A non-Bayesian approach to decision-making with irreversibilities

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Abstract

In this paper, we reconsider the irreversibility theory in a non-Bayesian framework. First, we propose three definitions in order to make the difference between the effects of the irreversibilities and of the information which were mixed in the standard definition of the "irreversibility effect" proposed by Arrow-Fisher [1] and Henry [10]. The agent faces total uncertainty and uses the Max-min criterion. We consider two types of model. The former deals with decisional irreversibilities while the latter considers also accumulation process. Our results are similar to the ones of the literature. Yet, we notice important quantitative differences. The Max-min criterion does not lead necessarily to more flexible decision than Expected Utility and there is no probability distributions that could produce the Max-min decisions.

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1 Introduction

Decision makers have to face many problems with uncertainty, irreversibilities and information. We just have to mention problems such as global warming, the "crazy" cow crisis, or the introduction of genetically modified organisms... How those problems have to be managed? Do decision makers have to take measures immediately in spite of uncertainties or to wait for more information? Should we be more careful because of irreversibilities? Many environmental problems can be formalised in the irreversibility theory framework. It can enable us to answer some questions of this type. This theory gives an interesting concept, which is the "irreversibility effect", shown by Arrow and Fisher [1] and Henry [10]. The standard definition of this "irreversibility effect" is the following : "*to a finer information structure, must be associated a less irreversible current decision*". Such a

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result is against the learn then act. The wait of information must not lead us to put back environmental preservation decisions.

In this work, we consider a particular type of irreversibility : a current decision c will be said to be less irreversible than a decision c' if the set of future possible choices following decision c contains the set of future possible choices following decision c'. This definition corresponds to the impossibility of a flashback. For example, it is impossible to decrease a CO_2 stock in the atmosphere. Formally, it corresponds to an infinite adjustment cost. It is an hard irreversibility. A more wide type of irreversibility would be to consider adjustment but with finite cost.

The lighting brought by the irreversibility theory is adequate,because environmental issues have three 3 main features of the decisional irreversibility problems :

1. *Uncertainty* : consequences (costs and benefits) of the current decision are uncertain. For example, we do not know what the consequences of greenhouse gas emissions will be...

2. *Uncertainty can possibly be solved by the time :* research enables to understand causalities better. Concerning global warming, models of climate become finer.

3. *Sequential decisions* : a succession of decisions has to be taken. Their consequences are more or less irreversible for the future. In the global warming issue, we have to choose the greenhouse gas emissions level, which will accumulate in the atmosphere.

What are the results of the literature? The "irreversibility effect" was proved by Arrow and Fisher [1]and Henry [10] then by Freixas and Laffont [7]in a more general context. Nevertheless,those models have specific features, that is the intertemporal separability : the current decision does not appear in the utility function, it only defines the constraint of future decision. But in the considered environmental issues, a stock phenomenom exists. This one generates intertemporal externalities. So, it seems more realistic to suppose that current decision is an argument of the future utility function. However, results do not allow to generalise the "irreversibility effect".

We must add another limit. The theoretical framework of this literature is only the Bayesian one : agents are endowed with probabilities and the information process is built on the Bayes rule. These hypothesis are not realistic for our problems : uncertainty on the costs of global warming... Moreover supposing that the decision maker maximises an expected utility with subjective probabilities is not more satisfactory. The Ellsberg's paradox [6]showed that with ambiguity most people do not conform to this model.

Our aim is to reconsider those decision problems with irreversibilities in a non-Bayesian framework. We propose to extend the analyse on two points.

1. *Uncertainty not entirely probabilised* : we will consider cases in which the agent would have ambiguous beliefs, for example family of probabilities, instead of having a unique probabilities distribution. It is more realistic. For instance, if we appeal to experts

to assess subjective probabilities, their valuation would be for sure different. One could compute a mean, i.e. one could use the Bayesian framework. But we would loose some elements : the different assessings reflect the ambiguity of knowledge at a certain time. We can keep the family of probabilities in using the Gilboa and Schmeidler'model [8] (Max-min criterion of the expected utility valued on the family).

2. *Information structure reflecting the ambiguity decrease* : In such a situation, new informations can lead to reduce ambiguity, i.e. a restriction of the family of probabilities. Information processes considered by Chassagnon and alii [3]and Chateauneuf and alii [4]are built on this idea. Dynamic optimisation is possible in the Gilboa and Schmeidler'model [8] with such processes, whereas non-Bayesian models are confronted to dynamic consistency problems with partitional informations.

Here we are in the first steps. We consider total uncertainty and simple information processes. So we propose an extreme case which permits to show clearly the differences with Bayesian framework. Under total uncertainty, the Gilboa and Schmeidler'model [8] is similar to the Max-min criterion. In conclusion, we discuss on the extend of the obtained results for more general processes. In a theoretical point of view, our analyse permits to test the range of the irreversibility effect and to study the extend of the literature results. In an operational point of view, it enables to establish the consequences of the choosen decision criterion .

Besides, we propose to make clear the irreversibility effect. Indeed, we propose three definitions in order to make the difference between the effects of the irreversibilities and of the information which were mixed in the standard definition of the "irreversibility effect".

We proceed as follow. In the second section, we present the general formulation of the model. We show that it permits us to deal with specific models treated in the literature. We try and clarify the notion of "irreversibility effect" and we propose a new typology. Technical results of the general model are in the appendix. In the third section, we analyse models with intertemporal separability (Arrow and Fisher [1], Henry [10], Freixas and Laffont [7]...). Our results extend those of the literature to a non-Bayesian framework. We consider quantitative differences between a total uncertainty and a probabilistic situation. Surprisingly, the Max-min criterion does not lead necessarely to make more careful choices. In the fourth section, we consider models without intertemporal separability (Gollier and alii [9], Kolstad [11], Ulph and Ulph [12]). We focus on the Ulph-Ulph model. Our results are similar to those of the literature. However, our revisiting of the "irreversibility effect" makes clear the part of the irreversibility constraint. We show that informationnal effect is not systematic but informationnal irreversibility effect holds : as it were, if more information does not imply necessarily to be more precautionnous, on the other hand irreversibilities always expand informationnal effect when it exists. In the fifth section, we conclude on the theoretical and operationnal applications of those results and we discuss on their generalisation.

2 The model

The general model we propose is simplified at the most so that difference of results between our approach and a bayesian approach appear clearly.

2.1 The utility function and features of the problem

We consider a sequential decision-making problem in which the payoff function is :

$$U_1(c_1) + U_2(c_{2,}\delta c_1 + c_2, \theta)$$

where $c_1 \geq 0$ denotes the choice in period 1, c_2 in period 2, $\delta \geq 0$, is a decay factor and θ the state of the world.

We formalize the characteristics of the problem in this way :

1. *Uncertainty :* we suppose 2 possible states of the world : $\Omega = \{\theta_1, \theta_2\}$, the agent faces a total uncertainty situation, that is to say he considers that all probability distributions are possible. In part 3, we will compare the results with the Bayesian ones where the agent would be endowed in period 1 with probabilistic beliefs $(p, 1-p)$ on Ω.

2. *Information :* for comparative statics , we will consider 2 structures of information. We will compare the case of perfect learning (the decision maker learns the true state of the world prior to the choice of c_2), with the case of no learning (the decision maker still does not know which state of the world will arise).

3. *Irreversibility :* in our models, we study a hard irreversibility constraint : a decision in first period is said to be more irreversible than an other decision if it restricts the set of choices in period 2. Irreversibility is captured by the constraint $c_2 \geq 0$. Then, if we consider the value $\delta c_1 + c_2$ from which the utility of period 2 depends, it can take only the values greater than δc_1 with the constraint $c_2 \geq 0$. So the choice of c_1 is said to be a more irreversible decision than the choice of c_1' if $c_1 > c_1'$.

The shape of the utility function enables us to analyse as special cases several models of the litterature.

- *Decision with irreversibility in the choice of the level of development*

The seminal works considered payoffs functions of the form : $V_1(x) + V_2(y, \theta)$ with the constraint set $y \geq x$. The variable x can be viewed as the level of development reached in period 1, y the level of development in period 2. The irreversibility constraint reflects the impossibility of a flashback, θ the uncertainty of the rentability of the project.Supposing $c_1 = x$ and $c_2 = y - x$, conducts to have $V_1(c_1) + V_2(c_1 + c_2, \theta)$ with the irreversibility constraint $c_2 \geq 0$.It is well a special case of our function considered. Nevertheless, the choice in period 1 appears in the payoff function of period 2 through a stock effect but it is the choice in period 2 which is no more a source of utility.

- *The optimal level of greenhouse gas emissions.*

The global warming issue can be treated by those models :

- there is an uncertainty about the consequences of greenhouse gas emissions.
- information comes with the pass of time.
- greenhouse gas emissions accumulates in the atmosphere, it is impossible te depollute.

Recent works have analysed this question supposing the intertemporal utility function such as :

Gollier and alii [9] take : $U_1(c_1) + U_2(c_2 - \theta(\delta c_1 + c_2))$ subject to $c_1 \geq 0, c_2 \geq 0$. c_1 and c_2 are the greenhouse gas emissions in period 1 and in period 2, δ is a decay factor, it represents the stock of greenhouse gases that survives from one period to the next. θ is the random variable, it determines how damaging the stock of emissions is.

Ulph and Ulph [12], consider a payoff function with separability in period 2 between utility of consumption and the damage : $U_1(c_1) + U_2(c_2) - \theta D(\delta c_1 + c_2)$ with the constraints $c_1 \geq 0, c_2 \geq 0$, the same meanings of the variables and $D(\delta c_1 + c_2)$ denotes the damage caused by the greenhouse gas emissions. D is an increasing and convex function. We will analyse a generalisation of this model considering a damage function $D(\delta c_1 + c_2, \theta)$. With the function $\theta D(\delta c_1 + c_2)$, uncertainty is only on the intensity of damage. There is no uncertainty on the mecanism of the relation stock-damage. A more general function can include this type of incertainty, which corresponds to the uncertainties nowadays.

It concerns 2 special cases of our decision problem. In these models, the irreversibility problem is compounded by the intertemporal externalities. Indeed the payoff function has now 2 arguments.

2.2 What is the irreversibility effect?

The definition of irreversibility effect introduced originally in the framework of decisionnal irreversibility problem seems us inadequate in a more general framework. That is the reason why we give another typology. Let $c_1^{I,AI}, c_1^{F,AI}$ be respectively the optimal choices in period 1 in the case of no learning and irreversibilities, and in the case of no learning without irreversibilities and let $c_1^{I,IP}$, $c_1^{F,IP}$ be respectively the optimal choices in period 1 in the case of perfect learning and irreversibilities, and in the case of perfect learning without irreversibilities.

