Capital versus Labor Income Taxation with Heterogeneous Agents

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Abstract

We investigate the welfare implications of eliminating a proportional capital income tax for a model economy in which heterogeneous households face labor income risk and trade only one asset. Labor taxes rises at the time of the reform to maintain long run budget balance. Our stochastic process for labor earnings is consistent with empirical estimates of earnings risk, and also implies a distribution of asset holdings across households closely resembling that in the United States.

We find that a vast majority of households prefers the *status quo* to the tax reform. This finding is interesting in light of the fact that our reform would be optimal if we abstracted from heterogeneity and assumed a representative agent. Initial household productivity and initial household wealth are independently important in determining a particular household's expected gain or loss, in constrast to a complete markets economy in which only the ratio of asset to labor income matters.

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1. Introduction

This paper explores the relation between what is taxed and who is taxed. In the representative agent framework, a common finding is that the optimal tax program involves zero taxation of capital income in the long run (see Chamley 1986, Judd 1985, or the recent paper by Atkeson, Chari and Kehoe 1999). However, representative agent models abstract from the fact that in practice an increased reliance on labor taxation is likely to be regressive, since low income households receive a large fraction of their income from labor relative to the fraction they receive from asset income.[1] Thus reducing the tax rate on capital income will increase the poor's share of the tax burden initially, even though in the long run all households will benefit from the higher pre-tax income associated with an increase in the capital stock.

The goal of this paper is to quantitatively assess the distributional implications of tax reform within a calibrated model of the U.S. economy. The model economy is populated by a large number of infinitely-lived households who face uninsurable idiosyncratic labor income risk. Households can achieve a degree of consumption smoothing by adjusting precautionary holdings of a single asset, provided they do not violate an exogenous borrowing limit. Because productivity shocks are uncorrelated across households, the model generates endogenous distributions of income and wealth.

There is a government which finances constant expenditure by levying proportional taxes on labor and asset income, and by issuing debt. The tax reforms we consider are permanent unanticipated changes in the capital income tax rate. We require these reforms to be sustainable in the sense that following a tax change, the time path for government debt is non-explosive. Thus at the same time that the capital income tax rate is changed, the labor income tax rate is also adjusted to ensure that the economy eventually converges to a steady state in which government debt is constant and finite.

We are interested in the relative quantitative importance of increased productive efficiency versus redistribution between households receiving a relatively large fraction of income from assets (who we will refer to as the wealth-rich) and households who receive a relatively large fraction of income from labor (the wealth-poor). We therefore calibrate the household earnings process to satisfy two criteria: (i) the persistence and variance of earnings shocks are consistent with estimates from the Panel Study of Income Dynamics, and (ii) the wealth distribution generated endogenously by the model closely resembles that observed

[1] Diaz Gimenez, Quadrini and Rios-Rull 1997 give a breakdown of sources of income by income level for U.S. households in the 1992 Survey of Consumer Finances.

in the United States.

In order to understand the importance of our asset market structure for the effects of tax changes, we compare the predictions of the incomplete markets economy to those of an economy in which markets are complete, but on which we impose the initial steady state wealth distribution from the incomplete markets economy. There are several reasons why we expect the welfare effects of tax changes to be sensitive to the assumed market structure.

First, note that because different households in the incomplete markets economy experience different paths for earnings, we will observe mobility within the income and wealth distributions.[2] For example, a household that experiences an increase in labor productivity will immediately jump further up the income distribution, and by increasing savings typically move upwards in the wealth distribution. If we assume that the process for household productivity is stationary, then households' expected earnings and wealth will converge to the economy-wide average in the long run. Thus households who initially depend heavily on labor income can expect to receive a larger fraction of income from wealth in the future, and may therefore favor reducing capital income taxation even if this would increase their total tax bill in the short run. By contrast, when markets are complete, the ranking of households by wealth is fixed through time.

A second consideration is that when markets are incomplete, the aggregate capital stock in the pre-reform steady state is enlarged by precautionary savings. Households are willing to hold assets despite an equilibrium return that is lower than the rate of time preference because by adjusting asset holdings they can achieve a time path for consumption that is smoother than that for labor income (see Aiyagari 1994). The welfare implications of this phenomenon are unclear. On the one hand, a higher pre-reform capital stock reduces the potential efficiency gains from reducing capital income taxes. On the other hand, the associated increase in the capital stock should both improve households' ability to smooth consumption, and at the same time reduce the idiosyncratic uncertainty households face by increasing the average ratio of post-tax asset to labor income.

Related Literature

In a seminal paper, Judd (1985) studies tax reforms for an economy in which households differ in their initial capital holdings, under the assumption that asset markets are complete. He shows that agents with below average wealth will desire an immediate permanent capital income tax increase if the current tax rate is sufficiently low. In a calibrated model with two types of household, Garcia-Mila, Marcet and Ventura (1995) examine the trade-off when reducing capital taxes

[2]For data on wealth and earnings mobility in the U.S., see Dias-Gimenez et. al. 1997.

between the efficiency gains in terms of increased production, and the losses experienced by the wealth-poor type as a result of higher labor taxation. Their main finding is that the redistribution effect typically dominates, and thus that a reduction in capital taxes leaves the wealth-poor type worse off. However, they also assume complete markets, so it is not clear that their conclusions will still obtain in an economy which allows for mobility within the income and wealth distributions.

Using an overlapping generations (OLG) framework, Auerbach and Kotlikoff (1987) find that switching from a general income tax to a labor income tax mainly benefits the current old (who receive a high fraction of income from wealth) while the policy imposes large welfare costs on the current young generations (whose income consists mainly of labor earnings). The wealth distribution in the Auerbach and Kotlikoff model is endogenous, as in our incomplete markets economy, but savings behavior is driven by life-cycle rather than precautionary motives.

Conesa and Krueger (1999) analyze social security reform in an OLG economy in which households face uninsurable shocks that lead to heterogeneity within age cohorts, as well as across generations. Households contemplate a switch from a pay-as-you-go social security system to a fully funded one, which involves reducing labor income taxation (the pay-roll tax) and at the same time confiscating all claims to the social security system. Although the reform does involve intra-generational redistribution, Conesa and Krueger's results are driven primarily by inter-generational redistribution: most of the losers are middle aged and elderly. We choose to abstract from inter-generational issues for two reasons: they are likely to be less important for the issue of factor taxation than for social security reform, and there is a large literature on optimal factor taxation in the infinite horizon setting that is a useful reference point for thinking about the welfare implications of tax reform.

Aiyagari (1995) defines an optimal tax problem for an incomplete markets economy similar to ours in which he does not constrain tax rates to be time-invariant following the initial reform. He argues that if the optimal tax program converges, then the tax rate of capital income in that steady state is positive.[3] While the reforms we consider are unlikely to be solutions to the unconstrained optimal tax problem, our approach has the advantage that it enables us to explicitly characterize both transition following the reform and the steady state to which the economy eventually converges.

Findings

[3] Judd (1985) in an economy with heterogeneous agents but complete markets finds the optimal long run tax rate on capital income to be zero.

The reform we primarily focus on involves moving from the current calibrated U.S. capital income tax rate (39.7 percent) to a capital income tax rate of zero. Eliminating capital income taxation is a natural benchmark, since our assumption that labor is inelastically supplied means that this policy is in the class of optimal tax reforms for a representative agent economy. We compute the expected welfare gain for the representative agent in a complete markets economy, and find it to be equivalent to a permanent 1.07 percent increase in consumption. This is a large gain relative, for example, to previous estimates of the likely benefits of eliminating business cycles.

When households differ, however, the welfare effects of the same policy change vary greatly depending on initial household wealth and productivity. None of the tax changes we consider are Pareto improving.[4] Moreover the majority of households lose from eliminating capital income taxation: 73 percent of households lose in the incomplete markets economy, while 72 percent do so in the complete markets economy. The average change in expected utility is equivalent to a permanent 0.88 percent fall in consumption in the incomplete markets economy. Households with higher initial wealth are more likely to be winners, and on average gain more, in both economies.

The rest of the paper is organized as follows. Section 2 outlines the economic environment. Section 3 presents the results, and Section 4 concludes.

2. The Models

We consider two model economies: one in which households have access to a single savings instrument and face a no-borrowing constraint, and a second in which households can trade a complete set of state contingent claims.

Both economies are populated by a large number of *ex ante* identical and infinitely lived households. Households supply labor inelastically and maximize the expected discounted utility from consumption. In aggregate, household savings decisions determine the evolution of the aggregate capital stock, which in turn determines aggregate output and the return to saving. There is a government which finances constant government spending by issuing one period debt and levying taxes. From the households' perspective, debt and capital are perfect substitutes since the one period return to both is risk free, and there are no transaction costs.

[4] This is not the case in Chamley (1998), who considers tax changes pre-announced far in advance, so that a household's expected position within the income / wealth distribution at the time of the tax change is independent of its current position. Thus Chamley is able to characterize tax reforms that leave all households better off. Note that households in our economy expect to be average roughly two hundred years into the future (see figure 5).

An equilibrium condition is that aggregate asset holdings at each date must equal the sum of the capital stock and the stock of outstanding government debt. To focus on the effects of tax changes, we abstract from other sources of aggregate risk.

We assume that households face idiosyncratic labor productivity shocks. In the incomplete markets model economy, markets which in principle could allow complete insurance against this risk do not exist. Instead there is a single risk-free savings instrument which enables households to partially self-insure by accumulating precautionary asset holdings, as in Aiyagari (1994) and Aiyagari and McGrattan (1998). An important assumption is that no borrowing is permitted. This limits the ability of low-wealth households to smooth consumption in the face of falls in their disposable income.

In the complete markets economy, by contrast, households can perfectly insure against idiosyncratic productivity shocks, and choose complete insurance in equilibrium. Thus we can think of the complete markets economy as a world in which households make consumption and savings decisions as though household productivity were constant through time and identical across households. Since the momentary utility function is such that the Engel curve is linear in lifetime wealth, the evolution of aggregate variables in equilibrium does not depend on the distribution of wealth at the date of a tax change (see Chatterjee 1994). However, since we assume that households cannot insure against the (zero probability) event of a tax reform, the welfare implications of tax reform in our complete markets economy will be sensitive to the shape of the initial wealth distribution.

We now give a more formal description of the incomplete markets economy. The complete markets economy may be viewed as a special case of the incomplete markets economy in which all productivity levels are the same, and this economy-wide household productivity level is normalized to 1.

The environment

Each infinitely-lived household supplies $\overline{n}$ labor hours per period. Household i's productivity at date t is denoted by e_{it} and takes one of three possible values, $e_{it} \in E = \{e_l, e_m, e_h\}$. Productivity is assumed to be i.i.d. across agents and to evolve according to a first-order Markov process. The 3×3 transition probability matrix for productivity is denoted π_e. Let $e^t = \{e_0, e_1, ..., e_t\} \in E^t \subset \prod_t E$ denote a history of productivity shocks up to date t, and let $\rho(e^t|e_0)$ denote the date 0 conditional probability that a household assigns to history e^t given e_0.

Let $c_i(e^t)$ denote the consumption of household i given productivity history

e^t. Household i's expected discounted lifetime utility is given by

$$\sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t \rho(e^t|e_0) u(c_i(e^t)) \tag{2.1}$$

where the momentary utility function is

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

and $\gamma > 0$ is the coefficient of relative risk aversion.

