EFFICIENCY PROPERTIES OF RATIONAL EXPECTATIONS EQUILIBRIA WITH ASYMMETRIC INFORMATION[*]

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ABSTRACT

We analyze the welfare properties of rational expectations equilibria (REE) in economies with asymmetrically informed agents and incomplete markets. We ask whether a planner can improve upon an equilibrium allocation, using an individually rational and incentive compatible mechanism, and subject to the same asset constraints as agents. For an REE that reveals any information at all, the planner can generically bring about an interim Pareto improvement even conditional on the information that is available to agents in equilibrium. He can do so by altering prices while keeping their informational content fixed. Furthermore, for any partially revealing equilibrium, the planner can generically effect an ex post Pareto improvement by providing more information to agents, while controlling for price effects.

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1. Introduction

**This introduction is incomplete. References to the literature have yet to be added.

In this paper we examine the welfare properties of rational expectations equilibria (REE) in economies with incomplete markets and asymmetric information. Our point of departure is a paper on the subject by La®ont (1985). La®ont considers a class of economies in which rational expectations equilibria can be implemented by an incentive compatible mechanism. He then evaluates their performance relative to such mechanisms. In particular, he provides an example of a fully revealing equilibrium that is interim ine±cient, and a partially revealing equilibrium that is ex post ine±cient. He concludes that rational expectations equilibria are not necessarily e±cient with respect to the amount of information transmitted | they may reveal too much or too little.

We show that such examples are generic. Rational expectations equilibria, whether fully or partially revealing, are ine±cient in the appropriate sense, for a generic subset of agents' endowments. More strikingly, an REE that reveals any information at all is generically interim ine±cient conditional on the amount of information it transmits. One way to interpret this result is as follows. An equilibrium price function induces a partition of the state space. Agents are subject to a budget constraint in each cell of this partition, restricting wealth transfers across cells. Equilibrium prices across cells are generically set ine±ciently. A planner can improve upon the allocation by altering prices across cells (while preserving their informational content) and choosing portfolios of the given assets that respect the altered budget constraints for each agent in each cell.

Thus we may say that rational expectations equilibria are typically price ine±cient. Are they also typically informationally ine±cient, i.e. ine±cient with respect to the information they reveal? We show that partially revealing equilibria are generically informationally ex post ine±cient.

To sum up, we provide a fairly complete characterization of the welfare properties of rational expectations equilibria. We generalize known examples of ine±ciency to theorems on generic ine±ciency. More importantly, the results we report are not only more general, but also tighter, which leads us to a better understanding of why equilibria are ine±cient. We carry out a careful analysis of the constraints that equilibrium allocations satisfy, and identify the particular constraints that are a source of ine±ciency. We are able to isolate three possible sources of ine±ciency. One source is a pecuniary externality which has nothing to do with information, public or private. The second source of ine±ciency is informational, but is not due to asymmetry of information; rather, it arises when an REE reveals information that has "negative value" or does not reveal information that has "positive value." Finally, inefficiency may be due to asymmetry of information.

2. The Economy

We consider a two-period economy with a single physical consumption good. There are finitely many types of agents with a continuum of each type. A typical agent is indexed by $(h;\tau)$, where $h \in H$ (with $\#H = H$; H finite) and $\tau \in [0;1]$. The aggregate uncertainty in the economy is described by the random variables $\tilde{s}$ and $\tilde{t}$, taking values in the finite sets S and T respectively. We assume that $\tilde{s}$ and $\tilde{t}$ are independent (as will become clear shortly, this entails no loss of generality). At date 0, agent $(h;\tau)$ observes a signal $\tilde{s}^{h\tau}$ taking values in the finite set S^h. The agent's type h is publicly observable, but the signal $\tilde{s}^{h\tau}$ is private information. A generic element of the sets S, T, and S^h is denoted, respectively, by s, t, and s^h. Let $\#S = S$, $\#T = T$, $\#S^h = S^h$, and $\sum_h S^h$ by $\overline{S}$. There are at least two "agent-types," i.e. $\overline{S} \geq 2$. The signals $\{\tilde{s}^{h\tau}\}$ are independent of $\tilde{t}$ but may be correlated with $\tilde{s}$. We assume that $\tilde{t}$ has full support, as does the joint distribution of $(\tilde{s}^{h\tau};\tilde{s})$, for every h.[1] We also assume the following (π denotes probabilities):

Assumption 1.

(i) $\pi(\tilde{s}^{h\tau} = s^h) = \pi(\tilde{s}^{h\tau'} = s^h) \qquad \forall h \in H;\ s^h \in S^h;\ \text{and } \tau;\tau' \in [0;1].$

(ii) $\pi(\tilde{s}^{h\tau} = s^h;\ \tilde{s}^{h'\tau'} = s^{h'} \mid s) = \pi(\tilde{s}^{h\tau} = s^h \mid s)\pi(\tilde{s}^{h'\tau'} = s^{h'} \mid s)$

$\forall (h';\tau') \neq (h;\tau);\ s^h \in S^h;\ s^{h'} \in S^{h'};\ s \in S.$

(iii) $\forall s;s' \in S\ (s \neq s');\ \exists s^h \in S^h$ (for some h) s.t. $\pi\{\tilde{s}^{h\tau} = s^h \mid s\} \neq \pi\{\tilde{s}^{h\tau} = s^h \mid s'\}.$

In other words, for any given type h the private signals of the agents have the same distribution. Also, conditional on $\tilde{s}$, agents' private signals are independent across $(h;\tau)$.[2] Informally, we can think of the signal $\tilde{s}^{h\tau}$ as containing a common-value component insofar as it is informative about $\tilde{s}$, and an idiosyncratic component $(\tilde{s}^{h\tau} \mid \tilde{s} = s)$. Assumptions 1(i) and 1(ii) imply that, for every type h, the idiosyncratic components are i.i.d. across τ. Finally, 1(iii) implies that $\tilde{s}$ can be inferred by observing $\tilde{s}^{h\tau}$ for every $(h;\tau)$, except possibly

[1] The full support restrictions are imposed for notational convenience and allow our results to be stated cleanly.

[2] We sidestep the technical issues associated with a continuum of independent random variables. These can be dealt with. See, for example, Al-Najjar (1995) and Sun (1998).

for a set of agents of Lebesgue measure zero. Assumption 1(iii) is in fact just a nontriviality condition that justifies our interpretation of $\tilde{s}$ as the component of the aggregate uncertainty that can be inferred from the private information of agents.

Agents of type h have a von Neumann-Morgenstern utility function u^h. For the purpose of smooth analysis, we make the following assumption:

Assumption 2.

(i) u^h is twice continuously differentiable.

(ii) $Du^h > 0$ and $D^2u^h < 0$.

(iii) $\lim_{c \to 0} Du^h(c) = \infty$.

The endowment of an agent of type h is a random variable $\omega^h : S^h \times S \times T \to \mathbb{R}_+$. Thus, within the same type, agents' endowments differ only insofar as they observe different private signals. We parameterize economies by agents' endowments

$$\omega := [\omega^h(s^h; s; t)]_{h \in H; s^h \in S^h; s \in S; t \in T} \in \mathbb{R}^{\bar{S}ST}.$$

By "generically" we mean "for an open subset of $\mathbb{R}^{\bar{S}ST}$ of full Lebesgue measure."

Agents can modify their state-contingent consumption by trading (at date 0, after observing their private signals) J assets ($J \geq 2$) whose payoff is $r : S \times T \to \mathbb{R}^J$.[3] A portfolio $y \in \mathbb{R}^J$ results in a payoff $r \cdot y$. At date 1, all uncertainty is resolved, assets pay off, and agents consume. Since we have a single-good economy, portfolios uniquely determine consumption.

We assume that there is an asset, say asset J, whose payoff vector for any s, over the state space T, is nonnegative and nonzero, i.e. for every $s \in S$, $r_J(s; t) \geq 0$, for all $t \in T$, and $r_J(s; t) > 0$, for some $t \in T$. Together with the monotonicity assumption on utility functions, this ensures that the equilibrium price of asset J is positive. It also guarantees that budget constraints are satisfied with equality. Finally, we denote by R_s the asset payoff matrix conditional on state s, i.e.

$$R_s := \begin{pmatrix} \vdots \\ r(s;t)^\top \\ \vdots \end{pmatrix}_{t \in T}$$

[3] Agents receive their private signals $\{\tilde{s}^h\}$ before they trade. At the time of trade, any remaining uncertainty in an agent's endowment is completely captured by the aggregate variables $\tilde{s}$ and $\tilde{t}$. Hence we restrict attention to contracts that depend only on aggregate uncertainty as is standard in the rational expectations literature.

where $^\top$ denotes "transpose." By default all vectors are column vectors, unless transposed.

To summarize the information structure: endowments and asset payoffs are uncertain, and this uncertainty is parameterized by the random variables $\{\tilde{s}^{h\tau}\}_{h\in H,\tau\in[0,1]}$, $\tilde{s}$, and $\tilde{t}$. The idiosyncratic component of $\tilde{s}^{h\tau}$ affects only the endowment of agent (h,τ). The random variables $\tilde{s}$ and $\tilde{t}$ describe the common aggregate uncertainty that affects endowments and asset payoffs. Furthermore, $\tilde{s}$ can be perfectly inferred from $\{\tilde{s}^{h\tau}\}_{h,\tau}$, while $\tilde{t}$ captures any residual uncertainty, given the pooled information of all agents. This residual uncertainty gives rise to the potential for gains from trade, even if $\tilde{s}$ is fully revealed.[4]

Our informational assumptions generalize those of Laffont (1985). Agents in our economy are "informationally small" in the following sense: an individual's private signal is informative about the aggregate uncertainty $\tilde{s}$, but $\tilde{s}$ can be fully inferred from the pooled information of the other agents (see also Gul and Postlewaite (1992)). This implies that REE allocations are incentive compatible (see Lemma 4.1 below). Moreover, the presence of an idiosyncratic component in the agent's signal, affecting only his own endowment but not the payoff of assets traded, ensures that generically an REE allocation is strict (Bayesian-Nash) implementable.[5]

Our description of private information is fairly general, and allows us to consider various standard cases in a unified framework. In particular, we can have two types U and I, who are respectively completely uninformed and (almost) perfectly informed. This case arises if $H = \{U, I\}$, S^U is a singleton, and there is a signal $s^I \in S^I$ associated with each state s such that $\pi(s|s^I) = 1 - \epsilon$, where ϵ is a small positive number.[6] On the other hand, if S^h is the same for all h and, for any given s, $\pi(s|\tilde{s}^{h\tau})$ is the same for all (h,τ), then agents are symmetrically informed (ex ante).

