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Optimal Fiscal Policy in Overlapping Generations Models

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ABSTRACT

This paper analyzes optimal taxation problems in overlapping generation economies with production where agents live I periods. The primal approach is used to characterize the optimal …scal policy in steady state and along the transition path to some steady state. The key …ndings is that under certain assumptions (complete set of instruments and separability of the utility function) capital taxes are zero along the transition path to the steady state after two periods. This result is an equivalent version of Chamley (1986) with OG. With additional assumptions it can be shown that non-separable utility functions satisfy the zero capital taxes result in steady state.

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1. Introduction

This paper analyzes the optimal …scal policy in overlapping generation economies with production where agents live I periods. The primal approach is used to characterize the optimal taxes in steady state and along the transition path to some steady state. The basic idea is to transform the government problem of choosing the optimal taxes, into a simple programming problem of choosing allocations subject to some constraints.

The key …ndings is that if the set of taxes is complete and the utility is homothetic and separable, then capital taxes are zero along the transition path to the steady state after two periods. This result is an equivalent version of Chamley (1986) with overlapping generations. With additional assumptions in the discount factor and endowment of e¢ciency units, it can be shown that non-separable utility functions satisfy the zero capital taxes result in steady state, but not during the transition path. This is due to the fact that from the government point of view, under this assumptions, the overlapping generation economy is equivalent to an in…nitely lived economy.

Optimal taxation literature tries to answer how taxes should be set in an e¢cient way (in a constrained e¢cient sense). For this purpose the government is introduced as an active agent in the economy that chooses optimally the …scal policy according to some objective function. In the last decade the optimal taxation literature, that started with Ramsey (1927) seminal paper, has take over analysis of policy design in macroeconomic models. The primal approach developed by Atkinson and Stiglitz (1980) for static economies and for Lucas and Stokey (1983) for dynamic economies has been very successful for the analysis of the optimal …scal policy. Most of the recent literature has focused in the optimal …scal design in in…nitely lived economies, see Chari et al. (1991, 1994 and 1998), Jones, Manuelli and Rossi (1993), Aiyagari (1995) and others.

The …rst paper that introduces the optimal taxation problem in a two period overlapping generation economy is Pestieau (1974). He analyzes the optimal …nancing of a public investment. An important contribution in the framework of the OG model is Atkinson and Sandmo (1980). They study the optimal …nancing of non-productive public consumption expenditure. In this environment if the government has access to a full set of tax instruments, that include lump-sum taxes, the optimal policy can achieve full e¢cient allocations. Both papers limit their analysis to steady state solutions. More recently, Escolano (1992) uses an optimal taxation approach to quantify the ine¢ciencies of the …scal system of the United States economy. He shows that under certain restrictions in the policy instruments, the …scal system can not be consider as suboptimal, so positive taxes on capital may be optimal. Atkeson et al. (1999) in a two period OG also show that capital taxes are zero in steady state.

The scope of this paper is to provide a general framework for analyzing optimal taxation issues in large economies where agents diverge in their age. As a …rst step, this paper proofs some results in characterizing the optimal …scal policy specially along the transition path to some steady state.

The paper is organized as follows. In section 2 we introduce the basic environment and de…ne a competitive equilibrium with distortions. Section 3 describes the government problem of choosing the optimal …scal policy, and section 4 introduces the primal approach for solving optimal taxation problems. Section 5 derives the zero capital tax for the transition path. Section 6 derives the basic results for the steady state, and section 7 introduces the optimal …scal problem in an environment with incomplete set taxes. Finally, section 8 concludes.

2. Environment

The model is a standard production economy with two goods, consumption-capital good and leisure. Agents live I periods and each cohort is populated by a continuum of households. For simplicity we assume that population is stationary and its total size is normalize to one.

