)}a_t(i) = \beta E_t\left[\frac{U_{1-N}\left(t+1\right)R_{t+2}}{W_{t+1}\left(1-\tau_{t+1}^n\right)}\left[a_{t+1}(i) + D_t(i)\left(1-\tau_t^d\right)\right]\right]$$

As usual, given that there are complete markets all marginal rates of substitutions are equalized across agents and they are equal to the price of the Arrow securities. It will be useful to rewrite the last equation as an asset pricing equation:

$$a_t(i) = E_t\left[z_{t+1,t}\left(a_{t+1}(i) + D_t(i)\left(1-\tau_t^d\right)\right)\right]$$

4 Government

We assume that the government has to finance a strictly positive (contingent-claim) sequence of purchases G_t, where

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}$$

As part of the definition of equilibrium, we will assume that the government, acting competitively, takes the prices of the consumption varieties as given and chooses the demand for variety i, $g_t\left(i\right)$, so as to minimize the expenditure to achieve a certain level of G_t. The aggregate level of government purchases depends on the realization of the state of the economy σ_t and is taken as exogenous, at least by the fiscal and monetary authorities. Thus expenditure minimization implies that the government's demand for variety i is given by

$$\frac{g_t\left(i\right)}{G_t} = \left(\frac{P_t\left(i\right)}{P_t}\right)^{-\theta}$$

5 Production sector

The production sector of the economy consist of two broad sectors. There is an intermediate goods sector with firms that produce competitively an intermediate input that can be used locally or can be traded in international markets with labor as the sole input. On the other hand, there is a sector of final goods characterized by a continuum of establishments indexed by some point in the unit interval, $i \in [0,1]$, each producing a differentiated final good, called of variety i. The market structure in the final goods sector is monopolistic competition as developed by Dixit and Stiglitz (1977). Each firm in this sector uses three type of inputs: labor and two intermediate inputs, both traded in international markets. The country has the technology to produce only one of the intermediate inputs.

5.1 Intermediate goods sector

Output of the home produced intermediate input is given by the linear production function

$$Q_t^x = s_t^x (N_t - N_t^y)$$

where s_t^x is a technology shock and N_t^y is aggregate labor used in the final goods sectors. The zero profit condition implies that

$$P_t^x = \frac{W_t}{s_t^x} \tag{5}$$

must hold in equilibrium.

5.2 Final goods sector

The technology to produce final goods of variety $i \in [0,1]$ is given by the production function

$$y_t(i) = F\left(n_t^y(i), q_t^x(i), q_t^m(i), s_t^y\right)$$

where $y_t(i)$ is output of the variety i, and $n_t^y(i)$, $q_t^x(i)$ and $q_t^m(i)$ are the demand for inputs by the i^{th} firm: labor, home produced and imported intermediate inputs respectively. s_t^y is an aggregate technology shock common across varieties, and the production function F is linearly homogeneous in the inputs, concave and satisfies standard Inada conditions.

Since technology is constant return to scale, the cost function is linear in output,

$$C\left(W_t, P_t^x, P_t^m, s_t^y, y_t\left(i\right)\right) = \varphi\left(W_t, P_t^x, P_t^m, s_t^y\right) y_t\left(i\right)$$

where φ is positive, globally concave, increasing and linearly homogeneous in prices and decreasing in s_t^y. Note that all firms have the same cost function.

Furthermore, using the zero profit condition in the intermediate input sector we can write the marginal cost function as

$$MC_t = \varphi\left(W_t, P_t^x, P_t^m, s_t^y\right) = W_t \varphi\left(1, \frac{P_t^x}{W_t}, \frac{P_t^m}{W_t}, s_t^y\right) = W_t \varphi\left(1, \frac{1}{s_t^x}, \frac{1}{s_t^x} \frac{P_t^m}{P_t^x}, s_t^y\right)$$

$>$From purchasing power parity we obtain that $\frac{P_t^x}{P_t^m} = \frac{P_t^{x*}}{P_t^{m*}} \equiv d_t^*$. Then the marginal cost function of each firm in the final goods sector is given by and defining $\phi\left(s_t^x, s_t^y, d_t^*\right)$, with $\frac{\partial \phi_t}{\partial s_t^x} < 0$, $\frac{\partial \phi_t}{\partial s_t^y} < 0$ and $\frac{\partial \phi_t}{\partial d_t^*} < 0$, we conclude that in any equilibrium, the marginal cost of each firm in the final goods sector will be given by

$$MC_t = W_t \phi\left(s_t^x, s_t^y, d_t^*\right) \equiv W_t \phi_t$$

where $\phi\left(s_t^x, s_t^y, d_t^*\right) \equiv \varphi\left(1, \frac{1}{s_t^x}, \frac{1}{s_t^x d_t^*}, s_t^y\right)$. The ϕ function only depends on exogenous shocks and satisfies$\frac{\partial \phi_t}{\partial s_t^x} < 0$, $\frac{\partial \phi_t}{\partial s_t^y} < 0$ and $\frac{\partial \phi_t}{\partial d_t^*} < 0$.

Even though technological possibilities are the same for each firm, we will assume that a fraction γ of the final goods producers are constrained to set prices one period in advance, called sticky firms, while the remaining fraction of firms are allowed to set state contingent prices, called flexible firms. As discussed in the introduction, we agree that this is, indeed, an ad-hoc assumption as are all other price setting restrictions found in the literature. Also, it is clear that the fraction γ should be endogeneized. However we will show below that the optimal policy is independent of this critical parameter, and therefore, potential movements on γ driven by different government's actions will not affect the normative claims we will derive.

5.2.1 Flexible firms

The pricing equation for stocks implies that the problem of maximizing the value of a single monopolistic flexible firm can be written as the following dynamic programing problem

$$a_t(i) = \max_{P_t(i)} E_t\left[z_{t+1,t}\left(D_t(i)\left(1-\tau_t^d\right) + a_{t+1}(i)\right)\right]$$

therefore, as long as $\left(1-\tau_t^d\right) E_t\left[z_{t+1,t}\right] \geq 0$, the problem reduces to the static optimization program

$$\max_{P_t(i)} D_t(i) = \max_{P_t(i)} \left[P_t(i) - MC_t\right] y_t(i)$$

subject to the demand function

$$\frac{y_t(i)}{Y_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{6}$$

obtained from the households and government problems, where $Y_t = C_t + G_t$.

The optimal pricing rule is,

$$P_t^F = P_t^F(i) = \frac{\theta}{\theta - 1} MC_t = \frac{\theta}{\theta - 1} W_t \phi_t$$

where the superscript F is used to denote a flexible prices firm. Flexible firms set a common price, a constant mark-up over their common marginal cost.