Let $a_{it-1} \in \mathcal{A} = \mathrm{R}_+$ denote household i's asset holdings at the start of period t, corresponding to a savings decision taken at $t-1$. The real return at t to one unit of the asset purchased at $t-1$ is r_{t-1}. The real return to supplying one unit of effective labor at date t is w_t. Asset and labor income are taxed proportionally at constant rates τ^k and τ^n respectively. The budget constraint of household i is given by

$$c_i(e^t) + a_{it} = \left[1 + \left(1 - \tau^k\right) r_{t-1}\right] a_{t-1} + (1 - \tau^n) w_{t-1} e_{it} \overline{n}. \tag{2.2}$$

Let the distribution of households across productivity and asset holdings at date t be described by $\psi_t(\cdot)$, a probability measure defined on $\mathcal{S}$, the set of all subsets of $E \times \mathcal{A}$. Aggregate effective per capita labor supply is constant across dates and states (given a law of large numbers) and equal to $\overline{n}$:

$$N_t = \int e_{it} \overline{n} d\psi_t(\cdot) = \overline{n}.$$

Aggregate asset holdings at the start of period t are denoted A_{t-1} where

$$A_{t-1} = \int a_{it-1} \psi_t(\cdot).$$

Real per capita government spending is constant and equal to G. Government debt issued at date t is denoted B_t and is assumed to be risk-free; the government guarantees the one period real return between t and $t+1$ at the start of period t. Debt evolves according to

$$B_t + \tau^k r_{t-1} A_{t-1} + \tau^n w_t \overline{n} = \left[1 + \left(1 - \tau^k\right) r_{t-1}\right] B_{t-1} + G.$$

Aggregate per capita output at t, Y_t, is produced according to a Cobb-Douglas technology from aggregate per capita capital at date t, K_{t-1}, and aggregate per capita labor supply:

$$Y_t = K_{t-1}^{\alpha} \overline{n}^{1-\alpha},$$

where $\alpha \in [0, 1]$.

Output can be transformed into future capital, consumption and government spending according to

$$C_t + G + K_t - (1 - \delta) K_{t-1} = Y_t$$

where $\delta \in [0, 1]$ is the rate of depreciation.

Product and factor markets are assumed to be competitive. This and the absence of aggregate productivity shocks implies a certain one period real return to saving in the form of capital.[5] Since the real one period return to debt is also known in advance (the government guarantees it), in equilibrium the two assets must pay the same real return. This is why it is not necessary to specify the division between capital and bonds in an individual's portfolio.

The households' problem

In the pre-tax reform steady state, households take as given constant prices $\overline{r}$ and $\overline{w}$, and constant tax rates $\overline{\tau^k}$ and $\overline{\tau^n}$. Otherwise the steady state household problem is identical to the post reform problem, which we now describe.

Let $t = 0$ denote the date of the tax change. At the start of period 0, a pair of new permanent tax rates τ^k and τ^n is announced and households re-optimize. This involves choosing $c_i(e^t)$ $\forall t$ and $\forall e^t \in E^t$ where $c_i(e^t)$ solves the following problem.

Maximize 2.1 such that

1. e_{i0} and a_{i-1} are given.

2. $\forall t$ and $\forall e^t \in E^t$ the household's budget constraint (eq. 2.2) is satisfied, consumption is non-negative, and the borrowing constraint for asset holdings is satisfied, $a_{it} \in \mathcal{A}$.

3. Prices are given by sequences $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$, and tax rates are τ^k and τ^n.

[5] Of course, prior to the tax reform households expectations over future after-tax interest rates are incorrect; we make the standard assumption in this type of exercise that the reform is assumed to be a zero probability event.

4. $\forall t$ and $\forall e^t \in E^t$ the conditional probability of household productivity history $\rho(e^t|e_0)$ is consistent with the transition probability matrix π_e. For example, suppose e^t is any history of length t such that $e_t = e_i$. Then if e^{t+1} is the history e^t followed by $e_{t+1} = e_j$

$$\rho(e^{t+1}|e_0) = \rho(e^t|e_0) \times \pi_e(i,j) \qquad \forall e_j \in E.$$

Equilibrium

A post-reform equilibrium for this economy is a pair of constant tax rates τ^k and τ^n and sequences of pre-tax prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$ such that when all households take prices and taxes as given and solve their maximization problems, the markets for capital, labor and output clear, and government debt is stationary. A formal definition of equilibrium is given in appendix A.2.

2.1. Parameterization

The model period is one year. All parameter values used are reported in yearly terms in table 1. The parameters relating to aggregate production and preferences are set to standard values. Capital's share in the Cobb-Douglas production function is 0.36 and the depreciation rate is 0.1. The risk aversion parameter γ is set to 1, implying logarithmic utility, and the discount factor β is 0.96.

The household productivity process

The main question addressed in the paper is how the presence of heterogeneity changes the welfare implications of tax reform, and the approach taken is to generate heterogeneity endogenously as a consequence of households receiving uninsurable idiosyncratic productivity shocks. Thus the specification of the process for these shocks is critical, since the choices here will determine how different households are in equilibrium, and therefore how differently they experience changes in fiscal policy. Broadly speaking there are two desiderata for the income process. The first is that the labor income uncertainty households experience is consistent with empirical estimates from panel data, so that the model is able to deliver appropriate time series variability in household income and consumption, and plausible levels of aggregate precautionary saving. The second is that the model economy generates realistic heterogeneity in terms of the distributions of labor and capital income, so that the tax reform involves a realistic redistribution of the tax burden.

We assume a three state process for productivity, since we found this to be the smallest number of states required to match overall wealth concentration and at the same time reproduce the fact that in the data the wealth-poorest two quintiles hold a positive fraction of total wealth. To reduce the number of free parameters, we assume that households cannot move between the high and low productivity levels directly, that the fraction of high productivity households equals the fraction of low productivity households, and that the probabilities of moving from the medium productivity state into either of the others are the same. These assumptions constitute four restrictions on the transition probability matrix, π_e. Since each row must add up to 1, we are left with two independent transition probabilities, p and q, where $p = \pi_e(e_h, e_h)$ and $q = \pi_e(e_m, e_m)$, and where p and q jointly define π_e as follows.

$$\pi_e = \begin{bmatrix} p & 1-p & 0 \\ \frac{1-q}{2} & q & \frac{1-q}{2} \\ 0 & 1-p & p \end{bmatrix} \tag{2.3}$$

Assuming that average productivity equals 1, the total number of free parameters is four: transition probabilities p and q, and two of the three values for productivity.

Various authors have estimated stochastic AR(1) processes for logged labor productivity using data from the PSID. Such a process may be summarized by the serial correlation coefficient, ρ, and the standard deviation of the innovation term, σ. Allowing for the presence of measurement error and the effects of observable characteristics such as education and age, work by Card (1991), Flodén and Lindé (1999), Hubbard, Skinner and Zeldes (1995) and Storesletten, Telmer and Yaron (1999) indicates a ρ in the range 0.88 to 0.96, and a σ in the range 0.12 to 0.25.[6] We therefore impose two restrictions on our finite state Markov process for productivity: (i) that the first order autocorrelation coefficient equals 0.9, and (ii) that the variance for productivity is $0.05/(1-0.9^2)$, corresponding to a standard deviation for the innovation term in the continuous representation of 0.224.

To generate realistic heterogeneity, we require that the Markov process for productivity be such that when the model economy is simulated, on average it reproduces certain features of the wealth distribution recently observed in the United States.[7,8] Given the two restrictions above, the number of remaining free

[6] Heaton and Lucas (1996) allow for permanent but unobservable household-specific effects, and find a much lower ρ of 0.53, and a σ of 0.25.

[7] This approach was pioneered by Castaneda et. al. (1998).

[8] In an earlier version of the paper we experimented with including the Gini coefficient for earnings as one of our targets. We abandoned this approach for two reasons. First, while esti-

parameters is two, and we therefore seek to match two properties of the empirical asset holding distribution: (i) the Gini coefficient, and (ii) the fraction of aggregate wealth held by the two poorest quintiles of the population. The first criterion ensures a realistic overall wealth distribution. The second criterion is designed to capture the bottom tail of the wealth distribution, and we include it because we expect that the households most likely to lose from reducing capital taxation are those with below average wealth. Using data from the 1992 Survey of Consumer Finances, Diaz-Gimenez, Quadrini and Rios-Rull (1997) report a wealth Gini of 0.78, and find that the two poorest quintiles of the distribution combined hold 1.35 percent of total wealth.[9]

Then calibration procedure is described in more detail in appendix A.1. To our initial surprise, we were able to find parameter values that satisfy all four criteria. This finding is interesting in light of the debate as to whether uninsurable fluctuations in earnings can account for U.S. households' wealth accumulation patterns (see Quadrini and Rios Rull 1997). Table 4 provides a detailed comparison between the asset holding distribution observed in the data, and the steady state pre-reform distribution implied by the calibrated incomplete markets model.

The values for productivity in the parameter set that matches our four targets are widely and asymmetrically spaced. The ratios between the productivity values in table 1 are

$$\frac{e_h}{e_m} = 5.09, \; \frac{e_m}{e_l} = 4.66.$$

The two transition probabilities are

$$p = 0.9, \;\; q = 0.988$$

which imply that at any point in time 5.25 percent of households have the high productivity level and the same percentage have the low productivity level.

Fiscal policy parameters

All remaining parameters relate to fiscal policy. The initial tax rates are calibrated to match the actual tax rates in the U.S. Since we are interested in the extent to which tax reform shifts the tax burden across households, we calibrate to average rather than marginal tax rates. Using the method outlined in Mendoza,

mates of the wealth Gini are stable across different data sources, estimates of Gini coefficients for earnings and income differ substantially. For example, Quadrini (1999) reports a Gini coefficient for income of 0.45 using PSID data, compared to 0.57 using SCF data. Second, in the model we abstract from various types of observable heterogeneity, such as differences in education and age, that we believe are essential for explaining the observed distribution of earnings. This is why our model generates a Gini coefficient for earnings of only 0.21.

[9] Kennickell and Woodburn (1999) report a wealth Gini of 0.788 for the 1995 SCF data.

Razin and Tesar (1994) we calculate average tax rates for the United States, the United Kingdom, France and Germany using OECD data. These are presented in table 2. For the period 1990-96, the capital income tax rate in the U.S. averaged 39.7 percent, while the labor income tax rate averaged 26.9 percent.

Constant government debt B in the pre-reform steady state is set to match the 67 percent debt / GDP ratio observed in post-war U.S data. Initial constant government spending G is set to ensure budget balance and is therefore not an independent parameter choice. However, the implied ratio of government spending to annual output is 0.20 (see table 3) which is close to the U.S. average of 0.19 between 1990-96.

2.2. Solution method

While techniques for solving for steady states in models with incomplete markets and heterogenous agents are fairly well established, less work has been done on developing methods for solving for transition between steady states in economies with production and incomplete markets. Exceptions are Huggett (1997) and Conesa and Krueger (1999). We describe our approach in appendix A.3.

2.3. Welfare measures

Our measure of welfare gains and losses is standard, and we now describe it for the incomplete markets economy (the complete markets economy is treated analogously).