The standard definition of the irreversibility effect consists in comparing $c_1^{I,AI}$ and $c_1^{I,IP}$, that is to say the effect of an improvment of information when there are irreversibilities constraints. In decisional irreversibility problems, under standard conditions on the form of utility functions, we have $c_1^{I,AI} \geq c_1^{I,IP}$ in a Bayesian framework. That means that an improvment of expected information leads to a more flexible decision in period 1. This effect does not exist without irreversibility constraint. Indeed, we have $c_1^{F,AI} = c_1^{F,IP}$. Introducing intertemporal externalities makes the problem more complex, since $c_1^{F,AI}$ differs from $c_1^{F,IP}$. Indeed, without the irreversibility constraint, there is no more restriction on the reachable values of $\delta c_1 + c_2$, but it means, for example, that for reaching a fixed goal for the value of the stock $\delta c_1 + c_2$ we will have to diminish c_2 if we increase c_1 which can induce a decrease of utility in period 2. Thus, even though hard irreversibilities are remove, there are still adjustments costs. In that manner, even without hard irreversibility constraint, $c_1 > c_1'$ means again that decision c_1 is more irreversible than decision c_1'.

Gollier and alii [9] call $c_1^{F,AI} \geq c_1^{F,IP}$ a precautionnary effect. We call "informationnal effect". A situation in which we would have $c_1^{I,AI} = c_1^{F,AI} \geq c_1^{I,IP} = c_1^{F,IP}$ is possible in a case where irreversibility constraint would not bite for the optimal choice in period 2. Can we call this an irreversibility effect, whereas the effect is not relied to the irreversibilities constraint? Obviously, no. Information plays a part that we have to distinguish from the irreversibility one. The typology suggested enables to avoid confusion of the different effects.

Definition 1 : *There is a pure irreversibility effect if for a given information structure, irreversibility constraint induces more flexible decisions in period 1 (i.e :* $c_1^{I,AI} \leq c_1^{F,AI}$ *and* $c_1^{I,IP} \leq c_1^{F,IP}$ *)*

This effect is not usually studied in the literature[1] whereas it corresponds to the more natural idea of an "irreversibility effect". The irreversibility constraint introduces a trade off for the decision maker between his first period aims and hisdecisionnal flexibility in period 2. We have to check that the trade off induces the decision maker to be more precautionnous, this one willing to relax his constraints of choice in period 2.

Definition 2 : *There is an informationnal effect if without irreversibility constraint, a finer information structure induces a less irreversible decision in period 1 (i.e :* $c_1^{F,AI} \geq c_1^{F,IP}$ *)*

Sometimes, we call informationnal effect in case of the irreversibility constraint to mean that $c_1^{I,AI} \geq c_1^{I,IP}$, which we usually denotes by "irreversibility effect".

[1] It seems perhaps too obvious?

Definition 3 : *There is an informationnal irreversibility effect if irreversibility accentuates the informationnel effect : if* $c_1^{F,AI} \geq c_1^{F,IP}$ *then* $c_1^{I,AI} \geq c_1^{I,IP}$

It is a second order effect : the introduction of the irreversibility constraint maintains the informationnal effect we had without irreversibility[2]. When $c_1^{F,AI} = c_1^{F,IP}$, informationnal irreversibility effect is equivalent to the "irreversibility effect" indicated in the literature. Thus the informationnal irreversibility effect has been demostrated in the Bayesian framework for the decisionnal irreversibilities. This definition is ours, so there is no result concerning it for the model without intertemporal separability.

2.3 Implementation of the Max-min criterion.

At the beginning,the decision maker faces a total uncertainty. Then, we suppose he assesses the possible options in accordance with the Max-min criterion. In view of the information structure and the irreversibilities constraints, he will have to face one of the four following sequential decision problems :

- *No learning and irreversible decision.*

When the DM makes her choice in period 2, she has no additional information and her optimal choices will amount to : $\underset{c_2\geq 0}{Max}\{\underset{\theta_i}{Min}\{U_2(c_2, \delta c_1{+}c_2, \theta_i)\}\}$. In fact, her maximization main of her temporal utility leads her to choose and value her optimal plan in this way :

$$\underset{c_1\geq 0}{Max}\{U_1(c_1^{I,AI}) + \underset{c_2\geq 0}{Max}\{\underset{\theta_i}{Min}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}\}\}$$

In a Bayesian framework where the agent would have probabilised beliefs $(p, 1-p)$ on $\{\theta_1, \theta_2\}$, the maximization problem would be :

$$\underset{c_1\geq 0}{Max}\{U_1(c_1^{I,AI}) + \underset{c_2\geq 0}{Max}\{pU_2(c_2, \delta c_1 + c_2, \theta_1) + (1-p)U_2(c_2, \delta c_1 + c_2, \theta_2)\}\}$$

- *Perfect learning and irreversible decision.*

  At the beginning of period 2, the DM knows the true state of the world θ_i and her optimal choice will be to maximize $\underset{c_2\geq 0}{Max}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}$. He anticipates his optimal choice in period 2 but he doesn't know which information he will receive. So he assesses ex ante the value of his choice in period 2 by : $\underset{\theta_i}{Min}\{\underset{c_2\geq 0}{Max}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}\}$ and in fact, his maximization goal of intertemporal utility leads him to choose et value his optimal choice like this :

$$\underset{c_1\geq 0}{Max}\{U_1(c_1^{I,AI}) + \underset{\theta_i}{Min}\{\underset{c_2\geq 0}{Max}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}\}\}$$

[2] We would have liked to quantify this "increase", but we can't prove , for example, that $(c_1^{I,AI} - c_1^{I,IP}) \geq (c_1^{F,AI} - c_1^{F,IP})$.

In a Bayesian framework, we would have :

$$\underset{c_1 \geq 0}{Max}\{U_1(c_1) + \{p \underset{c_2 \geq 0}{Max} U_2(c_2, \delta c_1 + c_2, \theta_1) + (1-p) \underset{c_2 \geq 0}{Max} U_2(c_2, \delta c_1 + c_2, \theta_2)\}\}$$

- *No learning and reversible decision.*

The decision and valuation is similar to the first case excepted that there is no more irreversibility constraint $c_2 \geq 0$. So the criterion is :

$$\underset{c_1 \geq 0}{Max}\{U_1(c_1) + \underset{c_2}{Max}\{\underset{\theta_i}{Min}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}\}\}$$

and in a Bayesian framework :

$$\underset{c_1 \geq 0}{Max}\{U_1(c_1) + \underset{c_2}{Max}\{pU_2(c_2, \delta c_1 + c_2, \theta_1) + (1-p)U_2(c_2, \delta c_1 + c_2, \theta_2)\}\}$$

- *Perfect learning and reversible decision.*

In relation to the second situation, the only difference is there is no ireversibility constraint too, hence : $\underset{c_1 \geq 0}{Max}\{U_1(c_1) + \underset{\theta_i}{Min}\ \{\underset{c_2}{Max}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}\}\}$

and in a Bayesian framework :

$$\underset{c_1 \geq 0}{Max}\{U_1(c_1) + \{p \underset{c_2}{Max} U_2(c_2, \delta c_1 + c_2, \theta_1) + (1-p) \underset{c_2}{Max} U_2(c_2, \delta c_1 + c_2, \theta_2)\}\}$$

In a total uncertainty framework, the Max-min criterion hasn't problems such as temporal inconsistency and it permits to apply the techniques of dynamic optimisation. Indeed, we can realize that solving decision trees by backward induction or by a strategic form would lead to the same optimal choice and to the same valuation. For example, that is the case for *Perfect learning and irreversible decisions.* We can see that :

$\underset{c_1 \geq 0}{Max}\{U_1(c_1) + \underset{\theta_i}{Min}\ \{\underset{c_2 \geq 0}{Max}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}\}\} = \underset{c_1 \geq 0,\, c_{2i} \geq 0}{Max}\{\underset{\theta_i}{Min}\ \{U_1(c_1) + U_2(c_{2i}, \delta c_1 + c_{2i}, \theta_i)\}\}$,

the right hand-side term represents the decision criterion to solve the problem under strategic form.

3 Irreversibilities in decision-making.

Here we use the specific formulation of the intertemporal utility function :

$$U_1(c_1) + U_2(c_2, \delta c_1 + c_2, \theta_i) = V_1(c_1) + V_2(c_1 + c_2, \theta_i)$$

The utility function in period 2 depends only on the stock $s = \delta c_1 + c_2$ with $\delta = 1$. Consider the following conditions :

Hypothesis 1-bis[3] **:**V_1 *and* V_2 *are strictly concave.*

Hypothesis 5-bis : *If* $V_2(c_1 + c_2, \theta_1) = V_2(c_1 + c_2, \theta_2)$ *then* $\frac{\partial V_2(c_1+c_2,\theta_1)}{\partial s} \neq \frac{\partial V_2(c_1+c_2,\theta_2)}{\partial s}$

It is a condition of differentiation on the states θ_i.

Hypothesis 6 : V_2 *reaches a maximum in* s_i *in the state* θ_i[4].

The next result shows that in our framework, we find the same results as ones in the Bayesian framework.

Proposition 1 : *Under the hypothesis 1-bis, 5-bis, 6* [5]
The irreversibility constraint induces less irreversible decisions in period 1 (pure irreversibility effect) i.e : $c_1^{F,AI} \geq c_1^{I,AI}$ *and* $c_1^{F,IP} \geq c_1^{I,IP}$.

Without irreversibility constraint, information structure has no influence on the choice in period 1 (informationnal effect is nil) i.e : $c_1^{F,AI} = c_1^{F,IP}$
With irreversibility constraint, perfect learning induces less irreversible decisions in period 1 than no learning : the informationnal irreversibility effect holds, $c_1^{I,AI} \geq c_1^{I,IP}$.

This "result" shows us the robustness of the "irreversibility effect" to the uncertainty situation we are confronted to. Broadly and whatever the uncertainty is, it seems that irreversibilities have a twofold effect. We have to prove those results in a wider extent. In conclusion, we give the feeling of such a kind of generalisation.
Qualitatively, results are the same whatever uncertainty is. We wonder wether it is the same quantitavely. We could think that being in a total uncertainty situation leads the agent to be all the more precautionnous, since he has a very pessimistic criterion[6]. But it is not the case. The next result points it :

Proposition 2 :*For some decision problems we can find a probability p such as* $c_1^{I,AI} > c_1^{I,AI}(p)$ *and/or* $c_1^{I,IP} > c_1^{I,IP}(p)$.

