EQUITY, OPTIONS AND EFFICIENCY IN THE PRESENCE OF MORAL HAZARD

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1. Introduction

Economists have long held two opposing views on the merits of the stock market and the associated corporate form of organization. On the one hand the stock market permits the substantial production risks of society to be diversified among many investors: this view underlies the capital asset pricing model (CAPM) which forms the basis for much of the modern theory of finance. On the other hand, the traditional view of classical economists, revived in modern times by Berle and Means (1932), Jensen and Meckling (1976) and the ensuing agency-cost literature, has emphasized the negative effect on incentives of the separation of ownership and control implied by the corporate form of organization. The object of this paper is to provide a theoretical framework for reconciling these two perspectives, by showing circumstances under which the stock market can provide an optimal trade-off between the beneficial effect of risk sharing and the distortive effect on incentives. We argue furthermore that, when capital markets have become sufficiently developed by the introduction of a rich array of associated options markets, incentive structures can be created using these markets which compensate for the reduced ownership shares of top executives, so that agency costs can be eliminated, permitting a Pareto optimum to be achieved by the combined trading of equity and options.

To capture the dual role of capital markets in affecting risk sharing and incentives, we consider a simple general equilibrium model of a finance economy with production. In the spirit of Knight (1921) we model the firm as an entity arising from the organizational ability, foresight and initiative of an *entrepreneur*. The activity of a firm consists in combining entrepreneurial effort and physical input (the value of capital and non-managerial labor) at an initial date: this gives rise to a random profit stream at the next date. In addition to entrepreneurs there is another class of agents which we call *investors*: they have initial wealth at date 0 but no productive opportunities. In the spirit of the principal-agent literature, we assume that the effort of entrepreneurs is not observable and that the risks to which firms are exposed are sufficiently complex to make the writing and enforcement of contracts contingent on states unfeasible (states of nature are unverifiable). Under these conditions, markets for channeling capital from investors to firms and for sharing risks must either be non-contingent or based on the realized outputs of firms. This class of financial markets includes the bond and equity markets, which have a long tradition, and the much more recently introduced markets for options on equity contracts.[1] We consider two scenarios, corresponding roughly with the different stages in the evolution of the capital markets: in the first the available

[1] While the New York Stock Exchange was established in 1772, the first organized market for trading options is of very recent origin — the Chicago Board Options Exchange was opened in 1973.

securities are limited in their ability to deal simultaneously with risk sharing and incentives; in the second the structure is very rich — more precisely it satisfies the condition which (in Section 4) is called *complete spanning with redundancy.*[2] In both settings the market structure is assumed to include both (default-free) debt and equity.

Thus entrepreneurs can obtain funds for financing their capital investment by drawing on their own initial wealth, by selling shares of their firms or by issuing debt; they can diversify their risks by buying shares of other firms and, when possible, by buying or selling options on their equity and that of other firms. Since arrangements for financing typically have to be made before production can take place, we assume that the trades on the financial markets are made *before* the entrepreneurs choose the level of effort to invest in their firms. Under these circumstances trades on the financial markets will influence the effort that entrepreneurs invest in their firms. To take the simplest example, if the markets consist solely of equity and bonds, and an entrepreneur finances his venture by selling most of the shares of his firm, he will not have much incentive to invest effort, since most of the payoff from his effort goes directly to outside shareholders.

Since investors know that the effort that an entrepreneur invests in running his firm (and hence the profit that it generates) is conditioned by his prior financing decision, they will go to considerable length to acquire the pertinent information regarding these financial decisions. In Section 2 we argue that most of the information required by investors to deduce the pattern of behavior of entrepreneurs is at least approximately available in US capital markets without prohibitive costs. We thus propose a concept of equilibrium based on two ideas. First, investors acquire the information about the financial decisions of the entrepreneurs and use this information to estimate the effort that entrepreneurs will exert. Second, rational entrepreneurs take this fact into account: this is formalized by the concept of *price perceptions.* To decide whether an investment-financing plan is optimal, an entrepreneur needs to evaluate what would happen if he were to change this plan: his price perceptions describe how he perceives that the price of his equity (and associated options) would react to any such change of plan. The price perceptions are assumed to be *rational* (i.e. entrepreneurs think that investors will correctly deduce from their investment-financing decision what their effort and the associated output of their firm will be) and *competitive* (entrepreneurs cannot affect the way the market prices risks i.e. the state prices implicit in the equilibrium prices of the securities). Putting these ideas together leads to the concept of a *stock*

[2] In this paper, since financial markets have to provide not only risk sharing but also the appropriate incentives, the distinction between the two scenarios does not reduce to the standard one between incomplete and complete markets: for example a debt and equity structure which is complete in the usual sense of spanning all possible income streams, does not satisfy the condition of complete spanning with redundancy.

market equilibrium with rational, competitive price perceptions (an RCPP equilibrium).

The financing decision of an entrepreneur affects both his incentives to invest effort in his firm and his ability to diversify risks. Creating greater incentives can typically only be made at the cost of creating greater risks. Such a trade-off between incentives and risk sharing has been extensively studied in the principal-agent literature.[3] In the setting that we consider however, there is no principal who directly designs a contract to induce entrepreneurs to behave in an optimal way: whatever incentive schemes there are must somehow be created by the markets. It is thus natural to ask whether the capital markets can create payoff functions for the agents which lead to a socially optimal balance between incentives and risk sharing.

This question is answered in two steps. We first consider market structures which are limited in their ability to share risks and provide incentives and examine if an RCPP equilibrium leads to the best allocation which can be obtained with such a market structure. The appropriate concept for studying this question is the concept of *constrained Pareto optimality* in which a benevolent planner replaces the agents as the single Olympian decision-maker choosing the variables — capital inputs and portfolios of financial securities — which agents would otherwise choose on the markets. We show that an RCPP equilibrium is constrained Pareto optimal. The non-observability of the actions of the entrepreneurs creates an externality, since entrepreneurs choose their effort levels in their own interests without taking into account the interests of outside shareholders. This externality is explicitly taken into account by the planner when choosing the financial variables of the agents. The crucial ingredient in an RCPP equilibrium, which leads each entrepreneur to make the same choice as the planner, is the *rational price perception function*: each entrepreneur realizes that the market value of his firm depends on the effort that investors expect from him, and in seeking to maximize the market value of his firm the entrepreneur is induced to align his interests with those of the outside shareholders of his firm. In short the perception function leads each entrepreneur to internalize the externality involved in his decision.

Thus, for any financial structure, an RCPP equilibrium functions well, exploiting optimally the existing financial markets. This leads naturally to the second question: is it possible to obtain first-best optimality with a sufficiently rich market structure? As noted by Ross (1976), introducing options on existing equity contracts is the lowest transaction-cost method for increasing the spanning opportunities of the capital markets. This observation has been strikingly confirmed by the subsequent developments on US capital markets: the number and volume of trades in such securities has grown dramatically in the last twenty years. Along with greatly refined hedging

[3] See e.g. Ross (1973), Holmström (1979), Rogerson (1985), Jewitt (1988), and Kreps (1990, Chapter 16).

opportunities, options provide a powerful and convenient substitute for direct ownership as a way of aligning the interests of top managers of a firm with those of outside shareholders. In this latter role, in the last ten years options have come to represent almost 40 % of the compensation of CEOs in the largest companies in the US (see Murphy (1998)). In the last section of the paper we study if theory can justify practice, i.e. if a rich menu of options permits both optimal risk sharing and optimal incentives to be achieved. The result is essentially positive, modulo technical difficulties. Options are extremely efficient instruments for increasing spanning and providing managers with incentives, but they have the unfortunate technical drawback of introducing non-convexities into the model. In the absence of options, the effort-choice problem of an entrepreneur is convex: when options are introduced the problem becomes non-convex and has a complicated structure because of the changes of regime induced by options going in and out of the money when effort is changed. Replacing the effort-choice problem by its first-order condition leads to what we call in Section 4 a weak-RCPP equilibrium. Under the condition of complete spanning with redundancy — which is satisfied when there are options with striking prices between the possible values of the firms' outputs — every weak-RCPP equilibrium is Pareto optimal. Analysis of examples suggests that with appropriate striking prices for the options, weak-RCPP equilibria are RCPP equilibria.[4]

The idea that financial decisions of agents transmit information about their characteristics or actions which are not directly observable or knowable by the market, has been extensively explored in the finance literature. Concepts of equilibrium based on this idea and the idea of rational expectations have been used in many partial equilibrium models: for *adverse selection* in the signaling models of Ross (1977), and Leland and Pyle (1977), and the subsequent literature (see Harris and Raviv (1992) for a survey); for problems of *moral hazard* by Jensen and Meckling (1976), Grossman and Hart (1982), and Brander and Spencer (1989). These concepts of equilibrium are close in spirit to an RCPP equilibrium. This paper differs from these latter contributions in that it provides a framework in which the risk-sharing function of financial markets and their disciplining role in attenuating the agency costs of firms can be studied simultaneously.

More recently, some authors (Lisboa (1998), Kocherlakota (1998)) have studied general equilibrium models with moral hazard, under the assumption of observability of trades. Since these models are generalizations of the principal-agent model with fixed outcomes, the moral hazard and risk-sharing problems cannot be simultaneously fully resolved and only second-best outcomes can

[4]In the standard model studied in the principal-agent literature it is assumed that the outcomes are fixed and that effort influences the probability of these outcomes. Sufficient conditions for the agent's effort-choice problem to be convex at the second-best optimum have been given by Rogerson (1985) and Jewitt (1988). At the moment we have not found similar assumptions which ensure that the first-order conditions are sufficient for characterizing the optimal effort, for the case where effort influences outcomes and options are used for creating incentives.

be obtained. Bisin-Gottardi (1997) have also studied a class of general equilibrium models with asymmetric information: they exhibit a variety of restrictions on agents' trades which make it possible to establish existence of an equilibrium, a problem which we examine only briefly in Section 4.

The paper is organized as follows. Section 2 presents the basic model of a stock-market economy with moral hazard and introduces the concept of an RCPP equilibrium. Section 3 analyzes the constrained Pareto optimality of an RCPP equilibrium. Section 4 studies the conditions under which a first-best optimum can be obtained.

2. Equilibrium with Rational Competitive Price Perceptions

Consider a two-period one-good economy with production in which investment of both capital and effort at date 0 is required to generate output at date 1, the output at date 1 being uncertain. There are two types of agents in the economy, *entrepreneurs* and *investors*: $\mathcal{I}_1 \neq \emptyset$ is the set of entrepreneurs, $\mathcal{I}_2 \neq \emptyset$ the set of investors and $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2$ is the set of all agents, which is assumed to be finite.[5] Every agent $i \in \mathcal{I}$ has an initial wealth ω_0^i at date 0. If agent i is an entrepreneur, then by investing capital $\kappa^i \in \mathbb{R}_+$ (an amount of the good (income)) and effort $e^i \in \mathbb{R}_+$ at date 0 he can obtain the uncertain stream of income

$$\boldsymbol{F}^i(\kappa^i, e^i) = (F_1^i(\kappa^i, e^i), \ldots, F_S^i(\kappa^i, e^i))$$

at date 1, where $\mathcal{S} = \{1, \ldots, S\}$ is the set of states of nature describing the uncertainty and $\boldsymbol{F}^i : \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+^S$ is a concave, differentiable, non-decreasing function, with $\boldsymbol{F}^i(\kappa^i, 0) = \boldsymbol{F}^i(0, e^i) = \mathbf{0}$. Investors are agents who do not undertake productive ventures, so if $i \in \mathcal{I}_2$, then $\boldsymbol{F}^i(\kappa^i, e^i) \equiv \mathbf{0}$.

Each agent has a utility function $U^i : \mathbb{R}_+^{S+1} \times \mathbb{R}_+ \longrightarrow \mathbb{R}$, where $U^i(\boldsymbol{x}^i, e^i)$ is the utility associated with the consumption stream $\boldsymbol{x}^i = (x_0^i, x_1^i, \ldots, x_S^i)$ and the effort level e^i. The utility function is assumed to be separable[6]

$$U^i(\boldsymbol{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(x_1^i, \ldots, x_S^i) - c^i(e^i)$$

[5] Sets are denoted by calligraphic letters: the same letter in roman denotes the cardinality of the set, e.g. $\mathcal{I} = \{1, \ldots, I\}$. Vectors, matrices and vector-valued functions are written in boldface.

[6] The separability in date 0 consumption x_0^i significantly simplifies the definition of an RCPP equilibrium in which the choice of date 0 consumption (and the financial variables) of an entrepreneur is based on a computation by backward induction of his choice of effort and consumption at date 1. The separability in effort is not an essential simplifying assumption and can be dropped without affecting any results: it is adopted to make the cost-benefit analysis of an entrepreneur's effort decision more intuitive.

where the functions u_0^i, u_1^i are differentiable, strictly concave, increasing, and c^i is differentiable, convex, increasing, with $c^i(0) = 0$.

To ensure that the technology of each entrepreneur i is sufficiently productive to make it worthwhile to operate, we assume that, as soon as entrepreneur i invests some positive effort in his firm, the marginal productivity of capital at zero is infinite. More precisely we assume that for all $i \in \mathcal{I}_1$ there is a smooth path $e^i : [0,1] \longrightarrow \mathbb{R}_+$ with $e^i(0) = 0$ and $e^{i\prime}(t) > 0$ such that

$$\lim_{t \to 0} c^{i\prime}(e^i(t))e^{i\prime}(t) < \infty \quad \text{and} \quad \lim_{t \to 0} \frac{\partial \boldsymbol{F}_s^i}{\partial z^i}\Big(t, e^i(t)\Big) = \infty, \quad \text{for some } s \in \mathcal{S}.$$

It is easy to see[7] that this assumptiom implies that for all $\boldsymbol{x}^i = (x_0^i, \boldsymbol{x}_1^i) \in \mathbb{R}_+^{S+1}$ with $x_0^i > 0$, there exist $(\kappa^i, e^i) \gg 0$ such that

$$u_0^i(x_0^i - \kappa^i) + u_1^i(\boldsymbol{x}_\mathbf{1}^i + \boldsymbol{F}^i(\kappa^i, e^i)) - c^i(e^i) > u^i(x_0^i) + u^i(\boldsymbol{x}_\mathbf{1}^i)$$

so that even if there were no market to finance the capital investment, entrepreneur i would choose to produce. To bound the economy we assume that, for any positive level of capital input, the marginal cost of effort eventually exceeds its marginal product

$$\frac{\partial F^i(\kappa^i, e^i)}{\partial e^i} \to 0, \quad \text{and} \quad c^{i\prime}(e^i) \to \infty \ \text{ when } e^i \to \infty, \quad i \in \mathcal{I}_1$$

This implies that, for a given level of capital, the effort chosen by entrepreneur i will always remain bounded.

Market Structure. The analysis of the paper rests on two basic assumptions. The first is that *effort is not observable*, so that contracts cannot be written contingent on the effort invested by entrepreneurs in their firms. This would not create a problem without the second assumption: *states of nature are not verifiable*, so that the enforcement of contracts contingent on states is not feasible. For if the uncertain contingencies to which businesses are exposed were sufficiently simple

[7] Let ΔU^i denote the difference in entrepreneur i's utility between investing $(\kappa^i, e^i(\kappa^i))$ and investing $(0,0)$, where $e^i(\cdot)$ is the function just defined, and let $\kappa^i \leq \min\,\{x_0^i/2, 1\}$. Then

$$\Delta U^i = -\int_0^{\kappa^i} u_0^{i\prime}(x_0^i - t)dt - \int_0^{\kappa^i} c^{i\prime}(e^i(t))e^{i\prime}(t)dt + \int_0^{\kappa^i} \nabla u_1^i(\boldsymbol{x}_\mathbf{1}^i + \boldsymbol{F}^i(t, e^i(t)))\cdot(D_{\kappa^i}\boldsymbol{F}^i(t, e^i(t)) + D_{e^i}\boldsymbol{F}^i(t, e^i(t))e^{i\prime}(t))dt$$

Set $k^i = u_0^{i\prime}(x_0^i/2)$, $K_s^i = \min_{t \in [0,1]} \frac{\partial u_1^i}{\partial x_s^i}(x_\mathbf{1}^i + \boldsymbol{F}^i(t, e^i(t))$, $s \in \mathcal{S}$, $\boldsymbol{K}^i = (K_1^i, \ldots, K_S^i)$. Then,

$$\Delta U^i \geq \int_0^{\kappa^i} (\boldsymbol{K^i} \cdot D_{\kappa^i}\boldsymbol{F}^i(t, e^i(t)) - k^i - c^{i\prime}(e^i(t))e^{i\prime}(t)dt$$

where the right side of the inequality is positive for $\kappa^i > 0$ sufficiently small.

to make the writing and enforcement of state contingent contracts enforceable, then, without loss of generality, it could be assumed that there is a complete set of Arrow securities: in such an economy an efficient allocation can be obtained by letting each entrepreneur be the sole proprietor of his firm and using Arrow securities to share the productive risks (see Section 4). But the only contracts which are assumed to be enforceable are either non-contingent contracts such as debt, or contracts contingent on the realized outputs of firms, namely equity contracts and options on equity.