$>$From now on, we will focus on competitive equilibria for which the relative price $z_{t+1,t}$ is positive. If this is the case, the constraint for positive production reduces to $\tau_t^d \leq 1$. What do we require for $z_{t+1,t}$ to be non-negative? From the household's problem and using that in any equilibria the gross nominal interest rate and the nominal wages are positive (otherwise firms do not optimize), we require $\left(1 - \tau_t^n\right) / \left(1 - \tau_{t+1}^n\right) \geq 0$. And of course, the relevant case is $1 - \tau_t^n \leq 0$ for all t and states of the world. This the natural case to work with, since otherwise, the government in at least one period is giving transfers through labor subsidies. But that will affect the intertemporal Euler equation. Since the government has access to non-negative lump-sum transfers, it will be better to transfer resources through using the latter instrument instead of labor subsidies.

5.2.2 Sticky firms

Sticky firms must set prices one period in advance. Even though at time t the firms are constrained in terms of the prices at which they can sell, they are free to choose any level of output. Hence, at time t, and given a previously chosen price, they do choose quantities to maximize profits. That problem is given by

$$\max_{y_t(i)} \left[P_t(i) - MC_t\right] y_t(i) \left(1 - \tau_t^d\right)$$

subject to

$$0 \leq y(i)_t \leq Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\theta}$$

The solution is to set $y_t(i) = Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\theta}$ as long as $[P_t(i) - MC_t]\left(1 - \tau_t^d\right) \geq 0$, and $y_t(i) = 0$ otherwise. That is, as long as they do not make negative profits, they will produce all that is demanded at the given price.

Prices must be chosen at $t-1$, then the manager's problem will be to maximize the expected value, as of time $t-1$ of the firm's stock price, or

$$\max_{P_t(i)} E_{t-1} a_t(i) = \max_{P_t(i)} E_{t-1}\left[E_t z_{t+1,t}\left(D_t(i)\left(1 - \tau_t^d\right) + a_{t+1}(i)\right)\right]$$

or

$$\max_{P_t(i)} E_{t-1}\left[z_{t+1,t}\left(P_t(i) - MC_t\right) y_t(i)\left(1 - \tau_t^d\right)\right]$$

subject to (6).

Taking the first order condition, using the intertemporal Euler equation to replace the contingent price $z_{t+1,t}$ and rearranging we find that the pricing rule for sticky firms is given by

$$P_t^S(i) = P_t^S = \frac{\theta}{(\theta - 1)} E_{t-1}\left[\omega_t MC_t\right]$$

where

$$\omega_t = \frac{\frac{U_{1-N}(t)}{W_t(1-\tau_t^n)} Y_t P_t^\theta \left(1 - \tau_t^d\right)}{E_{t-1}\left[\frac{U_{1-N}(t)}{W_t(1-\tau_t^n)} Y_t P_t^\theta \left(1 - \tau_t^d\right)\right]}$$

and S is used to denote a sticky prices firm.

As $G_t > 0$, feasibility requires sticky prices firms to produce positive amounts of output every period. This is the case when $\left[P_t^S - MC_t\right]\left(1 - \tau_t^d\right) \geq 0$ which can equivalently be written as

$$\left[\frac{P_t^S}{W_t} - \phi_t\right](1 - \tau_t^d) \geq 0$$

6 Foreign sector

The equation that determines the evolution of net holdings of foreign assets, measured in foreign currency, is given by

$$B_{t+1}^* + E_t Z_{t+1}^* z_{t+1,t}^* = B_t^* R_t^* + Z_t^* + P_t^{x*} TB_t$$

where TB_t is the trade balance measured in units of the home produced intermediate good, B_{t+1}^* is the amount of non-contingent foreign bonds held domestically between t and $t+1$, and R_t^* is the foreign interest rate between $t-1$ and t.

Let $\mathcal{Z}_t^* \equiv z_{t,t-1}^* z_{t-1,t-2}^* ... z_{1,0}^*$ be the price of one unit of good at time t contingent on a particular realization of the history of shocks $\sigma^t = (\sigma_t, \sigma_{t-1}, ..., \sigma_1 | \sigma_0)$ as of time zero. Multiplying the foreign assets accumulation equation by $\mathcal{Z}_t^*$, we have

$$\mathcal{Z}_t^* \left[B_t^* R_t^* + Z_t^* + P_t^{x*} TB_t - B_{t+1}^* - E_t Z_{t+1}^* z_{t+1,t}^* \right] = 0$$

But using that $z_{t+1,t}^* = \beta P_t^{x*} / P_{t+1}^{x*}$, then $\mathcal{Z}_t^* = \beta^t P_0^{x*} / P_t^{x*}$.

Introducing the last condition in place of $\mathcal{Z}_t^*$ above, summing for all periods and histories σ^t, and assuming away Ponzi schemes, we obtain the foreign sector budget constraint

$$E_0 \sum_{t=0}^{\infty} \beta^t TB_t = -\frac{R_0^* B_0^*}{P_0^{*x}} \tag{7}$$

where B_0^* is the initial stock of foreign assets held domestically. Hereafter we will assume that $B_0^* = 0$.

7 Equilibrium

7.1 Equilibrium with price setting frictions:

Given the state of the economy σ_t, an equilibrium is a set of prices

$$\{P_t, P_t(i)\ , W_t, z_{t+1,t}, R_{t+1}, a_t(i), \varepsilon_t, P_t^x, P_t^m\}$$

for $i \in [0,1]$ given $P_0(i)$ if i is a sticky prices firm, a set of allocations

$$\left\{ \begin{array}{c} C_t, c_t(i), N_t, A_{t+1}(i), Z_{t+1}^h, Z_{t+1}^g, Z_{t+1}^*, B_{t+1}^h, B_{t+1}^g, B_{t+1}^*, G_t, g_t(i), Y_t, \\ y_t(i), n_t(i), q_t^x(i), q_t^m(i), Q_t^x \end{array} \right\}$$

for $i \in [0,1]$, and a set of policy instruments

$$\left\{ \tau_t^c, \tau_t^n, \tau_t^d, \tau_t^*, T_t, M_t \right\}$$

given A_0, Z_0^h, Z_0^g, Z_0^*, B_0^h, B_0^g, B_0^* and the foreign prices $\left\{ z_{t+1,t}^* P_t^{*x}, P_t^{*m} \right\}$ such that :

1. Given prices $\{P_t, P_t(i)\ , W_t, z_{t+1,t}, R_{t+1}, a_t(i)\}$ and policy instruments $\left\{ \tau_t^c, \tau_t^n, \tau_t^d \right\}$, the allocations

$$\left\{ C_t, c_t(i), N_t, A_{t+1}(i), Z_{t+1}^h, M_t, B_{t+1}^h \right\}$$

solve the household's problem.

2. Given prices

$$\{P_t, W_t, z_{t+1,t}, P_t^x, P_t^m\}$$

and the quantity $\{Y_t\}$; the prices $\{P_t(i)\}$ and the quantities $\{y_t(i), n_t(i), q_t^x(i), q_t^m(i)\}$ solve the firm's problem as stated in section 5, depending on the price setting restriction on the particular firm.