In the appendix, a numerical example is given for the first situation. It is built on the following fact : the constraint can bite in a Bayesian framework for some c_1 more little than in a total uncertainty situation. That's why $c_1^{I,AI} > c_1^{I,AI}(p)$ is possible.

In the next results, we show that it is particular cases. It could be said that the Max-min criterion leads to be more careful than in a probabilised situation.

[3]The main numbering of the hypothesis is used for the general model in appendix.
[4]Monotony is not worth be analyzed because the constraints would always bite, and there would be no informationnal effects.
[5]Without hypothesis, finite optimal solutions are not certain.
[6]He could use the Hurwicz rule which weights the Max and the Min.

Proposition 3 : *Under the conditions 1-bis, 5-bis and 6,*

a) In case of perfect learning, if irreversibilities constraints does not bite in the two states of the world for the optimal solution then $c_1^{I,AI} = c_1^{I,AI}(p) = c_1^{F,AI} = c_1^{F,AI}(p) = c_1^{I,IP} = c_1^{I,IP}(p) = c_1^{F,IP} = c_1^{F,IP}(p) \, \forall p \in [0,1]$.

b) In case of perfect learning, if irreversibilities constraints bite in the two states of the world for the optimal solution then $c_1^{I,AI} = c_1^{I,IP}$ *and* $c_1^{I,AI}(p) = c_1^{I,IP}(p) \, \forall p \in [0,1]$.

c) *In case of perfect learning, if irreversibilities constraints bite in one state of the world for the optimal solution and* $\frac{dV_1(c_1^{I,IP})}{dc_1} \neq 0$[7], *then* $\forall p \in]0,1[\; c_1^{I,IP} < c_1^{I,IP}(p)$.

The case a) is possible only if V_1 is not monotonous, the optimal condition is then $\frac{dV_1(c_1^{I,AI})}{dc_1} = 0$. It is as if the agent didn't have to consider the second period and were interested only in this well-being in period 1 : total uncertainty and probabilistic situation, irreversibilities or not, information... it is not important.

In the case b), information possibilities have no effect ont the agent's choice. We can observe $c_1^{I,IP} > c_1^{I,IP}(p)$ only in this situation. It comes from the fact that max-min criterion keeps the worst state of the world in tems of value, but the marginal damage can be lower in this state than in the other. A Bayesian process weights marginal damages in the two states of the world. So, it can lead to a greater valuation of marginal damages.

A positive informationnal irreversibility effect can only exist in the situation c). In this case, the Max-min criterion induces the agent to be more careful in case of perfect learning.

In total uncertainty, processing with the Bayesian framework is not innocuous. We must bear in mind that it is tha Max-min criterion which permits to assure the best utility level in the worst situation and whatever the situation is. On the other hand, as shown by the next result, there is no probability distributions that could produce the Max-min decisions.

Proposition 4 : *If there is a positive informationnal irreversibility effect in case of total uncertainty,then there is no* $p \in [0,1]$ *such that* $c_1^{I,IP} = c_1^{I,IP}(p)$ *and* $c_1^{I,AI} = c_1^{I,AI}(p)$.

In this case where irreversibilities and information play a part, the Max-min criterion differs from a probability 1 on the bad state of the world. Indeed, one can not identify a state of the world which would be bad in all circumstances.

[7] So it is certain that the state for which the constraint bites, is the worst state of the world on optimum.

4 Intertemporal externalities.

In this section, we focus on the Ulph and Ulph model[12]. As the Gollier and alii model[9], it has a specific structure : there is a worst state of the world whatever the circumstances are. It is a restriction to our analyze. But the Ulph and Ulph model is easy to generalise.

4.1 The Ulph-Ulph model.

Decisionnal irreversibilities model permits us to formalize few environmental problems. We can not in particular formalize problems with stock phenomenom inducing intertemporal externalities (stock of greenhouse gas for example...). Ulph and alii [12] model this. They take as intertemporal utility function :

$$U_1(c_1) + U_2(c_{2,}\delta c_1 + c_2, \theta) = U_1(c_1) + V_2(c_2) - \theta D(\delta c_1 + c_2)$$

The random variable θ denotes uncertainty, it determines how damaging the stock of emmissions is. We consider 2 states of the world : θ_1 and θ_2,such as $\theta_1 < \theta_2$. In this model, θ_2 is the worst state of the world and whatever the structure information is, the Max-min criterion is equivalent to maximize :

$$U_1(c_1) + V_2(c_2) - \theta_2 D(\delta c_1 + c_2)$$

So the next result :

Proposition 5 : *If $U_2(c_{2,}\delta c_1 + c_2, \theta)$ is $V_2(c_2) - \theta D(\delta c_1 + c_2)$ then the irreversibility constraint leads to less irreversible decisions in period 1 (pure irreversibility effect) i.e.: $c_1^{F,AI} \geq c_1^{I,AI}$ and $c_1^{F,IP} \geq c_1^{I,IP}$ but information structure has no influence on choices in period 1 i.e.:$c_1^{F,AI} = c_1^{F,IP}$ and $c_1^{I,AI} = c_1^{I,IP}$.*

What are the results in the Bayesian framework ?

• The irreversibility constraint induces less irreversible decisions in period 1, i.e. $c_1^{F,AI}(p) \geq c_1^{I,AI}(p)$ and $c_1^{F,IP}(p) \geq c_1^{I,IP}(p)$. Pure irreversibility effect holds.

• On the other hand, it is difficult to rule on the informationnal effect. It is clear however that for some decisions problems this effect does not hold, i.e. $c_1^{F,AI}(p) \leq c_1^{F,IP}(p)$. That is the case particulary when the damage D and the utility function V_2 are quadratics.

• With irreversibilities, there is no general result. Ulph-Ulph give some sufficient conditions to have $c_1^{I,AI}(p) \geq c_1^{I,IP}(p)$:
- if the constraint bites in the case of no learning for the expected state, then a finer information structure leads to less irreversible decisions in period 1.
- if the constraint bites in case of no learning for the best state of the world then there is no effect of information structure.

Comparing choices in period 1 in the two situations of uncertainty enables us to see : whatever the theoretical framework is, the pur irreversibility effect holds. The irreversibility constraint leads to make choices more flexible in period 1.
In a bayesian framework results concerning informationnal effect are ambigous. The Maxmin criterion gives a clear result : information structure has no effect on choice in period 1.

The question is then : would those effects be relevant if the bad state of the world changed?

4.2 A generalization of the Ulph and Ulph model.

It is interesting to analyse the case in which the worst state of the world changes. Thus we consider a decision problem such as :

$$U_1(c_1) + U_2(c_{2,}\delta c_1 + c_2, \theta) = U_1(c_1) + V_2(c_2) - D(\delta c_1 + c_2,\, \theta)$$

The previous model considered only the case with an uncertainty about the importance of damage. We generalise the possibilities of uncertainty. Consider the following conditions :

Hypothesis 1-ter :U_1 *and* V_2 *are strictly concave,* V_2 *is an increasing function,* D *is an increasing convex function.*

Hypothesis 5-ter : *If* $D(\delta c_1 + c_2, \theta_1) = D(\delta c_1 + c_2, \theta_2)$ *then* $\frac{\partial D(\delta c_1 + c_2, \theta_1)}{\partial s} \neq \frac{\partial D(\delta c_1 + c_2, \theta_2)}{\partial s}$

The results are :

Proposition 6 : *Under the conditions 1-ter and 5-ter,*
Pure irreversibility effect holds i.e. $c_1^{F,AI} \geq c_1^{I,AI}$ *and* $c_1^{F,IP} \geq c_1^{I,IP}$

Without irreversibility constraint, perfect learning induces no necessarely more flexible decisions in period 1 than no learning.

The informationnal irreversibility effect is confirmed, i.e. if $c_1^{F,AI} \geq c_1^{F,IP}$ *then* $c_1^{I,AI} \geq c_1^{I,IP}$

Results are qualitatively the same in a situation of probabilistic uncertainty.

Proposition 7 : *Under the hypothesis 1-ter et 5-ter,*
The pure irreversibility effecet holds i.e. $\forall p \in [0,1]$ $c_1^{F,AI}(p) \geq c_1^{I,AI}(p)$ *and* $c_1^{F,IP}(p) \geq c_1^{I,IP}(p)$

Without irreversibility constraint, perfect learning induces no necessarely more flexible decisions in period 1 than no learning.

The informationnal irreversibility effect is confirmed, i.e. if $c_1^{F,AI}(p) \geq c_1^{F,IP}(p)$ *then* $c_1^{I,AI}(p) \geq c_1^{I,IP}(p)$

The sufficient condition to have $c_1^{I,AI}(p) \geq c_1^{I,IP}(p)$ is identical to the Ulph-Ulph 's one (cf proof of the proposition) : if the constraint bites in the case if no learning, then a finer structure information induces more flexible decisions in period 1.

The last result of proposition 7 means that informationnal irreversibility effect holds. It is a positive result in favour of our typology suggested to distinguish the part of information and the part of irreversibilities[8].

We shall not analyse quantitative differences between the Max-min criterion and the Bayesian framework. We think they are similar to those identified for decisionnal irreversibilities.

5 Conclusion

We proposed a very simple models in terms of information structure and states of the world set. Nevertheless the results enable to clarify the part of externalities. Moreover ,they do not depend on the nature of uncertainty. According to informationnal irreversibility effect and informationnal effect , irreversibilities must lead to be more careful.

It would be logical to extend those results, i.e. to consider more general information and uncertainty situation.

First, in a Bayesian framework with Blackwell information structure, we think the informationnal irreversibility effect must hold in the Ulph-Ulph generalised model. Then, the generalisation in a non Bayesian framework implies to precise the adopted formalisation of uncertainty and the information structure we would consider. Total uncertainty is a special case means we have no quantitative presumption on the risk level. Despite the lack of data, we can often have some clues.In this way, we can consider a family of probabilities on the state of the world and implement a Max-min criterion of expected utility[9]. Total uncertainty and Max-min criterion, probabilistic uncertainty and expected utility are two special cases of this general formalisation.. In this framework, Chassagnon and Vergnaud [3] and Chateauneuf and Vergnaud [4] proposed to consider an information

[8]On the contrary, there was a confusion with the standard definition of the irreversibility effect. In their article, Gollier and alii [9] noted that their sufficient conditions were similar for the informationnal effect (precautionary effect in their terms) and the irreversibility effect.

[9]For example, the family of probabilities can correspond to the subjective probabilities distributions of the consulted experts for a problem.

process with ambiguity reduction : when information arrives, the family of prior is reduced. They proposed a consistent definition of information structure and demonstrated we can apply the dynamic optimisation. They explained the partial order of these information structures. In our model, no learning and perfect learning in total uncertainty are two information structures of this type.[10]. Turning to this general formalization, results will probably be generalized.

