A Simple Dynamic General Equilibrium Analysis

of the Trade-off Between Fixed and Floating Exchange rates[1]

by

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Abstract

　　　　This paper provides a complete analytical characterization of the positive and normative effects of alternative exchange rate regimes in a simple two-country sticky-price dynamic general equilibrium model with money, technology, and government spending shocks. A central question addressed is whether fixing the exchange rate prevents macroeconomic adjustment in relative prices from occurring, in face of shocks. In the model, the exchange rate regime has implications for both the *volatility* and *mean* of macroeconomic aggregates. But the effects of the exchange rate regime depend upon both the stance of monetary policy and the way in which the exchange rate is pegged. With a passive monetary policy, a cooperative pegged exchange rate regime has *no implications* for macroeconomic volatility, relative to a floating regime, but implies a *higher mean* level of employment, capital stock, and real GDP.   When monetary policy is determined optimally however, a fixed exchange rate regime leads to *higher employment volatility* and a *lower mean* level of employment and real GDP. Therefore, whether fixing the exchange rate involves a welfare cost depends critically upon the flexibility of monetary policy in responding to macroeconomic shocks.

Section 1. Introduction

Much of the debate surrounding the single currency in Europe concerned the costs of sacrificing the nominal exchange rate as a macroeconomic adjustment device. Based on the older literature on the size of the *optimal currency area*[2] writers such as Eichengreen (1992) stressed the importance of exchange rate flexibility in dealing with country specific disturbances. Indeed Feldstein (1997) regards the sacrifice of exchange rate adjustment as the most critical drawback of the single currency in Europe[3].

Despite the theory of optimal currency areas, there seems little clear evidence that European exchange rate movements have been instrumental in adjustment to macroeconomic shocks (see for instance, Gordon 1999), and indeed it has been conjectured that the very criteria for the existence of an optimal currency area themselves might be endogenous, adjusting in response to a single currency so as to remove the necessity for exchange rate adjustment (Frankel and Rose (1998)). Furthermore, it has been suggested that the elimination of national currencies may give rise to longer run dynamic efficiencies, which may increase national income across Europe, or possibly increase average growth rates (EMU 1990).

This paper employs some recent developments in the analysis of exchange rates in stochastic general equilibrium settings in order to address the questions arising from the discussion of the previous paragraphs. First, under what circumstances is exchange rate flexibility useful in offsetting country specific macroeconomic disturbances such as productivity or demand shocks? Does fixing the exchange rate entail a sacrifice in terms of efficient adjustment to shocks? Second, what are the possible long run dynamic effects of the single currency area? Can a fixed exchange rate system give rise to long run benefits in terms of a higher average level of GDP, or even a higher rate of growth of GDP?

The paper is constructed around a fairly standard dynamic general equilibrium two country model. Prices are sticky, in the manner of Obstfeld and Rogoff (1995). We make specific functional form assumptions to allow for a complete closed form solution, so that the influence of all stochastic shocks on both the mean and variance of endogenous variables can be analyzed explicitly. We follow recent literature such as Obstfeld and Rogoff (1998), Bachetta and Van Wincoop (1999), and Devereux and Engel (1998) in exploring the link between the stochastic characteristics of the economy and the average level of prices that are pre-set by firms. This gives a mechanism by which the exchange rate regime affects not just the *variance* of

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[2] See, for instance, Mundell (1961), and McKinnon (1963).
[3] See De Grauwe (1994) for a survey of the arguments for and against a common currency.

international macroeconomic variables, but also their *mean* levels. Unlike the aforementioned papers, however, the present paper introduces a dynamic structure to the economy in an essential way, by allowing for endogenous capital accumulation in an infinite horizon environment[4]. Another important difference with the analysis of Bachetta and Van Wincoop (1998) and Devereux and Engel (1998) is that the present paper does not assume deviations from the law of one price due to `pricing-to-market' or local-currency pricing. Since the main arguments about the adjustment benefits of exchange rate flexibility focus on the role of the exchange rate in altering relative prices, it important to conduct the evaluation in a model in which the exchange rate has a direct affect on relative prices.

Our results can be briefly summarized. First, with respect to role of the exchange rate as a macroeconomic adjustment device, we find that in this model, the exchange rate does not help the economy to adjust to country specific productivity or demand shocks. This is somewhat surprising, as our model is an extension of a fairly straightforward sticky price model of exchange rates (such as Obstfeld and Rogoff (1995) or Corsetti and Pesenti (1998)) In the absence of price stickiness, country specific productivity and demand shocks would require terms of trade adjustment. Intuitively, one would expect that in order to achieve terms of trade adjustment in an environment of sticky prices, the exchange rate must be allowed to fluctuate. But the intuition for the effects of country specific productivity or demand shocks in a flexible price world economy does not necessarily carry over to the sticky price world, even when the exchange rate is flexible. When prices are pre-set, productivity shocks will have no affect on output, which is demand determined. The exchange rate does not respond to a country specific productivity shock, even under floating exchange rates. Furthermore, although demand shocks (government spending shocks in our example) do affect GDP, they do not affect the exchange rate either. Therefore, the exchange rate plays no role in the response of the economy to productivity or demand shocks. Consequently, fixing the exchange rate does not remove the economy's ability to adjust to these shocks.

The exchange rate regime will in general have effects on the economy nevertheless, because it does have implications for the cross-country correlation of money shocks. In a floating exchange rate regime, independent money shocks generate exchange rate volatility. Under a fixed exchange rate, money shocks must be identical across countries. We find that the effects of a fixed exchange rate regime depend sensitively upon the type of *monetary policy coordination* that is used to fix the exchange rate. When exchange rates are fixed in a *one sided*

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[4] An important early study was the pioneering papers of Helpman (1981), and Helpman and Razin (1982).

peg, which entails one country following the monetary policy of the other, output volatility tends to be higher with a fixed exchange rate regime, but the mean level of output is unchanged from a floating regime. When a *cooperative peg* is used however, output volatility is unaffected by the move to a fixed regimes, while mean output is higher. In fact, we can show that in the basic model, a cooperative peg unambiguously welfare-dominates a floating exchange rate regime.

These results suggest that a fixed exchange rate regime, in particular a cooperative peg, has no negative implications for macroeconomic performance at all, and in fact may be more desirable than a floating exchange rate regime.

While these results are somewhat sensitive to the particular model specification, they do point to the fact that the standard intuition regarding the adjustment benefits of floating exchange rates has to be heavily qualified. A more serious problem with the above intuition however is that it takes monetary policy as being exogenous. One of the key aspects of defending a pegged exchange rate regime is that it eliminates the possibility for using independent monetary policy as a macroeconomic tool. In a later section, we amend the basic model to allow for an optimal monetary policy rule. We show that when monetary policy is used to respond to demand or supply shocks in an optimal way, the economy will behave in a manner that imitates the flexible price equilibrium. But this critically requires country-specific cyclical behavior in monetary policy. Exchange rate adjustment now becomes a central mechanism for the use of optimal monetary policy. If exchange rates were fixed, then a second best monetary policy rule can be devised, but it requires giving up on exchange rate adjustment. That is costly, and so we may conclude that if we are interested not just in passive monetary policy regimes, but activist monetary policy that responds to macroeconomic shocks, then inevitably a fixed exchange rate regime has welfare costs. Moreover, in the case of optimal monetary policy setting, fixed exchange rates must lead to a lower *mean* level of output.

The rest of the paper is structured as follows. Section 2 presents the basic model with money and technology shocks. Section 3 develops the results under flexible prices. Section 4 analyzes the case of floating exchange rates with sticky prices, while section 5 looks at the pegged exchange rate case. Section 6 analyzes welfare across the different regimes. Section 7 derives the results under optimal monetary policy. Section 8 extends the model to allow for fiscal policy shocks. Some conclusions follow.

Section 2. A two-country model

We develop a model in which the properties of exchange rate regimes in dynamic economy can be compared. The model has two countries, which we denote "home" and "foreign". As shown below, extension to more than two countries is straightforward. Within each country, there exist consumers, firms and a government. Government issues fiat money. Initially we will abstract from government spending.

We assume that there is continuum of goods varieties in the world economy of measure 1, and that the relative size of the home and foreign economy's share of these goods is n and $1-n$ respectively. We choose units so that the population of the home and foreign country is also n and $1-n$, respectively.

Let the state of the world at time t be defined as z_t. In each period t, there is a finite set of possible states of the world. Let z^t denote the history of realized states between time 0 and t, i.e. $z^t = \{z_0, z_1, ... z_t\}$. The probability of any history, z_t, is denoted by $\pi(z^t)$.

We may just describe the details of the model for the home country economy. The conditions for the foreign country are analogously defined in all cases, except where stated.