3. The prices $\{W_t, P_t^x\}$ satisfy the free entry condition in the intermediate inputs sector.

4. Given the prices $\{P_t, P_t(i)\}$ and the quantities $\{G_t\}$ the government chooses $\{g_t(i)\}$ so as to minimize expenditures, as stated in section 4.

5. There are no arbitrage opportunities in international capital markets.

6. The foreign sector feasibility constraint is satisfied.

7. All markets clear. Of particular importance are the following conditions:

$$c_t(i) + g_t(i) = y_t(i), \quad \text{for all } i \tag{8}$$

$$A_t(i) = 1,\ B_t^h = B_t^g,\ Z_t^h = Z_t^g,\ B_t^* = B_t^g,\ Z_t^* = Z_t^g$$

These market clearing conditions assume, without loss of generality, that no domestic denominated assets is held abroad and that household do not hold foreign currency denominated assets.

The following proposition will prove useful for solving the Ramsey problem below:

Proposition 1 *In equilibrium the following condition must hold*

$$\gamma y_t^S + (1-\gamma) y_t^F = \phi(s_t^x, s_t^y, d_t^*)^{-1}\left[N_t - \frac{TB_t}{s_t^x}\right]$$

where S and F refer to sticky and flexible respectively. Proof: Appendix

8 Implementability conditions

As is usually done in the literature on optimal taxation, the strategy we will follow is to replace all the agents' first order conditions into the intertemporal budget constraint and a no arbitrage equation in international capital markets in order to capture the optimality conditions characterizing the equilibrium in two equations. In other words, we will rewrite the intertemporal budget constraint and the no-arbitrage condition in terms of home allocations, policy instruments and foreign prices.

To roughly capture the idea of taxing capital movements, we will assume that foreign assets receipts are taxed at the proportional rate $\tau^*_{t+1,t}$. We are introducing state contingent taxes, equivalent to state contingent prices. Given the current state, we assume that the government is able to choose a tax on foreign assets contingent on tomorrow's state. We will need this instrument in order to implement the optimal allocation. No arbitrage between foreign and home contingent claims requires that

$$z^*_{t+1,t} = z_{t+1,t}\frac{\varepsilon_{t+1}}{\varepsilon_t}\left(1-\tau^*_{t+1,t}\right)$$

The last equation tells us that given the equilibrium prices $z^*_{t+1,t}$, $z_{t+1,t}$ and ε_t, at period t, it must be the case that in the next period the exchange rate and/or the tax rate on foreign assets must adjust so as to make the no-arbitrage condition hold. In particular, if $\tau^*_{t+1,t}$ does not move, then the equilibrium exchange rate will have to adjust. Interestingly, notice that the government could manipulate the exchange rate by an appropriate choice of the tax rate $\tau^*_{t+1,t}$.

Our task now is to get rid off prices and write the last equation in terms of allocations, foreign prices and policy instruments. Using the pricing equation on foreign assets - pinned down by the linearity of foreign investors, the first order condition with respect to contingent claims, the zero profit condition in the home produced intermediate input sector and purchasing power parity one can show that the last equation can be written as

$$R_{t+1} = \frac{U_{1-N}(t+1)}{U_{1-N}(t)}\left(\frac{1-\tau^n_t}{1-\tau^n_{t+1}}\right)R_{t+2}\frac{s^x_t}{s^x_{t+1}}\left(1-\tau^*_{t+1,t}\right) \tag{9}$$

After imposing the equilibrium condition $A_t(i) = 1$ for all i, letting $D_t \equiv \int_0^1 d_t(i)\,di$, and $\mathcal{Z}_t = z_{t,t-1}z_{t-1,t-2}...z_{1,0}$, we can integrate forward (4) to obtain the intertemporal budget constraint

$$E_0\sum_{t=0}^{\infty}\mathcal{Z}_{t+1}\left\{P_tC_t(1+\tau^c_t) - T_t - W_tN_t(1-\tau^n_t) - D_t\left(1-\tau^d_t\right) + M_t\left(\frac{\mathcal{Z}_t}{\mathcal{Z}_{t+1}}-1\right)\right\} = 0$$

As mentioned earlier and as done in Lucas and Stokey (1983), we will introduce all the first order condition from the households' problem and the firms' problem into this equation. In the appendix we show that after doing all the necessary replacements we obtain

$$E_0 \sum_{t=1}^{\infty} \beta^t \left(U_{Ct} C_t - U_{1-N_t} \left[N_t + \left(N_t - \frac{TB_t}{s_t^x} \right) \frac{\left(1-\tau_t^d\right)}{\left(1-\tau_t^n\right)(\theta-1)} + \frac{T_t}{W_t \left(1-\tau_t^n\right)} \right] \right) - \tag{10}$$

$$\frac{U_{1-N_0} \gamma y_0^S}{W_0} \left(P_0^S - P_0^F\right) \left(\frac{1-\tau_0^d}{1-\tau_0^n} \right) + \frac{T_0}{W_0 \left(1-\tau_0^n\right)} = 0$$

The last equation summarizes the optimal behavior of the households and the firms. Any allocation that can be implemented by some fiscal and monetary policy must satisfy the market clearing conditions, the international capital market constraint (9), the foreign sector budget constraint (7) and the implementability condition (10). Conversely, any allocation that satisfies market clearing, (9), (7) and (10), can be implemented by some particular fiscal and monetary policy.

Interestingly, notice that the last equation *does not* include the velocity shocks, and this in turns implies that the optimal policy will be independent of v_t. Also, note that γ, the fraction of sticky firms appears only in the last term.

In a flexible prices economy the last equation becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(U_{Ct} C_t - U_{1-N_t} \left[N_t + \left(N_t - \frac{TB_t}{s_t^x} \right) \frac{\left(1-\tau_t^d\right)}{\left(1-\tau_t^n\right)(\theta-1)} + \frac{T_t}{W_t \left(1-\tau_t^n\right)} \right] \right) = 0 \tag{11}$$

Note that the only difference between (10) and the last equation is in the last term (i.e. the first period). It is sometime argued that with sticky prices the government has an additional degree of freedom to improve the allocation with respect to that of a flexible prices economy. We will argue below, however, that in our framework, this is not true: the additional degree of freedom given by the first period real wage in the sticky prices economy is redundant. I HAVE TO CHECK THIS, SINCE I THINK THAT THE SAME ARGUMENT THAT WE USE BELOW, USING THE NON-NEGATIVE LUMP-SUM TRANSFERS, CAN ALSO BE USED HERE: This result depends, however, on our particular price setting assumption. In particular, if at any moment in time we have firms with prices fixed by more than one period in a staggered fashion, it is not that straightforward to obtain the equivalence result mentioned above. In particular, more policy instruments are necessary to do it. Thus, in cases where there is a distribution of firms with the prices fixed over time, and if the set of policy instruments is small enough, there will be, indeed, an additional degree of freedom in the sticky prices economy relative to the flexible prices economy.

The following proposition states that any allocation that can be implemented by some government policy, can be implemented by another policy that taxes all dividends in all periods and states of the world.