Trading contracts contingent on realized output when there is no separate market for "effort" is liable however to lead to inefficiencies. For typically entrepreneurs need funds to finance their firms. If they only have access to borrowing (debt), and if there are penalties for default, then they will have to restrict the amount they borrow for fear of bad contingencies, and in addition will be exposed to rather risky leveraged positions. The traditional remedy lies in introducing the possibility of financing by issuing equity: this additional source of funds, permits entrepreneurs to share the risks involved in their productive activity with investors, and opens up the possibility for all agents to diversify their risks. But selling ownership shares of his firm has negative incentive effects on the entrepreneur, since any increment to effort is no longer rewarded by the full value of its marginal product. Debt and equity contracts however constitute only the first stage in the development of financial markets to meet the financing needs of firms; when financial markets become more sophisticated, a second stage consists in the introduction and systematic use of derivative securities. Such securities serve two roles: they increase the risk-sharing possibilities of agents (the span of the markets) and provide instruments for creating incentives for managers of firms. Holding an option which is worthless unless the profit of the firm exceeds the striking price of the option provides a strong incentive for a manager to exert the extra effort needed to assure that the profit stream is likely to surpass this level. Thus the use of options can potentially restore some, or even all, of the incentives of entrepreneurs lost in reducing their equity shares to finance their firms.

The financial contracts which the agents in the economy can trade are thus taken to be: first, the default-free bond with (date 0) price q_0 and payoff stream $\mathbf{1} = (1, \ldots, 1)$ at date 1; second for each firm $i \in \mathcal{I}_1$, its equity contract with price q_y^i and date 1 payoff stream $\boldsymbol{y}^i = (y_1^i, \ldots, y_S^i)$; finally for each firm a family of derivative securities on its equity, consisting of call options with different striking prices. Note that there is no loss of generality in restricting the analysis to this class of options, since the payoff of a put option can always be reconstructed from a suitable portfolio of the equity, the bond and a call option with the same striking price. Moreover with a finite state space, put and call options generate the maximum dimensional subspace that can be spanned by securities whose dividend streams are non-linear functions of the payoff of the underlying equity (Ross [1976, Theorem 2]). Thus let $\mathcal{J}^i \subset \mathbb{N}$ denote the index set for the call options on the equity of firm i, and let $\boldsymbol{\tau}^i = (\tau^i_j)_{j \in \mathcal{J}^i}$ denote the vector of associated striking prices, with $\boldsymbol{\tau}^i \in \mathbb{R}^{J^i}_+$. The call option (i, j) — the j^{th} option of firm i — has the price q^i_j at date 0 and the payoff stream $\boldsymbol{R^i_j} = (R^i_{j,1} \ldots, R^i_{j,S})$ across the states at date 1 given by

$$R^i_{j,s} = \max \{y^i_s - \tau^i_j, 0\}, \quad s \in \mathcal{S}$$

where $y^i_s = F^i_s(\kappa^i, e^i)$ denotes the output of firm i in state s. When it is important to stress that the choice of (κ^i, e^i) influences the payoff of the equity and thus of the option, we use the notation

$$R^i_{j,s}(\kappa^i, e^i) = \max \{F^i_s(\kappa^i, e^i) - \tau^i_j, 0\}, \quad s \in \mathcal{S}$$

Let $\boldsymbol{R}^i$ (or $\boldsymbol{R}^i(\kappa^i, e^i)$) denote the $S \times J^i$ matrix of payoffs of the J^i options of firm i, $\boldsymbol{R}^i_s$ the row vector of payoffs of the J^i options on firm i in state s and $\boldsymbol{R}^i_j$ the column vector of payoffs of option j across the states. Let $\mathcal{J} = \cup_{i \in \mathcal{I}_1} \mathcal{J}^i$ denote the set of all options and let $\boldsymbol{\tau} = (\boldsymbol{\tau}^i)_{i \in \mathcal{I}_1}$ denote the associated striking prices. The economy with characteristics $\boldsymbol{U} = (U^i)_{i \in \mathcal{I}}$, $\boldsymbol{\omega}_0 = (\omega^i_0)_{i \in \mathcal{I}}$, $\boldsymbol{F} = (\boldsymbol{F}^i)_{i \in \mathcal{I}_1}$ for the agents, and with a market structure composed of the riskless bond, the equity contracts of the I_1 firms, and the set of options $\mathcal{J}$ with striking prices $\boldsymbol{\tau}$, will be denoted $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$. In such an economy, we let $\boldsymbol{q}_y = (q^i_y)_{i \in \mathcal{I}_1}$ denote the vector of equity prices, $\boldsymbol{q}^i_c$ the vector of prices of the J^i call options of firm i, $\boldsymbol{q}^i = (q^i_y, \boldsymbol{q}^i_c)$ the vector of prices which are influenced by the actions of entrepreneur i, and $\boldsymbol{q} = (q_0, (\boldsymbol{q}^i)_{i \in \mathcal{I}})$ the vector of all security prices.

Budget Constraints. To simplify the analysis we assume that the penalty for default for individual agents and for bankruptcy[8] by firms is infinite so that the personal debt of an entrepreneur and the debt incurred by his firm are both default-free debt. With no default and no bankruptcy there is no loss of generality in assuming that the entrepreneur is personally responsible for the debt of his firm.

At date 0 entrepreneur i decides on the amount of capital κ^i to invest in his firm, on the amount to borrow ξ^i_0 (if $\xi^i_0 > 0$, lend if $\xi^i_0 < 0$), and on the share $(1 - \theta^i_i)$ of his firm to sell. He also purchases shares θ^i_k of other firms $k \neq i$, as well as amounts $\xi^i_{k,j}$ of the options of these firms $(j \in \mathcal{J}^k, k \neq i)$ to diversify his risks. The purchase of a portfolio of options $(\xi^i_{i,j})_{j \in \mathcal{J}^i}$ on his own equity contract

[8] When the set of options on firm i is sufficiently rich, the no-bankruptcy assumption is not restrictive, since with the possibility of bankruptcy the equity contract becomes a *call* option and its debt contract a *put* option on firm i's profit. The payoffs of these contracts can be reconstructed with the existing securities. The notation of this paper is much simplified by not introducing explicitly the possibility of bankruptcy, and thus taking the payoff of an equity contract to be the vector $(y^i_s), s \in \mathcal{S}$.

serves as an incentive device to "bond" his personal interest to those of the outside shareholders of his firm (as we shall see shortly). Let $\boldsymbol{\theta}^i = (\theta^i_k)_{k\in\mathcal{I}_1}$, denote the equity portfolio, $\boldsymbol{\xi}^i_k = (\xi^i_{k,j})_{j\in\mathcal{J}^k}$ the portfolio of options of firm k and $\boldsymbol{\xi}^i = (\xi^i_0, (\xi^i_k)_{k\in\mathcal{I}_1})$ the portfolio of all securities in zero net supply (bond and options) held by agent i. If entrepreneur i anticipates the production $(\boldsymbol{y}^k)$ of other entrepreneurs, then a choice of the financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, in conjunction with a choice of effort e^i, leads to a vector of consumption $\boldsymbol{x}^i = (x^i_0, x^i_1, \dots, x^i_S)$ given by

$$x^i_0 = \omega^i_0 - q_0\xi^i_0 - \sum_{k\neq i} q^k_y\theta^i_k \; - \; \sum_{k\neq i} \boldsymbol{q}^k_c\,\boldsymbol{\xi}^i_k + q^i_y(1-\theta^i_i) - \boldsymbol{q}^i_c\boldsymbol{\xi}^i_i - \kappa^i \tag{1}$$

$$x^i_s = \xi^i_0 + \sum_{k\neq i}\theta^i_k y^k_s + \sum_{k\neq i}\boldsymbol{R}^k_s\boldsymbol{\xi}^i_k + F^i_s(\kappa^i, e^i)\theta^i_i + \boldsymbol{R}^i_s(\kappa^i, e^i)\boldsymbol{\xi}^i_i, \quad s\in\mathcal{S} \tag{2}$$

If agent i is an investor, then the budget equations are the same with $\kappa^i = 0, e^i = 0, \boldsymbol{F}^i = \boldsymbol{0}, q^i_y = q^i_j = 0$, so that the terms related to his own "firm" are just dummy variables.[9]

It is clear from equations (2) that the date 1 reward of an entrepreneur for his effort depends on his choice of financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. This captures the idea that the capital structure of a firm (in particular the inside equity and options held by the manager, and the firm's debt) affects the performance of its management. Since financing arrangements must be in place before a firm can become operational, we assume that the choice of effort e^i by an entrepreneur is made after the financial decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ have been determined. To make this sequential structure of decision making explicit, it is useful to introduce the following timing of the agents' decisions. Date 0 is divided into two subperiods, $0_1, 0_2$. In subperiod 0_1 entrepreneurs use the financial markets to obtain the capital required to set up their firms and diversify their risks: in the second subperiod 0_2, after the investment and financing decisions have been made, firms become "operational" and entrepreneurs decide on the appropriate effort to invest in running their firms. At date 1 "nature" chooses a state of the world $s \in \mathcal{S}$: production takes place and profit is realized.

Optimal Effort. After entrepreneur i has chosen his financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ in subperiod 0_1 (in a way that we study later), in subperiod 0_2 he chooses the effort level e^i which maximizes $u^i_1(\boldsymbol{x}^i_{\mathbf{1}}) - c^i(e^i)$, where $\boldsymbol{x}^i_{\mathbf{1}} = (x^i_1, \dots, x^i_S)$ is the date 1 consumption stream given by (2). Entrepreneur i's financial variables are of two kinds: *inside* variables (those internal to the firm) which directly affect the payoff (reward) of the entrepreneur from his effort, and the *outside* variables (external to the firm) which determine the income that agent i gets independently of his effort. $(\kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i)$ are the inside financial variables which determine his *inside income* (the last two

[9]When we need unified notation for both types of agents, entrepreneurs and investors, we adopt the convention that for $k \in \mathcal{I}_2$, $\theta^i_k = 0$ if $i \neq k$, $\theta^k_k = 1$ and $\mathcal{J}^k = \emptyset$.

terms in (2)), while $(\xi_0^i, (\theta_k^i, \boldsymbol{\xi}_k^i)_{k \neq i})$ are the outside variables which determine his *outside income* $\boldsymbol{m}^i = (m_1^i, \dots, m_S^i)$ defined by

$$\boldsymbol{m}^i = \mathbf{1}\, \xi_0^i + \sum_{k \neq i} (\boldsymbol{y}^k\, \theta_k^i + \boldsymbol{R}^k \boldsymbol{\xi}_k^i) \tag{3}$$

namely the first three terms in (2). Define the *effort correspondence* of entrepreneur i

$$\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i) = \arg\max_{e^i \geq 0} \left\{ u_1^i(\boldsymbol{x}_\mathbf{1}^i) - c^i(e^i) \,\middle|\, \boldsymbol{x}_\mathbf{1}^i = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i \right\} \tag{E}$$

Since we have assumed that the marginal productivity of effort tends to zero when effort tends to infinity while its marginal cost tends to infinity, the problem (E) has a maximum for some finite value of e^i and the correspondence $\tilde{e}^i$ is well-defined on the domain $\mathcal{D}^i \subset \mathbb{R}_+^S \times \mathbb{R} \times \mathbb{R}^J \times \mathbb{R}^{J^i}$ consisting of the variables $(\boldsymbol{m}^i, \kappa^i, \theta_i^i, (\xi_{i,j}^i)_{j \in \mathcal{J}^i})$ such that $\boldsymbol{m}^i + \theta_i^i \boldsymbol{F}^i(\kappa^i, e^i) + \sum_{j \in \mathcal{J}^i} \xi_{i,j}^i \boldsymbol{R}_j^i(\kappa^i, e^i) \in \mathbb{R}_{++}^S$ for some $e^i > 0$. In the special case where there are no options ($\mathcal{J}^i = \emptyset$), the assumptions of strict concavity of u_1^i, convexity of c^i and concavity of $\boldsymbol{F}^i$ ensure that the solution to (E) is unique, so that $\tilde{e}^i$ is a function on $\mathcal{D}^i$. When $\mathcal{J}^i \neq \emptyset$, the payoffs of the options introduce a non-convexity into the constraint set in E, and the solution of the maximum problem may not be unique: in this case $\tilde{e}^i$ is a correspondence defined on $\mathcal{D}^i$.

RCPP Equilibrium. Consider an investor[10] who is thinking of buying either the equity or options of firm i. To anticipate what the firm's profit will be, the investor needs to anticipate the entrepreneur's inputs (κ^i, e^i). In this model we assume that the capital input κ^i is observable, while the effort e^i is not. However, as we have seen, e^i can be deduced if the entrepreneur's characteristics $(u_1^i, \boldsymbol{F}^i, c^i)$ and his financial variables, or more precisely his outside income $\boldsymbol{m}^i$ and the inside financial variables $(\kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$, are known: in the analysis that follows we assume that investors do indeed have access to this information and hence can deduce the effort e^i that the entrepreneur will invest in his firm.

In practice there is an important distinction between accessibility of information regarding the inside financial variables $(\kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$ and information regarding the outside wealth $\boldsymbol{m}^i$ and characteristics $(u_1^i, \boldsymbol{F}^i, c^i)$ of a firm's manager. Disclosure rules of the Securities and Exchange Commission require that proxy statements of publicly traded firms contain information regarding capital projects of the firm, as well as the equity and options holdings of the top management. Thus the assumption

[10] In the discussion that follows we use the term "investor" in an extended sense: it refers not only to agents in $\mathcal{I}_2$ but also to any agent who buys securities for which he is not in an insider position. Thus for example entrepreneur k buying shares of firm i with $k \neq i$ is considered as an investor on firm i's equity market.

that inside variables are known by investors conforms with the regulations of capital markets in the US.

More detailed information regarding the characteristics of the firm and its manager are less directly accessible, and it is essentially the job of security analysts to gain access to this type of information. While this information may not be available with the precision required by the model, analysts will however in the course of scrutinizing the earnings prospects of the firms they follow, acquire a good knowledge of the characteristics of the firms and their top management. Analysts who have followed the careers of top executives are likely to have a good estimate of the magnitude of their personal wealth and hence can impute at least the orders of magnitude of their outside incomes. Past performance gives information on their ability — which in the model is included in the function $\boldsymbol{F}^i$ — and their motivation and ability to take risks — in the model, the functions u_1^i and c^i. The information collected by analysts spreads to investors through advisory services and the recommendations given by large brokerage companies. The assumption that the characteristics and financial trades of the entrepreneurs are known by all agents is thus the theoretical limit of a situation in which both the rules of disclosure and the activity of professionals in financial services result in a large amount of information being available to investors in the market.

If entrepreneurs' financial trades are known to investors, if investors make optimal use of this information to anticipate the outputs of firms, and in this way come to decide on the prices they are prepared to pay for the equity and options of the firms, then it seems reasonable to suppose that entrepreneurs will come to understand this. Hence our second assumption regarding anticipations: entrepreneurs are aware that investors will use their financial decisions as "signals" of the effort that they will exert in their firms. The next step is to incorporate these two assumptions — namely that (1) investors use the available information (the financial variables) to correctly anticipate the firms' outputs, (2) entrepreneurs understand this — into a concept of equilibrium.

The description of an equilibrium thus consists of two parts. The first is the standard part which enumerates the *actions* of the agents and the *prices* of the securities; the second part describes the entrepreneurs' *perceptions* of the way their financial decisions affect the price that the "market" will pay for the securities — equity and options — based on the profit of their firm. Let $\tilde{\boldsymbol{Q}}^i = (\tilde{Q}_y^i, \tilde{\boldsymbol{Q}}_c^i) = (\tilde{Q}_y^i, (\tilde{Q}_j^i)_{j\in\mathcal{J}^i})$ denote the price perception of entrepreneur i where

$$\tilde{Q}_\beta^i : \mathbb{R}_+ \times \mathbb{R}^I \times \mathbb{R}^J \longrightarrow \mathbb{R}_+,\ \beta = y \text{ or } j,\ j \in \mathcal{J}^i$$

is the price that entrepreneur i expects for security β (his equity, or an option on his equity) if he chooses the financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. Let $\tilde{\boldsymbol{Q}} = (\tilde{\boldsymbol{Q}}^1, \ldots, \tilde{\boldsymbol{Q}}_1^I)$ denote the price perceptions of all entrepreneurs.

It is useful to define the following date 1 payoff matrices associated with the different securities in the economy. Let $\boldsymbol{V}^0 = (1, \ldots, 1)^T$ be the payoff of the riskless bond and, for a vector $\boldsymbol{y} = (\boldsymbol{y}^i)_{i \in \mathcal{I}_1}$, let $\boldsymbol{V}^i(\boldsymbol{y}) = [\boldsymbol{y}^i, \boldsymbol{R}^i(\boldsymbol{y}^i)]$ denote the $S \times (1 + J^i)$ matrix of payoffs of the securities of firm i. $\boldsymbol{V}(\boldsymbol{y}) = [\boldsymbol{V}^0, \boldsymbol{V}^1(\boldsymbol{y}), \ldots, \boldsymbol{V}^{I_1}(\boldsymbol{y})]$ denotes the $S \times [1 + (1 + J^1) + \ldots + (1 + J^{I_1})]$ payoff matrix of all the securities and $\boldsymbol{V}_{-i}(\boldsymbol{y}) = [\boldsymbol{V}^0, \ldots, \boldsymbol{V}^{i-1}(\boldsymbol{y}),\, \boldsymbol{V}^{i+1}(\boldsymbol{y}) \ldots \boldsymbol{V}^{I_1}(\boldsymbol{y})]$ is the payoff matrix of all securities other than those of firm i. The associated subspaces of $\mathbb{R}^S$ generated by the columns of the above matrices are denoted by $\mathcal{V}^0, \mathcal{V}^i(\boldsymbol{y}), \mathcal{V}(\boldsymbol{y})$ and $\mathcal{V}_{-i}(\boldsymbol{y})$ respectively: we call $\mathcal{V}^i(\boldsymbol{y})$ the *firm i-subspace* and $\mathcal{V}(\boldsymbol{y})$ the *marketed subspace* at $\boldsymbol{y}$.