Consumers

Assume that preferences are identical across countries. In the home country, consumers have preferences given by

$$(1) \qquad EU = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t \pi(z^t) U\left(c(z^t), \frac{m(z^t)}{P(z^t)}, (1-h(z^t)) \right)$$

where $c(z^t) = c_h(z^t)^n c_f(z^t)^{1-n}$, $c_h(z^t) = \left(n^{-1/\lambda} \int_0^n c(i, z^t)^{1-1/\lambda} di \right)^{\lambda/(\lambda-1)}$, and

$$c_f(z^t) = \left((1-n)^{-1/\lambda} \int_n^1 c(i, z^t)^{1-1/\lambda} di \right)^{\lambda/(\lambda-1)}.$$

In addition, we assume the specific functional form given by

$$U(c, \frac{m}{P}, 1-h) = \ln c + \chi \ln\left(\frac{m}{P} \right) + \eta \ln(1-h).$$

The consumer derives utility from a composite consumption good $c(z^t)$, real home country money balances $\frac{m(z^t)}{P(z^t)}$, and leisure, where $h(z^t)$ represents hours worked. The

composite consumption good is broken up into two sub-composites, representing home and foreign goods consumption, with a unit elasticity of substitution between the two. Within each country-specific sub-composite, there is an elasticity of substitution of λ between any two consumption goods.

The price index is defined as

(2) $\qquad P(z^t) = P_h(z^t)^n (S(z^t)P_f(z^t))^{1-n}$,

where $S(z^t)$ is the exchange rate, and the sub-price indices are defined as

$$P_h(z^t) = \left[\frac{1}{n} \int_0^n p_h(i,z^t)^{1-\lambda} \, di \right]^{1/(1-\lambda)} \qquad P_f(z^t) = \left[\frac{1}{1-n} \int_n^1 p_f(i,z^t)^{1-\lambda} \, di \right]^{1/(1-\lambda)}.$$

The consumer price index in the home economy depends on the composite price of home goods $P_h(z^t)$, and the exchange rate times the composite price of foreign goods, where $P_f(z^t)$ is the foreign goods price, expressed in foreign currency. Note that the foreign consumer price index will be analogously defined as $P^*(z^t) = \left(\dfrac{P_h(z^t)}{S(z^t)} \right)^n P_f(z^t)^{1-n}$, and therefore purchasing power parity must hold at all times in this economy.

The representative consumer in the home country receives income from wages and the return on physical capital holdings, profits from the ownership of domestic firms, income from international bond holdings and existing money balances, and receives transfers and/or pays taxes to the domestic government. Households then consume, accumulate capital and money balances and purchase new assets.

Therefore the home consumer's budget constraint is written as

(3) $\quad P(z^t)c(z^t) + m(z^t) + q(z^t)B(z^t) + P(z^t)v(z^t) =$

$W(z^t)h(z^t) + R(z^t)k(z^{t-1}) + \Pi(z^t) + m(z^{t-1}) + B(z^{t-1}) + TR(z^t)$,

where

(4) $\quad k(z^t) = v(z^t)$.

The home consumer purchases home-currency denominated nominal bonds at price $q(z^t)$. $v(z^t)$ represents a composite investment good, which requires the same basket of goods as the consumer goods, and which forms next period's capital holdings, given by $k(z^t)$.

The consumer also receives net transfers $TR(z^t)$ from the government, and nominal domestic currency profits $\Pi(z^t)$. $R(z^t)$ denotes the nominal rental return on a unit of capital.

The consumer's optimal consumption, money holdings, investment, and labor supply may be described by the following familiar conditions.

(5) $q(z^t)c(z^t)^{-1} = \beta \sum_{z^{t+1}} \pi(z^{t+1}) \dfrac{P(z^t)}{P(z^{t+1})} c(z^{t+1})^{-1},$

(6) $\dfrac{m(z^t)}{P(z^t)} = \varsigma c(z^t) \ \dfrac{1+i(z^t)}{i(z^t)},$

(7) $\dfrac{\eta}{1-h(z^t)} = \dfrac{W(z^t)}{P(z^t)c(z^t)},$

(8) $c(z^t)^{-1} = \beta \sum_{z^{t+1}} \pi(z^{t+1}) c(z^{t+1})^{-1} \dfrac{R(z^{t+1})}{P(z^{t+1})}.$

Equation (5) describes the choice of inter-temporal consumption smoothing, while (6) gives the implied demand for money of the consumer. The term $i(z^t)$ represents the nominal interest rate, where $\dfrac{1}{1+i(z^t)} = q(z^t)$. Equation (7) describes the labor supply choice, while (8) results from the optimal choice of the investment good.

In a symmetric equilibrium, all households in the same country will have the same consumption, money holdings, investment, and labor supply. To reflect this, in what follows we will denote these variables by capital letters.

Consumption insurance

There is a key feature of this class of models that has been pointed out by Corsetti and Pesenti (1998). If we begin at an initial date with zero net foreign bonds outstanding, then, with a unit elasticity of substitution between the consumption of home and foreign goods, and in the presence of purchasing power parity, the current account will always be zero, and equilibrium consumption is equalized across countries. The Appendix demonstrates this proposition for our economy. Thus, in all states of the world, we have

(9) $C(z^t) = C^*(z^t).$

The intuitive reason for this pooling property is the same as that pointed out by Cole and Obstfeld, (1991). When countries specialize in production, and there is a unit elasticity of substitution between home and foreign goods, an increase in domestic production is exactly offset by a term of trade deterioration, generating a terms of trade improvement in the foreign country. The end result is that income rises by identical amounts in both countries[5]

Government

Governments in each country print money and levy taxes, and purchase goods to produce a composite government consumption good. We assume the government does not issue bonds, and so must always balance its within period budget. It is assumed that the government composite good is produced using the same aggregator that private consumption and investment goods use. The home country government budget constraint is then

(10) $M(z^t) - M(z^{t-1}) = P(z^t)G(z^t) + TR(z^t)$,

where $G(z^t)$ represents the government composite good. We initially set this to zero. Section 7 below examines the effects of shocks to government spending on our main results.

Firms

Firms in each country hire capital and labor to produce output. For each home and foreign good, there is a separate, price-setting firm. The number of firms is sufficiently large that each firm ignores the impact of its pricing decision on the aggregate price index for that variety. A home firm of variety i, has production function given by

$$y(i, z^t) = \theta(z^t)k(i, z^t)^\alpha h(i, z^t)^{1-\alpha},$$

where $k(i, z^t)$ is capital usage and $h(i, z^t)$ is labor usage. $\theta(z^t)$ is a country-specific technology shock.

All firms will choose factor bundles to minimize costs. Thus, we must have

(11) $W(z^t) = MC(z^t)(1-\alpha)\dfrac{y(i, j, z^t)}{h(i, j, z^t)}$,

(12) $R(z^t) = MC(z^t)\alpha\dfrac{y(i, j, z^t)}{k(i, j, z^t)}$,

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[5] Corsetti and Pesenti (1998) show that this property still applies in a sticky price environment.

where $MC(z^t)$ is nominal marginal cost, which must be equal for all firms within the home economy.

Pricing

We assume that firms must set nominal prices one period in advance[6]. Prices are set to maximize profits, where profits are evaluated using the marginal utility of money of the firm owners. Thus, the home firm at time $t-1$ chooses its price to maximize

$$\sum_{z^t} \pi(z^t)\beta \frac{P(z^{t-1})C(z^{t-1})}{P(z^t)C(z^t)} \Big(p(i,z^t)x(i,z^t) - MC(z^t)x(i,z^t) \Big),$$

where $x(i,z^t)$ is the demand for firm $i's$ good. From the properties of the consumer preferences, it is easy to see that $x(i,z^t)$ is given by

$$(13) \qquad x(i,z^t) = \left(\frac{p(i,z^t)}{P_h(z^t)} \right)^{-\lambda} \frac{P(z^t)(C(z^t) + nV(z^t) + (1-n)V^*(z^t))}{P_h(z^t)}.$$

Expression (13) reflects the fact that consumption is equalized across countries, and that PPP holds. In a symmetric equilibrium, the optimal price set by all firms in the home country will be identical, and equal to $P_h(z^{t-1})$

$$(14) \qquad P_h(z^{t-1}) = \frac{\lambda}{\lambda-1} \frac{\sum_{z^t} \pi(z^t)(\dfrac{C(z^t) + nV(z^t) + (1-n)V^*(z^t)}{C(z^t)} MC(z^t))}{\sum_{z^t} \pi(z^t)(\dfrac{C(z^t) + nV(z^t) + (1-n)V^*(z^t)}{C(z^t)})}.$$

In general the optimal price will not be set simply as a markup over expected marginal cost, but will depend on the covariance between marginal cost and demand. The foreign firm sets its price in an analogous fashion.