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[4] Most of the literature on general equilibrium with rational expectations (for example, Radner (1979) and Allen (1981); also Laffont (1985)) does not consider asset trading specifically, assuming instead that utilities are state-dependent and trading takes place in spot commodity markets. Our framework reduces to this one if we define utilities directly over assets and reinterpret assets as commodities.

[5] Generic strict implementation implies that the incentive constraints are generically not binding in the neighborhood of an REE allocation (see Fact 2 in the Appendix). An alternative modeling choice would be to consider an economy in which private information is non-exclusive in the sense of Postlewaite and Schmeidler (1986). However, such an informational assumption is needlessly strong for our purposes. It implies that incentive constraints do not restrict the set of feasible allocations. In our setup, on the other hand, incentive constraints cannot be ignored because private information about idiosyncratic uncertainty is exclusive.

[6] If ϵ is zero, our full support assumption is violated. We reiterate, however, that this assumption is made merely for convenience.

3. Rational Expectations Equilibrium

In the economy described above, a consumption and portfolio allocation is described by functions $c^h : S^h \times S \times T \to \mathbb{R}_+$ and $y^h : S^h \times S \to \mathbb{R}^J$, for each type h. We will often refer to an allocation simply by specifying portfolios, since portfolios uniquely determine consumption[7]: $c^h = \omega^h + r \cdot y^h$. Using the law of large numbers, the aggregate portfolio of agents of type h in state s is

$$\int_{[0,1]} y^h(s^{h\tau}; s)\, d\tau = \sum_{s^h \in S^h} \pi(s^h \mid s)\, y^h(s^h; s).$$

A price function is a map $p : S \to P$, where $P := \mathbb{R}^{J-1} \times \{1\}$. Note that we normalize the price of asset J to one: $p_J(s) = 1$, for every s.

Definition 1. A rational expectations equilibrium (REE) consists of an allocation $\{y^h\}$, and a price function $p : S \to P$, satisfying the following two conditions:

(AO) Agent optimization: $\forall h \in H$ and $s^h \in S^h$, $y^h(s^h; \cdot)$ solves

$$\max_{y^h(s^h;\cdot):S \to \mathbb{R}^J} E u^h[\omega^h(s^h; \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot y^h(s^h; \cdot) \mid s^h; p]$$

subject to

$$p(s) \cdot y^h(s^h; s) = 0 \qquad \forall h \in H;\ s^h \in S^h;\ s \in S. \qquad (BC_p)$$

(RF) Resource feasibility: $\forall s \in S$,

$$\sum_{h; s^h} \pi(s^h \mid s)\, y^h(s^h; s) = 0.$$

This is the standard definition of an REE with asymmetric information (for example, as in Radner (1979)).[8] Agents know the equilibrium price function and this allows them to make inferences from prices. Taking budget constraints to be equalities is without loss of generality given our assumption that one of the assets (asset J) has a nonzero nonnegative payoff and utility functions are increasing.

An REE is fully revealing if $p(s) \neq p(s')$ for $s \neq s'$ $(s, s' \in S)$; otherwise it is partially revealing. To describe the information revealed by prices, it is convenient to associate with

[7] Note that we restrict attention to symmetric allocations wherein agents of the same type who observe the same signal hold the same portfolio.

[8] Except that we consider asset trading explicitly. See footnote 4.

any price function $p : S \to P$, the partition S^p of S induced by p. A generic element of S^p is denoted by S^p_s, which is the cell of S^p that contains s. Let S^p be the number of cells in S^p, and S^p_s the number of states in S^p_s. (AO) implies that equilibrium portfolios satisfy

(M_{S^p}) Measurability: $\forall h \in H$; and $s^h \in S^h$; $y^h(s^h; \cdot)$ is p-measurable.

This constraint depends on the price function only through the partition induced by it; hence it is indexed by S^p instead of p.

Since the state space S is finite, a partially revealing REE generically does not exist (Pietra and Siconolfi (1998)). Hence we cannot make any generic welfare statements for these equilibria. This leads us to consider a broader class of equilibria, that include rational expectations equilibria, and exist generically.

Definition 2. A pseudo-rational expectations equilibrium (P-REE) consists of an allocation $\{y^h\}$, and a price function $p : S \to P$, satisfying (AO) and

$$\sum_{s' \in S^p_s} \pi(s') \sum_{h; s^h} \pi(s^h \mid s')\, y^h(s^h; s) = 0 \qquad \forall S^p_s \in S^p:$$

A P-REE differs from an REE in that resource feasibility is required to hold only on average within cells of the partition S^p, rather than for every $s \in S$. Note that $y^h(s^h; s)$ is invariant with respect to s within any cell of the partition S^p. We can equivalently restate the above condition as follows:

(RF_{S^p}) Resource feasibility given S^p: $\forall S^p_s \in S^p$,

$$\sum_{h; s^h} \pi(s^h; S^p_s)\, y^h(s^h; s) = 0:$$

Generic existence of a P-REE follows from standard arguments:

Lemma 3.1. For any given partition of S, there generically exists a P-REE such that the equilibrium price function induces this partition.

The definition of a fully or partially revealing P-REE is analogous to that of a fully or partially revealing REE. In the fully revealing case, REE and P-REE are identical. On the other hand, while a partially revealing REE is partially revealing P-REE, the converse is in general not true.

Lemma 3.2. Any REE is a P-REE. Also, a fully revealing P-REE is a fully revealing REE.

In what follows, we will provide a characterization of the efficiency properties of P-REE. Working with P-REE rather than REE is only necessary for generic existence. Our inefficiency results do not depend on this construction.

4. Efficiency Criteria

For a given set of constraints (P), we say that an allocation $\{c^h\}$ is (P)-constrained ex ante efficient if there does not exist an allocation $\{\hat{c}^h\}$ satisfying (P) such that $Eu^h(\hat{c}^h) \geq Eu^h(c^h)$ for every h, with strict inequality for some h. It is (P)-constrained interim efficient if there does not exist an allocation $\{\hat{c}^h\}$ satisfying (P) such that $Eu^h(\hat{c}^h \mid \tilde{s}^{h\tau} = s^h) \geq Eu^h(c^h \mid \tilde{s}^{h\tau} = s^h)$ for every $h \in H$ and every $s^h \in S^h$, with strict inequality for some $h; s^h$. An allocation is (P)-constrained S^p-posterior efficient if there does not exist an allocation $\{\hat{c}^h\}$ satisfying (P) such that $Eu^h(\hat{c}^h \mid \tilde{s}^{h\tau} = s^h; \tilde{s} \in S^p_s) \geq Eu^h(c^h \mid \tilde{s}^{h\tau} = s^h; \tilde{s} \in S^p_s)$ for every $h; s^h; S^p_s$, with strict inequality for some $h; s^h; S^p_s$. Similarly, we define ex post efficiency wherein the utility of a type h agent is evaluated conditional on $(\tilde{s}^{h\tau}; \tilde{s})$.

We impose the following restrictions on the set of attainable allocations. First, an attainable allocation must satisfy resource feasibility. Second, we assume that exchange is voluntary — agents cannot be forced below their autarky utility level. Third, as is standard for economies with private information (Holmström and Myerson (1983)), incentive constraints must be satisfied.

In an REE the net trade of an agent depends not only on own his private information but also on the information of other agents (unless the equilibrium is completely nonrevealing). More generally, an important feature of a common-value environment, such as the one studied in this paper, is that an agent's consumption may depend on common information that the agent himself is not endowed with. Thus the allocation rule conveys some information to the agent. Unless this information is made available to the agent when he evaluates the allocation, renegotiation opportunities may arise. A necessary condition for an allocation rule to be renegotiation-proof is that the information used by the allocation rule be disclosed to the agents (see Forges (1994a, 1994b)). Furthermore, this disclosure constraint must be satisfied if the allocation rule is to be implemented in a decentralized way. In an REE, for example, the market mechanism provides agents with the information on which their portfolios depend. Hence we impose the information disclosure constraint on the set of attainable allocations.[9] Formally, given a price function $p: S \to P$ (not necessarily an REE price function), a p-measurable allocation rule must satisfy the following two constraints:

(IR_{S^p}) Individual rationality: $\forall h \in H$, $s^h \in S^h$, and $S^p_{\bar{s}} \in S^p$;

$$Eu^h(\omega^h + r \cdot y^h \mid \tilde{s}^h = s^h, \bar{s} \in S^p_{\bar{s}}) \geq Eu^h(\omega^h \mid \tilde{s}^h = s^h, \bar{s} \in S^p_{\bar{s}}).$$

(IC_{S^p}) Incentive compatibility[10]: $\forall h \in H$, $s^h, \hat{s}^h \in S^h$, and $S^p_{\bar{s}} \in S^p$,

$$Eu^h[\omega^h(s^h, \bar{s}, \bar{t}) + r(\bar{s}, \bar{t}) \cdot y^h(s^h, \bar{s}) \mid \tilde{s}^h = s^h, \bar{s} \in S^p_{\bar{s}}]$$

$$\geq Eu^h[\omega^h(s^h, \bar{s}, \bar{t}) + r(\bar{s}, \bar{t}) \cdot y^h(\hat{s}^h, \bar{s}) \mid \tilde{s}^h = s^h, \bar{s} \in S^p_{\bar{s}}].$$

As with the measurability constraint (M_{S^p}), the (IR_{S^p}) and (IC_{S^p}) constraints depend on the price function only through the partition induced by it. Both (IR_{S^p}) and (IC_{S^p}) become tighter as S^p becomes finer. In particular, the information disclosure constraint restricts the set of allocations by tightening the individual rationality and incentive constraints — an allocation that is individually rational or incentive compatible given the signals agents observe privately may no longer be so once they have the information that the allocation rule itself reveals.

The resource feasibility constraint we impose on a p-measurable allocation is (RF_{S^p}), the weaker feasibility condition that appears in the definition of a P-REE. As we have argued before, this is for technical reasons only. It ensures that a P-REE allocation is attainable. For the case of a fully revealing price function, (RF_{S^p}) reduces to exact feasibility (RF).

To sum up, the set of attainable allocations are portfolios $\{y^h\}$ satisfying (RF_{S^p}), (M_{S^p}), (IR_{S^p}), and (IC_{S^p}), for some function p. Under our assumptions, equilibrium allocations are attainable:

Lemma 4.1. Consider a P-REE with price function p. Then the equilibrium allocation satisfies (RF_{S^p}), (M_{S^p}), (IR_{S^p}), and (IC_{S^p}).

[9] More precisely the information about the aggregate uncertainty that is used by the allocation rule must be made public. An agent's portfolio may also depend on the idiosyncratic component of his private signal, but he does not have to be provided with this information since he is endowed with it.