A vector of prices $\boldsymbol{q}$ which prices the basic securities in the model (the columns of the matrix $\boldsymbol{V}(\boldsymbol{y})$) leads to a valuation of every income stream in the marketed subspace $v_q : \mathcal{V}(\boldsymbol{y}) \to \mathbb{R}$ defined by

$$v_q(\boldsymbol{m}) = q_0 \xi_0 + \boldsymbol{q}_y \boldsymbol{\theta} + \sum_{i \in \mathcal{I}_1} \boldsymbol{q}_c^i \, \boldsymbol{\xi}_i$$

where $\boldsymbol{z} = (\xi_0, \boldsymbol{\theta}, (\boldsymbol{\xi}_i)_{i \in \mathcal{I}_1})$ is any portfolio such that $\boldsymbol{m} = \boldsymbol{V}(\boldsymbol{y})\boldsymbol{z}$. The valuation v_q is well-defined if the vector of prices $\boldsymbol{q}$ does not offer any arbitrage opportunities — a property which is equivalent to the existence of a strictly positive vector $\boldsymbol{\pi} \in \mathbb{R}^S$ such that $\boldsymbol{\pi}\boldsymbol{V}(\boldsymbol{y}) = \boldsymbol{q}$ (see Magill-Quinzii [1996a, section 9]). Since we have assumed that there are investors ($\mathcal{I}_2 \neq \emptyset$) who can take advantage of arbitrage opportunities, any vector of equilibrium prices for the securities must be arbitrage free, and thus admit an associated vector of state prices.

Definition 1. A *financial market equilibrium with price perceptions* $\tilde{\boldsymbol{Q}}$ for the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$ is a triple

$$((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}), \bar{\boldsymbol{q}}\,; \tilde{\boldsymbol{Q}})$$

consisting of actions, prices and price perceptions such that

(i) for each agent $i \in \mathcal{I}$ the action $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ maximizes $U^i(\boldsymbol{x}^i, e^i)$ among consumption-effort streams such that

$$\begin{aligned} x_0^i &= \omega_0^i - v_{\bar{q}}(\boldsymbol{m}^i) + \tilde{\boldsymbol{Q}}_y^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)(1 - \theta_i^i) - \tilde{\boldsymbol{Q}}_c^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)\boldsymbol{\xi}_i^i - \kappa^i \\ \boldsymbol{x}_1^i &= \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i \end{aligned}$$

for $\kappa^i \in \mathbb{R}_+$ and $(\boldsymbol{\theta}^i, \boldsymbol{\xi}^i) \in \mathbb{R}^{I_1} \times \mathbb{R}^J$, with $\boldsymbol{m}^i = \boldsymbol{1}\, \xi_0^i + \sum_{k \neq i}(\bar{\boldsymbol{y}}^k \theta_k^i + \bar{\boldsymbol{R}}^k \boldsymbol{\xi}_k^i)$

(ii) $\bar{\boldsymbol{q}}^i = \tilde{\boldsymbol{Q}}^i(\bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i), i \in \mathcal{I}_1$

(iii) $\sum_{i \in \mathcal{I}} \bar{\theta}_k^i = 1, k \in \mathcal{I}_1$ (iv) $\sum_{i \in \mathcal{I}} \xi_0^i = 0$ (v) $\sum_{i \in \mathcal{I}} \bar{\boldsymbol{\xi}}_k^i = \boldsymbol{0},\ k \in \mathcal{I}_1$.

Note that this definition introduces some obvious notation: $\bar{\boldsymbol{y}}^k = \boldsymbol{F}^k(\bar{\kappa}^k, \bar{e}^k)$ is the equilibrium output of firm k and $\bar{\boldsymbol{R}}_j^k$ is the payoff of the j^{th} option on firm k when its output is $\bar{\boldsymbol{y}}^k$.

In an equilibrium with price perceptions, each entrepreneur takes the production plans and the prices of the securities of other entrepreneurs' firms as given, correctly anticipating the effort they invest in their firms. He chooses his own actions, anticipating that those which are observable (his financial decisions) will influence the prices of his securities in the way indicated by the function $\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. By (ii), the price perceptions are consistent with the observed equilibrium prices $\bar{\boldsymbol{q}}^i$ for each firm, and by (iii)-(v) the security markets clear.

Without more precise assumptions on the price perceptions $\tilde{\boldsymbol{Q}}^i$, the definition of equilibrium given so far only incorporates the first assumption that we discussed above — namely that investors have correct expectations — but it does not yet explicitly incorporate the second — namely that entrepreneurs are fully aware of this fact. To form his anticipations $\tilde{\boldsymbol{Q}}^i$, entrepreneur i needs to predict:

(a) the *output* of his firm that investors expect if they observe $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$

(b) how the market will *price* this expected output and the associated options of his firm.

For part (a) we use the assumption that entrepreneur i knows that investors will deduce from the observation of $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ what his likely effort $e^i \in \tilde{e}^i$ will be, and hence what the likely output $\boldsymbol{F}^i(\kappa^i, e^i)$ of his firm will be. For part (b) we assume that the entrepreneur is, like an investor, a price-taker in the market for risky income streams. This price-taking assumption for price perceptions can be formalized as follows. If $\boldsymbol{m} \in \mathbb{R}^S$ is a potential income stream in $\mathcal{V}(\bar{\boldsymbol{y}})$, then its anticipated value is $v_{\bar{q}}(\boldsymbol{m}) = \sum_{s\in\mathcal{S}} \pi_s m_s$, where $\boldsymbol{\pi} \in \mathbb{R}^S_{++}$ is any vector of state prices satisfying $\boldsymbol{\pi} V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$. As long as the entrepreneur envisions alternative production plans lying in the marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, he does not perceive the possibility of affecting the state prices implicit in the equilibrium prices $\bar{\boldsymbol{q}}$. While the price-taking assumption leads to a well-defined valuation of income streams in the marketed subspace, it does not extend in any natural way to income streams outside of the marketed subspace: for if $\boldsymbol{m} \notin \mathcal{V}(\bar{\boldsymbol{y}})$, the value $\sum_{s\in\mathcal{S}} \pi_s m_s$ can change when the vector of state prices satisfying $\boldsymbol{\pi} V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ is changed, so that the valuation of the stream $\boldsymbol{m}$ is no longer well-defined.[11] To stay within a framework that permits the competitive assumption to be retained without raising conceptual difficulties, we introduce the assumption of partial spanning.

Definition 2. We say that there is *partial spanning* (PS) at $\bar{\boldsymbol{y}}$ if for all $i \in \mathcal{I}_1$, for all $(\kappa^i, e^i) \in \mathbb{R}^2_+$

[11] This problem has been extensively discussed in the literature on equilibrium in a production economy with incomplete markets (see Ekern-Wilson [1974], Radner [1974], Drèze [1974], Grossman-Hart [1979], or the exposition in Magill-Quinzii [1996a, chapter 6].)

and $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e)$, the firm i-subspace $\mathcal{V}^i(\boldsymbol{y})$ is contained in the marketed subspace at $\bar{\boldsymbol{y}}$, i.e. $\mathcal{V}^i(\boldsymbol{y}) \subset \mathcal{V}(\bar{\boldsymbol{y}})$.

The partial spanning assumption is classical in the literature (see references in footnote 10): it means that a firm cannot create a "new security", i.e. an income stream which is not in the existing marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, by changing its production plan. *With partial spanning the market prices of the securities are sufficient signals to value all possible alternative production plans of any firm and its associated options.*

Definition 3. A *financial market equilibrium with rational competitive price perceptions* (RCPP) is an equilibrium $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}), \bar{\boldsymbol{q}}\,; \tilde{\boldsymbol{Q}})$ with price perceptions such that:

(i) PS holds at $\bar{\boldsymbol{y}}$

(ii) for each $i \in \mathcal{I}_1$ the price perceptions are given by

$$\tilde{\boldsymbol{Q}}^i_y(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i) = \bar{\boldsymbol{\pi}} \boldsymbol{F}^i(\kappa^i, \hat{e}^i), \quad \tilde{Q}^i_j(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i) = \bar{\boldsymbol{\pi}} \boldsymbol{R}^i_j(\kappa^i, \hat{e}^i),\ j \in \mathcal{J}^i$$

for any $\bar{\boldsymbol{\pi}} \in \mathbb{R}^S_{++}$ such that $\bar{\boldsymbol{\pi}} V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ and $\hat{e}^i \in \tilde{e}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ which maximizes

$$\bar{\boldsymbol{\pi}} \boldsymbol{F}^i(\kappa^i, e^i)(1 - \bar{\theta}^i_i) - \bar{\boldsymbol{\pi}} \boldsymbol{R}^i(\kappa^i, e^i) \bar{\boldsymbol{\xi}}^i_i$$

Note that we use the notation $\tilde{e}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ instead of $\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i)$, since $\boldsymbol{m}^i$ is a function of the financial variables $(\theta^i_k, \boldsymbol{\xi}^i_k, k \neq i)$ given by (3). To check if his equilibrium financial decisions $(\bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ are optimal, entrepreneur i considers alternative decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, recognizing that investors are rational and will deduce from $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ what his associated optimal effort will be — namely the solution of the optimal effort problem (E) if it is unique, or if it is multivalued, the solution which yields the highest date 0 income for entrepreneur i (recall that $u^i_1(\boldsymbol{x}^i_{\mathbf{1}}) - c^i(e^i)$ has the same value for each of the solutions). This is the "rational" part of his anticipations. To evaluate the prices $\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ that he would get for the alternative output or that he would pay for the options on his firm, he uses any state price vector $\bar{\boldsymbol{\pi}}$ compatible with the equilibrium vector of security prices $\bar{\boldsymbol{q}}$. This is the "competitive" part of his expectations, which requires that PS hold at equilibrium.

PS is automatically satisfied if the financial markets are complete at equilibrium (rank $V(\bar{\boldsymbol{y}}) = S$), but it can also be satisfied when the markets are incomplete as shown by the following examples.

Example 1. The financial markets are simple: they consist solely of the bond and equity markets, so that $\mathcal{J}^i = \emptyset$ for all $i \in \mathcal{I}_1$. The production function of each firm has a factor structure:

$\boldsymbol{F}^i(\kappa^i, e^i) = f^i(\kappa^i, e^i)\boldsymbol{\eta}^i$ where $f^i : \mathbb{R}^2_+ \longrightarrow \mathbb{R}$ is a concave increasing function and $\boldsymbol{\eta}^i \in \mathbb{R}^S_+$ is a fixed vector, characterizing the risk structure of the firm. Then PS is satisfied if $f^i(\bar{\kappa}^i, \bar{e}^i) > 0$ for all $i \in \mathcal{I}_1$. This case is studied in detail in Magill-Quinzii (1996b).

Example 2. The financial securities consist of the riskless bond, equity and options on each firm. Suppose the uncertainty (shocks) affecting the production in the economy is decomposed into a product of $\mathcal{I}_1$ spaces

$$\mathcal{S} = \mathcal{S}^1 \times \ldots \times \mathcal{S}^{I_1} = \{1, \ldots, S^1\} \times \ldots \times \{1, \ldots, S^{I_1}\}$$

so that a state of nature is an I_1 -triple $s = (s^1, \ldots, s^{I_1})$ where s^i is the shock experienced by firm i. Then for any pair of states $s = (s^1, \ldots, s^{I_1}) \in \mathcal{S}, \hat{s} = (\hat{s}^1, \ldots, \hat{s}^{I_1}) \in \mathcal{S}$ with $s^i = \hat{s}^i$, $F^i_s(\kappa^i, e^i) = F^i_{\hat{s}}(\kappa^i, e^i)$ for all $(\kappa^i, e^i) \in \mathbb{R}^2_+$. If the vector $\boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$ takes on S^i different values for the S^i individual states of firm i, and if there are options with striking prices in between the S^i different values taken by the output of firm i, for each firm $i \in \mathcal{I}_1$, then PS is satisfied.

3. Constrained Optimality of RCPP

The concept of an RCPP equilibrium is a natural way of describing market behavior in a production economy with moral hazard in which agents are informed. To get a feel for how natural this concept is we turn to a study of its normative properties. At the first stage of development, when the contracts traded consist solely of the bond and the equity of firms, there is a clear trade-off between incentives and risk sharing. Entrepreneurs who want to finance their investment without incurring a large debt (which would put them in an in inordinately risky situation) can choose to finance some of their investment by issuing equity, thus opening the way to risk sharing and diversification. But issuing equity means they no longer receive the full marginal benefit of their effort, so their incentives to exert effort are diminished. Do markets induce entrepreneurs to make the optimal trade-off between incentives and risk sharing in their choice of debt and equity?

At a more mature stage of development, in addition to the bond and equity markets, options on the firms' profits (equity) are introduced. Such contracts not only augment the opportunities for risk sharing, but also permit the introduction of non-linear reward schedules for entrepreneurs: non-linear schedules incorporate "high powered" incentives which can help to solve the moral-hazard problem induced by the reduced equity shares of entrepreneurs. If the entrepreneur receives a larger share of output when the firm's realized output is high than when it is low, then he will (typically) be induced to increase effort, to increase the likelihood of a high realization of output.

Such an incentive scheme can be obtained by adding options to his share of equity: but would an entrepreneur choose to buy options to increase his incentives in this way, given that the income stream received from his firm will tend to be more risky? In short, *do market-induced choices of bonds, equity and options by entrepreneurs and investors lead to the best possible use of these instruments*?

To answer this question we consider another way of arriving at an allocation where a "planner" — rather than the agents — chooses the financial variables, and examine if the planner could obtain a better allocation (in the Pareto sense) than that achieved in a RCPP equilibrium. Such a comparison only makes sense if the planner faces the same problem of unobservability of effort of the entrepreneurs and is restricted to the same opportunities for risk sharing as those available to the agents with the system of financial markets. In particular the planner cannot dictate effort levels to entrepreneurs — rather, these effort levels are chosen optimally by the entrepreneurs who take the reward structure given by the debt-equity-option choice of the planner and the effort levels of other agents (and hence their outputs) as given.

Definition 4. An allocation $(\boldsymbol{x}, \boldsymbol{e}) \in \mathbb{R}_+^{(S+1)I} \times \mathbb{R}_+^I$ is *constrained feasible* if there exist inputs and portfolios $(\boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi}) \in \mathbb{R}_+^I \times \mathbb{R}^{2I} \times \mathbb{R}^{IJ}$ such that

(i) $\sum_{i\in\mathcal{I}} \xi_0^i = 0$ (ii) $\sum_{i\in\mathcal{I}} \theta_k^i = 1,\ k \in \mathcal{I}$ (iii) $\sum_{i\in\mathcal{I}} \boldsymbol{\xi}_k^i = \boldsymbol{0},\ k \in \mathcal{I}$

(iv) $\sum_{i\in\mathcal{I}} x_0^i = \sum_{i\in\mathcal{I}} \omega_0^i - \sum_{i\in\mathcal{I}} \kappa^i$

(v) $\boldsymbol{x}_{\mathbf{1}}^i = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i,\ i \in \mathcal{I}$

(vi) $\boldsymbol{m}^i = \mathbf{1}\xi_0^i + \sum_{k\neq i}(\boldsymbol{F}^k(\kappa^k, e^k)\theta_k^i + \boldsymbol{R}^k(\kappa^k, e^k)\boldsymbol{\xi}_k^i),\ i \in \mathcal{I}$

(vii) $e^i \in \tilde{e}^i(\boldsymbol{m}^i, \theta_i^i, \boldsymbol{\xi}_i^i),\ i \in \mathcal{I}$

An allocation $(\boldsymbol{x}, \boldsymbol{e})$ is *constrained Pareto optimal* (CPO), if it is constrained-feasible, and if there does not exist any alternative constrained feasible allocation $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{e}})$ such that $U^i(\hat{\boldsymbol{x}}^i, \hat{e}^i) \geq U^i(\boldsymbol{x}^i, e^i)$, $i \in \mathcal{I}$, with strict inequality for at least one i.

(i)-(iii) are the feasibility constraints for the planner's choice of financial variables $(\boldsymbol{\theta}, \boldsymbol{\xi})$. Constraint (iv) indicates that the planner does not need to respect a system of prices for the securities and the associated date 0 budget constraint implied for each agent: it is in this sense that the planner replaces the "market". (v) and (vi) indicate that the planner's choice of date 1 consumption streams, and hence risk sharing, for the agents respects the existing structure of the financial securities. (vii) are the incentive constraints which reflect the fact that the choice of effort is made by entrepreneur i (and not the planner), and is the one that is optimal given the financial variables attributed to him, and given the effort levels of other agents (since agent i takes $\boldsymbol{m}^i$ as given).

The following proposition shows that despite the fact that a planner chooses the financial variables $(\kappa, \boldsymbol{\theta}, \boldsymbol{\xi})$ fully aware of their consequences for the choices of effort by entrepreneurs and of the effect of each entrepreneur's effort on the consumption of other agents (the outside shareholders), he cannot improve on an RCPP equilibrium allocation arising from the self-interested choices of agents co-ordinated by the financial markets, provided we invoke the following strengthening of the partial spanning assumption.

Definition 5. We say that there is *strong partial spanning* (SPS) at $\bar{\boldsymbol{y}}$ if for all $(\kappa, e) \in \mathbb{R}^{2I_1}$ and $\boldsymbol{y} = (\boldsymbol{F}^k(\kappa^k, e^k)_{k \in \mathcal{I}_1})$, $\mathcal{V}_{-i}(\boldsymbol{y}) \subset \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ for all $i \in \mathcal{I}_1$.