Proposition 2 *If the allocation* $\{c_t^S, c_t^F, n_t^S, n_t^F, N_t, TB\}$ *is implemented by some government policy* $\left\{\tau_t^c, \tau_t^n, \tau_t^d, \tau_{t+1,t}^*, T_t, M_t, \varepsilon_t\right\}$*, then there is a government policy* $\left\{\tilde{\tau}_t^c, \tilde{\tau}_t^n, 1, \tilde{\tau}_{t+1,t}^*, \tilde{T}_t, \tilde{M}_t, \tilde{\varepsilon}_t\right\}$ *with* $\tilde{\tau}_t^d = 1$ *that implements the same allocation.*

The last proposition says that any allocation implemented by some particular government policy can be implemented by another policy that taxes dividends in all periods and states of the world. In particular, we can restrict attention to government policies that tax all dividends. To see why the last proposition is true, suppose that the government sets a particular policy with $\tau_t^d(\sigma_t) < 1$ for some period and some state of the world. Since a change in dividend taxes *only* affects the equilibrium through changes in the present value budget constraint of the household, the same allocation emerges as a competitive equilibrium if we set $\tau_t^d = 1$ in all periods and states of the world and rebate the difference in the present value of dividend taxes to the household through lump-sum transfers.

Corollary 3 *The set of implementable allocations is the same in a flexible prices economy and in an economy with price setting frictions, for any degree of price stickiness* $\gamma \in [0, 1]$.

This is a straightforward corollary to the last proposition and follows because the set of implementable allocations between the sticky prices economy and the flexible prices economy differs only in the constraints (10) and (11); and we showed that if the government is able to give resources through non-negative lump-sum transfers, then both constraints are the same.

In order to make the problem interesting, we will assume that the government will have to use distorting taxes to finance its expenditures. We introduce the following assumption:

Assumption: *In all equilibria that we will focus on, the present value of dividends is strictly smaller than the present value of government purchases.*

The last assumption implies that the government must use other distorting taxes to finance its expenditures. This in turns imply that in any optimal policy the government will set lump-sum transfers to zero in all periods and states of the world. Taking this into account, the constraint (10) becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U_{Ct} C_t - U_{1-N_t} N_t\right] = 0 \tag{12}$$

9 Solution to the Ramsey problem

In this section we characterize the optimal allocation. Given a stochastic sequence of government purchases, the government's problem is to choose an element from the set of implementable competitive equilibria to maximize the representative household's utility. In other words, the Ramsey problem is to maximize, by choice of $\{c_t^S, c_t^F, n_t^S, n_t^F, N_t, TB\}_{t=0}^{\infty}$,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t, 1 - N_t\right) \right\}$$

subject to (7), (8), (9), (12) and

$$C_t = \left(\gamma \left(c_t^S\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)\left(c_t^F\right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

Our strategy will consist in solving a relaxed problem and then showing that the solution to the relaxed problem can be implemented in the more restricted environment. First of all, disregard the international capital market constraint (12). Second, if market clearing is satisfied, clearly it must be true that

$$\gamma\left(c_t^S + g_t^S\right) + (1-\gamma)\left(c_t^F + g_t^F\right) = \gamma y_t^S + (1-\gamma)\, y_t^F$$

must hold (but not necessarily vice-versa.) Using proposition 1, we can rewrite the last condition as

$$\gamma\left(c_t^S + g_t^S\right) + (1-\gamma)\left(c_t^F + g_t^F\right) = \phi_t^{-1}\left[N_t - \frac{TB_t}{s_t^x}\right] \tag{13}$$

The relaxed problem is to maximize, by choice of $\{c_t^S, c_t^F, N_t, TB\}_{t=0}^{\infty}$ the agent's utility subject to (13), (7) and (12).

The first order conditions are

$$\begin{aligned}
\left[U_{Ct}\left(1+\lambda\right)+\lambda\left(C_tU_{CCt}-U_{C,1-Nt}N_t\right)\right]C_t^{\frac{1}{\theta}}\left(c_t^S\right)^{\frac{-1}{\theta}} &= \xi_t \\
\left[U_{Ct}\left(1+\lambda\right)+\lambda\left(C_tU_{CCt}-U_{C,1-Nt}N_t\right)\right]C_t^{\frac{1}{\theta}}\left(c_t^F\right)^{\frac{-1}{\theta}} &= \xi_t \\
U_{1-Nt}\left(1+\lambda\right)+\lambda\left[C_tU_{1-N,Ct}-U_{1-N,1-Nt}N_t\right] &= \xi_t\phi_t^{-1} \\
\eta &= \frac{\xi_t\phi_t^{-1}}{s_t^x}
\end{aligned}$$

where λ, η and $\beta^t\xi_t$ are the Lagrange multipliers on the constraints (12), (13) and (7) respectively. Note that the optimal allocation is independent of γ, the fraction of sticky firms in the economy and of the velocity shocks v_t.

Now, from the first two conditions we find that it is optimal to set

$$c_t^S = c_t^F = C_t$$

>From the household's first order conditions, we have

$$\frac{c_t^S}{c_t^F} = \left(\frac{P_t^S}{P_t^F}\right)^{-\theta}$$

thus, the optimal policy implies $P_t^S = P_t^F = P_t$: the government must manipulate the policy instruments to equalize the state contingent price with the previously chosen sticky price. Since $P_t^S = P_t^F$ implies $g_t^S = g_t^F = G_t$, then

$$c_t^S + g_t^S = c_t^F + g_t^F$$

therefore

$$y_t^S = y_t^F = Y_t = \phi_t^{-1}\left[N_t - \frac{TB_t}{s_t^x}\right]$$

and then the market clearing conditions (8) are satisfied for all i. To show that the solution to the relaxed problem is the solution of the Ramsey problem, we must make sure that the international capital market constraint (9) is satisfied for all histories and for all periods. We can always manipulate the taxes on foreign securities so that (9) is satisfied evaluated at any sequence of labor taxes, productivity shocks and the consumption and labor allocation obtained from the relaxed problem. This shows that the solution to the relaxed problem is indeed, the solution to the Ramsey problem.