[10] Note that we have assumed that the type of an agent (indexed by $h \in H$) is public information, so that the incentive constraint only applies within types.

Proof. This is immediate from the definition of a P-REE. The incentive constraint holds by a revealed preference argument (see Laffont (1985), Proposition 2.2). ∎

It is useful to consider the restrictions on attainable allocations, and in particular on equilibrium allocations, that are implied by each constraint individually. The measurability constraint (M_{S^p}) is vacuous if markets are interim complete (i.e. complete with respect to the state space $S \times T$). With interim incomplete markets, (M_{S^p}) becomes weaker as p becomes more revealing (and is no longer binding when p is fully revealing). More information has the effect of increasing the asset span, and is equivalent to the introduction of new securities. The potentially positive value of information for the resource allocation problem is thus reflected by the measurability constraint. It is this constraint that underlies the ex post inefficiency of a partially revealing equilibrium (Propositions 6.1 and 6.3).

On the other hand, (IR_{S^p}) and (IC_{S^p}) capture one channel through which information can have negative value. As noted above, these constraints become tighter as p becomes more revealing. This can be a source of interim inefficiency of a fully revealing equilibrium (or more generally of an equilibrium that reveals some information), as is illustrated by the following example:

Example 1: The Hirshleifer effect I

The aggregate uncertainty is described as follows: $S = \{s_1; s_2\}$ and $T = \{\bar{t}\}$, with $\pi(s_1) = \pi(s_2) = \frac{1}{2}$. There are two types: $H = \{1; 2\}$. Agents are symmetrically informed ex ante: $S^1 = S^2 = \{\hat{s}_1; \hat{s}_2\}$ with $\pi(s_1 | \hat{s}_1) = \pi(s_2 | \hat{s}_2) = \frac{1}{2} + \epsilon$, where ϵ is a small positive number. Agents' endowments depend only on the aggregate state s and are given by $\omega^1(s_1) = \omega^2(s_2) = \omega_H$ and $\omega^1(s_2) = \omega^2(s_1) = \omega_L$, with $\omega_H > \omega_L$: Asset markets are complete, so that the measurability constraint is irrelevant. In a fully revealing rational expectations equilibrium, there is no trade. For sufficiently small ϵ, the equilibrium allocation is interim Pareto dominated by the allocation in which all agents consume their ex ante expected endowment, $\frac{1}{2}(\omega_H + \omega_L)$. Thus information has negative value in equilibrium, as in Hirshleifer's (1971) original example with a public information signal. More precisely, the REE is (RF, IR_{S^q}, IC_{S^q})-constrained interim inefficient, where $S^q = \{S\}$ is the partition associated with any nonrevealing price function q.[11] It is, however, (IR_{S^p})-constrained ex ante efficient, where $S^p = \{\{s_1\}; \{s_2\}\}$ is the partition induced by the equilibrium price

[11] Since there is no idiosyncratic uncertainty, incentive constraints do not impose any restriction.

function p. Hence we can identify the source of the ine±ciency of the market outcome as the ex post individual rationality constraint (IR_{S^p}), which must be satis¯ed in equilibrium, as opposed to the weaker interim individual rationality constraint (IR_{S^q}) that applies to a nonrevealing allocation rule. k

This example is rather special in one respect. To see this consider the following:

Example 2: The Hirshleifer e®ect II

We modify Example 1 by introducing residual uncertainty $T = \mathrm{f}t_1; t_2\mathrm{g}$, with $¼(t_1) = \frac{3}{4}$ and $¼(t_2) = \frac{1}{4}$. Asset markets are complete. Agents' private information is as in Example 1. Their endowments are $!^1(s_1; t_1) = !^1(s_2; t_2) = !^2(s_1; t_2) = !^2(s_2; t_1) = !_H$ and $!^1(s_1; t_2) = !^1(s_2; t_1) = !^2(s_1; t_1) = !^2(s_2; t_2) = !_L$. There is a fully revealing rational expectations equilibrium (with price function p) in which agents of type 1 consume $\frac{3}{4}!_H + \frac{1}{4}!_L$ in state s_1, and $\frac{1}{4}!_H + \frac{3}{4}!_L$ in state s_2, while the consumption of agents of type 2 is the reverse across states. Thus agents are able to smooth consumption across the residual uncertainty parameterized by T, but not across the uncertainty described by S. For su±ciently small ², the equilibrium allocation can be interim Pareto dominated by transferring a small quantity ´ from type 1 to type 2 agents in state s_1, and doing the opposite transfer in state s_2. As in Example 1, the equilibrium allocation is (RF, IR_{S^q}, IC_{S^q})-constrained interim ine±cient, where q is a nonrevealing price function. But in this case, if ´ is su±ciently small, the dominating allocation satis¯es the ex post individual rationality constraints of the agents. The equilibrium allocation is, therefore, ine±cient in a stronger sense: it is (RF, IR_{S^p}, IC_{S^p})-constrained interim ine±cient. It is possible to bring about a Pareto improvement by using a fully revealing allocation rule. In other words, the fully revealing REE is ine±cient even conditional on the information it transmits. k

In Examples 1 and 2 we see that one source of interim ine±ciency of an REE is that revelation of information restricts the transfers of wealth that agents can achieve across the states S. In Example 1 the constraint (IR_{S^p}) captures this ine±ciency: it is possible to improve upon the equilibrium allocation only by weakening this constraint. In Example 2, on the other hand, (IR_{S^p}) is not binding. To understand the source of ine±ciency in this case, we need to identify a tighter constraint that the equilibrium allocation satis¯es and with respect to which it is e±cient. This turns out to be the budget constraint (BC_p):

Proposition 4.2. A P-REE with price function p is (RF_{S^p}, M_{S^p}, BC_p)-constrained ex ante e±cient.

Proof. Consider an REE $(p; \{c^h; y^h\})$. If $\{c^h\}$ is not $(RF_{S^p}, M_{S^p}, BC_p)$-constrained ex ante efficient, there exists an allocation $\{\hat{c}^h\}$ which satisfies (RF_{S^p}), (M_{S^p}), and (BC_p), with $Eu^h(\hat{c}^h) > Eu^h(c^h)$ for some h. But this means that the REE allocation $\{c^h\}$ violates the agent optimization condition (AO), a contradiction. ∎

Thus the constraint (BC_p) is tight enough to capture the inefficiency of any P-REE, in the sense that a planner subject only to (RF_{S^p}) and (M_{S^p}) can improve upon a P-REE allocation only by violating (BC_p), i.e. by implementing wealth transfers across states that are not budget-feasible at equilibrium prices. In Example 2, in particular, the negative value of information revelation in equilibrium is manifested through the imposition of multiple budget constraints on agents, one constraint for each cell of the partition induced by the price function.

On the other hand, if agents' welfare is evaluated conditionally on the information revealed in equilibrium, then competitive equilibria are efficient (relative to the existing asset structure):

Proposition 4.3. A P-REE with price function p is (RF_{S^p}, M_{S^p})-constrained S^p-posterior efficient.

Proof. Consider a P-REE with price function p and portfolio allocation $\{y^h\}$. For each cell of the partition S^p induced by the price function, we can associate a subeconomy in which agents condition on being in that cell (as well as on any private information they may have). In a typical subeconomy corresponding to the cell S^p_s, the indirect utility over portfolios (in $\mathbb{R}^J$) of agent $(h; \tau)$ is

$$V^h_{S^p_s}(y; \mathbf{s}^{h\tau}) := Eu^h[\omega^h + r \cdot y \,|\, \mathbf{s}^{h\tau}; S^p_s]:$$

For this subeconomy consider a (symmetric) competitive equilibrium $(\bar{p} \in P; \{\bar{y}^h : S^h \to \mathbb{R}^J\})$ wherein, for every $s^h \in S^h$,

$$\bar{y}^h(s^h) \in \arg\max_{y \in \mathbb{R}^J} V^h_{S^p_s}(y; s^h) \quad \text{s.t.} \quad \bar{p} \cdot y = 0;$$

and asset markets clear:

$$\sum_{h; s^h} \pi(s^h; S^p_s)\, \bar{y}^h(s^h) = 0:$$

Since $(p; \{y^h\})$ is an REE for the overall economy, $(p(s); \{y^h(s^h; s)\})$ is an equilibrium in the subeconomy associated with the cell S^p_s. Furthermore, along the lines of the first welfare theorem, the equilibrium in this subeconomy is Pareto e±cient relative to preferences $fV^h_{S^p_s}g$. Hence the REE is S^p-posterior e±cient in the set of allocations satisfying (RF_{S^p}) and (M_{S^p}).

■

The following is immediate:

Corollary 4.4. A fully revealing REE is (RF)-constrained ex post e±cient.

The above proposition and corollary generalize Proposition 2.3 in La®ont (1985).

These results identify restricted notions of e±ciency that are satis¯ed by rational expectations equilibria. We will now show that these restrictions are quite tight | if we relax them, rational expectations equilibria are typically ine±cient. We proceed by identifying necessary conditions satis¯ed by an REE allocation on the one hand and by a constrained e±cient (in the appropriate sense) allocation on the other. We then prove that, generically, these conditions cannot hold simultaneously.

5. Price Ine±ciency of REE

In our setup there is some resolution of uncertainty before trading takes place (the private signals $fs^{h¿}g$ and the information about the aggregate state S that is revealed by prices). While asset markets allow agents to trade risks, subject to the incompleteness of markets, that are resolved after the trading stage, no asset is available to reallocate income across states that are resolved at the initial stage, and attainable allocations depend on asset prices. The economy can thus be viewed as an incomplete markets economy with two periods and multiple goods, but no assets traded at the initial date. As shown by Hart's (1975) well-known example, this economy may have Pareto-ranked equilibria. We will show that indeed this source ine±ciency is present in the setup under consideration and leads to the (generic) constrained ine±ciency of competitive equilibria when agents' welfare is evaluated at the ex ante or interim stage (in the latter case when some information is revealed in equilibrium).[12] More precisely, we will show that competitive equilibrium allocations can be improved upon, even if we ¯x the amount of information revealed in equilibrium.

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[12] As argued by HolmstrÄm and Myerson (1983), the notion of interim e±ciency appears more appropriate in situations of incomplete information such as the one under consideration (rather than ex ante e±ciency, since the economy begins after the agents' private information is revealed to them).

We should note that this result does not rely on the presence of asymmetric information, and point to the presence of a "price inefficiency" also in situations associated with what is commonly known as the Hirshleifer effect.

As a preliminary step we state the following result:

Proposition 5.1. A P-REE with price function p, such that $S^p > 1$, is generically (RF_{S^p}, M_{S^p})-constrained interim inefficient (and, therefore, (RF_{S^p}, M_{S^p})-constrained ex ante inefficient).