SPS ensures that there is partial spanning for every subset of $I_1 - 1$ firms. Note that even if markets were complete, SPS would not automatically be satisfied. It holds if the securities based on the outputs of any subset of $I_1 - 1$ firms suffice to complete the markets, or if each firm spans its own subspace, as in Examples 1 and 2. SPS implies PS: if firm i cannot create an income stream which lies outside $\mathcal{V}^{-k}(\bar{\boldsymbol{y}})$ for $k \neq i$, it cannot create an income stream lying outside $\mathcal{V}(\bar{\boldsymbol{y}})$.

Proposition 1. (RCPP is CPO) *If an RCPP equilibrium* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}), \bar{\boldsymbol{q}}\,; \tilde{\boldsymbol{Q}})$ *of the economy* $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$ *satisfies SPS at* $\bar{\boldsymbol{y}}$, *then* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{e}})$ *is constrained Pareto optimal.*

Proof. Suppose the equilibrium allocation $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}})$ is not CPO, then there exists a constrained feasible allocation $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{e}, \boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi})$ satisfying (i)-(vii), with $\boldsymbol{y} = \boldsymbol{F}^i(\kappa^i, e^i)$ for $i \in \mathcal{I}$, such that $U^i(\boldsymbol{x}^i, e^i) \geq U^i(\bar{\boldsymbol{x}}^i, \bar{e}^i)$, for all $i \in \mathcal{I}$, with strict inequality for some i. Since SPS holds, agent i could, in the equilibrium situation, have chosen the outside variables $(\hat{\xi}^i_0, (\hat{\theta}^i_k, \hat{\boldsymbol{\xi}}^i_k)_{k \neq i})$ to obtain the same outside income $\boldsymbol{m}^i$ as the one chosen by the planner

$$\begin{aligned} \boldsymbol{m}^i &= \boldsymbol{1}\xi^i_0 + \sum_{k \neq i} (\boldsymbol{F}^k(\kappa^k, e^k)\theta^i_k + \boldsymbol{R}^k(\kappa^k, e^k)\boldsymbol{\xi}^i_k) \\ &= \boldsymbol{1}\hat{\xi}^i_0 + \sum_{k \neq i} (\boldsymbol{F}^k(\bar{\kappa}^k, \bar{e}^k)\hat{\theta}^i_k + \boldsymbol{R}^k(\bar{\kappa}^k, \bar{e}^k)\hat{\boldsymbol{\xi}}^i_k) \end{aligned} \tag{4}$$

He could also have chosen the same inside variables as the planner $(\hat{\kappa}^i, \hat{\theta}^i_i, \hat{\boldsymbol{\xi}}^i_i) = (\kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i)$, in which case, by (vii) in Definition 4, the effort level e^i would have been optimal. With the prices and price perceptions at equilibrium, the choice of financial variables $(\hat{\kappa}^i, \hat{\boldsymbol{\theta}}^i, \hat{\boldsymbol{\xi}}^i)$ would have led to a date 0 consumption $\hat{x}^i_0$. Since $U^i(\boldsymbol{x}^i, e^i) \geq U^i(\bar{\boldsymbol{x}}^i, \bar{e}^i)$ $\forall\ i \in \mathcal{I}$, with strict inequality for some i, if $x^i_0 < \hat{x}^i_0$ for some agent indifferent between $(\boldsymbol{x}^i, e^i)$ and $(\bar{\boldsymbol{x}}^i, \bar{e}^i)$ or if $x^i_0 \leq \hat{x}^i_0$ for any agent who strictly prefers $(\boldsymbol{x}^i, e^i)$ to $(\bar{\boldsymbol{x}}^i, \bar{e}^i)$ then the optimality of an agent's equilibrium consumption would be contradicted. Thus

$$x_0^i \;\geq\; \omega_0^i - q_0\hat{\xi}_0^i - \sum_{k\neq i}(q_y^k\hat{\theta}_k^i + \boldsymbol{q}_c^k\hat{\xi}_k^i) + \tilde{Q}_y^i(\hat{\kappa}^i, \hat{\boldsymbol{\theta}}^i, \hat{\boldsymbol{\xi}}^i)(1-\hat{\theta}_i^i) - \tilde{\boldsymbol{Q}}_c^i(\hat{\kappa}^i, \hat{\boldsymbol{\theta}}^i, \hat{\boldsymbol{\xi}}^i)\hat{\boldsymbol{\xi}}_i^i$$

for all $i \in \mathcal{I}$, with strict inequality for some i. (Note that the RHS of the above inequality is $\hat{x}_0^i$.) Since $(\boldsymbol{m}^i, \hat{\kappa}^i, \hat{\theta}_i^i, \hat{\boldsymbol{\xi}}_i^i) = (\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$ and $e^i \in \tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$, (ii) of Definition 2 implies

$$\tilde{Q}_y^i(\hat{\kappa}^i, \hat{\boldsymbol{\theta}}^i, \hat{\boldsymbol{\xi}}_i^i)(1-\hat{\theta}_i^i) - \tilde{\boldsymbol{Q}}_c^i(\hat{\kappa}^i, \hat{\boldsymbol{\theta}}^i, \hat{\boldsymbol{\xi}}_i^i)\hat{\boldsymbol{\xi}}_i^i \geq\; \boldsymbol{\pi}\boldsymbol{F}^i(\kappa^i, e^i)(1-\theta_i^i) - \boldsymbol{\pi}\boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i$$

for any $\boldsymbol{\pi}$ such that $\boldsymbol{\pi}V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$. By (4)

$$\begin{aligned}
\boldsymbol{\pi}\boldsymbol{m}^i &= \boldsymbol{\pi}\mathbf{1}\,\xi_0^i + \sum_{k\neq i}(\boldsymbol{\pi}\boldsymbol{F}^k(\kappa^k, e^k)\,\theta_k^i + \boldsymbol{\pi}\boldsymbol{R}^k(\kappa^k, e^k)\boldsymbol{\xi}_k^i) \\
&= \boldsymbol{\pi}\mathbf{1}\,\hat{\xi}_0^i + \sum_{k\neq i}(\boldsymbol{\pi}\boldsymbol{F}^k(\bar{\kappa}^k, e^k)\,\hat{\theta}_k^i + \boldsymbol{\pi}\boldsymbol{R}^k(\bar{\kappa}^k, \bar{e}^k)\hat{\boldsymbol{\xi}}_k^i) \\
&= \bar{q}_0\,\hat{\xi}_0^i + \sum_{k\neq i}(\bar{q}_y^k\,\hat{\theta}_k^i + \bar{\boldsymbol{q}}_c^k\hat{\boldsymbol{\xi}}_k^i)
\end{aligned}$$

Thus

$$x_0^i \geq \omega_0^i - \boldsymbol{\pi}\mathbf{1}\xi_0^i - \sum_{k\neq i}(\boldsymbol{\pi}\boldsymbol{F}^k(\kappa^k, e^k)\,\theta_k^i + \boldsymbol{\pi}\boldsymbol{R}^k(\kappa^k, e^k)\boldsymbol{\xi}_k^i) + \boldsymbol{\pi}\boldsymbol{F}^i(\kappa^i, e^i)(1-\theta_i^i) - \boldsymbol{\pi}\boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i$$

for all $i \in \mathcal{I}$, with a strict inequality for some i. Since $(\boldsymbol{\theta}, \boldsymbol{\xi})$ satisfy (i)–(iii) of Definition 4, summing over i yields

$$\sum_{i\in\mathcal{I}} x_0^i > \sum_{i\in\mathcal{I}} \omega_0^i - \sum_{i\in\mathcal{I}} \kappa^i$$

which contradicts (iv) of Definition 4. Thus it is not possible for a planner to improve on an RCPP equilibrium allocation. △

The choice of financial variables $(\boldsymbol{\theta}, \boldsymbol{\xi})$ creates a reward structure for each entrepreneur, namely a *contract* linking his payoff to the performance of his firm and the rest of the economy

$$\phi^i(y) = \theta_i^i y^i + \sum_{j\in\mathcal{J}^i} \xi_{i,j}^i \,\max\,\{y^i - \tau_j^i, 0\} + m^i(y^{-i}), \quad i \in \mathcal{I}_1$$

where $y = (y^i, y^{-i})$, y^i being the random output of the firm and y^{-i} the random output of all other firms. $\mathcal{J} = \bigcup_{i\in\mathcal{I}_1} \mathcal{J}^i$ determines the richness of the incentive structure over and above the basic equity contracts. If $\mathcal{J} = \emptyset$, the market and the planner are restricted to linear contracts, while if $\mathcal{J} \neq \emptyset$ the admissible contracts are nonlinear (piecewise linear): the larger the sets $\mathcal{J}^i$, the larger the admissible class of piecewise linear functions.

The CPO problem, which amounts to choosing optimally the investment, risk and incentive structure for the economy, is a generalized *principal-agent problem*, in which the planner (the principal) chooses the investment in each firm and the (constrained) optimal contract for each entrepreneur and investor in the economy. When agents rationally anticipate in the way described by an RCPP equilibrium, then Proposition 1 asserts that *a system of markets is capable of solving the principal-agent problem.* The basic driving force for this optimality property of an RCPP equilibrium is that the social effect of each entrepreneur's choice of capital and reward structure — in particular the effect on outside investors — is transmitted to the entrepreneur through the rational price perceptions. A way of better understanding the forces which lead agents to optimally co-ordinate their actions is to study the first-order (i.e. the marginal) conditions that must be satisfied at a CPO and to show how these end up being achieved at an RCPP equilibrium through the effect of the perception functions $\tilde{\boldsymbol{Q}}$.

First-order Conditions. Let $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{e}, \boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi})$ be a CPO allocation such that: (i) the striking prices are strictly between the values $F_s^i(\kappa^i, e^i), s \in \mathcal{S}$, so that the payoffs $\boldsymbol{R}_j^i(\boldsymbol{y}^i)$ of the options are locally differentiable; (ii) each agent's consumption vector x^i is strictly positive; (iii) each entrepreneur i's effort level e^i is a locally differentiable selection of the effort correspondence $\tilde{e}^i$, which with a slight abuse of notation we denote by $\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$. This CPO allocation is an extremum of the social welfare function

$$\sum_{i \in \mathcal{I}} \nu^i (u_0^i(x_0^i) + u_1^i(\boldsymbol{x}_{\boldsymbol{1}}^i) - c^i(e^i))$$

subject to the constraints (i)-(vii) in Definition 4, for some vector of relative weights $\boldsymbol{\nu} \in \mathbb{R}_+^I$, $\boldsymbol{\nu} \neq \boldsymbol{0}$. It must therefore satisfy the FOC for this constrained maximum problem. To express the cost of each constraint in units of date 0 consumption, we divide all the multipliers by the multiplier induced by the resource availability constraint (iv) at date 0. Let $(q_0, q_y^k, \boldsymbol{q}_c^k, 1, \boldsymbol{\pi}^i, \boldsymbol{\mu}^i, \epsilon^i)$ denote the resulting normalized multipliers associated with the constraints (i)-(vii). For each $s \in \mathcal{S}$ and $i \in \mathcal{I}$, let $\mathcal{J}_s^i \subset \mathcal{J}^i$ denote the subset of options which are "in the money" at the CPO in state s, i.e. $j \in \mathcal{J}_s^i$ implies $F_s^i(\kappa^i, e^i) > \tau_j^i$. The first-order conditions with respect to the variables $(\boldsymbol{x}^i, e^i, \boldsymbol{m}^i, \kappa^i, \boldsymbol{\xi}^i, \boldsymbol{\theta}^i)$ are:

$$\frac{\partial u_1^i / \partial x_s^i}{u_0^{i\prime}} = \pi_s^i, \quad s \in \mathcal{S} \tag{5}$$

$$\frac{c^{i\prime}}{u_0^{i\prime}} = \left[\sum_{s \in \mathcal{S}} \pi_s^i \left(\theta_i^i + \sum_{j \in \mathcal{J}_s^i} \xi_{i,j}^i \right) + \sum_{k \neq i} \sum_{s \in \mathcal{S}} \mu_s^k \left(\theta_i^k + \sum_{j \in \mathcal{J}_s^i} \xi_{i,j}^k \right) \right] \frac{\partial F_s^i}{\partial e^i} - \epsilon^i \tag{6}$$

$$\mu_s^i = \pi_s^i + \epsilon^i \frac{\partial \tilde{e}^i}{\partial m_s^i}, \quad s \in \mathcal{S} \tag{7}$$

$$1 = \left[\sum_{s\in\mathcal{S}} \pi_s^i \left(\theta_i^i + \sum_{j\in\mathcal{J}_s^i} \xi_{i,j}^i \right) + \sum_{k\neq i} \sum_{s\in\mathcal{S}} \mu_s^k \left(\theta_{i+}^k \sum_{j\in\mathcal{J}_s^i} \xi_{i,j}^k \right) \right] \frac{\partial F_s^i}{\partial \kappa^i} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \kappa^i} \tag{8}$$

$$\begin{aligned} q_0 &= \sum_{s\in\mathcal{S}} \mu_s^i & (9)\\ q_j^k &= \sum_{s\in\mathcal{S}} \mu_s^i R_{j,s}^k & (10)\\ q_j^i &= \sum_{s\in\mathcal{S}} \pi_s^i R_{j,s}^i + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \xi_{i,j}^i} & (11)\\ q_y^k &= \sum_{s\in\mathcal{S}} \mu_s^i F_s^k & (12)\\ q_y^i &= \sum_{s\in\mathcal{S}} \pi_s^i F_s^i + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \theta_i^i} & (13) \end{aligned}$$

To these equations should be added the FOC for the choice of optimal effort by entrepreneur i

$$\frac{c^{i\prime}(e^i)}{u_0^{i\prime}} = \sum_{s\in\mathcal{S}} \pi_s^i \left(\theta_i^i + \sum_{j\in\mathcal{J}_s^i} \xi_{i,j}^i \right) \frac{\partial F_s^i}{\partial e^i} \tag{14}$$

where we have divided both sides by $u_0^{i\prime}$. (14) is just the marginal way of expressing the incentive constraint $e^i = \tilde{e}^i(\cdot)$ in (vii). Using (6) and (14) gives

$$\epsilon^i = \sum_{k\neq i} \sum_{s\in\mathcal{S}} \mu_s^k \left(\theta_i^k + \sum_{j\in\mathcal{J}_s^i} \xi_{i,j}^k \right) \frac{\partial F_s^i}{\partial e^i} \tag{15}$$

Note that for an investor $\frac{\partial F_s^i}{\partial e^i} = 0$, $\frac{\partial F_s^i}{\partial \kappa^i} = 0$ which implies $\epsilon^i = 0$ and $\mu_s^i = \pi_s^i$.

Economic Interpretation of FOC. Equation (5) defines the present-value vector $\pi^i = (\pi_1^i, \dots, \pi_S^i)$ of agent i: for any date 1 income stream $\boldsymbol{v} = (v_1, \dots, v_S)$, $\boldsymbol{\pi}^i \boldsymbol{v}$ is the present value to agent i of the income stream $\boldsymbol{v}$ (i.e. what he is prepared to pay for it at date 0). The components of the vector $\boldsymbol{q} = (q_0, q_y^k, \boldsymbol{q}_c^k, k \in \mathcal{I})$ are the shadow prices of the securities i.e. the social gain from giving one (marginal) unit of the relevant security to any agent in the economy. μ_s^i is the social gain from giving one more unit of income to entrepreneur i in state s: in most models this social gain would coincide with the private gain π_s^i, but in this setting, giving more income to entrepreneur i influences his effort, and thus has a consequence on other agents (equity or option holders of firm i), which creates a discrepancy between social and private benefit. ϵ^i is the social value of an additional unit of effort by entrepreneur i; by (6) ϵ^i is the difference between the social marginal benefit — namely the (marginal) benefit to entrepreneur i *plus* the benefit to every "outside investor" holding either the equity or options of firm i — and the social marginal cost, which here coincides with the private cost $c^{i\prime}/u_0^{i\prime}$, since entrepreneur i is the only one to bear the cost of his effort. Since effort is chosen optimally by entrepreneur i, by the "envelope theorem", or more precisely by the FOC (14), the welfare effect on the entrepreneur of a marginal change in his effort is zero. This explains why (6) and (14) lead to (15), namely *that the social value of an additional unit of effort by entrepreneur i is the value to agents other than himself of the additional output that his effort would create.*[12] As soon as $\theta_i^k \neq 0$ or $\boldsymbol{\xi}_i^k \neq 0$ for some $k \neq i$, a marginal increment of effort by agent i has an *external effect* on agent k which is not taken into account when entrepreneur i makes his effort decision. ϵ^i, which is the cost of the incentive constraint (vii), is the sum of these external effects, and is in essence the cost of separating the ownership and control of firm i. This cost is explicitly taken into account by the planner when he chooses the financial variables $(\boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi})$. Equations (8)-(13), i.e. the first-order conditions with respect to the financial variables $(\boldsymbol{\xi}, \boldsymbol{\theta})$, express the limited sense in which there must be equalization of marginal rates of substitution to achieve a CPO allocation, full equalization (in the general case) being prevented by the fact that income can only be distributed indirectly using securities, and that the incentive constraints of the agents must be satisfied. Equations (8)-(13) require that the social marginal cost of each security equal its social marginal benefit, the latter being a sum of two terms, one direct the other indirect: the direct effect is the private benefit to an agent of the security's income stream, and the indirect effect is the social cost of the reduced effort made by agent i as a result of this increment to his income stream. For the outside variables $(\xi_0, \xi_{k,j}^i, \theta_k^i)$ the indirect effect is taken into account by μ_s^i, for the inside variables $(\xi_{i,j}^i, \theta_i^i)$ it depends on the specific way in which the variable affects the entrepreneur's effort. The FOC (8) for the capital stock κ^i of firm i differs in that an increment to κ^i affects all agents holding one of the securities of firm i.[13]

[12] Note that the benefit to these agents $k \neq i$ is evaluated using μ_s^k rather than π_s^k and thus when k is an entrepreneur incorporates the incentive cost of giving him a marginal increment of income in state s.