Let $c_i^R(e^t)$ be equilibrium consumption at date t for household i given productivity history e^t in the case in which there is a tax reform at date 0, and let $c_i^{NR}(e^t)$ be the same thing in the case of no tax reform. The welfare gain for household i as a result of the reform is defined as the percentage increase in consumption that gives the household the same utility when the reform is implemented as when there is no reform but consumption is increased at each date by that amount. Thus the welfare gain is the Δ_i that solves the following equation.

$$\sum_{t=0}^{\infty} \beta^t u(c_i^R(e^t)) = \sum_{t=0}^{\infty} \beta^t u\left((1+\Delta_i)c_i^{NR}(e^t)\right). \tag{2.4}$$

In the complete markets economy, a household's welfare gain or loss is a known function of initial household wealth. In the incomplete markets case, initially identical households experience different subsequent productivity histories, and therefore experience the reform differently.

In table 5 we report the average *ex post* welfare gain by initial wealth quintile and by initial productivity, where the initial steady state distribution over individual states is used to assign households to groups. In table 6 we report the expected welfare gain or loss for households with various initial combinations of wealth and productivity. These numbers are computed by first creating a large artificial population each member of which has shares the initial wealth and productivity level of interest. The economy is then simulated forward (using the equilibrium sequence for interest rates) under both scenarios for fiscal policy.

We would like to be able to assess whether the changes in welfare that result from the tax reform occur because the tax reform affects the efficiency of production at the aggregate level, or because it involves a redistribution of existing resources. One way to control for the redistributional effects of reform is to consider the lifetime utility households would enjoy if they got to consume the same fraction of aggregate consumption in every date and state in the case of reform as they do without reform.

To this end, let C_t^R $\left(C_t^{NR}\right)$ denote aggregate consumption at date t in the case of reform (no reform),

$$C_t^j = \int c_i^j(e^t)\psi_t(\cdot) \qquad j \in \{R, NR\}.$$

Let $\widehat{c}_i(e^t)$ denote the hypothetical value for i's consumption in case of reform if the household got to consume the same fraction of aggregate consumption as in the case of no reform. Thus

$$\widehat{c}_i(e^t) = \alpha_{it} C_t^R \tag{2.5}$$

where

$$\alpha_{it} = \frac{c_i^{NR}(e^t)}{C_t^{NR}}. \tag{2.6}$$

The *efficiency gain* for household i as a result of the reform is defined as the Δ_i^e that satisfies.

$$\sum_{t=0}^{\infty} \beta^t u(\widehat{c}_i(e^t)) = \sum_{t=0}^{\infty} \beta^t u((1+\Delta_i^e) c_i^{NR}(e^t)). \tag{2.7}$$

The *distributional gain* for household i as a result of the reform is the Δ_i^d that satisfies.

$$\sum_{t=0}^{\infty} \beta^t u(c_i^R(e^t)) = \sum_{t=0}^{\infty} \beta^t u((1+\Delta_i^d)\widehat{c}_i(e^t)). \tag{2.8}$$

Comparing (2.4), (2.7) and (2.8) it is straightforward to show that the total welfare gain for household i is the product of the efficiency gain and the distributional gain:

$$(1+\Delta_i) = (1+\Delta_i^e)\left(1+\Delta_i^d\right).$$

A slightly less trivial result, formalized in the following proposition, is that with logarithmic utility, the efficiency gains are the same for all households and equal to the efficiency gain of a household with average consumption.

Proposition 2.1. *If $u(c) = \log(c)$, then for both market structures (i) $\Delta_i^e = \Delta^e$ for all i, and (ii) Δ^e is such that*

$$\sum_{t=0}^{\infty} \beta^t u(C_t^R) = \sum_{t=0}^{\infty} \beta^t u((1 + \Delta^e) C_t^{NR}) \tag{2.9}$$

Proof. See appendix A.4 ■

A tax reform is efficient if the efficiency gain is positive, that is if $\Delta^e > 0$.[10]

3. Results

In discussing our results, we focus primarily on the benchmark tax reform in which capital taxes are eliminated. These results are reported in tables 3 to 6 and in figures 1 to 7. In subsection 3.3 we discuss alternative tax reforms.

3.1. Findings

1. Both capital and government debt increase during transition, but the increases are smaller in the incomplete markets economy.

2. Eliminating capital income taxation reduces wealth concentration in the long run. In the complete markets economy, for example, a household with zero wealth at the time of the tax reform has 7.4 percent of mean wealth in the steady state to which the economy eventually converges.

3. When markets are complete, a household with mean wealth (the representative agent) sees a welfare gain of 1.07 percent in permanent consumption from eliminating the capital tax.[11]

4. Eliminating capital income taxation leads to a positive efficiency gain in both market structures. The efficiency gain is smaller when markets are incomplete.

[10] Of course, efficiency does not imply that the reform leaves everyone better off, since households typically do not consume the same fractions of aggregate consumption in the no refrom economy as in the reform economy: Δ_i^d is typically non-zero.

[11] This is large compared, for example, to previous estimates of the benefits of eliminating business cycles.

5. For both parameterizations and market structures, the majority of households lose *ex post* from the tax reform. In the incomplete markets economy 73.4 percent of households lose, while 71.7 percent lose in the complete markets economy.

6. The welfare effects of reform vary greatly depending on initial household wealth and productivity. In a representative sample of 9,600 households, the *ex post* change in welfare in the incomplete markets economy ranges from the equivalent of a 3.7 percent permanent fall in consumption to a 13.6 percent increase.

7. The average change in welfare for the entire population is negative: -0.44 percent of consumption in the complete markets economy versus -0.88 percent of consumption in the incomplete markets economy.

8. Households with higher initial wealth are more likely to be winners. On average, households in the lower three wealth quintiles lose around 3 percent in terms of constant consumption while those in the top quintile gain around 6 percent.

9. Controlling for initial wealth, in the incomplete markets economy households with initially higher productivity are less affected one way or the other by the reform.

10. The asset market structure does not appear to greatly affect the general results, and in particular the main conclusions survive across market structures: most households lose from eliminating capital income taxation, and the average welfare change is negative.

3.2. Interpretation

Aggregate variables

Rios-Rull (1994) shows that in calibrated model economies aggregate variables can behave in a similar manner under complete and incomplete market structures. In our complete markets economy, the long run effect of eliminating capital taxes is to increase capital, output, consumption and government debt. The dynamics for aggregate variables are very similar in the incomplete markets economy (see figure 1). The fact that the capital stock is always larger when markets are incomplete reflects the fact that households accumulate precautionary savings when they are unable to purchase insurance. As the capital stock (and government debt) increases during transition, so do per capita asset holdings. Thus the typical household in the incomplete markets economy has more wealth to use to smooth consumption in response to income shocks, and the demand for precautionary savings falls. This is why the increase in the capital stock is smaller in the incomplete markets economy.

The reason why government debt increases more in the complete markets economy is that the capital stock increases by a greater amount in transition. Thus for any given labor income tax the difference between revenue from taxation immediately after the reform and revenue in the eventual steady state is relatively larger. If the increase in government revenue during transition is larger when markets are complete, total government outlays must also increase by a larger amount to ensure a stationary path for debt. Since real government spending is assumed fixed, higher outlays must come from a larger increase in interest payments on debt, which implicitly corresponds to a larger increase in the stock of debt.

Why does it take roughly 40 years for the capital stock to approach the new steady state level? One reason is that the total increase in the capital stock is large: 32 percent in the complete markets economy, for example. With an initial capital to output ratio of 2.13 this increase amounts to 68.2 percent of initial GDP, while initial aggregate consumption is only 58.4 percent of GDP (see table 3). With mild consumption smoothing (log-utility), the optimal plan for a household with average wealth is to gradually increase asset holdings such that only after about 10 years does consumption exceed the initial steady state level (see figure 2)

Welfare

In a representative agent setting with exogenous labor supply, labor income taxation is effectively lump-sum and eliminating capital income taxation is in the class of optimal tax reforms. Eliminating the capital income tax means that in the long run the capital stock increases to a level at which the household's intertemporal marginal rate of substitution between consumption at different dates is equated to the marginal rate of transformation in production between those dates. In the economy with complete markets but heterogeneity with respect to initial wealth, the efficiency gain from tax reform is equal to the welfare gain of a household that has mean wealth at the time of the reform (see tables 6 and 7).

When markets are incomplete, the efficiency gain from reform remains positive but is much smaller than in the complete markets case. There are several reasons for this. First, the increase in the capital stock is smaller in the incomplete markets economy, which reduces the potential efficiency gain. Second, precautionary saving means that the initial capital stock is higher when markets are incomplete, and consequently the marginal benefit from an in increase in the stock is smaller. Finally, Aiyagari (1995) argues that if the optimal tax program in this type of economy converges to a steady state, then the optimal tax rate on capital in that steady state is positive. Although our tax reform is not likely to be optimal, this finding nonetheless suggests that eliminating capital taxation, as we do, leads to excessive capital accumulation from an efficiency point of view.

A second factor determining who gains and who loses is that emphasized by Garcia-Mila et. al. (1996). If households differ in the initial fractions of their income they receive from asset holdings versus labor supply, then eliminating capital income taxation effectively shifts the burden of taxation away from households who receive a large fraction of their income from capital and towards those who receive a large fraction from labor. An implicit assumption here, of course, is that markets to insure against the redistributive effects of future tax changes do not exist; if they did all households would share equally in the efficiency gains associated with the reform.[12]

Figures 4 and 6 show how tax payments change after the reform for different households depending on initial wealth. Households with low wealth (and in the incomplete markets case high productivity) see the largest initial increase in their tax bills. In the complete markets economy the fraction of income received from wealth is strictly increasing in wealth. There is a certain value for wealth such that all richer households benefit from the tax reform, while all poorer households lose (see figure 7). When markets are complete, the indifferent household lies in the 72^{nd} percentile of the initial wealth distribution and has 70.4 percent of mean per capita wealth. The household with median wealth has only 4.6 percent of mean wealth. This is one measure of how far away we are from finding a majority in favor of eliminating capital income taxes.

The redistributive effect of changing the balance between labor and capital taxation is also present in the incomplete markets economy. In addition to shifting the tax burden, however, the increase in capital stock increases the share of capital income of total income. With our assumption of no aggregate risk, the return on capital is known upon investment, and the uncertainty households face about future income is reduced. The reduced riskiness in the economy, reduces the demand for precautionary spending, which enables households to increase its consumption. This makes more households favor the tax reform.

The third column of table 5 shows the average initial ratio of asset to labor income for households in different parts of the initial wealth and productivity distributions. When households are ranked by initial wealth it appears that the

[12] One might therefore argue that it is misleading to label the economy in which labor income risk can be perfectly insured the "complete markets" economy.

ratio of asset to labor income is closely related to the expected welfare gain. When households are ranked by initial productivity, however, the low productivity type initially receive the largest fraction of income from wealth, yet they appear to fare worst as a result of the tax change, both on average and in terms of the fraction of households *ex post* worse off. Thus it is clear that in this economy, while the redistribution of the tax burden on impact still plays a role, tax reform must also affect the welfare calculus in subtler ways.

The effect of market incompleteness

Luck is one of several factors not present in the complete markets analysis that come into play when considering who gains and who loses from tax reform in an environment with uncertainty and missing markets. Two households with identical wealth and productivity at the time of the shock might view the reform very differently *ex post* depending on their subsequent fortune. From figure 7 it is clear that there is considerable variation in the experienced welfare gain of households with identical initial wealth. For example, in our sample population with $9,600$ households, the poorest household to gain *ex post* had 6.6 percent of mean initial wealth, while the richest household to lose started with 200.5 percent of mean initial wealth.