Since, as we argued above, markets open only "after" some information is revealed (by the price function p) it is, generically, possible to bring about a Pareto improvement by reallocating risks relative to such information. This result is fairly obvious and so is the proof, which is omitted. It is in fact the analogue of results showing that equilibria with incomplete markets are Pareto inefficient generically. The way the planner is able to improve is essentially by "completing" the market.

Proposition 5.1 generalizes Laffont's (1985) Proposition 3.2 on the possibility of interim inefficiency of fully revealing equilibria. We show, however, that a stronger form of inefficiency holds in the present setup: it is possible to improve even without "being able to complete the market." In other words, the situation illustrated by Hart's example is generic. The source of inefficiency is the presence of a price externality due to the fact that with incomplete markets the set attainable allocations depends on prices (at each information node; in our setup at each element of S^p.) It can then be shown that an improvement can be achieved by inducing a change in prices, but maintaining the same structure of constraints. Accordingly, we introduce the following:

(BC_{S^p}) Information-preserving budget constraints: $\forall h \in H$; $s^h \in S^h$, and $s \in S$;

$$q(s) \cdot y^h(s^h; s) = 0$$

for some function $q : S \to P$, such that $S^q = S^p$.

We restrict a p-measurable allocation rule to satisfy (BC_{S^p}) in addition to (RF_{S^p}), (M_{S^p}), (IR_{S^p}), and (IC_{S^p}). By analyzing (BC_{S^p}) separately from the other constraints, we are able to separate the informational role of prices from their allocative function.

Let $J_{S^p_s}$ be the number of linearly independent assets in the subeconomy corresponding to the cell S^p_s. We assume that $J_{S^p_s} \geq 2$ (note that this rules out Example 1), for every $S^p_s \in$

S^p, and de¯ne $J_{S^p} := \sum_{S^p_s \in S^p} J_{S^p_s}$: We continue to use the notation p for the price function and y^h for portfolios with the understanding that these pertain only to the nonredundant assets in the subeconomy under consideration.

Proposition 5.2. A P-REE with price function p is generically (RF_{S^p}, M_{S^p}, IR_{S^p}, IC_{S^p}, BC_{S^p})-constrained interim ine±cient, provided $J_{S^p_s} \leq \overline{S} \leq J_{S^p} - S^p$, for every $S^p_s \in S^p$. It is generically (RF_{S^p}, M_{S^p}, IR_{S^p}, IC_{S^p}, BC_{S^p})-constrained ex ante ine±cient, provided $J_{S^p_s} \leq \overline{S}$, for every $S^p_s \in S^p$, and $H \leq J_{S^p} - S^p$.

The proof is in the Appendix. Note that the condition for generic interim ine±ciency in the above proposition cannot hold for a nonrevealing P-REE (since in this case $J_{S^p_s} = J_{S^p} = J$, and $S^p = 1$). Indeed, from Proposition 4.3 it follows that a nonrevealing P-REE is (RF_{S^p}, M_{S^p})-constrained interim e±cient. The condition for generic ex ante ine±ciency, however, does cover the case of nonrevealing P-REE.

Since there is no trading at the ex ante stage, using ex ante e±ciency as the welfare criterion may be too strong. This point has been made by Holmström and Myerson (1983). However it is only for the case of completely nonrevealing equilibria that we invoke ex ante e±ciency. The crucial feature of our environment is that agents have private information when they enter the market. The source of interim ine±ciency is that the process of trade itself reveals some information. This (interim) ine±ciency result will survive even if agents can trade at the ex ante stage provided asset markets are incomplete with respect to S^p.

Proposition 5.2 shows that rational expectations equilibria are interim ine±cient even conditional on the amount of information transmitted, and with the planner subject to budget constraints in each subeconomy comparable to those that apply in equilibrium.

The price ine±ciency result may be thought of as an example of the \folk theorem" that an equilibrium allocation with competitive agents is generically ine±cient whenever the agents face constraints that depend on endogenous variables, such as prices, in addition to the usual budget constraint. Other examples have been studied by Stiglitz (). The closest in spirit to our paper is Geanakoplos and Polemarchakis (1986) (G-P) who demonstrate generic ine±ciency with incomplete markets and many goods. The culprit in their paper, as in ours, is a \pecuniary externality" that arises because competitive agents ignore the e®ect of their actions on equilibrium prices. However, the planner's problem in the two cases is quite di®erent. In G-P the planner is free to reallocate income across states (albeit by using the existing assets) and subsequently agents trade to (an ex post e±cient) competitive equilibrium. The planner in this paper cannot do any reallocation at the ex ante stage. He can only alter prices, and choose a portfolio allocation that is budget-feasible for the agents at these prices. This allocation is not ex post optimal for the agents, nor is it ex post e±cient. It seems reasonable to conjecture that if we allow a prior round of trade with asset markets that are su±ciently incomplete, a G-P type of result will hold in our setup.

The point is then similar to the one considered by Stiglitz (1982) and later formalized by G-P. The main issue concerns the precise de¯nition of what \inducing" a price change means. What Stiglitz (and G-P) consider is a reallocation of agents' income across nodes, or state-contingent transfers and taxes. These are moreover required to lie in the asset span, or to be attainable via the existing asset structure. The change in prices is then generated by equilibrium in spot commodity markets. Hence the allocations which can be so achieved are ex post Pareto optimal.

In our setup, the notion of constrained optimality formalized by G-P does not apply since there are no assets (similarly for Hart's example, for that matter). We develop di®erent notion of constrained optimality which is intended to capture another way to \induce" a change in spot prices (in our setup asset prices at the intermediate date of trade). In particular we show that generically a Pareto improvement can be achieved with no transfer of income across nodes, but by \moving" prices away from their competitive values. For instance, we could think of perturbing prices away from their equilibrium values and clear then markets via some prespeci¯ed rationing scheme (or other mechanism). In this way the feasible allocations (achievable in this way) are no longer ex post optimal (as they are in G-P) but still we will show that an improvement can be generically achieved by trading o® some ex post ine±ciency with some gains from trade achieved by changes in the attainable set induced by changes in prices. To keep things simple we will consider here the case in which the planner can choose prices at each node and reallocate commodities within each state s subject to a budget constraint. Hence no transfer of income can take place (at those prices). We show that generically competitive equilibria are interim (and hence ex ante) ine±cient if the number of agent-types is not too large.

We conjecture however that a similar result holds (with a tighter bound on the number of agents, possibly) even if the planner can only change prices and the allocation is then determined by some prespeci¯ed rationing scheme.

On the other hand, if we were to allow for a round of trade before the uncertainty resolved by p is revealed (as well as after that), then an improvement could also be achieved by reallocating portfolios at the initial round of trade, by an application of G-P argument.

6. Informational Inefficiency of REE

The generic inefficiency result of the previous section holds even if markets are complete. The inefficiency results of this section rely on market incompleteness.

Example 3: Adverse selection

**To be added. k

When we consider the possibility of improving upon a P-REE by using more information, we do not wish to exploit the fact that the resource feasibility constraint (RF_{S^p}) is weaker than exact feasibility state by state. We require that any deviation from the equilibrium portfolio allocation does satisfy exact feasibility. Formally, given a P-REE with portfolio allocation $\{y^h\}$, we restrict the set of attainable allocations $\{y^h + \Delta y^h\}$ to satisfy

$$(\overline{RF}) \qquad \sum_{h;s^h} \pi(s^h|s)\,\Delta y^h(s^h;s) = 0; \qquad \forall s \in S.$$

Clearly ($\overline{RF}$) is a stronger restriction that (RF_{S^p}). We do not impose exact feasibility on $\{y^h + \Delta y^h\}$, for then the P-REE allocation $\{y^h\}$ itself would be unattainable in general.

Recall that R_s is the asset payoff matrix conditional on state s. For $\hat{s} \in S^p_s$, let

$$R_{S^p_s;\hat{s}} := \begin{pmatrix} \begin{pmatrix} \vdots \\ R_{s'} \\ \vdots \end{pmatrix}_{s' \in S^p_s} & \vdots & \begin{matrix} 0 \\ \vdots \\ R_{\hat{s}} \\ \vdots \\ 0 \end{matrix} \end{pmatrix}.$$

Proposition 6.1. A P-REE with price function p is generically ($\overline{RF}$, IR_{S^p}, IC_{S^p}, BC_p)-constrained ex post inefficient, if $\bar{S} \geq 3$, and provided there is a cell S^p_s, and a state $\hat{s} \in S^p_s$, such that rank $(R_{S^p_s;\hat{s}}) \geq J_{S^p_s} + 3$.

The proof is in the Appendix. This result generalizes Laffont's (1985) Proposition 4.2 on the possibility of ex post inefficiency of partially revealing equilibria.

A P-REE satisfies (RF_{S^p}), (M_{S^p}), (IR_{S^p}), (IC_{S^p}), and (BC_p). The proposition states that the planner can (generically) bring about a Pareto improvement ex post (and hence also ex ante and interim) by relaxing (M_{S^p}), the constraint that the portfolios be measurable with respect to the price function. Thus the planner improves upon the equilibrium allocation by using more information to construct portfolios. This amounts to adding new securities that increase the rank of the asset payoff matrix in the subeconomy S_s^p by at least 3, as the condition, rank $(R_{S_s^p;\hat{s}}) \geq J_{S_s^p} + 3$, says. For this to be possible, asset markets must be sufficiently incomplete, and the P-REE must be partially revealing.

Note that the additional information used by the planner is not made available to the agents themselves. If this information had to made public, a Pareto improvement may not be possible without violating the individual rationality and incentive constraints.