There is a representative firm that produces aggregate output Y_t using a constant returns to scale production function $F(K_t; L_t)$, using capital K_t and labor L_t as primary inputs (measured in efficiency units). This technology is constant returns to scale, monotone, strictly concave and satisfies the Inada conditions. Capital depreciates each period at a constant rate $\delta \in (0; 1)$; there is no exogenous technological change. Competitive factor markets ensure that factors are paid to its marginal product,

(1) $$r_t = F_{K_t} - \delta$$

(2) $$w_t = F_{L_t}$$

Households in this economy have standard preferences defined over consumption and leisure and are represented by a time separable utility function:

(3) $$\sum_{i=1}^{I} \beta^{i-1} U(c_t^i; \ell_t^i); \qquad \forall t$$

where c_t^i and ℓ_t^i denote consumption and leisure of a household of age i at time t, and $\beta > 0$ is the subjective discount factor. The utility function $u(\cdot)$ is C^2; strictly increasing in consumption, decreasing in labor, strictly concave and satisfies Inada conditions. The agents are endowed at each period of a unit of divisible time and an age specific vector of efficiency units $\varepsilon = (\varepsilon^1 ::: ; \varepsilon^I)$; that is assumed time invariant. Therefore at each period agents will decide how much to consume, save, and how many effective units of labor supply to firms. Households face a sequence of budget constraints:

$$(4) \qquad a_t^1 = a_t^{I+1} = 0; \quad \forall t$$

$$(5) \qquad (1 + \tau_{c_{i,t}})c_t^i + a_{t+1}^{i+1} = (1 - \tau_{\ell_{i,t}})w_t \varepsilon^i \ell_t^i + (1 + r_t(1 - \theta_{i,t}))a_t^i \qquad i \in [1, I]; \quad \forall t$$

the first equation imposes that agents born and die with zero wealth, so agents can not die indebted nor leave bequest. Let r_t be the net return of asset holdings and w_t is the wage rate per efficiency units of labor. Households can accumulate wealth a_{t+1}^i in two forms, lending capital to firms and buying government debt of one period maturity. Let $\tau_{c_{i,t}}$; $\theta_{k_{i,t}}$ and $\tau_{\ell_{i,t}}$ be an age specific consumption capital and labor tax respectively.[1] I assume that the government can perfectly discriminate agents by age on their tax payments, and can monitor any side trade done by agents. Therefore, I rule out any possibility of collusion among agents on their investing decisions. Latter on we will see that dropping this assumption and imposing the same taxes to all individuals will have important implications in the optimal policy. Intertemporal trade between generations is allowed. The capital stock and debt at period $t = 0$ is owned by the initial generations.

The government in this economy finances an exogenous sequence of public expenditure $\{G_t\}_{t=0}^{\infty}$, and redistributes resources between generations using taxes and debt. The government budget constraint is:

$$(6) \qquad \tau_{c_{i,t}} \sum_{i=1}^{I} c_t^i + w_t \sum_{i=1}^{I} \tau_{\ell_{i,t}} \ell_t^i + r_t \sum_{i=1}^{I} \theta_{i,t} k_t^i + B_{t+1} = G_t + R_t B_t; \qquad \forall t$$

[1] In order to rule out corner solutions on the investment decisions of households, it is necessary impose a tax on the return of debt. Otherwise agents with low capital tax would invest all their savings in capital and none in debt, and the other way around. With this implicit tax on the return of debt the arbitrage condition will holds for all agents. Abusing notation I will assume that net returns on capital and debt are the same for each generation, but different across generations.

On the left hand side we have government revenues from age specific taxes on consumption, labor and capital and debt; on the right hand side we have government expenditure in public consumption and payroll on debt. The government expenditure is assumed to be a fixed fraction of output, $G_t = \bar{g} Y_t$. The economy resource constraint is given by:

$$\sum_{i=1}^{I} c_t^i + K_{t+1} + G_t = F(K_t, L_t) + (1 - \delta) K_t, \qquad \forall t \tag{7}$$

Definition 1. Let $\pi = \{\{\tau_{c_{i,t}}, \tau_{\ell_{i,t}}, \theta_{i,t}\}_{i=1}^{I}\}_{t=0}^{\infty}$ be the infinite sequence of policies, and let Ψ be the set of all feasible policies. We denote by $\Pi \subset \Psi$ the subset of policies for which competitive equilibrium exists.

Definition 2. Given a policy rule π, and public consumption $\bar{g}$, a competitive equilibrium for this economy is a sequence of individual allocations $x = \{\{c_t^i, \ell_t^i\}_{i=1}^{I}, \{a_{t+1}^i\}_{i=2}^{I}\}_{t=0}^{\infty}$ and prices $\{r_t, w_t, \{R_t^i\}_{i=2}^{I}\}_{t=0}^{\infty}$, such that, the consumers maximize (3) subject to (4) and (5). In the production sector (1) and (2) holds. Markets clear and feasibility is satisfied.

