"A Theory of Size and Product Diversity of Marketplaces with Application to the Trade Show Arena"

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ISBM Report 2-2000

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A Theory of Size and Product Diversity of Marketplaces with Application to the Trade Show Arena

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Version: March 9th 2000

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Abstract

Markets involve the exchange of information and products between buyers and sellers in marketplaces that are created by market organizers. This paper develops a theory to explain the differences in the size (number of participants) and diversity (range of products displayed) across these marketplaces. We assume that successful transactions require information transmission between parties calling for investment in time and effort. Two key factors affect this process of information interchange: diminishing marginal returns to effort, which encourages diversification and congestion cost resulting from participant overload. We study a sequential model of interaction between buyers, sellers and marketplace organizers. Organizers choose the number and nature of marketplaces to organize and set entry fees, while buyers and sellers make participation and effort allocation decisions. We show that participants' breadth of product interest, their buying and selling intensities (i.e. how frequently they are likely to engage in future product transactions) as well as the technological innovativeness of the industry have important influences on the size and range of product diversity in the marketplace. We apply this model to the industrial trade show arena to explain differences in trade show types (horizontal with broad product focus vs. vertical with narrow product focus) across industries. Empirical tests of several propositions derived from our model provide an encouraging degree of support for our theory. Our analysis identifies several industries that appear to be underserved by one type of show or the other, suggesting possible future opportunities for organizers.

Keywords: Marketplaces, Trade Shows, Product Diversity, Game Theory

1. Introduction

Buyers, sellers and purchase influencers exchange products and information in a wide variety of forums, including shopping centers, supermarkets, professional meetings, exhibitions and trade shows as well as, increasingly in electronic, on-line marketplaces. While the act of actual exchange of products is often the ultimate interaction goal, the information exchange that precedes the actual exchange receives much less careful study. Our goal in this paper is to show that the process of effort and time allocation towards receiving and distributing information has major implications for the number of marketplace participants and the diversity of product offerings in those marketplaces.

As an example of one forum where a large proportion of the participants' time is spent on information exchange, consider exhibitions and trade shows. In a typical trade show attendees spend considerable time researching the costs/benefits of current and future solutions to their needs and exhibitors invest comparable time disseminating the costs/benefits of their proposed solutions to prospective customers. These shows constitute an important communication and exchange medium in the business marketplace, accounting for nearly 20% of the communications mix dollar (Business Marketing, 1999). According to one estimate (Kotler, 1999), companies annually spend more than $15 billion on trade shows in the United States alone; moreover, these shows generate more than $70 billion in sales annually there. Trade shows are no less important on the global scene; for instance, in 1989 there were 169 international shows held in Germany with 124,000 exhibitors and over ten million attendees. During the same year, 220 international shows were organized in the UK with 61,000 exhibitors and 4.6 million attendees (Hansen, 1999).

But trade shows don't just happen. Someone must organize them and position them in the market, setting prices (entry fees) and communication programs to encourage exhibitors and attendees to participate. Optimal fees should maximize the organizer's profit by drawing the optimal number of participants while giving the right incentives for participants: i.e., no actual participant should make more (net) profit by staying out, and no uninterested potential participant should make more (net) profit by participating (at those fees). But, what determines these optimal numbers? We can develop some insights into this issue by taking a closer look at the act of information exchange.

Gathering of information and its dissemination, like many other economic activities, are likely to experience diminishing marginal returns to investment in time: after some interchange neither a buyer nor a seller is likely to get much more out of additional interaction. Since diminishing marginal returns encourages spreading of resources across alternatives, it would seem that marketplaces should be filled with as many buyers and sellers as possible. However, with more participants comes congestion, a dissipation of participants' effort that could have been spent in information exchange. This observation suggests that a market should be divided up among a number of marketplaces, the size of each of which should be governed by the tradeoff between participants' desire for diversity and the cost of congestion.

Thus, even with only (multiple varieties of) a single product one might expect multiple marketplaces to exist; however, additional considerations emerge when we consider a market with multiple products. In this case, a marketplace organizer has the choice between organizing marketplaces where all products are displayed, or he may organize trading platforms where only a few are traded. Given this choice, what kind of product coverage should the organizer opt for? The heterogeneity of product interests across buyers and sellers clearly is a determining factor here, but in this paper we raise a more limited question: assuming that both buyers and sellers are homogenous, what other factors would drive us toward narrow versus broad trading platforms? Can we define operational variables that provide guidance for the organizers about which marketplaces might currently be 'too narrow' or 'too broad'?

To our knowledge, these issues are raised here for the first time. In the Economics of Industrial Organization literature, product diversity (or differentiation) in markets has seen considerable attention, following the seminal models of Hotelling (1929) in the context of oligopoly and Dixit-Stiglitz (1977) in the context of monopolistic competition (see Beath and Katsoulacos (1991) for an overview). However, we emphasize that the notion of product differentiation in *markets* is unrelated to the notion of product diversity in *marketplaces*. A market or an industry may have great product diversity and yet, each marketplace may feature a very narrow set of products. Similarly, congestion in the product markets is a well-studied phenomenon but usually in the context of externalities that a customer experiences in the presence of other customers (e.g. see Reitman, 1991, and the references cited there). However, in our framework, congestion acts through its negative impact on information transmission (and hence matching of needs), thus affecting producers in equal measure.

We proceed as follows. While we have couched our discussion broadly, we focus on the specifics of the industrial exhibition or trade show here both for concreteness and to provide an opportunity for empirical testing. Thus, Section 2 discusses trade shows and provides some interesting observations that we seek to understand. In Section 3 we set up our model in terms of the interaction that takes place among three types of agents: attendees, exhibitors and show organizers. We then specify a production function and a congestion cost function. In Section 4, we first deal with a one-product (vertical) show, identify a plausible equilibrium and calculate the total surplus the show generates. We then do the same for a multi-product (horizontal) show and develop a key proposition comparing the profits from a vertical show with those from a horizontal show. In Section 5 we develop the theory that determines the size of shows as well as the numbers of horizontal and vertical shows that one can expect to see in an industry (and, thus, their relative proportions). Section 6 uses this theory to study the influence of various industry characteristics on the proportion of horizontal shows, resulting in several testable hypotheses. In Section 7 we conduct an empirical investigation of these hypotheses, providing support for our theory. Our analysis also develops some useful managerial guidelines by pointing out which industries, according to our theory, are currently over or underserved by horizontal (versus vertical) shows. Section 8 discusses the implications of our findings, model limitations and directions for further research. A glossary lists all notations used in the text while proofs of the propositions are available in Dasgupta, Lilien and Wu (2000) or on line at www.smeal.psu.edu/isbm.

2. Trade Show Facts and Puzzles

One type of marketplace where the issues raised above are clearly relevant involves trade shows or (equivalently here) industrial exhibitions. Trade shows serve as excellent conduits of information exchange as well as selling platforms for new and upcoming products. Trade shows have been traditionally categorized as vertical or horizontal based on market coverage (Gopalakrishna and Lilien, 1994). Typically, a vertical show involves a narrow range of products and attracts visitors specifically interested in those products. In contrast, a horizontal show usually involves a much broader range of products and a more diverse audience. For example, attendees at the Association of Operating Room Nurses show, a vertical show, are almost all operating room nurses, and exhibitors display products that are used almost exclusively in the operating room. The National Design Engineering Show is a horizontal show in which firms demonstrate products ranging from mechanical components, electrical and electronic components, plastics, elastomers, to CAD/CAM systems (Gopalakrishna and Williams, 1992).

Exhibit 1, prepared by Exhibit Surveys Inc., details the number of vertical shows, horizontal shows and the proportion of horizontal shows for a number of different industries during the period from 1985 to 1991. There appears to be no simple pattern here: why are 319 of the 322 shows in the Communications industry vertical, 61 of the 64 shows in the Food Processing and Distribution industry horizontal while the Chemical industry splits almost evenly with 20 vertical and 23 horizontal shows?

Even the status of existing shows evolve, with some merging and becoming more horizontal and others splitting into more narrow shows. For instance, Conexpo-Con/Agg '96, the largest US trade show in the construction industry, resulted from a merger of Conexpo (owned by Construction Industry Manufacturers Association), the largest construction show, and Con/Agg (owned by the National Aggregates Association and the National Ready Mixed Concrete Association), one of the premier vertical shows specializing in concrete and aggregates in the United States. The new, horizontal show attracted more than 1250 exhibitors and 100,000 attendees, and covered 1.25 million square feet of exhibiting area, making it bigger than the last Conexpo and Con/Agg combined[1]. On the other hand, Comdex, the largest (horizontal) computer show in the US, has seen threats by key exhibitors to split and form more narrowly focused shows[2].

This diversity and structural evolution of show type has seen little academic attention. Most trade show studies emphasize lead (or sales) generation as the key goal, providing little motivation for the existence of horizontal shows (Gopalakrishna and William 1992, Gopalakrishna and Lilien 1993, Gopalakrishna and Lilien 1994, Dekimpe et al. 1996). These studies measure the performance of the show by how efficiently the show generates leads or sales and generally assume homogeneous objectives on the parts of both exhibitors and attendees. The studies all indicate that vertical shows outperform horizontal shows in terms of selling efficiency, calling into question the economic rationale for horizontal shows. So, why do

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[1] Show Daily of Conexpo and Con/Agg, '96, page 18.

[2] Jim Carlton, “Comdex Complaint As Big Computer Show Opens, Key Exhibitors Grumble About Costs”, W*all Street Journal*, 11/14/1994, Page A1.

both types of shows exist and why does the proportion of show types vary so widely across industries?

One way to address this question is to recognize that most measures of lead generation, being rooted to benefits taking place in the present or near future, do not capture the full economic gains that may result from information interchange and be realized in the more distant future. This is not a new idea; the literature has recognized the existence of these dual motives of participants in a show. For instance, Kerin & Cron (1987) have looked at the functions of trade shows from the exhibitors' point of view and showed that there are two categories of roles that trade shows play: selling functions and non-selling functions. Some exhibitors have objectives that are primarily sales oriented, such as generating leads, while others have non-sales oriented objectives such as gathering competitive intelligence and enhancing company image. Following Kerin and Cron’s approach, Hansen (1999) reconsidered the multidimensionality of trade show performances. He proposed a two-dimensional framework of trade show performance: an outcome-based dimension, which includes the sales-related activities, and a behavior-based dimension which includes nonsales-related activities, such as information-gathering activities, image-building activities, motivation activities, and relationship-building activities. Both papers focus solely on exhibitors, but Hansen (1999) recognizes that this dichotomy of focus may be relevant for attendees as well.

The above discussion seems to suggest that the alleged inefficiency of horizontal shows is solely a result of researchers’ inability to quantify the *long-run* economic output of non-selling and information-gathering activities. However, this explanation holds only if horizontal shows experience more non-selling/information gathering activity than do their vertical counterparts. But are there a priori reasons why a wider diversity of product in a marketplace should necessarily be correlated with a longer time horizon for transaction of the participants? One of the goals of this paper is to show that indeed, there are such reasons. Thus, we provide an explanation for the observation made by several authors on the superiority of vertical shows in terms of lead generation.

One might wonder why horizontal shows, with more products and typically larger number of participants, will not always dominate vertical shows in terms of total surplus generation. One reason for the absence of such a clear dominance is the differences in organizational costs: vertical shows, appealing to a narrower audience, should cost less to organize than a horizontal show. However, there is another factor at work: exhibitors and attendees may have different interests in different product categories either because of firm or job-responsibility differences (purchasing agents vs. R&D engineers; smaller firms vs. large firms) or because of industry differences (Williams et al, 1994). For example, in the computer industry, a hardware manufacturer such as Intel may focus its attention on hardware; however, Dell, another manufacturer, in its role as an attendee, may not only be concerned about hardware but also about the software it is considering bundling with its computers. Now, returns to effort in a potential exchange must account for efforts expended by both parties: if attendees and exhibitors are interested in different kinds of products, or if the range of product interests is narrow for one group and broad for the other, a mismatch arises. The allocation of effort across different products then becomes a strategic consideration and, in equilibrium, the total gross surplus generated in a horizontal show may not always dominate that of a vertical show or vice versa. Thus we posit that the proportion of horizontal shows in an industry is the result of a complex interaction between: a) the degree of diminishing marginal returns associated with the information interchange process, b) the extent of future transaction interests of both parties, c) the breadth of product interests of both parties and their compatibility and d) the fixed costs associated with organizing shows.

3. The Model Setup: Benefits and Costs of Participation

In this section we specify the sequence of events in our model, and two of its key constructs — the function relating the time/effort investment by participants into benefits (the production function) and a deterrent factor for markets of unmanageable size, the congestion cost. Although our model applies beyond the trade show arena, for concreteness we will refer to the marketplace as a 'show' and the two categories of participants in the marketplace as 'attendees' and 'exhibitors'.

A. Timing: Our model is based on the interaction between three types of agents in the marketplace: attendees, exhibitors (assumed homogeneous within category) and (potential) marketplace organizer(s). We will also assume there is either one product category (X) or two product categories (X plus Y) under consideration. The sequence of events in the model is as follows:

1. Each (potential) organizer decides how many shows of each type to set up (X and/or Y, vertical shows or X&Y, the horizontal show). They then invest in the fixed factors, like

show space, publicity and so forth that do not vary with the number of participants, for the shows they have decided to set up. We assume that the prices of these factors will rise as the demand for them increases.

2. Next, the organizers decide on (and announce) the fees they plan to charge attendees and exhibitors that participate in any show they organize.

3. Then, each individual from a large pool of identical potential attendees decides which show to attend (if at all) and each individual from a large pool of identical exhibitors (simultaneously) decides whether to participate in that show. The organizer of a show realizes the total of the entry fees minus his variable costs.

4. Upon arriving at a show, each attendee decides how to allocate his available effort or time among the exhibitors present. For a horizontal show, he further decides how to allocate effort toward particular exhibitors within different product categories. Exhibitors make similar effort allocation decisions.

Participants see benefits in interaction through a production function and see disincentives to interaction through a congestion cost function. We specify these functions next.

B. Production Function: A simple way to model how efforts by an attendee-exhibitor pair get translated into the benefits they seek is to develop a production function where efforts appear as inputs and the (expected) benefit appears as the output. Such a function should have the following properties:

a. If effort expended by either party is zero, then no matter how much effort the other party expends, the output (or benefit) is zero.
b. The marginal return to effort of any party is increasing in the effort expended by the other party and
c. There are diminishing marginal returns to effort for both parties, thus giving rise to 'variety seeking' behavior.

A commonly used specification with these properties is the Cobb-Douglas production function:

(3.1) $U = kr^{-1}\lambda v(pq)^{\alpha}$

where U is the expected benefit to a party, k is a positive constant, r is the (common) rate of interest used by both parties, λ is the 'future transaction intensity factor' of the pair, v is the net value generated to the party per unit transaction in the exhibitor's product, p is the effort invested by the exhibitor, q is the effort invested by the attendee and α is a parameter that captures how fast marginal returns (to joint effort) diminish. We further motivate the formulation and explain the terms in (3.1) next.

Within a product category (X or Y), we consider each exhibitor as a producer of a specific "brand, " where each brand has a slightly different formulation that the attendees unaware of are a priori. Each attendee has slightly different needs, characterized by an idiosyncratic taste parameter that governs whether a specific exhibitor's brand fulfils those needs. The purpose of information exchange is for the attendee to explain his needs to the to the exhibitor and for the exhibitor to explore how well the features of his brand meet that attendee's needs. The joint investment of effort raises the likelihood of a match of needs with features. To see the importance of this process, consider for example, the way DuPont communicates to its potential customers for engineering polymers, stressing the need for deep and continued interaction *prior* to developing a purchase commitment:

> Today you need more from your suppliers than just materials. You need a resource that is willing and able to join in at the earliest stages of the product development process. One that can carry a project from concept through design, component analysis, material selection, prototyping, testing, quality control, and even commercialization. You need a fully fledged partner.
>
> Source: *http://www.dupont.com/enggpolymers*/

Specifically, we assume that if the attendee expends effort level p and the exhibitor expends effort q in the interchange, then $k(pq)^{\alpha}$ reflects the ex-ante likelihood of a match. More effort permits the attendee to reveal more about his needs and the exhibitor to explore more ways to satisfy those needs, making a match more likely. The constant of proportionality k can be product specific.

If a match is made, we assume that it will lead to a sequence of purchase transactions in the future occurring at random points of time following (roughly) a Poisson process with intensity λ. Thus, the expected time until the first transaction from the time of the show (and the expected length of time between any two subsequent transactions) is λ^{-1}. We assume that λ depends on both the 'buying intensity' of the attendee and the 'selling intensity' of the exhibitor. "Buying intensity' is an attendee characteristic that roughly captures the frequency of occurrence of the need that the exhibitor's product satisfies. 'Selling intensity' is an exhibitor characteristic that summarizes several aspects of the seller's situation (capacity utilization, need for inventory clearance, financial situation etc.) that influence how heavily the exhibitor pressures attendees to convert their needs into more immediate orders and also how quickly the exhibitor can deliver on such orders. We expect both buying intensity and selling intensity to make actual product transactions more frequent on average, increasing λ.

Each time an attendee actually engages in a unit transaction of a brand, he receives a certain gross benefit – or 'unit value'. Delivering each unit of that transaction costs an exhibitor a certain amount – or 'unit cost' (irrespective of the brand). The difference between the unit value and the unit cost is the surplus generated per unit transaction (specific to that product class). We assume that all brands charge the same 'unit price'. Then the attendee's net value per transaction in the product is v_a = unit value – unit price, and the exhibitor's net profit per transaction is v_e = unit price – unit cost. We can now interpret equation (3.1): if the (continuous) rate of interest used by all parties is r, then the expected net present value of the (random) payoff stream for an attendee is $\lambda v_a / r$ conditional on a match having taken place. Ex-ante, then, that exhibitor's expected benefit is given by $kr^{-1}\lambda\, v_a(pq)^{\alpha}$. Similarly, the exhibitor's expected benefit is given by $kr^{-1}\lambda\, v_e(pq)^{\alpha}$. Henceforth, we will call the expression $kr^{-1}\lambda\, v_e$ the exhibitor's *product valuation parameter* (which is product class specific) and use the symbol β to refer to it. Similarly we will use the symbol γ to stand for $kr^{-1}\lambda\, v_a$, the attendee's product valuation parameter. We investigate the roles that the various components of β and γ play in Section 6.

Note that in (3.1) we do not use separate exponents for p and q as that would complicate the analysis without adding much insight. Also, α must be less than 1.0 to guarantee diminishing marginal returns; we will require it to be less than 0.5 to ensure that if *both* the attendee and the exhibitor double their efforts, benefits increase by less than 100%. Finally, note that for markets dealing with new and technologically innovative products α is likely to be small, as the first few units of time-investment are likely to have a high learning content and yield much more benefit than will later investments.

C. Congestion Costs: While both attendees and exhibitors must allocate time and effort to each other, not all such available time is potentially used; some of it is lost as a consequence of congestion costs. We use the term *congestion cost* to refer to any unproductive usage of participants' time. For attendees, a major component of attendees' costs stems from other attendees queuing up at booths, a quantity increasing in the number of attendees, n_a. Another unproductive use of time is associated with moving from booth to booth, suggesting that the cost should also be increasing in n_e, the number of exhibitors (but less than linearly, as booths are usually spread out in an exhibition hall and not arranged in a row). There are also switching costs associated with changing the frame of reference/conversation when moving from one exhibitor to another. Finally, note that the total congestion cost must stay bounded by total time available. Thus, if the total available time is normalized to 1, it is perhaps not unreasonable to model it by means of a function such as:

(3.2) $\quad$ *Congestion Cost (for Attendees) = 1 – Exp* $(-\ \varphi_a(n_a\ ,\ n_e))$,

where φ_a is increasing in both arguments. Similarly, exhibitors are also likely to suffer from congestion costs, particularly because of the mind-frame switching effect, though conceivably they may suffer less than the attendees do. Letting the total available time for an exhibitor be also normalized by 1, we postulate that

(3.3) $\quad$ *Congestion Cost (for exhibitors) = 1 – Exp* $(-\ \varphi_e\ (n_a\ ,\ n_e))$,

where φ_e is another function (weakly) increasing in both arguments.

Let the symbols T_a and T_e denote available *productive* time (i.e. time net of congestion cost) for attendees and exhibitors respectively. As Sections 4 and 5 will show, what matters for our analysis is the product T_aT_e, which can be seen from (3.2) and (3.3) to be of the form: $Exp\ \{-\ (\varphi_a(n_a\ ,\ n_e) + \varphi_e\ (n_a\ ,\ n_e)\ )\}$. For simplicity, we will assume a linear, first order approximation to the argument of the exponential function, i.e. we will assume:

(3.4) $\quad$ $T_aT_e = Exp\ \{-\ (k_a\ n_e + k_e\ n_a\ )\}$,

where k_a and k_e are positive parameters.

We reiterate that the individual expressions for T_a and T_e are unimportant as long as their product is expressible in the above form (thus it is possible to allow one of them to be unaffected or, in the extreme, be decreasing in either n_a or n_e). Also, note that for the purpose of our theory, we need formulation (3.4) to hold only in the vicinity of equilibrium values of n_a and n_e. Indeed, one can easily conceive of extreme instances where the product of the productive times for the two parties will actually go up with an influx of more attendees. For instance, consider the situation where there are ten attendees and a thousand exhibitors, resulting in most exhibitors being idle most of the times. It is reasonable to assume that the introduction of another ten attendees will not adversely affect the other attendees' productive time(s) much, but will raise the exhibitors' productive time(s) quite a bit. However, such scenarios never occur in practice and therefore, are unlikely to be relevant for equilibrium analysis.