Example 4

The economy is the same as in Example 2, except that markets are incomplete. The asset payoff matrix is

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

where the states of the world are ordered as $\{(s_1; t_1); (s_1; t_2); (s_2; t_1); (s_2; t_2)\}$. Thus markets are complete with respect to s, and with respect to t, conditional on s. Both types of agent have the same utility function: $u(c) = ac - \frac{1}{2}c^2$, where a is a parameter that is sufficiently large to ensure that marginal utility is increasing over the relevant range of consumption. This economy has a nonrevealing P-REE in which the price of each asset is equal to its expected payoff, i.e. $p_1 = p_2 = \frac{1}{2}$, and $p_3 = \frac{3}{4}$: Agents are able to smooth consumption across the states s_1 and s_2 by trading the first two securities. Agents of type 1 sell asset 1 and buy asset 2, thus transferring consumption from s_1 to s_2, while agents of type 2 take the opposite side of this trade. However, due to the incompleteness of markets, agents are not able to smooth consumption within the two subeconomies indexed by s_1 and s_2. Agents wish to trade asset 3 in opposite directions in the two subeconomies, but they cannot distinguish these subeconomies in a nonrevealing equilibrium. Indeed, the equilibrium amount of trade in asset 3 goes to zero as ϵ tends to zero. By making the allocation of asset 3 contingent on the state s, it is possible to bring about a Pareto improvement in each subeconomy. To be precise, the P-REE allocation is ($\overline{RF}$, IR_{S^p}, IC_{S^p})-constrained ex post inefficient. Note, however, that a Pareto improvement can only be brought about by using an allocation rule that reveals additional information to agents. If the true state (s_1 or s_2) is revealed to agents, a feasible allocation must respect the ex post individual rationality constraints of all agents. Consider, for instance, the subeconomy indexed by s_1. An ex post efficient allocation smoothes consumption completely across t_1 and t_2, so when we look for a Pareto improvement in this subeconomy, we may restrict attention to allocations of consumption that are nonrandom for each agent. Suppose a is very large, so that agents are close to risk neutral. Then a Pareto improving allocation must assign consumption to each agent-type that is close to the equilibrium expected consumption of that agent-type (or better). In the subeconomy under consideration, at the P-REE, the expected consumption of agent-type $(1; \hat{s}_2)$ is

$$E(c^1(\hat{s}_2)) = (\frac{3}{4}\omega_H + \frac{1}{4}\omega_L) - \frac{1}{4}(\omega_H - \omega_L) - \frac{\epsilon}{2}(\omega_H + \omega_L) - 2a\epsilon.$$

The maximum consumption that a Pareto improving allocation can assign to this agent $E(c^1(\hat{s}_2)) + \eta$, where η can be made arbitrarily small by choosing a sufficiently large. But for small η, this agent-type is better off consuming his endowment (which gives him expected utility $\frac{3}{4}\omega_H + \frac{1}{4}\omega_L$). Thus the P-REE allocation is $(\overline{RF}, IR_{S^q})$-constrained ex post efficient, where S^q is the partition $\{\{s_1\}, \{s_2\}\}$. ∎

In Example 4, we see the two countervailing effects of more information. A fully revealing allocation rule induces the Hirshleifer effect, which shrinks the set of feasible allocations through a tighter individual rationality constraint. At the same time, there is a positive spanning effect, captured by a weaker measurability constraint, that expands the set of attainable allocations. In this example, the first effect dominates.

Since the individual rationality and incentive constraints are generically not binding at an equilibrium, we should expect to be able to make a local improvement, by using only a small amount of extra information, which is made public. The problem is that with the partition representation of information on a finite set S, no change in information is "small."

In order to rectify this problem, we proceed with the following construction. We index the elements of S by $i = 1, \ldots, S$, and analogously define the set $\Sigma := \{\sigma_1, \ldots, \sigma_S\}$. Let $\tilde{\sigma}$ be a random variable, taking values in Σ, that is independent of $\bar{t}$ and, conditionally on $\bar{s}$, also of $\bar{s}^{h\tau}$, for all (h, τ), (i.e. $\pi(\bar{s}^{h\tau}, \tilde{\sigma} | \bar{s}) = \pi(\bar{s}^{h\tau} | \bar{s})\pi(\tilde{\sigma} | \bar{s})$), with $\pi(\sigma_j) = \pi(s_j)$ for every j. We control for prices by defining $p : \Sigma \to P$, with $p(\sigma_j) = p(s_j)$ for every j.

Given a P-REE with price function p and portfolio allocation $\{y^h\}$, consider the set of consumption allocations $\{\bar{c}^h\}$ satisfying:

$(\overline{RF}_\sigma)$ Resource feasibility: $\forall h \in H$,

$$\bar{c}^h = \omega^h + r \cdot (y^h + \Delta y^h)$$

and, $\forall s \in S, \sigma \in \Sigma$,

$$\sum_{h,s^h} \pi(s^h | s)\, \Delta y^h(s^h, \sigma) = 0.$$

(M_σ) Measurability: $\forall h \in H$, and $s^h \in S^h$, $y^h(s^h, \tilde{s}) + \Delta y^h(s^h, \tilde{\sigma})$ is $\tilde{\sigma}$-measurable.

(IR_σ) Individual rationality: $\forall h \in H$, $s^h \in S^h$, and $\sigma \in \Sigma$,

$$Eu^h(\bar{c}^h | \tilde{s}^h = s^h, \tilde{\sigma} = \sigma) \geq Eu^h(\omega^h | \tilde{s}^h = s^h, \tilde{\sigma} = \sigma).$$

(IC_σ) Incentive compatibility: $\forall h \in H$, $s^h, \hat{s}^h \in S^h$, and $\sigma \in \Sigma$,

$$Eu^h\left[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot [y^h(s^h, \tilde{s}) + \Delta y^h(s^h, \sigma)] \,|\, \tilde{s}^h = s^h, \tilde{\sigma} = \sigma\right]$$
$$\geq Eu^h\left[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot [y^h(\hat{s}^h, \tilde{s}) + \Delta y^h(\hat{s}^h, \sigma)] \,|\, \tilde{s}^h = \hat{s}^h, \tilde{\sigma} = \sigma\right].$$

$(BC_{p,\sigma})$ Budget constraints: $\forall h \in H$, $s^h \in S^h$, and $\sigma \in \Sigma$,

$$p(\sigma) \cdot \Delta y^h(s^h, \sigma) = 0.$$

Conditions (M_σ), (IR_σ), and (IC_σ) are essentially the same as (M_{S^p}), (IR_{S^p}), and (IC_{S^p}) respectively, except that $\tilde{\sigma}$ replaces p. Conditions $(\overline{RF}_\sigma)$ and $(BC_{p,\sigma})$ are the analogues of $(\overline{RF})$ and (BC_p) respectively. The set of attainable allocations are those that satisfy $(\overline{RF}_\sigma)$, (M_σ), (IR_σ), (IC_σ), and $(BC_{p,\sigma})$ for some $\tilde{\sigma}$.

We parameterize the random variable $\tilde{\sigma}$ by $\Pi := \{\pi_{ij}\}$, where $\pi_{ij} := \pi(s_i, \sigma_j)$, with

$$\sum_j \pi_{ij} = \pi(s_i) \quad \forall i; \qquad \text{and} \qquad \sum_i \pi_{ij} = \pi(s_j) \quad \forall j. \tag{1}$$

The notation $i \in S_j^p$ is shorthand for $s_i \in S_{s_j}^p$. The following is easily verified (we simply choose $\tilde{\sigma}$ to have the same information content as p):

Lemma 6.2. Suppose p is a P-REE price function. Then the equilibrium allocation satisfies $(\overline{RF}_\sigma)$, (IR_σ), (IC_σ), and $(BC_{p,\sigma})$, with $\{\Delta y^h\} = 0$ and the following choice of Π:

$$\pi_{ij} = \begin{cases} \dfrac{\pi(s_i)\pi(s_j)}{\sum_{k \in S_j^p} \pi(s_k)} & \text{if } i \in S_j^p; \\ 0 & \text{if } i \notin S_j^p. \end{cases}$$

For a fully revealing REE

$$\pi_{ij} = \begin{cases} \pi(s_i) & \text{if } i = j; \\ 0 & \text{if } i \neq j. \end{cases}$$

Perturbing the probabilities Π allows us to perturb the information of agents in a smooth way. In doing so, we do not change the support of σ, nor do we change the dependence of p on σ.

We now show that a partially revealing P-REE is generically ex post inefficient, even if feasible allocations are restricted by the information disclosure constraint. For ease of exposition, we state and prove the result for the case of a nonrevealing P-REE (with $S_s^p = S$). The extension to the general partially revealing case is immediate.

Proposition 6.3. A nonrevealing P-REE with price function p is generically ($\overline{RF}_\sigma$, M_σ, IR_σ, IC_σ, $BC_{p;\sigma}$)-constrained ex post inefficient provided $\overline{S} \geq S + 2$, and provided there are states s_m and s_n in S, such that $\min[\mathrm{rank}(R_{S;s_m}); \mathrm{rank}(R_{S;s_n})] \geq J + 2(S + 2)$.

The proof is in the Appendix.

First we need to introduce some more notation. Let

$$c^h_{s^h,s} := [\omega^h(s^h,s,t) + r(s,t)\cdot y^h(s^h,s)]_{t\in T}$$

be the vector of state-contingent consumption of agent (h,τ), conditional on the information $(\tilde{s}^{h\tau} = s^h,\ \tilde{s} = s)$. Define the function $U^h_{s^h,s} : \mathbb{R}^T_+ \to \mathbb{R}$ as follows:

$$U^h_{s^h,s}(c^h_{s^h,s}) = \pi(s^h,s) \sum_{t\in T} \pi(t)\, u^h[\omega^h(s^h,s,t) + r(s,t)\cdot y^h(s^h,s)].$$

Thus $U^h_{s^h,s}(c^h_{s^h,s})$ is the expected utility (up to a multiplicative constant) of agent (h,τ) conditional on $(\tilde{s}^{h\tau} = s^h,\ \tilde{s} = s)$. For ease of notation we often drop the argument $c^h_{s^h,s}$.

Recall that $J_{S^p_s}$ is the number of linearly independent assets in the subeconomy associated with S^p_s. Given a price function p, we adopt the convention of disregarding redundant assets in each subeconomy, so that

$$R_{S^p_s} := \begin{pmatrix} \vdots \\ R_{s'} \\ \vdots \end{pmatrix}_{s'\in S^p_s}$$

has full column rank $J_{S^p_s}$ (we always retain the J-th asset which serves as numeraire).[13]

The agent optimality condition (AO) can be restated as follows: for every $h \in H, s^h \in S^h$, and $S^p_s \in S^p$, $y^h(s^h,s)$ maximizes $\sum_{s'\in S^p_s} U^h_{s^h,s'}(c^h_{s^h,s'})$ subject to $p(s)\cdot y^h(s^h,s) = 0$, and subject to $y^h(s^h,\cdot)$ being p-measurable. Under Assumption 2, the solutions of (AO) are then characterized by the following system of first order conditions:

$$\sum_{s'\in S^p_s} R^\top_{s'} DU^h_{s^h,s'} - \lambda^h(s^h,s)\,p(s) = 0, \qquad \forall h\in H, s^h\in S^h, s\in S; \qquad (A.1)$$

$$p(s)\cdot y^h(s^h,s) = 0, \qquad \forall h\in H, s^h\in S^h, s\in S; \qquad (A.2)$$

where $\lambda^h(s^h,\cdot) : S \to \mathbb{R}$ is a p-measurable function. By Walras' law, for each $S^p_s \in S^p$, the market-clearing equation for one asset is redundant. Hence, the resource feasibility condition can be written as

$$\sum_{h,s^h} \pi(s^h,S^p_s)\,\tilde{y}^h(s^h,s) = 0, \qquad \forall s\in S; \qquad (A.3)$$

[13] Note that the submatrix $R_{s'}$ of $R_{S^p_s}$ has $J_{S^p_s}$ columns corresponding the assets that are linearly independent in the subeconomy S^p_s; thus $R_{s'}$ does not have full column rank in general.

where $\hat{y}^h(s^h; s)$ is the vector obtained from $y^h(s^h; s)$ by deleting the last element.