Market Clearing

Within a country, all firms use the same capital labor ratio. Therefore we may aggregate across firms and sectors to define the aggregate output in the home economy as

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[6] It would be possible to introduce more persistent price rigidity, along the lines of Calvo (1983) or Taylor (1979). But the results would be *qualitatively* similar, and we would no longer obtain a complete analytical solution to the model.

$$Y(z^t) = \theta(z^t)K(z^{t-1})^\alpha H(z^t)^{1-\alpha}.$$

Output must equal aggregate demand for the home. Total demand comes from the demand for consumption and investment of home and foreign consumers. Thus

$$(15) \quad \theta(z^t)K(z^{t-1})^\alpha H(z^t)^{1-\alpha} = \frac{P(z^t)}{P_h(z^{t-1})}[C(z^t) + nV(z^t) + (1-n)V^*(z^t)].$$

A similar market clearing equation holds for the foreign country;

$$(16) \quad \theta^*(z^t)K(z^{t-1})^{*\alpha} H(z^t)^{*1-\alpha} = \frac{P*(z^t)}{P_f(z^{t-1})}[C(z^t) + nV(z^t) + (1-n)V^*(z^t)].$$

Equilibrium

We may characterize the equilibrium of the two-country economy by collecting the equations set out above. The equilibrium is the sequence $C(z^t), C(z^t)^*, H(z^t), H(z^t)^*$

$q(z^t), q^*(z^t), K(z^t), K^*(z^t), P_h(z^t), P_f(z^t), S(z^t), MC(z^t), MC^*(z^t), W(z^t), W^*(z^t),$

$R(z^t), R^*(z^t)$ that is a solution of the equations (5)-(8), and (11)-(14), and their counterparts for the foreign economy, as well as (9), (15) and (16). The model is sufficiently simple that we can solve it in closed form. However, it is necessary to give an explicit description of the structure of the shock processes. We take

$z_t = \{M_t, M_t^*, \theta_t, \theta_t^*\}$

Moreover, money supply in each country is a random walk in logs, so that for the home country;

$$(17) \qquad M_t = \exp(m_t) \qquad\qquad m_t = m_{t-1} + u_t$$

where u_t is a mean zero, i.i.d. shock to the money supply with variance σ_u^2. In section 4 we will allow for money supply to be explicitly targeted on technology shock realizations.

We assume that the technology shocks follow the process

$$(18) \qquad \theta_t = \exp(v_t) \qquad\qquad v_t = \rho v_{t-1} + \varepsilon_t$$

where ε_t is a mean zero i.i.d shock to technology with variance σ_ε^2.

To keep the model as symmetric as possible, we assume that the foreign country money and technology shocks take on identical variances to the home country shocks.

The Exchange Rate Regime

Under floating exchange rates the shocks to the money supply in each country may be independent of one another. But under fixed exchange rates, money supplies are adjusted by one or both countries so as to keep the exchange rate constant. It is possible to fix an exchange rate

by fiscal policies or other instruments, but we will focus solely on a monetary rule for fixing the exchange rate. In our symmetric setup, this requires that $M_t = \xi M_t^*$, where ξ is a constant term. In practice we set $\xi = 1$.

Section 3. Solution of the model under Flexible Prices

We may solve the model in a series of steps. First, for comparison purposes, we describe the equilibrium of the two country economy that would obtain were all prices completely flexible. This is then used as a benchmark to compare against the properties of the sticky price economies, in the case of fixed or floating exchange rates.

With flexible prices, full monetary neutrality obtains, and the evolution of consumption and investment is independent of monetary polices or the exchange rate regime. Then

$P_h(z^t) = \dfrac{\lambda}{\lambda - 1} MC(z^t)$ must hold; that is, price must equal ex-post marginal cost, adjusted for

the monopolistic competitive markup. With this, we may combine the factor pricing condition (15), the optimal investment condition (8), and the two market clearing conditions (14) and (15)

to obtain the following solutions for investment, the terms of trade $\left(\dfrac{P_{ht}}{S_t P_{ft}} \right)$, and consumption.

Because capital is constructed from the output of both countries, and because expected future income is pooled as described above, the current cost of capital and the expected return to investment is identical in both countries, so their investment rates are equal. Thus, we have $K_t = K_t^*$. The economy under flexible prices is then characterized by[7]:

(19) $K_{t+1} = \beta\alpha\tilde{\lambda}\,\theta_t^{n}\theta_t^{*(1-n)} K_t^{\alpha} H_t^{(1-\alpha)}$,

(20) $\dfrac{P_{ht}}{S_t P_{ft}} = \dfrac{\theta_t^*}{\theta_t}$,

(21) $C_t = \dfrac{(1 - \beta\alpha\tilde{\lambda})}{\beta\alpha\tilde{\lambda}} K_{t+1}$,

(22) $H_t = H_t^* = \dfrac{(\lambda - 1)(1 - \alpha)}{((\lambda - 1)(1 - \alpha) + \lambda\eta(1 - \beta\alpha\tilde{\lambda})}$,

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[7] In what follows, we omit the state contingent notation, since that configuration of shocks has now been explicitly defined.

where $\tilde{\lambda} = \frac{(\lambda - 1)}{\lambda}$. Investment is a constant fraction of real GDP (in the home country case

this is $\frac{P_{ht}}{P} \theta_t K_t^\alpha H_t^{1-\alpha}$, which is equal to that of the foreign country; i.e. $\frac{P_{ft}}{P_t^*} \theta_t^* K_t^\alpha H_t^{1-\alpha}$).

Given the assumption that home and foreign consumption goods have a unit elasticity of substitution, the relative price of home goods is inversely proportional to the relative size of home country total factor productivity. Finally, since consumption is a constant fraction of real GDP, movements in the real wage are reflected proportionally in consumption. Therefore wealth and substitution effects of wage increases on employment actually cancel out, and equilibrium employment is actually constant.

We now contrast this to the case of the sticky price economy.

Section 4: Floating exchange rates

Under pre-set prices, the allocation described by (19)-(22) cannot in general be attained, since each firm passively adjusts the production of its good to the amount demanded at the preset price. Shocks to the money supply will affect aggregate demand, and there is no longer money neutrality. One initial result that is very useful is that the nominal interest rate is constant, in equilibrium. Thus, when the money supply follows the process given by (16), neither money shocks nor technology shocks have any affect on the nominal interest rate. This result holds whether prices are sticky or not, and follow from the assumption of a random walk process (in logs) for the money supply, combined with the form of the money demand function (6). The result is established rigorously in the Appendix[8]. Thus, neither disturbance will alter the nominal interest rate. In equilibrium the nominal interest rate is constant, equal to

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[8] Intuitively, we may note that the nominal interest rate satisfies

$$\frac{1}{(1+i_t)} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

An unanticipated permanent home country money shock will raise current consumption, increasing the price level (through exchange rate depreciation) by less than in proportion to the shock. In the next period, prices will rise by more and consumption by less (in the economy without capital, prices in the next period would rise by the full amount of the money shock, and consumption in the next period would be unchanged), but overall, the proportional response of nominal consumption will be equal to the proportional rise in the money supply. Alternatively, a technology shock has no impact on current consumption or the price level, but will lead to a rise in next periods consumption and a fall in next periods price level which in net leaves $P_{t+1} C_{t+1}$ unchanged.

(23) $\quad \dfrac{1}{(1+i)} = \beta\mu$,

where $\mu = E_t \exp(u_t)$.

The exchange rate

      Using (23), it follows from the money market clearing condition (5) (and its foreign country counterpart) the risk sharing condition (9), and the PPP condition that

(24) $\quad S_t = \dfrac{M(1-\beta\mu)}{M_t^{*}(1-\beta\mu^{*})}$.

The nominal exchange rate depends only on relative money supplies, not on technology shocks.

Consumption and Investment

      Using (24) and the money market clearing conditions, it follows in addition that

(25) $\quad C_t = \dfrac{M_t^{n} M_t^{*(1-n)}}{P_{ht}^{n} P_{ft}^{1-n}} (1-\beta\mu)^{n}(1-\beta\mu_t^{*})^{1-n}$.

Given preset prices, consumption within any period depends only on money shocks. Consumption is proportional to a country -weighted geometric average of home and foreign money. The intuition is quite clear from a glance at the money market equilibrium condition (5). An increase in the money supply in the home country will generate an exchange rate depreciation. The depreciation raises the home country price level, mitigating the consumption effects of the money shock, but reduces the foreign country price level, therefore increasing foreign consumption.

      Since capital is subject to full depreciation, the trade-off between consumption and investment is actually the same as (21) above. That is, investment is proportional to consumption in each country

(26) $\quad K_{t+1} = \dfrac{\beta\alpha\tilde{\lambda}}{1-\beta\alpha\tilde{\lambda}} C_t$.

      Intuitively, even though consumption, employment and output are not at their flexible-price optimal level, the consumer is still able to smooth consumption optimally over time using bond markets and physical capital. The optimal rule is therefore to divide income between consumption and investment in a constant proportion.