4. Equilibria for Vertical and Horizontal Shows

Now that we have developed the conditions for our model setup, we next investigate how those assumptions and functions determine the structure (p's, q's and resulting economic returns) associated with different show types. We thus consider the nature of equilibria within a trade show marketplace, focusing on the effort allocation decisions by attendees and exhibitors. We investigate these equilibria for a vertical show first and then for a horizontal show, assuming that exhibitors and attendees have paid their entry fees and that n_e exhibitors and n_a attendees have already gathered there.

A. *<u>Vertical Shows</u>*: Assume exhibitor i allocates effort p_{ij} toward attendee j and let attendee j allocate effort q_{ji} toward exhibitor i in a vertical show. The allocations of efforts (calculated net of congestion costs incurred) that the attendee and the exhibitor make are the critical, strategic decisions. Since each attendee j divides up his available time T_a among various exhibitor i's and each exhibitor i divides up his available time T_e among various attendee j's, we must have:

(4.1) $$\sum_{j=1,..n_a} p_{ij} = T_e \qquad i = 1,...,n_e \text{, and}$$

(4.2) $$\sum_{i=1,...,n_e} q_{ji} = T_a \qquad j = 1,...,n_a \text{.}$$

Assume that the expenditure of a unit of effort by each party results in a net benefit of β for an exhibitor, and γ for an attendee. Then exhibitor i makes $\beta(p_{ij}\, q_{ji})^{\alpha}$ and attendee j makes $\gamma(p_{ij}\, q_{ji})^{\alpha}$ through their joint meeting. We now have used our model to define a game between $n_a + n_e$ agents and we seek the (Nash) equilibrium efforts and the equilibrium payoffs.

A natural and intuitively appealing equilibrium for this game is the symmetric equilibrium where each attendee allocates (available) effort equally among all exhibitors, and vice versa, each exhibitor allocates effort equally among all attendees. However, this is not necessarily the unique symmetric equilibrium: consider a show where there are two exhibitors E1 and E2 and two attendees A1 and A2. Then, an equilibrium is $p_{11} = T_e, p_{12} = 0, p_{21} = 0, p_{22} = T_e, q_{11} = T_a, q_{12} = 0, q_{21} = 0, q_{22} = T_a$. In other words, in this equilibrium E1 and A1 work with each other as do E2 and A2. A1 does not spend any effort toward E2 justified by the fact that E2 does not spend any effort towards A1, and E2 is justified in his not spending effort towards A1 since E2 sees none coming from A1. The same story works for A2 and E1 as well.

Extending the spirit of this example, one can construct numerous equilibria when there are a large number of exhibitors and attendees.

We will, nevertheless, focus on the 'equal division' equilibrium; we justify this choice using a selection criterion that is in the spirit of Selten's (1975) trembling-hand perfection criterion (for related definitions and their equivalence see Fudenberg and Tirole (1993) or van Damme (1987)). Intuitively, for a Nash Equilibrium to be trembling hand perfect, not only should a player's strategy choice be best response to the strategy profile chosen by the others, but in addition it should continue to be a near-best response when the others' strategies are perturbed a little. This additional constraint imparts stability to the equilibrium, ruling out certain dubious equilibria such as ones where weakly dominant strategies are employed. Formally, in a finite-player game where each player has a finite number of strategies, a perfect equilibrium is defined as the limit of ε - constrained equilibria as ε goes to zero. An ε - constrained equilibrium is, on the other hand, a mixed strategy profile where each player *i* puts at least $\varepsilon(s_i)$ weight on each of his pure strategy options s_i with $0 < \varepsilon(s_i) \leq \varepsilon$, and which, (subject to this minimum weight restriction) is a best response to itself. There is no established definition of trembling hand perfection where players' strategy spaces are not finite; however in a similar spirit we define a 'perfect-like' equilibrium here as follows:

Definition: A (pure strategy) Nash equilibrium profile (p_{ij}, q_{ij}) is *perfect-like* if there exists strictly positive sequences $p_{ij}^n, q_{ji}^n, \varepsilon_{ij}^n, \varepsilon_{ji}^n$ such that

(4.3) 1) $p_{ij}^n \xrightarrow{n} p_{ij}$, $q_{ji}^n \xrightarrow{n} q_{ji}$

2) $\varepsilon_{ij}^n \xrightarrow{n} 0$, $\varepsilon_{ji}^n \xrightarrow{n} 0$

3) $p_{ij}^n \geq \varepsilon_{ij}^n$, $q_{ji}^n \geq \varepsilon_{ji}^n$

and 4) subject to the restriction in 3), (p_{ij}^n, q_{ji}^n) is a best reaction to itself.

As we will show, this criterion rules out equilibria where one of the effort levels is zero. Although perfect-likeness might sound complex, it does capture the observation that when attendees go to a show, they generally at least scan *all* the exhibits. Technically, the positivity of all efforts enables us to use first order conditions for an interior optimum for each player's problem, resulting in the intutively appealing 'equal division' equilibrium. We thus have the following proposition.

Proposition 1. *In the vertical show game, there is a unique (pure strategy) perfect-like equilibrium where for all i,* $p_{ij} = T_e / n_a$ *, j = 1, …,* n_a *and for all j,* $q_{ji} = T_a / n_e$*, i = 1, …,* n_e*.*

We note that in this equilibrium each exhibitor makes

(4.4) $\beta\, n_a^{1-\alpha} n_e^{-\alpha} (T_a T_e)^{\alpha}$

while each attendee makes

(4.5) $\gamma\, n_e^{1-\alpha} n_a^{-\alpha} (T_a T_e)^{\alpha}$.

Thus the total surplus generated from the show is

(4.6) $(\beta + \gamma)(n_a n_e)^{(1-\alpha)} (T_a T_e)^{\alpha}$.

B. *Horizontal Shows*. Now consider a horizontal show with two product categories X and Y. As before, let n_a and n_e be the number of attendees and exhibitors. Let the net benefit for an exhibitor when each party invests unit effort toward the X product be given by β_x and that for the Y product be given by β_y. Similarly, let γ_x and γ_y represent each attendee's product valuation parameters for a unit expenditure of (joint) effort in X and Y respectively. Each attendee will now have to decide not only how to allocate efforts among exhibitors but also how to split the effort he allocates to each exhibitor into effort directed towards X and that directed towards Y. A similar decision problem will have to be solved by exhibitors.

Let p_{ijx} and p_{ijy} represent efforts allocated by exhibitor i toward attendee j in discussing X and Y respectively. Similarly, let q_{jix} and q_{jiy} represent the effort allocated by attendee j toward exhibitor i concerning products X and Y respectively. The following identities must hold:

(4.7) $\sum_{j=1,\ldots n_a} p_{ijx} + \sum_{j=1,\ldots,n_a} p_{ijy} = T_e \quad \forall\ i = 1,\ldots,n_e$

and

(4.8) $\sum_{i=1,\ldots n_e} q_{jix} + \sum_{i=1,\ldots,n_e} q_{jiy} = T_a \quad \forall\ j = 1,\ldots,n_a$

In his meeting with attendee j exhibitor i makes

(4.9) $\beta_x (p_{ijx} q_{jix})^{\alpha} + \beta_y (p_{ijy} q_{ijy})^{\alpha}$

while attendee j makes

(4.10) $\gamma_x (p_{ijx} q_{jix})^{\alpha} + \gamma_y (p_{ijy} q_{ijy})^{\alpha}$.

As in the vertical trade show case, we can identify a perfect-like equilibrium, where each exhibitor allocates effort equally among all attendees, and each attendee allocates effort equally among all exhibitors; moreover each exhibitor allocates effort p_x towards X per attendee and an effort p_y towards Y per attendee. Similarly attendees, vis-à-vis each exhibitor, spend efforts q_x and q_y for products X and Y respectively. The following proposition gives their equilibrium values:

Proposition 2. *The unique perfect-like equilibrium in a horizontal show is then:*

(4.11) $$p_{ijx} = p_x = \frac{\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}}}{\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}} + \beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}} \; (T_e / n_a)$$

(4.12) $$p_{ijy} = p_y = \frac{\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}}{\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}} + \beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}} \; (T_e / n_a)$$

(4.13) $$q_{jix} = q_x = \frac{\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}}}{\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}} + \beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}} \; (T_a / n_e)$$

(4.14) $$q_{jiy} = q_y = \frac{\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}}{\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}} + \beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}} \; (T_a / n_e)$$

for all i and j.

Equations (4.11)-(4.14) give us a glimpse into how in a perfect-like equilibrium, effort levels are chosen to resolve possibly conflicting product interests between exhibitors and attendees. To see this consider the ratios β_y / β_x and γ_y / γ_x. In industries where these ratios are similar, there is a parity of product interest between attendees and exhibitors. In the event these ratios are exactly equal, the above formulas show that both attendees and exhibitors will divide their efforts between products in the same ratio. On the other hand, in industries where these ratios are dissimilar, there is a disparity of interest between the parties. This may happen for instance, when exhibitors are more interested in 'pushing' product X as opposed to product Y (perhaps because it is cheap to produce) while attendees get more value from product Y rather than X. Allocated efforts in such (mismatched) cases respect both one's own preferences as well as the preferences of the other party, but one's own preferences are given more weight than those of the other party. To see this from the exhibitor's point of view, we can use (4.11) and (4.12) to note that the ratio p_y / p_x is given by the product of $(\beta_y / \beta_x)^{(1-\alpha)/(1-2\alpha)}$ and $(\gamma_y / \gamma_x)^{\alpha / (1-2\alpha)}$. Since $(1-\alpha) > \alpha$, the point follows. These ratios are important constructs in our model. Later in section 6, we will also see how they affect the preponderance of horizontal shows over vertical shows.

We will work with the equilibrium values given in (4.11) – (4.14) in calculating all profit and surplus expressions. Note that each exhibitor, in a horizontal show, makes

$$(4.15)\quad \frac{\left(\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}}+\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}\right)}{\left(\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}}+\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}\right)^{\alpha}\left(\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}}+\beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\right)^{\alpha}}\, n_a^{1-\alpha} n_e^{-\alpha}(T_aT_e)^{\alpha}$$

while each attendee makes

$$(4.16)\quad \frac{\left(\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}}+\beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\right)}{\left(\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}}+\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}\right)^{\alpha}\left(\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}}+\beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\right)^{\alpha}}\, n_e^{1-\alpha} n_a^{-\alpha}(T_aT_e)^{\alpha}$$

Hence the total surplus generated in the show (n_e times (4.15) plus n_a times (4.16)) is

$$(4.17)\quad \frac{\left(\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}}+\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}\right)+\left(\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}}+\beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\right)}{\left(\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}}+\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}\right)^{\alpha}\left(\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}}+\beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\right)^{\alpha}}\,(n_an_e)^{1-\alpha}(T_aT_e)^{\alpha}$$

The expressions in (4.6) and (4.17) will be important in subsequent analysis as they will determine the relative incentives for show organizers to hold vertical versus horizontal show.

C. Results with a Stackelberg Setup: An alternate way of justifying this allocation of efforts is to consider a slightly different game. In this alternative version attendees first select their effort levels. The exhibitors, upon observing the actions of the attendees, then allocate their efforts among the attendees (a reasonable representation of the actual sequence of actions at a show). The difference between this set up and the previous one is similar to the difference between Cournot and Stackelberg game set-ups in the oligopoly literature, and one might question whether these different setups lead to radically different equilibria. For a vertical show, we can show that in the sequential game, given the reaction of the exhibitors, the marginal returns for each attendee with each exhibitor goes to infinity when effort levels go to zero, ensuring positivity of all effort levels. The rest follows easily. We thus have:

Proposition 3. *The unique equilibrium of the sequential game (where attendees allocate efforts first and then exhibitors allocate efforts (having observed attendees' decisions) is the symmetric equal division equilibrium.*

However, the sequential horizontal trade show version of the game, where attendees move first and then the exhibitors move, does *not* produce exactly the same outcome as the simultaneous version. Unlike for the simultaneous version, for at least some parameters, the sequential version can result in equilibria where no party spends any efforts for a particular product, although for large enough n_a this cannot happen. While the equilibrium outcomes are different, they do tend toward each other asymptotically as n_a increases:

Proposition 4. *Suppose that in the sequential horizontal game there exists a symmetric equilibrium where for all i and j,* $p_{ijx} = p_x > 0$, $p_{ijy} = p_y > 0$, $q_{jix} = q_x > 0$, *and* $q_{jiy} = q_y > 0$. *Then, while the equilibrium values of* p_x, p_y, q_x, q_y *are different from those corresponding to the (perfect-like) equilibrium values of the simultaneous version, the two sets of values converge as* n_a *becomes large.*

5. Size, Diversity and Number of Trade Shows—the Industry Perspective

Now that we have investigated the effort level (and profit) that arises for a show in equilibrium, we turn our attention to two key questions: (a) how many exhibitors and attendees should we expect at a show? and (b) how many horizontal and vertical shows are most appropriate for an industry? To address these questions, we focus on the role of the middleman in bringing buyers and sellers together, or in creating marketing distribution channels, a field of study that has had a long history in the fields of Marketing and Management Science (see for instance Balderston (1958) or the seminal book by Baligh and Richartz (1967)). A fundamental insight of this literature is that middlemen perform a useful role by reducing 'contact costs'- thus if there are *m* buyers and *n* sellers, normally there will be *mn* contacts between them, but if they all go through a middleman there need be only $m+n$ contacts. Trade show organizers clearly perform this role in bringing attendees and exhibitors to one venue so that they do not have to travel to each others' locations to display or learn about products. However, what this analysis does not address is the *endogeneity* of the number of buyers and sellers to be brought in one venue. In Section 1 and Section 3, we noted the benefits and costs of large numbers of participants in a show (or in the context of distribution, channels). In the first subsection of this section we will argue that by internalizing the net benefits of the participants in a show, the organizers will levy the 'correct' congestion tolls to resolve the tradeoffs between costs and benefits optimally. This analysis involves a particular show; so we must also determine how many shows there should be in an industry and what proportion of them should be horizontal. That is addressed in the second subsection.

A. Targeting the Right Number of Participants by Setting Entry Fees. Assume an infinite population of potential attendees and exhibitors who are potential visitors to (at most one of) the shows about to take place (Our infinite population assumption is for analytic convenience only and the results hold with “very large” populations). Throughout this section, for simplicity, we treat the numbers of these participants as continuous variables. The organizer of each show decides to charge certain entry fees to each attendee and to each exhibitor, which he announces upfront. After knowing these fees, each potential exhibitor or attendee decides to attend (enter) or not. The gross benefits the exhibitor and the participant realize are the show-generated values in (4.4), (4.5), (4.15) and (4.16); if they anticipate these benefits to be less than the entry fees, they do not enter. The net profit that the organizer receives is the sum of all the entry fees minus variable costs. We will assume, for simplicity, that variable costs are linear in n_a and n_e; i.e. each attendee costs the organizer an amount c_a and each exhibitor costs the organizer c_e.

Since attendees who enter a show can’t make (strictly) more than zero profit (otherwise those who do not enter would see profit from entering and are, hence, acting sub-optimally), the organizer realizes the entire surplus generated by a show. Thus each show’s net profit function is given by total surplus generated minus variable costs, which, using (3.4), (4.6) and (4.17) is

(5.1) $$\pi \equiv R\,(n_a n_e)^{(1-\alpha)}\,Exp(-\alpha\,(k_a\, n_e + k_e\, n_a)) - c_a\, n_a - c_e\, n_e\,,$$

where R *is the surplus generated by an attendee-exhibitor pair if they each had one unit of time to invest towards the other*. The unit surplus, R, depends on the format of the show. For a vertical show,

(5.2) $$R = \beta + \gamma := R_v$$

and for a horizontal show,

(5.3) $$R = \frac{\left(\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}} + \beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}\right) + \left(\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}} + \beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\right)}{\left(\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}} + \beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}\right)^{\alpha}\left(\beta_x^{\frac{\alpha}{1-2\alpha}}\gamma_x^{\frac{1-\alpha}{1-2\alpha}} + \beta_y^{\frac{\alpha}{1-2\alpha}}\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\right)^{\alpha}} := R_h\,.$$

Now notice that the show organizer’s problem of choosing entry fees can be reduced to the problem of choosing n_a and n_e to maximize π. Given the optimal values of n_a and n_e, he can determine the entry fees by setting these fees equal to the attendee and exhibitor surpluses generated at these n_a and n_e values.

This optimization problem has a solution yielding positive profits, provided the unit surplus, *R*, is larger than a certain critical value. The relevant first order conditions are:

(5.4) $$R(n_a n_e)^{(1-\alpha)} Exp\left(-\alpha\,(k_a n_e + k_e n_a)\right)\left(\frac{1-\alpha}{n_a} - \alpha k_e\right) - c_a = 0$$

(5.5) $$R(n_a n_e)^{(1-\alpha)} Exp\left(-\alpha\,(k_a n_e + k_e n_a)\right)\left(\frac{1-\alpha}{n_e} - \alpha k_a\right) - c_e = 0$$

Using (5.4) and (5.5) and the second order conditions, we can perform comparative statics exercises which yield the next observation.

Observation 1. *Optimal values of n_a and n_e increase with a rise in the unit surplus R, and decrease with the rise of any of the four parameters k_e , k_a , c_a or c_e .*

Observation 1, together with equations (5.4) and (5.5) provides us with an important insight. It tells us that no matter how high the return from interaction (i.e., the unit surplus *R*) is, or no matter how low the variable cost parameters (c_a and c_e) are, the optimized solutions n_a^* and n_e^* must, respectively, be less than $\hat{n}_a$ and $\hat{n}_e$, where $\hat{n}_a = \frac{(1-\alpha)}{\alpha k_e}$ and $\hat{n}_e = \frac{(1-\alpha)}{\alpha k_a}$ (these are the limiting solutions to (5.4) and (5.5) as *R* goes to infinity and/or c_a and c_e go to zero). *Thus, congestion cost will eventually neutralize the benefit of a bigger marketplace as the larger number of marketplace participants eventually provide a physical limit to marketplace size*. For market organizers, the implication is that when considering raising marketplace size to cover fixed costs, congestion will eventually become an economic as well as a physical limit. One reason many electronic marketplaces are so successful in attracting volume is that in these marketplaces participants rarely feel the congestion cost associated with the presence of other participants, if the bandwidth of these sites is not a constraint.

Different unit surpluses for horizontal and vertical shows (R_v and R_h) produce different incentives for organizing horizontal versus vertical shows because they give different (optimized) profits realized from those shows. Hence it is important to know how *R* influences π^*, the optimized value of the net profit. Although π^* will increase with *R*, we must understand the elasticity properties of π^* (*R*) to make a meaningful statement about how changes in buying/selling interests for all products will affect the organizer's incentive to organize a vertical versus a horizontal show. The next proposition gives a critical result that we will use to derive such a statement later:

Proposition 5.

a) π^* *is zero up to a critical value of R, after which it is strictly increasing in R.*

b) π^* *is convex, asymptoting to a straight line with a positive slope and negative intercept.*

c) The elasticity of the π^* *curve is decreasing in R.*

Thus, the π^* curve as a function of R, for fixed α, k_a, k_e, c_a and c_e, looks like as represented in Exhibit 2, with the dashed line its asymptote. The intercept of the asymptote is given by (the negative of) the variable cost at the critical values $\hat{n}_a$ and $\hat{n}_e$, and its slope is given by $(\hat{n}_a\ \hat{n}_e)^{(1-\alpha)}\ Exp\ (-\ \alpha(k_e\ \hat{n}_a + k_a\ \hat{n}_e))$.

Let π_h^* and π_v^* represent the optimal profits from a horizontal and a vertical show respectively (net of variable but gross of fixed costs). A corollary to Proposition 5 that gives us a useful result is:

Corollary 1. *Suppose* α, k_a, k_e, c_a and c_e *are fixed. If in addition,* R_v *is fixed (and hence* π_v^* *fixed),* π_h^* / π_v^* *is an increasing function of* R_h.

Hence, given fixed vertical show unit surplus R_v, we only need to investigate how market factors affect R_h in order to understand incentives to organize horizontal versus vertical shows.

B. Determination of the Number and Mix of Vertical and Horizontal Shows: So far we know what determines the relative profits of a horizontal and a vertical show. One can produce several arguments why the ratio of these (optimized) profits should directly influence the proportion of horizontal shows in an industry. We present one such argument here, while noting that there are other plausible setups leading to the same conclusion[3].

Let N_v and N_h represent the number of vertical and horizontal shows in an industry. In our framework, in equilibrium these numbers depend on the fixed costs as well as the market structure of the organizing industry. Fixed costs arise from rental of show space, pre-show promotion and advertising, commitments to entertainment events, staff commitments and the

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[3] For instance, assume a fixed number of organizers in an industry each of whom can organize just one show and each of whom has a logarithmic utility function *U*(.) for money. Also assume that the actual utilities that a particular organizer will obtain from a show (horizontal or vertical) will be *U*(.) of the corresponding show's profit (π_h^* or π_v^*) plus a random disturbance term (e.g. *Actual Utility* (*Horizontal*) = $ln\,\pi_h^* + \xi$). If the random disturbance term (ξ) follows IID extreme value distribution, then following standard random utility based choice model arguments (see e.g., McFadden (1974)) one can show that the proportion of horizontal shows in the industry will be given by the ratio $\pi_h^* / (\pi_h^* + \pi_v^*)$. While this setup gives an explanation for the proportions of show types, we prefer our setup as it provides insight into the number of shows of each type as well.

like. A simple way to characterize these fixed costs is to assume that a vertical show requires one unit of a 'vertical fixed factor' and a horizontal show requires one unit of a 'horizontal fixed factor,' where a unit factor comprises a unit vector of the fixed cost components. Organizers purchase these factors and their prices are influenced by the demands of both factors. Letting P_v and P_h denote the prices of these fixed factors, we postulate simple, linear price-quantity relationships:

(5.6) $P_v = m_1 N_v + m_2 N_h$

(5.7) $P_h = m_3 N_h + m_4 N_v$

We also assume that effect of own demand on price exceeds the effect of cross demand. That is m_1 and m_3 are assumed to be larger than m_2 and m_4; more precisely, Min(m_1 , m_3) > Max(m_2 , m_4). Our assumptions imply that the factors for the two types of shows are not identical (in which case m_1 would equal m_2 and m_3 would equal m_4), but there is a certain amount of substitutability between them. For example, vertical and horizontal shows might compete for the same convention center, but might use different media to reach prospective participants and attempt to hire different types of entertainers.