When markets are incomplete, households move around within the income and wealth distributions. Contrary to the complete markets case, the ratio of household capital to labor income at the time of the shock is no longer a sufficient statistic for predicting a household's expected gain or loss from the reform. Households with low wealth but high labor productivity face the largest percentage increase in their total tax bill initially (see figure 6). However, since they wish to accumulate wealth quickly (see figure 5) they are well placed to take advantage of the increase in the after-tax return to saving, and after a few years expect to pay less tax than they would have in the absence of reform. Even so, they expect to lose from the reform, because in the long run the after tax return on capital returns towards its pre-reform level. This suggest that removing the capital income tax do not primary benefit future capital holders but is rather a large windfall gain for current wealthy.

Figure 6 shows that households wishing and able to dis-save (those with high wealth but low productivity) pay lower taxes initially but in the future expect to pay more as they become increasingly dependent on highly-taxed labor income. This is why these households increase saving in the short run (relative to the no-reform case). The households that expect to fare worst are those who start out with low productivity and zero wealth. These households pay more tax initially, and do not want to accumulate wealth quickly since they expect higher labor productivity in the future. Thus they do not benefit from the temporarily high after-tax return to saving.

In the long run all households are the same, in that their expectations of future income and wealth are independent of their current income and wealth. Convergence of expected productivity is illustrated in figure 6, and accounts for the convergence in expected consumption and wealth paths for different types of households seen in figure 5. This might lead one to expect that the distributional effects highlighted by Garcia-Mila et. al. would be swamped by efficiency gains in an incomplete markets setting. There are two reasons why this is not what we find. First, productivity shocks are very persistent relative to the households' rate of time preference. Second, the initial distribution of wealth is so skewed that tax reform involves substantial redistribution even in the short run.

The importance of initial household wealth as a predictor of how much a household has to gain from tax reform is clear from table 6 and figure 7, which show the expected welfare gain by initial asset holdings. Not surprisingly, wealthy households have most to gain from eliminating taxes on capital income. The importance of initial productivity is more complicated. The last panel in figure 7 shows that households with high productivity are less affected one way or the other by the tax reform. High productivity households with very low wealth lose initially from the increase in labor taxes, but they lose less than low productivity households since they plan to accumulate un-taxed wealth quickly. High-wealth high-productivity households gain less than equally wealthy low productivity households, since they receive a larger fraction of their income from labor, and are therefore hit harder by the increase in the labor income tax. From table 6, a low productivity household with mean wealth expects a gain of 1.51 percent of consumption while the high productivity household only expects a gain of 0.72 percent.

A final factor that comes into play when markets are incomplete is that the tax reform affects the frequency with which the no-borrowing constraint binds, both in transition and in the eventual steady state. Households with both low wealth and low productivity (and who are therefore likely to be borrowing-constrained) are pushed even harder onto their borrowing constraint when the tax on labor income increases in order to offset the loss in capital income tax revenue. At the same time, in the new steady state aggregate capital and debt are both higher than in the initial steady state, and the after-tax return on asset holdings is also higher. This means that the opportunity cost of accumulating a buffer-stock of savings is reduced, and that in the long run all households are better able to smooth consumption (in addition to having higher consumption on average).

3.3. Alternative tax reforms

We investigate two alternative tax reforms. In the first we consider a smaller decrease in the capital income tax rate. In particular, we set the post reform capital income tax to 25.6 percent, which is the average of our estimates of the French and German average tax rates on capital income in the 1990's (see table 2). We call this the European tax reform. In our second alternative reform we increase the capital income tax rate to the U.K. level of 47.7 percent. We call this the U.K. tax reform. A comparison of the welfare implications of the different reforms is presented in table 7.

European tax reform

Not surprisingly, the results here are qualitatively very similar to the benchmark reform, though quantitatively the welfare changes are of a smaller magnitude. A striking finding is that the number of households that gain is more or less unchanged, even though the decrease in the capital tax rate is much smaller. In the incomplete markets economy, for example, 71.3 percent of households lose from the tax reform, compared to 73.4 percent under the benchmark reform. The reason the number of households that gain is almost unchanged is that the bottom three quintiles of the wealth distribution hold very little wealth. Thus any reduction is capital taxes will leave most households paying more tax in the short run and consequently, we conjecture, worse off.

Another important aspect to note is that the efficiency gain in the incomplete market economy is larger under the European tax reform than under the benchmark reform. This suggests that overaccumulation of capital do occur when the capital tax is removed, and that the efficient long run capital income tax is positive even when tax reforms are restricted to once and for all changes.

U.K. tax reform

When we increase the capital tax rate to the estimated U.K. level, we find the economy experiences an efficiency loss of 0.17 percent, but that most households are left better off. However, the gains are small, and the average welfare effect is negative. This suggests that the results of decreasing the capital tax (the European tax reform) and increasing the capital tax (the U.K. tax reforms) is not symmetric. While 71.3 percent of households lose *ex post* as a result of the tax decrease, and 67.8 percent gain from the tax increase, in both economies the unconditional expected change in welfare is negative. We interpret these findings as indicating that the marginal productive efficiency cost of capital income taxation is increasing in the tax rate.

3.4. Alternative parameterizations

In all the tax reforms described above, the high persistence of productivity shocks relative to households' rate of time preference reduces the importance of income mobility in determining welfare gains. We therefore recompute the effects of the benchmark tax reform using a productivity process with less persistence to investigate the extent to which this reduces the cost of the reform for low wealth and low productivity households. Heaton and Lucas (1996) allow for permanent but unobservable household-specific effects when estimating stochastic AR(1) processes for logged labor productivity. Their estimates for the autocorrelation coefficient and the variance for productivity are 0.53, and $0.251^2/(1-0.53^2)$ respectively.

As an alternative to our benchmark parameterization we therefore consider an alternative in which the household productivity process has the Heaton and Lucas persistence and variance. In this case we are unable to reproduce the degree of wealth concentration observed in the U.S. We therefore assume that the productivity shocks are equally spaced, and that the fractions of households in each state are as in the benchmark parameterization. Given these assumptions, the wealth Gini in the pre-reform steady state is 0.45 and the poorest 40 percent of households hold 11.5 percent of total wealth. Thus the model now generates much less wealth concentration than under the benchmark process. The results are presented in table 8.

We find that a greater fraction of households benefit from eliminating capital income taxes under this parameterization: in both economies approximately 50 percent of households are better off *ex post* as a result of the reform. The average change in welfare is now positive. As under the benchmark parameterization, the welfare implications of reform in the complete and incomplete markets economies remain very similar. This suggests that what is most important for the welfare implications of tax reform is not the degree of earnings mobility implied by the productivity process, but rather the degree of concentration in wealth prior to the reform.

4. Conclusions

The main conclusion we take from this paper is that changing the balance between capital and labor income taxation is likely to have very large distributional implications. Reducing taxes on capital income in our model *does* stimulate investment, raising output and consumption for all households in the long run. However, the short run cost in the form of higher labor taxes is too heavy a price to pay for all except the wealth-richest households. This finding survives even if markets are complete and idiosyncratic earnings risk is fully insurable.

The welfare implications of tax reform vary dramatically with the shape of the initial wealth distribution. In a representative agent economy the reform we consider is optimal. In a parameterization in which we reproduce the highly concentrated distribution of wealth observed in the U.S., over 70 percent of households are made worse off by eliminating capital income taxation. Thus our quantitative modelling exercise suggests that for understanding the welfare implications of this type of fiscal reform, it is important to think hard about the right way to model observed heterogeneity.

In future work we plan to experiment with a wider range of possible labor / capital tax combinations, and to find the reform in the class of once and for all changes in tax rates that maximizes a utilitarian social welfare function. We shall also investigate whether there exists a social welfare function with weights inversely proportional to initial wealth that rationalizes the current balance between labor and capital taxation.

References

[1] Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving". *Quarterly Journal of Economics* **109(3)**, pp. 659-684.

[2] Aiyagari, S. Rao. 1995. "Optimal Capital Iincome Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting". *Journal of Political Economy* **103(6)**, pp. 1158-1175.

[3] Aiyagari, S. Rao. and Ellen R. McGrattan. 1998. "The Optimum Quantity of Debt". *Journal of Monetary Economics* **42**, pp. 447-469.

[4] Atkeson Andrew, V. V. Chari and Patrick J. Kehoe. 1999."Taxing Capital Income: A Bad Idea". *Quarterly Review (Summer)*. Federal Reserve Bank of Minneapolis.

[5] Auerbach Alan J. and Laurence J. Kotlikoff. 1987. "Dynamic fiscal policy". Cambridge University Press, Cambridge

[6] Card, David. 1991. "Intertemporal Labor Supply: An Assesment". *NBER Working Paper* No. 3602.

[7] Castaneda, Ana, Javier Diaz-Gimenez, and Jose-Victor Rios-Rull. 1998. "Earnings and Wealth Inequality and Income Taxation", mimeo, University of Pennsylvania.

[8] Chamley, Christophe. 1986. “Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives”, *Econometrica* **54(3)**. pp. 607-622

[9] Chamley, Christophe. 1998. “Capital Income Taxation, Wealth Distribution and Borrowing Constraints”. mimeo.

[10] Chatterjee, Satyajit. 1994. “Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model”, Journal of Public Economics 54, pp. 97-119.

[11] Conesa, Juan C. and Dirk Krueger. 1999. “Social Security Reform with Heterogenous Agents”, *Review of Economic Dynamics*, forthcoming.

[12] Diaz-Gimenez, Javier, Vincenzo Quadrini, and Jose-Victor Rios-Rull. 1997. “Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income and Wealth”, *Quarterly Review (Spring).* Federal Reserve Bank of Minneapolis.

[13] Flodén, Martin and Jesper Lindé. 1999. “Idiosyncratic Risk in the U.S. and Sweden: Is There a Role for Government Insurance?”, mimeo, Stockholm School of Economics.

[14] Garcia-Mila, Teresa, Albert Marcet, and Eva Ventura. 1995, “Supply Side Interventions and Redistribution”, UPF Working Paper 115.

[15] Heaton, John and Deborah Lucas. 1996. “Evaluating the Effect of Incomplete Markets on Risk Sharing and Assets Pricing”, *Journal of Political Economy* **104(3)**, pp. 443-487.

[16] Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes. 1995. “Precautionary Saving and Social Insurance”, *Journal of Polictical Economy* **103(2)**, pp. 360-399.

[17] Huggett, Mark. 1997. “The one-sector growth model with idiosyncratic shocks: Steady states and dynamics”, *Journal of Monetary Economics* **39(3)**, pp. 385-404.

[18] Judd, Kenneth L. 1985. “Redistributive Taxation in a Simple Perfect Forsight Model”, *Journal of Public Economics* **28**, pp. 59–83.

[19] Judd, Kenneth L. 1987. “The Welfare Cost of Factor Taxation in a Perfect Foresight Model”, *Journal of Political Economy* **95**

[20] Kennickell, Arthur B. and R. Louise Woodburn. 1997. "Consistent Weight Design for the 1989, 1992 and 1995 SCFs, and the Distribution of Wealth", *Review-of-Income-and-Wealth*, **45(2)**, pp. 193-215.

[21] Mendoza, Enrique G., Assaf Razin and Linda L. Tesar. 1994. "Effective Tax Rates in Macroeconomics. Cross-country Estimates of Tax Rates on Factor Income and Consumption," *Journal of Monetary Economics,* **34**, pp. 297-323.

[22] Quadrini, Vincenzo. 1999. "Entrepreneurship, Saving and Social Mobility", *Review-of-Economic Dynamics,* forthcoming.