Employment

Given (25) and (26), it is clear that investment rates are equalized across countries, and therefore so must be real GDP. Note from (25) that, for given prices P_{ht} and P_{ft}, consumption, investment, and real GDP are independent of technology shocks. Output is determined residually given the aggregate demand by consumers and investors and equation (15). Since capital is fixed, endogenous movements in output correspond to movements in employment. We may derive the solution for home employment by combining (25), (26), and the market clearing equation (15), to obtain

$$(27) \qquad H_t = \left(\frac{P_t C_t}{P_{ht}\theta_t K_t^{\alpha}(1-\beta\alpha\tilde{\lambda})} \right)^{\frac{1}{1-\alpha}} = \left(\frac{(1-\beta\mu)}{(1-\beta\alpha\tilde{\lambda})} \frac{M_t}{P_{ht}\theta_t K_t^{\alpha}} \right)^{\frac{1}{1-\alpha}}.$$

Employment is no longer constant, in an equilibrium with sticky prices. Moreover, while consumption and investment depends upon both home and foreign money shocks, employment depends only on domestic money shocks. This is due to the fact that home country output is determined by both the scale of world demand, and the relative price of the home country good. There is an expenditure `level' effect, and an `expenditure switching' effect. A home country money shock will raise consumption and investment demand for the home good according to (25) and (26), in proportion to *n* times the percentage increase in home money. But it also leads to an exchange rate depreciation, and a fall in the relative price of the home country good in proportion to *1-n* times the increase in the money supply. The full impact leads output to increase in direct proportion to the money shock, so that employment must rise in proportion to $1/_{1-\alpha}$ times the increase in the money stock. On the other hand, for the foreign economy, the level and expenditure switching effects of a domestic money shock cancel out, so that foreign output and employment is unchanged.

Note also that according to (27), a current technology shock reduces employment. This follows since output is demand determined, so a rise in total factor productivity cannot increase production. Therefore, firms will reduce their labor demand, in response to a technology shock expansion (this has been pointed out in the closed economy context by Gali 1998).

Conditional Variances

Table 1 documents the conditional (on the current capital stock) variance of consumption, investment, and employment, in the economy with floating exchange rates. Consumption variance is less than employment variance. In addition, consumption variance is independent of the variance of technology shocks.

Table 1 Conditional variance of consumption, investment, and employment under floating exchange rates	
Consumption (investment) Variance (log)	$(n^2 + (1-n)^2)\sigma_u^2$
Employment variance (log)	$\dfrac{1}{(1-\alpha)^2}\left(\sigma_u^2 + \sigma_\varepsilon^2\right)$

Price Determination

The solution for consumption, investment, and employment takes as given the pre-set prices P_{ht}, P_{ft}. But these prices must be determined optimally, ex ante, using condition (14). Recognizing that consumption and investment are in proportion to one another allows this condition to be significantly simplified. We get

$$P_{ht-1} = \frac{\lambda\eta(1-\beta\alpha\tilde{\lambda})}{(\lambda-1)(1-\alpha)} E_{t-1}\left(\frac{H(z^t)}{(1-H(z^t))} P_{ht-1}\right).$$

Substituting for the definition of employment from (27) gives

$$(28) \quad 1 = \frac{\lambda\eta(1-\beta\alpha\tilde{\lambda})}{(\lambda-1)(1-\alpha)} E_{t-1}\left(\frac{\left(\varphi\dfrac{M_{t-1}\exp(u_t)}{\theta_{t-1}^\rho \exp(\varepsilon_t)K_t^\alpha P_{ht}}\right)^{1/1-\alpha}}{\left(1-\left(\varphi\dfrac{M_{t-1}\exp(u_t)}{\theta_{t-1}^\rho \exp(\varepsilon_t)K_t^\alpha P_{ht}}\right)^{1/1-\alpha}\right)}\right),$$

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where $\varphi = (1-\beta\mu) \big/ (1-\beta\alpha\tilde{\lambda})$. Examining (28), it is clear that the equilibrium pre-set price

P_{ht} will be proportional to $\left(\varphi \dfrac{M_{t-1}}{\theta_{t-1}^{\rho} K_t^{\alpha}} \right)$, which is already in the period t-1 information set. We

may thus characterize the solution for P_{ht} as

(29) $\quad P_{ht} = \Theta\left(\varphi \dfrac{M_{t-1}}{\theta_{t-1}^{\rho} K_t^{\alpha}} \right),$

where Θ satisfies the following condition

(30) $\quad 1 = \dfrac{\lambda\eta(1-\beta\alpha\tilde{\lambda})}{(\lambda-1)(1-\alpha)} E_{t-1}\left[\dfrac{\left(\dfrac{\exp(u_t - \varepsilon_t)}{\Theta} \right)^{1/1-\alpha}}{\left(1 - \left(\dfrac{\exp(u_t - \varepsilon_t)}{\Theta} \right)^{1/1-\alpha} \right)} \right],$

If $\sigma_t^2 = \sigma_\varepsilon^2 = 0$, then $\Theta = (\overline{H})^{-(1-\alpha)}$ would hold. Thus, (30) implicitly describes also the

determination of expected employment in the home country. In general however, the value of

Θ is going to depend on the distribution of $\exp(u_t - \varepsilon_t)$. Figure 1 illustrates how equation

(30) determines $E_{t-1}(H_t \big/ (1-H_t))$. Since this is a convex function of H_t, an increase in the

variance of H_t must reduce $E_{t-1}(H_t)$, in order to keep $E_{t-1}(H_t \big/ (1-H_t))$ constant. The model

therefore implies that there is a negative relationship between the *variance* of employment, and

the *mean* employment level.

The intuitive explanation of this relationship is that a rise in the variability of

employment, generated either by monetary shocks or technology shocks, will increase expected

marginal costs facing firms. This will lead them to set higher prices, conditional on the

predicted values of money, technology, and the capital stock. Thus, an increase in the variance

of $\exp(u_t - \varepsilon_t)$ will raise Θ. An increase in the mean price level will reduce the mean value

of employment implied by equation (27).

The value of employment and the pre-set price level for the foreign economy can likewise be calculated, using identical procedures. We reach the same conclusions; foreign money and technology uncertainty biases down the mean level of employment[9].

Dynamics

Now using the solution for prices, we may describe the full dynamic path of consumption, the capital stock, and output in each country. Substituting (29) into the equation for investment, (26), we obtain

(31) $\qquad K_{t+1} = \beta\alpha\tilde{\lambda}\exp(nu_t)\exp((1-n)u_t^*)\theta_{t-1}^{n\rho}\theta_{t-1}^{*(1-n)\rho}\left(\dfrac{1}{\Theta}\right)K_t^{\alpha}.$

Investment is affected by technology shocks only with a one period delay. Within any period, a technological improvement has no impact on consumption, investment, or output. But prices are adjusted after one period. A persistent technology shock, for instance in the home country, will lead to a fall in home country prices in the next period, which allows for an increase in consumption and investment for both countries. The rise in foreign consumption is achieved through a terms of trade deterioration for the home economy.

Since (31) depends upon Θ, it follows that the unconditional mean of the capital stock, consumption, and output are also affected by the volatility of money and technology[10]. The unconditional mean level of $\ln K$ is given by:

$$k = 1/(1-\alpha)\left(\ln(\beta\alpha\tilde{\lambda}) - \ln(\Theta)\right).$$

This is also negatively related to the volatility of money and technology shocks. The same holds true for consumption and output.

Also from (31), we may derive the unconditional variance of (the log of) consumption and investment and real GDP in the home (and foreign) economy. It is written as

$$\frac{1}{1-\alpha^2}\left(n^2 + (1-n)^2\right)\sigma_u^2 + \frac{\rho^2}{(1-\alpha\rho)^2}\left(1 + \frac{\alpha^2}{(1-\alpha^2)} + \frac{\rho^2}{(1-\rho^2)}\right)\sigma_\varepsilon^2.$$

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[9] The relationship between the volatility and mean of macroeconomic aggregates has been shown in a different context by Devereux and Engel (1998).

[10] The absolute level of the capital stock is influenced by volatility in two ways. First, a higher volatility of (for instance) domestic money raises the expectation of $\exp(nu_t)$, which increases the capital stock. But in addition, a higher volatility of money raised Θ, which reduces the capital stock. The net result of monetary volatility on the level of K_{t+1} is ambiguous.

Real GDP variance depends upon the variance of both money shocks and technology shocks. Note that the smaller is the persistence in technology shocks (i.e. the smaller is ρ), the smaller is the influence of technology shocks on output variance. If technology shocks were entirely transitory (ρ =0), output variance would be unaffected by technology shocks at all. This makes sense, since if there is no persistence in the technology shock, there is no ex post readjustment of prices to reflect the shock.

The unconditional variance of employment is in fact the same as the unconditional variance in Table 1. This is because employment is unaffected by anticipated movements in the capital stock and in technology.

Section 5: Fixed exchange rates

Since prices are pre-set, it would seem that the exchange rate regime would have a significant effect on allocations and welfare in this economy. More generally, following the discussion of the introduction, we will investigate whether the decision to fix exchange rates involves a sacrifice due to the inability of relative prices to adjust in response to supply shocks.