We now analyze how N_v and N_h arise under different market structures. Assume first that the organizing industry is a monopoly. Then the sole show organizer solves an optimizing problem of the following sort:

(5.8) *Maximize* $\pi_v^* N_v + \pi_h^* N_h - (m_1 N_v + m_2 N_h) N_v - (m_3 N_h + m_4 N_v) N_h.$

The first order conditions for this optimization problem enable us to solve for the optimal numbers of each type of show. The ratio of the optimized profits plays a critical role in determining the proportion of horizontal shows (p^h), as the following key proposition shows:

Proposition 6*. Under the condition of a monopolistic organization industry,*

a) p^h *is 0 (i.e.* N_h *is 0) if* π_h^* / π_v^* *is less than or equal to* $(m_2 + m_4) / 2m_1$.

b) p^h *increases with a rise in* π_h^* / π_v^* *as long as the latter is within*

$((m_2 + m_4) / 2m_1 , 2m_3 / (m_2 + m_4))$.

c) p^h *is 1 (i.e.* N_v *is 0) if* π_h^* / π_v^* *equals or exceeds* $2m_3 / (m_2 + m_4)$.

Consider next a competitive market structure. Under perfect competition, entry of new shows will occur as long as the average revenue from a show, π^*, stays above its average cost, which in our case is the price of the corresponding fixed factor. This 'zero profit condition' allows us to determine the dependence of p^h on the ratio of the optimized profits from the two types of the shows which we describe in the next proposition.

Proposition 7. *Under the condition of a competitive organization industry,*

a) p^h *is 0 (i.e.* N_h *is 0) if* π_h^* / π_v^* *is less than or equal to* m_4 / m_1.

b) p^h *increases with a rise in* π_h^* / π_v^* *as long as the latter is within (* m_4 / m_1 , m_3 / m_2).

c) p^h *is 1 (i.e.* N_v *is 0) if* π_h^* / π_v^* *equals or exceeds* m_3 / m_2.

Thus, we observe that under both types of market structure, p^h responds positively to the ratio π_h^* / π_v^*. In the next section we investigate what factors influence this profit ratio, leading directly to six testable hypotheses that we investigate in the section that follows.

6. Key Drivers of Horizontal vs. Vertical Shows

Our model setup permits two types of vertical shows: X and Y. Equation (4.6) shows that a vertical X show dominates a vertical Y show if $\beta_x + \gamma_x > \beta_y + \gamma_y$ and vice versa. Without loss of generality, we assume that X is the more popular product, so the competition is between a vertical X and a horizontal show. To keep our analysis tractable we also assume that for both attendees and exhibitors X is the more valuable product, i.e. $\beta_x > \beta_y$ and $\gamma_x > \gamma_y$. This is a reasonable assumption; after all, it is the sellers' job to match interests with those of buyers. We formally justify this assumption later.

One important construct that we will make use of here is the level of the breadth of product interests across the two products, both for attendees and exhibitors. In our framework, if an exhibitor's β_x value is 10, his breadth of product breadth interest is higher when his β_y is 9 than when it is 1. Hence we will use $b = \beta_y / \beta_x$ as a measure of breadth of product interest for an exhibitor. Similarly, $c = \gamma_y / \gamma_x$ captures breadth of product breadth interest for attendees. In industries where *b* and *c* are very different, there is disparity of breadth of product interest between attendees and exhibitors.

From both theoretical and empirical viewpoints we now argue, that *b* and *c* are unlikely to be very different. The theoretical argument follows from the following expressions

$$(6.1) \quad \begin{aligned} \beta_x &= (\theta_e S_x)\lambda_x k_x \quad & \gamma_x &= (\theta_a S_x)\lambda_x k_x \\ \beta_y &= (\theta_e S_y)\lambda_y k_y \quad & \gamma_y &= (\theta_a S_y)\lambda_y k_y \end{aligned}$$

where S_x, S_y are the total surplus (attendee's valuation minus exhibitor's cost) for a unit transaction in X and Y respectively; θ_e and θ_a are the shares of this surplus for the exhibitor and the attendee respectively ($\theta_e + \theta_a = 1$), λ_x and λ_y are the transaction intensity factors for the exhibitor-attendee pair for the two products and k_x, k_y are technology parameters for the production function (see equation 3.1). Then simple division confirms that both *b* and *c* are given by $S_y k_y \lambda_x / S_x k_x \lambda_y$ and hence are equal.

For an empirical counterpart of this closeness argument, consider Exhibit 4, where we describe data compiled on breadth of exhibitor and attendee interests across industries, and the hypothesis that this difference (averaging 0.238 on a 1-7 Likert scale) is not different from zero cannot be rejected. From (6.1) we also see that if θ_e is close to θ_a then β_x is likely to be close to γ_x as well (and β_y should be close to γ_y). Indeed, if the bargaining abilities of the two parties (captured by their shares of the surplus) are not very different, then these shares of surplus are likely to be close. Hence, if the attendees prefer X to Y so will the exhibitors and vice versa.

With the aid of Propositions 5, 6 and 7 we now investigate the factors that influence p^h, the proportion of horizontal shows in an industry. The parameters of our model will affect this ratio in a number of subtle ways, but all will work through their influence on the movement of R_h relative to R_v and/or the movement of the π^* curve itself (and hence the movement of π_h^* / π_v^*). We will present theoretical propositions and related proofs, and will also provide numerical simulations showing that our results hold even for many plausible values of the model parameters outside the conditions identified in the propositions.

To provide background for our next four propositions, consider Exhibit 3 which provides R_h / R_v ratios for various values of our model parameters. Holding γ_x at a (normalized) value of 1, we present the numerical values of R_h / R_v for various *b*'s and *c*'s ranging from 0.2 to 1 (in steps of .2). The other parameter values we chose were all possible combinations of $\beta_x = 1, 3, 5$ and $\alpha = .05, .25, .45$. It is unlikely that the benefit that attendees (exhibitors) get out of a transaction is likely to be more than a small multiple of the benefit the attendees (exhibitors) get; so we limited our simulations to β_x less than 5.

Although, it is difficult to draw firm conclusions from Exhibit 3 data alone, it seems that R_h is typically at least as large as R_v. The only cases where this condition is violated involve very large values of *c* compared to *b* and very high values of α, conditions which we have previously argued, are unlikely. Further, if *either* b is close to c *or* if β_x is close to γ_x, R_h will always be larger than R_v. The following observation covers both cases.

Observation 2. *R_h strictly exceeds R_v in an open neighborhood of parameters satisfying*

(6.2) $(\beta_x - \gamma_x)(b - c) \geq 0$.

Note that this is only a sufficient condition. Also note that if one assumes that β_x (slightly) exceeds γ_x (i.e. exhibitors see more benefit than attendees per meeting for the X good), this condition is likely to be satisfied, as our data show that exhibitors have (slightly) higher breadth of product interest than do attendees.

We now develop propositions on drivers of the ratio of horizontal to vertical shows. For the first two propositions, the change in the relevant parameter will not affect R_v (i.e. $\beta_x + \gamma_x$), so by Corollary 1 we only need to examine the effect of the relevant parameter on R_h. For the third, both R_v and R_h change. Finally for the fourth, R_v remains fixed while R_h changes, but in addition, the π^* curve itself shifts.

A. The Impact of Breadth of Product Interest of Exhibitors and Attendees. Horizontal shows address a broader set of products than do vertical shows, and one might expect to see that as the breadth of interest in products increases (as reflected in increases in the b and c parameters), we would see more of such shows. Specifically, we investigate what happens to R_h, if keeping β_x, γ_x and α fixed, we change b and c. Exhibit 3 suggests that a rise in b or c generally raises R_h (R_v remains fixed), although some slight exceptions can be found. For instance when α = .45, β_x = 3, γ_x = 1, and c = 1, raising b from .2 to .4 actually lowers R_h. These occurrences seem surprising as it means that if we hold the attendees' product valuations fixed, and do not change the valuation of the preferred product for an exhibitor, an increase in the exhibitors' less preferred product's valuation can hurt the revenues and hence profits for a horizontal show. This result can be understood by noting that such a change may end up producing new effort allocation schemes by participants in accordance to the equilibrium results of Section 4. The change in effort allocations may, in turn, decrease total (equilibrium) surplus and hence, total revenues. However this situation should be the exception rather than the rule as the table and the following proposition suggest.

Proposition 8. *In an open neighborhood of the set of parameters that satisfy condition (6.2), R_h (and, hence π_h^* / π_v^* because R_v is fixed) is strictly increasing in b or c. Therefore, breadth of product interest has a* ***positive*** *impact on the proportion of horizontal shows, p^h.*

B. *The Impact of Difference of Breadth of Product Interest Between Exhibitors and Attendees*. Next we ask how the difference in the breadth of product interest between attendees and exhibitors affects the proportion of horizontal shows. If these interests are too disparate, we might expect a "coordination problem" that is harmful for surplus generation. More formally we ask whether, keeping the sum of *b* and *c* fixed, it helps the horizontal show proportion if *b* and *c* are closer to each other. The answer is clear when β_x is close to γ_x or when *b* is close to *c*:

Proposition 9. *Suppose β_x and γ_x are sufficiently close. Then*

a) If b > c, the following holds:

(6.3) $\partial R_h / \partial b - \partial R_h / \partial c < 0$.

Vice versa, if c > b, the inequality above is reversed.

b) If b = c, irrespective of β_x and γ_x, the following holds:

(6.4) $\partial R_h / \partial b - \partial R_h / \partial c = 0$.

c) If b > c, and b is close to c, the following holds:

(6.5) $\partial R_h / \partial b - \partial R_h / \partial c < 0$.

Vice versa, if c > b, and they are close, the inequality above is reversed.

Thus, when β_x is close to γ_x or when b is close to c, an increase in | b-c|, keeping the sum b+ c fixed, decreases R_h (hence decreases π_h^* / π_v^* because R_v is fixed). Therefore, the absolute difference in range of product breadth between exhibitors and attendees has a ***negative*** *impact on the proportion of horizontal shows, p^h.*

C. *The Impact of Selling/Buying Intensity of Exhibitors and Attendees*. Next we ask what happens if all four of the product valuation parameters (β_x, β_y, γ_x, γ_y) decrease proportionally, (signaling, in effect, a rise in non-selling or information gathering propensity or λ)? In the proof of the next proposition we show that the answer relates to the elasticity of the $\pi^*(R)$ curve: higher *R* values result in lower elasticity. Hence, a rise in both non-selling and non-buying interests lowers π_v^* relatively more than π_h^*, thus benefiting the proportion of horizontal shows.

Proposition 10. *Suppose R_h is larger than R_v. Then, a percentage decrease in all the beta and gamma values (β_x, β_y, γ_x, γ_y) increases π_h^* / π_v^*. Therefore, increase in non-selling interest for exhibitors and non-buying interest for attendees has a* ***positive*** *effect on the proportion of horizontal shows, p^h.*

Proposition 10 helps explain why in industries with relatively more horizontal than vertical shows, we are also likely to see participants who have more non-selling and non-buying interests and thus, a smaller transaction intensity factor λ. In a related manner, one can see why on average, vertical shows will be more closely associated with lead generation, given that the latter measure is more likely to track immediate rather than distant (product) transactions.

D. The Impact of Technological Innovativeness in an Industry. Shows in fast moving, innovative industries have more technical issues to communicate, and typically display many state-of-the-art products which are completely new and unfamiliar to the majority of attendees. When one starts from ground zero there is a lot to learn, which implies that the rate of information transmission for the first few units of joint effort will be high *relative* to the same rate for the last few units of available effort. Thus α (a measure of diminishing marginal returns and hence, gains to diversification), will be lower for such industries.

How will α affect p^h? There are two different diversification effects to consider here. First, there is the '*product diversification effect*'. Sharper diminishing marginal returns are likely to provide more benefit when there are more products to allocate efforts to. Analytically, this relates to how α will change R_h (without affecting R_v). We see from Exhibit 3 that generally, a lower α raises R_h , which is conducive to horizontal shows.

Lemma 1. *Fixing the betas and gammas, a lowering of α raises R_h (hence raises π_h^* / π_v^* because R_v is fixed) in an open neighborhood of parameter values satisfying* (6.2).

The second effect to consider is the '*participant diversification effect*'. Intuitively, with more participants among whom to spread efforts, the gains to diversification due to changes in marginal returns will have a greater impact. Analytically, this effect refers to the movement of the π^* curve itself as α changes. Under certain simplifying and plausible approximations one can show that this effect is also conducive to horizontal shows.

Lemma 2. *Suppose R_1 and R_2 are large fixed values such that the asymptote to the π^* curve approximates the curve itself closely at these values. Also, suppose $R_1 > R_2$ and k_a and k_e are small (it suffices to assume $k_a k_e < e^{-2}$). Then, $\pi^*(R_1) / \pi^*(R_2)$ increases as α decreases.*

Combining lemmas 1 and 2, we have:

Proposition 11. *Under the assumptions made in Lemmas 1 and 2, a lowering of α raises π_h^* / π_v^*. Therefore, the technological innovativeness of an industry has a* ***positive*** *impact on the proportion of horizontal shows, p^h.*

We reiterate that we derive propositions 8-11 (like Observation 2) using certain *sufficient* conditions. We have argued that these sufficient conditions are likely to hold, the conclusions are nevertheless true for a much larger set of parameters (which we have not been able to characterize in closed, interpretable form). We now turn our attention to an empirical assessment of these propositions.

7. Empirical Analysis

A. *<u>Data Collection and Hypothesis Generation</u>*. To link our theoretical results to the Exhibit 1 patterns that motivated our key questions, we needed data on industry characteristics that related to our key model constructs, especially those highlighted in Propositions 9-12: breadth of product interest, selling/buying intensities of participants', and technological innovativeness. The Exhibit 1 database, representing 1152 trade shows in 21 industries provided our sample frame. In order to capture the other data, we needed evaluations that would be comparable across industries. For selling/buying intensities and breadth of interest, we needed experts who had trade show specific, cross industry knowledge. We selected two experts who were executives at a major trade show research firm and asked them individually to complete a questionnaire with the following 4 questions

On a 1-7 Likert scale (1=lowest, 7=highest):

1) *For an average attendee in each of the following industries, please rate his/her buying intensity at a typical show;*
2) *For an average attendee in each of the following industries, please rate his/her breadth of product interests at a typical show*
3) *For an average exhibitor in each of the following industry, please rate his/her selling intensity at a typical show*
4) *For an average exhibitor in each of the following industry, please rate his/her degree of interest in contacting attendees with different product interests*

When the two experts' answers differed significantly, we recycled their answers to them and asked them to resolve those differences. After two rounds their answers essentially converged and we averaged any (small) remaining differences.

To assess technological innovativeness, trade show knowledge was not an issue in selecting experts. We contacted 17 well-known experts in new product development and technological innovation and asked each to rate the 21 industries (again on a Likert 1-7 scale) on degree of innovativeness, defined as "the relative amount of new product development and new technical information that occurs per year within the industry". Here, the experts generally agreed after the second round: the median was quite close to the mean and all our results were robust to the use of either value.

Exhibit 4 summarizes the mean values of the experts' rankings of the different variables. A review of that Exhibit shows that the numbers in columns 4 vs. 5 tend to stay close together. Indeed, the mean difference between (4) and (5) is 0.28 with standard deviation of 1.01. We cannot reject the hypothesis that within-industry attendee/exhibitor differences in breadth of product interests are the same at the p = 0.001 level for either pair of variables. Hence, this preliminary look at the data confirms that the sufficient conditions we used to prove Observations 2,3 and Propositions 8-11 do hold (*b* is empirically very close to *c*). Hence, our data appear appropriate to justify their use to test the following hypotheses generated from the propositions in the last section:

A larger proportion of horizontal trade shows in an industry, p^h will be associated with:

H1. A larger breadth of product interests for attendees (ATPROD); (column 4, Exhibit 4). *Source: Proposition 8.*

H2. A larger breadth of product interests for exhibitors (EXPROD); (column 5, Exhibit 4). *Source: Proposition 8*.

H3. A smaller difference in breadth of product interests (DIFFPROD); (Absolute difference between columns 4 and 5, Exhibit 4). *Source: Proposition 9*.

H4. A smaller buying intensity of the attendees (ATBUY); (column 2, Exhibit 4) *Source: Proposition 10*.

H5. A smaller selling intensity of the exhibitors (EXBUY); (column 3, Exhibit 4*) Source: Proposition 10*.

H6. A higher level of industry technological innovativeness (TECH); (column 1, Exhibit 4). *Source: Proposition 11.*

B. Analysis/Results. We ran several regression models to test these hypotheses. In all cases, we weighted the observations by the total number of shows in the industry. The models run were: a) standard OLS, b) an OLS model with the dependent variable being a logistic transformation of doubly truncated *p*; i.e. Max {*a*, Min {*p*, *b*}} where *a* and *b* are the lower and upper truncation limits (we tried *a*=.01, *b*= .99; *a*=.05, *b*=.95 and *a*=.1, *b*=.9 as different sets of truncation limits) c) probit and d) logit. While all models generally gave consistent results, the standard logit model dominated the others in terms of fit. Henceforth, we use the term 'better fit' in the sense of lower AIC; the Akaike Information criterion (Akaike (1973), (1974)), where that AIC for a model is –2*Log Likelihood + 2*(number of parameters in the model).

Exhibit 5 presents our results. The first four columns in that Exhibit present the result of running the four models mentioned above (the truncation limits used for the version of the second model reported here are .01 and .99). As can be seen, the logit model gives the best fit. All models strongly reject the hypothesis that the explanatory variables are jointly insignificant; and all variables are individually highly significant with the hypothesized signs except TECH in the OLS model. Apart from this one exception all variables are significant at 5% or beyond and most are significant at the1% level and beyond. We also tested if disaggregating attendee and exhibitor characteristics provides a more powerful explanation of the data than in the aggregated case, a concern as these pairs of characteristics, do not differ by much. To check on the effect of disaggregation, we ran the logit model dropping ATBUY, EXBUY, ATPROD and EXPROD, but introducing TOTBUY (ATBUY+EXBUY) and TOTPROD (ATPROD+EXPROD), reported in the fifth column of Exhibit 5. The results favor the disaggregated version. Finally, we ran one set of regressions using SDIFPROD instead of DIFFPROD (a scaled version of DIFFPROD; defined as DIFFPROD divided by TOTPROD), and found the fit to be slightly better with DIFFPROD.

To summarize the results of this section, *our empirical analysis confirms all 6 hypotheses, H1 through H6, at significance levels 5% or better*.

C. Predictive Validation and Discussion. In order to test the predictive validity of our model, we used a jackknife approach: for each industry we used the data from the other twenty industries as the calibration sample and predicted the proportion of horizontal shows for this hold out industry. The residual is the difference between the empirical proportion of that industry's shows that are horizontal and the predicted proportion, based on the coefficients of the logistic analysis from the calibration sample. Exhibit 6 gives the results (along with full sample fitted values and residuals).

From Exhibit 6 we note that, from a predictive validity perspective, our model does quite well. More than half our sample comes from the first two categories; yet our prediction error for Computers is only 8.4% while it is 0.4% for Communications. These compare to an average Mean Absolute Deviation (MAD) of 6.6 % for the sample as a whole (on an unweighted basis). Indeed, one of the industries—Packaging—appears to be an outlier. If we drop Packaging, we get a MAD of 2.5% for fitted values and 4.8% for predicted values.

One can look at the jackknife results from a different perspective. If our model characterizes the behavior of efficient industries well, then it appears that the Packaging and the Building and Construction industries may be overserved by horizontal shows (large positive residuals) while the Chemicals and Manufacturing industries appear to be underserved (large negative residuals). The two industries with the largest numbers of shows, namely Computers/Computer Applications and Communications appear to have about the right mix according to our model.

Recall that we have used expert judgment data to construct our predictor variables here. With such data it is always possible that if the experts believe in the theory (even if it is false) they will provide confirming evidence. Although we cannot dismiss this possibility out of hand, our hypotheses and models are sufficiently subtle to suggest that this is highly unlikely. For example, if one tries to infer just by looking at our data table in what direction an independent variable influences the dependent variable, one might be misled. Consider TECH. Since the samples are weighted by the number of shows in the industries, one might suppose that the two top industries by this category, namely Computers & Computer Applications and Communications will influence the results heavily since together they account for 61% of the shows in the sample. They also have the two highest scores in terms of TECH (6.58 and 6.32 respectively). Now, given that the proportions of horizontal shows in these two industries are small (17% and 1% respectively), one might conclude that TECH influences the dependent variable negatively. Yet, our analysis shows that *after controlling for other variables,* we find TECH has a positive influence on p^h. This instance suggests that our model has subtle contingencies that experts would have found difficult to (mentally) control for in giving their responses.

However, these empirical results are preliminary and depend on the judgment of a limited number of expert evaluators. Clearly more work is needed to evaluate these results more soundly, but this preliminary investigation at least supports our theoretical results in a rewarding manner. The results also suggest how such findings can be both of theoretical and managerial value.