Notice that we have not imposed the government budget constraint in the definition of equilibrium. If all the equilibrium conditions are satisfied but the government budget constraint, then Walras law ensures that this constraint also is satisfied.

Given the assumptions of concavity and monotonicity on the functional forms the first-order conditions are sufficient to characterize an interior solution.

3. Ramsey Equilibrium

Once we have defined a competitive equilibrium we want to focus our attention on the government problem, which has to choose optimally a policy π that maximize society's welfare (the utility of all generations), subject to constraints. This constraints imply, first that the government budget constraint has to be satis…ed in present value. Second, the additional constraint imposes that the optimal policy constitutes a competitive equilibrium with distortionary taxes. I assume that the government has access to a commitment technology that allows it in period 0; to bind itself choosing a one shot sequence of policies. This kind of commitment technology has been proved that might cause time-consistency problems, as it has been show by Stroz (1957), Kydland and Prescott (1977) and Chari et al. (1986). This is because the government may have incentives to deviate from the optimal policy once it has been announce. When the government does not have access to a commitment technology, it is necessary to design reputation and credible mechanisms to avoid time consistency problems, a reference in this literature are Chari and Kehoe (1990 and 93), Kotliko¤ (1988) and Tabellini (1991). Alternatively Klein and Rios-Rull (1999) have solved the problem with partial commitment. They …nd that the optimal capital taxes are very similar to the observed in the data.

The Ramsey equilibrium concept treats the government as a dynamic player that takes into account that changes in policies will a¤ect prices, allocations and hence government revenues. Notice that in this type of environment players time horizon does not coincide, while the government is an in…nitely lived player, households live I periods. Given that the government has to foresight agents' behavior is useful to describe allocations and prices using rules.[2] The reaction function of each agent is useful to understand how individual decisions change under changes in the policy.

[2] We do not make any assumption of uniqueness or continuity of the allocation rule and the price rule, allowing those be a correspondence. A reference for anomalies in models with taxes and externalities is Kehoe et al. (1992).

Definition 3. An allocation rule $x_\pi = x(\pi)$ is a sequence of functions that map policies $\pi \in \Pi$ into allocations x. A price rule $r_\pi = r(\pi)$, $w_\pi = w(\pi)$, $R_\pi = R(\pi)$ is a sequence of functions that map policies $\pi \in \Pi$ into prices r_π, w_π, R_π.

If we do not introduce any restriction on the optimal policies, the government has incentives to tax heavily the initial stock of capital. To avoid these problems we assume that the government takes as given the initial taxes on capital stock $\{\theta_{i,0}\}_{i=1}^{\omega}$, and consumption $\{\tau_{ci,0}\}_{i=1}^{\omega}$.

The government is benevolent and values the utility of all households in the economy from period 0 onwards. Its objective function will be the weighted sum of all generations. Therefore the government assigns a non negative weight to all generations. The infinite sequence is assumed to be bounded above by a positive constant $\Gamma < \infty$. Formally ω_t is the weight that the government assigns to the generation born at time t,

$$W(\{c_t^i, \ell_t^i\}) = \sum_{t=0} \sum_{i=1} \omega_{t+1-i} \left[\beta^{i-1} U(c_t^i, \ell_t^i) \right] \tag{8}$$

$$\sum_{t=-(I-1)} \omega_t \leq \Gamma \tag{9}$$

A particular case of this objective function that will be used latter on to derive some important results, imposes an exponential decreasing sequence of weight to all agents. This is important to characterize the optimal fiscal policy in steady state. Given the objective function we can proceed to describe the government problem.

Definition 4. Given an exogenous sequence of public expenditure $\{G_t\}_{t=0}^{\infty}$, a Ramsey equilibrium is a policy $\hat{\pi}$, an allocation rule $x_{\hat{\pi}}$ and a price rule $r_{\hat{\pi}}, w_{\hat{\pi}}, R_{\hat{\pi}}$ that satisfies:

(i) The policy π^* solves:

$$\max_{\pi} \sum_{t=0}^{\infty} \sum_{i=1}^{I} \omega_{t+1-i} \left[\beta^{i-1} U\left(c_t^i(\pi), \ell_t^i(\pi) \right) \right] \tag{10}$$

$$\tau_{c_{i,t}} \sum_{i=1}^{I} c_{i,t}(\pi) + w_t(\pi) \sum_{i=1}^{I} \tau_{\ell_{i,t}} \ell_{i,t}(\pi) + r_t(\pi) \sum_{i=1}^{I} \theta_{i,t} k_{i,t}(\pi) + B_{t+1}(\pi) = G_t + R_t(\pi) B_t(\pi); \quad \forall t \tag{11}$$

given allocations $x(\pi)$ and prices $r(\pi); w(\pi); R(\pi)$:

(ii) The policy π^*; the allocation rule $x(\pi)$ and the price rule $r(\pi); w(\pi); R(\pi)$ belong to the subset of policies that constitute a competitive equilibrium, $\pi^* \in \Pi$:

The concept of Ramsey equilibrium is equivalent to a Nash equilibrium, where the strategies for the government are the policies $\pi^* \in \Pi$; and households chose $x(\pi^*)$ taking as given competitive prices and the fiscal policy.

4. Primal Approach

We will use the primal approach to solve the government problem. The base line is to define a problem of choosing efficient allocations subject to some constraints that restrict the allocations to be supported as a competitive equilibrium with distortionary taxes. These constraints are given by feasibility and the implementability constraint. The implementability constraint takes into account that changes in the policy will affect agents' decisions, and is constructed by substituting the first-order conditions of the households and firms' problem in consumers budget constraints. Therefore all constraints depend on allocations. The following two propositions show equivalence between this problem and the standard Ramsey equilibrium problem.

Proposition 1. The allocations $x = \{\{c_t^i; \ell_t^i\}_{i=1}^{I}; \{a_{t+1}^i\}_{i=2}^{I}\}_{t=0}^{\infty}$ in a competitive equilibrium satisfy the resource constraint, and an implementability for each generation. The implementability constraint for the newborn generations,

$$\sum_{i=1}^{I} \beta^{i-1}\left(c^i_{t+i-1}U_{c^i_{t+i-1}} + \ell^i_{t+i-1}U_{\ell^i_{t+i-1}} \right) = 0; \qquad t \geq 0 \tag{12}$$

and for the initial old generations $s \in [2, I]$ (the s term will denote the initial generations) at $t = 0$ the implementability constraint has a shorter life-span and the initial endowment of wealth in capital and debt appears on the right hand side, together with the consumption taxes at time zero.

$$\sum_{i=s}^{I} \beta^{i-s}\left(c^i_{i-s}U_{c^i_{i-s}} + \ell^i_{i-s}U_{\ell^i_{i-s}} \right) = U_{c^s_0} \cdot a^s_0; \quad s = 2, \dots, I; \tag{13}$$

Proof. Is straightforward to see that any competitive equilibrium by definition satisfies the resource constraint. To derive the implementability constraint we find households first-order conditions with respect to $c^i_t, \ell^i_t, a^i_{t+1}$. Assuming an interior solution and being α_t the Lagrange multiplier of the intertemporal budget constraint we have:

$$\beta^i U_{c^i_t} = \alpha_t(1 + \tau_{c_{i,t}}); \qquad \forall t, i \tag{14}$$

$$\beta^i U_{\ell^i_t} = -\alpha_t(1 - \tau_{\ell_{i,t}})w_t; \qquad \forall t, i \tag{15}$$

and with respect to $a_{i+1,t+1}$:

$$\alpha_t = \alpha_{t+1}(1 + r_{t+1}(1 - \theta_{i+1,t+1})); \qquad \forall t, i \tag{16}$$

to derive the implementability constraint we have to multiply (14), (15) with its respective control variable and then add them up, we substitute households budget constraint and using (16) we can eliminate the asset holdings. The resulting expression is the implementability constraint for the new born. The initial old at $t = 0$ will have a wealth endowment, that appears on the right hand side of the implementability constraint. ■

In the standard representative agent economies the government only faces one implementability constraint, due to the fact that there is only one agent in the economy that lives infinite periods. In this case, there are infinite agents that live a finite number of periods, this implies that all agents have their implementability constraint that reflects that changes in the policy will affect their optimal decisions.

Proposition 2. Given the initial allocations and the initial policies, if there is a sequence of allocations that satisfies feasibility and the implementability constraint of all agents we can construct a sequence of policies and prices, that together with the allocations and the price system constitute a competitive equilibrium with distortionary taxes.