We have noted quantitative differences between the Max-min criterion and the bayesian framework : Max-min criterion leads to make more careful choices. Broadly, we can not say that Max-min criterion is equivalent to put a probability 1 on the "bad" state of the world, because we can not identify a state of the world which would be bad in all circumstances. There is no probability distributions that could produce the Max-min decisions. We shall obtain probably the same quantitative differences in the more general framework with family of probabilities. An intuitive definition of a more uncertain or more ambiguous would build on an increasing of the familty of probabilities In an operational point of view, proceeding with the Bayesian framework rather than the Max-min criterion when we have some quantitative data is less relevant because the alternative with family of probabilities exists.

6 Appendix

6.1 Optimality conditions in period 2 in the general model.

The comparative static results we look for are built on the comparison of the current decisions c_1, in view of the expected information structure and the irreversibilities constraints. First, we have to explicit the conditionnal choice in period 2. In this section, we study the hypothesis that permit to have optimization results in period 2 with good properties [11]. We consider that there is a unique solution for every optimisation problem considered. The next hypothesis assures it :

U_1 and U_2 are strictly concave (in their 2 first arguments for U_2).

6.1.1 Optimal choices in period 2 in the case of perfect learning.

Let's consider the case in which the agent is informed of the true state of the world θ_i. We note $I^{\theta_i}(c_1)$, $F^{\theta_i}(c_1)$ the optimal choices respectively in case of irreversibilities and without irreversibilities; and $J^I(c_1, \theta_i)$, $J^F(c_1, \theta_i)$ the corresponding value functions. Value functions give the utility of an optimal choice in period 2. So, we have :

[10] It justifies that in a first step we have only considered the extreme case of total uncertainty, which enables to see clearly the differences.

[11] The Max-min criterion makes the analyze particular because of points which can not be differentied.

$J^I(c_1,\theta_i) = U_2(I^{\theta_i}(c_1), \delta c_1 + I^{\theta_i}(c_1), \theta_i) = \underset{c_2 \geq 0}{Max}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}$ for $i = 1, 2$

$J^F(c_1,\theta_i) = U_2(F^{\theta_i}(c_1), \delta c_1 + F^{\theta_i}(c_1), \theta_i) = \underset{c_2}{Max}\{U_2(c_2, \delta c_1 + c_2, \theta_i)\}$ for $i = 1, 2$

If the irreversibility constraint prevents from choosing the optimal level, then the constraint bites as the next result shows it.

Lemma 1 *Under the condition 6.1,* $I^{\theta_i}(c_1) = 0 \Leftrightarrow F^{\theta_i}(c_1) \leq 0$ *and* $F^{\theta_i}(c_1) \geq 0 \Rightarrow I^{\theta_i}(c_1) = F^{\theta_i}(c_1)$ *for* $i = 1, 2$

How evolves the choice in period 1 with respect to c_1? It depends on the U_2 features. For this, let's introduce the following conditions.

U_2 is an increasing function in its first argument (i.e : $\frac{\partial U_2(c_2,\delta c_1+c_2,\theta_i)}{\partial c_2} \geq 0 \, \forall c_1,\, c_2,\, \theta_i$).

This condition could be relaxed.

$\forall c_1,\, c_2,\, \theta_i,\, \frac{\partial^2 U_2(c_2,\delta c_1+c_2,\theta_i)}{\partial c_2 \partial s} = 0.$

It amounts to supposing that increase in stock s has no effect on the marginal utility of consumption in period 2 and that there is separability between variables c_2 and s. As we will see this condition holds for the utility functions quoted before.

Lemma 2 *Under the conditions 6.1 and 6.1.1,* $I^{\theta_i}(c_1)$ *and* $F^{\theta_i}(c_1)$ *are decreasing functions in* $c_1^{I,AI}$ *with* $\frac{dI^{\theta_i}(c_1)}{dc_1} \geq -\delta$ *and* $\frac{dF^{\theta_i}(c_1)}{dc_1} \geq -\delta$*.*

Proof.

$F^{\theta_i}(c_1)$ is such that $\frac{\partial U_2(F^{\theta_i}(c_1),\delta c_1+F^{\theta_i}(c_1),\theta_i)}{\partial c_2} + \frac{\partial U_2(F^{\theta_i}(c_1),\delta c_1+F^{\theta_i}(c_1),\theta_i)}{\partial s} = 0$. If we differentiate this equality with respect to $c_1^{I,AI}$,we have $\frac{dF^{\theta_i}(c_1)}{dc_1} = -\delta . \frac{\frac{\partial^2 U_2}{\partial C_2 \partial s} + \frac{\partial^2 U_2}{\partial s^2}}{\frac{\partial^2 U_2}{\partial C_2^2} + 2\frac{\partial^2 U_2}{\partial C_2 \partial s} + \frac{\partial^2 U_2}{\partial s^2}} = -\delta . \frac{\frac{\partial^2 U_2}{\partial s^2}}{\frac{\partial^2 U_2}{\partial C_2^2} + \frac{\partial^2 U_2}{\partial s^2}} \leq 0$ given the hypothesis 6.1 too on the strict concavity of U_2. Note that $\frac{dF^{\theta_i}(c_1)}{dc_1} \geq -\delta$. ■

By the lemma 1, we deduce the results on $I^{\theta_i}(c_1)$ as well.

It seems to be logical that in period 2 it is optimal to compensate an increasing in c_1 by a decreasing in c_2. Note that the decreasing in c_2 is lower than the increasing in stock induced by the increasing in c_1.

Lemma 3 *Under the hypothesis 6.1, 6.1.1,* $J^I(c_1,\theta_i)$ *are* $J^F(c_1,\theta_i)$ *decreasing functions in* c_1 *and if moreover the condition 6.1.1 holds, then they are concave.*

Proof.

$\frac{dJ^I(c_1,\theta_i)}{dc_1} = \left[\frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial c_2} + \frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial s}\right] \frac{dI^{\theta_i}(c_1)}{dc_1} + \delta . \frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial s}$

Given optimality conditions, the first term of the right hand-side expression is null. Indeed, either the irreversibility constraint doesn't bite at the optimum and the term between brackets is nil, or it bites and so it is the second term which is nil. Since

$\frac{\partial U_2}{\partial c_2} \geq$ 0,in $I^{\theta_i}(c_1)$ then $\frac{\partial U_2}{\partial s} \leq 0$ and so

$\frac{dJ^I(c_1,\theta_i)}{dc_1} = \delta . \frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial s} \leq 0.$

$\frac{d^2J^I(c_1,\theta_i)}{dc_1^2} = \delta . \left[\frac{\partial^2 U_2}{\partial C_2 \partial s}\frac{dI^{\theta_i}(c_1)}{dc_1} + \left[\delta + \frac{dI^{\theta_i}(c_1)}{dc_1}\right]\frac{\partial^2 U_2}{\partial s^2}\right] = \delta\left[\delta + \frac{dI^{\theta_i}(c_1)}{dc_1}\right]\frac{\partial^2 U_2}{\partial s^2}$

and since $\delta + \frac{dI^{\theta_i}(c_1)}{dc_1} \geq 0$, the concavity of U_2 implies $\frac{d^2J^I(c_1,\theta_i)}{dc_1^2} \leq 0$. ■

Given the negative intertemporal externalities problems and the irreversibilities problems we formalise, it is natural to have negative effects of c_1 on the well-being in period 2.

Consider functions $J^I(c_1) = \underset{\theta_i}{Min}\, J^I(c_1,\theta_i)$ and $J^F(c_1) = \underset{\theta_i}{Min} J^F(c_1,\theta_i)$ which are the value functions that the agent anticipates when he expected to receive a perfect learning.

Lemma 4 *Under the conditions 6.1, 6.1.1 and 6.1.1,* $J^I(c_1)$ *and* $J^F(c_1)$ *are decreasing and concave functions in* c_1

Proof.
The decreasing of the functions $J^I(c_1)$ and $J^F(c_1)$ is due to the decreasing of the functions $J^I(c_1,\theta_i)$ and $J^F(c_1,\theta_i)$ and to their definition. They are continue too.
The concavity of the functions $J^I(c_1,\theta_i)$ and $J^F(c_1,\theta_i)$ implies the piecewise concavity (on the intervals where $J^I(c_1) = J^I(c_1,\theta_i)$ for the same θ_i and $J^F(c_1) = J^F(c_1,\theta_i)$ for the same θ_i).
In a point where $J^I(c_1) = J^I(c_1,\theta_i) = J^I(c_1,\theta_{-i})$,
$J^I(c_1') = J^I(c_1',\theta_i)$ on the left side of c_1, $J^I(c_1') = J^I(c_1',\theta_{-i})$ on the right of c_1,
then necessarily $0 \geq \frac{dJ^I(c_1,\theta_i)}{dc_1} \geq \frac{dJ^I(c_1,\theta_{-i})}{dc_1}$ which means that the derivative on the left side of $J^I(c_1)$ is higher than the derivative on the right side, it is the proof that $J^I(c_1)$ is concave on its entire field.
The proof is similar for $J^F(c_1)$ ■

Now we are sure that in period 1 the problem is to optimise the sum of two concave functions.One of them is strictly concave, so the optimal solution will be unique.

6.1.2 Optimal choices in period 2 in the case of no learning.

In the case of no learning, we note in the same way $I^{\theta_1,\theta_2}(c_1)$, $F^{\theta_1,\theta_2}(c_1)$, $J^I(c_1,\theta_1,\theta_2)$ and $J^F(c_1,\theta_1,\theta_2)$ the optimal choices and the value functions.

$J^I(c_1,\theta_1,\theta_2) = \underset{\theta_i}{min}\{U_2(I^{\theta_1,\theta_2}(c_1),\delta c_1+I^{\theta_1,\theta_2}(c_1),\theta_i)\} = \underset{c_2\geq 0}{Max}\{\underset{\theta_i}{Min}\{U_2(c_2,\delta c_1+c_2,\theta_i)\}\}$

$J^F(c_1,\theta_1,\theta_2) = \underset{\theta_i}{min}\{U_2(F^{\theta_1,\theta_2}(c_1),\delta c_1+F^{\theta_1,\theta_2}(c_1),\theta_i)\} = \underset{c_2}{Max}\{\underset{\theta_i}{Min}\{U_2(c_2,\delta c_1+c_2,\theta_i)\}\}$

The irreversibility constraint induces the same features on the optimal choices than ones in case if perfect learning.