[23] Organization for Economic Cooperation and Development. 1999. "Revenue Statistics of OECD Member Countries", and "National Accounts Detailed Tables: Vol. II". OECD Statistical Compendium ed. 01#1999. Organization for Economic Cooperation and Development. Paris.

[24] Quadrini, Vincenzo and Jose-Victor Rios-Rull. 1997. "Understanding the U.S. Distribution of Wealth", *Quarterly Review (Spring).* Federal Reserve Bank of Minneapolis.

[25] Ríos-Rull, José-Víctor. 1994. "On the Quantitative Importance of Market Incompleteness", *Journal of Monetary Economics* **34(3)**, pp. 463-496.

[26] Storesletten, Kjetil, Chris Telmer and Amir Yaron. 1999. "Asset Pricing with Idiosyncratic Risk and Overlapping Generations", mimeo, Carnegie Mellon University.

A. Appendix

A.1. Calibrating the productivity process

Consider the following AR(1) process for labor productivity

$$\ln e' = \rho \ln e + \varepsilon' \qquad \varepsilon \sim N(0, \sigma^2). \tag{A.1}$$

and note that

$$\rho = \frac{cov(\ln e', \ln e)}{var(\ln e)} \tag{A.2}$$

and

$$var(\ln e) = \frac{\sigma^2}{1-\rho^2} \tag{A.3}$$

Equations resembling (A.1) have been estimated on panel data. Our goal to approximate equation (A.1) by a 3-state Markov chain, preserving the estimated autocorrelation and variance of log productivity. Let e_i, $i = 1, 2, 3$ denote the three productivity levels in our Markov chain, and let π_i denote the constant proportion of households with each productivity level in the ergodic distribution associated with the transition probability matrix, π_e. Thus $\sum_i \pi_i = 1$. The matrix itself, reproduced here, defines the probabilities of moving between productivity levels as functions of two parameters, p and q.

$$\pi_e = \begin{bmatrix} p & 1-p & 0 \\ \frac{1-q}{2} & q & \frac{1-q}{2} \\ 0 & 1-p & p \end{bmatrix} \tag{A.4}$$

Given the symmetry of π_e, $\pi_1 = \pi_3$, and π_1 is related to p and q as follows.

$$\begin{aligned} \pi_1(1-p) &= \pi_2 \frac{1-q}{2} \\ &= (1-2\pi_1)\frac{1-q}{2} \end{aligned} \tag{A.5}$$

To enable comparison with the estimated process for log productivity, assume that mean (natural) log productivity equals 1.

$$\overline{\ln e} = \sum_i \pi_i \ln e_i = 0 \tag{A.6}$$

The variance and covariance of log productivity are given by

$$var(\ln e) = \sum_i \left(\ln e_i - \overline{\ln e}\right)^2 \tag{A.7}$$

and

$$cov(\ln e', \ln e) = \sum_i \left(\ln e_i' - \overline{\ln e}\right)\left(\ln e_i - \overline{\ln e}\right) \tag{A.8}$$

Let π_1 and e_2 be such that when the model economy is simulated, on average it reproduces the two chosen moments characterizing the wealth distribution as discussed in section 2.1. Once values for these parameters have been chosen, the goal is to adjust the remaining free parameters so that the process for log productivity inherits the properties estimated in the data. During this second stage, π_1 and e_2 are treated as exogenously fixed.

Since $\pi_3 = \pi_1$, and $\sum_i \pi_i = 1$, (A.5) can be rearranged to express q as a known function of p.

$$q = \frac{\pi_2 - 2\pi_1(1-p)}{\pi_2} \tag{A.9}$$

Equation (A.6) can be rearranged to give an expression for $\ln e_3$

$$\ln e_3 = -\frac{\pi_1 \ln e_1 + \pi_2 \ln e_2}{\pi_1} \tag{A.10}$$

Given π_1 and e_2, and expressions (A.9) and (A.10), the only remaining free parameters are p and e_1.

From (A.3) and (A.7), equating the variances of the discrete and continuous processes for log productivity implies that.

$$\sigma_e^2 = \left(1-\rho^2\right)\left(\pi_1 (\ln e_1)^2 + \pi_2 (\ln e_2)^2 + \pi_1 (\ln e_3)^2\right). \tag{A.11}$$

Substituting (A.10) into (A.11) then implies

$$2(\ln e_1)^2 + 2k \ln e_1 \ln e_2 + k(1+k)(\ln e_2)^2 - \frac{\sigma^2}{(1-\rho^2)\pi_1} = 0 \qquad \text{where } k = \frac{\pi_2}{\pi_1}$$

This is a quadratic equation that can be solved for $\ln e_1$. The relevant root is

$$\ln e_1 = \frac{-2k \ln e_2 - \sqrt{(2k \ln e_2)^2 - 4 \times 2 \times \left(k(1+k)(\ln e_2)^2 - \frac{\sigma^2}{(1-\rho^2)\pi_1}\right)}}{2 \times 2} \tag{A.12}$$

From (A.2), (A.7) and (A.8), equating the autocorrelation of the discrete and continuous processes for log productivity implies that

$$\rho = p + \frac{(-1+p)(\ln e_2)^2}{\pi_1 (\ln e_1)^2 + \pi_2 (\ln e_2)^2 + \pi_1 (\ln e_3)^2}. \tag{A.13}$$

Substituting in equation (A.11) this simplifies to

$$\rho = p + \frac{(-1+p)(1-\rho^2)(\ln e_2)^2}{\sigma^2} \tag{A.14}$$

Equation (A.14) can then be used to solve for p

$$p = \frac{\rho + \frac{(1-\rho^2)(\ln e_2)^2}{\sigma^2}}{1 + \frac{(1-\rho^2)(\ln e_2)^2}{\sigma^2}}. \tag{A.15}$$

A.2. Definition of equilibrium

We now describe the conditions that jointly characterize the equilibrium path of the incomplete markets economy following a tax reform at date $t = 0$.

Let the decision rules $c_i(e^t)$ together with the probabilities $\rho(e^t|e^0)$ jointly define functions $T_i : E^t \times \mathcal{S} \rightarrow \mathrm{R}_+$ such that $T_i\left(e^t, U\right)$ is the probability that household i after history e^t will have productivity and wealth in the set $U \in \mathcal{S}$ at date $t+1$. (Recall that $\mathcal{S}$ denotes the set of all subsets of $E \times \mathcal{A}$). Thus $\forall i$, $\forall e^t \in E^t$ and $\forall U \in \mathcal{S}$

$$T_i\left(e^t, U\right) = \sum_{e \in E} \sum_{e^t \in E^t} I_{\{(e, a_{it}) \in U\}} \times \rho(e^t|e_0) \times \pi_e\left(e_t, e\right)$$

where a_{it} is given by subtracting $c_i(e^t)$ from the right hand side of 2.2, and I is an indicator function.

An equilibrium is a pair of constant tax rates τ^k and τ^n and a sequence of measures $\{\psi_t(\cdot)\}_{t=0}^{\infty}$, prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$, aggregate capital, debt and asset holdings $\{K_t\}_{t=0}^{\infty}$, $\{B_t\}_{t=0}^{\infty}$ and $\{A_t\}_{t=0}^{\infty}$, and decision rules $\left\{c_i(e^t)\right\}_{t=0}^{\infty}$ $\forall i$ and $\forall e^t \in E^t$ such that $\forall t$, $\forall e^t \in E^t$, and $\forall i$:

1. $c_i(e^t)$ solves household i's maximization problem (described in the text) given $\{r_t\}_{t=0}^{\infty}$, $\{w_t\}_{t=0}^{\infty}$, and the pair of constant tax rates $\left\{\tau^k, \tau^n\right\}$.

2. $\psi_t(\cdot)$ is consistent with $T_i(\cdot,\cdot)$ in that for all $U \in \mathcal{S}$

$$\psi_t(U) = \int T_i\left(e^t, U\right) di.$$

3. The market for savings clears.

$$K_t + B_t = \int a_{it} d\psi_t(\cdot) = A_t.$$

4. Factor markets clear.

$$r_{t-1} = \alpha K_{t-1}^{\alpha-1} \overline{n}^{1-\alpha} - \delta.$$
$$w_t = (1-\alpha) K_{t-1}^{\alpha} \overline{n}^{-\alpha}.$$

5. The government budget constraint is satisfied and debt remains bounded.

$$B_t + \tau^k r_{t-1} A_{t-1} + \tau^n w_t \overline{n} = \left[1 + \left(1 - \tau^k\right) r_{t-1}\right] B_{t-1} + G.$$

6. The goods market clears.

$$C_t + G + K_t - (1-\delta) K_{t-1} = Y_t.$$

A.3. Solution algorithm

1. Solve for the initial steady state given the initial capital tax rate as follows.
    1. Guess a value for the capital stock (and thus implicitly for output).
    2. Compute the government spending G, such that given the labor tax τ^n, government debt B remains constant at the target ratio for debt to GDP.
    3. Simulate the economy to compute a stationary asset holding distribution.
    4. Check that aggregate household savings decisions equal aggregate capital plus aggregate debt.
    5. Adjust the guess for the capital stock and iterate until the market for savings clears.
2. Choose a new value for the capital tax τ^k. Assume this is announced before households make decisions in period 1.
3. Assume that the economy converges to a new steady state and that it is in this steady state in period T.
4. Guess a sequence $K_2...K_{T-1}$ for capital during transition.
5. Solve for the new proportional tax on labor τ^n such that given $K_2...K_{T-1}$ and τ^k, government debt is unchanged between $T-1$ and T. Compute the associated path for government debt, $B_2...B_T$.
6. Solve for the final steady state using the same procedure outlined in step one, taking as given tax rates τ^k and τ^n and G and B_T. Compute the capital stock in the new steady state, K_T.
7. Solve for household savings decisions in transition as follows.
    1. Start in period $T-1$.
    2. Assume that:
        1. capital today is K_{T-1} and capital tomorrow is K_T.
        2. consumption tomorrow (in period T) is given by the consumption function in the new steady state, $c_T(\cdot)$.
    3. Solve for the consumption decision rule at $T-1$ across the grid on individual wealth and productivity, $c_{T-1}(a, e : K_{T-1}, K_T, c_T(\cdot))$.

4. Move back one period to $T-2$, and solve for $c_{T-2}(a, e : K_{T-2}, K_{T-1}, c_{T-1}(\cdot))$.
5. Continue moving back until we have decision rule functions $c_i(a, e : K_i, K_{i+1}, c_{i+1}(\cdot))$, $i = 1...T-1$.

8. Now start updating the path of capital. The procedure below is a Gauss Seidel algorithm. The basic problem we have is one of finding a sequence of capital stocks such that when hosueholds optimize markets clear at every date and government debt eventually stabilizes at a finite level. A Newton Raphson approach would start by computing excess demand at every date before updating any values for capital in the sequence. The advantage of the Gauss Seidel method is that we update continuously.