One important issue to confront is just how the exchange rate is fixed. Any set of monetary policies that keeps the exchange rate constant is consistent with a fixed exchange rate. In the floating exchange rate environment described above, we assumed that the home and foreign country had independent randomness in their money supplies, so that exchange rates would in general fluctuate. We have assumed that in order to fix exchange rates the monetary authorities sets money supplies the same across countries. But there are many different ways to do this. One assumption is that the money supply process in each country follows a country weighted average of the money supply processes in a floating exchange rate regime. That is

(32) $M_t = M_t^* = M_{t-1} \exp(nu_t + (1-n)u_t^*)$.

This keeps the exchange rate constant and always equal to unity. Moreover, it maintains the variance of the world money supply equal to the variance of a country weighted average of money supplies in the floating exchange rate regime. But the variance of the (log) money supply within each country is actually smaller than under floating. This represents what might be called a `cooperative peg' exchange rate regime.

An alternative assumption is that the one country adjusts its money supply to follow the policy of the other country, so as to maintain the pegged rate. If we took the foreign country as the follower, then this would entail that it set its money supply equal to that of the home country, so that

(33) $M_t^* = M_t = M_{t-1} \exp(u_t)$.

This might be called a `one-sided peg' exchange rate regime. It implies that the variance of the money supply in each country is the same as under floating (since the monetary shocks had been assumed to have equal variance to begin with). But the variance of world money supply exceeds that of the country weighted average of money supplies under floating exchange rates.

The implications of a fixed exchange rate regime are obtained by simply imposing either the monetary rule (32) or (33) on to the solutions for consumption, investment and employment from (25)-(27). Table 2 illustrates the implications for the conditional variance of consumption, investment and employment, for the cooperative peg regime. Table 3 shows the same variables in the case of the one-sided peg.

A number of conclusions can be drawn from the Tables. First, in the cooperative peg, the conditional variance of consumption and investment is unaffected by the fixed exchange rate. It is easy to see why from (25). Whether the exchange rate is fixed or floats, consumption is determined by a geometric average of national money stocks. But in the cooperative peg, each countries money stock is a geometric average of the money stocks under floating exchange rates. Thus consumption (and investment) volatility is identical under the two regimes. On the other hand, since each individual country's money supply variance falls, the conditional variance of employment also falls. Therefore, while consumption and investment volatility remains unchanged, employment volatility falls[11].

For the one-sided peg regime, the situation is exactly the reverse. Because the variance of the world money supply is higher, consumption and investment volatility is higher in a fixed exchange rate regime. But employment volatility is unchanged, since the volatility of each individual national money supply is not changed by the move to fixed exchange rates (given our assumption of identical money variances).

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[11] Employment is determined by total demand (consumption and investment) and relative prices (the terms of trade). A fixed exchange rate leaves demand volatility unchanged, but reduces terms of trade volatility, and so reduces employment volatility.

Table 2	
Conditional variance, consumption, investment and employment under fixed exchange rates (cooperative peg)	
Consumption (investment) Variance (log)	$(n^2 + (1-n)^2)\sigma_u^2$
Employment variance (log)	$\dfrac{1}{(1-\alpha)^2}\left((n^2 + (1-n)^2)\sigma_u^2 + \sigma_\varepsilon^2\right)$

Table 3	
Conditional variance, consumption, investment and employment under fixed exchange rates (one-sided peg)	
Consumption (investment) Variance (log)	σ_u^2
Employment variance (log)	$\dfrac{1}{(1-\alpha)^2}\left(\sigma_u^2 + \sigma_\varepsilon^2\right)$

The exchange rate regime will also make a difference for the average level of prices. In the cooperative peg regime, the coefficient Θ is determined by the condition:

$$(34) \qquad 1 = \frac{\lambda\eta(1-\beta\alpha)}{(\lambda-1)(1-\alpha)} E_{t-1}\left(\frac{\left(\dfrac{\exp(nu_t + (1-n)u_t^* - \varepsilon_t)}{\Theta}\right)^{1/1-\alpha}}{\left(1 - \left(\dfrac{\exp(nu_t + (1-n)u_t^* - \varepsilon_t)}{\Theta}\right)^{1/1-\alpha}\right)}\right).$$

Given that money and supply shocks are independent, it is clear that the value of Θ implied by (34) is lower than that under floating exchange rates. Therefore, average prices are lower in both countries in a cooperative peg. By implication, average employment is higher. For the one-sided peg, the condition determining average prices is the same as (30). Since the variance of money for each country is unchanged, it follows that the value of Θ is unchanged.

Therefore, the fixed exchange rate under a one-sided peg has no implications for prices, and average employment is the same as under a floating regime.

In the cooperative peg, the dynamic process for the capital stock is identical to (31), except for the fact that the Θ term is lower. In the one-sided peg, the capital stock process is

$$(35) \qquad K_{t+1} = \beta\alpha\tilde{\lambda}\exp(u_t)\theta_{t-1}^{n\rho}\theta_{t-1}^{*(1-n)\rho}\left(\frac{1}{\Theta}\right)K_t^{\alpha}.$$

where Θ is the same as under floating exchange rates.

Using (35), following the logic of the previous section, it can be seen that the unconditional mean of (log) consumption, the capital stock, and real GDP are higher in a cooperative peg than under an floating regime. Therefore, moving from a floating exchange rate to a fixed exchange rate under a cooperative peg actually *increases average GDP*.

An important implication of these results is that the response of the economy to technology shocks *is independent of the exchange rate regime*. The conditional volatility of consumption and investment does not depend on technology variance. While the conditional variance of employment does depend on technology variance, the component of employment volatility that is explained by technology variance is independent of the exchange rate regime. Intuitively, with pre-set prices, consumption, investment and output are determined solely by aggregate demand shocks. Within the period within which prices are set, technology shocks only impact on employment. This means that the exchange rate regime cannot help in any way to improve the economy's adjustment to technology shocks. Even in the face of country specific technology shocks that in the flexible price environment would require terms of trade adjustment (according to (20)), floating exchange rates do not function to provide this adjustment potential. In the absence of monetary shocks, there is in fact no difference at all between the economy with floating exchange rates and fixed exchange rates.

More generally, it can be seen from (35) that the role of technology shocks in the unconditional volatility of real GDP is independent of whether the exchange rate is fixed or floating. Technology shocks affect output, consumption and investment only with a one period lag. But this property holds across all different exchange rate regimes.

Section 6. Welfare and exchange rate regimes

We may conduct a welfare comparison of the floating exchange rate regime with the two types of fixed exchange rate regimes. Welfare may be defined as the expected utility of the

representative individual (in either country). Define the *value function* for an individual as as function of initial capital and the technology shocks from one period ago[12];

$$V = V(K_t, \theta_{t-1} \theta_{t-1}^*)$$

Given the structure of the model, it is not surprising that we can solve for the exact form of the value function. It is given by

(36) $V = A + B \ln K_t + D_1 \ln(\theta_{t-1}) + D_2 \ln(\theta_{t-1}^*)$,

where A, B, D_1 and D_2 are constants, given by

$$A = \Omega_0 + \eta E_{t-1} \ln(1 - H_t) - \frac{1+\chi}{1-\beta\alpha} \ln \Theta,$$

$$B = \frac{(1+\chi)\alpha}{1-\beta\alpha},$$

$$D_1 = \frac{(1+\chi)\rho n}{(1-\beta\alpha)(1-\beta\rho)},$$

$$D_2 = \frac{(1+\chi)\rho(1-n)}{(1-\beta\alpha)(1-\beta\rho)}.$$

Given the structure of the economy, the exchange rate regime affects only the constant term A in the value function. Recall by (27) and (29), $H_t = \left(\frac{\exp(u_t - \varepsilon_t)}{\Theta} \right)^{\frac{1}{1-\alpha}}$.

An important feature of (36) is that it does not involve the markup term $\tilde{\lambda}$. The markup pricing rule (14) involves a distortion which biases down average employment. A social planner would choose Θ so that equilibrium employment would be given by

$$\hat{H} = \frac{(1+\chi)(1-\alpha)}{(1+\chi)(1-\alpha) + \eta(1-\beta\alpha)}.$$

This always exceeds $\overline{H}$, the flexible price equilibrium employment level. The planner would like too eliminate the effects of the markup on employment. In addition, the planner would take into account the affect of employment on equilibrium real money balances [13].

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[12] In the presence of the constraint on ex ante price setting, it is natural to define welfare as a function of state variables in the date t-1 information set. In addition, the value function does not depend on the money stock since the anticipated value of the money stock has no effect on welfare.

[13] This second inefficiency is due to the fact that the presence of a positive nominal interest rate implies a deviation from the Friedman rule in monetary policy. Higher employment would increase consumption and real balances, moving the economy closer to the Friedman rule. Under our specification, however, the Friedman rule (or zero nominal interest rate) would imply an infinite level of real money balances.