[13] (8) can be written as

$$1 = \sum_{k \in \mathcal{I}} \pi_s^k \Big(\theta_i^k + \sum_{j \in \mathcal{J}_s^i} \xi_{i,j}^k \Big) \frac{\partial F_s^i}{\partial \kappa^i} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \kappa^i} + \sum_{k \neq i} \epsilon^k \sum_{s \in \mathcal{S}} \frac{\partial \tilde{e}^k}{\partial m_s^k} \Delta m_s^k$$

with $\Delta m_s^k = (\theta_i^k + \sum_{j \in \mathcal{J}_s^i} \xi_{i,j}^k) \partial F_s^i / \partial \kappa^i$. The left side is the social marginal cost of capital, which, given the normalization, is 1. The right side is the net marginal benefit of an additional unit of capital: its first term is the direct marginal benefit to all agents holding securities of firm i from the increased output. The last two terms are the indirect effects: for entrepreneur i changing κ^i affects the marginal product of his effort by $\partial \tilde{e}^i / \partial \kappa^i$, which

How the FOC for CPO are Achieved at Equilibrium. Since an RCPP equilibrium is constrained Pareto optimal, in such an equilibrium entrepreneurs must — just like the planner in a CPO problem — be induced to take into account the external effect of their effort on the welfare of others, namely the terms involving ϵ^i in equations (5)-(13). How is this effect transmitted to entrepreneurs?

The first point to note is that entrepreneur i raises money by selling a share $(1-\theta^i_i)$ of his equity and is thus concerned with the valuation $q^i_y = \bar{\boldsymbol{\pi}}\, \boldsymbol{y}^i$ that investors will assign to his firm. The assumption of competition implies that he doesn't perceive any effect of his actions on the vector of state prices $\bar{\boldsymbol{\pi}}$; the assumption of rationality implies that he perceives that the output $\boldsymbol{y}^i$ that investors anticipate from his firm is influenced by his choice of financial variables. Actually since the entrepreneur can typically raise the value of his equity by holding options—to convince investors that it is in his interest to make a high effort—the net proceeds from selling equity is $q^i_y(1-\theta^i_i) - \sum_{j\in\mathcal{J}^i} q^i_j \xi^i_{i,j}$, and it is this net value which is of concern to entrepreneur i. When he considers alternative financing decisions, he knows that investors will anticipate the output $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, \tilde{e}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i))$ and that this anticipation on their part will translate into the net proceeds for him $\bar{\boldsymbol{\pi}}\boldsymbol{y}^i(1-\theta^i_i) - \bar{\boldsymbol{\pi}}\boldsymbol{R}^i(\boldsymbol{y}^i)\boldsymbol{\xi}^{\boldsymbol{i}}_{\boldsymbol{i}}$ at date 0. It is his concern for the value of the equity that he sells, net of the cost of options, which leads the entrepreneur to take into account the interests of outside investors when he chooses his financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. The exact mechanism by which entrepreneur's internalize the external effect of their effort on other investors can best be seen by studying the first order conditions for each entrepreneur's maximum problem in an RCPP equilibrium and comparing them with the FOC for a CPO allocation. Consider the maximum problem of agent i in Definition 1 (i). Let $\boldsymbol{\lambda}^i = (\lambda^i_0, \lambda^i_1, \ldots, \lambda^i_S) \in \mathbb{R}^{S+1}_+$ denote the vector of multipliers induced by the $S+1$ budget constraints: the normalized vector

$$\bar{\boldsymbol{\pi}}^i = \frac{1}{\bar{\lambda}^i_0}(\bar{\lambda}^i_1, \ldots, \bar{\lambda}^i_S) = (\bar{\pi}^i_1, \ldots, \bar{\pi}^i_S)$$

is the present-value vector of agent i at the equilibrium. The FOC are

$$\frac{\partial u^i_1/\partial x^i_s}{u^{i\prime}_0} = \bar{\pi}^i_s, \quad s \in S \tag{16}$$

$$\frac{c^{i\prime}}{u^{i\prime}_0} = \sum_{s\in S} \bar{\pi}^i_s \left(\bar{\theta}^i_i + \sum_{j\in\mathcal{J}^i_s} \bar{\xi}^i_{i,j}\right) \frac{\partial F^i_s}{\partial e^i} \tag{17}$$

has the social benefit $\epsilon^i(\partial\tilde{e}^i/\partial\kappa^i)$; for agent $k \neq i$ the increased output of firm i changes his outside income by $\Delta m^k_s = (\theta^k_i + \sum_{j\in\mathcal{J}^i_s} \xi^k_{i,j})\partial F^i_s/\partial\kappa^i$ in state $s \in \mathcal{S}$, and this in turn "reduces" (changes) the effort of agent k by $\Delta e^k = \sum_{s\in\mathcal{S}} \dfrac{\partial\tilde{e}^k}{\partial m^k_s}\Delta m^k_s$, which has the social "cost" $\epsilon^k \Delta e^k$.

$$1 \;=\; \sum_{s\in\mathcal{S}} \bar{\pi}_s^i \left(\bar{\theta}_i^i + \sum_{j\in\mathcal{J}_s^i} \bar{\xi}_{i,j}^i\right) \frac{\partial F_s^i}{\partial \kappa^i} + \frac{\partial \tilde{Q}_y^i}{\partial \kappa^i}(1-\bar{\theta}_i^i) - \frac{\partial \tilde{\boldsymbol{Q}}_c^i}{\partial \kappa^i}\bar{\boldsymbol{\xi}}_i^i \tag{18}$$

$$\bar{q}_\alpha \;=\; \sum_{s\in\mathcal{S}} \bar{\pi}_s^i v_s^\alpha \;+\; \frac{\partial \tilde{Q}_y^i}{\partial z_\alpha^i}(1-\bar{\theta}_i^i) \;-\; \frac{\partial \tilde{\boldsymbol{Q}}_c^i}{\partial z_\alpha^i}\bar{\boldsymbol{\xi}}_i^i \tag{19}$$

where α is an index denoting any one of the traded securities [$\alpha = 0$ (bond) or $\alpha = k$ (equity of firm k) or $\alpha = (k,j)$ (option j of firm k)], $v^\alpha \in \mathbb{R}^S$ is its dividend stream, and z_α^i is the appropriate component of agent i's portfolio $\boldsymbol{z}^i = (\boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. By paying attention to the way investors in the securities of firm i react to his financial decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, through the partial derivatives $\left(\frac{\partial Q^i}{\partial \kappa^i}, \frac{\partial Q^i}{\partial z_\alpha^i}, \alpha = 0, \ldots\right)$, entrepreneur i is led to take their interests into account. With the rational, competitive price perceptions $\tilde{\boldsymbol{Q}}^i$ given by Definition 2, these partial derivatives are

$$\frac{\partial \tilde{Q}_\beta^i}{\partial \kappa^i} \;=\; \sum_{x\in\mathcal{S}_\beta^i} \bar{\pi}_s \left(\frac{\partial F_s^i}{\partial \kappa^i} \;+\; \frac{\partial F_s^i}{\partial e^i}\frac{\partial \tilde{e}^i}{\partial \kappa^i}\right) \tag{20}$$

$$\frac{\partial \tilde{Q}_\beta^i}{\partial \theta_k^i} \;=\; \sum_{s\in\mathcal{S}_\beta^i} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}\left(\sum_{s'\in\mathcal{S}} \frac{\partial \tilde{e}^i}{\partial m_{s'}^i} y_{s'}^k\right), \; k \neq i \tag{21}$$

$$\frac{\partial \tilde{Q}_\beta^i}{\partial \xi_{k,j}^i} \;=\; \sum_{s^i\mathcal{S}_\beta^i} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}\left(\sum_{s'\in\mathcal{S}_j^k} \frac{\partial \tilde{e}^i}{\partial m_{s'}^i} (y_{s'}^k - \tau_j^k)\right), \; k \neq i, \quad j \in \mathcal{J}^k \tag{22}$$

$$\frac{\partial \tilde{Q}_\beta^i}{\partial \theta_i^i} \;=\; \sum_{s^i\mathcal{S}_\beta^i} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}\frac{\partial \tilde{e}^i}{\partial \theta_i^i} \tag{23}$$

$$\frac{\partial \tilde{Q}_\beta^i}{\partial \xi_{i,j}^i} \;=\; \sum_{s\in\mathcal{S}_\beta^i} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}\frac{\partial \tilde{e}^i}{\partial \xi_{i,j}^i} \tag{24}$$

where β is an index denoting one of the securities associated with firm i ($\beta = y$ or $\beta = j$, $j \in \mathcal{J}^i$), whose price is influenced by the action of entrepreneur i, and $\mathcal{S}_\beta^i$ is the subset of states in which security (i,β) has a positive payoff: thus $\mathcal{S}_\beta^i = \mathcal{S}$ if $\beta = y$ and $\mathcal{S}_\beta^i = \mathcal{S}_j^i = \{s \in \mathcal{S} \mid F_s^i(\bar{\kappa}^i, \bar{e}^i) > \tau_j^i\}$ if $\beta = j,\; j \in \mathcal{J}^i$.

For $i \in \mathcal{I}$, define

$$\epsilon^i \;=\; (1-\bar{\theta}_i^i)\sum_{s\in\mathcal{S}} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i} - \sum_{j\in\mathcal{J}^i} \bar{\xi}_{i,j}^i \sum_{s\in\mathcal{S}_j^i} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i} \tag{25}$$

$$\bar{\mu}_{s'}^i \;=\; \bar{\pi}_{s'}^i \;+\; \epsilon^i \frac{\partial \tilde{e}^i}{\partial m_{s'}^i}, \quad s' \in \mathcal{S} \tag{26}$$

Substituting (21) and (22) into equation (19) for a security α whose payoff is not directly influenced by the effort of agent i ($\alpha = (k, \beta)$ with $k \neq i$, $\beta = y$ or $\beta = j, j \in \mathcal{J}^k$) and using the expressions (25) and (26), we obtain

$$\bar{q}^k_\alpha = \sum_{s \in \mathcal{S}} \bar{\mu}^i_s v^\alpha_s$$

so that each agent equalizes the price of a security which influences his outside income with its present value under the modified present-value vector (26). Thus for an agent $k \neq i$ and a security for firm i ($\alpha = (i, \beta)$)

$$\bar{q}_\alpha = \bar{q}^i_\beta = \sum_{s \in \mathcal{S}} \bar{\mu}^k_s v^\alpha_s = \sum_{s \in \mathcal{S}} \bar{\pi}_s v^\alpha_s \tag{27}$$

where the second equality follows from the definition of $\bar{\boldsymbol{\pi}}$. Thus the valuations under the vectors $\bar{\boldsymbol{\pi}}$ and $\bar{\boldsymbol{\mu}}^k$ agree on the subspace $\mathcal{V}_{-k}(\bar{\boldsymbol{y}})$. A marginal change $\Delta\kappa^i$ in the input, or Δe^i in the effort of entrepreneur i, induces a change $\Delta y^i_s = \dfrac{\partial F^i_s}{\partial \kappa^i} \Delta\kappa^i$ or $\dfrac{\partial F^i_s}{\partial e^i} \Delta e^i$ in output in each state: this induces a change $\Delta \boldsymbol{y}^i$ in the payoff of equity and a change $\Delta \boldsymbol{R}^i_j$ in the payoff of option j where

$$\Delta R^i_{js} = \begin{cases} \dfrac{\partial F^i_s}{\partial \kappa^i} \ \Delta\kappa^i \text{ or } \dfrac{\partial F^i_s}{\partial e^i} \Delta e^i, & \text{if } s \in \mathcal{S}^i_j \\ 0 \quad \text{if} \quad s \notin \mathcal{S}^i_j \end{cases}$$

By SPS the changes $\Delta \boldsymbol{y}^i$ and $\Delta \boldsymbol{R}^i_j$ must lie in $\mathcal{V}_{-k}(\bar{\boldsymbol{y}})$ for all $k \neq i$. In view of (27)

$$\sum_{s \in \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial \kappa^i} = \sum_{s \in \mathcal{S}^i_\beta} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial \kappa^i}, \quad \sum_{s \in \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} = \sum_{s \in \mathcal{S}^i_\beta} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial e^i} \tag{28}$$

where we recall that $\mathcal{S}^i_\beta = \mathcal{S}$ when $\beta = y$. Since

$$1 - \bar{\theta}^i_i = \sum_{k \neq i} \bar{\theta}^i_k, \quad \bar{\xi}^i_{i,j} = -\sum_{k \neq i} \bar{\xi}^k_{i,j} \tag{29}$$

using (28), ϵ^i in (25) can be written as

$$\begin{aligned} \epsilon^i &= \sum_{k \neq i} \bar{\theta}^k_i \sum_{s \in \mathcal{S}} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial e^i} + \sum_{k \neq i} \sum_{j \in \mathcal{J}^i} \bar{\xi}^k_{i,j} \sum_{s \in \mathcal{S}^i_j} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial e^i} \\ &= \sum_{k \neq i} \sum_{s \in \mathcal{S}} \bar{\mu}^k_s \left(\bar{\theta}^k_i + \sum_{j \in \mathcal{J}^i_s} \bar{\xi}^k_{i,j} \right) \frac{\partial F^i_s}{\partial e^i} \end{aligned} \tag{30}$$

which is the same as (15). Substituting (20)-(24) into (17)-(19), using (28) and (30), gives the FOC (6)-(13) for a CPO.

It is interesting to note that when ϵ^i is defined by (25), and price perceptions satisfy (20)-(24), then for any change dz^i_α in the portfolio of entrepreneur i

$$\frac{\partial}{\partial z^i_\alpha}\left(\tilde{Q}^i_y(1-\bar{\theta}^i_i) - \tilde{\boldsymbol{Q}}^i_c \bar{\boldsymbol{\xi}}^i_i\right) \;=\; \epsilon^i \frac{\partial \tilde{e}^i}{\partial z^i_\alpha}$$

Thus in an RCPP equilibrium *an entrepreneur acting purely in his own self interest is made aware of the value of his effort* (ϵ^i) *through the change in the date 0 income earned from the sale of his equity (net of options)*, arising from a change Δe^i in the effort that investors expect from him. The optimality property of an RCPP equilibrium is then explained by equality (30): market clearing and the common valuation of the traded securities imply that *the private value* ϵ^i *of his effort to entrepreneur i given by (25) coincides with the social value of his effort to investors holding securities of firm i, given by the right side of (30).* In short, with sophisticated participants, the security markets ensure that self-interested behavior leads to a (constrained) socially optimal outcome.

4. RCPP and Pareto Optimality

In the previous section we showed that a financial market (RCPP) equilibrium is always constrained Pareto optimal, that is, given the constraints imposed by the limited possibilities for risk sharing and the unobservability of effort of entrepreneurs, the system of markets does as well as a "benevolent" planner. A more developed capital market in essence means a capital market with a richer system of derivative securities. Is it possible by adjoining a rich enough system of options on the underlying equity contracts to achieve a Pareto optimal allocation? In short can the addition of enough options ensure that the risk-sharing and incentive constraints are not binding? To study this question recall

Definition 6. An allocation $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}, \boldsymbol{e}) \in \mathbb{R}^{I(S+1)}_+ \times \mathbb{R}^{IS}_+ \times \mathbb{R}^I_+ \times \mathbb{R}^I_+$ with $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e^i),\ i \in \mathcal{I}$ is *feasible* if

$$\sum_{i\in\mathcal{I}} x^i_0 \;\leq\; \sum_{i\in\mathcal{I}} \omega^i_0 - \sum_{i\in\mathcal{I}} \kappa^i, \qquad \sum_{i\in\mathcal{I}} \boldsymbol{x}^i_{\boldsymbol{1}} \;\leqq\; \sum_{i\in\mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i) \tag{31}$$

An allocation $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{\kappa}^*, \boldsymbol{e}^*)$ is Pareto optimal, if it is feasible and if there does not exist another feasible allocation $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}, \boldsymbol{e})$ such that $U^i(\boldsymbol{x}^i, e^i) \;\geq\; U^i(\boldsymbol{x}^{i*}, e^{i*})$, $\forall\ i \in \mathcal{I}$, with strict inequality for at least one i.