8. Conclusions

We have argued that given certain key characteristics of potential participants, each marketplace has an optimal size and structure. While motivated to a degree from observations from the trade show marketplace and corroborated to a rewarding extent from an empirical analysis in that arena, we have used little institutional structure specific to that arena in developing our main results. Any market characterized by time-consuming search processes for information leading to 'matches' between brands and needs can be subject to this type of analysis. We have been unable to locate other studies involving the three sets of market participants we study here and the costs and benefits of their interaction as the factors that lead to marketplace structure and formation. Hence our results are best treated as exploratory. Below, we discuss the restrictiveness of some of our modeling choices and how these invite further research, while pointing out why most of our results are likely to hold under more general frameworks.

Our model depends on specific functional forms of production functions and congestion costs. While the functions we chose were quite specific, our main results derive from the (ultimately) decreasing returns to additional interaction balanced against an increasingly costly congestion effect. Admittedly, the fact that for Cobb-Douglas functions marginal returns to effort is infinity at zero effort levels is what gives us the 'equal-division' results in Section 4; however, as long as the marginal returns to effort at zero high enough (and the cost parameters are non-negligible) we should obtain similar results. Also note that the congestion cost function for a participant could be generalized to include as its arguments both total number of participants as well as the number of participants actually visited. These features will complicate the analysis substantially, but in all likelihood, will not alter the general conclusion that marketplaces will have limited sizes determined by product valuation parameters, extent of diminishing marginal returns (positive impact), and cost coefficients (negative impact).

We looked at two products in a marketplace and framed a horizontal show as one that had both of those products. In a two-product world, breadth of product interest is easy to define as the ratio of the two product valuation parameters. In a multiple product world there is no such obvious summary statistic; and hence we are unsure about the formats Propositions 8 and 9 are likely to assume in such a case. But the fundamental insight here is that if the two sides of a market are interested in different things, it may not pay the organizer to allow them to pursue their own interests – which is why a vertical show, by forcing all participants to concentrate on a narrow range of products may avoid this harmful conflict of interest.

An assumption that is critical to our model is that of both attendee and exhibitor homogeneity in terms of product valuations, although we have incorporated heterogeneity of brand preferences. As the segmentation literature shows (Wedel and Kamakura, 1998), one can provide an explanation for almost any market structure by appealing to heterogeneity arguments. While that is certainly true, the insight that our results provide is that these different market structures can be derived even under the conditions of homogeneity. While adding heterogeneity to the model would clearly enrich the structure, we have not been able to develop results with heterogeneous populations that provided any clear insights.

Our results were also driven by the specific cost functions that we specified. The reader may question why variable costs are linear in their arguments while fixed costs are not. We conjecture that if marginal variable costs were increasing, the propositions relating to limiting size and the shape of the π^* function will remain unchanged. Also, future empirical work should consider industry specific factors that affect these costs, and hence the mix between horizontal and vertical shows.

Finally, while we are encouraged by our empirical results, we have noted that the data that support these results are based on judgmental data from experts. It would be desirable to validate these findings using data gathered from actual as well potential show participants. Such data should also be able to pick up the effects of heterogeneity in the participant pool on variables of interest.

As with any theoretical model, at the cost of a certain amount of abstraction we have been able to gain considerable insight. It may well be that other richer model structures can extend these findings or lead to others in this or in related marketplaces. Indeed, the booming electronic analogy of our development here is the portals marketplace, with Chemdex.com (1999) and Plastics.net examples of vertical portals, Vertical.net an example of a portal organizer that helps set up vertical portals, and Industry.net an example of a horizontal portal.. We hope that an extension of our model framework will permit a better understanding of such developments, extending our findings from the marketplace to marketspace.

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LIST OF NOTATION USED IN THE TEXT
(in order of appearance)

Notation	Definition
X, Y	Different product categories
U	Benefit accrued to a party due to effort investment (by both parties)
k	A constant in the production function
r	(Common) rate of interest used by attendees and exhibitors
λ	Transaction intensity factor
v	(Net) value to a party per unit transaction in the exhibitor's product
p	Effort expended by exhibitor in the interaction
q	Effort expended by attendee in the interaction
α	A parameter in the production function that captures the extent of diminishing marginal returns
β	Product valuation parameter for exhibitor (product of v, ρ, k)
γ	Product valuation parameter for attendee (product of v, ρ, k)
n_a	Number of attendees in a show
n_e	Number of exhibitors in a show
ϕ_a	A function appearing in attendees' congestion cost function
ϕ_e	A function appearing in exhibitors' congestion cost function
T_a	Total time available to attendees (net of congestion cost) for investment in information gathering
T_e	Total time available to exhibitors (net of congestion cost) for investment in information dissemination
k_a	Parameter determining how n_e affects T_aT_e
k_e	Parameter determining how n_e affects T_aT_e
p_{ij}	Effort expended by exhibitor i toward attendee j (in a vertical show)
q_{ji}	Effort expended by attendee j toward exhibitor i (in a vertical show)
β_x, β_y	Product valuation parameter for exhibitors specific to products X and Y respectively
γ_x, γ_y	Product valuation parameters for attendees specific to products X and Y respectively
p_{ijx} , p_{ijy}	Effort expended by exhibitor i toward attendee j in disseminating information about products X and Y respectively (in a horizontal show)

q_{jix} , q_{jiy}	Effort expended by exhibitor *i* toward attendee *j* in disseminating information about products X and Y respectively (in a horizontal show)
c_a	Variable cost for each extra attendee
c_e	Variable cost for each extra exhibitor
π	Organizer's profit function net of variable but gross of fixed costs (depends on n_a, n_e, R, α, k_a, k_e , c_a , c_e)
R	Surplus generated by an attendee-exhibitor pair if they each had one unit of time to devote to the other
R_v	R for a vertical show
R_h	R for a horizontal show
n_a^*	Optimal number of exhibitors in a show
n_e^*	Optimal number of attendees in a show
$\hat{n}_a$	An endogenously determined limit on the optimal number of attendees
$\hat{n}_e$	An endogenously determined limit on the optimal number of exhibitors
π^*	A function that takes R as an argument and generates optimized profits for that R by choosing the best n_a and n_e (parametrized on α, k_a, k_e, c_a and c_e)
π_h^*	Optimized profits (gross of fixed but net of variable costs) for a horizontal show (π^* (R_h))
π_v^*	Optimized profits (gross of fixed but net of variable costs) for a vertical show (π^*(R_v))
N_v	Number of vertical shows in an industry
N_h	Number of horizontal shows in an industry
P_v	Price of 'vertical fixed cost factor'
P_h	Price of 'horizontal fixed cost factor'
m_1 , m_2	Demand function parameters for the 'vertical fixed factor'
m_3 , m_4	Demand function parameters for the 'horizontal fixed factor'
p^h	Proportion of horizontal shows in an industry
b	Product breadth interest parameter for exhibitors
c	Product breadth interest for attendees
S_x, S_y	Surplus generated in 1 unit of *product* exchange for the X and Y goods respectively
θ_e,θ_a	Exhibitor and attendee shares of this surplus

Group	Industry	Number of Vertical Shows	Number of Horizontal Shows	Total Number of Shows	Proportion of Horizontal Shows
Over 100 Shows	**Computers & Computer Applications**	379	78	457	17%
	Communications	319	3	322	1%
50-100 Shows	**Engineering**	3	93	96	97%
	Medical & Health Care	90	4	94	4%
	Housing	80	0	80	0%
	Food Processing & Distribution	3	61	64	95%
	Electrical & Electronics	52	7	59	12%
15-50 Shows	**Paint**	49	0	49	0%
	Nursing	48	0	48	0%
	Automotive & Trucking	3	44	47	94%
	Chemical	20	23	43	53%
	Plastics	0	33	33	100%
	Radio, TV & Cable	32	1	33	3%
	Energy	0	30	30	100%
	Building & Construction	15	10	25	40%
	Restaurants & Food Service	0	24	24	100%
	Manufacturing	6	15	21	71%
	Photographic	21	0	21	0%
	Welding	17	0	17	0%
	Packaging	0	15	15	100%
	Education	15	0	15	0%
	Total	**1152**	**441**	**1593**	**28%**

Exhibit 1. A Selection of Industries and the Associated Number and Distribution of Trade Show Types (1985-1991), Showing the Wide Diversity in the Proportion of Horizontal Shows

Source: Exhibit Surveys Inc. Note: industries with fewer than 15 trade shows have been omitted from this chart due the instability of the proportions when the total number of shows in the industry is that low.

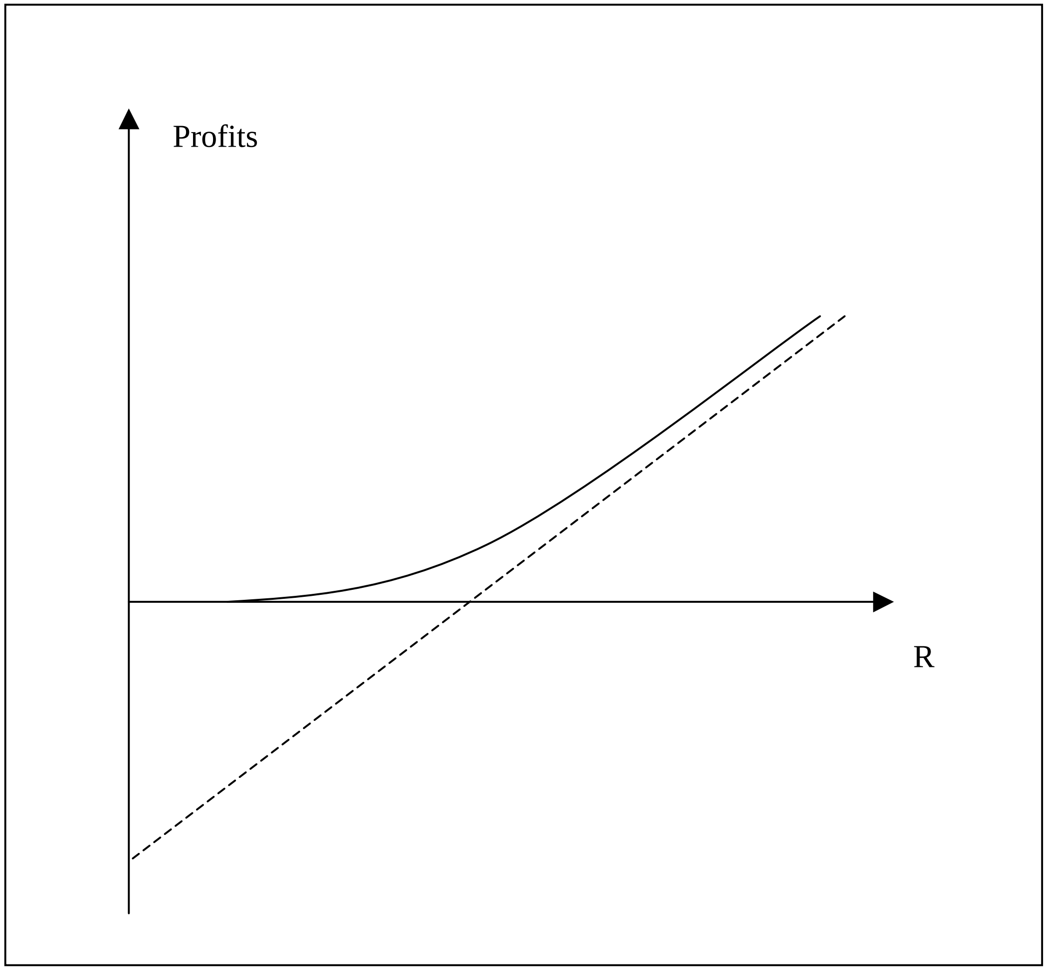


Exhibit 2: How Show Organizer's Profits Increase with R, the Unit Surplus of an Attendee-Exhibitor Pair

	α=.05					**α=.25**					**α=.45**				
b\c	**0.200**	**0.400**	**0.600**	**0.800**	**1.000**	**0.200**	**0.400**	**0.600**	**0.800**	**1.000**	**0.200**	**0.400**	**0.600**	**0.800**	**1.000**
β_X=1															
0.200	1.149	1.232	1.316	1.401	1.487	1.019	1.042	1.068	1.098	1.132	1.000	1.000	1.000	1.000	1.000
0.400	1.232	1.319	1.408	1.496	1.585	1.042	1.077	1.116	1.159	1.206	1.000	1.000	1.000	1.000	1.001
0.600	1.316	1.408	1.498	1.589	1.679	1.068	1.116	1.166	1.219	1.276	1.000	1.000	1.001	1.003	1.008
0.800	1.401	1.496	1.589	1.681	1.773	1.098	1.159	1.219	1.281	1.344	1.000	1.000	1.003	1.010	1.029
1.000	1.487	1.585	1.679	1.773	1.866	1.132	1.206	1.276	1.344	1.414	1.000	1.001	1.008	1.029	1.072
β_X=3															
0.200	1.149	1.189	1.230	1.270	1.311	1.020	1.028	1.036	1.043	1.052	1.000	1.000	1.000	1.000	1.000
0.400	1.275	1.320	1.363	1.406	1.449	1.055	1.077	1.094	1.110	1.127	1.000	1.000	1.000	1.000	0.999
0.600	1.402	1.452	1.498	1.543	1.587	1.101	1.138	1.166	1.191	1.215	1.000	1.000	1.001	1.001	0.999
0.800	1.532	1.586	1.634	1.681	1.726	1.153	1.208	1.248	1.281	1.311	1.000	1.001	1.004	1.010	1.015
1.000	1.662	1.721	1.771	1.819	1.866	1.212	1.286	1.337	1.378	1.414	1.000	1.004	1.017	1.043	1.072
β_X=5															
0.200	1.149	1.176	1.201	1.227	1.253	1.020	1.024	1.025	1.025	1.025	1.000	1.000	1.000	1.000	1.000
0.400	1.289	1.320	1.348	1.376	1.404	1.060	1.077	1.087	1.094	1.101	1.000	1.000	1.000	1.000	0.998
0.600	1.431	1.467	1.498	1.528	1.557	1.111	1.145	1.166	1.181	1.194	1.000	1.000	1.001	1.000	0.995
0.800	1.575	1.616	1.649	1.681	1.711	1.171	1.225	1.257	1.281	1.300	1.000	1.001	1.005	1.010	1.010
1.000	1.721	1.766	1.802	1.835	1.866	1.238	1.312	1.357	1.389	1.414	1.000	1.004	1.020	1.048	1.072

Exhibit 3. Values of the R_h / R_v ratio when varying α, β_X, b and c, showing how unlikely it is that that ratio is less than 1.0

Industry	(1) Technological Innovativeness	(2) Buying Interest (Attendee)	(3) Selling Interest (Exhibitor)	(4) Breadth of Product Interest (Attendee)	(5) Breadth of Product Interest (Exhibitor)
Computers & Computer Applications	6.58	6.00	6.00	3.00	4.00
Communications	6.32	5.00	6.00	2.00	3.00
Engineering	5.00	2.00	3.00	6.00	5.50
Medical and Health Care	5.08	5.00	4.00	2.00	2.00
Housing	2.66	5.00	4.50	2.50	3.50
Food Processing & Distribution	3.11	2.50	4.00	4.50	4.50
Electrical & Electronics	5.58	3.00	5.00	2.00	2.00
Paint	2.26	6.00	3.00	2.00	3.00
Nursing	2.37	3.00	5.00	2.00	4.00
Automotive & Trucking	3.26	2.00	3.00	4.00	4.50
Chemical	3.71	3.50	4.00	3.50	4.00
Plastics	3.82	3.00	4.00	4.50	4.50
Radio TV & Cable	3.53	6.00	4.00	2.00	4.00
Energy	2.61	3.00	3.00	5.00	5.50
Building Construction	2.53	3.00	5.00	5.00	3.00
Restaurants & Food Service	2.45	2.00	2.00	4.50	5.00
Manufacturing	3.79	3.00	5.00	4.00	4.00
Photographic	4.26	6.00	5.00	2.50	2.50
Welding	2.32	5.00	6.00	3.00	3.00
Packaging	3.03	5.00	4.50	4.50	4.00
Education	2.76	6.00	4.00	4.00	2.00
Mean	3.67	4.05	4.29	3.45	3.69
Standard Deviation	1.33	1.51	1.09	1.25	1.07

Exhibit 4. Mean Expert Evaluations of Various Characteristics of Both the Trade Shows and the Associated Industries Noted in Exhibit 1.

Note: Column 1 is the mean from 17 expert evaluations while the other columns represent the consensus judgement after 2 rounds from our two trade show research experts.

	(1) OLS	(2) OLS with truncated Logistic p^h	(3) Probit	(4) Logit	(5) Logit	(6) Logit
CONSTANT	-.0422*	-6.8850**	-3.3790**	-7.1664**	-6.5413**	-6.7088**
	(.0197)	(.2101)	(.9249)	(1.0225)	(.9249)	(.9884)
ATBUY	-.0599**	-.1732**	-.1828**	-.2831*		-.2979*
	(.0025)	(.0268)	(.0664)	(.1427)		(.1456)
EXBUY	-.0208**	-.2313**	-.2030*	-.5861*		-.5066*
	(.0038)	(.0408)	(.1019)	(.2381)		(.2400)
ATPROD	.1268**	1.2559**	.8333**	1.9384**		1.7089**
	(.0041)	(.0439)	(.1060)	(.2715)		(.2597)
EXPROD	.1284**	1.0867**	.4375**	.6252**		.7156**
	(.0046)	(.0485)	(.0988)	(.2123)		(.2101)
DIFFPROD	-.1681**	-1.5564**	-.6661**	-1.2558**	-1.0695**	
	(.0057)	(.0605)	(.1276)	(.2768)	(.2470)	
TECH	-.0039	.1246**	.1705**	.5625**	.3215**	.4885**
	(.0024)	(.0254)	(.0641)	(.1745)	(.1221)	(.1682)
TOTBUY					-.8115**	
					(.1979)	
TOTPROD					2.4992**	
					(.1825)	
SDIFPROD						-8.9116**
						(1.8562)
-Log Likelihood	1558.84	2209.84	395.36	388.29	393.33	387.01
Wald Statistics (df)	22534(6)	12736(6)	482.07(6)	318.61(6)	327.37(4)	318.02(6)
Significance level	.0000	.0000	.0000	.0000	.0000	.0000

Exhibit 5: Results of (weighted) OLS , PROBIT and LOGIT Models, with Dependent Variable p^h, the Proportion of Horizontal Show in an Industry.

(Standard errors are in parenthesis. Note that * denotes significance at 5 %, and ** at 1%). TECH= Exhibit 4, Column 1; ATBUY = Exhibit 4, column 2; EXBUY= Exhibit 4, column 3; TOTBUY = Exhibit 4, Column 2 + Column 3; ATPROD = Exhibit 4, column 4; EXPROD = Exhibit 4, column 5; TOTPROD =Exhibit 4, Column 4 +Column 5; DIFFPROD= Exhibit 4, Abs(Column 4-Column5); SDIFFPROD = DIFFPROD/TOTPROD.

Industry	(1) Actual Proportion Horizontal (p)	(2) Estimated Proportion (Full Sample) (phat1)	(3) = (1) –(2) Difference (discr1)	(4) Predicted Proportion (Jackknife: phat2)	(5)=(1)-(4) Difference (discr2)
Computers and Computer Applications	17.00%	18.13%	-1.13%	8.60%	8.40%
Communications	1.00%	0.57%	0.43%	1.40%	-0.40%
Engineering	97.00%	99.99%	-2.99%	100.00%	-3.00%
Medical and Health Care	4.00%	5.46%	-1.46%	8.59%	-4.59%
Housing	0.00%	1.04%	-1.04%	3.36%	-3.36%
Food Processing and Distribution	95.00%	96.35%	-1.35%	95.03%	-0.03%
Electrical and Electronics	12.00%	3.38%	8.62%	2.22%	9.78%
Paint	0.00%	0.53%	-0.53%	0.69%	-0.69%
Nursing	0.00%	0.05%	-0.05%	0.41%	-0.41%
Automotive and Trucking	94.00%	93.57%	0.43%	88.63%	5.37%
Chemicals	53.00%	59.43%	-6.43%	59.11%	-6.11%
Plastics	100.00%	97.73%	2.27%	95.42%	4.58%
Radio TV and Cable	3.00%	0.27%	2.73%	0.23%	2.77%
Energy	100.00%	99.46%	0.54%	98.44%	1.56%
Building and Construction	40.00%	39.02%	0.98%	30.31%	9.69%
Restaurants & Food Services	100.00%	99.01%	0.99%	97.20%	2.80%
Manufacturing	71.00%	78.20%	-7.20%	84.70%	-13.70%
Photographic	0.00%	4.74%	-4.74%	6.38%	-6.38%
Welding	0.00%	2.08%	-2.08%	10.93%	-10.93%
Packaging	100.00%	84.22%	15.78%	58.15%	41.85%
Education	0.00%	3.76%	-3.76%	2.42%	-2.42%
Unweighted MAD			3.1%		6.6%

Exhibit 6. Predictive Validity Analysis of the Exhibit 5 (Last Column) Logit Model

Note: p is the actual proportion of horizontal shows by industry, phat1 and discr1 are the fitted values and residuals for the logit model (using full sample), phat2 and discr2 are the fitted values and residuals (using the jackknife approach).