Lemma 5 *Under the hypothesis 6.1,*
$I^{\theta_1,\theta_2}(c_1) = 0 \Leftrightarrow F^{\theta_1,\theta_2}(c_1) \leq 0$ *and* $F^{\theta_1,\theta_2}(c_1) \geq 0 \Rightarrow I^{\theta_1,\theta_2}(c_1) = F^{\theta_1,\theta_2}(c_1)$ *for* $i = 1, 2$

The Max-min criterion we use in this total uncertainty framework has particular consequences[12] on the optimal choice in period 2 in the case of no learning :

- either it corresponds to one of the optimal choices in one of the states of the world,
- or it is between the two possible optimal choices in perfect learning.

Lemma 6 *Under the hypothesis 6.1,*
(i) If $J^I(c_1,\theta_1) \leq U_2(I^{\theta_1}(c_1), \delta c_1 + I^{\theta_1}(c_1), \theta_2)$ *(resp.* $J^F(c_1,\theta_1) \leq U_2(F^{\theta_1}(c_1), \delta c_1 + F^{\theta_1}(c_1), \theta_2)$*) then* $I^{\theta_1,\theta_2}(c_1) = I^{\theta_1}(c_1)$ *and* $J^I(c_1,\theta_1,\theta_2) = J^I(c_1,\theta_1) = \underset{\theta_i}{min} J^I(c_1,\theta_i)$ *(resp.* $F^{\theta_1,\theta_2}(c_1) = F^{\theta_1}(c_1)$ *and* $J^F(c_1,\theta_1,\theta_2) = J^F(c_1,\theta_1) = \underset{\theta_i}{min} J^F(c_1,\theta_i)$*)*

(ii) If $J^I(c_1,\theta_2) \leq U_2(I^{\theta_2}(c_1), \delta c_1 + I^{\theta_2}(c_1), \theta_1)$ *(resp.* $J^F(c_1,\theta_2) \leq U_2(F^{\theta_2}(c_1), \delta c_1 + F^{\theta_2}(c_1), \theta_1)$*) then* $I^{\theta_1,\theta_2}(c_1) = I^{\theta_2}(c_1)$ *and* $J^I(c_1,\theta_1,\theta_2) = J^I(c_1,\theta_2) = \underset{\theta_i}{min} J^I(c_1,\theta_i)$ *(resp.* $F^{\theta_1,\theta_2}(c_1) = F^{\theta_2}(c_1)$ *and* $J^F(c_1,\theta_1,\theta_2) = J^F(c_1,\theta_2) = \underset{\theta_i}{min} J^F(c_1,\theta_i)$*)*

(iii) In the other situations, $\underset{\theta_i}{min} I^{\theta_i}(c_1) < I^{\theta_1,\theta_2}(c_1) < \underset{\theta_i}{\max} I^{\theta_i}(c_1)$ *(resp.*$\underset{\theta_i}{min} F^{\theta_i}(c_1) < F^{\theta_1,\theta_2}(c_1) < \underset{\theta_i}{\max} F^{\theta_i}(c_1)$ *) and*
$J^I(c_1,\theta_1,\theta_2) = U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_1) = U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_2) < \underset{\theta_i}{min} J^I(c_1,\theta_i)$ *(resp.* $J^F(c_1,\theta_1,\theta_2) = U_2(F^{\theta_1,\theta_2}(c_1), \delta c_1 + F^{\theta_1,\theta_2}(c_1), \theta_1) = U_2(F^{\theta_1,\theta_2}(c_1), \delta c_1 + F^{\theta_1,\theta_2}(c_1), \theta_2) < \underset{\theta_i}{min} J^F(c_1,\theta_i)$*)*

Proof. $(i) \forall c_2 \geq 0 \ \underset{\theta_i}{min} U_2(I^{\theta_1}(c_1), \delta c_1 + I^{\theta_1}(c_1), \theta_i) = U_2(I^{\theta_1}(c_1), \delta c_1 + I^{\theta_1}(c_1), \theta_1)$
$\geq U_2(c_2, \delta c_1 + c_2, \theta_1) \geq \underset{\theta_i}{min} U_2(c_2, \delta c_1 + c_2, \theta_i)$
that shows that $I^{\theta_1,\theta_2}(c_1) = I^{\theta_1}(c_1)$ et $J^I(c_1,\theta_1,\theta_2) = J^I(c_1,\theta_1)$. Besides, $J^I(c_1,\theta_2) = U_2(I^{\theta_2}(c_1), \delta c_1 + I^{\theta_2}(c_1), \theta_2) \geq U_2(I^{\theta_1}(c_1), \delta c_1 + I^{\theta_1}(c_1), \theta_2)$
$\geq U_2(I^{\theta_1}(c_1), \delta c_1 + I^{\theta_1}(c_1), \theta_1) = J^I(c_1,\theta_1)$

(ii) idem

(iii) Reasoning by absurde, if we had for instance $U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_1) < U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_2)$, since we aren't in the case (i), $I^{\theta_1,\theta_2}(c_1) \neq I^{\theta_1}(c_1)$ .There is c_2 standing between $I^{\theta_1,\theta_2}(c_1)$ and $I^{\theta_1}(c_1)$, $c_2 \neq I^{\theta_1,\theta_2}(c_1)$, such as $U_2(c_2, \delta c_1 + c_2, \theta_1) \leq U_2(c_2, \delta c_1 + c_2, \theta_2)$ and the strict concavity of U_2 implies $U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_1) < U_2(c_2, \delta c_1 + c_2, \theta_2)$. Moreover, since $U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_1) = U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_2)$ and we are neither in the case (i), nor in the case (ii), we have necessarily

[12] A comparer avec le cas Bayésien, où le choix de seconde période en absence d'information se situe entre les deux choix optimaux possibles en information parfaite.

$I^{\theta_1,\theta_2}(c_1) \neq I^{\theta_1}(c_1)$ and $I^{\theta_1,\theta_2}(c_1) \neq I^{\theta_2}(c_1)$. Thus, $J^I(c_1,\theta_1,\theta_2) = U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_i) < U_2(I^{\theta_i}(c_1), \delta c_1 + I^{\theta_i}(c_1), \theta_i) = J^I(c_1,\theta_i)$, $\forall \theta_i$.
The hypothesis 6.1 implies $\underset{\theta_i}{min} I^{\theta_i}(c_1) < I^{\theta_1,\theta_2}(c_1) < \underset{\theta_i}{\max} I^{\theta_i}(c_1)$ too.
(Demonstration for flexible decisions is identical). ■

In the two first cases, there is a "bad" state of the world. The Max-min criterion leads in the case of no learning to focus on this state and to make the corresponding optimal choice. In the third case, the optimal choice is a compromise solution.
What is the effect of the decision in period 1 on the well-being in period 2.?
To see this, consider the following conditions.

Marginal utility with respect to c_2 is independent of θ_i (i.e $\frac{\partial U_2(c_2,\delta c_1+c_2,\theta_i)}{\partial c_2}$ is independent of θ_i).

For instance, it seems natural for the problem of global warming : the utility obtained directly by the consumption in period 2 doesn't depend on the state of the world.

If $U_2(c_2, \delta c_1 + c_2, \theta_1) = U_2(c_2, \delta c_1 + c_2, \theta_2)$ then $\frac{\partial U_2(c_2,\delta c_1+c_2,\theta_1)}{\partial s} \neq \frac{\partial U_2(c_2,\delta c_1+c_2,\theta_2)}{\partial s}$

It is a differentiation hypothesis : if the well-being is similar in the two state of the world, nevertheless the marginal impact of the stock s is different.

Lemma 7 *Under the conditions 6.1, 6.1.1, 6.1.1, 6.1.2 and 6.1.2, then* $J^I(c_1,\theta_1,\theta_2)$ *and* $J^F(c_1,\theta_1,\theta_2)$ *are decreasing and concave functions in* c_1.

Proof. Results of lemma 6 show that points for which we can not derivate are possible of the $J^I(c_1,\theta_1,\theta_2)$ function because of the non derivability of $I^{\theta_1,\theta_2}(c_1)$ function. Firstly, let's show it is a decreasing function, then a piecewise concave function.
If we are in the case (i) or (ii) given in the lemma 6, then the decreasing and the concavity are deduced from the decreasing and the concavity demonstrated in the lemma3.
If we are in the case (iii) given in the lemma 6, then
$J^I(c_1,\theta_1,\theta_2) = U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_1) = U_2(I^{\theta_1,\theta_2}(c_1), \delta c_1 + I^{\theta_1,\theta_2}(c_1), \theta_2)$.
If we differentiate with respect to $c_1^{I,AI}$, we obtain :
$\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} = \frac{\partial U_2(I^{\theta_1,\theta_2}(c_1),\delta c_1+I^{\theta_1,\theta_2}(c_1),\theta_1)}{\partial c_2} . \frac{dI^{\theta_1,\theta_2}(c_1)}{dc_1} + (\frac{\partial U_2(I^{\theta_1,\theta_2}(c_1),\delta c_1+I^{\theta_1,\theta_2}(c_1),\theta_1)}{\partial s})(\frac{dI^{\theta_1,\theta_2}(c_1^{I,AI})}{dc_1} + \delta)$
$= \frac{\partial U_2(I^{\theta_1,\theta_2}(c_1),\delta c_1+I^{\theta_1,\theta_2}(c_1),\theta_2)}{\partial c_2} . \frac{dI^{\theta_1,\theta_2}(c_1)}{dc_1} + (\frac{\partial U_2(I^{\theta_1,\theta_2}(c_1),\delta c_1+I^{\theta_1,\theta_2}(c_1),\theta_2)}{\partial s})(\frac{dI^{\theta_1,\theta_2}(c_1)}{dc_1} + \delta)$.
Given hypothesis 6.1.2 and 6.1.2, this equality is true only if $\frac{dI^{\theta_1,\theta_2}(c_1)}{dc_1} = -\delta$. (We have demonstrated that when $U_2(c_2, \delta c_1 + c_2, \theta_1) = U_2(c_2, \delta c_1 + c_2, \theta_2)$ then $\delta c_1 + c_2 = \alpha$ where α is a constant).
Finally, we have $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} = -\delta . \frac{\partial U_2(I^{\theta_1,\theta_2}(c_1),\delta c_1+I^{\theta_1,\theta_2}(c_1),\theta_1)}{\partial c_2} \leq 0$.
Moreover $\frac{d^2 J^I(c_1,\theta_1,\theta_2)}{dc_1^2} = \delta^2 \frac{\partial^2 U_2(I^{\theta_1,\theta_2}(c_1),\delta c_1+I^{\theta_1,\theta_2}(c_1),\theta_1)}{\partial^2 c_2} < 0$ from the hypothesis 6.1.
Until now we have supposed differentiability. We note non differentiability points when we go from case (i) or (ii) to case (iii), that is to say when $\exists\, i$ such as $U_2(I^{\theta_i}(c_1), \delta c_1 +$