If financial contracts could be written contingent on the states of nature, so that they have payoffs independent of the agents' actions, and if such contracts were complete — in short, if there were a complete set of Arrow securities — then there would be a simple way of obtaining a Pareto optimal allocation, despite the non-observability of effort. It would suffice to let each entrepreneur be the *sole proprietor* of his firm so that he has both the full marginal benefit and cost of his effort and there is no distortion of incentives. It is useful to make this statement precise by introducing the concept of a sole-proprietorship equilibrium with Arrow securities, or equivalently with contingent markets, since it serves as the *reference concept* for studying when contracts based on observable outputs of the firms can lead to Pareto optimality. Letting the price of income at date 0 be normalized to 1 and letting π_s denote the price (at date 0) of the Arrow security for state s (which delivers one unit of income in state $s \in \mathcal{S}$), the budget set of agent i with Arrow securities and sole proprietorship is given by

$$B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} \;\middle|\; \begin{array}{c} x_0^i = \omega_0^i - \boldsymbol{\pi}\boldsymbol{\zeta}^i - \kappa^i \\ \boldsymbol{x}_{\mathbf{1}}^i = \boldsymbol{\zeta}^i + \boldsymbol{F}^i(\kappa^i, e^i) \\ (\kappa^i, \boldsymbol{\zeta}^i) \in \mathbb{R}_+ \times \mathbb{R}^S \end{array} \right\}$$

where $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_S)$ and $\boldsymbol{\zeta}^i = (\zeta_1^i, \ldots, \zeta_S^i)$ is agent i' portfolio of the Arrow securities. As usual, the $S+1$ budget constraints with Arrow securities can be reduced to a single budget constraint, i.e. the set $B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i)$ can be written as

$$B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} \mid x_0^i + \boldsymbol{\pi}\boldsymbol{x}_{\mathbf{1}}^i = \omega_0^i + \boldsymbol{\pi}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i \right\} \tag{32}$$

This is the budget set of an agent with contingent markets for income and sole proprietorship.

Definition 6. $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is *a sole-proprietorship equilibrium with contingent markets* (SPCM) if

(i) $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i) \in \arg\max \left\{ U^i(\boldsymbol{x}^i, e^i) \;\middle|\; (\boldsymbol{x}^i, e^i) \in B(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i) \right\}$ and $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i), \; i \in \mathcal{I}$

(ii) $\sum_{i \in \mathcal{I}} x_0^i = \sum_{i \in \mathcal{I}} \omega_0^i - \sum_{i \in \mathcal{I}} \kappa^i, \qquad \sum_{i \in \mathcal{I}} \boldsymbol{x}_{\mathbf{1}}^i = \sum_{i \in \mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i)$

An SPCM is not precisely an Arrow-Debreu equilibrium, since there are $S + 1 + I_1$ "goods" in the economy — the $S+1$ incomes at dates 0 and 1, and the I_1 effort levels of the entrepreneurs — but there are only $S+1$ markets. Despite the absence of the I_1 markets for the effort levels of entrepreneurs, the first and second welfare theorems — as well as the existence of equilibrium — are satisfied by SPCM equilibria. This is due to the following two properties of "Robinson Crusoe" economies:

(i) An agent who is both a producer and a consumer in a convex economy can be split into two "personalities": an entrepreneur who maximizes profit and a consumer who takes the profit as given and maximizes utility over his budget set (see Magill-Quinzii (1996a) for an account of this property in a general framework).

(ii) Agent i as the entrepreneur running firm i buys the input "effort for firm i" from only one agent, himself as a consumer. The market for effort e^i can thus be "internalized" in the joint consumer-producer maximum problem of agent i in a SPCM. Any other ownership structure of the firm would fail to lead to Pareto optimality in the absence of a market for effort

Proposition 2. (Properties of SPCM equilibrium)

(i) *For any* $\boldsymbol{\omega}_0 \in \mathbb{R}^I_+, \omega_0 \neq 0$, *there exists an SPCM equilibrium.*

(ii) *If* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ *is an SPCM equilibrium, then the allocation* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ *is Pareto optimal.*

(iii) *For any Pareto optimal allocation* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ *there exist incomes* $\boldsymbol{\omega}_0 \in \mathbb{R}^I$ *and state prices* $\bar{\boldsymbol{\pi}} \in \mathbb{R}^S_{++}$ *such that* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ *is an SPCM equilibrium.*

Proof. The existence proof is standard. To prove the equivalence between PO allocations and SPCM equilibria in the differentiable case it suffices to note that the FOC for Pareto optimality are the same as the FOC for the maximum problems of the agents in an SPCM equilibrium:

$$\frac{\partial u_1^i(\bar{\boldsymbol{x}}_1^i)}{\partial x_s^i} / u_0^{i'}(\bar{x}_0^i) = \bar{\pi}_s, \quad c^{i'}(e^i) = \sum_{s \in \mathcal{S}} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}(\bar{\kappa}^i, \bar{e}^i), \quad 1 = \sum_{s \in \mathcal{S}} \bar{\pi}_s \frac{\partial F_s^i}{\partial \kappa^i}(\bar{\kappa}^i, \bar{e}^i), \quad i \in \mathcal{I}$$

In both cases the problems are convex so that the FOC are necessary and sufficient. Differentiability assumptions are not required for the results of Proposition 2. We leave it to the reader to adapt the standard arguments in the non-differentiable case. △

If an RCPP equilibrium is to lead to a Pareto optimum, then the equilibrium needs to mimick an SPCM equilibrium: to this end it is useful to write the budget set of an RCPP equilibrium in a form that brings it closer to the SPCM budget set. This can be done in two steps as follows. Incorporating Definition 3(ii) of price perceptions directly into agent i's date 0 budget equation (in Definition 1(i)), we can write the budget set in an RCPP equilibrium as

$$\mathbb{B}(\boldsymbol{q}, \omega_0^i, \boldsymbol{F}^i, \boldsymbol{y}) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}^{S+2}_+ \;\middle|\; \begin{array}{l} x_0^i = \omega_0^i + v_q(\boldsymbol{m}^i) + \boldsymbol{\pi}\boldsymbol{F}(\kappa^i, e^i)(1 - \theta_i^i) - \boldsymbol{\pi}\boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i - \kappa^i \\ \boldsymbol{x}_1^i = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i \\ e^i \in \tilde{e}^i(\boldsymbol{m}^i, \theta_i^i, \boldsymbol{\xi}_i^i) \\ \boldsymbol{m}^i \in \mathcal{V}_{-i}(\boldsymbol{y}), (\theta_i^i, \boldsymbol{\xi}_i^i, \kappa^i) \in \mathbb{R} \times \mathbb{R}^{J^i} \times \mathbb{R}_+ \end{array} \right\} \tag{33}$$

where $\boldsymbol{\pi} \in \mathbb{R}^S_{++}$ satisfies the no-arbitrage condition $\boldsymbol{\pi V}(\boldsymbol{y}) = \boldsymbol{q}$. Note that if we multiply the date 1 budget equation for state s by π_s and add, we obtain $\boldsymbol{\pi x^i_1}$, namely the present value of agent i's date 1 consumption stream: since by no-arbitrage $\boldsymbol{\pi m}^i = v_q(\boldsymbol{m}^i)$, adding the present value of the date 1 equations to the date 0 equation gives the equivalent budget set[14]

$$\mathcal{B}(\pi, \omega^i_0, \boldsymbol{F}^i, \boldsymbol{y}) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}^{S+2}_+ \;\middle|\; \begin{array}{l} x^i_0 + \boldsymbol{\pi x^i_1} = \omega^i_0 + \boldsymbol{\pi F}^i(\kappa^i, e^i) - \kappa^i \\ \boldsymbol{x^i_1} = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta^i_i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}^i_i \\ e^i \in \tilde{e}^i(\boldsymbol{m}^i, \theta^i_i, \boldsymbol{\xi}^i_i) \\ \boldsymbol{m}^i \in \mathcal{V}_{-i}(y), (\theta^i_i, \boldsymbol{\xi}^i_i, \kappa^i) \in \mathbb{R} \times \mathbb{R}^{J^i} \times \mathbb{R}_+ \end{array} \right\} \tag{34}$$

in which the date 0 equation is the budget equation of an SPCM equilibrium. If the budget set $\mathcal{B}(\boldsymbol{\pi})$ in (34) is to reduce to the budget set $B(\boldsymbol{\pi})$ in (32), then neither the date 1 equations nor the effort constraint $e^i \in \tilde{e}^i$ must be binding. Since the date 1 constraints can be written as $x^i_{\boldsymbol{1}} \in \mathcal{V}_{-i}(\boldsymbol{y}) + \mathcal{V}^i(\kappa^i, e^i)$, these constraints are not binding if $\mathcal{V}_{-i}(\boldsymbol{y}) + \mathcal{V}^i(\kappa^i, e^i) = \mathbb{R}^S$ for the relevant choices of (κ^i, e^i). Up to non-convexities, the constraint $e^i \in \tilde{e}^i$ will not be binding if the cost ϵ^i of the constraint that effort is optimal for entrepreneur i after the financial variables are chosen, is zero. By equation (25), $\epsilon^i = 0$ if and only if

$$\sum_{s \in \mathcal{S}} \pi_s \frac{\partial F^i_s}{\partial e^i} = \sum_{s \in \mathcal{S}} \pi_s (\theta^i_i + \sum_{j \in \mathcal{J}^i_s} \xi^i_{i,j}) \frac{\partial F^i_s}{\partial e^i} \tag{35}$$

that is, if the combined holdings of equity and options of his own firm are such that entrepreneur i receives the full value of the marginal product of his effort, as in an SPCM equilibrium. The difference between an RCPP and an SPCM equilibrium is that in the system with equity and options, an entrepreneur can receive the full value of the marginal product of his effort without appropriating the full value of the output of his firm as in an SPCM equilibrium.

If options can serve to complete the markets and to provide entrepreneurs with the appropriate incentives, they do however introduce a technical difficulty into the analysis: because of their non-linear payoffs, the problem (E) of optimal effort of an entrepreneur is *non-convex*, so that the FOC for optimal effort is not equivalent to $e^i \in \tilde{e}^i(\boldsymbol{m}^i, \theta^i_i, \boldsymbol{\xi}^i_i)$. To be able to relate RCPP equilibria to SPCM equilibria, we begin by weakening the concept of an RCPP equilibrium to a weak-RCPP equilibrium in which the optimal-effort condition $e^i \in \tilde{e}^i$ is replaced by the requirement e^i satisfies the FOC for optimal effort. We discuss in Example 4 below the relationship between a weak-RCPP

[14] i.e. $\mathbb{B}(\boldsymbol{q}, \omega^i_0, F^i, y) = \mathcal{B}(\boldsymbol{\pi}, \omega^i_0.F^i, y)$. A reader familiar with *Theory of Incomplete Markets* (Magill-Quinzii (1996a), section 10) will note that the budget set $\mathcal{B}(\boldsymbol{\pi})$ written in terms of the vector of state prices $\boldsymbol{\pi}$, is the equivalent for this model of the budget set of an agent in a *no-arbitrage equilibrium.*

equilibrium and a "true" RCPP equilibrium. For the analysis that follows it will be more convenient to use the form (34) rather than (33) for an agent's budget set and to define the equilibrium in terms of state prices $\boldsymbol{\pi}$: the standard concept expressed in terms of security prices can be recovered from the formula $\bar{\boldsymbol{q}} = \bar{\boldsymbol{\pi}}\boldsymbol{V}(\bar{\boldsymbol{y}})$.

Definition 7. $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}; \bar{\boldsymbol{\pi}})$ is a *weak*-RCPP equilibrium if

(i) for each agent $i \in \mathcal{I}$, the action $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ maximizes $U^i(\boldsymbol{x}^i, e^i)$ over the budget set

$$\mathcal{B}'(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i, \bar{\boldsymbol{y}}) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} \;\middle|\; \begin{array}{l} x_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{x}_{\boldsymbol{1}}^i = \omega_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i \\ \boldsymbol{x}_{\boldsymbol{1}}^i = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i \\ c^{i\prime}(e^i) = \sum_{s\in\mathcal{S}} \pi_s^i(\boldsymbol{x}^i)\left(\theta_i^i + \sum_{j\in\mathcal{J}_s^i} \xi_{i,j}^i\right) \dfrac{\partial F_s^i}{\partial e^i}(\kappa^i, e^i) \\ \boldsymbol{m}^i = \boldsymbol{1}\xi_0^i + \sum_{k\neq i}\left(\bar{\boldsymbol{y}}^k\theta_k^i + \bar{\boldsymbol{R}}^k\boldsymbol{\xi}_k^i\right) \\ (\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i) \in \mathbb{R}_+ \times \mathbb{R}^{I_1} \times \mathbb{R}^J \end{array} \right\} \tag{36}$$

(ii) $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i), \quad i \in \mathcal{I}_1$

(iii) $\sum_{i\in\mathcal{I}} \bar{\theta}_k^i = 1,\ k \in \mathcal{I}_1$ (iv) $\sum_{i\in\mathcal{I}} \bar{\xi}_0^i = 0$ (v) $\sum_{i\in\mathcal{I}} \boldsymbol{\xi}_k^i = \boldsymbol{0}, \quad k \in \mathcal{I}_1$

The possibility of making the FOC for optimal effort into non-binding constraints comes from the following property of options: if the subspace $\mathcal{V}^i(\bar{\boldsymbol{y}}^i)$ spanned by firm i's securities is of maximum possible dimension given the vector of outputs $\bar{\boldsymbol{y}}^i$, i.e. if there are options with striking prices lying between the distinct values of output across the states, then $\boldsymbol{1} \in \mathcal{V}^i(\bar{\boldsymbol{y}}^i)$. For example, suppose that there are 4 states, that $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i) = (a, b, b, c)$ with $a > b > c > 0$ and that there are two options with striking prices τ_1, τ_2 such that $a > \tau_1 > b > \tau_2 > c$. Then

$$\boldsymbol{V}^i(\bar{\boldsymbol{y}}^i) \;=\; \begin{bmatrix} a & a-\tau_1 & a-\tau_2 \\ b & 0 & b-\tau_2 \\ b & 0 & b-\tau_2 \\ c & 0 & 0 \end{bmatrix}$$

The columns of the matrix are linearly independent and generate the subspace $\mathcal{V}^i(\bar{\boldsymbol{y}}^i) = \{\boldsymbol{v} \in \mathbb{R}^4 \mid v_2 = v_3\}$ which contains $\boldsymbol{1}$. Because of this redundancy, any vector $\boldsymbol{v} \in \mathcal{V}^i(\bar{\boldsymbol{y}}^i)$ can be written in many different ways as

$$\boldsymbol{v} = \lambda_0\boldsymbol{1} + \lambda_1\boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i) + \mu_1\boldsymbol{R}_1^i(\bar{\kappa}^i, \bar{e}^i) + \mu_2\boldsymbol{R}_2^i(\bar{\kappa}^i, \bar{e}^i)$$

If $\boldsymbol{v}$ is an income stream received by entrepreneur i, then the choice of (λ, μ) which is irrelevant when $(\bar{\kappa}^i, \bar{e}^i)$ are fixed, i.e. from the spanning point of view, is important when they are varied, i.e.

in deciding whether $(\bar{\kappa}^i, \bar{e}^i)$ is optimal: λ_0 determines the outside income independent of agent i's effort, while $(\lambda_1, \mu_1, \mu_2)$ directly affect the marginal benefit accruing from one more unit of effort.

Definition 8. We say that there is *complete spanning with redundancy* (CSR) at $\bar{\boldsymbol{y}}$ if

(i) $\mathcal{V}(\bar{\boldsymbol{y}}) = \mathbb{R}^S$ (complete markets)

(ii) $\mathbf{1} \in \mathcal{V}^i(\bar{\boldsymbol{y}}), \quad \forall\, i \in \mathcal{I}_1$ (redundancy).

Proposition 3. (SPCM implies weak-RCPP) *Let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ be an SPCM equilibrium. If the security structure of the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$ is such that CSR is satisfied at $\bar{\boldsymbol{y}}$, then there exist portfolios $(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}})$ such that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}; \bar{\boldsymbol{\pi}})$ is a weak-RCPP equilibrium.*

Proof. It is clear that for an entrepreneur $i \in \mathcal{I}_1$ the budget set (36) is contained in the SPCM budget set i.e. $\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i, \bar{\boldsymbol{y}}) \subset B(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i)$, while for an investor $i \in \mathcal{I}_2$ the two budget sets concide, since for $i \in \mathcal{I}_2$, $\mathcal{V}_{-i}(\bar{\boldsymbol{y}}) = \mathcal{V}(\bar{\boldsymbol{y}}) = \mathbb{R}^S$. Suppose for the moment that we can show that the optimal choice $(\bar{\boldsymbol{x}}^i, \bar{\kappa}^i, \bar{e}^i)$ of each entrepreneur $i \in \mathcal{I}_1$ in the larger budget set B can also be obtained in $\mathcal{B}'$ with a portfolio $(\bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$: then clearly $(\bar{\boldsymbol{x}}^i, \bar{\kappa}^i, \bar{e}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ is optimal in $\mathcal{B}'$. The proof of Proposition 3 can then be completed by the following argument. Choose one investor and call him agent I: such an investor exists since $\mathcal{I}_2 \neq \emptyset$. For each of the other investors $i \in \mathcal{I}_2, i \neq I$ choose a portfolio $(\bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ such that $\bar{\boldsymbol{x}}^i_{\mathbf{1}} = \boldsymbol{V}(\bar{\boldsymbol{y}})(\bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)^T$: such a portfolio exists since the markets are complete. For agent I choose

$$\bar{\boldsymbol{\theta}}^I = 1 - \sum_{\substack{i \in \mathcal{I}_1 \cup \mathcal{I}_2 \\ i \neq I}} \bar{\boldsymbol{\theta}}^i, \qquad \bar{\boldsymbol{\xi}}^I = - \sum_{\substack{i \in \mathcal{I}_1 \cup \mathcal{I}_2 \\ i \neq I}} \bar{\boldsymbol{\xi}}^i$$

This ensures that the market clearing equations hold for the securities. Since for all $i \neq I$, $\bar{\boldsymbol{x}}^i_{\mathbf{1}} = \boldsymbol{V}(\bar{\boldsymbol{y}})(\bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)^T$ and since $V(\bar{\boldsymbol{y}})(\sum_{i \in \mathcal{I}} \bar{\boldsymbol{\theta}}^i, \sum_{i \in \mathcal{I}} \bar{\boldsymbol{\xi}}^i)^T = \sum_{i \in \mathcal{I}_1} \bar{\boldsymbol{y}}^i$, $\boldsymbol{V}(\bar{\boldsymbol{y}})(\bar{\boldsymbol{\theta}}^I, \bar{\boldsymbol{\xi}}^I)^T = \sum_{i \in \mathcal{I}_1} \bar{\boldsymbol{y}}^i - \sum_{i \neq I} \bar{\boldsymbol{x}}^i_{\mathbf{1}}$. By the date 1 market clearing equations in an SPCM, $\bar{\boldsymbol{x}}^I_{\mathbf{1}} = \boldsymbol{V}(\bar{\boldsymbol{y}})(\bar{\boldsymbol{\theta}}^I, \bar{\boldsymbol{\xi}}^I)^T$. Since the date 0 constraint in $\mathcal{B}'$ is the same as the budget constraint in B, for each investor $\bar{\boldsymbol{x}}^i \in \mathcal{B}'$ and is thus optimal in this set.