The endogenous variables that describe a P-REE in the cell S^p_s are

$$\xi_{S^p_s} := [y^h(s^h; s);\ \lambda^h(s^h; s);\ \hat{p}(s)]_{h\in H; s^h\in S^h} \quad \in \mathbb{R}^{\bar{S} J_{S^p_s}} \times \mathbb{R}^{\bar{S}} \times \mathbb{R}^{J_{S^p_s} - 1};$$

where $\hat{p}(s)$ is the vector obtained by deleting the last element of $p(s)$. Note that $\xi_{S^p_s}$ is invariant with respect to s in the cell S^p_s. Thus

$$\xi := [\xi_{S^p_s}]_{S^p_s \in S^p} \quad \in \mathbb{R}^{\bar{S} J_{S^p}} \times \mathbb{R}^{\bar{S} S^p} \times \mathbb{R}^{J_{S^p} - S^p}$$

is a complete specification of the endogenous variables of the equations (A.1)–(A.3). We denote the equations (A.1) by $f(\xi; \omega) = 0$, and the equations (A.2)–(A.3) by $g(\xi; \omega) = 0$. Then, ξ is a P-REE if and only if

$$F(\xi; \omega) := \begin{pmatrix} f(\xi; \omega) \\ g(\xi; \omega) \end{pmatrix} = 0.$$

This system has $\bar{S} J_{S^p} + \bar{S} S^p + J_{S^p} - S^p$ equations, which is equal to the dimension of ξ. We denote the components of F corresponding to the cell S^p_s by $F_{S^p_s}(\xi_{S^p_s}; \omega_{S^p_s})$, where

$$\omega_{S^p_s} := [\omega^h(s^h; s'; t)]_{h\in H; s^h\in S^h; s'\in S^p_s; t\in T} \quad \in \mathbb{R}^{\bar{S} S^p_s T}_{++}.$$

The functions $f_{S^p_s}$ and $g_{S^p_s}$ are defined analogously.

We denote by $\mathrm{diag}_{a\in A}[z(a; b)]$ the (block) diagonal matrix with typical entry $z(a; b)$, where a varies across the diagonal entries, while b is fixed; $\mathrm{diag}_{a\in A}[z(b)]$ is the diagonal matrix with the term $z(b)$ repeated $\#A$ times. Finally, we define $y_{S^p_s} := \{y^h(s^h; s)\}_{h\in H; s^h\in S^h}$ and $y := [y_{S^p_s}]_{S^p_s\in S^p}$.

It is easily seen that $D_{\xi;\omega} F$ has a diagonal structure:

$$D_{\xi;\omega} F = \mathrm{diag}_{S^p_s \in S^p}[D_{\xi_{S^p_s};\omega_{S^p_s}} F_{S^p_s}]$$

Furthermore,

$$D_{\xi_{S^p_s};\omega_{S^p_s}} F_{S^p_s} = \begin{pmatrix} D_{\xi_{S^p_s}} f_{S^p_s} & D_{\omega_{S^p_s}} f_{S^p_s} \\ D_{\xi_{S^p_s}} g_{S^p_s} & 0 \end{pmatrix};$$

$$D_{\omega_{S^p_s}} f_{S^p_s} = \mathrm{diag}_{h; s^h}[\{\dots R^\top_{s'} D^2 U^h_{s^h; s'} \dots\}_{s'\in S^p_s}];$$

and

$$D_{\xi_{S^p_s}} g_{S^p_s} = \begin{pmatrix} \mathrm{diag}_{h;s^h}[p(s)^\top] & \vdots & 0 & \vdots & Y^\top_{S^p_s} \\ \cdots\cdots\cdots\cdots & & \vdots & & \cdots \\ \{\cdots \pi(s^h; S^p_s)\, \hat{\Gamma}^\top \cdots\}_{h;s^h} & \vdots & 0 & \vdots & 0 \end{pmatrix};$$

where

$$Y_{S^p_s} := [\cdots \hat{y}^h(s^h; s) \cdots]_{h;s^h}$$

is the $((J_{S^p_s} - 1) \times \bar{S})$ matrix of agents' portfolios in the cell S^p_s, and $\hat{\Gamma}$ is the $(J_{S^p_s} \times (J_{S^p_s} - 1))$ matrix defined by

$$\hat{\Gamma} := \begin{pmatrix} I_{(J_{S^p_s} - 1)} \\ \cdots \\ 0 \end{pmatrix}.$$

The following two results can be established using standard arguments (see, for instance, Citanna, Kajii, and Villanacci (1998)):

Fact 1. The matrices $D_{\omega_{S^p_s}} f_{S^p_s}$ and $D_{\xi_{S^p_s}} g_{S^p_s}$ have full row rank. Hence, so do $D_{\xi_{S^p_s};\omega_{S^p_s}} F_{S^p_s}$ and $D_{\xi;\omega} F$. Also, $D_\omega f$ and $D_y g$ have full row rank.

Fact 2. At a P-REE with price function p, the constraints (IR_{S^p}) and (IC_{S^p}) are generically satisfied with strict inequality, for every $h \in H$, $s^h; \hat{s}^h \in S^h$ $(s^h \neq \hat{s}^h)$, and $S^p_s \in S^p$.

In the proofs of Propositions 5.2, 6.1, and 6.3, we restrict endowments to be in the generic subset for which Fact 2 holds. The next lemma is a preliminary step to proving Proposition 5.2.

Lemma A.1. Consider a P-REE with portfolio allocation $\{y^h\}$. Generically, for every $S^p_s \in S^p$, $Y_{S^p_s}$ has full row rank $(J_{S^p_s} - 1)$, provided $\bar{S} \geq J_{S^p_s}$.

Proof. Fix a partition S^p and a cell S^p_s of this partition. We will show that generically, at a P-REE that induces the partition S^p, there is no solution δ to the equations $\delta^\top \hat{Y}_{S^p_s} = 0$ and $\delta \cdot \delta = 1$, where $\hat{Y}_{S^p_s}$ is obtained from $Y_{S^p_s}$ by deleting its first column. Consider the equation system

$$\Gamma_{S^p_s}(\xi_{S^p_s}; \delta; \omega_{S^p_s}) := \begin{pmatrix} F_{S^p_s}(\xi_{S^p_s}; \omega_{S^p_s}) \\ \delta^\top \hat{Y}_{S^p_s} \\ \delta \cdot \delta - 1 \end{pmatrix} = 0.$$

Its Jacobian is

$$D_{\xi_{S^p_s},\delta,\omega_{S^p_s}}\Gamma_{S^p_s} = \begin{pmatrix} D_{\xi_{S^p_s}} f_{S^p_s} & 0 & \vdots & D_{\omega_{S^p_s}} f_{S^p_s} \\ \cdots\cdots & \cdots & & \cdots\cdots \\ D_{\xi_{S^p_s}} g_{S^p_s} & 0 & \vdots & 0 \\ D_{\xi_{S^p_s}}(\delta^\top \hat\gamma_{S^p_s}) & \hat\gamma^\top_{S^p_s} & \vdots & 0 \\ 0 & 2\delta^\top & \vdots & 0 \end{pmatrix}. \tag{A.4}$$

Note that the matrix

$$D_{y_{S^p_s}} \begin{pmatrix} g_{S^p_s} \\ \delta^\top \hat\gamma_{S^p_s} \end{pmatrix} = \begin{pmatrix} \mathrm{diag}_{\,h;s^h}[p(s)^\top] \\ \{\ldots \pi(s^h;S^p_s)\,\mu^\top \ldots\}_{h;s^h} \\ \bigl(0 \quad \mathrm{diag}_{\,\{s^2,\ldots,s^{\bar S}\}}[(\delta^\top\ 0)]\bigr) \end{pmatrix}$$

is row-equivalent to

$$\begin{pmatrix} p(s)^\top & 0 & \ldots & 0 \\ \pi(s^1;S^p_s)\,\mu^\top & \pi(s^2;S^p_s)\,\mu^\top & \ldots & \pi(s^{\bar S};S^p_s)\,\mu^\top \\ 0 & p(s)^\top & \ldots & 0 \\ 0 & (\delta^\top\ 0) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & p(s)^\top \\ 0 & 0 & \ldots & (\delta^\top\ 0) \end{pmatrix}; \tag{A.5}$$

where we have relabelled the set $\times_h S^h$ as $\{s^1, s^2, \ldots, s^{\bar S}\}$. Since $p_J(s) = 1$ and, at any zero of $\Gamma_{S^p_s}$, $\delta \neq 0$, the matrix (A.5) has full row rank, and hence so does the lower left block of $D_{\xi_{S^p_s},\delta,\omega_{S^p_s}}\Gamma_{S^p_s}$ (as partitioned in (A.4)). Furthermore, by Fact 1, $D_{\omega_{S^p_s}} f_{S^p_s}$ has full row rank. Hence, the whole matrix $D_{\xi_{S^p_s},\delta,\omega_{S^p_s}}\Gamma_{S^p_s}$ has full row rank. By the transversality theorem, for $\omega_{S^p_s}$ in a generic subset $E_{S^p_s}$ of $\mathbb{R}^{\bar S S^p_s T}_{++}$, the same is true for $D_{\xi_{S^p_s},\delta}\Gamma_{S^p_s}$ at all zeros of $\Gamma_{S^p_s}(\xi_{S^p_s}, \delta, \omega_{S^p_s})$. But this system has more independent equations, $(\bar S J_{S^p_s} + \bar S + J_{S^p_s} - 1) + \bar S$, than unknowns, $(\bar S J_{S^p_s} + \bar S + J_{S^p_s} - 1) + J_{S^p_s} - 1$, since, by hypothesis, $\bar S > J_{S^p_s} - 1$. So $\Gamma_{S^p_s}(\xi_{S^p_s}, \delta, \omega_{S^p_s}) = 0$ has no solution, for any $\omega_{S^p_s} \in E_{S^p_s}$.