   1. Take the initial steady state distribution over wealth and productivity and use $c_1(a, e : K_1, K_2, c_2(\cdot))$ to compute the implied joint distribution in period 2.
   2. Compute the value for aggregate capital in the second period of transition, $\widehat{K_2}$ that is implied by $c_1(a, e : K_1, K_2, c_2(\cdot))$. This is given by aggregate savings minus B_2.
   3. Compare K_2 (the value for capital in period 2 that was used to compute household savings decisions) and compare it to $\widehat{K_2}$. Set $K_2 = K_2 + \phi\left(\widehat{K_2} - K_2\right)$ where $0 < \phi < 1$.
   4. Recompute τ^n and the sequence for government debt.
   5. Recompute $c_2(a, e : K_2, K_3, c_3(\cdot))$ and $c_1(a, e : K_1, K_2, c_2(\cdot))$.
   6. Using the initial steady state distribution over wealth and productivity, simulate the economy forward two periods with savings rules given by $c_1(a, e : K_1, K_2, c_2(\cdot))$ and $c_2(a, e : K_2, K_3, c_3(\cdot))$.to compute the implied value for $\widehat{K_3}$.
   7. Given $\widehat{K_3}$, adjust K_3, and recompute τ^n, the sequence for government debt, and $c_3(\cdot)$, $c_2(\cdot)$ and $c_1(\cdot)$.
   8. Iterate forward until we have updated $K_2...K_{T-1}$,

9. If the new sequence for capital is the same as the old, we have found the equilibrium path. Otherwise go back to step 5, resolve for the new labor tax given the updated capital sequence, and proceed.

10. Once the sequence for capital has converged, check whether T is sufficient by increasing T and checking whether the equilibrium path is affected. In all experiments T has been set to 80, implying that the aggregate capital stock converges to its new steady state level with 80 years.

A.4. Efficiency

In this appendix we prove proposition 2.1. To show that all households experience the same efficiency gain, let Δ^e be such that equation (2.9) is satisfied given aggregate consumption streams $\left\{C_t^R\right\}_{t=0}^{\infty}$ and $\left\{C_t^{NR}\right\}_{t=0}^{\infty}$. Equation (2.9) then implies that

$$\sum_{t=0}^{\infty} \beta^t \log(C_{it}^R) = \sum_{t=0}^{\infty} \beta^t \log(1+\Delta^e) + \sum_{t=0}^{\infty} \beta^t \log(C_t^{NR}) \tag{A.16}$$

For all i, let Δ_i^e satisfy equation (2.7) given $\{\widehat{c}_i(e^t)\}_{t=0}^{\infty}$ and $\left\{c_i^{NR}(e^t)\right\}_{t=0}^{\infty}$. Then for any i, substituting equations (2.5) and (2.6) into (2.7) gives

$$\sum_{t=0}^{\infty} \beta^t u(\alpha_{it} C_t^R) = \sum_{t=0}^{\infty} \beta^t u((1+\Delta_i^e)\alpha_{it} C_t^{NR})$$

which may be rewritten as

$$\sum_{t=0}^{\infty} \beta^t \log(\alpha_{it}) + \sum_{t=0}^{\infty} \beta^t \log(C_t^R) = \sum_{t=0}^{\infty} \beta^t \log(1+\Delta_i^e) + \sum_{t=0}^{\infty} \beta^t \log(\alpha_{it}) + \sum_{t=0}^{\infty} \beta^t \log(C_t^{NR}) \tag{A.17}$$

Comparing equations (A.16) and (A.17) we see that

$$\sum_{t=0}^{\infty} \beta^t \log(1+\Delta_i^e) = \sum_{t=0}^{\infty} \beta^t \log(1+\Delta^e).$$

Thus for all i, $\Delta_i^e = \Delta^e$.

Table 1: Parameter values (yearly basis)

		Market structure	
		Incomplete	Complete
Aggregate production	α	0.36	
	δ	0.1	
Individual productivity	e_h	4.334	1.0
	e_m	0.852	1.0
	e_l	0.183	1.0
	$\pi(e_h \vert e_h)$	0.900	
	$\pi(e_m \vert e_m)$	0.988	
	$\pi(e_l \vert e_l)$	0.900	
Preferences	γ	1.0	
	β	0.96	
Government	B/Y	0.67	
	τ^n	0.269	
	τ^k	0.397	
	$\tau^{k/}$	0.000	

Table 2: Average tax rates (percent)[*]

	United States	United Kingdom	France	Germany
1965-1996				
Consumption tax	5.7	15.0	21.3	15.7
Labor income tax	23.6	26.4	42.7	37.8
Capital income tax	40.1	54.1	24.1	26.6
1990-1996				
Consumption tax	5.4	16.8	19.4	16.5
Labor income tax	26.9	24.3	48.8	42.1
Capital income tax	39.7	47.7	25.0	26.2

[*] These figures are computed using the method described by Mendoza, Razin and Tesar (1994) and OECD (1999) data.

Table 3: Aggregate properties of initial and final steady states benchmark tax reform

		Market structure	
		Incomplete	Complete
τ^k	initial	0.397*	0.397*
	final	0.000*	0.000*
τ^n	initial	0.269*	0.269*
	final	0.334	0.343
G/Y	initial	0.200	0.203
	final	0.186	0.183
B/Y	initial	0.670*	0.670*
	final	0.823	0.858
K/Y	initial	2.34	2.13
	final	2.68	2.54
C/Y	initial	0.565	0.584
	final	0.546	0.562
Y	initial	0.528	0.500
	final	0.570	0.553
r (% post-tax)	initial	3.23	4.17
	final	3.42	4.17
post-tax asset to labor income ratio	initial	0.21	0.25
	final	0.28	0.34

* Starred values indicate exogenous parameters.

Table 4: Distributional properties of initial and final steady states benchmark tax reform

	Data*	Market structure	
	U.S. 1992	Incomplete	Complete
Asset holding distribution		Initial steady state	
Gini	0.78	0.78	0.78
99-100%	29.6	11.6	11.6
90-100%	66.1	60.2	60.2
80-100%	79.5	83.9	83.9
0-40%	1.35	1.35	1.35
Earnings Gini	0.63	0.21	0.00
Wealth – earnings correlation	0.23	0.34	0.00
Asset holding distribution		Final steady state	
Gini	0.78	0.74	0.72
99-100%	29.6	10.1	10.8
90-100%	66.1	55.4	56.5
80-100%	79.5	79.0	79.2
0-40%	1.35	1.81	4.21
Earnings Gini	0.63	0.21	0.00
Wealth – earnings correlation	0.23	0.31	0.00

* The data column is taken from Diaz-Gimenez et. al. (1997) whose data source is the 1992 Survey of Consumer Finances.

Table 5: Welfare results, benchmark tax reform

		Fraction of population (%)	Average post-tax asset to labor income ratio	Average gain from reform (% of period consumption)	Fraction within group that gains (%)
Incomplete Markets Economy					
Wealth quintile	Q1	20	0.01	-3.25	0.0
	Q2	20	0.01	-3.17	0.0
	Q3	20	0.01	-3.09	0.1
	Q4	20	0.18	-0.64	33.4
	Q5	20	0.96	5.77	99.9
Productivity	Low	5.25	0.70	-1.57	20.2
	Medium	89.50	0.21	-1.12	23.7
	High	5.25	0.19	4.00	84.5
	Entire population	100	0.23	-0.88	26.7
Complete Markets Economy					
Wealth quintile	Q1	20	0.01	-3.04	0.0
	Q2	20	0.01	-2.97	0.0
	Q3	20	0.01	-2.88	0.0
	Q4	20	0.17	-0.24	41.7
	Q5	20	1.05	6.92	100.0
	Entire population	100	0.25	-0.44	28.3

Table 6: Average gain from reform (fraction in group that gains in parentheses) benchmark tax reform

		Wealth Zero	*Wealth* Median	*Wealth* Mean
Incomplete Markets Economy				
Productivity	Low	-3.49 (0.0)	-2.96 (0.0)	1.51 (94.5)
	Medium	-3.39 (0.0)	-3.16 (0.0)	0.51 (86.8)
	High	-1.58 (1.5)	-1.46 (1.9)	0.72 (83.9)
Complete Markets Economy				
		-3.18	-2.95	1.07

Table 7: Comparison of alternative tax reforms

		Average gain from reform (% of period consumption) Fraction within group that gains (%) in parentheses		
		New $\tau^k = 0$	New $\tau^k = 25.6$	New $\tau^k = 47.7$
		Incomplete Markets Economy		
Wealth quintile	Q1	-3.25 (0.0)	-1.00 (0.0)	0.27 (99.1)
	Q3	-3.09 (0.1)	-0.94 (0.3)	0.25 (98.2)
	Q5	5.77 (99.9)	2.28 (100.0)	-0.92 (0.0)
Productivity	Low	-1.57 (20.2)	-0.42 (22.0)	0.06 (76.4)
	Medium	-1.12 (23.7)	-0.23 (25.5)	-0.01 (71.1)
	High	4.00 (84.5)	1.69 (89.9)	-0.72 (3.2)
Entire population	Welfare gain	-0.88 (26.7)	-0.14 (28.7)	-0.04 (67.8)
	Efficiency gain	0.12	0.22	-0.17
	Distributional gain	-1.00	-0.36	0.13
		Complete Markets Economy		
Wealth quintile	Q1	-3.04 (0.0)	-0.75 (0.0)	0.09 (100.0)
	Q3	-2.88 (0.0)	-0.69 (0.0)	0.07 (100.0)
	Q5	6.92 (100.0)	2.80 (100.0)	-1.15 (0.0)
Entire population	Welfare gain	-0.44 (28.3)	0.18 (31.8)	-0.23 (60.7)
	Efficiency gain	1.07	0.72	-0.42
	Distributional gain	-1.51	-0.54	0.19

Table 8: Comparison of alternative parameterizations. New $\tau^k = 0$

		Average gain from reform (% of period consumption) Fraction within group that gains (%) in parentheses Benchmark	Heaton & Lucas
	Incomplete Markets Economy		
Wealth quintile	Q1	-3.25 (0.0)	-2.37 (0.0)
	Q3	-3.09 (0.1)	0.07 (53.2)
	Q5	5.77 (99.9)	4.64 (100.0)
	Low	-1.57 (20.2)	0.08 (45.4)
Productivity	Medium	-1.12 (23.7)	0.55 (50.2)
	High	4.00 (84.5)	1.43 (65.9)
Entire population	Welfare gain	-0.88 (26.7)	0.57 (50.7)
	Efficiency gain	0.12	0.93
	Distributional gain	-1.00	-0.36
	Complete Markets Economy		
Wealth quintile	Q1	-3.04 (0.0)	-2.34 (0.0)
	Q3	-2.88 (0.0)	0.18 (61.6)
	Q5	6.92 (100.0)	4.67 (100.0)
Entire population	Welfare gain	-0.44 (28.3)	0.63 (52.3)
	Efficiency gain	1.07	1.07
	Distributional gain	-1.51	-0.44

Figure 1: Paths for aggregate capital, debt and tax revenue. Benchmark tax reform