Using (36) we can draw two conclusions. *First, in welfare terms, a one-sided peg and a floating exchange rate are identical.* This follows because the one-sided peg leaves the term Θ unchanged, and because it leaves the variance of H_t unchanged, relative to the floating exchange rate regime. Second, we may conclude that *the cooperative peg delivers higher welfare than under a floating exchange rate.* This follows because a) a cooperative peg reduces the volatility of employment, thereby increasing welfare directly through the second term in *A*, and b) a cooperative peg reduces Θ, thereby raising employment above its initial, inefficiently low level under floating exchange rates.

Section 7. Optimal monetary policy

The previous section showed that exchange rate adjustment had no role to play in the response of the economy to technology shocks. Since prices are pre-set, technology shocks can only influence output with a one period lag. But clearly this involves an efficiency loss for the world economy. In a flexible price economy, a home country technology shock would reduce home country prices, increase income both at home and in the foreign country, and raise consumption and investment. In the fixed price economy, the technology shock is fully absorbed by a fall in domestic employment, with no affect on income, or consumption and investment. Given the existence of price rigidity, it is natural to ask whether there is a role for monetary policy. In principle, we would expect that a pro-cyclical monetary policy could be used to expand aggregate demand in face of a technology shock that raises potential aggregate supply. To investigate this, we take the conjectured monetary rule given by

(37) $\quad M_t = M_{t-1} \exp(\varepsilon_t)$.

Incorporating this rule would make it impossible to conduct our analysis. Therefore, we abstract away from the Friedman rule. One way to think about it, following Obstfeld and Rogoff (1995, 1998), is to conduct welfare analysis only for the special case where $\chi \rightarrow 0$.

The monetary authority in each country is assumed to set the money supply in proportion to the current technology innovation[14]. Since the money supply is still a random walk (in logs), the solution procedure used above still applies. Substituting this into (25) and (26), we see that this monetary rule will lead consumption and investment to replicate that of the flexible price economy. Moreover, from (27), we see that this rule will exactly stabilize employment at the level of the flexible price economy[15],[16]. It follows that the rule will achieve two ends. First, it ensures that the economy displays the same volatility as in the flexible price economy. But because it leads to an increase in average employment, it will lead to a fall in average prices, leading to a rise in expected employment, consumption, and investment in the economy also.

Thus, the optimal monetary policy rule not only stabilizes employment in face of technology shocks, it also increases the *average* level of employment in the home and foreign economy. Using the same analysis as section 4, it follows that mean GDP is also higher under this monetary rule.

It is also apparent that not only does (37) replicate the flexible price economy, but it is the optimal monetary rule within all class of feedback rules which depend on the technology shocks. This follows because any monetary rule other than (37) would fail to stabilize employment. Thus, the volatility of employment would be higher, and the average level of employment would be lower. By (36), this would imply lower welfare than the rule (37).

A critical aspect of (37) however is that it must operate through exchange rate movements. The exchange rate under the rule (36) will be given by

$$S_t = \frac{M_0 \exp(\varepsilon_t)}{M_0 \exp(\varepsilon_t^*)}.$$

Unless technology shocks are perfectly correlated across countries, the adjustment to technology shocks under the optimal monetary policy rule will require exchange rate adjustment. Intuitively,

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[14] For simplicity omit the possibility that there are errors in the monetary rule caused by intrinsic monetary volatility such as in section 4. It is easy to see how this would affect the conclusions given below.

[15] Note that employment is still at an inefficiently low value in this economy, due to the monopolistic competition and markup pricing. But this could not be alleviated by use of monetary policy. It is clear from equation (30) that the most that monetary policy can do is to eliminate the effects of uncertainty in technology (and money) on average prices (and so average employment). Once the flexible price level of employment has been attained, any further monetary rule could only reduce employment below $\overline{H}$, by introducing more uncertainty.

A further important factor is the time consistency of monetary policy. It is well known that the distortions from monopolistic competition give rise to an incentive for surprise inflation in this economy (e.g. Betts and Devereux (1999)). We ignore these complications here by use of a fixed monetary rule given by (36).

a technology shock in the home economy requires that the relative price of the home good should fall. A compensating monetary expansion in the home economy leads to an exchange rate depreciation, which achieves the required terms of trade deterioration. For the foreign economy, their exchange rate *appreciation* leads to a rise in real money balances and consumption. At the same time, the terms of trade improvement leads to a rise in real income. The end result is that consumption and investment in both the home and foreign economy rises.

Now let us ask what is the effect of fixing the exchange rate? It is immediately clear that the optimal rule (37) is inconsistent with a fixed exchange rate. The optimal rule entails country specific monetary accommodation of technology disturbances. But under a fixed exchange rate regime, all monetary policy movements have to be exactly coordinated in the home and foreign economy. There can be no country specific monetary policy movements. Thus, loss of exchange rate flexibility has real consequences in an economy where optimal monetary policy can be employed.

Is there a limited monetary policy rule within a fixed exchange rate system that improves upon the environment without activist monetary policy? The optimal rule requires that each country's monetary policy expand to raise aggregate demand in face of technology shocks. A compromise rule, consistent with fixed exchange rates, is that the world money supply expands in response to a technology shock in either country. Take the rule, followed by each country, given by

$$(38) \qquad M_t = M_{t-1} \exp(n\varepsilon_t) \exp((1-n)\varepsilon_t^*) \, .$$

Substituting this rule into (25) and (26), we see that this rule ensures that consumption and investment respond to a technology shock in the same way as in the flexible price economy. Thus, even under fixed exchange rates, an optimal monetary rule can ensure that output, consumption and investment variance is identical to that under floating exchange rates (with an optimal monetary rule), and identical to that under flexible prices. But from (27), it is clear that the rule given by (38) does not stabilize employment. Employment (in the home economy) is now given by

$$(39) \qquad H_t = \left(\frac{(1-\beta\mu)}{(1-\beta\alpha)} \frac{M_0 \exp((1-n)(\varepsilon_t^* - \varepsilon_t))}{P_{ht}\theta_{t-1}^{\rho}K_t^{\alpha}} \right)^{\frac{1}{1-\alpha}} \, .$$

---

[16] A recent paper by Obstfeld and Rogoff (1999) makes a similar point regarding the optimality of the flexible price equilibrium as a target for a monetary rule.

While the monetary rule can ensure that real GDP in each country responds efficiently to technology shocks, the equilibrium response of employment requires terms of trade adjustment, which cannot occur under a fixed exchange rate. For instance, a technology shock in the home economy requires that real GDP increase in both the home and foreign countries. This much can be achieved with a world monetary expansion. But in the floating exchange rate economy (with the optimal monetary policy), the increase in real GDP at home is achieved by a rise in production in proportion to the technology shock, and a fall in the terms of trade, while in the foreign economy it is achieved by no change in production, and an improvement in the terms of trade.

With a fixed exchange rate, the terms of trade cannot change. Therefore, production must increase in both economies[17]. This means that production rises by too little in the home economy (as employment partially falls in response to a home technology shock, in place of the terms of trade deterioration that would take place under the optimal rule (37)), and too much in the foreign economy (as foreign employment increases in response to the technology shock, in place of the terms of trade improvement that would take place under (37)).

As a corollary, we see that the fixed exchange rate economy will have higher prices than the floating exchange rate economy. Using the monetary rule (38) in the condition (29), it is clear that the value of Θ under fixed exchange rates exceeds that under floating exchange rates. It follows that average employment, real GDP, and welfare is lower under a fixed exchange rate.

Section 8. Introducing Fiscal Policy Shocks

In this section we very briefly sketch out the way in which government spending shocks can be introduced into the model. The central question is whether the presence of shocks to government spending alter the main features of the trade-off between fixed and floating exchange rates outlined in previous sections. The answer is no. Even with government spending shocks as separate demand side disturbances, we reach the same conclusions.

Assume that each government chooses its spending in a similar to that of the private sector. Thus, home country firms face the demand from governments of both countries given by

$$(39) \qquad g(i, z^t) = \left(\frac{p_h(i, z^t)}{P_h(z^t)} \right)^{-\lambda} (G_h(z^t) + G_f(z^t)),$$

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[17] Note the real GDP of the home economy is equal to production y_t times the terms of trade P_{ht}/P_t.

$$(40) \qquad g(i, z^t) = \left(\frac{p_f(i, z^t)}{P_f(z^t)} \right)^{-\lambda} (G_h^*(z^t) + G_f^*(z^t)).$$

Here $G_h(z^t)$ is government spending of the home government on home country goods, $G_h^*(z^t)$ is home country government spending on foreign goods, etc.

Government spending shocks are country specific[18]. We assume that

$$G_h(z^t) = G_f(z^t) = g_t \, y(z^t) \,,$$

$$G_h^*(z^t) = G_f^*(z^t) = g_t^* \, y^*(z^t).$$

where $0 \le g_t \le 1 \ 0 \le g_t^* \le 1$, and $g_t = 1 - \exp(\upsilon_t) \quad g_t^* = 1 - \exp(\upsilon_t^*)$.

Government spending on home goods is a random linear function of home output, and similarly for government spending on foreign output. To maintain analytical simplicity, we assume government spending shocks are i.i.d. across time and countries. Qualitatively, these assumptions have no impact on the results.