Appendix 1 (Proofs of All Results in Section 4)

Proof of Proposition 1:

<u>Step 1</u> (Positivity of all efforts in any perfect-like equilibrium)

Let ((p_{ij} , q_{ji})) be a perfect-like equilibrium. Suppose for a particular exhibitor $\bar{i}$ and attendee $\bar{j}$, $p_{\bar{i}\bar{j}} = q_{\bar{j}\bar{i}}$ = 0 (Note: If any one of these is 0, the other must be 0). Note also, there must exist i' and j' such that $p_{\bar{i}j'} > 0$ and $q_{\bar{j}i'}$ > 0 (i.e. it cannot be that in an equilibrium an exhibitor receives no attention from any attendee or vice versa). Now since ((p_{ij} , q_{ji})) is perfect-like, $\exists$ $p^n_{ij}, q^n_{ji}, \varepsilon^n_{ij}, \varepsilon^n_{ji} (i = 1,..., n_e, j = 1,..., n_a)$ such that conditions (4.3) hold. In particular for $\bar{i}$ this means (($p^n_{\bar{i}j}$)) solve the following maximization problem[4]

Max $\quad {}_{j} \left(p^n_{\bar{i}j} q^n_{j\bar{i}}\right)^{\alpha}$

st $\quad p^n_{\bar{i}j} \geq \varepsilon^n_{\bar{i}j} (> 0) \quad j = 1,...n_a$

and $\quad T_e = \ {}_{j} \ p^n_{\bar{i}j}$

Letting the Lagrangian multipliers on the constraints to be $\mu^n_{\bar{i}j}$ and $\lambda^n_{\bar{i}}$, we have the following f.o.c. :

$$\alpha (p^n_{\bar{i}j})^{\alpha-1} \ (q^n_{j\bar{i}})^{\alpha} + \mu^n_{\bar{i}j} - \lambda^n_{\bar{i}} = 0 \quad j = 1,...n_a \qquad \text{(A1.1)}$$

Now note that $\lambda^n_{\bar{i}}$ must go to some $\lambda_{\bar{i}} > 0$ as $n \to \infty$. To see this, observe that for $j = j'$, for n large $\mu^n_{\bar{i}j}$ must be = 0 and given $p^n_{\bar{i}j}, \to p_{\bar{i}j}, > 0$, $q^n_{j'\bar{i}} \to q_{j'\bar{i}} > 0$, the assertion must hold. Now consider (1) when $j = \bar{j}$. On noting that $\mu^n_{\bar{i}\bar{j}} \geq 0$, , we observe that $(p^n_{\bar{i}\bar{j}})^{\alpha-1} (q^n_{\bar{j}\bar{i}})^{\alpha}$ is bounded from above for large enough n, say by M_1. An exactly analogous argument considering $\bar{j}'s$ problem tells us that $(q^n_{\bar{j}\bar{i}})^{\alpha-1} (p^n_{\bar{i}\bar{j}})^{\alpha}$ is bounded from above for large enough n, say by M_2.

Hence we have

$$(p^n_{\bar{i}\bar{j}})^{\alpha-1} (q^n_{\bar{j}\bar{i}})^{\alpha} \leq \mathrm{M}_1 \qquad \text{(A1.2)}$$

$$(q^n_{\bar{j}\bar{i}})^{\alpha-1} (p^n_{\bar{i}\bar{j}})^{\alpha} \leq \mathrm{M}_2 \qquad \text{(A1.3)}$$

Taking logs,

$$(\alpha - 1) \ \ln \ p^n_{\bar{i}\bar{j}} + \alpha \ \ln \ q^n_{\bar{j}\bar{i}} \leq \ln \mathrm{M}_1 \qquad \text{(A1.4)}$$

---

[4] We have omitted consideration of the valuation parameters β and γ here as their exclusion makes no difference to the analysis.

$$(\alpha - 1)\ \ln\ q^{n}_{\overline{ji}} + \alpha\ \ln\ p^{n}_{\overline{ij}} \le \ln \mathrm{M}_2 \qquad \text{(A1.5)}$$

From (A1.4), we have

$$\alpha \ln q^{n}_{\overline{ji}} \le \ln \mathrm{M}_1 + (1-\alpha)\ln p^{n}_{\overline{ij}} \qquad \text{(A1.6)}$$

But from (A1.5), we obtain

$$(1-\alpha)\ln p^{n}_{\overline{ij}} \le \tfrac{(1-\alpha)}{\alpha}\ln \mathrm{M}_2 + \tfrac{(1-\alpha)}{a}\ln q^{n}_{\overline{ji}} \qquad \text{(A1.7)}$$

Adding (A1.6) and (A1.7) we get

$$\tfrac{\alpha^2-(1-\alpha)^2}{\alpha}\ \ln\ q^{n}_{\overline{ji}} \le \ln \mathrm{M}_1 + \tfrac{(1-\alpha)}{\alpha}\ln \mathrm{M}_2 \qquad \text{(A1.8)}$$

But $\because\ \alpha < 1-\alpha$, and $\ln\ q^{n}_{\overline{ji}} \to -\infty$ ($\because q^{n}_{\overline{ji}} \to q_{\overline{ji}} (=0)$) this is an impossibility.

Hence, $q_{ji} > 0$ and $p_{ij} > 0\ \ \forall i, j$.

<u>Step 2</u> (Uniqueness of the equal-division equilibrium)

Given that all equilibrium effort values are strictly positive, the f.o.c. for an interior optimum holds for each exhibitor *i* whose problem is

$$\text{Max} \quad \sum_j p_{ij}^{\alpha} q_{ji}^{\alpha}$$

$$\text{st} \quad \sum_j p_{ij} = \mathrm{T}_e \qquad \text{(A1.9)}$$

It may be easily verified that this implies

$$\frac{p_{ij}}{p_{ij'}} = \frac{q_{ji}^{\frac{\alpha}{1-\alpha}}}{q_{j'i}^{\frac{\alpha}{1-\alpha}}} \qquad \text{(A1.10)}$$

which implies

$$p_{ij} = \mathrm{E}_i q_{ji}^{\theta} \qquad \text{(A1.11)}$$

where $\theta = \frac{\alpha}{1-\alpha}$ and E_i is an exhibitor *i* specific constant. Similarly, by considering the attendee's problems we can derive

$$q_{ji} = \mathrm{A}_j p_{ij}^{\theta} \qquad \text{(A1.12)}$$

where A_j is an attendee *j* specific constant.

Now from (A1.11) and (A1.12) we have

$$p_{ij} = \mathrm{E}_i \mathrm{A}_j^{\theta} p_{ij}^{\theta^2} \qquad \text{(A1.13)}$$

$$\text{or} \quad p_{ij}^{1-\theta^2} = \mathrm{E}_i \mathrm{A}_j^{\theta} \qquad \text{(A1.14)}$$

$$\text{or,} \quad p_{ij} = E_i^{\frac{1}{1-\theta^2}} A_j^{\frac{\theta}{1-\theta^2}} \qquad \text{(A1.15)}$$

(Note: $1-\theta^2 \ne 0$). Hence summing over *j* we have:

$$\sum_j p_{ij} = \mathrm{E}_i^{\frac{1}{1-\theta^2}} \sum_j \mathrm{A}_j^{\frac{\theta}{1-\theta^2}} \qquad \text{(A1.16)}$$

But since the LHS of (A1.16) is also $= T_e$ which is independent of *i*, E_i must be equal to $\mathrm{E}\ \forall i$. Similarly one can show that $\mathrm{A}_j = \mathrm{A}\forall j$. This implies $p_{ij} = p_{ij'}\ \ \forall i, j, j'$. A similar analysis for attendees show $q_{ji} = q_{ji'}$ $\forall i, j, j'$. Hence etc…

Proof of Proposition 2:

This proof follows along the lines of the proof of Proposition 1 and hence we only sketch the major points in the argument.

<u>Step 1</u> (Positivity)

Suppose $((p_{ijx,} p_{ijy,} q_{jix,} q_{jiy}))$ is a perfect-like equilibrium. We wish to prove that all its components are strictly positive. Suppose not. Wlog, let $p_{\bar{i}\bar{j}x} = q_{\bar{j}\bar{i}x} = 0$. Now $\exists$ sequences $((p^n_{ijx}, p^n_{ijy}, q^n_{jix}, q^n_{jiy}, \in^n_{ijx}, \in^n_{ijy}, \in^n_{jix}, \in^n_{jiy}))$, satisfying conditions like those in (4.3). In particular, $((p^n_{\bar{i}jx}, p^n_{\bar{i}jy}))$ solve

$$\text{Max} \quad \beta_x \sum_j (p^n_{\bar{i}jx})^\alpha + (q^n_{j\bar{i}x})^\alpha + \beta_\gamma \sum_j (p^n_{\bar{i}jy})^\alpha (q^n_{j\bar{i}y})^\alpha \tag{A1.17}$$

$$\text{st} \quad p^n_{\bar{i}jx} \geq \in^n_{\bar{i}jx} \ (>0) \tag{A1.18}$$

$$p^n_{\bar{i}jy} \geq \in^n_{\bar{i}jy} \ (>0) \tag{A1.19}$$

$$\sum_j p^n_{\bar{i}jx} + \sum_j p^n_{\bar{i}jy} = T_e \tag{A1.20}$$

This gives 2 sets of f.o.c.

$$\alpha\beta_x (p^n_{\bar{i}jx})^{\alpha-1} (q^n_{j\bar{i}x})^\alpha + \mu^n_{\bar{i}jx} - \lambda^n_i = 0 \ \forall j \tag{A1.21}$$

$$\alpha\beta_\gamma (p^n_{\bar{i}jy})^{\alpha-1} (q^n_{j\bar{i}y})^\alpha + \mu^n_{\bar{i}jy} - \lambda^n_i = 0 \ \forall j \tag{A1.22}$$

Now, as before we argue that in equilibrium, it must be case that either $p_{\bar{i}jx} > 0, q_{\bar{i}jx} > 0$ for some *j* <u>or</u> $p_{\bar{i}jy} > 0, q_{\bar{i}jy} > 0$ for some *j* [5]. Wlog let $p_{\bar{i}j'y} > 0, q_{\bar{i}j'y} > 0$ [6]

Hence, eventually (i.e. for large *n*) $\mu_{\bar{i}jy} = 0$ and λ^n_i must tend to a positive constant say $\hat{\lambda}_i$.

Now consider (A1.21) for $j = \bar{j}$. Because $\mu_{\bar{i}\bar{j}x} \geq 0$, it is clear that for large *n*

$$(p^n_{\bar{i}\bar{j}x})^{\alpha-1} (q^n_{\bar{j}\bar{i}x})^\alpha \leq M_1 \tag{A1.23}$$

where M_1 is some positive constant. Similarly, by considering $\bar{j}$'s problem one can verify that for another positive constant M_2, the following holds:

$$(q^n_{\bar{j}\bar{i}x})^{\alpha-1} (p^n_{\bar{i}\bar{j}x})^\alpha \leq M_2 \tag{A1.24}$$

---

[5] i must receive positive effort toward some product by some one.

[6] This is without loss of generality because if $p_{\bar{i}j'x} > 0$ and $q_{j'\bar{i}x} > 0$ we may infer that $\lambda^n_i \to \hat{\lambda} > 0$ which is all that we need.

Now, (A1.23) & (A1.24) together with the conditions $p_{\bar{i}\bar{j}x} \to 0 \;\; q_{\bar{j}\bar{i}x} \to 0$ lead to a contradiction as can be shown following the argument outlined in the last few lines of Step 1 in the Proof of Proposition 1.

<u>Step 2</u> (Uniqueness)

$\because ((p_{ijx}, p_{ijy}))_{j=1}^{n_a}$ is a best reaction by *i* against others' strategic choices they

$$\text{Max} \quad \beta_x \sum_j \left(p_{ijx}\right)^{\alpha}\left(q_{jix}\right)^{\alpha} + \beta_y \sum_j \left(p_{ijy}\right)^{\alpha}\left(q_{jiy}\right)^{\alpha} \tag{A1.25}$$

$$\text{st} \quad \sum_j p_{ijx} + \sum_j p_{ijy} = T_e \tag{A1.26}$$

This implies

$$\alpha\beta_x\left(p_{ijx}\right)^{\alpha-1}\left(q_{jix}\right)^{\alpha} = \alpha\beta_y\left(p_{ijy}\right)^{\alpha-1}\left(q_{jiy}\right)^{\alpha} = \lambda_i \tag{A1.27}$$

Where λ_i is the Lagrangian multiplier on the constraint.

This implies

$$p_{ijx} = E_i \beta_x^{\frac{1}{1-\alpha}}\left(q_{jix}\right)^{\theta}, \tag{A1.28}$$

$$p_{ijy} = E_i \beta_y^{\frac{1}{1-\alpha}}\left(q_{jiy}\right)^{\theta} \tag{A1.29}$$

where $\theta = \alpha/_{1-\alpha}$ & E_i is an exhibitor *i* specific constant.

Similarly, considering attendee *j*'s problem we get

$$q_{jix} = A_j\left(\gamma_x\right)^{\frac{1}{1-\alpha}}\left(p_{ijx}\right)^{\theta}, \tag{A1.30}$$

$$q_{jiy} = A_j\left(\gamma_y\right)^{\frac{1}{1-\alpha}}\left(p_{ijy}\right)^{\theta} \tag{A1.31}$$

where A_j is an attendee *j* specific constant.

Equations (A1.28) & (A1.30) give

$$p_{ijx} = E_i \beta_x^{\frac{1}{1-\alpha}} A_j^{\theta} \gamma_e^{\frac{\theta}{1-\alpha}}\left(p_{ijx}\right)^{\theta^2} \tag{A1.32}$$

$$\text{or} \quad p_{ijx} = \left[E_i A_j^{\theta}\right]^{\frac{1}{1-\theta^2}}\left[\beta_x^{\frac{1}{1-\alpha}} \gamma_x^{\frac{\theta}{1-\alpha}}\right]^{\frac{1}{1-\theta^2}} \tag{A1.33}$$

Similarly, one can show using (A1.29) and (A1.31)

$$p_{ijy} = \left[E_i A_j^{\theta}\right]^{\frac{1}{1-\theta^2}}\left[\beta_y^{\frac{1}{1-\alpha}} \gamma_x^{\frac{\theta}{1-\alpha}}\right]^{\frac{1}{1-\theta^2}} \tag{A1.34}$$

Note that $\because \theta^2 \neq 1$ these transformations are permissible.

$$\text{Let} \quad \left[\beta_x^{\frac{1}{1-\alpha}} \gamma_x^{\frac{\theta}{1-\alpha}}\right]^{\frac{1}{1-\theta^2}} = \beta_x^{\frac{1-\alpha}{1-2\alpha}} \gamma_x^{\frac{\alpha}{1-2\alpha}} = C \tag{A1.35}$$

$$\text{and} \quad \left[\beta_y^{\frac{1}{1-\alpha}} \gamma_y^{\frac{\theta}{1-\alpha}}\right]^{\frac{1}{1-\theta^2}} = \beta_y^{\frac{1-\alpha}{1-2\alpha}} \gamma_y^{\frac{\alpha}{1-2\alpha}} = D \tag{A1.36}$$

$$\text{Then} \quad p_{ijx} + p_{ijy} = (C+D) E_i^{\frac{1}{1-\theta^2}} A_j^{\frac{\theta}{1-\theta^2}} \tag{A1.37}$$

$$\text{But since} \quad \sum_j (p_{ijx} + p_{ijy}) = T_e = (C+D) E_i^{\frac{1}{1-\theta^2}} \sum_j A_j^{\frac{\theta}{1-\theta^2}} \;\; \forall i \tag{A1.38}$$

it follows that $E_i = E \;\; \forall i$

A similar argument from attendees' perspectives show $A_j = A \;\; \forall j$.

It follows that

$$p_{ijx} = p_x = \frac{C}{C+D}\left(\frac{T_e}{n_\alpha}\right) = \frac{\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}}}{\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}} + \beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}}\left(\frac{T_e}{n_\alpha}\right) \quad \text{(A1.39)}$$

and $$p_{ijy} = p_y = \frac{D}{C+D}\left(\frac{T_e}{n_\alpha}\right) = \frac{\beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}}{\beta_x^{\frac{1-\alpha}{1-2\alpha}}\gamma_x^{\frac{\alpha}{1-2\alpha}} + \beta_y^{\frac{1-\alpha}{1-2\alpha}}\gamma_y^{\frac{\alpha}{1-2\alpha}}}\left(\frac{T_e}{n_\alpha}\right) \quad \text{(A1.40)}$$

Analogous arguments show

$$q_{jix} = q_x = \frac{\gamma_x^{\frac{1-\alpha}{1-2\alpha}}\beta_x^{\frac{\alpha}{1-2\alpha}}}{\gamma_x^{\frac{1-\alpha}{1-2\alpha}}\beta_x^{\frac{\alpha}{1-2\alpha}} + \gamma_y^{\frac{1-\alpha}{1-2\alpha}}\beta_y^{\frac{\alpha}{1-2\alpha}}}\left(\frac{T_a}{n_e}\right) \quad \text{(A1.41)}$$

and $$q_{jiy} = q_y = \frac{\gamma_y^{\frac{1-\alpha}{1-2\alpha}}\beta_y^{\frac{\alpha}{1-2\alpha}}}{\gamma_x^{\frac{1-\alpha}{1-2\alpha}}\beta_x^{\frac{\alpha}{1-2\alpha}} + \gamma_y^{\frac{1-\alpha}{1-2\alpha}}\beta_y^{\frac{\alpha}{1-2\alpha}}}\left(\frac{T_a}{n_e}\right) \quad \text{(A1.42)}$$

Proof of Proposition 3:

Consider the subgame where attendees have already allocated efforts ((q_{ji})). At this point exhibitor *i* solves the problem

Max $$\beta(p_{ij})^\alpha (q_{ji})^\alpha \quad \text{(A1.43)}$$

st $$\sum_j p_{ij} = T_e \quad \text{(A1.44)}$$

Straightforward analysis reveals[7] that optimal p_{ji}'s are given by

$$p_{ij} = \frac{T_e}{\sum_k (q_{ki})^{\frac{\alpha}{1-\alpha}}}(q_{ji})^{\frac{\alpha}{1-\alpha}} \quad \text{(A1.45)}$$

and hence payoff to attendee *j* (from interacting with exhibitor *i* only) is

$$u_{ji} = \gamma\left[\frac{T_e}{\sum_k (q_{ki})^{\frac{\alpha}{1-\alpha}}}(q_{ji})^{\frac{\alpha}{1-\alpha}}\right]^\alpha \quad \text{(A1.46)}$$

Hence, we can think of the 'sequential' game as a game played among n_a attendees with attendee *j's* strategy variables being $((q_{ji}))_{i=1}^{n_e}$ and his (derived) payoff function being $\sum_{i=1}^{n_e} u_{ji}$.

Straightforward differentiation reveals that u_{ji} is concave and increasing in q_{ji} ; moreover, $\left.\frac{\partial u_{ji}}{\partial q_{ji}}\right|_{q_{ji}=0} = \infty$ (irrespective of the values of q_{ki}, $k \neq j$). Hence we know that we will obtain only interior equilibria where f.o.c.'s for interior optima will apply in calculating best reaction functions.

[7] See equation (A1. 10) in the proof of Proposition 1.

Now consider any 2 attendees *j* and *j'*. Since *j* maximizes $\sum_i u_{ji}$ subject to $\sum_i q_{ji} = T_a$,

the following hold:

$$\gamma T_e^{\alpha}\left[\left(\sum_k q_{ki}^{\theta}\right)\theta q_{ji}^{\theta-1} - \alpha\left(\sum_k q_{ki}^{\theta}\right)^{-\alpha-1}\theta q_{ji}^{2\theta-1}\right] = \gamma T_e^{\alpha}\left[\left(\sum_k q_{ki'}^{\theta}\right)\theta q_{ji'}^{\theta-1} - \alpha\left(\sum_k q_{ki'}^{\theta}\right)^{-\alpha-1}\theta q_{ji'}^{2\theta-1}\right] \forall i, i'.$$

(A1.47)

or $$X_i^{-\alpha} q_{ji}^{\theta-1}\left[1-\frac{q_{ji}^{\theta}}{X_i}\right] = X_{i'}^{-\alpha} q_{ji'}^{\theta-1}\left[1-\frac{q_{ji}^{\theta}}{X_{i'}}\right] \quad \forall\, i, i'. \tag{A1.48}$$

where $\theta = \dfrac{\alpha}{1-\alpha}$, $\quad X_i = \sum_k q_{ki}^{\theta}$.

A similar analysis done on the part of attendee *j'* shows:

$$X_i^{-\alpha} q_{j'i}^{\theta-1}\left[1-\frac{q_{j'i}^{\theta}}{X_i}\right] = X_{i'}^{-\alpha} q_{j'i'}^{\theta-1}\left[1-\frac{q_{j'i'}^{\theta}}{X_{i'}}\right] \quad \forall\, i, i' \tag{A1.49}$$

Dividing (6) by (7) we have

$$\left(\frac{q_{ji}}{q_{j'i}}\right)^{\theta-1}\left(\frac{X_i - \alpha q_{ji}^{\theta}}{X_i - \alpha q_{j'i}^{\theta}}\right) = \left(\frac{q_{ji'}}{q_{j'i'}}\right)^{\theta-1}\left(\frac{X_i - \alpha q_{ji'}^{\theta}}{X_i - \alpha q_{j'i'}^{\theta}}\right) \tag{A1.50}$$

Now note that $\theta > 0$ and $\theta < 1 (\because \alpha < 1/2)$.

Hence (A1.50) $\Rightarrow q_{ji} > q_{j'i} \leftrightarrow q_{ji'} > q_{j'i'}$ (A1.51)

But $\because$ this holds for all *i*, *i'* and $\sum_i q_{ji} = \sum_i q_{j'i} = T_a$, this means

$$q_{ji} = q_{j'i} \ \forall i . \tag{A1.52}$$

Thus we know that for any exhibitor *i*, there is no difference in effort received across attendees. What remains to prove is that each exhibitor gets equal effort from each attendee. We will show that if this is not the case, the marginal utility of effort of a particular attendee will not be constant across exhibitors - a precondition for equilibrium.