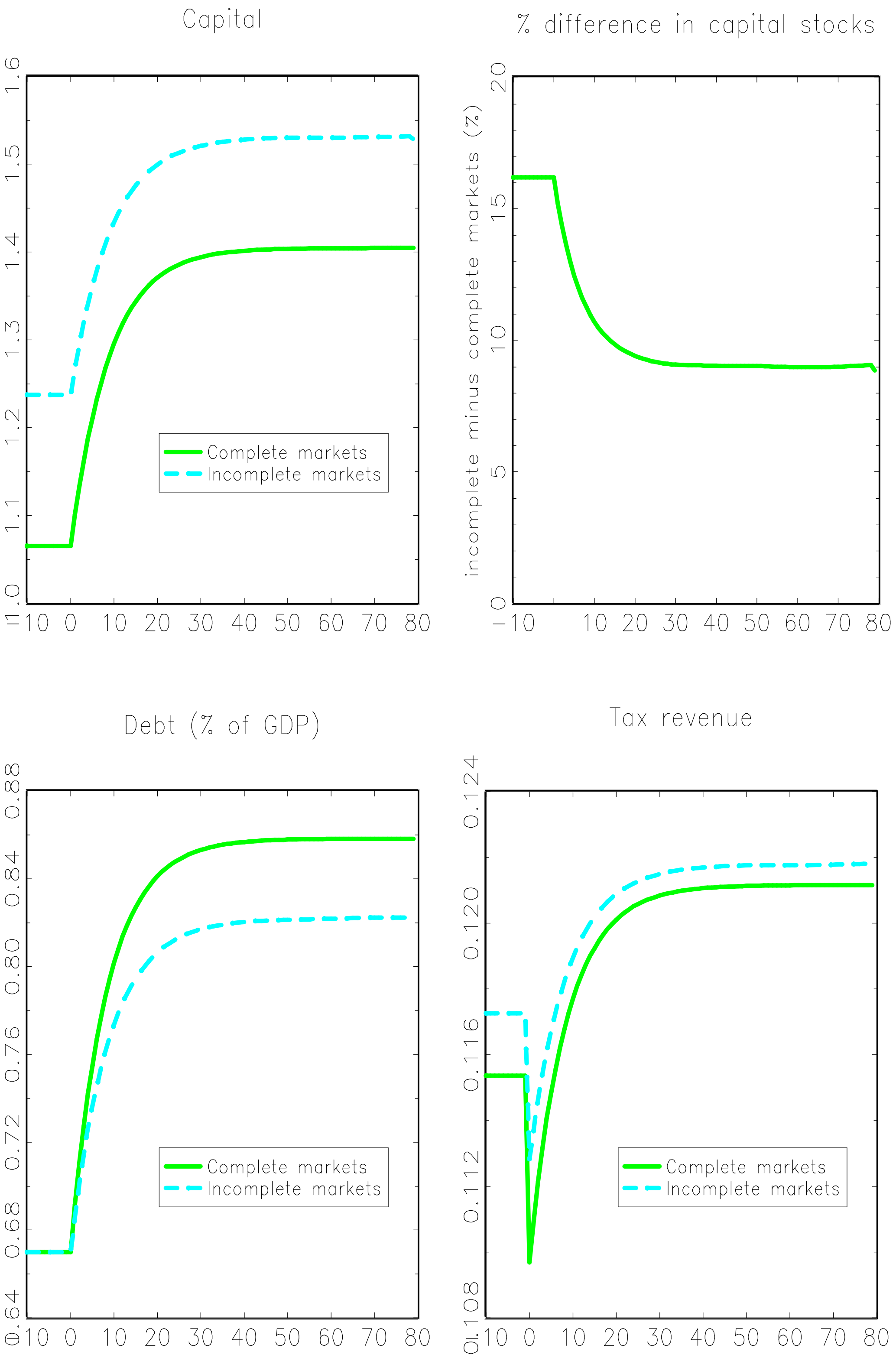

Figure 2: Paths for aggregate consumption, investment, government spending and output. Benchmark tax reform

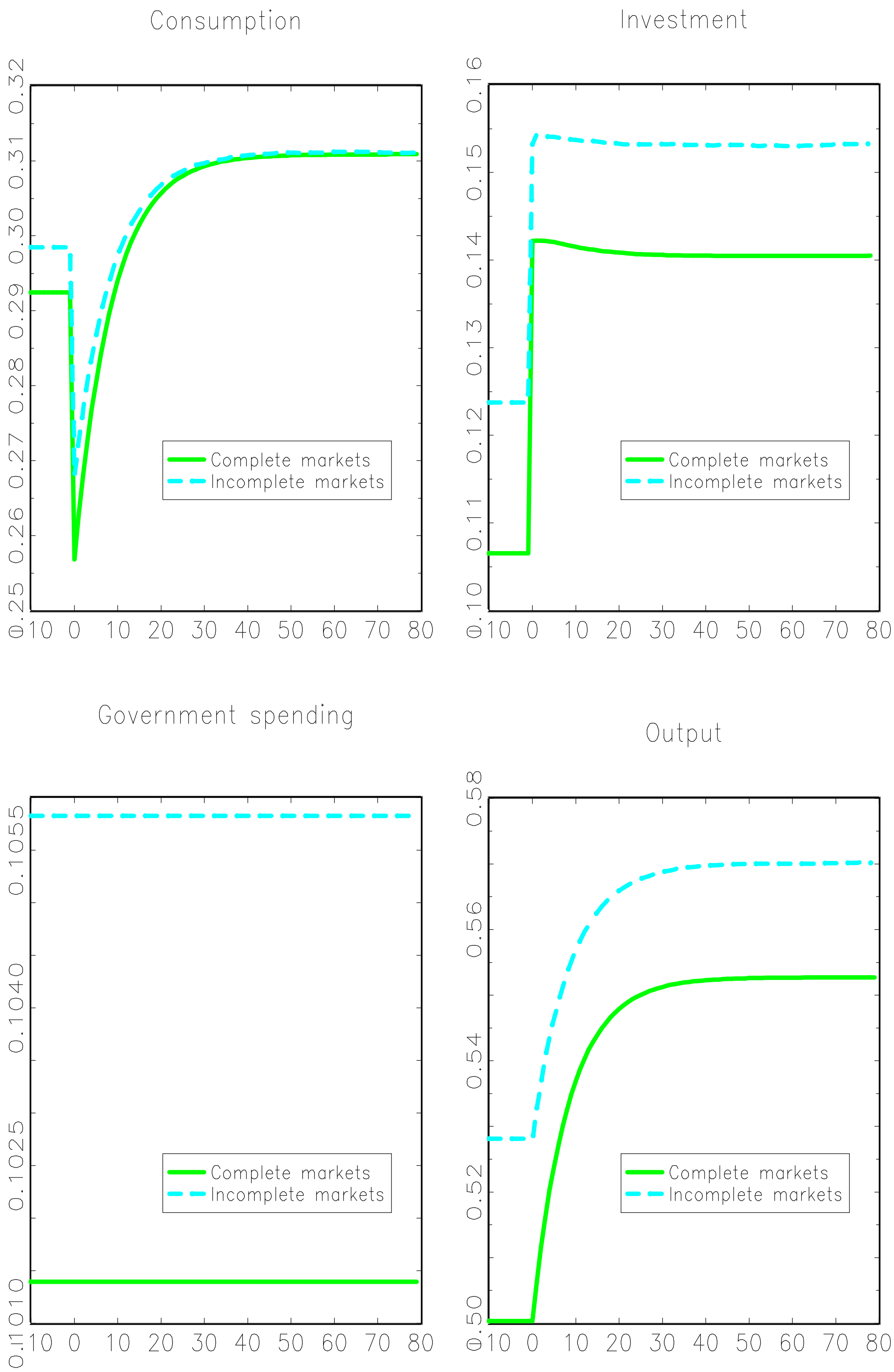

Figure 3: Complete markets economy. Consumption and asset holdings by type. Benchmark tax reform

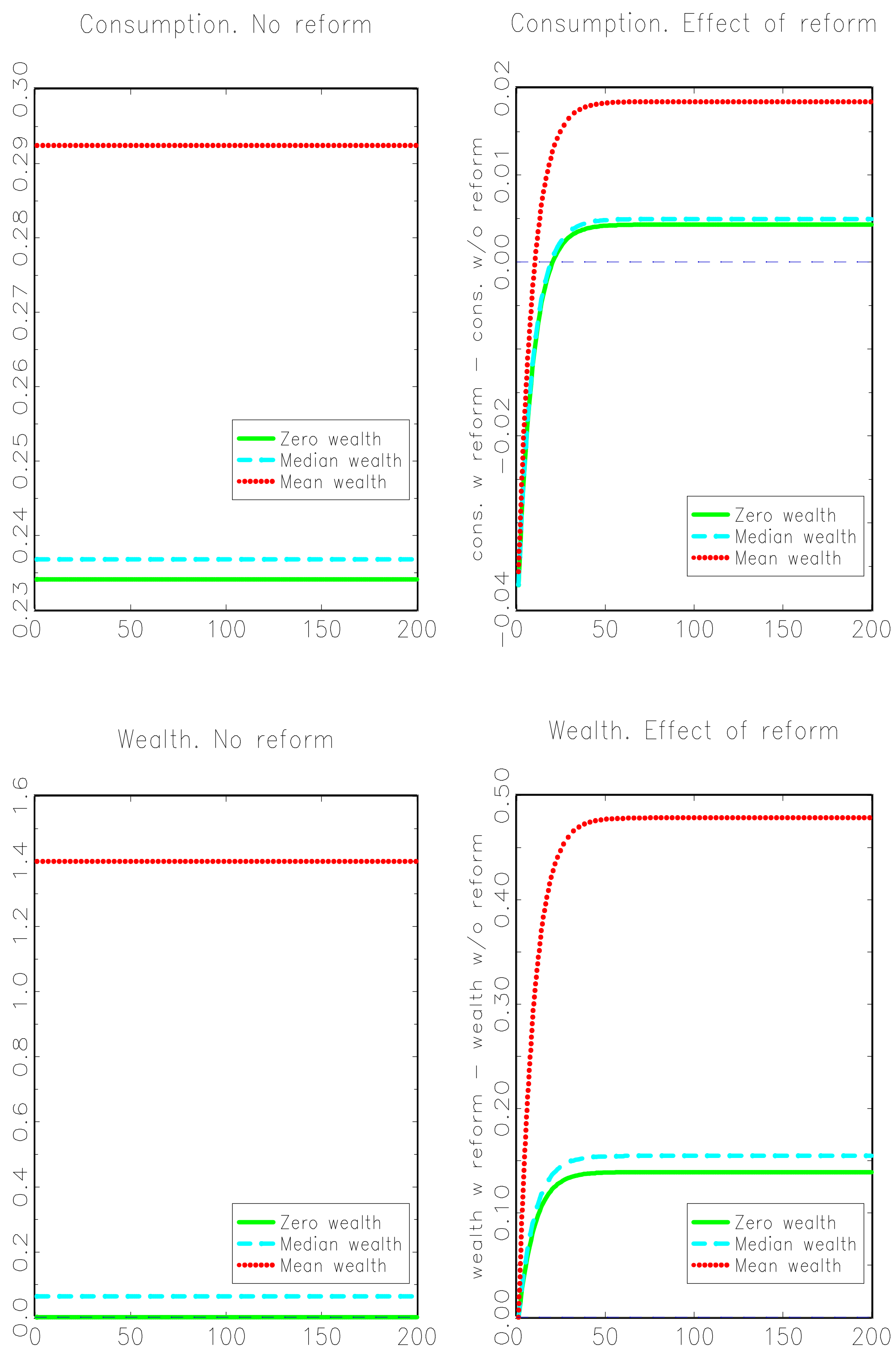

Figure 4: Complete markets economy. Tax payments and productivity by type. Benchmark tax reform