Since the Cartesian product of generic sets is generic in the product space, it follows that for a generic subset $E := \times_{S^p_s \in S^p} E_{S^p_s}$ of $\mathbb{R}^{\bar S S T}_{++}$ there is no solution to $\Gamma_{S^p_s}(\xi_{S^p_s}, \delta, \omega_{S^p_s}) = 0$ for any $S^p_s \in S^p$. This establishes the result. ∎

Proof of Proposition 5.2. We restrict endowments to be in the generic subset for which the rank condition of Lemma A.1 holds. An (RF_{S^p}, M_{S^p}, IR_{S^p}, IC_{S^p}, BC_{S^p})-constrained interim efficient allocation $\{y^h\}$ solves the program

$$\max \sum_{h;s^h} \mu^h(s^h) \sum_{s \in S} U^h_{s^h;s}(c^h_{s^h;s})$$

subject to the constraints (RF_{S^p}), (M_{S^p}), (IR_{S^p}), (IC_{S^p}) and (BC_{S^p}), for some strictly positive weights $\mu := \{\mu^h(s^h)\}_{h;s^h}$. By Fact 2, at a P-REE, the constraints (IR_{S^p}) and (IC_{S^p}) are not binding. Therefore, necessary conditions for a P-REE allocation to be (RF_{S^p}, M_{S^p}, IR_{S^p}, IC_{S^p}, BC_{S^p})-constrained interim efficient are that there exist strictly positive weights μ and a function q with $S^q = S^p$ such that

$$\mu^h(s^h) \sum_{s' \in S^p_s} R^{\top}_{s'} DU^h_{s^h;s'} = \nu(s)\,\pi(s^h; S^p_s) + \gamma^h(s^h;s)\,q(s); \qquad \forall h \in H; s^h \in S^h; s \in S; \tag{A.6}$$

$$q(s) \cdot y^h(s^h;s) = 0; \qquad \forall h \in H; s^h \in S^h; s \in S; \tag{A.7}$$

$$\sum_{h;s^h} \gamma^h(s^h;s)\,\hat{y}^h(s^h;s) = 0; \qquad \forall s \in S; \tag{A.8}$$

for some p-measurable functions $\nu : S \to \bigcup_{S^p_s \in S^p} \mathbb{R}^{J_{S^p_s}}$ (with $\nu(s) \in \mathbb{R}^{J_{S^p_s}}$ for every s), and $\gamma^h(s^h;\cdot) : S \to \mathbb{R}$.

From (A.2), (A.7), and the fact that $Y_{S^p_s}$ has full row rank, we get $q = p$. Then it follows from (A.1) and (A.6) that

$$\mu^h(s^h)\lambda^h(s^h;s)p(s) = \nu(s)\,\pi(s^h; S^p_s) + \gamma^h(s^h;s)\,p(s); \qquad \forall h \in H; s^h \in S^h; s \in S.$$

Since, for every s, $p_J(s) = 1$ and, from Walras' law, $\nu_J(s) = 0$; we obtain

$$\mu^h(s^h)\lambda^h(s^h;s) = \gamma^h(s^h;s). \tag{A.9}$$

Multiplying both sides of (A.9) by $\hat{y}^h(s^h;s)$, summing over $(h;s^h)$, and using (A.8), we get:

$$\phi(\xi;\mu;\omega) := \sum_{h;s^h} \mu^h(s^h)\lambda^h(s^h;s)\hat{y}^h(s^h;s) = 0 \qquad \forall s \in S. \tag{A.10}$$

Since $\lambda^h(s^h;\cdot)$ and $\hat{y}^h(s^h;\cdot)$ are p-measurable, (A.10) consists of $(J_{S^p} - S^p)$ distinct equations.

If a P-REE is (RF_{S^p}, M_{S^p}, IR_{S^p}, IC_{S^p}, BC_{S^p})-constrained interim efficient, it follows from the foregoing analysis that

$$\Phi(\xi;\mu;\omega) := \begin{pmatrix} F(\xi;\omega) \\ \phi(\xi;\mu;\omega) \end{pmatrix} = 0.$$

The Jacobian of Φ, $D_{\xi,\mu,\omega}\Phi$, is row/column-equivalent to the block triangular matrix

$$\begin{pmatrix} D_{y,\hat p} f & D_{\lambda,\mu} f & D_{\omega} f \\ D_{y,\hat p}\phi & D_{\lambda,\mu}\phi & 0 \\ D_{y,\hat p} g & 0 & 0 \end{pmatrix};$$

where the subscripts $\hat p$ and λ are used to denote derivatives with respect to $\{\hat p(s)\}_{S^p_s \in S^p}$ and $\{\lambda^h(s^h;s)\}_{s^h \in S^h; S^p_s \in S^p}$ respectively. Now $D_\lambda \phi = \mathrm{diag}_{S^p_s \in S^p}[\prod_{S^p_s} \mathrm{diag}_{h;s^h}(\mu^h(s^h))]$, which has full row rank. From Fact 1, $D_\omega f$ and $D_y g$ also have full row rank. Therefore, $D_{\xi,\mu,\omega}\Phi$ has full row rank and, by the transversality theorem, for a generic subset of endowments so does $D_{\xi,\mu}\Phi$, at every solution of $\Phi(\xi;\mu;\omega) = 0$: But then this set of solutions must be empty, since $J_{S^p} - S^p > \bar S - 1$ implies that the system $\Phi(\xi;\mu;\omega) = 0$ has more independent equations than unknowns. (Note that we can normalize one of the weights μ to be one.)

The argument for generic ex ante inefficiency is analogous, the only difference being that the weights $\{\mu^h\}$ are invariant with respect to s^h. ∎

Proof of Proposition 6.1. Consider a P-REE with price function p and portfolio allocation $\{y^h\}$. If it is ($\overline{RF}$, IR_{S^p}, IC_{S^p}, BC_p)-constrained ex post efficient, then in particular it is ex post efficient in the subeconomy S^p_s and, therefore, $\Delta y := \{\Delta y^h(s^h;s')\}_{h\in H; s^h \in S^h; s' \in S^p_s} = 0$ is a solution to the following program, for some strictly positive weights $\{\mu^h(s^h;s')\}_{h\in H; s^h\in S^h; s'\in S^p_s}$. (Note that Fact 2 allows us to ignore (IR_{S^p}) and (IC_{S^p}).)

$$\max_{\Delta y} \sum_{h;s^h} \sum_{s' \in S^p_s} \mu^h(s^h;s') U^h_{s^h;s'}(\bar c^h_{s^h;s'})$$

subject to

$$\bar c^h_{s^h;s'} = [\omega^h(s^h;s';t) + r(s';t)\cdot(y^h(s^h;s) + \Delta y^h(s^h;s'))]_{t\in T}; \qquad \forall h \in H; s^h \in S^h; s' \in S^p_s;$$

$$\sum_{h;s^h} \pi(s^h|s')\,\Delta y^h(s^h;s') = 0; \qquad \forall s' \in S^p_s;$$

$$p(s)\cdot\Delta y^h(s^h;s') = 0; \qquad \forall h \in H; s^h \in S^h; s' \in S^p_s:$$

The first order conditions, evaluated at $\Delta y = 0$, give us

$$\mu^h(s^h;s') R^\top_{s'} DU^h_{s^h;s'} = \nu(s')\,\pi(s^h|s') + \gamma^h(s^h;s')\,p(s); \qquad \forall h \in H; s^h \in S^h; s' \in S^p_s;$$

for some functions $\nu : S^p_s \to \mathbb{R}^{J_{S^p_s}}$, and $\gamma^h : S^h \times S^p_s \to \mathbb{R}$: In particular, this implies that, for any given $s' \in S^p_s$, the marginal utility vectors $R^\top_{s'} DU^h_{s^h;s'}$ lie in the two-dimensional subspace of $R^{J_{S^p_s}}$ spanned by $\nu(s')$ and $p(s)$, for every $(h;s^h)$.

Now consider three pairs $(h; s^h)$, identifying three agent-types, indexed by $\{h_1; h_2; h_3\}$, and define

$$\psi(\xi; \eta; \omega) := \eta_1 R_{\hat{s}}^{\top} DU_{\hat{s}}^{h_1} + \eta_2 R_{\hat{s}}^{\top} DU_{\hat{s}}^{h_2} + \eta_3 R_{\hat{s}}^{\top} DU_{\hat{s}}^{h_3} = 0; \qquad \eta \in \mathbb{R}^3;$$

for some $\hat{s} \in S_s^p$ such that rank $(R_{S_s^p;\hat{s}}) \geq J_{S_s^p} + 3$. Then, a necessary condition for the P-REE allocation to be $(\overline{RF}, IR_{S^p}, IC_{S^p}, BC_p)$-constrained ex post efficient is that

$$\Psi(\xi; \eta; \omega) := \begin{pmatrix} F(\xi; \omega) \\ \psi(\xi; \eta; \omega) \\ \eta \cdot \eta - 1 \end{pmatrix} = 0;$$

for some $\eta \in \mathbb{R}^3$. The Jacobian, $D_{\xi;\eta;\omega}\Psi$, is row-equivalent to

$$\begin{pmatrix} D_\xi f & 0 & \vdots & D_\omega f \\ D_\xi \psi & D_\eta \psi & \vdots & D_\omega \psi \\ \cdots & \cdots & & \cdots \\ 0 & 2\eta^{\top} & \vdots & 0 \\ D_\xi g & 0 & \vdots & 0 \end{pmatrix};$$

and $D_\omega \begin{bmatrix} f \\ \psi \end{bmatrix}$ is given, up to a permutation of columns, by

$$\begin{pmatrix} (\ldots R_{s'}^{\top} D^2 U_{s'}^{h_1} \ldots)_{s' \in S_s^p} & \vdots & 0 & \vdots & 0 & \vdots & 0 \\ 0 & \vdots & (\ldots R_{s'}^{\top} D^2 U_{s'}^{h_2} \ldots)_{s' \in S_s^p} & \vdots & 0 & \vdots & 0 \\ 0 & \vdots & 0 & \vdots & (\ldots R_{s'}^{\top} D^2 U_{s'}^{h_3} \ldots)_{s' \in S_s^p} & \vdots & 0 \\ 0 & \vdots & 0 & \vdots & 0 & \vdots & * \\ 0 \ldots \eta_1 R_{\hat{s}}^{\top} D^2 U_{\hat{s}}^{h_1} \ldots 0 & \vdots & 0 \ldots \eta_2 R_{\hat{s}}^{\top} D^2 U_{\hat{s}}^{h_2} \ldots 0 & \vdots & 0 \ldots \eta_3 R_{\hat{s}}^{\top} D^2 U_{\hat{s}}^{h_3} \ldots 0 & \vdots & 0 \end{pmatrix}:$$