Let $q_{ji}^{\theta} = x_{ji}$ and $X_{-ji} = \sum_{k \neq j} x_{ki}$. (A1.53)

Now $$u_{ji} = A\frac{x_{ji}}{(X_{-ji} + x_{ji})^{\alpha}} \tag{A1.54}$$

where *A* is a constant.

and $$\frac{\partial u_{ji}}{\partial q_{ji}} = A\left[(X_{-ji} + x_{ji})^{-\alpha} - \alpha x_{ji}(X_{-ji} + x_{ji})^{-\alpha-1}\right]\left(\tfrac{\alpha}{1-\alpha}\right)\left(q_{ji}\right)^{\frac{2\alpha-1}{1-\alpha}} \tag{A1.55}$$

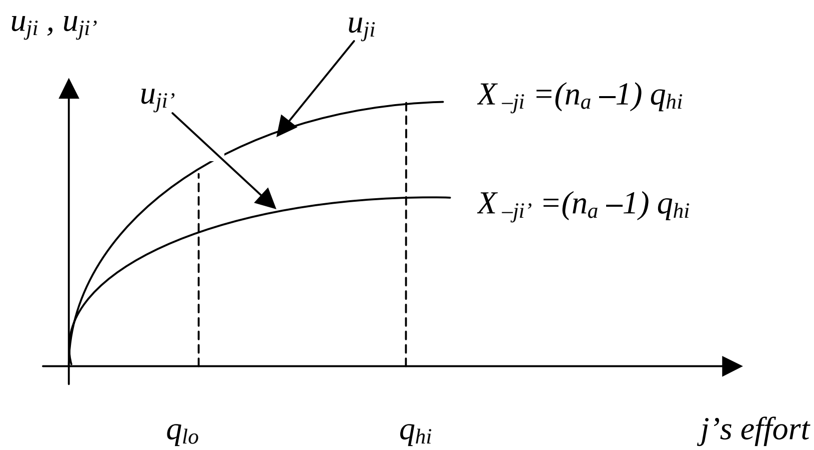
and hence

$$\frac{\partial}{\partial X_{-ji}}\left(\frac{\partial u_{ji}}{\partial q_{ji}}\right) = A\left[-\alpha(X_{-ji}+x_{ji})^{-\alpha-1} + \alpha(1+\alpha)\,x_{ji}\,(X_{-ji}+x_{ji})^{-\alpha-2}\right]\left(\tfrac{\alpha}{1-\alpha}\right)\left(q_{ji}\right)^{\frac{2\alpha-1}{1-\alpha}}$$

(A1.56)

Thus, $\operatorname{sgn}\left[\frac{\partial}{\partial X_{-ji}}\left(\frac{\partial u_{ji}}{\partial q_{ji}}\right)\right] = \operatorname{sgn}\left[(1+\alpha)x_{ji} - \left(X_{ji}+x_{ji}\right)\right] = \operatorname{sgn}\left(\alpha x_{ji} - X_{-ji}\right)$ (A1.57)

Now consider 2 exhibitors *i* and *i*'. Suppose <u>all</u> attendees spend (equal) low effort with *i*, say q_{o}, and (equal) high effort with *i'*, say q_{hi} . The diagram below portrays u_{ji} and $u_{ji'}$ curves, as functions of *j*'s own effort levels. The lower curve represents $u_{ji'}$ while the upper curve represents u_{ji}. This should make sense since *j* gets more for his efforts when others are investing less.



Now, $\left.\frac{\partial u_{ji}}{\partial q_{ji}}\right|_{q_{lo}} > \left.\frac{\partial u_{ji'}}{\partial q_{ji'}}\right|_{q_{lo}}$. (A1.58)

Algebraically this follows because $\alpha q_{o} < (n_a - 1)x \;\; \forall x \in (q_{o}, q_{hi})$ and (A1.57).
Geometrically, this means that the slope of the upper curve is larger than the slope of the lower curve at q_{lo}.

Further, $\left.\frac{\partial u_{ji}}{\partial qji}\right|_{q_{lo}} > \left.\frac{\partial u_{ji'}}{\partial q_{ji'}}\right|_{q_{hi}}$ (A1.59)

because of the concavity of $u_{ji'}$ (the lower curve).
This implies

$\left.\frac{\partial u_{ji}}{\partial q_{ji}}\right|_{q_{lo}} \neq \left.\frac{\partial u_{ji'}}{\partial q_{ji'}}\right|_{q_{hi}}$ which is impossible since the marginal returns *j* earns at both *i* and *i*' must be the same if it was to play best reaction to others' effort choices. Hence…

Proof of Proposition 4

Recall from the Proof of Proposition 2 that an exhibitor *i*, encountering a vector of efforts $((q_{jix}, q_{jiy}))_{j=1,\ldots n_a}$ from the attendees chooses his own efforts $((p_{ijx}, q_{ijy}))_{j=1,\ldots n_a}$ so as to satisfy the f.o.c.

$$\alpha\beta_x\left(p_{ijx}\right)^{\alpha-1}\left(q_{jix}\right)^{\alpha} = \alpha\beta_y\left(p_{ijy}\right)^{\alpha-1}\left(q_{jiy}\right)^{\alpha} = \lambda \qquad \text{(A1.60)}$$

where λ is the multiplier on his total effort constraint (assuming positive valued decision variables). From this the following are easily deduced.

$$p_{ijx} = \frac{\beta_x^{\frac{1}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}}}{\sum_j \beta_x^{\frac{1}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}} + \sum_j \beta_y^{\frac{1}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}}} T_e \qquad \text{(A1.61)}$$

$$\& \; p_{ijy} = \frac{\beta_y^{\frac{1}{1-\alpha}} q_{jiy}^{\frac{\alpha}{1-\alpha}}}{\sum_j \beta_x^{\frac{1}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}} + \sum_j \beta_y^{\frac{1}{1-\alpha}} q_{jiy}^{\frac{\alpha}{1-\alpha}}} T_e \qquad \text{(A1.62)}$$

Hence,

$$(p_{ijx} q_{jix})^{\alpha} = \frac{\beta_x^{\frac{\alpha}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}}}{\left(\sum_j \beta_x^{\frac{1}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}} + \sum_j \beta_y^{\frac{1}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}}\right)^{\alpha}} T_e^{\alpha} \qquad \text{(A1.63)}$$

and $$\left(p_{ijy} q_{jiy}\right)^{\alpha} = \frac{\beta_y^{\frac{\alpha}{1-\alpha}} q_{jiy}^{\frac{\alpha}{1-\alpha}}}{\left(\sum_j \beta_x^{\frac{1}{1-\alpha}} q_{jix}^{\frac{\alpha}{1-\alpha}} + \sum_j \beta_y^{\frac{1}{1-\alpha}} q_{jiy}^{\frac{\alpha}{1-\alpha}}\right)^{\alpha}} T_e^{a} \qquad \text{(A1.64)}$$

Hence, attendee *j*'s overall payoff is

$$q_j = \sum_i \frac{\gamma_x \beta_x^{\frac{\alpha}{1-\alpha}} q_{jix}^{\frac{1}{1-\alpha}} + \gamma_y \beta_y^{\frac{\alpha}{1-\alpha}} q_{jiy}^{\frac{1}{1-\alpha}}}{\left(\sum_k \beta_x^{\frac{1}{1-\alpha}} q_{kix}^{\frac{\alpha}{1-\alpha}} + \sum_k \beta_y^{\frac{1}{1-\alpha}} q_{kiy}^{\frac{\alpha}{1-\alpha}} \right)} T_e^{\alpha} \tag{A1.65}$$

His problem is to maximize $((q_{kix}, q_{kiy}))_{k=1,\ldots n_a; k\neq j}^{i=1,\ldots n_e}$ given the efforts of other attendees.

At this stage, we will invoke the symmetry assumption. If a symmetric equilibrium is to exist where equilibrium values of $q_{kix} = q_x \forall k, i$ and $q_{kiy} = q_y \forall k, i,$ that implies the solution to the problem

$$\text{Max} \quad \sum_i \frac{\eta_x (q_{jix})^{\theta} + \eta_y (q_{jiy})^{\theta}}{\left\{ (n_a - 1)\left(\xi_x q_x^{\theta} + \xi_y q_y^{\theta}\right) + \left(\xi_x q_{jix}^{\theta} + \xi_y q_{jiiy}^{\theta}\right)\right\}^{\alpha}} \tag{A1.66}$$

$$\text{st} \quad \sum_i q_{jix} + q_{jiy} = T_a \tag{A1.67}$$

is: $q_{jix} = q_x \ \forall i,$ $q_{jiy} = q_y \forall i,$ where $\eta_x = \gamma_x \beta_x^{\frac{\alpha}{1-\alpha}}$, $\eta_y = \gamma_y \beta_y^{\frac{\alpha}{1-\alpha}}$, $\xi_x = \beta_x^{\frac{1}{1-\alpha}}$, $\xi_y = \beta_y^{\frac{1}{1-\alpha}}$ and $\theta = \dfrac{\alpha}{1-\alpha}$.

If we write out the f.o.c. for the problem given in (A1.66) and (A1.67) and substitute $q_{jix} = q_x \forall i$ and $q_{jiy} = q_y \forall i$, we get (after some manipulation)

$$\eta_x q_x^{\theta-1} - \frac{\left(\eta_x q_x^{\theta} + \eta_y q_y^{\theta}\right)\alpha \xi_x q_x^{\theta-1}}{n_a \left(\xi_x q_x^{\theta} + \xi_y q_y^{\theta}\right)} = \eta_y q_y^{\theta-1} - \frac{\left(\eta_x q_x^{\theta} + \eta_y q_y^{\theta}\right)\alpha \xi_y q_y^{\theta-1}}{n_a \left(\xi_x q_x^{\theta} + \xi_y q_y^{\theta}\right)} \tag{A1.68}$$

or

$$\left(\frac{q_y}{q_x}\right)^{1-\theta} \left[\frac{\eta_x}{\eta_x q_x^{\theta} + \eta_y q_y^{\theta}} - \frac{\alpha \xi_x}{n_a \left(\xi_x q_x^{\theta} + \xi_y q_y^{\theta}\right)} \right] = \left[\frac{\eta_y}{\eta_x q_x^{\theta} + \eta_y q_y^{\theta}} - \frac{\alpha \xi_y}{n_a \left(\xi_x q_x^{\theta} + \xi_y q_y^{\theta}\right)} \right] \tag{A1.69}$$

Since $q_x + q_y = \dfrac{T_a}{n_e}$, $\xi_x q_x^{\theta} + \xi_y q_y^{\theta}$ is always bounded (both from above and below). Hence, as $n_a \to \infty$, $\dfrac{1}{n_a (\xi_x q_x^{\theta} + \xi_y q_y^{\theta})} \to 0$. This implies as $n_a \to \infty$, $\left(\dfrac{q_y}{q_x}\right)^{1-\theta} \to \left(\dfrac{\eta_y}{\eta_x}\right)$

$$\text{or} \quad \frac{q_y}{q_x} \to \left(\frac{\eta_y}{\eta_x}\right)^{\frac{1}{1-\theta}} = \left(\frac{\gamma_y \beta_y^{\frac{\alpha}{1-\alpha}}}{\gamma_x \beta_x^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1-\alpha}{1-2\alpha}} = \frac{\gamma_y^{\frac{1-\alpha}{1-2\alpha}} \beta_y^{\frac{\alpha}{1-2\alpha}}}{\gamma_x^{\frac{1-\alpha}{1-2\alpha}} \beta_x^{\frac{\alpha}{1-2\alpha}}} \tag{A1.70}$$

This implies as $n_a \to \infty$

$$q_x \to \frac{\gamma_x^{\frac{1-\alpha}{1-2\alpha}} \beta_x^{\frac{\alpha}{1-2\alpha}}}{\gamma_x^{\frac{1-\alpha}{1-2\alpha}} \beta_x^{\frac{\alpha}{1-2\alpha}} + \gamma_y^{\frac{1-\alpha}{1-2\alpha}} \beta_y^{\frac{\alpha}{1-2\alpha}}} \left(\frac{T_a}{\eta_e} \right) \quad \text{(A1.71)}$$

$$\& \quad q_y \to \frac{\gamma_y^{\frac{1-\alpha}{1-2\alpha}} \beta_y^{\frac{\alpha}{1-2\alpha}}}{\gamma_x^{\frac{1-\alpha}{1-2\alpha}} \beta_x^{\frac{\alpha}{1-2\alpha}} + \gamma_y^{\frac{1-\alpha}{1-2\alpha}} \beta_y^{\frac{\alpha}{1-2\alpha}}} \left(\frac{T_a}{\eta_e} \right) \quad \text{(A1.72)}$$

These equilibrium effort values coincide with those obtained in the simultaneous version, and using these expressions, the exhibitor effort values are also seen to coincide with those obtained in the simultaneous version.

Appendix 2 (Proofs of All Results in Section 5)

Proof of Observation 1:

Note that the f.o.c. for the organizer's maximization problem are

$$\frac{\partial \pi}{\partial n_a} = \psi(n_a, n_e)\left(\frac{1-\alpha}{n_a} - \alpha k_e \right) - c_a = 0 \quad \text{(A2.1)}$$

$$\frac{\partial \pi}{\partial n_e} = \psi(n_a, n_e)\left(\frac{1-\alpha}{n_e} - \alpha k_a \right) - c_e = 0 \quad \text{(A2.2)}$$

where $\psi(n_a, n_e) = R(n_a n_e)^{1-\alpha} Exp(-\alpha((k_a n_e + k_e n_a)))$ (A2.3)

Note that since ψ is always positive when n_a and n_e are positive, it follows that $\frac{1-\alpha}{n_a} > \alpha k_e$ and $\frac{1-\alpha}{n_e} > \alpha k_a$.

In what follows we will need the s.o.c. as well; hence we'll need the second derivatives. Straightforward differentiation and some manipulations show:

$$\frac{\partial^2 \pi}{\partial n_a^2} = \psi \left[\left(\frac{1-\alpha}{n_a} - \alpha k_e \right)^2 - \frac{1-\alpha}{n_a^2} \right] \quad \text{(A2.4)}$$

$$\frac{\partial^2 \pi}{\partial n_e^2} = \psi \left[\left(\frac{1-\alpha}{n_e} - \alpha k_a \right)^2 - \frac{1-\alpha}{n_e^{\,2}} \right] \quad \text{(A2.5)}$$

and $\quad \dfrac{\partial^2 \pi}{\partial n_a \partial n_e} = \psi\left[\left(\dfrac{1-\alpha}{n_e} - \alpha k_a\right)\left(\dfrac{1-\alpha}{n_e} - \alpha k_e\right)\right]$ (A2.6)

The s.o.c. requires that the Hessian is negative define to which requires the following to hold:

$$\frac{\partial^2 \pi}{\partial n_a^2} < 0 \tag{A2.7}$$

$$\frac{\partial^2 \pi}{\partial n_e^2} < 0 \tag{A2.8}$$

$$\left(\frac{\partial^2 \pi}{\partial n_a^2}\right)\left(\frac{\partial^2 \pi}{\partial n_e^2}\right) > \left(\frac{\partial^2 \pi}{\partial n_a \partial n_e}\right)^2 \tag{A2.9}$$

While the first two requirements are easily seen to be met since $(1-\alpha)^{1/2} > (1-\alpha)$,
the last condition implies (at the optimum):

$$\left[\left(\frac{1-\alpha}{n_a} - \alpha k_e\right)^2 - \frac{1-\alpha}{n_a^2}\right]\left[\left(\frac{1-2}{n_e} - \alpha k_a\right)^2 - \frac{1-\alpha}{n_e^2}\right] > \left[\left(\frac{1-\alpha}{n_e} - \alpha k_a\right)\frac{1-\alpha}{n_a} - \alpha k_e\right]^2$$

(A2.10)

We will use this inequality extensively in the rest of this proof.

$\underline{\dfrac{\partial n_a^*}{\partial R} > 0, \dfrac{\partial n_e^*}{\partial R} > 0}$:

To demonstrate these, we totally differentiate (A2.1) and (A2.2) w.r.t. R to obtain

$$\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)\left[\frac{\psi}{R} + \left\{\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)\psi\frac{\partial n_a^*}{\partial R} + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)\psi\frac{\partial n_e^*}{\partial R}\right\}\right]$$
$$-\psi\left(\frac{1-\alpha}{n_a^{*2}}\right)\frac{\partial n_a^*}{\partial R} = 0 \tag{A2.11}$$

and

$$\left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)\left[\frac{\psi}{R} + \left\{\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)\psi\frac{\partial n_a^*}{\partial R} + \left(\frac{1-\alpha}{n_e} - \alpha k_a\right)\psi\frac{\partial n_e^*}{\partial R}\right\}\right]$$
$$-\psi\left(\frac{1-\alpha}{n_e^{*2}}\right)\frac{\partial n_e^*}{\partial R} = 0 \tag{A2.12}$$

Rearranging the above two in the from of 2 equations in the 2 unknowns $\dfrac{\partial n_a^*}{\partial R}$ and $\dfrac{\partial n_e^*}{\partial R}$ we have:

$$\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)^2-\left(\frac{1-\alpha}{n_a^{*2}}\right)\right\}\frac{\partial n_a^*}{\partial R}+\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)\right\}\frac{\partial n_e^*}{\partial R}=-\frac{1}{R}\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)$$

(A2.13)

$$\left\{\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)\left(\frac{1-\alpha}{n_a^*}\right)\right\}\frac{\partial n_a^*}{\partial R}+\left\{\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)^2-\left(\frac{1-\alpha}{n_e^{*2}}\right)\right\}\frac{\partial n_e^*}{\partial R}=-\frac{1}{R}\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)$$

(A2.14)

Noting that $\frac{1-\alpha}{n_a^*}-\alpha\kappa_e>0, \frac{1-\alpha}{n_e^*}-\alpha\kappa_a>0$ (because of the f.o.c.) and

$\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)^2-\left(\frac{1-\alpha}{n_a^{*2}}\right)<0, \left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)^2-\frac{1-\alpha}{n_e^{*2}}<0$ (because of (A2.7) and (A2.8)), we observe that the first equation represents a positively sloped line with positive x intercept and negative y intercept whereas the second equation represents a positively sloped line w/negative *X* intercept & positive *Y* intercept (where $\frac{\partial n_a^*}{\partial R}$ is on the abcissa, and $\frac{\partial n_e^*}{\partial R}$ is on the ordinate). Also because of (A2.10) the slope of the former is more than the slope of the latter; hence they must intersect in the first quadrant implying

$$\frac{\partial n_a^*}{\partial R}>0, \frac{\partial n_e^*}{\partial R}>0.$$

<u>$\frac{\partial n_a^*}{\partial k_a}<0, \frac{\partial n_e^*}{\partial k_a}<0, \frac{\partial n_a^*}{\partial k_e}<0, \frac{\partial n_e^*}{\partial k_e}<0$:</u>

We now differentiate (A2.1) and (A2.2) w.r.t. k_a to obtain:

$$\left(\frac{1-\alpha}{n_a^*}-\alpha\kappa_e\right)\left[-\psi\alpha n_e^*+\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)\psi\frac{\partial n_a^*}{\partial k_a}+\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)\psi\frac{\partial n_e^*}{\partial k_a}\right\}\right]$$

$$-\psi\left(\frac{1-\alpha}{n_a^{*2}}\right)\frac{\partial n_a^*}{\partial k_a} = 0 \qquad \text{(A2.15)}$$

$$\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)\left[-\psi\alpha n_e^*+\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)\psi\frac{\partial n_a^*}{\partial k_a}+\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)\psi\frac{\partial n_e^*}{\partial k_a}\right\}\right]$$

$$-\frac{1-\alpha}{n_e^{*2}}\frac{\partial n_e^*}{\partial k_a}-\psi\alpha=0 \qquad \text{(A2.16)}$$

Rearranging, we have

$$\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)^2-\left(\frac{1-\alpha}{n_a^{*2}}\right)\right\}\frac{\partial n_a^*}{\partial \kappa_a}+\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right)\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)\right\}\frac{\partial n_e^*}{\partial k_a}$$

$$=\alpha n_e^*\left(\frac{1-\alpha}{n_a^*}-\alpha k_e\right) \quad \text{(A2.17)}$$

and $$\left\{\left(\frac{1-\alpha}{n_e^*}-\alpha k_e\right)\left(\frac{1-\alpha}{n_a^{\cdot}}-\alpha k_e\right)\right\}\frac{\partial n_a^*}{\partial k_a}+\left\{\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)^2-\left(\frac{1-\alpha}{n_e^{*2}}\right)\right\}\frac{\partial n_e^*}{\partial k_a}$$

$$=\alpha+\alpha n_e^*\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right) \quad \text{(A2.18)}$$

In the $\frac{\partial n_a^*}{\partial k_a}-\frac{\partial n_e^*}{\partial k_a}$ space ($\frac{\partial n_a^*}{\partial k_a}$ is on the abcissa, $\frac{\partial n_e^*}{\partial \kappa_a}$ is on the ordinate), the first equation represents a positively sloped line w/negative x-intercept and positive y intercept. Contrarily, the second equation represents a (positively sloped) line with positive x – intercept & negative y intercept. The first line's slope is larger than the second; so the two lines intersect each other in the 3rd quadrant implying $\frac{\partial n_a^*}{\partial \kappa_a}<0, \frac{\partial n_e^*}{\partial \kappa_a}<0$.

Similarly totally differentiate (A2.1) & (A2.2) w.r.t. k_e, and following the above reasoning, one can show $\frac{\partial n_a^*}{\alpha \kappa_e}<0, \frac{\partial n_e^*}{\partial \kappa_e}<0$. The details are omitted.