The (infinite) set of equations that determines the Ramsey allocation is given by

$$U_{Ct}\left(1+\lambda\right)+\lambda\left(C_tU_{CCt}-U_{C,1-Nt}N_t\right) = \eta s_t^x\phi\left(s_t^x, s_t^y, d_t^*\right) \tag{14}$$

$$U_{1-Nt}\left(1+\lambda\right)+\lambda\left[C_tU_{1-N,Ct}-U_{1-N,1-Nt}N_t\right]=\eta s_t^x \tag{15}$$

$$C_t+G_t=\phi\left(s_t^x,s_t^y,d_t^*\right)^{-1}\left[N_t-\frac{TB_t}{s_t^x}\right] \tag{16}$$

$$E_0\sum_{t=0}^{\infty}\beta^t\left(U_{Ct}C_t-U_{1-N_t}N_t\right)=0$$

and

$$E_0\sum_{t=0}^{\infty}\beta^tTB_t=0$$

Recall from the firm's problem that ϕ_t is decreasing in s_t^x, s_t^x and d_t^*, and $s_t^x\phi_t$ is increasing in s_t^x.[1]

9.0.1 Example

To obtain more concrete results we will assume that preferences are separable in consumption and labor:

$$E_0\sum_{t=0}^{\infty}\beta^t\left[U\left(C_t\right)-V\left(N_t\right)\right]$$

where H is strictly increasing and strictly concave, and V is strictly increasing and strictly convex. Equations (14) and (15) are replaced by

$$U'\left(C_t\right)\left[1+\lambda+\lambda\frac{U''\left(C_t\right)C_t}{U'\left(C_t\right)}\right]=\eta s_t^x\phi\left(s_t^x,s_t^y,d_t^*\right) \tag{17}$$

$$V'\left(N_t\right)\left[1+\lambda+\lambda\frac{V''\left(N_t\right)N_t}{V'\left(N_t\right)}\right]=\eta s_t^x \tag{18}$$

Assuming that the ratios

$$\frac{V''\left(N_t\right)N_t}{V'\left(N_t\right)}\text{ and }\frac{U''\left(C_t\right)C_t}{U'\left(C_t\right)}$$

are roughly constant[2], we can draw the following conclusions:[3]

[1] $s_t^x\phi\left(s_t^x,s_t^y,d_t^*\right)=\varphi\left(s_t^x,1,\frac{1}{d_t^*},s_t^y\right)$, and φ is increasing in its three first arguments and decreasing in its fourth argument.

[2] In fact, we need the weaker condition that the left hand side of (17) to be decreasing in C_t, and the left hand side of (18) to be increasing in N_t.

[3] When considering comparative statics we mean de following: for example, when analyzing a higher shock to the final goods technology we want to analyze the effects on the optimal allocations and policy instruments such that for two states σ_t and σ_t', $s^y\left(\sigma_t\right)>s^y\left(\sigma_t'\right)$, $s^x\left(\sigma_t\right)=s^x\left(\sigma_t'\right)$, $P^{x*}\left(\sigma_t\right)=P^{x*}\left(\sigma_t'\right)$, $P^{m*}\left(\sigma_t\right)=P^{m*}\left(\sigma_t'\right)$ and $G\left(\sigma_t\right)=G\left(\sigma_t'\right)$ given σ_{t-1}.

- Expenditure shocks are completely insured against in the international capital markets.

- Shocks to the intermediate input production function alone, reduce consumption and increase labor.

- Shocks to the final good production function and/or shocks to the terms of trade, d_t^* increase consumption and does not affect labor.

- The Ramsey allocation is independent of the velocity shocks v_t.

- The fraction of firms that must set prices in advance, γ, is irrelevant in the allocation.

It is interesting the conclusion that shocks to the intermediate inputs sector increase labor and reduces consumption. This shocks determine when it is good to export and when it is not. Consider, for instance, an increase in productivity in the intermediate goods sector and assume that goverment consumption does not change. It is easy to see from equation (16) that the trade balance must increase for otherwise feasibility would not hold for the higher shock.

Increases in the final goods technology and increases in the terms of trade increase consumption but does not affect labor; even though consumption and leisure are normal goods, the first remains unchanged when the term of trade or technology in the final goods sector change. Also notice that this shocks have ambiguous effects on the trade balance.

9.1 Decentralization

As explained above, the existence of sticky prices does not expand the set of allocations that can be implemented as equilibria. Now we will discuss particular policies that implement any given allocation. Recall that under the optimal policy $P_t = P_t^S = P_t^F$. Given a competitive equilibrium allocation the conditions that determine the price system associated with it and the government policies that decentralize it are

$$R_{t+1} = \frac{U_{1-N}(t+1)}{U_{1-N}(t)} \left(\frac{1-\tau_t^n}{1-\tau_{t+1}^n} \right) R_{t+2} \frac{s_t^x}{s_{t+1}^x} \left(1-\tau_{t+1,t}^*\right)$$

$$\frac{U_C(t)}{U_{1-N}(t)} = \frac{P_t(1+\tau_t^c)}{W_t(1-\tau_t^n)} \frac{(R_{t+1}+v_t-1)}{v_t}$$

$$z_{t+1,t} = \beta \frac{U_{1-N}(t+1) R_{t+2}}{U_{1-N}(t) R_{t+1}} \frac{W_t (1-\tau_t^n)}{W_{t+1} (1-\tau_{t+1}^n)}$$

$$R_{t+1} = \frac{1}{E_t [z_{t+1,t}]}$$

$$P_t^x = \varepsilon_t P_t^{x*}$$

$$W_t = P_t^x s_t^x$$

$$P_t C_t (1+\tau_t^c) = M_{t-1} \mu_{t-1} \delta_t v_t$$

$$P_t = \frac{\theta}{\theta - 1} W_t \phi_t$$

$$P_t = \frac{\theta}{\theta - 1} E_{t-1} [\omega_t W_t \phi_t]$$

In this model we have some redundant instruments; in particular, we can set any value for the nominal interest rate and labor taxes and being able to implement the Ramsey allocation. Consequently we will focus on equilibria where the Friedman Rule holds, i.e. let $R_{t+1} = 1$, and for which labor taxes are zero for all t and state of the economy. We choose this particular normalization to avoid carrying additional notation. As is known (see Carlstrom and Fuerst, 1995), when the gross nominal interest rate equals one, the cash-in-advance constraint does not bind, and the model can have multiple equilibria. We will disregard this subtlety.

The international capital market constraint becomes

$$\frac{U_{1-N}(t)}{s_t^x} = \frac{U_{1-N}(t+1)}{s_{t+1}^x} \left(1 - \tau_{t+1,t}^*\right) \tag{19}$$

and the rest of the system can be written as follows:

$$\frac{U_C(t)}{U_{1-N}(t)} = (1+\tau_t^c) \frac{\theta}{\theta-1} \phi_t \tag{20}$$

$$U_C(t) C_t = \frac{1}{\mu_t} \beta E_t \left[\frac{U_C(t+1) C_{t+1}}{\delta_{t+1}} \right] \tag{21}$$

$$\frac{C_t (1+\tau_t^c)}{E_{t-1} [C_t (1+\tau_t^c)]} = \frac{\delta_t v_t}{E_{t-1} [\delta_t v_t]} \tag{22}$$

$$s_t^x \phi_t P_t^{x*} \zeta_t = E_{t-1} [\omega_t s_t^x \phi_t P_t^{x*} \zeta_t] \tag{23}$$

Given the Ramsey allocation and the current state, (19) determines the (contingent) tax on foreign assets $\tau_{t+1,t}^*$. Consumption taxes are pinned down from the intratemporal condition (20). Regarding monetary policy, given the allocation and consumption taxes, (22) pins down the unexpected component of the money supply, δ_t. And given this function, equation (21) pins down the expected money growth μ_t that induces the Friedman rule. Finally, (23) is used to solve for the unexpected exchange rate shock ζ_t. Expected changes in the exchange rate are irrelevant.