At any zero of Ψ, η is nonzero; without loss of generality, let η_1 be nonzero. Then

$$\begin{pmatrix} (\ldots R_{s'}^{\top} D^2 U_{s'}^{h_1} \ldots)_{s' \in S_s^p} \\ 0 \ldots \eta_1 R_{\hat{s}}^{\top} D^2 U_{\hat{s}}^{h_1} \ldots 0 \end{pmatrix}$$

has the same row rank as $R_{S_s^p;\hat{s}}^{\top}$, which is by assumption at least $J_{S_s^p} + 3$. Therefore,

$$\text{row rank} \left[D_\omega \begin{pmatrix} f \\ \psi \end{pmatrix} \right] \geq \text{row rank}(D_\omega f) + 3:$$

Since $D_\xi g$ has full row rank (by Fact 1), at any zero of Ψ,

$$\text{row rank}(D_{\xi;\eta;\omega}\Psi) \geq \text{row rank}(D_{\xi;\omega} F) + 4:$$

In other words, relative to the equilibrium equations, the equation system $\Psi = 0$ has three additional unknowns ($\eta \in \mathbb{R}^3$) and at least four additional (locally) independent equations. Generically, therefore, the system has no solution. ∎

Proof of Proposition 6.3. Without loss of generality, assume that $n = S$. Consider a P-REE with price function p and portfolio allocation $\{y^h\}$. Let $\Delta y := \{\Delta y^h(s^h,\sigma_j)\}_{h,s^h,j}$, and

$$W := \sum_{h,s^h,t,i,j} \mu^h(s^h,s_i)\,\pi(s^h|s_i)\,\pi_{ij}\,\pi(t)\,u^h[\omega^h(s^h,s_i,t)+r(s_i,t)\cdot y^h(s^h,s_i)+r(s_i,t)\cdot\Delta y^h(s^h,\sigma_j)].$$

The P-REE allocation is ($\overline{RF}_\sigma$, IR_σ, IC_σ, $BC_{p,\sigma}$)-constrained ex post efficient only if

$$\pi_{ij} = \pi(s_i)\pi(s_j) \quad \forall i,j \qquad \text{and} \qquad \Delta y = 0 \tag{A.11}$$

solves

$$\max_{\Pi,\Delta y} W \qquad \text{subject to} \qquad (\overline{RF}_\sigma);\, VC_{p,\sigma};\ \text{and (1);} \tag{A.12}$$

for some strictly positive weights $\{\mu^h(s^h,s_i)\}_{h,s^h,i}$. Fact 2 allows us to ignore the (IR_{S^p}) and (IC_{S^p}) constraints.

It can easily be verified that, if the weights $\{\mu^h\}$ are chosen to be invariant with respect to s_i, the first order conditions for (A.12) are satisfied at (A.11). The second order necessary condition is that the Hessian D^2W (evaluated at (A.11)) be negative semidefinite when restricted to the directions that satisfy the constraints of (A.12). The directions for π_{ij} and $\Delta y^h(s^h,\sigma_j)$ are respectively denoted by $\alpha_{ij} \in \mathbb{R}$ and $\beta^h(s^h,\sigma_j) \in \mathbb{R}^J$. Let

$$\alpha := \begin{pmatrix} \vdots \\ \{\alpha_{ij}\}_j \\ \vdots \end{pmatrix}_i ; \qquad \text{and} \qquad \beta := \begin{pmatrix} \vdots \\ \{\beta^h(s^h,\sigma_j)\}_{h,s^h} \\ \vdots \end{pmatrix}_j .$$

The directions satisfying the constraints of (A.12) are solutions to

$$\zeta_1(\alpha) := \begin{pmatrix} 1\cdots1 & 0\cdots0 & \cdots & 0\cdots0 & 0\cdots0 \\ 0\cdots0 & 1\cdots1 & \cdots & 0\cdots0 & 0\cdots0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0\cdots0 & 0\cdots0 & \cdots & 1\cdots1 & 0\cdots0 \\ I_S & I_S & \cdots & I_S & I_S \end{pmatrix} \alpha = 0; \tag{A.13}$$

and

$$\zeta_2(\beta) := \begin{pmatrix} \operatorname{diag}_j \begin{bmatrix} [\ldots \pi(s^h \mid s_1)\, \hat{\Gamma}^\top \ldots]_{h;s^h} \\ \vdots \\ [\ldots \pi(s^h \mid s_S)\, \hat{\Gamma}^\top \ldots]_{h;s^h} \end{bmatrix} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \operatorname{diag}_{h;s^h;j}(p^\top) \end{pmatrix} \beta = 0; \tag{A.14}$$

where we have suppressed the dependence of p on s, since p is constant. Thus the second order condition for (A.12) can be stated as follows:

$$(\alpha^\top \quad \beta^\top)\, D^2 W \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \le 0 \tag{A.15}$$

for all α and β satisfying

$$\zeta(\alpha; \beta) := \begin{pmatrix} \zeta_1(\alpha) \\ \zeta_2(\beta) \end{pmatrix} = 0.$$

The set of zeros of ζ_1 and ζ_2 are subspaces of dimension $S(S - 2) + 1$ and $S(J - 1)(\bar{S} - S)$ respectively. Therefore, the set of zeros of ζ is nonempty. We assume for simplicity that the equations in (A.14) are linearly independent (if not, our argument goes through by deleting redundant equations).

Note that

$$D^2 W = \begin{pmatrix} 0 & D^2_{\Pi;\Delta y} W \\ (D^2_{\Pi;\Delta y} W)^\top & D^2_{\Delta y;\Delta y} W \end{pmatrix}.$$

Therefore,

$$(\alpha^\top \quad \beta^\top)\, D^2 W \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 2\alpha^\top (D^2_{\Pi;\Delta y} W)\beta + \beta^\top (D^2_{\Delta y;\Delta y} W)\beta. \tag{A.16}$$

If the first term of (A.16) is nonzero for some $\alpha; \beta$, this term can be made positive and arbitrarily large by appropriately rescaling α (which is always possible by (A.13)), without affecting the second term of (A.16). This will result in a violation of (A.15). Hence, for (A.11) to be a solution to (A.12), we must have

$$v(\alpha; \beta) := \alpha^\top (D^2_{\Pi;\Delta y} W)\beta = 0$$

for all $\alpha; \beta$ satisfying $\zeta(\alpha; \beta) = 0$ (with $D^2_{\Pi;\Delta y} W$ evaluated at (A.11)). Equivalently, the set of solutions to

$$\Upsilon(\alpha; \beta) := \begin{pmatrix} v(\alpha; \beta) \\ \zeta(\alpha; \beta) \end{pmatrix} = 0$$

and to $\zeta(\alpha;\beta) = 0$ must coincide. This implies that $D_{\alpha;\beta}\Upsilon$ does not have full row rank at any zero of Υ. (Suppose not, i.e. suppose there is a zero of Υ, $(\alpha^*;\beta^*)$, at which $D_{\alpha;\beta}\Upsilon$ has full row rank. Then, by the local submersion theorem, $D_{\alpha;\beta}\Upsilon$ has full row rank on a neighborhood N of $(\alpha^*;\beta^*)$. Let Υ_N and ζ_N be the restriction to N of Υ and ζ respectively. Then, zero is a regular value of Υ_N and ζ_N. By the preimage theorem, the set of solutions to $\Upsilon_N(\alpha;\beta) = 0$ is either empty or is a manifold of dimension one less than the manifold of the set of solutions to $\zeta_N(\alpha;\beta) = 0$. In other words, Υ and ζ do not have the same zeros.)

Straightforward computations yield

$$v(\alpha;\beta) = \sum_{h;s^h;i;j} \frac{\mu^h(s^h;s_i)}{\pi(s_i)}\alpha_{ij}\beta^h(s^h;\sigma_j)^\top (R^\top_{s_i} DU^h_{s^h;s_i});$$

and

$$D_\alpha v = \left(\cdots \sum_{h;s^h} \frac{\mu^h(s^h;s_i)}{\pi(s_i)}\beta^h(s^h;\sigma_j)^\top (R^\top_{s_i} DU^h_{s^h;s_i})\cdots\right)_{i;j}:$$

From the foregoing argument, $D_{\alpha;\beta}v$ lies in the row space of $D_{\alpha;\beta}\zeta$, at every zero of ζ. In particular, $D_\alpha v$ is spanned by the rows of $D_\alpha\zeta$, i.e. there exist $a \in \mathbb{R}^{S-1}\times\{0\}$, and $b \in \mathbb{R}^S$, such that

$$\sum_{h;s^h} \frac{\mu^h(s^h;s_i)}{\pi(s_i)}\beta^h(s^h;\sigma_j)^\top (R^\top_{s_i} DU^h_{s^h;s_i}) = a_i + b_j; \qquad \forall i;j: \qquad (A.17)$$

Using (A.17) to substitute for a_i and b_j, we obtain:

$$\sum_{h;s^h} [\beta^h(s^h;\sigma_j) - \beta^h(s^h;\sigma_S)]^\top \left[\frac{\mu^h(s^h;s_i)}{\pi(s_i)}(R^\top_{s_i} DU^h_{s^h;s_i}) - \frac{\mu^h(s^h;s_S)}{\pi(s_S)}(R^\top_{s_S} DU^h_{s^h;s_S})\right] = 0;$$

$$\forall i;j = 1;\ldots;S-1: \qquad (A.18)$$

This condition must hold for all β satisfying (A.14). Since (A.14) and (A.18) are both linear in β, the coefficients of β in (A.18) are linearly dependent on those in (A.14), i.e. there exist $c_i \in \mathbb{R}^{S-1}\times\{0\}$, and $d^h(s^h) \in \mathbb{R}$, such that

$$\frac{\mu^h(s^h;s_m)}{\pi(s_m)}(R^\top_{s_m} DU^h_{s^h;s_m}) - \frac{\mu^h(s^h;s_S)}{\pi(s_S)}(R^\top_{s_S} DU^h_{s^h;s_S}) = \sum_{i=1}^{S-1} \pi(s^h|s_i)c_i + d^h(s^h)p; \qquad \forall h;s^h:$$

(A:19)

This implies that the vectors on the left hand side of (A.19) lie in an $(S+1)$-dimensional subspace of R^J (the one spanned by $\{c_i\}$ and p), for every $(h;s^h)$. By an immediate reformulation of the argument in the proof of Proposition 6.1, we can show that, generically, the vectors $\{R^\top_{s_m}U^h_{s^h;s_m}; R^\top_{s_S}U^h_{s^h;s_S}\}$ are linearly independent across $S+2$ agent-types. Therefore, generically, condition (A.19) cannot hold. ∎

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