$\frac{\partial n_a^*}{\partial C_a}<0, \frac{\partial n_e^*}{\partial C_a}<0, \frac{\partial n_a^*}{\partial C_e}<0, \frac{\partial n_e^*}{\partial C_e}<0$:

The steps here are completely analogous to those of previous exercises. For instance, the following equations obtain in determining $\frac{\partial n_a^*}{\partial C_a}$ and $\frac{\partial n_e^*}{\partial C_a}$:

$$\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha\kappa_e\right)^2-\left(\frac{1-\alpha}{n_a^{*2}}\right)\right\}\frac{\partial n_a^*}{n_a^{*2}}+\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha\kappa_e\right)\left(\frac{1-\alpha}{n_e^*}-\alpha\kappa_a\right)\right\}\frac{\partial n_e^*}{\partial R}=\frac{1}{\psi} \quad \text{(A2.19)}$$

$$\left\{\left(\frac{1-\alpha}{n_a^*}-\alpha\kappa_e\right)\left(\frac{1-\alpha}{n_e^*}-\alpha\kappa_a\right)\right\}\frac{\partial n_a^*}{\partial C_a}+\left\{\left(\frac{1-\alpha}{n_e^*}-\alpha\kappa_a\right)-\left[\frac{1-\alpha}{n_e^{*2}}\right]\right\}\frac{\partial n_e^*}{\partial R}=0 \quad \text{(A2.20)}$$

From these the negativity of $\frac{\partial n_a^*}{\partial C_a}$ and $\frac{\partial n_e^*}{\partial C_a}$ follow easily. The explanations for the negativity of $\frac{\partial n_a^*}{\partial c_e}$ and $\frac{\partial n_e^*}{\partial c_e}$ follow similarly the differentiation of the f.o.c. w.r.t. c_e.

Proof of Proposition 5:

(a) We show that if R is too small π, the profit function in equation (5.1) must be nonpositive for *all* nonnegative values of n_a and n_e. At the same time, *for any fixed* n_a *and* n_e, since the function increases without bounds as R increases, it follows from continuity considerations, that there will be a critical value such that if and only if R was strictly above this value, optimized profits (π^*) will be strictly positive.

Let $R \le \min\left\{c_a\left(\frac{\alpha k_e}{1-\alpha}\right)^{1-\alpha} Exp(\alpha-1),\, c_e\left(\frac{\alpha \kappa_a}{1-\alpha}\right)^{1-\alpha} Exp(\alpha-1)\right\}$ (A2.21)

We claim that for such an R, π evaluated at any n_a and n_e is nonpositive. Suppose not. Suppose there exist some n_a and n_e values for which π is strictly positive. Wlog let $n_a \le n_e$.

Now $\pi = R\left(n_a n_e\right)^{1-\alpha} Exp\left(-\alpha\left(k_a n_e + k_e n_a\right)\right) - c_a n_a - c_e n_e$ (A2.22)

$\le \quad R\ n_e^{2-2\alpha} Exp\left(-\alpha k_a n_e\right) - c_e n_e$ (A2.23)

Hence, we will arrive at a contradiction if the above expression is $\le 0 \quad \forall n_e$.

This is equivalent to showing

$R\, n_e^{1-\alpha} Exp\left(-\alpha k_a n_e\right) \le c_e \quad \forall n_e$ (A2.24)

As can be easily checked, the left hand side of (A2.25) is maximized at $n_e = \frac{1-\alpha}{\alpha k_a}$. Hence the lhs of (A2.24) is $\le R\left(\frac{1-\alpha}{\alpha k_a}\right)^{1-\alpha} Exp(1-\alpha)$ which is less than c_e given that

$R \le c_e\left(\frac{\alpha k_e}{1-\alpha}\right)^{1-\alpha} Exp(\alpha-1)$.

Similarly one can show that π is ≤ 0 when $n_e \le n_a$ using the fact that

$R \le c_a\left(\frac{\alpha k_e}{1-\alpha}\right)^{1-\alpha} Exp(\alpha-1)$.

To see that π^* is strictly increasing in R beyond the above-mentioned critical value we need only differentiate it using envelope theorem:

$$\frac{\partial \pi^*}{\partial R} = \left(n_a^* n_e^*\right)^{1-\alpha} Exp(-\alpha(k_a n_e + k_e n_a)) > 0 \qquad \text{(A2.25)}$$

(b) Let $\phi = \left(n_a^* n_e^*\right)^{1-\alpha} Exp(-\alpha(k_a n_e + k_e n_a))$ (A2.26)

Now $\frac{\partial^2 \pi^*}{\partial R^2} = \phi\left\{\left(\frac{1-\alpha}{n_a^*} - \alpha\kappa_e\right)\frac{\partial n_a^*}{\partial R} + \left(\frac{1-\alpha}{n_e^*} - \alpha\kappa_a\right)\frac{\partial n_e^*}{\partial R}\right\} \quad > 0$ (A2.27)

$\because \frac{\partial n_a^*}{\partial R} > 0, \frac{\partial n_e^*}{\partial R} > 0$. This shows that π^* is convex in *R*.

To see that π^* asymptotes to a positively sloped line with negative intercept, notice that the f.o.c.'s can be written as

$$\phi\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right) = \frac{c_a}{R} \quad \text{(A2.28)}$$

and

$$\phi\left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right) = \frac{c_e}{R} \quad \text{(A2.29)}$$

Now as *R* increases, n_a and n_e increase (Observation 1). At the same time, since the bracketed quantities in (A2.28) and (A2.29) must always stay positive (as we have argued before), $n_a^* \leq \frac{1-\alpha}{\alpha k_e}$ and $n_e^* \leq \frac{1-\alpha}{\alpha k_a}$. Also notice that ϕ goes to zero only if either n_a or n_e goes to zero or infinity (this follows from the fact that the function $x^{1-\alpha} Exp(-k\alpha x)$ goes to zero only if either *x* goes to zero or to infinity as can be easily checked). Hence as *R* goes to zero, it must be the case that the bracketed terms in (A2.28) and (A2.29) must go to zero. Thus, as $R \to \infty, n_a^*, n_e^*$ go to critical values $\hat{n}_a, \hat{n}_e$ where

$$\hat{n}_a = \frac{1-\alpha}{\alpha k_e} \quad \text{(A2.30)}$$

and

$$\hat{n}_e = \frac{1-\alpha}{\alpha k_a} \quad \text{(A2.31)}$$

This shows that as $R \to \infty$, π^* goes to $R(\hat{n}_a \hat{n}_e)^{1-\alpha} Exp(-\alpha(k_a n_e + k_e n_a)) - c_a \hat{n}_a - c_e \hat{n}_e$. As a function of *R*, this is a straight line with positive slope and negative intercept as claimed.

(c) We need to prove

$$\frac{\partial}{\partial R}\left(\frac{\partial \pi^*}{\partial R} \cdot \frac{R}{\pi^*}\right) < 0 \quad \text{(A2.32)}$$

or $\pi^*\left(\frac{\partial^2 \pi^*}{\partial R^2} R + \frac{\partial \pi^*}{\partial R}\right) < R\left(\frac{\partial \pi^*}{\partial R}\right)^2$ (A2.33) or $$\pi^* R \frac{\partial^2 \pi^*}{\partial R^2} + \pi^* \phi < (R\phi)\phi \tag{A2.34}$$

or $$\pi^* R \frac{\partial \phi}{\partial R} + \pi^* \phi < \left(\pi^* + c_a n_a^* + c_e n_e^*\right)\phi \tag{A2.35}$$

or $$\pi^* R \frac{\partial \phi}{\partial R} < \left(c_a n_a^* + c_e n_e^*\right)\phi \tag{A2.36}$$

Now using (A2.28) and (A2.29) to replace c_a and c_e and using (A2.25), (A2.26) and the definition of π^*, we see that we need to prove

$$\left(R\phi - R\phi\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)n_a^* - R\phi\left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)n_e^*\right) * \phi\left\{\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)\frac{\partial n_a^*}{\partial R} + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)\frac{\partial n_e^*}{\partial R}\right\}R$$

$$< R\phi\left\{\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)n_a^* + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)n_e^*\right\}\phi \tag{A2.37}$$

Canceling $R\phi^2$ from both sides, this reduces to proving

$$\left[1 - \left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)n_a^* - \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)n_e^*\right] * \left[\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)R\frac{\partial n_a^*}{\partial R} + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)R\frac{\partial n_e^*}{\partial R}\right]$$

$$< \left[\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)n_a^* + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)n_e^*\right] \tag{A2.38}$$

We now use equations (A2.13) and (A2.14) from the proof of the previous proposition to derive expressions for $\frac{\partial n_a^*}{\partial R}$ and $\frac{\partial n_e^*}{\partial R}$. As can be easily checked,

$$\frac{\partial n_a^*}{\partial R} = \frac{1}{R}\frac{\left(\frac{1-\alpha}{n_a^*} - \alpha\kappa_e\right)\left(\frac{1-\alpha}{n_e^{*2}}\right)}{\Delta} \tag{A2.39}$$

and

$$\frac{\partial n_e^*}{\partial R} = \frac{1}{R} * \frac{\left(\frac{1-\alpha}{n_e^*} - \alpha\kappa_a\right)\left(\frac{1-\alpha}{n_a^{*2}}\right)}{\Delta} \tag{A2.40}$$

where

$$\Delta = \left[\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)^2 - \frac{1-\alpha}{n_a^{*2}}\right]\left[\left(\frac{1-\alpha}{n_e^*} - \alpha\kappa_a\right)^2 - \frac{1-\alpha}{n_e^{*2}}\right]$$

$$- \left[\left(\frac{1-\alpha}{n_e^*} - \alpha\kappa_a\right)\left(\frac{1-\alpha}{n_a^*} - \alpha\kappa_e\right)\right]^2 \qquad \text{(A2.41)}$$

$$= \left[\left(\frac{1-\alpha}{n_a^{*2}}\right)\left(\frac{1-\alpha}{n_e^{*2}}\right) - \left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)^2\left(\frac{1-\alpha}{n_e^{*2}}\right) - \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)^2\left(\frac{1-\alpha}{n_a^{*2}}\right)\right] \qquad \text{(A2.42)}$$

> 0, because of s.o.c.

Using (A2.39) and (A2.40) in (A2.38), the task reduces to proving

$$\left[1 - \left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)n_a^* - \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)n_a^*\right]$$

$$* \left[\left(\frac{1-\alpha}{n_a^*}\alpha k_e\right)^2\left(\frac{1-\alpha}{n_e^{*2}}\right) + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)^2\left(\frac{1-\alpha}{n_a^{*2}}\right)\right]$$

$$< \left[\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)n_a^* + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)n_e^*\right]$$

$$* \left[\left(\frac{1-\alpha}{n_a^{*2}}\right)\left(\frac{1-\alpha}{n_e^{*2}}\right) - \left(\frac{1-\alpha}{n_a^*} - \alpha\kappa_e\right)^2\left(\frac{1-\alpha}{n_e^{*2}}\right) - \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)^2\left(\frac{1-\alpha}{n_a^{*2}}\right)\right] \qquad \text{(A2.43)}$$

which holds if

$$\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)^2\left(\frac{1-\alpha}{n_e^{*2}}\right) + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)^2\left(\frac{1-\alpha}{n_a^{*2}}\right)$$

$$< \left(\frac{1-\alpha}{n_a^{*2}}\right)\left(\frac{1-\alpha}{n_e^{*2}}\right)\left[\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)n_a^* + \left(\frac{1-\alpha}{n_e^*} - \alpha k_a\right)n_e^*\right] \qquad \text{(A2.44)}$$

which in turn holds if

$$\left(\frac{1-\alpha}{n_e^{*2}}\right)\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right)\left[\left(\frac{1-\alpha}{n_a^*} - \alpha k_e\right) - n_a^*\left(\frac{1-\alpha}{n_a^{*2}}\right)\right]$$

$$+\left(\frac{1-\alpha}{n_a^{*2}}\right)\left(\frac{1-\alpha}{n_e^*}-\alpha k_a\right)\left[\left(\frac{1-\alpha}{n_a^*}-\alpha k_a\right)-n_e^*\left(\frac{1-\alpha}{n_e^{*2}}\right)\right]<0 \quad \text{(A2.45)}$$

which clearly holds.

Proof of Proposition 6:

We write down the Kuhn-Tucker conditions for the optimization problem in (5.8) which are necessary and sufficient because of concavity of the objective and convexity of the domain. These are:

$$\pi_v^* - 2m_1 N_v^* - (m_2 + m_4) N_h^* \le 0 \quad \text{(A2.46)}$$

$$(\pi_v^* - 2m_1 N_v^* - (m_2 + m_4) N_h^*) N_v^* = 0 \quad \text{(A2.47)}$$

$$\pi_h^* - 2m_3 N_h^* - (m_2 + m_4) N_v^* \le 0 \quad \text{(A2.48)}$$

$$(\pi_h^* - 2m_3 N_h^* - (m_2 + m_4) N_v^*) N_h^* = 0 \quad \text{(A2.49)}$$

Case A) Suppose $\frac{\pi_h^*}{\pi_v^*} \le \frac{(m_2 + m_4)}{2m_1}$ or, $2m_1 \pi_h^* \le (m_2 + m_4)\pi_v^*$ (A2.50)

Then N_h^* must be 0. If not, from (A2.49) we have

$$\pi_h^* - 2m_3 N_h^* - (m_2 + m_4) N_v^* = 0 \quad \text{(A2.51)}$$

and hence $2m_1 \pi_h^* = 4m_1 m_3 N_h^* + 2m_1 (m_2 + m_4) N_v^*$ (A2.52)

But from (A2.46),

$$(m_2 + m_4)\pi_v^* \le 2m_1 (m_2 + m_4) N_v^* - (m_2 + m_4)^2 N_h^* \quad \text{(A2.53)}$$

Combining (A2.50), (A2.52) and (A2.53), we have

$$4m_1 m_3 N_h^* < (m_2 + m_4)^2 N_h^* \quad \text{(A2.54)}$$

which is impossible given Min(m_1 , m_3) > Max(m_2 , m_4). Hence, $p^h = 0$.

Case C) Suppose $\frac{\pi_h^*}{\pi_v^*} \ge \frac{2m_3}{(m_2 + m_4)}$ or $(m_2 + m_4)\pi_h^* \ge 2m_3 \pi_v^*$ (A2.55)

Then N_v^* must be 0. If not, from (A2.47) we have

$$2m_3 \pi_v^* = 4m_1 m_3 N_v^* + 2m_3 (m_2 + m_4) N_h^* \quad \text{(A2.56)}$$

But (A2.48) gives

$$(m_2 + m_4)\pi_h^* \le 2m_3 (m_2 + m_4) N_h^* + (m_2 + m_4)^2 N_v^* \quad \text{(A2.57)}$$

Combining (A2.55), (A2.56) & (A2.57), we again get a contradiction to the assumption Min(m_1 , m_3) > Max(m_2 , m_4). Hence, $p^h = 1$.

Case B) In this case, it may be checked that

$$N_v^* = \frac{2m_3\pi_v^* - (m_2 + m_4)\pi_h^*}{4m_1m_3 - (m_2 + m_4)^2} \tag{A2.58}$$

$$N_h^* = \frac{2m_1\pi_h^* - (m_2 + m_4)\pi_v^*}{4m_1m_3 - (m_2 + m_4)^2} \tag{A2.59}$$

satisfy all the K-T conditions.

Furthermore,

$$\frac{N_h^*}{N_v^*} = \frac{2m_1\pi_h^* - (m_2 + m_4)\pi_v^*}{2m_3\pi_v^* - (m_2 + m_4)\pi_h^*} \tag{A2.60}$$

$$= \frac{2m_1 \dfrac{\pi_h^*}{\pi_v^*} - (m_2 + m_4)}{2m_3 - (m_2 + m_4)\dfrac{\pi_h^*}{\pi_v^*}} \tag{A2.61}$$

which is clearly increasing in $\frac{\pi_h^*}{\pi_v^*}$. Hence p^h increases with $\frac{\pi_h^*}{\pi_v^*}$.

Proof of Proposition 7:

Case A) Suppose $\frac{\pi_h^*}{\pi_v^*} \le \frac{m_4}{m_1}$ or, $m_1\pi_h^* \le m_4\pi_v^*$ (A2.62)

Then, we claim that N_h^* must be 0. Suppose not. Then, in the horizontal factor market, price of the factor must equal π_h^* (to prevent further entry), while in the vertical factor market the price must be greater than or equal to π_v^* (thus allowing for the possibility that no vertical show is organized). These considerations yield the following two equations:

$$\pi_h^* = m_3N_h^* + m_4N_v^* \tag{A2.63}$$

$$\pi_v^* \le m_1N_v^* + m_2N_h^* \tag{A2.64}$$

Multiplying (A2.63) by m_1 and (A2.64) by m_4 and using (A2.62) we see that

$$m_1 m_3 N_h^* + m_1 m_4 N_v^* \leq m_1 m_4 N_v^* + m_2 m_4 N_h^* \quad \text{(A2.65)}$$

But the above contradicts Min(m_1 , m_3) > Max(m_2 , m_4). Hence N_h^* must be 0 and so should $p^{h.}$

Case C) The proof of this is similar to that of case A. Now, N_v^* must be zero; otherwise equality between factor cost and revenues must hold in the vertical market while factor cost should be at least as large as revenues in the horizontal market. These, along with the stated condition $\frac{\pi_h^*}{\pi_v^*} \geq \frac{m_3}{m_2}$ will then generate a contradiction to the assumption Min(m_1 , m_3) > Max(m_2 , m_4).

Case B) We argue that in this case both N_v^* and N_h^* must be strictly positive. Clearly, both cannot be zero, or entry will occur in both horizontal and vertical arena. Under the stated conditions, it is also impossible to have one of them assume zero value and the other a positive value. Suppose, for instance, N_v^* is zero, while N_h^* is positive. Then,
We must have:

$$\pi_v^* \leq m_2 N_h^* \quad \text{(A2.66)}$$

and

$$\pi_h^* = m_3 N_h^* \quad \text{(A2.67)}$$

From the above two equations we get $\frac{\pi_h^*}{\pi_v^*} \geq \frac{m_3}{m_2}$, which contradicts the stated assumption for this case. Similarly, we can disprove the possibility that N_h^* is zero while N_v^* is positive. Given the positivity of both, we can use the equality of revenue and costs in both markets to write:

$$\pi_v^* = m_1 N_v^* + m_2 N_h^* \quad \text{(A2.68)}$$

and

$$\pi_h^* = m_3 N_h^* + m_4 N_v^* \quad \text{(A2.69)}$$

These can be solved to obtain

$$N_v^* = \frac{m_3 \pi_v^* - m_2 \pi_h^*}{m_1 m_3 - m_2 m_4} \quad \text{(A2.70)}$$

$$N_h^* = \frac{m_1 \pi_h^* - m_4 \pi_v^*}{m_1 m_3 - m_2 m_4} \quad \text{(A2.71)}$$

Hence, $$\frac{N_h^*}{N_v^*} = \frac{m_1 \frac{\pi_h^*}{\pi_v^*} - m_4}{m_3 - m_2 \frac{\pi_h^*}{\pi_v^*}} \quad \text{(A2.72)}$$

As the above expression clearly increases with the $\frac{\pi_h^*}{\pi_v^*}$ ratio, so must p^h.

Appendix 3 (Proofs of All Results in Section 6)

First, we state a few useful facts which we will use in the proofs that follow.

Fact 1. Let $A = 1 + b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}}$ (A3.1)

and $B = 1 + b^{\frac{\alpha}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}}$ (A3.2)

(Recall: $b = \beta_y / \beta_x$, $c = \gamma_y / \gamma_x$).

Then $R_H = \beta_x \frac{A^{1-\alpha}}{B^{\alpha}} + \gamma_x \frac{B^{1-\alpha}}{A^{\alpha}}$

Proving this is a simple matter of manipulating the expression for R_h in (5.3).

Fact 2.

$A \in (1, 2)$; $B \in (1, 2)$.

This follows from $b \in (0, 1)$ and $c \in (0, 1)$.

Fact 3.

$b \geq c \leftrightarrow A \geq B$

This follows trivially from the definitions of *A* and *B*.

Fact 4.

$b \geq c \leftrightarrow bB \geq Ac$

This follows since $bB = b + b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}}$

& $Ac = c + b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}}$

Fact 5.

$$\underset{b,c}{Max} \frac{A}{B} = \underset{b,c}{Max} \frac{B}{A} < \frac{1-\alpha}{\alpha}$$

Proof. Given that $b^{\frac{1-\alpha}{1-2\alpha}} > b^{\frac{\alpha}{1-2\alpha}}$, clearly to maximize A/B, c should be set at 1. Now, the f.o.c. for this maximization problem gives $\frac{A}{B} = \frac{1-\alpha}{\alpha} b.$ Since, $b < 1$, the result follows.

Proof of Observation 2:

We separately deal w/ the cases i) $\beta_x = \gamma_x$, ii) $b = c$ and iii) $(\beta_x - \gamma_x)(b-c) > 0.$

Case i) In this case $R_v = 2\beta_x$ & $R_H = \beta_x \left(\frac{A^{1-\alpha}}{B^\alpha} + \frac{B^{1-\alpha}}{A^\alpha} \right)$

Hence it suffices to show $\frac{A^{1-\alpha}}{B^\alpha} + \frac{B^{1-\alpha}}{A^\alpha} > 2$ (A3.3)

or to show $A + B > 2A^\alpha B^\alpha$ (A3.4)

But $A + B \geq 2A^{1/2} B^{1/2}$ (A3.5)

$\because A > 1$, $B > 1$ (Fact 2), $\alpha < 1/2$, it follows that $A^{1/2} B^{1/2} > A^\alpha B^\alpha$. Hence proved.

Case ii) Suppose $b = c$

In this case $R_H = \beta_x \left(1 + b^{\frac{1}{1-2\alpha}}\right)^{1-2\alpha} + \gamma_x \left(1 + b^{\frac{1}{1-2\alpha}}\right)^{1-2\alpha}$ (A3.6)

$= (\beta_x + \gamma_x)\left(1 + b^{\frac{1}{1-2\alpha}}\right)^{1-2\alpha}$ (A3.7)

whereas $R_v = \beta_x + \gamma_x$ (A3.8)

Hence it suffices to show $\left(1 + b^{\frac{1}{1-2\alpha}}\right)^{1-2\alpha} > 1$, which of course, holds since $\alpha < 1/2$.
Hence proved.