Proof. The supporting prices are determine by the firms first-order conditions:

$$r_t = F_{K_t} - \delta; \qquad \forall t; \tag{17}$$

$$w_t = F_{L_t}; \qquad \forall t; \tag{18}$$

The optimal specific taxes from $t \geq 0$ can be recovered by using the first-order conditions of the consumers problem and its budget constraint:

$$\frac{U_{c_t^i}}{U_{\ell_t^i}} = -\frac{(1+\tau_{ci;t})}{(1-\tau_{\ell_{i;t}}) \cdot w_t}; \qquad \forall t; i \tag{19}$$

$$\frac{U_{c_t^i}}{(1+\tau_{c_{i;t}})} = \beta \frac{U_{c_{t+1}^{i+1}}}{(1+\tau_{c_{i+1;t+1}})}(1 + r_{t+1}(1-\theta_{i+1;t+1})); \qquad \forall t; i \tag{20}$$

substituting the equilibrium prices $\{r_t; w_t; \{R_{i;t}\}_{i=1}^{I}\}_{t=0}^{\infty}$ and the optimal allocations x constitute a system of equations from were we can obtain the optimal policy. Notice that if feasibility and the implementability constraint (households' first-order conditions and budget constraint) are satisfied, then by Walras law the government budget constraint is also satisfied. From the consumer budget constraint we obtain the asset holding consistent with prices and taxes. The difference between the aggregate level of assets and the capital stock gives the aggregate level of debt. ■

The optimal policy associated with the Ramsey allocations is not unique, the decentralization of the Ramsey allocation problem is conditioned by set of instruments that are available for the government.

Corollary 1. Given an exogenous sequence of public expenditure $\{\bar{g}_t\}_{t=0}^{\infty}$; the initial distribution of wealth $a_{i,0}$; if $\pi = \{\{\tau_{c_{i,t}}, \tau_{\ell_{i,t}}, \theta_{i,t}\}_{i=1}^{I}\}_{t=0}^{\infty}$ is the optimal policy associated to an efficient allocation $x = \{\{c_t^i, \ell_t^i\}_{i=1}^{I}, \{a_{t+1}^i\}_{i=2}^{I}\}_{t=0}^{\infty}$; then there exists some $\pi' = \{\{\tau'_{c_{i,t}}, \tau'_{\ell_{i,t}}, \theta'_{i,t}\}_{i=1}^{I}\}_{t=0}^{\infty}$ that support the same allocation.

In this case we have redundancy of instruments, because the number of equations at each period is $3 \times I$ and the number of instruments is $4 \times I$. The redundancy of instruments depends on the number of equations that need to satisfy the equilibrium conditions and the number of fiscal instruments. The primal approach implies implement the wedges between the marginal rates of substitution and marginal rates of transformation, but it does not prescribe any particular type of instruments. This implies that the optimal policy can be supported as a competitive equilibrium under a variety of tax schemes. The unique requirement to decentralize the economy is have a complete set of instruments, that means that there are enough instruments to equate all wedges. We will see later on the implications of relaxing this assumption. For simplicity, the analysis will be restricted to optimal policies where only taxes on production factors $\pi = \{\{\tau_{\ell_{i,t}}, \theta_{i,t}\}_{i=1}^{I}\}_{t=0}^{\infty}$ and debt B_{t+1} are available.[3]

Proposition 3. The allocation on the Ramsey equilibrium (RE) solves the Ramsey allocation problem (RAP):

$$\max_{\{\{c_t^i, \ell_t^i\}_{i=1}^{I}, \{a_{t+1}^i\}_{i=2}^{I}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{i=1}^{I} \omega_{t+1-i} \left[\beta^{i-1} U(c_t^i, \ell_t^i) \right] \tag{21}$$

$$\sum_{i=1}^{I} c_{i,t} + K_{t+1} + G_t = F(K_t, L_t) + (1-\delta)K_t, \qquad \forall t \tag{22}$$

$$\sum_{i=1}^{I} \beta^{i-1} \left(c_{t+i-1}^i U_{c_{t+i-1}^i} + \ell_{t+i-1}^i U_{\ell_{t+i-1}^i} \right) = 0, \qquad t \geq 0 \tag{23}$$

$$\sum_{i=s}^{I} \beta^{i-s} \left(c_{i-s}^i U_{c_{i-s}^i} + \ell_{i-s}^i U_{\ell_{i-s}^i} \right) = U_{c_0^s} \cdot a_0^s, \quad s = 2, \ldots, I; \tag{24}$$

where the initial distribution of wealth $a_{s,0}$ for $s \in (2, I)$ is given.