It only remains to prove that for each entrepreneur $i \in \mathcal{I}_1$ there exists a portfolio $(\bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ such that $(\bar{\boldsymbol{x}}^i, \bar{e}^i; \bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \boldsymbol{\xi}^i)$ lies in the budget set $\mathcal{B}'$. Since $\mathcal{V}(\bar{\boldsymbol{y}}) = \mathcal{V}_{-i}(\bar{\boldsymbol{y}}) + \mathcal{V}^i(\bar{\boldsymbol{y}}) = \mathbb{R}^S$, there exist $\tilde{\boldsymbol{m}}^i \in \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ and $(\tilde{\theta}^i_i, \tilde{\boldsymbol{\xi}}^i_i)$ such that $\bar{\boldsymbol{x}}^i_{\mathbf{1}} = \tilde{\boldsymbol{m}}^i + \bar{\boldsymbol{y}}^i \tilde{\theta}^i_i + \bar{\boldsymbol{R}}^i \tilde{\xi}^i_i$. Since $\mathbf{1} \in \mathcal{V}^i(\bar{\boldsymbol{y}})$, there exist coefficients $(\alpha^i_0, \alpha^i_1, \ldots, \alpha^i_{J^i})$ such that $\mathbf{1} = \alpha^i_0 \bar{\boldsymbol{y}}^i + \sum_{j \in \mathcal{J}^i} \alpha^i_j \bar{\boldsymbol{R}}^i_j$. Thus for any $\gamma \in \mathbb{R}$

$$\bar{\boldsymbol{x}}^i_{\mathbf{1}} = \tilde{\boldsymbol{m}}^i - \gamma \mathbf{1} + \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)(\tilde{\theta}^i_i + \alpha^i_0 \gamma) + \sum_{j \in \mathcal{J}^i} \boldsymbol{R}^i(\bar{\kappa}^i, \bar{e}^i)(\tilde{\xi}^i_{i,j} + \gamma \alpha^i_j)$$

and $\tilde{\boldsymbol{m}}^i - \gamma \mathbf{1} \in \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ (since $\mathbf{1} \in \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$). The optimal-effort-FOC constraint is satisfied at $\bar{e}^i$ if

$$c^{i'}(\bar{e}^i) \;=\; \sum_{s \in \mathcal{S}} \pi_s^i(\bar{\boldsymbol{x}}^i) \left[(\tilde{\theta}_i^i + \alpha_0^i \gamma) \;+\; \sum_{j \in \mathcal{J}_s^i} (\tilde{\xi}_{ij}^i \;+\; \alpha_j^i \gamma) \right] \frac{\partial F_s^i}{\partial e^i}(\bar{\kappa}^i, \bar{e}^i)$$

which holds for some $\bar{\gamma} \in \mathbb{R}$ if $\sum_{s \in \mathcal{S}} \pi_s^i(\bar{\boldsymbol{x}}^i) \left(\alpha_0^i \;+\; \sum_{j \in \mathcal{J}_s} \alpha_j^i \right) \frac{\partial F_s^i}{\partial e^i}(\bar{\kappa}^i, \bar{e}^i) \neq 0$. This condition is generic since it is possible to perturb one coefficient α_j^i by perturbing the striking price of the option with the highest striking price which is in the money at $\bar{\boldsymbol{y}}^i$.[15] Let $\bar{\boldsymbol{m}}^i = \tilde{\boldsymbol{m}}^i - \bar{\gamma}\mathbf{1}$, $\bar{\theta}_i^i = \tilde{\theta}_i^i + \alpha_0^i \bar{\gamma}$, $\bar{\xi}_{ij}^i = \tilde{\xi}_{ij}^i \;+\; \alpha_j^i \bar{\gamma}$, and let $(\bar{\theta}_k^i, \bar{\boldsymbol{\xi}}_k^i)_{k \neq i}$ be such that $\bar{\boldsymbol{m}}^i = \sum_{k \neq i} (\bar{\boldsymbol{y}}^k \bar{\theta}_k^i + \bar{\boldsymbol{R}}^k \bar{\boldsymbol{\xi}}_k^i)$, then $(\bar{\boldsymbol{x}}^i, \bar{e}^i; \bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i) \in \mathcal{B}'$. △

Given the way that we have set up the model, the assumption CSR requires that markets can be completed using only options on individual firms' equity (so-called *simple* options) and this restricts the stochastic structure of firms' production functions. As Ross (1976) has pointed out in some cases it may be necessary to introduce complex options to complete the markets — a *complex* option being an option based on the payoff of a portfolio of equity contracts—such as an option on the S&P500 or the Dow Jones index (see Example 3 below).

Introducting complex options into the model would not in any essential way change the analysis and would only require additional notation—which is the reason for their omission.[16] The condition CSR can always be satisfied by having a complete set of simple options on each firm and appropriate complex options to complete the markets.[17]

Example 3. To illustrate the conditions involved in CSR suppose that there are 4 states and 3 firms and that firms' production functions have the following properties. Firm 1 is such that for all $(\kappa^1, e^1) \gg 0$ and $\boldsymbol{y}^1 = \boldsymbol{F}^1(\kappa^1, e^1)$, $y_1^1 = y_2^1 > y_3^1 = y_4^1$, while firm 2 is such that for all $(\kappa^2, e^2) \gg 0$ and $\boldsymbol{y}^2 = \boldsymbol{F}^2(\kappa^2, e^2)$, $y_1^2 = y_3^2 > y_2^2 = y_4^2$. This suggests a state space $\mathcal{S} = \{\alpha, \beta\} \times \{\gamma, \delta\}$, in which (α, β) are the good and bad states for firm 1, while (γ, δ) are the good and bad states for

[15] We do not make a generic statement in the Proposition since the striking price of an option is easily changed and the genericity is rather trivial.

[16] Complex options, which serve only a spanning role in the two-period model, are likely to have a more important role in a *multiperiod* model: for in such a setting, the payoff of a firm's option depends on the price of its equity instead of depending directly on its realized profit as in the two-period model. Complex options can help discriminate betwen the performance of a firm and the general state of the market, both of which affect its equity price.

[17] Ross (1976) has shown that markets can be completed by using *only* a family of complex options (i.e. with different striking prices) on a *single* portfolio of the equity contracts. He argued that it would be simpler to have options on such a single portoflio rather than the proliferation of simple options which is observed in US markets. The present analysis shows however that simple options play a role both for risk sharing and incentives and hence cannot readily be replaced by complex options on a single portfolio of equity contracts.

firm 2. Suppose firm 3 does well when firms 1 and 2 are in their good states, badly when they are both in their bad states, and has a "medium" profit otherwise, i.e. for all $(\kappa^3, e^3) \gg 0$ and $\boldsymbol{y}^3 = \boldsymbol{F}^3(\kappa^3, e^3)$, $y^3_1 > y^3_2 = y^3_3 > y^3_4$. Let $(\bar{\boldsymbol{y}}^i, \bar{\kappa}^i, \bar{e}^i)$, $i = 1, 2, 3$ be the production part of a Pareto optimal allocation with $\bar{\boldsymbol{y}} \gg 0$. If there is one option for firm 1 with striking price τ^1 such that $\bar{y}^1_H > \tau^1 > \bar{y}^1_L$ (where $\bar{y}^1_H = \bar{y}^1_1 = \bar{y}^1_2$, $\bar{y}^1_L = \bar{y}^1_3 = \bar{y}^1_4$), one option for firm 2 with striking price τ^2 such that $\bar{y}^2_H > \tau^2 > \bar{y}^2_L$ (with obvious notation) and 2 options for firm 3 with striking prices τ^3_1 and τ^3_2 such that $\bar{y}^3_H > \tau^3_1 > \bar{y}^3_M > \tau^3_2 > \bar{y}^3_L$, then the payoff matrix $\boldsymbol{V}(\bar{\boldsymbol{y}})$ is

$$\boldsymbol{V}(\bar{\boldsymbol{y}}) = \begin{bmatrix} 1 & \bar{y}^1_H & \bar{y}^1_H - \tau^1 & \bar{y}^2_H & \bar{y}^2_H - \tau^2 & \bar{y}^3_H & \bar{y}^3_H - \tau^3_1 & \bar{y}^3_H - \tau^3_2 \\ 1 & \bar{y}^1_H & \bar{y}^1_H - \tau^1 & \bar{y}^2_L & 0 & \bar{y}^3_M & 0 & \bar{y}^3_M - \tau^3_2 \\ 1 & \bar{y}^1_L & 0 & \bar{y}^2_H & \bar{y}^2_H - \tau^2 & \bar{y}^3_M & 0 & \bar{y}^3_M - \tau^3_2 \\ 1 & \bar{y}^1_L & 0 & \bar{y}^2_L & 0 & \bar{y}^3_L & 0 & 0 \end{bmatrix}$$

The securities of firm 1 span the 2-dimensional subspace $\mathcal{V}^1(\bar{\boldsymbol{y}}) = \{\boldsymbol{v} \in \mathbb{R}^4 \mid v_1 = v_2, v_3 = v_4\}$, the securities for firm 2 span the 2-dimensional subspace $\mathcal{V}^2(\bar{\boldsymbol{y}}) = \{\boldsymbol{v} \in \mathbb{R}^4 \mid v_1 = v_3, v_2 = v_4\}$ and the securities of firm 3 span the 3-dimensional subspace $\mathcal{V}^3(\bar{\boldsymbol{y}}) = \{\boldsymbol{v} \in \mathbb{R}^4 \mid v_2 = v_3\}$. Each of these subspaces contains $\mathbf{1}$ and $\mathcal{V}^0 + \mathcal{V}^1(\bar{\boldsymbol{y}}) + \mathcal{V}^2(\bar{\boldsymbol{y}}) + \mathcal{V}^3(\bar{\boldsymbol{y}}) = \mathbb{R}^4$ so that CSR is satisfied. However $\mathcal{V}^0 + \mathcal{V}^1(\bar{\boldsymbol{y}}) + \mathcal{V}^2(\bar{\boldsymbol{y}})$ is of dimension 3 (since $\mathcal{V}^0 \subset \mathcal{V}^1(\bar{\boldsymbol{y}})$ and $\mathcal{V}^0 \subset \mathcal{V}^2(\bar{\boldsymbol{y}})$): thus if firms 1 and 2 were the only firms, then simple options would not complete the markets. A complex option on a portfolio of the two equities (for example the portfolio $2\boldsymbol{y}^1 + \boldsymbol{y}^2$ with striking price τ such that $2\bar{y}^1_L + \bar{y}^2_H > \tau > 2\bar{y}^1_L + \bar{y}^2_L$ or such that $2\bar{y}^1_H + \bar{y}^2_H > \tau > 2\bar{y}^1_H + \bar{y}^2_L$) would complete the markets.

To use Proposition 3 to derive an existence theorem for weak-RCPP equilibria, we need to give sufficient conditions ensuring that CSR is satisfied at any SPCM equilibrium. To have the redundancy condition (ii) of Definition 8 satisfied it suffices to have the maximum number of linearly independent options on each firm. By Ross (1976), the complete markets condition (i) will be satisfied at $\bar{\boldsymbol{y}}$ (by introducing complex options if necessary) if $\bar{\boldsymbol{y}}$ distinguishes between the states, i.e. for each pair of states s and s' there is at least one firm for which $\bar{y}^i_s \neq \bar{y}^i_{s'}$. This leads to the following definitions.

Definition 9. We say that the state space $\mathcal{S}$ is *technological* if for any pair of states s and s' in $\mathcal{S}$, there exists a firm $i \in \mathcal{I}_1$ and inputs (κ^i, e^i) such that $F^i_s(\kappa^i, e^i) \neq F^i_{s'}(\kappa^i, e^i)$.

Definition 10. We say that the technology of firm i has the *no-crossing property* if $F^i_s(\kappa^i, e^i) \neq F^i_{s'}(\kappa^i, e^i)$ for some input pair (κ^i, e^i) implies that this property is satisfied for all $(\kappa^i, e^i) \gg 0$.

Since we have assumed that firms are sufficiently productive to be active in any equilibrium, the combination of a technological state space and the no-crossing property imply that at any SPCM equilibrium $\bar{\boldsymbol{y}}$ distinguishes between states. The no-crossing property is strong and could be weakened to the existence of only *isolated* crossing points at the cost of introducing generic arguments. The following result follows readily from Proposition 2(i) and 3.

Corollary 4. (Existence of weak-RCPP equilibrium) *If the state space is technological and the firms' technologies satisfy the no-crossing property, then for any* $\boldsymbol{\omega}_0 \in \mathbb{R}^I_+, \boldsymbol{\omega}_0 \neq 0$ *there is a security structure such that there exists a weak-RCPP equilibrium which is Pareto optimal for the economy* $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$.

In order to obtain the stronger result that any weak-RCPP equilibrium is Pareto optimal, stronger assumptions must be made to prevent a weak-RCPP equilibrium from getting "stuck" at an inefficient allocation because entrepreneurs cannot use options other than those currently active at the equilibrium. Thus the set $\mathcal{J}^i$ of potentially traded options must be sufficiently rich, even though typically at a given equilibrium only a few of these options will be actively traded: most will either be "out of the money" or duplicating existing traded options. The existence of many such potentially active options ensures that in a weak-RCPP equilibrium an entrepreneur can draw on a sufficiently rich set of financial strategies to be able to duplicate any of the "relevant" choices in the SPCM budget set $B(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i)$. To define what we mean by the "relevant choices", note that in an equilibrium with complete markets (whether it be an SPCM or a weak-RCPP equilibrium), every investor is subject to a standard Arrow-Debreu budget constraint. Since the economy has bounded resources and investors have monotonic preferences, any candidate equilibrium vector of state prices must be bounded away from zero. In short, for a given economy, there exists $\epsilon > 0$ such that any equilibrium price vector of an SPCM equilibrium or a weak-RCPP equilibrium with complete markets satisfies $\boldsymbol{\pi} \geq \epsilon \mathbf{1}$.

Definition 11. A plan $(\boldsymbol{x}^i, \kappa^i, e^i)$ is said to be *undominated* for agent i if there exists a vector of state prices $\boldsymbol{\pi} \geq \epsilon \mathbf{1}$, such that $(\boldsymbol{x}^i, \kappa^i, e^i)$ is optimal in $B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i)$. An input pair (κ^i, e^i) is undominated, if it is part of an undominated plan $(\boldsymbol{x}^i, \kappa^i, e^i)$.

Definition 12. We say that there is *complete spanning with redundancy at all undominated input pairs*(CSRU) if there exist subspaces $\mathcal{V}^1, \ldots, \mathcal{V}^{I_1}$ such that

(i) $\mathcal{V}^0 + \mathcal{V}^1 + \ldots + \mathcal{V}^{I_1} = \mathbb{R}^S$

(ii) $\mathbf{1} \in \mathcal{V}^i$, for all $i \in \mathcal{I}_1$

(iii) $\mathcal{V}^i(\kappa^i, e^i) \ \subset \ V^i$, for all $(\kappa^i, e^i) \subset \mathbb{R}^2_+$

(iv) $\mathcal{V}^i(\kappa^i, e^i) = \mathcal{V}^i$, for all undominated (κ^i, e^i)

The property CSRU is satisfied if the firms' technologies have the no-crossing property and the minimum of the differences between production levels across the states for all undominated input pairs is positive, i.e. for each $i \in I_1$, there exist α^i such that

$$\inf\left\{|F^i_s(\kappa^i, e^i) - F^i_{s'}(\kappa^i, e^i)| \,\Big|(\kappa^i, e^i) \text{ undominated and } (s, s') \text{ such that } F^i_s \neq F^i_{s'}\right\} > \alpha^i$$

If for each firm i the set of striking prices $\tau^i = (\tau^i_j)_{j \in J^i}$, contains all multiples of α^i as potential striking prices, then CSRU is satisfied.

Proposition 5. (Weak-RCPP is Pareto optimal) Let $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$ be an economy satisfying CSRU. If $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}; \bar{\boldsymbol{\pi}})$ is a weak-RCPP equilibrium at which markets are complete, then $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is an SPCM equillibrium and the allocation $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ is Pareto optimal.