The analysis of government spending shocks is derived in the Appendix. In what follows we briefly present the main results, confirming that the arguments established in the previous section also apply in the case of fiscal policy shocks.

The Flexible Price Economy

Under flexible prices, the economy with government shocks

$$(41) \qquad K_{t+1} = \beta \alpha \tilde{\lambda} (1 - g_t) \gamma \theta_t^n \theta_t^{*(1-n)} K_t^\alpha H_t^{n(1-\alpha)} H_t^{*(1-n)(1-\alpha)} \,,$$

$$(42) \qquad \frac{P_{ht}}{S_t P_{ft}} = \frac{\theta_t^*(1 - g_t^*)}{\theta_t(1 - g_t)} \,,$$

$$(43) \qquad C_t = \frac{(1 - \beta \alpha \tilde{\lambda} \gamma)}{\beta \alpha \tilde{\lambda} \gamma} K_{t+1} \,,$$

$$(44) \qquad H_t = \frac{(\lambda - 1)(1 - \alpha)}{((\lambda - 1)(1 - \alpha) + \lambda \eta (1 - g_t)(1 - \beta \alpha \tilde{\lambda} \gamma)} \,.$$

---

[18] With common government spending shocks, there would be no need for terms of trade/exchange rate adjustment at all.

where $\gamma = E_t\left(\dfrac{1}{1-g_t}\right) = E_t\left(\exp(\upsilon_{t+1})\right)$, which is a constant.

In the flexible price economy, country specific government spending shocks require terms of trade adjustment, according to (42). While consumption and investment are still equated across countries, employment is now time varying, and not equal across countries. A transitory domestic government spending shock will entail an increase in domestic employment.

Sticky Prices

With preset nominal prices, consumption will again be determined as in equation (25). Therefore, government spending does not directly affect consumption during the period of the spending increase. As a result, government spending does not affect the exchange rate. The consumption investment ratio is now

$$\frac{C_t}{K_t} = \frac{(1-\beta\alpha\tilde{\lambda}\gamma)(1-g_t)}{\beta\alpha\tilde{\lambda}\gamma}.$$

Employment (in the home country) is determined by the condition

$$H_t = \left(\frac{\exp(u_t - \varepsilon_t + \upsilon_t)}{\Theta}\right)^{\frac{1}{1-\alpha}}.$$

Employment is increasing in shocks to the money supply, decreasing in productivity shocks, and increasing in shocks to government spending. The term Θ, as before, is increasing in the volatility of monetary, technology, and government spending shocks.

When monetary policy is passive, a move from floating to fixed exchange rates looks very much the same as before. Under a cooperative peg regime, the volatility of consumption, investment and real GDP is unaffected, but the volatility of production and employment is reduced. As a result, the mean level of employment is higher under the cooperative peg. Under a one-sided peg, the volatility of consumption, investment, and real GDP is magnified, relative to a floating exchange rate, but the volatility and mean level of employment is unchanged. More importantly, the exchange rate regime has no relevance for the response of the economy to government spending shocks.

By contrast, with active monetary policy, there is an optimal monetary rule, given by

$$(45) \quad M_t = M_{t-1}\exp(\varepsilon_t - \upsilon_t)\left(1 + \frac{\eta\exp(-\upsilon_t)(1-\beta\alpha\tilde{\lambda}\gamma)}{\tilde{\lambda}(1-\alpha)}\right)^{-(1-\alpha)},$$

which will replicate the flexible price economy. The rule (45) is more complicated than that for technology shocks alone, because in face of government spending shocks, a monetary policy rule designed to replicate the flexible price economy should not stabilize employment. The first term in the rule (45) implies a negatively relationship between money and government spending shocks, while the second term implies a positive relationship between money and government spending shocks. The first term arises because consumption would fall in face of a government spending increase in the flexible price economy, so this fall in consumption is achieved by monetary contraction. But if the monetary contraction were as much as indicated by the first term, consumption would fall so much that domestic production and employment would be unaffected by the shock. The second term indicates that this monetary contraction is mitigated somewhat by the need for overall aggregate demand to increase, so as to increase employment towards the desired, flexible price level. Overall, however, since consumption must be reduced in response to a government spending shock, relative to the sticky price economy (where it does not move at all), it must the case that the optimal monetary rule is negatively related to government spending shocks.

Clearly, the rule given by (45) requires that the nominal exchange rate be free to move in response to the country specific optimal monetary rules. Under any type of pegged exchange rate arrangement, this monetary rule cannot be implemented. Therefore, the central message of the previous sections is upheld in the presence of fiscal policy shocks.

Section 9. Conclusions

This paper has examined the trade-off between fixed and floating exchange rates in a sticky-price dynamic, stochastic general equilibrium model with capital accumulation. The results suggest that the trade-off is quite at variance with much of the discussion in the policy literature. In our model, allowing the exchange rate to float does not help at all in the response to country- specific supply or demand policy shocks. In fact, fixed exchange rates may do better, by increasing employment and long run GDP, as well as welfare. But if the benchmark comparison is one where monetary policy can be `activist', adjusting efficiently in response to macroeconomic shocks, then giving up exchange rate flexibility will have real costs, both in terms of macroeconomic volatility and average long run GDP.

A number of qualifications should be made about the analysis. Very special functional forms are used in order to facilitate an analytical solution. In a more general model, it is not clear that the sharp conclusions of our analysis would remain in complete. For instance, with a

more general money demand schedule, interest rates would be variable, and non-monetary shocks would influence the exchange rate. But it is worthwhile to note that the functional forms used are very close to those employed in the quantitative International Real Business Cycle literature, so it is not clear how limiting the present model is. In general however, the special assumptions of the model make it difficult to assess the full quantitative effects of alternative exchange rate regimes. In particular, to properly assess the impact of monetary shocks on the economy, one would presumably need to allow for a more persistent degree of price rigidity, along the lines of Calvo (1983) or Taylor (1979). Extending our model to allow for this would make it necessary to employ numerical solution methods. This is left for future research.

Nonetheless, our analysis may throw some light on the debate about the costs and benefits of exchange rate flexibility. In this vein, we might interpret the results as providing a cautionary note about the adjustment properties of floating exchange rates.

Consumption Insurance

The goods market clearing condition for the home and foreign country's good are written as

(A1)

$$\theta(z^t)K(z^{t-1})^{\alpha}H(z^t)^{1-\alpha} = \frac{P(z^t)}{P_h(z^{t-1})}[nC(z^t)+(1-n)C*(z^t)+nV(z^t)+(1-n)V^*(z^t)]$$

. (A2)

$$\theta^*(z^t)K(z^{t-1})*^{\alpha}H(z^t)*^{1-\alpha} = \frac{P*(z^t)}{P_f(z^{t-1})}[nC(z^t)+(1-n)C*(z^t)+nV(z^t)+(1-n)V^*(z^t)]$$

Divide equation (A1) by equation (A2) to get

(A3) $$\frac{P_h(z^{t-1})}{P(z^t)}\theta(z^t)K(z^{t-1})^{\alpha}H(z^t)^{1-\alpha} = \frac{P_f(z^{t-1})}{P*(z^t)}\theta*(z^t)K(z^{t-1})*^{\alpha}H(z^t)*^{1-\alpha}$$

Equation (A3) says that real GDP is equated across countries. Now take the budget constraint (3) of the text, and substitute into it the money market equilibrium $m(z^t) = m(z^{t-1})+TR(z^t)$. Do the same for the foreign country. Using the bond market clearing condition $B(z^t)+B*(z^t) = 0$, we arrive at the two conditions

(A4) $$C(z^t)+V(z^t)+q(z^t)\frac{B(z^t)}{P(z^t)} = \frac{B(z^{t-1})}{P(z^t)}+\frac{P_h(z^{t-1})}{P(z^t)}\theta(z^t)K(z^{t-1})^{\alpha}H(z^t)^{1-\alpha}$$

(A5) $$C*(z^t)+V*(z^t)+q(z^t)\frac{B*(z^t)}{P*(z^t)S(z^t)}$$

$$= \frac{B*(z^{t-1})}{P*(z^t)S(z^t)}+\frac{P_f(z^{t-1})}{P*(z^t)S(z^t)}\theta*(z^t)K(z^{t-1})*^{\alpha}H(z^t)*^{1-\alpha}$$

Define the common value of real GDP to be $Y(z^t)$. Then, because each country faces a common world interest rate on bonds, and chooses an optimal pattern of consumption according to (5), we have

(A6)

$$\sum_{z^{t+1}}\pi(z^{t+1})\frac{P(z^t)C(z^t)}{P(z^{t+1})C(z^{t+1})} = \sum_{z^{t+1}}\pi(z^{t+1})\frac{P*(z^t)C*(z^t)S(z^{t+1})}{P*(z^{t+1})C*(z^{t+1})S(z^t)} = \sum_{z^{t+1}}\pi(z^{t+1})\frac{P(z^t)C*(z^t)}{P(z^{t+1})C*(z^{t+1})}$$

The last equality follows due to PPP.