The main difference between those two problems is that the RAP does not depend on taxes and prices. The existence of a solution depends on the properties of the implementability constraint. Except for the implementability constraint, this problem is equivalent to a growth model with a finite number of goods. To find a solution, is useful to redefine the objective function by introducing the implementability constraint on it, and its associated Lagrange multiplier as co-state variable. Let η_t be the Lagrange multiplier of the implementability constraint for the agent born in period t:[4]

$$V(c_t^i, \ell_t^i, \eta_t) = U(c_t^i, \ell_t^i) + \eta_t (c_t^i U_{c_t^i} + \ell_t^i U_{\ell_t^i}) \tag{25}$$

[3] The case where budget balanced is imposed at each period will not be studied in this paper. In this case will be necessary to use consumption taxes, otherwise the set of instruments will be incomplete, see Chari and Kehoe (1998) for a detail explanation.

[4] See Marcet and Marimon (1998) to see under what conditions we can expand the state space to include the Lagrange multiplier as a co-state variable.

The Ramsey allocation in Lagrangian terms can be written as:

$$\max_{x} \sum_{t=0}^{\infty} \sum_{i=1}^{\infty} \omega_{t+1-i} \left[\beta^{i-1} U(c_t^i, \ell_t^i) \right] - \sum_{s=2}^{\infty} \eta_{1-s} U_{c_0^s} \cdot a_0^s; \tag{26}$$

subject to resource constraint. The first-order conditions of an interior solution for this problem at $t > 0$;

$$\frac{V_{\ell_t^i}}{V_{c_t^i}} = -F_{L_t} \cdot \varepsilon^i; \qquad \forall i, t \tag{27}$$

$$\omega_{t+1-i} V_{c_t^i} = \omega_{t+2-i} V_{c_{t+1}^i} (1 - \delta + F_{K_{t+1}}); \qquad \forall i, t \tag{28}$$

$$V_{c_t^i} = \beta \cdot \frac{\omega_{t+2-i}}{\omega_{t+1-i}} V_{c_t^{i+1}}; \qquad \forall i, t \tag{29}$$

Equation (27)is the intratemporal condition between consumption and labor, note that the $V(\cdot)$ are different objects than the utility function. Equation (28)is the intertemporal condition that says how much capital is going to be invested next period according the government subjective valuation of future generations and the marginal productivity of capital. Equation (29) implies that the planner will assign resources according within two different generation depending on the ratio of relative weight of each generation.

For the s initial generations at $t = 0$; the first-order conditions are slightly different given that the incorporate an additional term. The intratemporal condition:

$$\frac{V_{\ell_0^s} - \eta_{1-s} \left[U_{c_0^s \ell_0^s} ((1 + r_o(1 - \theta_{s0}) a_0^s) + U_{c_0^s} F_{K_0 \ell_0^s} (1 - \theta_{s0}) a_0^s \right]}{V_{c_0^s} - \eta_{1-s} U_{c_0^s c_0^s} ((1 + r_o(1 - \theta_{s0}) a_0^s)} = -F_{L_0} \cdot \varepsilon^s; \tag{30}$$

and the redistributive condition,

$$V_{c_0^s} - \eta_{1-s} U_{c_0^s c_0^s} ((1 + r_o(1 - \theta_{s0}) a_0^s) = \beta \frac{\omega_{s+1}}{\omega_s} \left[V_{c_0^{s+1}} - \eta_{2-s} U_{c_0^{s+1} c_0^{s+1}} \left((1 + r_o(1 - \theta_{s+1,t}) a_0^{s+1} \right) \right];$$

We also have a transversality condition, but in this case it will not add sufficiency because the first-order conditions are only necessary to characterize the optimal solution.

$$\lim_{t \to \infty} \mu_t \cdot K_{t+1} = 0 \tag{31}$$

5. Particular Functional Forms

This section characterizes the optimal fiscal policy for some particular functional forms. Under the assumption of a complete set of fiscal instruments, certain types of utility function satisfy Chamley (1986) result.