Notice that, contrary to the Ramsey allocation, the policies that implements it do change with changes in velocity shocks v_t as shows equation (22), thus the unexpected money supply shock depends on the realization of the velocity shock. It must be mentioned, however, that if instead of choosing $R_{t+1} = 1$ we would have chosen some other policy, both in (20) and (21) would have appeared the velocity shocks v_t.

It is interesting to notice that in our model, in order to implement the Ramsey allocation, unexpected monetary and exchange rate policy *must* be managed independently. This conclusion does not hold in standard flexible prices models, where if you decide to manage the money supply, the exchange rate is determined endogenously in the model and cannot be freely manipulated. Or vice versa, if the government runs a particular exchange rate policy, the model endogenously determines the money supply.

As we noted above, neither the optimal allocation nor the policy that implements it depend on the fraction of firms that must set prices in advance. This is, indeed, an remarkable result.

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9.1.1 Example.

CAN WE DO THIS? I'M NOT SURE SINCE A CHANGE IN ONE SHOCK WILL HAVE AN EFFECT IN THE DISTRIBUTION OF TOMORROW'S SHOCK AND THEREFORE, TOMORROW'S ALLOCATION (CONSEQUENTLY, THE EXPECTED VALUE OF ANY FUNCTION WILL ALSO CHANGE.) WE DON'T KNOW HOW DOES IT CHANGE, AND THEREFORE I THINK THAT WE CANNOT DO THE SIMPLE ANALYSIS WE DID IN THE PREVIOUS VERSION OF THE PAPER.

SHOULD WE DO SOME NUMERICAL STUFF INSTEAD?

We will characterize the optimal policy in our example with separable utility from the last section, and analyze how it responds to the different shocks. That is, how the government should manage the monetary and fiscal policy in different scenarios about the shocks hitting the economy. Assume the stochastic processes are such that right hand side of (**??**) is the same for all $\sigma \in \Sigma$. For our analysis, we write it as

$$\frac{U_{1-Nt}}{s_t^x\left(1-\tau_t^n\right)P_t^{x*}}=k \tag{24}$$

Joining equations (24) and (**??**) we get

$$1+\tau^c\left(\sigma_t\right)=\frac{\theta-1}{\theta}\frac{U_{Ct}}{kP_t^{x*}s_t^x\phi_t} \tag{25}$$

Introducing the last equation into (**??**) the CIA equation becomes

$$\frac{U_{Ct}C_t}{P_t^{x*}s_t^x\phi_t}A=\delta\left(\sigma_t\right) \tag{26}$$

where A is constant as of time t. Now

$$\frac{\partial U_{Ct}C_t}{\partial C_t}=U_{Ct}\left[1+\frac{U_{CCt}C_t}{U_{Ct}}\right]$$

thus, $\frac{\partial U_{Ct}C_t}{\partial C_t}>(<)\ 0$ as long as $\left|\frac{U_{CCt}C_t}{U_{Ct}}\right|<(>)\ 1$. The optimal policy can be characterized as follows:

- **Shocks to the final goods technology,** $s^y\left(\sigma_t\right)$: consumption increases and labor remains constant. From (24) labor tax remains constant. From equation (23), the nominal exchange rate ζ_t goes up since ϕ_t goes down. Since U_{Ct} decreases but ϕ_t also decreases, the consumption tax rate can go up or down. Finally, if $\left|\frac{U_{CCt}C_t}{U_{Ct}}\right|<1$, then from (26) $\delta\left(\sigma_t\right)$ must go up. On the other hand, if $\left|\frac{U_{CCt}C_t}{U_{Ct}}\right|>1$, $\delta\left(\sigma_t\right)$ can go up or down. However, a decrease in $\delta\left(\sigma_t\right)$ can be considered as a strange case, since usually we expect the change in $\tau^c\left(\sigma_t\right)$ to be smaller than the change in C_t. Therefore, in general, expansionary monetary policy must follow. Ireland (1996) also obtains that expansionary monetary policy must be performed when shocks to final goods technology hit the economy, but in a model without distorting taxes.

- **Shocks to the home produced intermediate good technology,** $s^x\left(\sigma_t\right)$: labor increases and consumption decreases. >From (**??**) we must increase $\tau^n\left(\sigma_t\right)$ so as to keep the left hand side constant. Since $\phi_t s_t^x$ is increasing in s_t^x, we need to decrease the nominal exchange rate, $\zeta\left(\sigma_t\right)$. As before, we cannot determine how the consumption tax rate moves, since U_{Ct} increases but $s_t^x\phi_t$ also increases. If $\left|\frac{U_{CCt}C_t}{U_{Ct}}\right|>1$, then from (26) $\delta(\sigma_t)$ must go down. On the other hand, if $\left|\frac{U_{CCt}C_t}{U_{Ct}}\right| < 1$, $\delta(\sigma_t)$ can go up or down. However, using the same reasoning as above, we can consider an increase in $\delta(\sigma_t)$ as rare. Therefore, in general, contractionary monetary policy must follow.

- **Shocks to the terms of trade,** $d^*(\sigma_t)$: consumption increases and labor remains constant. The labor tax rate goes up or down as P_t^{x*} increases or decreases respectively. As with the final goods technology shocks, $\tau^c(\sigma_t)$ and $\delta(\sigma_t)$ can go up or down depending on the parameters, however as before, the more natural case is to increase the money supply. To derive the optimal exchange rate policy, note that $P_t^{x*}\phi_t = \varphi\left(P_t^{x*}\frac{P_t^{x*}}{s_t^x}, \frac{P_t^{m*}}{s_t^x}, s_t^y\right)$ so, if the increase in d_t^* is driven by an increase in P_t^{x*}, then $P_t^{x*}\phi_t$ increases and the exchange rate must decrease, while if the shock is caused by a decline in P_t^{m*}, $P_t^{x*}\phi_t$ increases and a revaluation must follow.

10 Concluding Remarks

In this paper we developed a simple model with varying degrees of price setting frictions and sought to characterize optimal fiscal, monetary and exchange rate policy following the dynamic Ramsey optimal taxation literature, and how those allocations and policy instruments should respond to different shocks hitting the economy.

A structural parameter, taken as given, is the fraction of firms that cannot set state contingent prices and are constrained to chose its prices one period in advance. A remarkable result is that neither the optimal allocation nor the optimal policies that implement it depends on that fundamental parameter. We also showed that, contrary to what is commonly thought, price stickiness does not expand the set of implementable allocations. In other words, the optimal allocation under sticky prices is the same as the optimal allocation under flexible prices. Furthermore, in order to implement the Ramsey allocation, the government must run active and independent monetary and exchange rate policy.