$I^{\theta_i}(c_1), \theta_1) = U_2(I^{\theta_i}(c_1), \delta c_1 + I^{\theta_i}(c_1), \theta_2)$ (i.e $\delta c_1 + I^{\theta_i}(c_1) = \alpha$) and $I^{\theta_1,\theta_2}(c_1) = I^{\theta_i}(c_1)$. In one hand, we have $\frac{dJ^I(c_1,\theta_i)}{dc_1} = \delta.\frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial s}$ and on the other hand $\frac{dU_2(\alpha-\delta c_1,\alpha),\theta_i)}{dc_1} = -\delta.\frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial c_2}$ in $(I^{\theta_i}(c_1), \delta c_1 + I^{\theta_i}(c_1))$.
Two cases are possible :

- either the irreversibility constraint does not bite in $I^{\theta_i}(c_1)$ and the optimality conditions are so $\frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial c_2} + \frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial s} = 0$ and so we have $\frac{dJ^I(c_1,\theta_i)}{dc_1} = \frac{dU_2(\alpha-\delta c_1,\alpha),\theta_i)}{dc_1}$ in $(I^{\theta_i}(c_1), \delta c_1 + I^{\theta_i}(c_1))$ that means that necessarily the right and left derivatives of $J^I(c_1, \theta_1, \theta_2)$ are equal in the non differentiability point,

- or the irreversibility constraint bites and in this case the decreasing of $I^{\theta_1,\theta_2}(c_1)$ and the irreversibility constraint imply that $I^{\theta_1,\theta_2}(c_1) = I^{\theta_i}(c_1) = 0$ on the left-side dicontinuuity point and that we are in the case (iii) on the left. The right derivative of $J^I(c_1, \theta_1, \theta_2)$ is equal to $\delta.\frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial s}$ and the left derivative is $-\delta.\frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial c_2}$.
The optimality constraint is then $\frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial c_2} + \frac{\partial U_2(I^{\theta_i}(c_1),\delta c_1+I^{\theta_i}(c_1),\theta_i)}{\partial s} \leq 0$ which shows that the left derivative is upper than the right derivative.
We have conditions on derivatives assuring the concavity of $J^I(c_1, \theta_1, \theta_2)$ on its all definition field. ■

Under our hypothesis, we have good properties for the maximization problem in period 1 in the case of no learning. Besides, the optimal solution is unique.

6.2 Proofs of propositions of decisionnal irreversibilities.

Hypothesis 6.1.1, 6.1.1 and 6.1.2 are of course confirmed. Hypothesis 1-bis and 5-bis imply that hypothesis 6.1 and 6.1.2 hold. one can use the previous demonstrated results for the general model.

Proof. of proposition 1.
Without irreversibilities constraint, the optimal choice in period 2 always gives the maximum reachable utility : we have $\frac{dJ^F(c_1,\theta_i)}{dc_1} = \frac{dJ^F(c_1,\theta_1,\theta_2)}{dc_1} = 0$. Results of lemma 4 and 7, hypothesis 6.1 lead to $c_1^{F,AI} = c_1^{F,IP}, c_1^{F,AI} \geq c_1^{I,AI}$ and $c_1^{F,IP} \geq c_1^{I,IP}$.
Lets's show that $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} \geq \frac{dJ^I(c_1)}{dc_1}$ (in case of differentiability). Either we can find $i = 1, 2$ such as $J^I(c_1, \theta_i) \leq U_2(I^{\theta_i}(c_1), \delta c_1 + I^{\theta_i}(c_1), \theta_{-i})$ and then $J^I(c_1) = J^I(c_1, \theta_1, \theta_2) = J^I(c_1, \theta_i)$ and in this case $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} = \frac{dJ^I(c_1)}{dc_1}$ .Or it doesn't exist such a i and if $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1}$ exists and then $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} = -\frac{\partial U_2}{\partial c_2} = 0$ (cf proof of lemma 7). Thus necessarily $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} \geq \frac{dJ^I(c_1)}{dc_1}$ and we deduce $c_1^{I,AI} \geq c_1^{I,IP}$. ■

Proof. of proposition 2.

we give an example in which $c_1^{I,AI} > c_1^{I,AI}(p)$. Given the utility functions $U_1(c_1) = 3.c_1 - c_1^2$ (maximum of U_1 in $c_1 = 3/2$) and $U_2(c_1+c_2,\theta_i) = 4.(c_1+c_2+\theta_i) - (c_1+c_2+\theta_i)^2$ with $\theta_1 = 0$ et $\theta_2 = 1$. Function $\underset{\theta_i}{Min}U_2(c_1+c_2,\theta_i)$ reaches its maximum in $c_1+c_2 = 3/2$, so $I^{\theta_1,\theta_2}(c_1) = 3/2 - c_1$ if $c_1 \leq 3/2$, $I^{\theta_1,\theta_2}(c_1) = 0$ otherwise and $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} = 0$ if $c_1 < 3/2$, $\frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} = \frac{\partial U_2(c_1,\theta_2)}{\partial c_1} = 2 - 2.c_1$ if $c_1 > 3/2$. Consequently, $c_1^{I,AI} = 3/2$. Or for example, for $p = 1/4$, function $1/4.U_2(c_1+c_2,\theta_1) + 3/4.U_2(c_1+c_2,\theta_2)$ reaches its maximum in $c_1+c_2 = 5/4$ and one can check that $c_1^{I,AI}(p) = 11/8 < c_1^{I,AI} = 3/2$. ■

Proof. of proposition 3.

a) So we are in the case where for instance, $\forall\theta_i\, I^{\theta_i}(c_1^{I,IP}) = F^{\theta_i}(c_1^{I,IP}) \geq 0$ with $\frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1} = \frac{dJ^F(c_1^{I,IP},\theta_i)}{dc_1} = 0$ and so $\frac{dV_1(c_1^{I,IP})}{dc_1} = \frac{dV_1(c_1^{F,IP})}{dc_1} = 0$. Consequently, $c_1^{I,IP} = c_1^{F,IP}$. On the other hand, we have seen in the demonstration of the lemma 6 we have ever $\underset{\theta_i}{min}I^{\theta_i}(c_1) < I^{\theta_1,\theta_2}(c_1) < \underset{\theta_i}{\max}I^{\theta_i}(c_1)$ and $\underset{\theta_i}{min}F^{\theta_i}(c_1) < F^{\theta_1,\theta_2}(c_1) < \underset{\theta_i}{\max}F^{\theta_i}(c_1)$ and consequently $I^{\theta_1,\theta_2}(c_1^{I,IP}) = F^{\theta_1,\theta_2}(c_1^{I,IP}) \geq 0$ with $\frac{dJ^I(c_1^{I,IP},\theta_1,\theta_2)}{dc_1} = \frac{dJ^F(c_1^{I,IP},\theta_1,\theta_2)}{dc_1} = 0$, and so $c_1^{I,IP}$ is a solution as well to the problems in case of no learning, given the unicity, in total $c_1^{I,AI} = c_1^{F,AI} = c_1^{I,IP} = c_1^{F,IP}$.

If we consider a bayesian situation, $p.\frac{dJ^I(c_1^{I,IP},\theta 1)}{dc_1} + (1-p).\frac{dJ^I(c_1^{I,IP},\theta_2)}{dc_1} = 0$ and so $c_1^{I,IP} = c_1^{I,IP}(p) = c_1^{F,IP}(p)$. Since irreversibility constraints don't bite, la fonction $p.V_2(c_1^{I,IP}+c_2,\theta_1) + (1-p).V_2(c_1^{I,IP}+c_2,\theta_2)$ reaches its maximum in $c_2 \geq 0$, and so $\frac{d\left[\underset{c_2}{Max}\left\{p.V_2(c_1^{I,IP}+c_2,\theta_1)+(1-p).V_2(c_1^{I,IP}+c_2,\theta_2)\right\}\right]}{dc_1} = 0$ we deduce then $c_1^{I,IP} = c_1^{I,AI}(p) = c_1^{F,AI}(p)$

b) Idem demonstration of the case a) we notice then $I^{\theta_i}(c_1^{I,IP}) = 0$, $\frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1} = 0$ and $\underset{\theta_i}{min}I^{\theta_i}(c_1) < I^{\theta_1,\theta_2}(c_1) < \underset{\theta_i}{\max}I^{\theta_i}(c_1) \Rightarrow I^{\theta_1,\theta_2}(c_1^{I,IP}) = 0$.

c) In the described situation, $\exists\theta_i$ such as $I^{\theta_i}(c_1^{I,IP}) = 0$, $I^{\theta_{-i}}(c_1^{I,IP}) \geq 0$,with $\frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1} < 0$ and $\frac{dJ^I(c_1^{I,IP},\theta_{-i})}{dc_1} = 0$, since $\frac{dV_1(c_1^{I,IP})}{dc_1} \neq 0$ but $\frac{dV_1(c_1^{I,IP})}{dc_1} + \frac{dJ^I(c_1^{I,IP})}{dc_1} = 0$, necessarely $J^I(c_1^{I,IP}) = J^I(c_1^{I,IP},\theta_i)$ and $\frac{dJ^I(c_1^{I,IP})}{dc_1} = \frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1}$. In a Bayesian framework, in case of perfect learning, in $c_1^{I,IP}$ the derivative of the value function in period 2 is thus $\forall\, p_i \in]0,1[$, $p_i.\frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1} + p_{-i}.\frac{dJ^I(c_1^{I,IP},\theta_{-i})}{dc_1} = p_i.\frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1} > \frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1}$ and consequently $\frac{dV_1(c_1^{I,IP})}{dc_1} + \frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1} > 0$ which shows that $c_1^{I,IP} < c_1^{I,IP}(p)$. ■

Proof. of proposition 4.