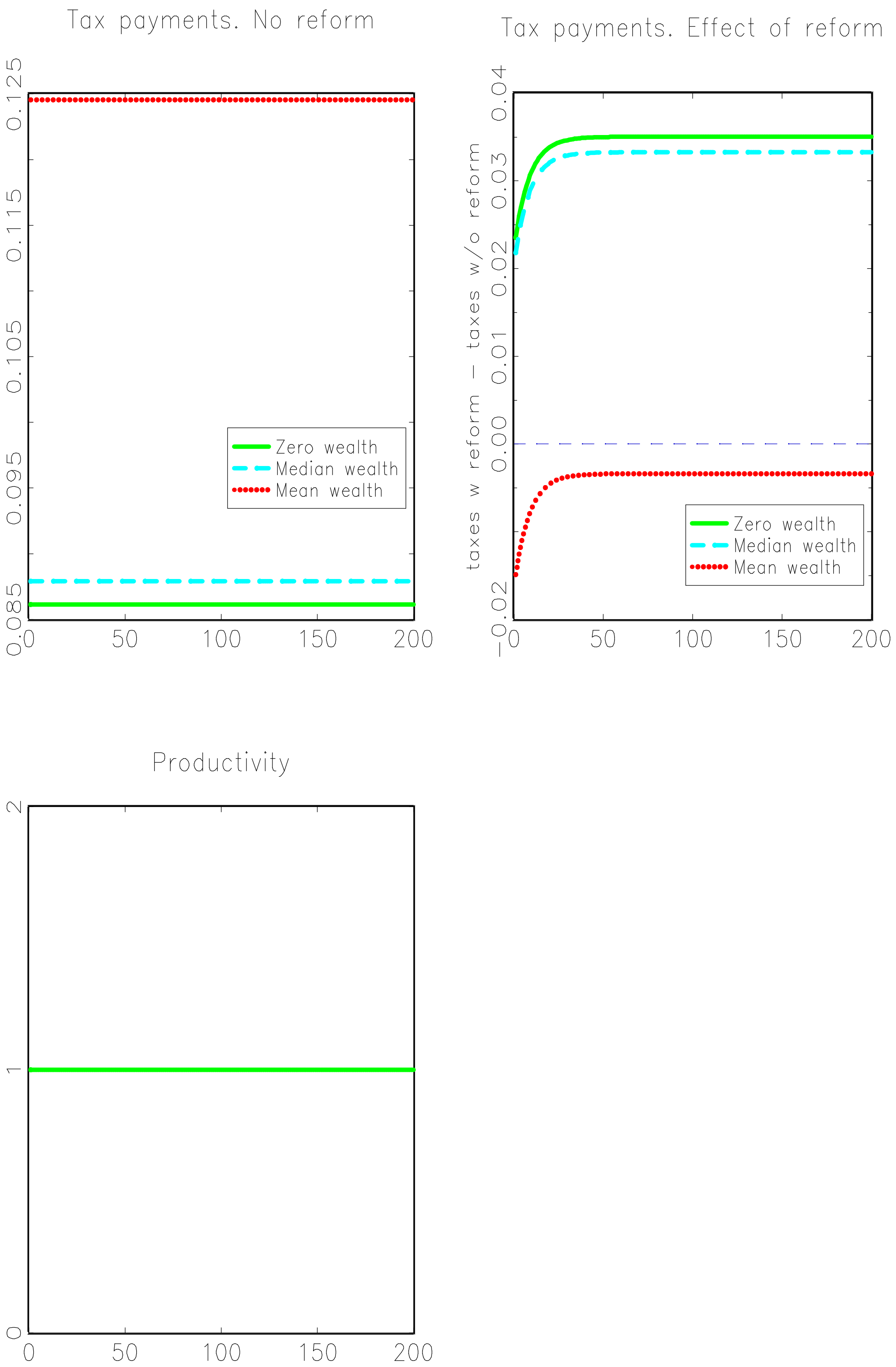

Figure 5: Incomplete markets economy. Consumption and asset holdings by type. Benchmark tax reform

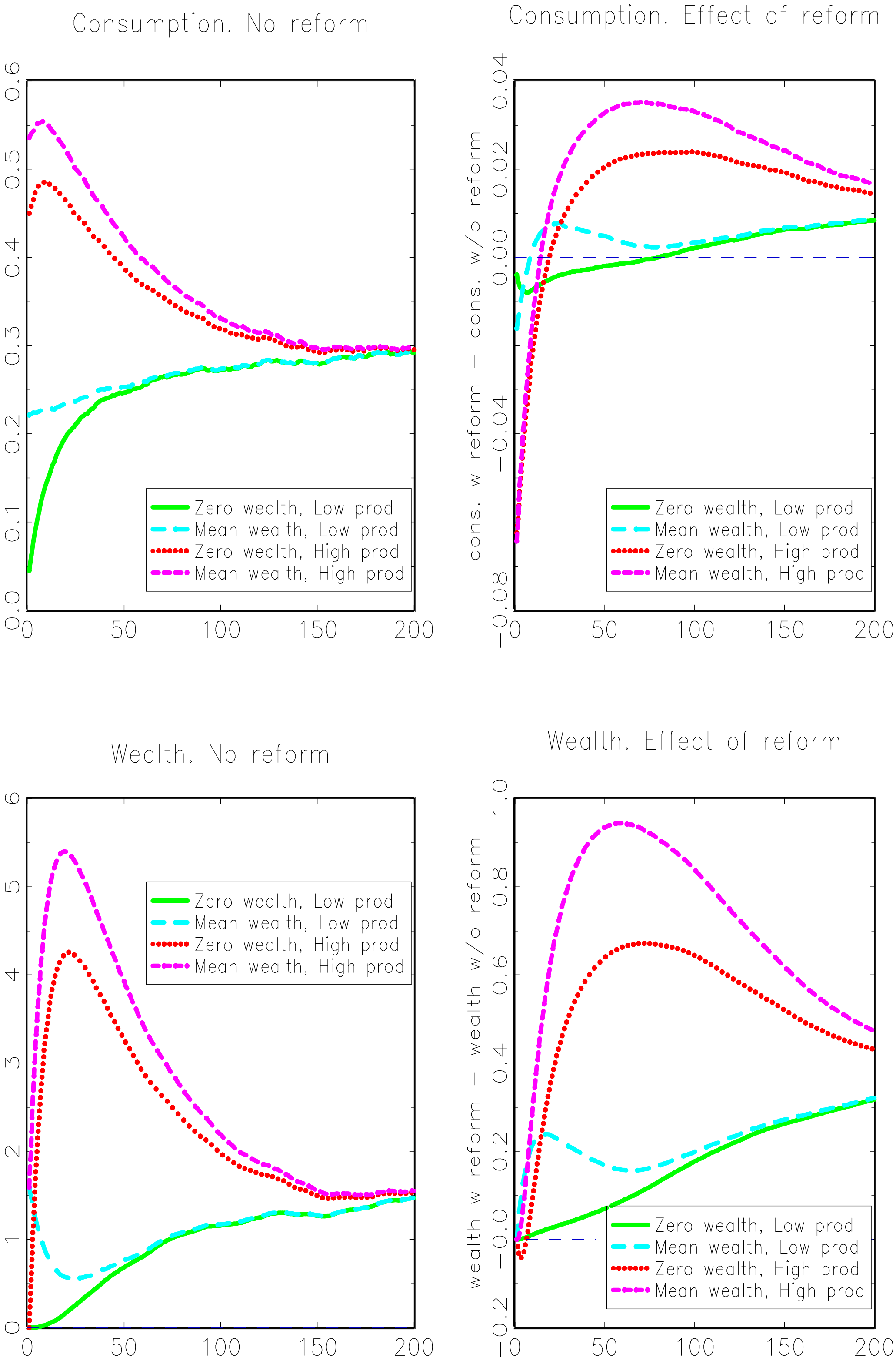

Figure 6: Incomplete markets economy. Tax payments and productivity by type. Benchmark tax reform

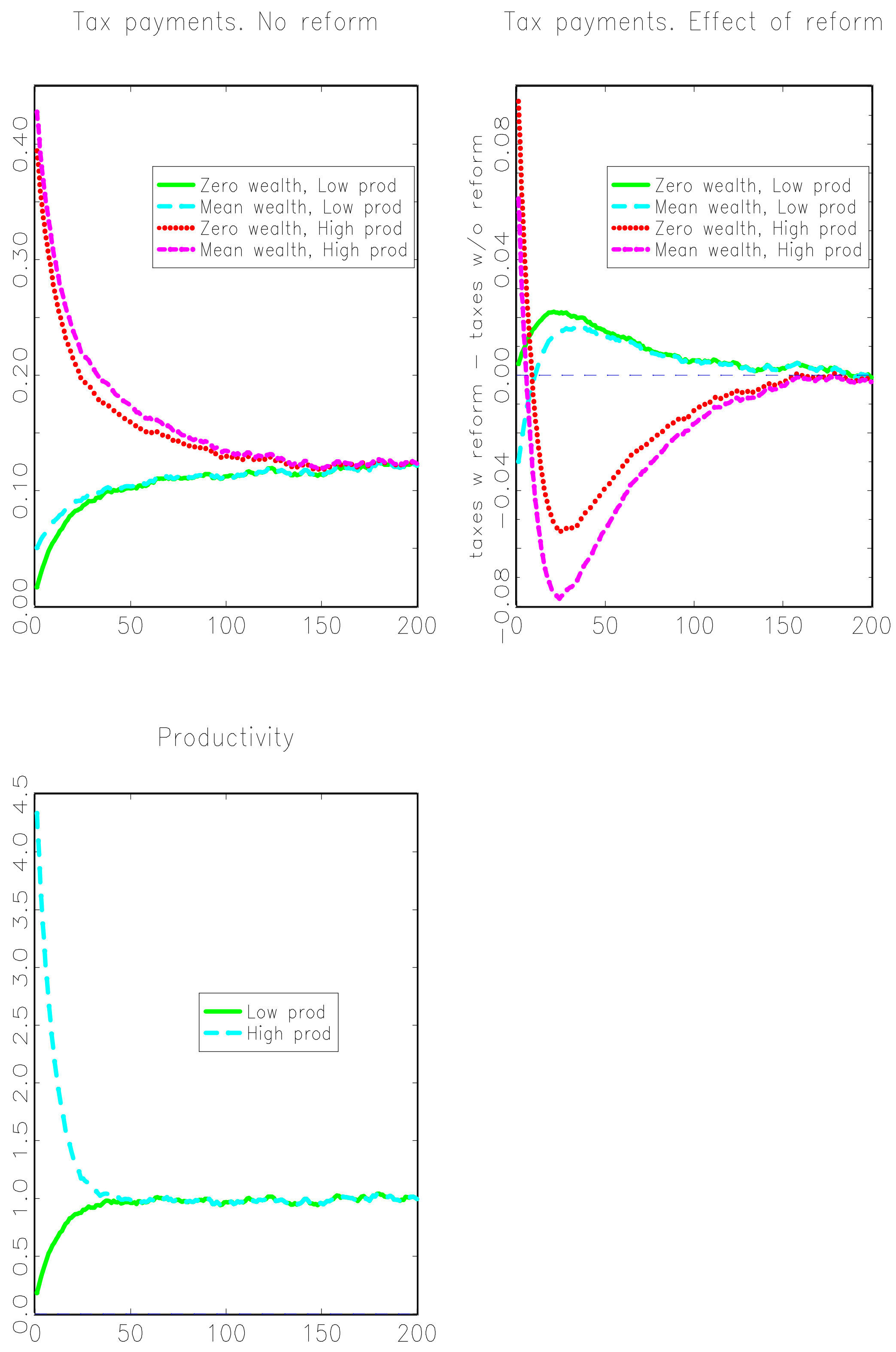

Figure 7: Distribution of gains and losses as equivalent % change in period consumption. Benchmark tax reform