Finally, we showed that the cyclical properties of optimal short run monetary policy depend on the nature of the shock driving the cycle. Shocks to the final goods (nontradable) technology call upon expansionary monetary policy and a devaluation of the exchange rate, shocks to the intermediate good (tradable) technology require contractionary monetary policy and a revaluation of the exchange rate, and finally, shocks to the term of trade asks for expansionary monetary policy, and the exchange rate depends on the composition of the shock: if it is driven by an increase in the price of exportable goods, the exchange rate must be revalued while if it is caused by a decrease in the price of importable goods, the exchange rate must be devalued.

Appendix

Proof of proposition 1: Cost minimization implies that $\frac{q_t^x}{n_t^y}$ and $\frac{q_t^m}{n_t^y}$ are function of s_t^y and the relative prices $\frac{P_t^x}{W_t}$, $\frac{P_t^m}{W_t}$. The zero profit condition in the intermediate goods sector and PPP imply that $\frac{W_t}{P_t^x} = s_t^x$ and $\frac{W_t}{P_t^m} = \frac{W_t}{P_t^x}\frac{P_t^x}{P_t^m} = s_t^x d_t^*$. Thus, the ratio of input demands are, in equilibrium, only function of s_t^x, s_t^y, and d_t^* and equal for all firms. Since $y_t(i) = F\left(n_t^y(i), q_t^x(i), q_t^m(i), s_t^y\right) = n_t^y(i) F\left(1, \frac{q_t^x}{n_t^y}(i), \frac{q_t^m}{n_t^y}(i), s_t^y\right)$, in any equilibrium

$$y_t(i) = n_t^y(i) f\left(s_t^x, s_t^y, d_t^*\right) \tag{A1}$$

where $f\left(s_t^x, s_t^y, d_t^*\right) \equiv F\left(1, \frac{q_t^x}{n_t^y}(i), \frac{q_t^m}{n_t^y}(i), s_t^y\right)$ is independent of i.

Given symmetry, each type of firms make the same choices (either flexible of sticky.) Thus, the trade balance in terms of the exportable input can be written as

$$TB_t = Q_t^x - q_t^x - \frac{P_t^m}{P_t^x} q_t^m = Q_t^x - \gamma\left(q_t^{xS} + \frac{q_t^{mS}}{d_t^*}\right) - (1-\gamma)\left(q_t^{xF} + \frac{q_t^{mF}}{d_t^*}\right)$$

Using the definition of Q_t^x,

$$\int_0^1 n_t(i)\, di = \gamma n_t^{yS} + (1-\gamma) n_t^{yF} = N_t - \frac{Q_t^x}{s_t^x}$$

we can write the trade balance condition as,

$$\gamma n_t^{yS}\left(1 + \frac{q_t^{xS}}{n_t^{yS}}\frac{1}{s_t^x} + \frac{q_t^{mS}}{n_t^{yS}}\frac{1}{s_t^x d_t^*}\right) + (1-\gamma) n_t^{yF}\left(1 + \frac{q_t^{xF}}{n_t^{yF}}\frac{1}{s_t^x} + \frac{q_t^{mF}}{n_t^{yF}}\frac{1}{s_t^x d_t^*}\right) = \left[N_t - \frac{TB_t}{s_t^x}\right]$$

Now, using $(A1)$

$$\gamma y_t^S \frac{\left(1 + \frac{q_t^{xS}}{n_t^{yS}}\frac{1}{s_t^x} + \frac{q_t^{xS}}{n_t^{yS}}\frac{1}{s_t^x d_t^*}\right)}{f\left(s_t^x, s_t^y, d_t^*\right)} + (1-\gamma) y_t^F \frac{\left(1 + \frac{q_t^{xF}}{n_t^{yF}}\frac{1}{s_t^x} + \frac{q_t^{mF}}{n_t^{yF}}\frac{1}{s_t^x d_t^*}\right)}{f\left(s_t^x, s_t^y, d_t^*\right)} = \left[N_t - \frac{TB_t}{s_t^x}\right] \tag{A2}$$

But from the definition of the cost function we obtain,

$$W n_t^y + P_t^x q_t^x + P_t^m q_t^m = W_t \phi_t y_t$$

therefore,

$$\begin{aligned}\phi_t &= \frac{W_t n_t^y + P_t^x q_t^x + P_t^m q_t^m}{W_t n_t^y f\left(s_t^x, s_t^m, d_t^*\right)} = \frac{P_t^x\left[\frac{W_t}{P_t^x} + \frac{q_t^x}{n_t^y} + \frac{P_t^m}{P_t^x}\frac{q_t^x}{n_t^y}\right]}{W_t f\left(s_t^x, s_t^m, d_t^*\right)} \\ &= \frac{\left[s_t^x + \frac{q_t^x}{n_t^y} + \frac{P_t^m}{P_t^x}\frac{q_t^x}{n_t^y}\right]}{s_t^x f\left(s_t^x, s_t^m, d_t^*\right)} = \frac{\left[1 + \frac{q_t^x}{n_t^y}\frac{1}{s_t^x} + \frac{q_t^x}{n_t^y}\frac{1}{s_t^x d_t^*}\right]}{f\left(s_t^x, s_t^m, d_t^*\right)}\end{aligned}$$

where we have used (5). Introducing the last equation into $(A2)$ completes the proof.■

Implementability condition: The intertemporal budget constraint is

$$E_0\sum_{t=0}^{\infty}\mathcal{Z}_{t+1}\left\{P_tC_t\left(1+\tau_t^c\right) - W_tN_t\left(1-\tau_t^n\right) - D_t\left(1-\tau_t^d\right) - T_t + M_t\left(\frac{\mathcal{Z}_t}{\mathcal{Z}_{t+1}} - 1\right)\right\} = 0$$

Using the cash in advance constraint at equality (i.e. $P_tC_t\left(1+\tau_t^c\right) = M_tv_t$) we obtain

$$E_0\sum_{t=0}^{\infty}\left\{\frac{P_tC_t\left(1+\tau_t^c\right)}{v_t}\left[\mathcal{Z}_t - \left(1-v_t\right)\mathcal{Z}_{t+1}\right] - \mathcal{Z}_{t+1}\left[W_tN_t\left(1-\tau_t^n\right) + D_t\left(1-\tau_t^d\right) + T_t\right]\right\} = 0$$