In $c_1^{I,IP}$, we are in a situation in which $\exists\theta_i$ such as $I^{\theta_i}(c_1^{I,IP}) = 0$, $I^{\theta_{-i}}(c_1^{I,IP}) \geq 0$,with $J^I(c_1^{I,IP}) = J^I(c_1^{I,IP},\theta_i)$ and $\frac{dJ^I(c_1^{I,IP})}{dc_1} = \frac{dJ^I(c_1^{I,IP},\theta_i)}{dc_1} < 0$ and to obtain $c_1^{I,IP}(p) = c_1^{I,IP}$ we must have $p_i = 1$. But then we have as well $c_1^{I,IP}(p) = c_1^{I,AI}(p)$ which implies $c_1^{I,AI}(p) \neq c_1^{I,AI}$ since $c_1^{I,IP} < c_1^{I,AI}$. ■

6.3 Proofs of propositions for intertemporal externalities.

Proof. of proposition 5 :

The only case is : $J^F(c_1,\theta_i) \leq U_2(F^{\theta_i}(c_1), \delta c_1 + F^{\theta_i}(c_1), \theta_j)$ with $\theta_j < \theta_i$.
Indeed, if we consider : $\theta_1 < \theta_2$, $J^F(c_1,\theta_2) \leq U_2(F^{\theta_2}(c_1), \delta c_1 + F^{\theta_2}(c_1), \theta_1)$ everytime.
So we have : $\underset{\theta_i}{min} J^F(c_1,\theta_i) = J^F(c_1,\theta_2) = J^F(c_1,\theta_1,\theta_2)$. The expected information structure has no effect on current decisions : $c_1^{F,AI} = c_1^{F,IP}$.
Concerning irreversible decisions, we have always : $J^I(c_1,\theta_2) \leq U_2(I^{\theta_2}(c_1), \delta c_1 + I^{\theta_2}(c_1), \theta_1)$ with $\theta_1 < \theta_2$.
So : $\underset{\theta_i}{min} J^I(c_1,\theta_i) = J^I(c_1,\theta_2) = J^I(c_1,\theta_1,\theta_2)$. Thus the expected information structure has no effect on current decisions : $c_1^{I,AI} = c_1^{I,IP}$
In case of perfect learning and no learning, since value functions are similar,
$\frac{dJ^F(c_1)}{dc_1} = \frac{dJ^F(c_1,\theta_1,\theta_2)}{dc_1} = -\delta\theta_2 D'(\delta c_1 + F^{\theta_2}(c_1), \theta_2)$
$\frac{dJ^I(c_1)}{dc_1} = \frac{dJ^I(c_1,\theta_1,\theta_2)}{dc_1} = -\delta\theta_2 D'(\delta c_1 + I^{\theta_2}(c_1), \theta_2)$
If $F^{\theta_2}(c_1) \geq 0, I^{\theta_2}(c_1) = F^{\theta_2}(c_1)$ and so $\frac{dJ^F(c_1)}{dc_1} = \frac{dJ^I(c_1)}{dc_1}$.
If $F^{\theta_2}(c_1) \leq 0, I^{\theta_2}(c_1) = 0$ and so $\frac{dJ^I(c_1)}{dc_1} = -\delta\theta_2 D'(\delta c_1, \theta_2)$ and $\frac{dJ^F(c_1)}{dc_1} = -\delta\theta_2 D'(\delta c_1 + F^{\theta_2}(c_1), \theta_2)$. $D'(\delta c_1 + F^{\theta_2}(c_1), \theta_2) \leq D'(\delta c_1, \theta_2)$ because $\delta c_1 + F^{\theta_2}(c_1) \leq \delta c_1$ and D is increasing, convex. Whatever c_1 is, we have $\frac{dJ^F(c_1)}{dc_1} \geq \frac{dJ^I(c_1)}{dc_1}$ and the U_1 concavity implies $c_1^{F,AI} \geq c_1^{I,AI}, c_1^{F,IP} \geq c_1^{I,IP}$ ■

Proof. of proposition 6 :

Firstly, let's remark that hypothesis $6.1, 6.1.1, 6.1.1, 6.1.2$ and $6.1.2$ hold.

a) The pur irreversibility effect : the case of perfect learning.

Let's consider at first the case of perfect learning :
Let θ_j (resp. θ_i) be the state for which value function is minimum in $c_1^{F,IP}$ in case of reversible decisions (resp. irreversible).
1) If $\theta_j = \theta_i$, then
- if $F^{\theta_j}(c_1^{F,IP}) \geq 0$,
$\frac{dJ^I(I^{\theta_i}(c_1^{F,IP}))}{dc_1} = \frac{dJ^I(c_1^{F,IP},\theta_j)}{dc_1} = -\delta.D'(\delta c_1^{F,IP} + I^{\theta_j}(c_1^{F,IP}), \theta_j) = -\delta.D'(\delta c_1^{F,IP} + F^{\theta_j}(c_1^{F,IP}), \theta_j) = \frac{dJ^F(c_1^{F,IP},\theta_j)}{dc_1}$ and so $c_1^{F,IP}$ is an optimum in the case of irreversible decisions too. Thus $c_1^{F,IP} = c_1^{I,IP}$.
- if $F^{\theta_j}(c_1^{F,IP}) < 0$, $I^{\theta_j}(c_1^{F,IP}) = 0$ then $\frac{dJ^I(I^{\theta_i}(c_1^{F,IP}))}{dc_1} = \frac{dJ^I(c_1^{F,IP},\theta_j)}{dc_1} = -\delta.D'(\delta c_1^{F,IP}, \theta_j) < -\delta.D'(\delta c_1^{F,IP} + F^{\theta_j}(c_1^{F,IP}), \theta_j) = \frac{dJ^F(c_1^{F,IP},\theta_j)}{dc_1}$.Thus $c_1^{F,IP} > c_1^{I,IP}$

2) If $\theta_j \neq \theta_i$, then necessarily
$F^{\theta_i}(c_1^{F,IP}) < 0$. So : $I^{\theta_i}(c_1^{F,IP}) = 0$
- If $I^{\theta_j}(c_1^{F,IP}) > 0$ then $F^{\theta_j}(c_1^{F,IP}) = I^{\theta_j}(c_1^{F,IP})$ and $F^{\theta_j}(c_1^{F,IP}) > I^{\theta_i}(c_1^{F,IP})$,
Since $\frac{dJ^F(F^{\theta_j}(c_1^{F,IP}),\theta_j)}{dc_1} = -\delta V_2'(F^{\theta_j}(c_1^{F,IP})) > -\delta V_2'(I^{\theta_i}(c_1^{F,IP})) \geq \frac{dJ^I(I^{\theta_i}(c_1^{F,IP}),\theta_i)}{dc_1} = \frac{dJ^I(I^{\theta_i}(c_1^{F,IP}))}{dc_1}$

then necessarily $c_1^{F,IP} \geq c_1^{I,IP}$

- If $I^{\theta_j}(c_1^{F,IP}) = 0$ and $F^{\theta_j}(c_1^{F,IP}) \leq 0$, then $\exists\ c_2\ \epsilon \left[F^{\theta_i}(c_1^{F,IP}); 0\right]$ such as $D(\delta c_1^{F,IP} + c_2, \theta_j) = D(\delta c_1^{F,IP} + c_2, \theta_i)$ with $D(\delta c_1^{F,IP} + c_2 + \epsilon, \theta_j) < D(\delta c_1^{F,IP} + c_2 + \epsilon, \theta_i)$ and $D(\delta c_1^{F,IP} + c_2 - \epsilon, \theta_j) < D(\delta c_1^{F,IP} + c_2 - \epsilon, \theta_i)$ for a small $\epsilon > 0$. So $D'(\delta c_1^{F,IP} + c_2, \theta_j) < D'(\delta c_1^{F,IP} + c_2, \theta_i)$

Either $c_2 \leq F^{\theta_j}(c_1^{F,IP}) \leq I^{\theta_i}(c_1^{F,IP})$ and so $\frac{dJ^F(F^{\theta_j}(c_1^{F,IP}),\theta_j)}{dc_1} = -\delta V_2'(F^{\theta_j}(c_1^{F,IP})) >$ $-\delta V_2'(F^{\theta_i}(c_1^{F,IP})) = -\delta D'(\delta c_1^{F,IP} + F^{\theta_i}(c_1^{F,IP}), \theta_i) > -\delta D'(\delta c_1^{F,IP} + I^{\theta_i}(c_1^{F,IP}), \theta_i) = \frac{dJ^I(c_1^{F,IP},\theta_i)}{dc_1}$ and so $c_1^{I,IP} < c_1^{F,IP}$.

Or $F^{\theta_j}(c_1^{F,IP}) \leq c_2$ and then $\frac{dJ^F(F^{\theta_j}(c_1^{F,IP}),\theta_j)}{dc_1} \geq -\delta D'(\delta c_1^{F,IP} + c_2, \theta_j) \geq -\delta D'(\delta c_1^{F,IP} + c_2, \theta_i) \geq -\delta D'(\delta c_1^{F,IP} + I^{\theta_i}(c_1^{F,IP}), \theta_i) = \frac{dJ^I(c_1^{F,IP},\theta_i)}{dc_1}$ thus $c_1^{I,IP} < c_1^{F,IP}$.

Now, let's consider the cas of no learning :

If $F^{\theta_1,\theta_2}(c_1^{F,AI}) \geq 0$, then $I^{\theta_1,\theta_2}(c_1^{F,AI}) = F^{\theta_1,\theta_2}(c_1^{F,AI})$, $\frac{dJ^I(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} = \frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1}$.and $c_1^{F,AI} = c_1^{I,AI}$.

If $F^{\theta_1,\theta_2}(c_1^{F,AI}) < 0$, then $I^{\theta_1,\theta_2}(c_1^{F,AI}) = 0$ and necessarily $\exists \theta_i$ such as $I^{\theta_i}(c_1^{F,AI}) = 0$, $J^I(c_1^{F,AI}, \theta_1, \theta_2) = J^I(c_1^{F,AI}, \theta_i)$ and $\frac{dJ^I(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} = \frac{dJ^I(c_1^{F,AI},\theta_i)}{dc_1} = -\delta.D'(\delta c_1^{F,AI}, \theta_i)$. On the other hand,

- either $\exists\, \theta_j$ such as $J^F(c_1^{F,AI}, \theta_1, \theta_2) = J^F(c_1^{F,AI}, \theta_j)$, $F^{\theta_1,\theta_2}(c_1^{F,AI}) = F^{\theta_j}(c_1^{F,AI})$ and $\frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} = \frac{dJ^F(c_1^{F,AI},\theta_j)}{dc_1} = -\delta.D'(\delta c_1^{F,AI} + F^{\theta_1,\theta_2}(c_1^{F,AI}), \theta_j)$ and if $\theta_j = \theta_i$ then $-\delta.D'(\delta c_1^{F,AI} + F^{\theta_1,\theta_2}(c_1^{F,AI}), \theta_j) > -\delta.D'(\delta c_1^{F,AI}, \theta_i)$, otherwise $\exists\, c_2 \in \left[F^{\theta_1,\theta_2}(c_1^{F,AI}), 0\right]$ such as $D(\delta c_1^{F,AI} + c_2, \theta_j) = D(\delta c_1^{F,AI} + c_2, \theta_i)$ and $D'(\delta c_1^{F,IP} + c_2, \theta_j) < D'(\delta c_1^{F,IP} + c_2, \theta_i)$ and so $-\delta.D'(\delta c_1^{F,AI} + F^{\theta_1,\theta_2}(c_1^{F,AI}), \theta_j) \geq -\delta.D'(\delta c_1^{F,AI} + c_2, \theta_j) > -\delta.D'(\delta c_1^{F,AI} + c_2, \theta_i) \geq -\delta.D'(\delta c_1^{\dagger}, \theta_i)$ which implies $c_1^{I,AI} \leq c_1^{F,AI}$,

- or $\frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} = -\delta.V_2'(F^{\theta_1,\theta_2}(c_1^{F,AI}))$ and $\exists\, \theta_j$ such as $F^{\theta_1,\theta_2}(c_1^{F,AI}) \geq F^{\theta_j}(c_1^{F,AI})$.