Proposition 4. If the utility function is of this type $u(c;\ell) = \frac{c^{1-\sigma}}{1-\sigma} + h(\ell)$; it can be shown that is no optimal tax capital accumulation decisions from period 2 onwards.[5]

Proof. In order to proof this result I proceed in two steps. First show that age specific capital taxes for the newborns are zero from period 1 onwards, and then that capital taxes for the initial old are zero from period 2 onwards. The Ramsey allocation first-order conditions for the newborns imply:

$$V_{c_t^i} = \beta V_{c_{t+1}^{i+1}}(1 - \delta + F_{K_{t+1}}); \qquad t \geq 0 \tag{32}$$

the competitive equilibrium first-order conditions for the newborns are given by:

$$U_{c_t^i} = \beta U_{c_{t+1}^{i+1}}(1 + r_{t+1}(1 - \tau_{i;t+1})); \qquad \forall t \tag{33}$$

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[5] Some examples of this utility function that satisfies the initial assumptions can be the followings:
(1)

$$U(\cdot) = \frac{c^{1-\sigma_1} - 1}{1-\sigma_1} + \frac{(1-\ell)^{1-\sigma_2} - 1}{1-\sigma_2};$$

notice that if $\sigma_1 = \sigma_2 = 1$; we have the logarithmic utility function, $U(\cdot) = \ln c + \ln(1-\ell)$; or alternatively,
(2)

$$U(\cdot) = \frac{c^{1-\sigma_1} - 1}{1-\sigma_1} - a\ell;$$

(3)

$$U(\cdot) = \frac{c^{1-\sigma_1} - 1}{1-\sigma_1} + \frac{\ell^{1+\sigma_2}}{1+\sigma_2};$$

for this type of utility functions is easy to see that satisfies:

$$(34)\qquad \frac{W_{c_t^i}}{W_{c_{t+1}^{i+1}}} = \frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}}, \qquad \forall t,$$

therefore the optimal policy implies set $\theta_{i,t+1} = 0$. So is optimal not distort capital accumulation for the newborn generations. Second, I need to show that the s old generation only will have positive taxes on capital on the first period. At $t = 0$, the Ramsey allocation first-order conditions for the initial generations:

$$(35)\qquad \hat{V}_{c_0^s} = \beta V_{c_1^{s+1}}(1 - \delta + F_{K_1}), \qquad s \in (2, I - 1)$$

where $\hat{V}_{c_0^s} = V_{c_0^s} - \eta_{1-s} U_{c_0^s c_0^s}\left(R^k_{s0}k^s_0 + R^b_{s0}b^s_0\right)$. The competitive equilibrium intertemporal equation imply:

$$(36)\qquad U_{c_0^s} = \beta U_{c_1^{s+1}}(1 + r_1(1 - \theta_{s,1})), \qquad s \in (2, I - 1)$$

Notice that there are s initial old, but only $I - 2$ have saving decisions. This type of utility function does not satisfy:

$$(37)\qquad \frac{\hat{V}_{c_0^s}}{V_{c_1^{s+1}}} \neq \frac{U_{c_0^s}}{U_{c_1^s}}, \qquad s \in (2, I - 1)$$

Hence the initial generations at $t = 0$ will have $\theta_{s,1} \neq 0$, but at $t = 1$ the Ramsey intertemporal imply,

$$(38)\qquad V_{c_1^s} = \beta V_{c_2^{s+1}}(1 - \delta + F_{K_2}), \qquad s \in (2, I - 2)$$

Hence $\theta_{s,2} = 0$. From this point onwards age specific capital taxes are zero for all agents. ■

This implies that from period $t \geq 2$, capital taxes will be zero for all agents, and different from zero in period 1. The initial capital taxes at period 0 are given. This result relies on the separability between consumption and leisure, homotheticity, and the ability of the government to tax differently all generations.

6. Steady State Analysis

The purpose of this section is analyze if there are other type of utility functions that might deliver the zero capital result as an asymptotic property but not along the transition path. In order to study the optimal fiscal policy in the steady state is necessary to have additional assumptions in the government objective function. Let's assume that the government assigns the same weight discounted by time to each generation utility function, formally γ^t is the weight that the government assigns to the generation born at time t; where $\gamma \in (0;1)$:

$$\sum_{t=-(I-1)}^{\infty} \gamma^t \leq \Gamma$$

where γ parameter indicates how much weight the government values future versus current generations, and Γ is a positive constant. Using this objective functions, the first-order conditions for steady state are,

$$(39) \quad \frac{1}{\gamma} = 1 - \delta + F_K$$

$$(40) \quad \frac{V_{\ell^i}}{V_{c^i}} = -F_L \cdot \epsilon^i; \qquad \forall i$$

$$(41) \quad V_{c^i} = \frac{\beta}{\gamma} V_{c^{i+1}}; \qquad \forall i$$

A feature of this model, is that if the economy converges to the steady state, this is independent of the initial conditions and the transition path, see Escolano (1992). The government discount factor determines the interest rate in steady state. For the general type of utility functions that are additively separable capital taxes are zero in steady state, because the new type of constraint optima allocations is a subset of the general welfare function. With non-separable utility functions Chamley (1986) result can be proved under additional assumptions on the government discount factor and the endowment of efficiency units.