Now, from the household's first order conditions we find that

$$\mathcal{Z}_t = \beta^t\frac{U_C\left(t\right)R_{t+1}}{P_t\left(1+\tau_t^c\right)\left(R_{t+1}+v_t-1\right)} \times \frac{P_0\left(1+\tau_0^c\right)\left(R_1+v_0-1\right)}{U_C\left(0\right)R_1}$$

then

$$E_0\sum_{t=0}^{\infty}\{\frac{P_tC_t\left(1+\tau_t^c\right)}{v_t}\left[\frac{\beta^tU_{Ct}}{P_t\left(1+\tau_t^c\right)}\left(\frac{v_tR_{t+1}}{R_{t+1}+v_t-1}\right) - \left(1-v_t\right)\frac{\beta^{t+1}U_{Ct+1}}{P_{t+1}\left(1+\tau_{t+1}^c\right)}\left(\frac{v_{t+1}R_{t+2}}{R_{t+2}+v_{t+1}-1}\right)\right]$$
$$\frac{\beta^{t+1}U_{Ct+1}}{P_{t+1}\left(1+\tau_{t+1}^c\right)}\left(\frac{v_{t+1}R_{t+2}}{R_{t+2}+v_{t+1}-1}\right)\left[W_tN_t\left(1-\tau_t^n\right) + D_t\left(1-\tau_t^d\right) + T_t\right]\} = 0$$

>From the law of iterated expectations, the intertemporal Euler equation, and the intratemporal condition we can show that the last condition becomes

$$E_0\sum_{t=0}^{\infty}\beta^t\left[U_{Ct}C_t - U_{1-Nt}\left(N_t + \frac{D_t}{W_t}\frac{\left(1-\tau_t^d\right)}{\left(1-\tau_t^n\right)} + \frac{T_t}{W_t\left(1-\tau_t^n\right)}\right)\right] = 0 \quad (*)$$

Finally, it rests to express the term $\frac{D_t}{W_t}$ in term of the allocations. To this end, note that total dividends are

$$D_t = \int_0^1 d_t\left(i\right)di = \gamma y_t^S\left(P_t^S - W_t\phi_t\right) + \left(1-\gamma\right)y_t^F\left(P_t^F - W_t\phi_t\right)$$

Dividing by W_t and using the pricing rule for flexible firms we find that

$$\frac{D_t}{W_t} = \gamma y_t^S\left(\frac{P_t^S}{W_t} - \phi_t\right) + \frac{\left(1-\gamma\right)y_t^F\phi_t}{\theta - 1} \tag{**}$$

As an intermediate step, we will show that for all $t \geq 1$,

$$E_0\left[\frac{U_{1-Nt}\left(1-\tau_t^d\right)}{\left(1-\tau_t^n\right)}\frac{D_t}{W_t}\right] = E_0\left[\frac{U_{1-Nt}\left(1-\tau_t^d\right)}{\left(1-\tau_t^n\right)\left(\theta-1\right)}\left[N_t - \frac{TB_t}{s_t^x}\right]\right]$$

Manipulating the first order condition of the sticky firms and using the law of iterated expectations we find that for all $t \geq 1$.

$$E_0\left(\frac{U_{Ct}(1-\tau_t^d)}{R_{t+1}\left(1+\tau_t^c\right)}y_t^S\frac{w_t}{w_t^S}\right) = \frac{\theta}{\theta-1}E_0\left(\frac{U_{Ct}(1-\tau_t^d)}{R_{t+1}\left(1+\tau_t^c\right)}y_t^S w_t\phi_t\right)$$

Use the intratemporal condition of the households' problem to write the above equation as

$$E_0\left(\frac{U_{1-Nt}(1-\tau_t^d)}{(1-\tau_t^n)}\frac{y_t^S}{w_t^S}\right) = \frac{\theta}{\theta-1}E_0\left(\frac{U_{1-Nt}(1-\tau_t^d)}{(1-\tau_t^n)}y_t^S\phi_t\right) \tag{A3}$$

>From (**) we obtain

$$E_0\left[U_{1-Nt}\frac{D_t(1-\tau_t^d)}{W_t\left(1-\tau_t^n\right)}\right] = E_0\left[\frac{U_{1-Nt}(1-\tau_t^d)}{(1-\tau_t^n)}\left(\gamma y_t^S\left(\frac{1}{w_t^S} - \phi_t\right) + \left(1-\gamma\right)y_t^F\phi_t\frac{1}{\theta-1}\right)\right]$$

Using (A3), proposition 1 and rearranging we obtain the result.

Now, plugging that result into (*) we find

$$U_{C0}C_0 - U_{1-N0}\left[N_0 + \left(\frac{1-\tau_0^d}{1-\tau_0^n}\right)\left(\gamma y_0^S\left(\frac{P_0^S}{W_0} - \phi_0\right) + \frac{\left(1-\gamma\right)y_0^F\phi_0}{\theta-1}\right) + \frac{T_0}{W_0\left(1-\tau_0^n\right)}\right] +$$
$$E_0\sum_{t=1}^{\infty}\beta^t\left(U_{Ct}C_t - U_{1-N_t}\left[N_t + \left(N_t - \frac{TB_t}{s_t^x}\right)\frac{\left(1-\tau_t^d\right)}{\left(1-\tau_t^n\right)\left(\theta-1\right)} + \frac{T_t}{W_t\left(1-\tau_t^n\right)}\right]\right) = 0$$

Recall that

$$
\begin{aligned}
\frac{D_0}{W_0} &= \gamma y_0^S \left(\frac{P_0^S}{W_0} - \phi_0 \right) + \frac{(1-\gamma)\, y_0^F \phi_0}{\theta - 1} \\
\frac{D_0}{W_0} &= \phi_0 \left[\gamma y_0^S \left(\frac{P_0^S}{W_0 \phi_0} - 1 \right) + \frac{(1-\gamma)\, y_0^F}{\theta - 1} \right]
\end{aligned}
$$

Introducing the pricing rule of flexible firms at $t = 0$, $W_0 \phi_0 = \left(\frac{\theta-1}{\theta}\right) P_0^F$, in the last condition we find

$$
\frac{D_0}{W_0} = \left(\frac{\theta}{\theta - 1} \right) \frac{\phi_0}{P_0^F} \gamma y_0^S \left(P_0^S - P_0^F \right) + \frac{\phi_0 \left(\gamma y_0^S + (1-\gamma)\, y_0^F \right)}{\theta - 1}
$$

but $\left(\frac{\theta}{\theta-1} \right) \frac{\phi_0}{P_0^F} = \frac{1}{W_0}$, and using proposition 1 on the last term, we obtain that

$$
\frac{D_0}{W_0} = \gamma y_0^S \left(\frac{P_0^S}{W_0} - \frac{P_0^F}{W_0} \right) + \frac{1}{\theta - 1} \left[N_0 - \frac{TB_0}{s_0^x} \right]
$$

therefore, the implementability condition can be written as

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left(U_{Ct} C_t - U_{1-N_t} \left[N_t + \left(N_t - \frac{TB_t}{s_t^x} \right) \frac{\left(1 - \tau_t^d\right)}{\left(1 - \tau_t^n\right)(\theta - 1)} + \frac{T_t}{W_t \left(1 - \tau_t^n\right)} \right] \right) -
$$

$$
\frac{U_{1-N_0} \gamma y_0^S}{W_0} \left(P_0^S - P_0^F \right) \left(\frac{1 - \tau_0^d}{1 - \tau_0^n} \right) + \frac{T_0}{W_0 \left(1 - \tau_0^n\right)} = 0
$$

and we are done.

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