If $\theta_j = \theta_i$ and thus $\frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} = -\delta.V_2'(F^{\theta_1,\theta_2}(c_1^{F,AI})) \geq -\delta.V_2'(F^{\theta_j}(c_1^{F,AI}))$ $= -\delta.D'(\delta c_1^{F,AI} + F^{\theta_j}(c_1^{F,AI}), \theta_j) > -\delta.D'(\delta c_1^{F,AI}, \theta_i) = \frac{dJ^I(c_1^{F,AI},\theta_1,\theta_2)}{dc_1}$ and so $c_1^{I,AI} \leq c_1^{F,AI}$.

If $\theta_j \neq \theta_i$, that means that $F^{\theta_i}(c_1^{F,AI}) \geq F^{\theta_1,\theta_2}(c_1^{F,AI}) \geq F^{\theta_j}(c_1^{F,AI})$ and necessarily $D(\delta c_1^{F,AI} + F^{\theta_1,\theta_2}(c_1^{F,AI}) + \epsilon, \theta_j) > D(\delta c_1^{F,AI} + F^{\theta_1,\theta_2}(c_1^{F,AI}) + \epsilon, \theta_i)$ for $\varepsilon > 0$ small (i.e : θ_j is the bad state of the world on the left side of $\delta c_1^{F,AI} + F^{\theta_1,\theta_2}(c_1^{F,AI})$).But that is θ_i the bad state of the world in $\delta c_1^{F,AI}$. Consequently $\exists\, c_2 \in \left[F^{\theta_1,\theta_2}(c_1^{F,AI}), 0\right]$ such as $D(\delta c_1^{F,AI} + c_2, \theta_j) = D(\delta c_1^{F,AI} + c_2, \theta_i)$ and $D'(\delta c_1^{F,IP} + c_2, \theta_j) < D'(\delta c_1^{F,IP} + c_2, \theta_i)$ So $\frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} \geq -\delta.D'(\delta c_1^{F,AI} + F^{\theta_j}(c_1^{F,AI}), \theta_j) > -\delta.D'(\delta c_1^{F,IP} + c_2, \theta_j) >$ $-\delta.D'(\delta c_1^{F,IP} + c_2, \theta_i) \geq -\delta.D'(\delta c_1^{F,AI}, \theta_i) = \frac{dJ^I(c_1^{F,AI},\theta_1,\theta_2)}{dc_1}$,thus $c_1^{I,AI} \leq c_1^{F,AI}$.

b) Informationnal effect does not hold necessarily.

- If $D(\delta c_1 + F^{\theta_i}(c_1), \theta_i) > D(\delta c_1^{F,AI} + F^{\theta_i}(c_1), \theta_{-i})$ in $c_1^{F,AI}$ or $c_1^{F,IP}$, then we are in the

case (i) or (ii) of lemma 6, thus $J^F(c_1) = J^F(c_1, \theta_i) = J^F(c_1, \theta_1, \theta_2)$
i.e. the information structure has no effect on the current decisions : $c_1^{F,AI} = c_1^{F,IP}$.

• Let's consider now the case (iii) of lemma 6 :

If in $c_1^{F,AI}$: $J^F(c_1^{F,AI}, \theta_i) < J^F(c_1^{F,AI}, \theta_{-i})$ and $F^{\theta_i}(c_1^{F,AI}) > F^{\theta_{-i}}(c_1^{F,AI})$ then $\frac{dJ^F(c_1^{F,AI})}{dc_1} = \frac{dJ^F(c_1^{F,AI},\theta_i)}{dc_1} = -\delta.V_2'(F^{\theta_i}(c_1^{F,AI}))$ and $\frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} = -V_2'(F^{\theta_1,\theta_2}(c_1^{F,AI}))$. Since $F^{\theta_1,\theta_2}(c_1^{F,AI}) < F^{\theta_i}(c_1^{F,AI})$ we have $\frac{dJ^F(c_1^{F,AI})}{dc_1} > \frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} \Rightarrow c_1^{F,IP} > c_1^{F,AI}$: informationnal effect does not hold. (one can construct examples to have this situation).
If in $c_1^{F,AI}$: $J^F(c_1^{F,AI}, \theta_i) < J^F(c_1^{F,AI}, \theta_{-i})$ and $F^{\theta_i}(c_1^{F,AI}) < F^{\theta_{-i}}(c_1^{F,AI})$, then $\frac{dJ^F(c_1^{F,AI})}{dc_1} < \frac{dJ^F(c_1^{F,AI},\theta_1,\theta_2)}{dc_1} \Rightarrow c_1^{F,IP} < c_1^{F,AI}$: informationnal effect holds.

We have demonstrated the case whre theres is not informationnal effect. One can produce a similar demonstration for the case of irreversible decisions and we have the same formal characterisation.

c) Irreversibility informationnal effect holds..

We proceed by a demonstration by the absurde. Suppose that the informationnal effect holds for the case of reversible decision but does not hold for the case of irreversible decisions. Let θ_i, be the state in which the value function is minimal for the case of irreversible decisions in $c_1^{I,IP}$. Given the formal characterisation in b) for the absence of informationnal effect, we must have $I^{\theta_i}(c_1^{I,IP}) > I^{\theta_1,\theta_2}(c_1^{I,IP}). \geq I^{\theta_{-i}}(c_1^{I,IP})$ and necessarily $I^{\theta_i}(c_1^{I,IP}) > 0$, so $I^{\theta_i}(c_1^{I,IP}) = F^{\theta_i}(c_1^{I,IP})$, which implies that in case of reversible decisions, in $c_1^{I,IP}$, the minimum is reached in θ_i.So $c_1^{I,IP} = c_1^{F,IP}$.
If $I^{\theta_1,\theta_2}(c_1^{I,IP}) > 0$, then $I^{\theta_1,\theta_2}(c_1^{I,IP}) = F^{\theta_1,\theta_2}(c_1^{I,IP})$ and we should have $c_1^{F,IP} > c_1^{F,AI}$ which is conflicting with the postulate of the beginning.
If $I^{\theta_1,\theta_2}(c_1^{I,IP}) = 0$, then $I^{\theta_1,\theta_2}(c_1^{I,IP}) = 0 = I^{\theta_{-i}}(c_1^{I,IP})$ and necessarily θ_{-i} is the "bad" state of the world in $c_1^{I,IP}$ which is contradictory with the postulate of the beginning.
In fact, $I^{\theta_i}(c_1^{I,IP}) > I^{\theta_1,\theta_2}(c_1^{I,IP}) \geq I^{\theta_{-i}}(c_1^{I,IP})$ does not hold. I.e. informationnal irreversibility effect necessarily holds.

■

Proof. of proposition 7

The reader can easily check the pure irreversibility effect.

The quadratic example proposed by Ulph et Ulph in order to demonstrate that informationnal effect isn't systematic is good for the general model.

Let's suppose $c_1^{F,AI}(p) \geq c_1^{F,IP}(p)$ which is similar to the fact that $-\delta.p.D'(\delta c_1 + F^{\theta_1}(c_1^{F,AI}(p)), \theta_1) - \delta.(1-p).D'(\delta c_1 + F^{\theta_2}(c_1^{F,AI}(p)), \theta_2)$
$\leq -\delta.p.D'(\delta c_1 + F^p(c_1^{F,AI}(p)), \theta_1) - \delta.(1-p).D'(\delta c_1 + F^p(c_1^{F,AI}(p)), \theta_2)$, where $F^p(c_1^{F,AI}(p))$ denotes

the optimal choice in period 2 in the cas of no learning. Either the constraint does not bite in case of no learning and then $c_1^{F,AI}(p) = c_1^{I,AI}(p) \geq c_1^{F,IP}(p) \geq c_1^{I,IP}(p)$. Or it bites and so $I^p(c_1^{F,AI}(p)) = 0$ and necessarily $I^p(c_1^{I,AI}(p)) = 0$ too. Since $\forall c_1^{I,AI}\ F^p(c_1^{I,AI}) \in \left[\min(F^{\theta_1}(c_1^{I,AI}), F^{\theta_2}(c_1^{I,AI})), \max(F^{\theta_1}(c_1^{I,AI}), F^{\theta_2}(c_1^{I,AI}))\right]$,necessarily $\min(I^{\theta_1}(c_1^{I,AI}(p)), I^{\theta_2}(c_1^{I,AI}(p)))$ 0. If $I^{\theta_i}(c_1^{I,AI}(p)) = \min(I^{\theta_1}(c_1^{I,AI}(p)), I^{\theta_2}(c_1^{I,AI}(p)))$ then $-D'(\delta c_1 + I^{\theta_i}(c_1^{I,AI}(p)), \theta_i) = -D'(\delta c_1 + I^p(c_1^{I,AI}(p)), \theta_i)$. If $I^{\theta_i}(c_1^{I,AI}(p)) = \max(I^{\theta_1}(c_1^{I,AI}(p)), I^{\theta_2}(c_1^{I,AI}(p))) \geq 0 = I^p(c_1^{I,AI}(p))$ then $-D'(\delta c_1 + I^{\theta_i}(c_1^{I,AI}(p)), \theta_i) \leq -D'(\delta c_1 + I^p(c_1^{I,AI}(p)), \theta_i)$. Consequently $-\delta.p.D'(\delta c_1 + I^{\theta_1}(c_1^{I,AI}(p)), \theta_1) - \delta.(1-p).D'(\delta c_1 + F^{\theta_2}(c_1^{I,AI}(p)), \theta_2)$ $\leq -\delta.p.D'(\delta c_1 + F^p(c_1^{I,AI}(p)), \theta_1) - \delta.(1-p).D'(\delta c_1 + F^p(c_1^{I,AI}(p)), \theta_2)$, which implies $c_1^{I,AI}(p) \geq c_1^{I,IP}(p)$. ■

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