Now assume that investment is equal across countries, so that $V(z^t) = V^*(z^t)$ (this will be verified later). Then substitute from (A4) and (A5) in the first and last expression in (A6) to get (A7)

$$\sum_{z^{t+1}} \pi(z^{t+1}) \frac{P(z^t)(Y(z^t) - V(z^t) - q(z^t)b(z^t) + b(z^{t-1}))}{P(z^{t+1})(Y(z^{t+1}) - V(z^{t+1}) - q(z^{t+1})b(z^{t+1}) + b(z^t))}$$

$$= \sum_{z^{t+1}} \pi(z^{t+1}) \frac{P(z^t)(Y(z^t) - V(z^t) + q(z^t)b(z^t) - b(z^{t-1}))}{P(z^{t+1})(Y(z^{t+1}) - V(z^{t+1}) + q(z^{t+1})b(z^{t+1}) - b(z^t))}$$

where $b(z^t) = \dfrac{B(z^t)}{P(z^t)}$. Beginning at $b(z^{t-1}) = 0$, the only solution to equation (A7) must imply

that $b(z^t) = b(z^{t+1}) = 0$. Thus, if there is no outstanding international debt at the beginning of time, then each country will have a zero current account balance. Then, from (A4) and (A5), it must be that consumption is equal across countries in all time periods.

It remains to verify that investment is the same in the home and foreign countries. To check this, note that investment is determined by the condition

$$1 = \beta \sum_{z^{t+1}} \pi(z^{t+1}) \frac{C(z^t)R(z^{t+1})}{C(z^{t+1})P(z^{t+1})}$$

Substitute in for $R(z^{t+1})$, and rearrange to get

$$(A8)\ V(z^t) = K(z^t) = \beta\alpha \sum_{z^{t+1}} \pi(z^{t+1}) \frac{C(z^t)Y(z^{t+1})}{C(z^{t+1})}$$

This gives home country investment. But the right hand side of (A8) is identical for the home and foreign country. Therefore investment is equal across countries.

Pricing

We assume that firms must set nominal prices one period in advance. The first order condition for the firm in setting the price $p(i, z^{t-1})$ is given by

$$\sum_{z^t} \pi(z^t)\beta \frac{P(z^{t-1})C(z^{t-1})}{P(z^t)C(z^t)} \left(x(i, z^t) - \lambda x(i, z^t) + MC(z^t)\lambda \frac{x(i, z^t)}{p(i, z^{t-1})} \right) = 0$$

First substitute in for the expression $x(i, z^t)$. Then impose a symmetric equilibrium so that $p(i, z^{t-1}) = p(z^{t-1})$. Rearranging, we arrive at the pricing equation (14) of the text.

Solution of the model under flexible prices

When prices are flexible, equation (A3) determines the endogenous terms of trade (given equal capital stocks across countries):

(A9) $\quad \dfrac{P_{ht}(z^t)}{S(z^t)P_f(z^t)} = \dfrac{\theta(z^t)H(z^t)^{1-\alpha}}{\theta*(z^t)H(z^t)*^{1-\alpha}}$

Now substitute (A9), (15) of the text, and the budget constraint (A4) into the investment equation, ((8) of the text), to get

(A10) $\quad \dfrac{1}{\theta(z^t)^n \theta*(z^t)^{1-n} K(z^{t-1})^\alpha H(z^t)^{(1-\alpha)n} H*(z^t)^{(1-\alpha)(1-n)} - K(z^t)}$

$= \beta \sum_{z^{t+1}} \pi(z^{t+1}) \dfrac{\alpha \tilde{\lambda} \theta(z^{t+1})^n \theta*(z^{t+1})^{1-n} K(z^t)^\alpha H(z^{t+1})^{(1-\alpha)n} H*(z^{t+1})^{(1-\alpha)(1-n)}}{\theta(z^{t+1})^n \theta*(z^{t+1})^{1-n} K(z^t)^\alpha H(z^{t+1})^{(1-\alpha)n} H*(z^{t+1})^{(1-\alpha)(1-n)} - K(z^{t+1})}$

From (A10), conjecture that investment is a constant linear function of real GDP, i.e.

$K(z^t) = \Gamma_0 \theta(z^t)^n \theta*(z^t)^{1-n} K(z^{t-1})^\alpha H(z^t)^{(1-\alpha)n} H*(z^t)^{(1-\alpha)(1-n)}$

where Γ_0 is the conjecture. Substituting this into (A10) establishes that $\Gamma_0 = \beta \alpha \tilde{\lambda}$.

Using this and (A4), it follows that consumption is given by equation (21) of the text. Substituting this into (7) establishes that employment is constant and given by (22). Finally, with constant employment, (A9) gives equation (20).

The model solution with sticky prices

Floating exchange rates

First we show that the nominal interest rate is constant under the specification of money demand given by (6). Taking equation (6) for the aggregate economy, substitute for the definition of the nominal interest rate, and for the money supply process (17), to get

(A11) $\quad \exp(m_t) = P_t C_t \dfrac{1}{(1 - \beta E_t \dfrac{P_t C_t}{P_{t+1} C_{t+1}})}$

Update (A11) for one period, then divide one equation by the other, and take expectations dated t, to get

(A12) $\quad E_t \exp(u_{t+1}) = E_t \dfrac{P_t C_t}{P_{t+1} C_{t+1}} \dfrac{\left(1 - \beta E_{t+1}\left(\left.\dfrac{P_{t+1}C_{t+1}}{}\right/ P_{t+2}C_{t+2}\right)\right)}{\left(1 - \beta E_t\left(\left.\dfrac{P_t C_t}{}\right/ P_{t+1}C_{t+1}\right)\right)}$

The left hand side of (A12) is constant (equal to μ) given (17). Since (A12) represents a difference equation in the term; $E_t \dfrac{P_t C_t}{P_{t+1} C_{t+1}}$. A non-explosive solution must imply that

$E_t \dfrac{P_t C_t}{P_{t+1} C_{t+1}}$ is constant and equal to μ.

Consumption is determined by equation (25). Then from equation (8) of the text, we may write

(A13) $\quad \dfrac{1}{C_t} = \beta \alpha \tilde{\lambda} E_t \dfrac{\dfrac{P_{ht+1}}{P_{t+1}} y_{t+1}}{C_{t+1} K_{t+1}}$

Conjecture that consumption is a constant linear function of real GDP, $\dfrac{P_{ht}}{P_t} y_t$.

Substituting this into (A13) leads to the investment rule given by $K_{t+1} = \beta \alpha \tilde{\lambda} \dfrac{P_{ht}}{P_t} y_t$.

Then the conjecture is established using (A4). The results in equation (26) of the text.

Given consumption and investment, domestic production is determined from the condition (15) of the text

(A14) $\quad y_t = \dfrac{P_t}{P_{ht}} (C_t + K_{t+1}) = \dfrac{P_t}{P_{ht}} \dfrac{C_t}{(1 - \beta \alpha \tilde{\lambda})}$

Substituting into (A14) for the definition of the production function gives the employment equation (27).

To derive equation (28), take the pricing equation (14), and substitute for the labor supply equation (7) and the factor pricing equation (14), to derive

$P_{ht} = \dfrac{\lambda \eta}{(\lambda - 1)(1 - \alpha)} E_{t-1} \left(\dfrac{P_t C_t H_t}{(1 - H_t) y_t} \right)$

Now since nominal consumption is a constant fraction $1 - \beta \alpha \tilde{\lambda}$ of nominal income, we obtain

(A15) $\quad 1 = \dfrac{\lambda \eta (1 - \beta \alpha \tilde{\lambda})}{(\lambda - 1)(1 - \alpha)} E_{t-1} \left(\dfrac{H_t}{(1 - H_t)} \right)$

Substituting for employment from (26) gives equation (28).

Fixed exchange rates

To obtain the solution of the model under fixed exchange rates, just take the analysis of (A11) –(A15) above, and use either the monetary rule (32) (for a cooperative peg regime) or (33) (for the one-sided peg regime).

Deriving the Value Function

To obtain the value function (36), note that the $V(K_t, \theta_{t-1} \theta_{t-1}^*)$ is defined by

$$V(K_t, \theta_{t-1} \theta_{t-1}^*) = \ln C_t + \chi \ln \frac{M_t}{P_t} + \eta \ln(1 - H_t) + \beta E_{t-1} V(K_{t+1}, \theta_t, \theta_t^*)$$

Substitute for the conjectured value function (36). Then substitute for the equilibrium values of consumption, real money balances, and employment. Then we may solve for the coefficients A, B, D_1, and D_2 given in Section 6.

The solution with fiscal policy shocks

In the presence of fiscal policy shocks, the solution must take account of government spending in the budget constraints, (A4) and (A5), and this leads to a different rule for the consumption to GDP ratio derived from (A13). In addition, the fiscal policy shocks enter into the equation determining expected employment; (A15), in a straightforward way as shown in the text.

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Figure 1