Case iii) We deal with the situation where $\beta_x > \gamma_x$ and $b > c$

The argument is symmetric in the case $\beta_x < \gamma_x$ & $b < c$.

If $\beta_x > \gamma_x$ & $b > c$, it follows that

$$R_H = \beta_x \frac{A^{1-\alpha}}{B^\alpha} + \gamma_x \frac{B^{1-\alpha}}{A^\alpha} > \beta_x + \gamma_x = R_v \tag{A3.9}$$

iff $\beta_x \left[\frac{A^{1-\alpha}}{B^\alpha} - 1 \right] > \gamma_x \left[1 - \frac{B^{1-\alpha}}{A^\alpha} \right]$ (A3.10)

(A3.10) follows if we can show

a) $\frac{A^{1-\alpha}}{B^\alpha} - 1 > 0$ (A3.11)

& b) $\frac{A^{1-\alpha}}{B^\alpha} - 1 > 1 - \frac{B^{1-\alpha}}{A^\alpha}$ (A3.12)

b) has already been shown to hold in dealing with Case i).
a) also holds $\because b > c \quad A > B$ (Fact 3) and $A > 1$ (Fact 2). Hence proved.

Now note that in each case, we have shown that under the stated conditions R_h strictly exceeds R_v. Hence, appealing to the continuity of the R_h & R_v functions (of their arguments *b, c,* β_x, γ_x – as long as they are strictly positive) we see that the inequality continues to hold in a nbd. of the set of parameters satisfying eqn. (6.2).

Proof of Proposition 8:

We show that $\frac{\partial R_h}{\partial b} > 0$ under the stated condition. The positivity of $\frac{\partial R_h}{\partial c}$ follows symmetrically. Now,

$$\frac{\partial R_h}{\partial b} = \beta_x \left\{ (1-\alpha) B^{-\alpha} A^{-\alpha} \left(\frac{1-\alpha}{1-2\alpha} \right) b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} - \alpha B^{-\alpha-1} A^{1-\alpha} \left(\frac{\alpha}{1-2\alpha} \right) b^{\frac{3\alpha-1}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}} \right\}$$
$$+ \gamma_x \left\{ (1-\alpha) B^{-\alpha} A^{-\alpha} \left(\frac{\alpha}{1-2\alpha} \right) b^{\frac{3\alpha-1}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}} - \alpha B^{1-\alpha} A^{-\alpha-1} \left(\frac{1-\alpha}{1-2\alpha} \right) b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} \right\} \qquad \text{(A3.13)}$$

<u>Case i)</u> $b = c$

If $b = c$, $A = B$. Then,

$$\frac{\partial R_h}{\partial b} = A^{-2\alpha} b^{\frac{2\alpha}{1-2\alpha}} \left[\beta_x \left\{ \frac{(1-\alpha)^2}{1-2\alpha} - \frac{\alpha^2}{1-2\alpha} \right\} + \gamma_x \left\{ \frac{(1-\alpha)\alpha}{1-2\alpha} - \frac{\alpha(1-\alpha)}{1-2\alpha} \right\} \right] > 0 \qquad \text{(A3.14)}$$

<u>Case ii)</u> $\beta_x = \gamma_x$

To show:

$$(1-\alpha)^2 B^{-\alpha} A^{-\alpha} b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} - \alpha^2 B^{-\alpha-1} A^{1-\alpha} b^{\frac{3\alpha-1}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}}$$
$$+ \alpha(1-\alpha) B^{-\alpha} A^{-\alpha} b^{\frac{3\alpha-1}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}} - \alpha(1-\alpha) B^{1-\alpha} A^{-\alpha-1} b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} > 0 \qquad \text{(A3.15)}$$

or equivalently,

$$(1-\alpha)^2 - \alpha^2 \frac{A}{B}\frac{c}{b} + \alpha(1-\alpha)\frac{c}{b} - \alpha(1-\alpha)\frac{B}{A} > 0 \qquad \text{(A3.16)}$$

But $\because \max \frac{A}{B} < \frac{1-\alpha}{\alpha}$ and $\max \frac{B}{A} < \frac{1-\alpha}{\alpha}$ (Fact 5), it suffices to show:

$$(1-\alpha)^2 - \alpha^2 \frac{1-\alpha}{\alpha} \cdot \frac{c}{b} + \alpha(1-\alpha)\frac{c}{b} - \alpha(1-\alpha)\frac{(1-\alpha)}{\alpha} \geq 0 \qquad \text{(A3.17)}$$

which, of course is true.

<u>Case iii)</u> $(\beta_x - \gamma_x)(b - c) > 0$

Suppose $b < c$ & $\beta_x < \gamma_x$. We need to show

$$b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} \left\{ \gamma_x \alpha(1-\alpha)\frac{c}{b} - \beta_x \alpha^2 \frac{Ac}{Bb} \right\} > b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{3\alpha-1}{1-2\alpha}} \left\{ \gamma_x \alpha(1-\alpha)\frac{B}{A} - \beta_x (1-\alpha)^2 \frac{c}{b} \right\} \qquad \text{(A3.18)}$$

Now if $b < c$

$$b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} > b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{3\alpha-1}{1-2\alpha}} \qquad \text{(A3.19)}$$

and if $\beta_x < \gamma_x$ additionally

$$\gamma_x \alpha(1-\alpha)\frac{c}{b} > \beta_x \alpha^2 \frac{Ac}{Bb} \qquad \text{(A3.20)}$$

and finally $b < c \rightarrow \quad \frac{c}{b} > \frac{B}{A}$ allows us to claim

$$\gamma_x \alpha(1-\alpha)\frac{c}{b} + \beta_x (1-\alpha)^2 \frac{c}{b} > \gamma_x \alpha(1-\alpha)\frac{B}{A} + \beta_x \alpha^2 \frac{A}{B}\frac{c}{b} \qquad \text{(A3.21)}$$

Combining (A3.19), (A3.20), and (A3.21), (A3.18) is seen to follow.
The case of $b > c$ & $\beta_x > \gamma_x$ is similar and omitted.

Proof of Proposition 9:

Suppose $\beta_x = \gamma_x$. Then,

$$\begin{aligned} \operatorname{sgn}\left[\frac{\partial R_h}{\partial b} - \frac{\partial R_h}{\partial c}\right] = \operatorname{sgn} \quad [& -\alpha B^{-\alpha-1} A^{1-\alpha} \left(\frac{\alpha}{1-2\alpha}\right) b^{\frac{3\alpha-1}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}} + \alpha B^{1-\alpha} A^{\alpha-1} \left(\frac{\alpha}{1-2\alpha}\right) b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{3\alpha-1}{1-2\alpha}} \\ & + \frac{(1-\alpha)\alpha}{1-2\alpha} B^{-\alpha} A^{-\alpha} b^{\frac{3\alpha-1}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}} - \alpha B^{1-\alpha} A^{-\alpha-1} \left(\frac{1-\alpha}{1-2\alpha}\right) b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} \\ & - \frac{(1-\alpha)\alpha}{1-2\alpha} B^{-\alpha} A^{-\alpha} b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{3\alpha-1}{1-2\alpha}} - \alpha B^{-\alpha-1} A^{1-\alpha} \left(\frac{1-\alpha}{1-2\alpha}\right) b^{\frac{\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}} \quad] \end{aligned} \qquad \text{(A3.22)}$$

The sign of the above expression on the other hand, is the same as that of

$$(1-\alpha)\left\{\frac{c}{b} - \frac{B}{A}\right\} - (1-\alpha)\left\{\frac{b}{c} - \frac{A}{B}\right\} + \alpha\left\{\frac{Bb}{Ac} - \frac{Ac}{Bb}\right\} \qquad \text{(A3.23)}$$

Clearly this is 0 if $b = c$ when $A = B$ and $Ac = Bb$.

Now suppose $b > c$. Then (A3.23) can be written as

$$-(1-\alpha)\frac{(Bb-Ac)(Ab+Bc)}{AbBc} + \alpha\frac{(Bb-Ac)(Bb+Ac)}{AbBc} \qquad \text{(A3.24)}$$

Noting that $Ab + Bc > Bb + Ac$ ($\because (A-B)b > (A-B)c$) and $(1-\alpha) > \alpha$, this is clearly seen to be negative. The negativity of $\frac{\partial R_h}{\partial c} - \frac{\partial R_h}{\partial b}$ when $c > b$ follows analogously.

To see $\frac{\partial R_h}{\partial b} - \frac{\partial R_h}{\partial c} = 0$ when *b* = *c*, one simply needs to look at expression (A3.14) where we have computed $\frac{\partial R_h}{\partial b}$. Noting that $\frac{\partial R_h}{\partial c}$ can be obtained from this expression by replacing *b* by *c* and *A* by *B* and since *A* = *B* when *b* = *c*, the result follows.

Finally, we show that when *b* ~ *c*:

$$b > c \qquad \frac{\partial R_h}{\partial b} - \frac{\partial R_h}{\partial c} < 0 \tag{A3.25}$$

and

$$b < c \qquad \frac{\partial R_h}{\partial b} - \frac{\partial R_h}{\partial c} > 0 \tag{A3.26}$$

Since, $\frac{\partial R_h}{\partial b} - \frac{\partial R_h}{\partial c} = 0$ when *b* = *c*, this is achieved by showing that the function R_h is concave in δ for δ small when *b* is replaced by *e* + δ and *c* by *e* - δ and *e* is arbitrary.

Now $R_h = \beta_x R_1(\alpha, e, \delta) + \gamma_x R_2(\alpha, e, \delta)$ where

$$R_1(\alpha, e, \delta) = \frac{\left[1 + (e+\delta)^{\frac{1-\alpha}{1-2\alpha}} (e-\delta)^{\frac{\alpha}{1-2\alpha}}\right]^{1-\alpha}}{\left[1 + (e+\delta)^{\frac{\alpha}{1-2\alpha}} (e-\delta)^{\frac{1-\alpha}{1-2\alpha}}\right]^{\alpha}} \tag{A3.27}$$

and

$$R_2(\alpha, e, \delta) = \frac{\left[1 + (e+\delta)^{\frac{\alpha}{1-2\alpha}} (e-\delta)^{\frac{1-\alpha}{1-2\alpha}}\right]^{1-\alpha}}{\left[1 + (e+\delta)^{\frac{1-\alpha}{1-2\alpha}} (e-\delta)^{\frac{\alpha}{1-2\alpha}}\right]^{\alpha}} \tag{A3.28}$$

Tedious differentiation and some manipulations show that the second derivative of both R_1 and R_2 evaluated at δ = 0 is $-2\alpha e^{-2+\frac{1}{1-\alpha}}(1+e^{\frac{1}{1-2\alpha}})^{-(1+2\alpha)}$ which is negative. Hence...

Proof of Proposition 10:

By looking at expressions (5.2) and (5.3) one can see that if all product evaluation parameters were to experience a lowering of *r* % because of a <u>decrease</u> in the transaction intensity factor, then both R_v and R_h will go down by *r* %. We need to prove that this will raise the π_h^* / π_v^* ratio.

This will hold if $\frac{\partial}{\partial r}\left(\frac{\pi^*(rR_h)}{\pi^*(rR_v)}\right)\Bigg|_{r=1} < 0$ (A3.29)

which in turn holds if $\pi^*(R_v)\frac{\partial \pi^*}{\partial R}\bigg|_{R_h} R_h < \pi^*(R_h)\frac{\partial \rho^*}{\partial R}\bigg|_{R_v} R_v$ (A3.30)

Or, if the elasticity of the π^* function at R_h is less than the corresponding elasticity at R_v. But if $R_h > R_v$, this will hold according to part (c) of Proposition 5.

Proof of Lemma 1:

Recalling that $R_H = \beta_x \frac{A^{1-\alpha}}{B^\alpha} + \gamma_x \frac{B^{1-\alpha}}{A^\alpha}$ where $A = 1 + b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}}$ and $B = 1 + b^{\frac{\alpha}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}}$, we obtain by direct differentiation

$$\frac{\partial R_H}{\partial \alpha} = \beta_x \frac{\partial}{\partial \alpha}\left\{\frac{A^{1-\alpha}}{B^\alpha}\right\} + \gamma_x \frac{\partial}{\partial \alpha}\left\{\frac{B^{1-\alpha}}{A^\alpha}\right\} \tag{A3.31}$$

$$= \beta_x \frac{(1-\alpha)B^\alpha A^{-\alpha}\frac{\partial A}{\partial \alpha} - \alpha A^{1-\alpha}B^{\alpha-1}\frac{\partial B}{\partial \alpha}}{B^{2\alpha}} + \gamma_x \frac{(1-\alpha)A^\alpha B^{-\alpha}\frac{\partial B}{\partial \alpha} - \alpha B^{1-\alpha}A^{\alpha-1}\frac{\partial A}{\partial \alpha}}{A^{2\alpha}} \tag{A3.32}$$

Now to calculate $\frac{\partial A}{\partial \alpha}$, we note that $A - 1 = b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}}$ (A3.33)

and hence $\ln\,(A-1) = \frac{1-\alpha}{1-2\alpha}\ln b + \frac{\alpha}{1-2\alpha}\ln c$ (A3.34)

Differentiating both sides w.r.t. α we get

$$\frac{\partial A/\partial \alpha}{A-1} = \frac{1}{(1-2\alpha)^2}\ln b + \frac{1}{(1-2\alpha)^2}\ln c \tag{A3.35}$$

which gives $\frac{\partial A}{\partial \alpha} = \left(b^{\frac{1-\alpha}{1-2\alpha}} c^{\frac{\alpha}{1-2\alpha}}\right)\frac{\ln b + \ln c}{(1-2\alpha)^2}$ (A3.36)

Similarly $\frac{\partial B}{\partial \alpha} = \left(b^{\frac{\alpha}{1-2\alpha}} c^{\frac{1-\alpha}{1-2\alpha}}\right)\frac{\ln b + \ln c}{(1-2\alpha)^2}$ (A3.37)

Using (A3.36) and (A3.37), (A3.32) can be written as

$$\frac{\partial R_h}{\partial \alpha} = \frac{\ln b + \ln c}{(1-2\alpha)^2}\left\{\beta_x\left[(1-\alpha)B^{-\alpha}A^{-\alpha}b^{\frac{1-\alpha}{1-2\alpha}}c^{\frac{\alpha}{1-2\alpha}} - \alpha A^{1-\alpha}B^{-\alpha-1}b^{\frac{\alpha}{1-2\alpha}}c^{\frac{1-\alpha}{1-2\alpha}}\right]\right.$$
$$\left. + \gamma_x\left[(1-\alpha)A^{-\alpha}B^{-\alpha}b^{\frac{\alpha}{1-2\alpha}}c^{\frac{1-\alpha}{1-2\alpha}} - \alpha B^{1-\alpha}A^{-\alpha-1}b^{\frac{1-\alpha}{1-2\alpha}}c^{\frac{\alpha}{1-2\alpha}}\right]\right\} \qquad \text{(A3.38)}$$

or $$\frac{\partial R_h}{\partial \alpha} = \frac{\ln b + \ln c}{(1-2\alpha)^2}B^{-\alpha}A^{-\alpha}b^{\frac{\alpha}{1-2\alpha}}c^{\frac{\alpha}{1-2\alpha}}\left\{\beta_x\left[(1-\alpha)b - \frac{\alpha Ac}{B}\right] + \gamma_x\left[(1-\alpha)c - \frac{\alpha Bb}{A}\right]\right\} \qquad \text{(A3.39)}$$

Now since $\ln b + \ln c < 0$ ($b < 1$, $c < 1$), it follows that $\frac{\partial R_h}{\partial \alpha} < 0$ if the curly bracketed term above is positive.

To show that this holds under (6.2), we consider the 3 cases i) $b = c$ ii) $\beta_x = \gamma_x$ and iii) $(\beta_x - \gamma_x)(b-c) > 0$ separately.

Case i) When $b = c$, $A = B$ & the curly bracketed term reduces to $\beta_x(1-2\alpha)b + \gamma_x(1-2\alpha)b$; which is clearly positive.

Case ii) Here, we need to show

$$(1-\alpha)(b+c) > \alpha\left(\frac{A}{B}c + \frac{B}{A}b\right) \qquad \text{(A3.40)}$$

which holds if $$b + c \geq \frac{A}{B}c + \frac{B}{A}b \qquad \text{(A3.41)}$$

or if $$(A-B)\left(\frac{b}{A} - \frac{c}{B}\right) \geq 0 \qquad \text{(A3.42)}$$

or if $$(A-B)(bB - Ac) \geq 0 \qquad \text{(A3.43)}$$

This holds since if $b \geq c$, $A \geq B$ and $bB \geq Ac$ (Facts 3 and 4) while if $b \leq c$, $A \leq B$ and $bB \leq Ac$.

Case iii) $$\beta_x\left\{(1-\alpha)b - \frac{\alpha Ac}{B}\right\} + \gamma_x\left\{(1-\alpha)c - \frac{\alpha Bb}{A}\right\}$$
$$\geq \frac{1}{2}\left[\beta_x\left\{b - \frac{Ac}{B}\right\} + \gamma_x\left\{c - \frac{Bb}{A}\right\}\right] \qquad \text{(A3.44)}$$

Now suppose $b > c$ and $\beta_x > \gamma_x$. Then the given expression is

$$\geq \frac{1}{2}\gamma_x\left[\frac{bB - Ac}{B} + \frac{Ac - bB}{A}\right]$$
$$\geq \frac{1}{2}\gamma_x(bB - Ac)\left[\frac{A-B}{AB}\right] \geq 0 \qquad \text{(A3.45)}$$

The case $b < c$ & $\beta_x < \gamma_x$ follows similarly.

Proof of Lemma 2:

We write the linear approximation of $\pi^*(R)$ as $m(\alpha)\, R + c\,(\alpha)$, where m is the slope and c is the intercept – both of which are functions of α (they are functions of other parameters too, but not explicitly stated as such since we deal with changes in α only in this lemma).
From the discussion in Section 5.A, on replacing the expressions for $\hat{n}_a$ and $\hat{n}_e$ in the asymptote to π^* we observe that:

$$m(\alpha) = \left[\left(\frac{1-\alpha}{\alpha}\right)^2 \frac{1}{k_a k_e e^2}\right]^{1-\alpha} \tag{A3.46}$$

$$c(\alpha) = -\left(\frac{c_a}{k_e} + \frac{c_e}{k_a}\right)\left(\frac{1-\alpha}{\alpha}\right) = -A\left(\frac{1-\alpha}{\alpha}\right) \tag{A3.47}$$

where A is a constant independent of α.

Now, let $\pi_1^*(\alpha) = m(\alpha)R_1 + c(\alpha)$ and $\pi_2^*(\alpha) = m(\alpha)R_2 + c(\alpha)$. We need to show that $\frac{\partial}{\partial\alpha}\left(\frac{\pi_1^*(\alpha)}{\pi_2^*(\alpha)}\right) < 0$ under the stated assumptions.

$$\operatorname{sgn}\left[\frac{\partial}{\partial\alpha}\left(\frac{\pi_1^*(\alpha)}{\pi_2^*(\alpha)}\right)\right] = \operatorname{sgn}\left[\left(m(\alpha)R_2 - c(\alpha)\right)\left(m'(\alpha)R_1 - c(\alpha)\right) - \left(m(\alpha)R_1 - c(\alpha)\right)\left(m'(\alpha)R_2 - c(\alpha)\right)\right] \tag{A3.48}$$

Now note that

$$\operatorname{sgn}\left[\frac{\partial}{\partial\alpha}\left(\frac{\pi_1^*(\alpha)}{\pi_2^*(\alpha)}\right)\right] = \operatorname{sgn}\left[\left(R_1 - R_2\right)\left(m(\alpha)c'(\alpha) - c(\alpha)m'(\alpha)\right)\right] \tag{A3.49}$$

Since $R_1 > R_2$, we will have proven the desired result if we can show that the intercept is more

elastic to changes in α than the slope is (note that a rise in α lowers both). Or if,

$$\frac{c'(\alpha)}{c(\alpha)} < \frac{m'(\alpha)}{m(\alpha)} \tag{A3.50}$$

Taking log of (A3.46) and differentiating w.r.t. α gives us the l.h.s. of (A3.50):

$$-\frac{1}{\alpha(1-\alpha)} \tag{A3.51}$$

Similarly, taking log of (A3.45) and differentiating w.r.t. α gives us the r.h.s. of (A3.50):

$$-\left[2\ln(1-\alpha)-2\ln\alpha-\ln(k_a k_e)-2+\frac{2}{\alpha}\right] \tag{A3.52}$$

Now, if $k_a k_e < e^{-2}$, then (A3.51) will be smaller than (A3.52) if the following hold: $2\ln(1-\alpha) > 2\ln\alpha$ and $\frac{2}{\alpha} > \frac{1}{\alpha(1-\alpha)}$. But both inequalities clearly hold since α is less than ½.

Proof of Proposition 11:

Suppose $\alpha_{lo} < \alpha_{hi}$. By Lemma 2, $\frac{\pi^*(R_h(\alpha_{lo});\alpha_{hi})}{\pi^*(R_v;\alpha_{hi})} > \frac{\pi^*(R_h(\alpha_{lo});\alpha_{lo})}{\pi^*(R_v;\alpha_{lo})}$ (note that R_v does not depend on α). Also by Lemma 1, and the increasing-ness of the π^* function , we have

$\frac{\pi^*(R_h(\alpha_{hi});\alpha_{hi})}{\pi^*(R_v;\alpha_{hi})} > \frac{\pi^*(R_h(\alpha_{lo});\alpha_{hi})}{\pi^*(R_v;\alpha_{hi})}$. Combining these, yields the desired result.