Proposition 5. If the utility function is of this type:

$$u(c;\ell) = \frac{c^{1-\sigma}}{1-\sigma} \cdot h(\ell) \tag{42}$$

and $\beta = \gamma$, and $\epsilon^1 = ... = \epsilon^I$; then capital taxes will be zero in steady state

Proof. The basic underling in this proof is that under this assumptions, from the planner point of view, this economy is equivalent to an infinite lived agents economy. If the planner and households discount factors are the same, then the redistributive condition implies that all agents will have the same marginal rates of substitution. That, does not mean that all agents will achieve the same allocation in terms of consumption and leisure because agents are endowed with different efficiency units of labor. The additional assumption of equal efficiency units across generations ensures that all households will achieve the same allocation in terms of consumption and leisure at each period. Hence this condition will imply that marginal rates of substitution between consumption and leisure will be equal across generations. Therefore this model perfectly behaves as a representative consumer economy, and reproduces Chamley (1986) result.

Formally this is straightforward by comparing the first-order conditions of the competitive equilibrium in steady state,

$$1 = \beta(1 + r(1 - \theta_i)) \tag{43}$$

with the Ramsey efficient allocation (under this assumptions $U_{c^i} = U_{c^{i+1}}$) :

$$1 = \beta(1 - \delta + F_k)$$

then it must be the case that $\mu_i = 0$; for all i:

7. Incomplete Set of Instruments

An important assumption in the previous section is that the government can perfectly discriminate taxes among di¤erent generations. Suppose the tax system does not allow tax rates in capital returns or labor income di¤er across consumers. Escolano (1992) analyses a similar model imposing this additional assumption, but uses the dual approach of optimal …scal policy. Adding this assumption imposes additional restriction on the Ramsey problem because restricts the set of instruments that the government can use to implement the e¢cient allocation. Let's consider …rst the restriction that taxes in capital returns must be equal across households, that is $\mu_i = \mu$ for all i. The additional restrictions that must be added in the Ramsey problem can be derived from the intertemporal …rst-order conditions in the competitive equilibrium,

$$(44) \qquad \frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}} = {}^{-}(1 + r_{t+1}(1 \ ¡ \ \mu_{t+1})); \qquad 8t; i$$

which implies that the right hand side is equal for all i; therefore any competitive equilibrium must satisfy

$$(45) \qquad \frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}} = ::: = \frac{U_{c_t^{I \ ¡ 1}}}{U_{c_{t+1}^I}}; \qquad 8t; \quad i = 1; :::; I$$

As in the previous case, this additional restrictions can be added in the government objective function, introducing the associate Lagrange multiplier as a co-state variable. Let $À_t^i$ be the Lagrange multiplier of this additional sequence of $I \ ¡ \ 1$ constraints for each period.

Now let's assume that taxes on labor income must be equal across consumers, $¿_{`_{i;t}} = ¿$ $8i; t$. Then, the additional restriction on allocations imply that any competitive equilibrium must satisfy:

$$\frac{U_{c^1}}{U_{\ell^1}} \cdot \varepsilon^1 = \ldots = \frac{U_{c^I}}{U_{\ell^I}} \cdot \varepsilon^I$$

The main conclusions is that with this additional assumption capital taxes are not generally zero in steady state, only for some specific government discount factor he obtains the standard result.

8. Conclusions

This paper introduces the theory of optimal taxation in economies were agents have a finite lifetime. This approach departs from the pioneer works of Auerbach and Kotlikoff (1987) and introduces the government in the economy as an active agent that chooses the optimal fiscal policy. The theory of optimal taxation is applied to answer how to determine the optimal fiscal policy in the transition path converging to the steady state. It can be shown that if the government has a complete set of taxes (age specific taxes) and the utility function is addtively separable, then capital taxes are zero in the transition path after the second period. More over, with additional assumptions on the discount factor and the endowment of efficiency units, it can be proved that with non-separable utility functions is not optimal tax capital in steady state.

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