Proof. In order to prove that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is an SPCM equilibrium, it suffices to prove that for each entrepreneur $(\bar{\boldsymbol{x}}^i, \bar{\kappa}^i, \bar{e}^i)$ is optimal in $B(\bar{\boldsymbol{\pi}}, \omega^i_0, \boldsymbol{F}^i)$. Since $(\bar{\boldsymbol{x}}, \bar{\kappa}^i, \bar{e}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ is optimal in $\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega^i_0, \boldsymbol{F}^i, \bar{\boldsymbol{y}})$, it suffices to prove that the optimal choice $(\boldsymbol{x}^i, \kappa^i, e^i)$ of agent i in $B(\bar{\boldsymbol{\pi}}, \omega^i_0, F^i)$ can be obtained with financial variables $(\boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ in $\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega^i_0, \boldsymbol{F}^i, \bar{\boldsymbol{y}})$. Using CSRU this can be shown by an argument similar to the proof of Proposition 3. △

Thus, under the assumption that options are brought into the market when they are needed, weak-RCPP equilibria are Pareto optimal. What remains to be studied is when a weak-RCPP is an RCPP equilibrium, i.e. when the first-order approach gives a correct result. This happens when the effort level satisfying the first-order conditions for problem (E) which maximizes the overall utility of entrepreneur i in the budget set (36) is also the solution to problem (E), i.e. maximizes the date 1 utility of the entrepreneur, net of the cost of effort, once the financial variables are fixed. We have examined a large family of economies satisfying the assumptions of Proposition 4, and in each case we found that there is a profile of striking prices (typically many profiles) for the options for which the weak-RCPP equilibrium is an RCPP equilibrium: there are also profiles for which weak-RCPP are not RCPP equilibria. The example that follows is typical and illustrates the apparent difficulty of establishing a general result due to the inherent non-convexities introduced by options.

Example 4. Consider an economy with two agents, an entrepreneur (agent 1) and an investor (agent 2), three states of nature of equal probability, the output across the states being given by the production function

$$\boldsymbol{F}(\kappa, e) = \sqrt{\kappa e}\,\boldsymbol{\eta} \quad \text{with} \quad \boldsymbol{\eta} = (25, 23, 20)$$

The utility functions are given by $U^i(\boldsymbol{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(\boldsymbol{x}_{\mathbf{1}}^i) - c(e^i)$, $i = 1, 2$, with

$$u_0^i(x_0^i) = \sqrt{a^i + x_0^i}, \quad u_1^i(\boldsymbol{x}_{\mathbf{1}}^i) = \frac{\delta}{3}\sum_{s=1}^{3}\sqrt{a^i + x_s^i}, \quad \delta = 0.9, \quad c(e^i) = (e^i)^2$$

and the initial wealth of each agent is $\omega_0^1 = 30$, $\omega_0^2 = 270$. When $a^1 = a^2 = 0$, the sole-proprietorship contingent-market equilibrium is given in the table below

SPCM Equilibrium									
	$\bar{x}_0^i$	$\bar{x}_1^i$	$\bar{x}_2^i$	$\bar{x}_3^i$	$\bar{\kappa}$	$\bar{e}$	$\bar{y}_1$	$\bar{y}_2$	$\bar{y}_3$
agent 1	65	124	114	99	112	1.86	360	331	288
agent 2	124	236	331	288					

To decentralize this Pareto optimum as a weak-RCPP equilibrium with a market structure consisting of equity with payoff $\bar{\boldsymbol{y}}$, a bond with payoff $\mathbf{1} = (1, 1, 1)$ and options with payoffs $\boldsymbol{R}_j(\bar{\boldsymbol{y}}) = \max\{\bar{y}_s - \tau_j, 0\}$, $j \in \mathcal{J}$, we need to solve the system of equations

$$\begin{aligned} \bar{\boldsymbol{x}}_{\mathbf{1}}^1 &= \theta^1\,\bar{y} + \textstyle\sum_{j\in\mathcal{J}} \xi_j^1 \boldsymbol{R}_{js}(\bar{\boldsymbol{y}}) + \xi_0 \mathbf{1} \\ c'(\bar{e}) &= \sum_{s=1}^{3} (\theta^1 + \textstyle\sum_{j\in\mathcal{J}_s} \xi_j^1) \dfrac{\partial u_1^1}{\partial x_s^1}(\bar{x}_s^i) \dfrac{\partial F_s}{\partial e}(\bar{\kappa}, \bar{e}) \end{aligned} \tag{37}$$

which admits a solution as soon as there is an option, call it option 1, with a striking price τ_1 such that $\bar{y}_2 < \tau_1 < \bar{y}_1$ and an second, option 2, with a striking price τ_2 such that $\bar{y}_3 < \tau_2 < \bar{y}_2$. For most values of (τ_1, τ_2), the solution of the system (37) is such that $\theta^1 > 1$ and ξ_0^1 is large and negative: this is more readily interpreted by introducing a third option, option 3, with striking price $\tau_3 < \bar{y}_3$, which has the same marginal effect as equity but automatically subtracts the income $\xi_3^1\,\tau_3$ from the income that the entrepreneur receives from the firm. For each combination $\tau = (\tau_1, \tau_2, \tau_3)$ of the striking prices satisfying the relevant inequalities and any θ,[18] there is a solution to the system of equations (37) which gives the financial variables $\boldsymbol{\xi}^1(\tau, \theta) = (\xi_j^1((\tau, \theta))_{j=0,\ldots,3}$ of a weak RCPP equilibrium.[19] To study if these equilibria are RCPP equilibria, we have computed the maximum

[18] The notation θ^1 is simplified to θ. The share of agent is $\theta^2 = 1 - \theta$.

[19] Since the Pareto optimal allocation can be decentralized as a weak-RCPP equilibrium for any value of θ, and in particular for $\theta = 0$, the results of this section are likely to extend to a more complicated (but also more realistic) model in which the initial owner of the firm and the manager are separate agents, as long as the reward schedule is chosen with the objective of maximizing the market value of the firm subject to the incentive constraint of the manager.

of the function

$$V^1_{(\tau,\theta)}(e) = u^1_1(\theta \boldsymbol{F}(\bar{\kappa}, e) + \sum_{j=1}^{3} \boldsymbol{R}_j(\bar{\kappa}, e)\, \xi^1_j(\tau,\theta) + \mathbf{1}\, \xi^1_0(\tau,\theta)) - c(e)$$

for different values of τ, fixing θ at 0.4.[20] Many, but not all, values of τ give a maximum at $\bar{e}$. Some lead to an optimal effort which is smaller and some to an optimal effort which is larger than $\bar{e}$. Figure 1 illustrates the three cases, and shows the incentive schedule $\phi(y)$ (i.e the date 1 consumption of agent 1 as a function of the realized output y) associated with the corresponding values of $\boldsymbol{\xi}^1(\tau,\theta)$. As the figure shows, the function $V^1_{(\tau,\theta)}(e)$ is far from being concave and has a complicated structure because of the changes of regime induced by options entering successively into the money in the different states. Varying the parameters of the model — for example the risk composition $\boldsymbol{\eta}$ of the firm's output or the parameters a^1 and a^2 of agents' risk aversion— leads in each case to the same conclusion: it is always possible to find striking prices which induce a reward structure such that the entrepreneur chooses the optimal level of effort $\bar{e}$, but not all striking prices work. If the entrepreneur is risk tolerant or if the technology is not very risky, the reward schedules which work tend to be increasing; when the investor is risk tolerant and insures the entrepreneur in the Pareto optimal allocation, the reward schedules have a decreasing portion as in Figure 1c. It is interesting to note that *the entrepreneur can be induced to make the optimal effort even when the investor is essentially risk neutral and bears all the risks of the economy.* For example with $\boldsymbol{\eta} = (25, 15, 10)$ and $a^1 = 0$, $a^2 = 10{,}000$, the SPCM equilibrium is

SPCM Equilibrium									
	$\bar{x}^i_0$	$\bar{x}^i_1$	$\bar{x}^i_2$	$\bar{x}^i_3$	$\bar{\kappa}$	$\bar{e}$	$\bar{y}_1$	$\bar{y}_2$	$\bar{y}_3$
agent 1	66	67	66	66	96	1.72	322	193	129
agent 2	137	254	127	63					

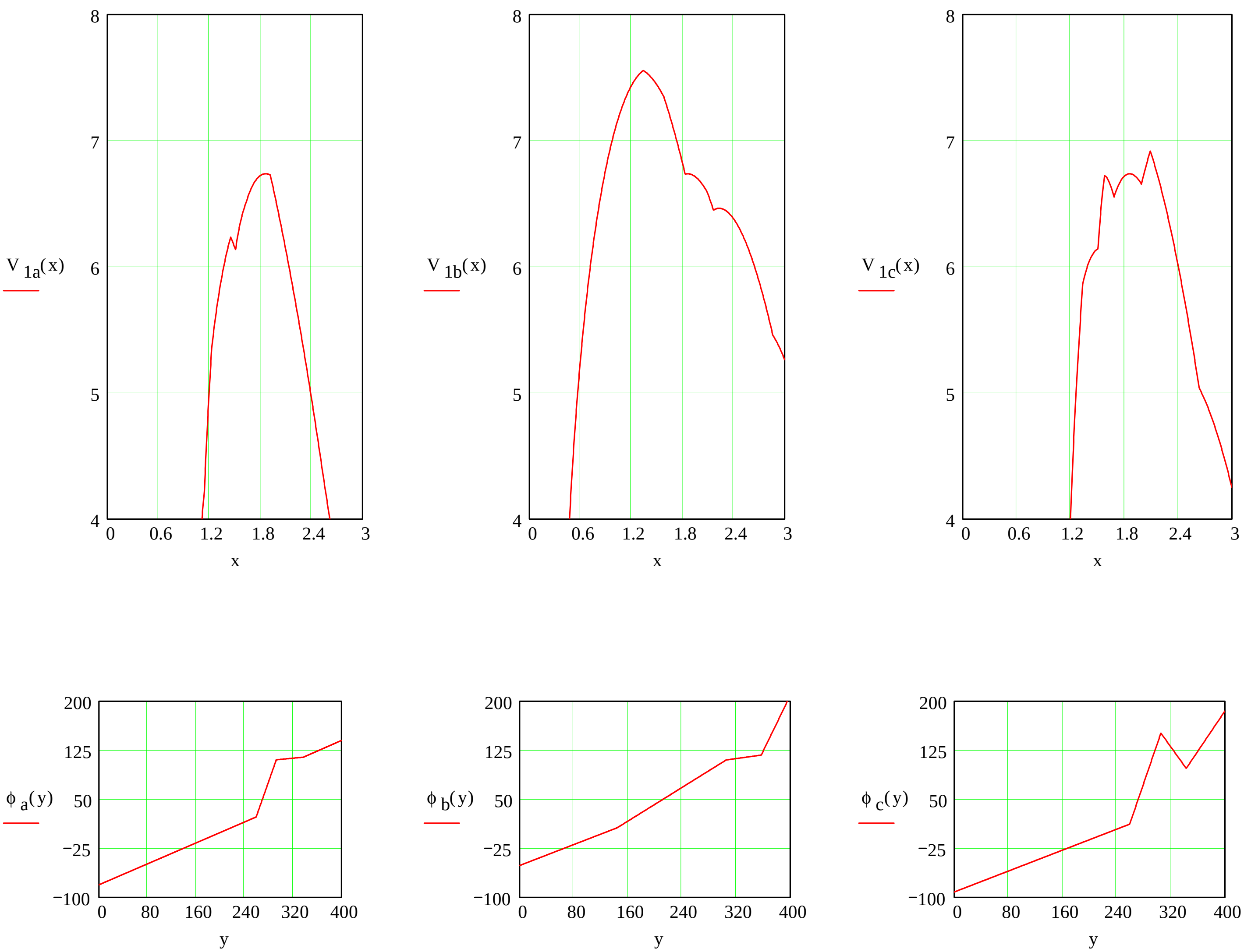
so that the entrepreneur's consumption is essentially constant across states. Achieving the first best allocation in this case, while impossible in the standard model where effort affects the probability of the outcomes but the outcomes are fixed[21], is possible in the present model where effort influences the outcome in each state. A reward schedule which induces the optimal effort of the entrepreneur in this case is shown in Figure 2.

The family of examples that we have studied is encouraging since it shows that options, which are now extensively used for incentive contracts of top executives, can lead to an efficient allocation

[20]We take a grid $\tau_{1_i} = \bar{y}_2 + (i/10)(\bar{y}_1 - \bar{y}_2)$, $\tau_{2_j} = \bar{y}_3 + (j/10)(\bar{y}_2 - \bar{y}_1)$, $\tau_{3_k} = (k/10)(\bar{y}_3)$, for $i, j, k \in \{1, \ldots, 9\}$.

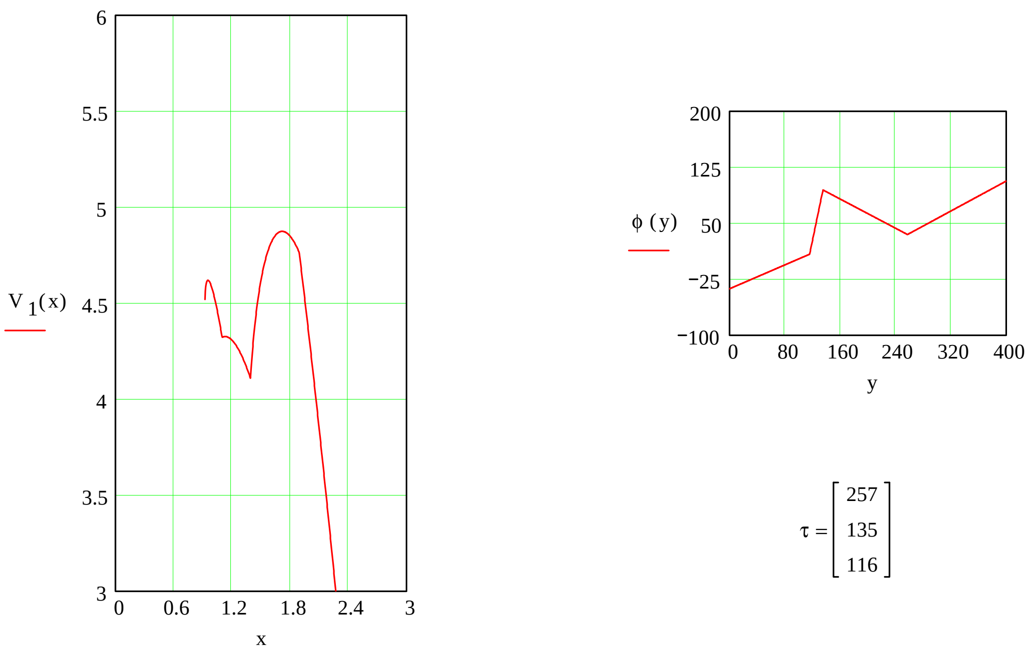
[21]This is the case most often studied in the principal-agent literature (see e.g. Kreps (1990, Chapter 16) for an exposition)

of risk and incentives. But it also shows that the result is sensitive to the exact form of the incentive package, which makes it hard to obtain general results. At the moment we have not succeeded in finding general conditions under which it can be proved that Pareto optimal allocations can be decentralized as RCPP equilibria, and we leave the question open for future research.



$$\tau_a = \begin{bmatrix} 337 \\ 292 \\ 259 \end{bmatrix} \qquad \tau_b = \begin{bmatrix} 357 \\ 305 \\ 144 \end{bmatrix} \qquad \tau_c = \begin{bmatrix} 343 \\ 305 \\ 259 \end{bmatrix}$$

Figure 1: The date 1 utility of the entrepreneur, net of his cost of effort, as a function of effort for the portfolio of options which solves the system (37), for different combinations of striking prices. In case (a) the optimal effort is at the Pareto optimal level e=1.86 and the weak-RCPP is an RCPP; with the striking prices of case (b) the effort optimal for agent 1 is below the Pareto optimal level, above in case (c).



$$\tau = \begin{bmatrix} 257 \\ 135 \\ 116 \end{bmatrix}$$

Figure 2: A reward schedule which induces the entrepreneur to choose the optimal effort in the case where the investor is risk neutral.

References

Arrow, K.J., "Le Role des Valeurs Boursières pour la Répartition la Meilleure des Risques", in *Econométrie, Colloques Internationaux du Centre National de la Recherche Scientifique*, 1953, *40*, pp. 41-47; English version: "The Role of Securities in the Optimal Allocation of Risk Bearing", *Review of Economic Studies*, 1964, *31*, pp. 91-96.

Berle, A.A. and G.C. Means, *The Modern Corporation and Private Property*, New York: Harcourt, Brace and World, 1932.

Bisin, A. and P. Gottardi, "General Competitive Analysis with Asymmetric Information", Working Paper, 1997.

Brander, J.A. and B.J. Spencer, "Moral Hazard and Limited Liability: Implications for the Theory of the Firm", *International Economic Review*, 1989, *30*, pp. 833-849.

Drèze, J.H., "Investment under Private Ownership: Optimality, Equilibrium and Stability", in *Allocation Under Uncertainty: Equilibrium and Optimality*, pp. 129-65, J.H. Drèze ed., New York: Wiley, 1974.

Ekern, S. and R. Wilson , "On The Theory of the Firm in an Economy with Incomplete Markets", *Bell Journal of Economics and Management Science*, 1974, *5*, pp. 171-180.

Grossman, S. and O. Hart, "A Theory of Competitive Equilibrium in Stock Market Economies", 1979, *Econometrica*, *47*, pp. 293-330.

Grossman, S. and O. Hart, "Corporate Financial Structure and Managerial Incentives", in: J. McCall, ed., *The Economics of Information and Uncertainty.* Chicago: University of Chicago Press, 1982, pp. 107-137.

Harris, M. and A. Raviv, "Financial Contracting Theory", in Jean-Jacques Laffont ed., *Advances in Economic Theory: Sixth World Congress, Volume II*, Cambridge: Cambridge University Press, 1992, pp. 64-150.

Holmstrőm. B., "Moral Hazard and Observability", *The Bell Journal of Economics*, 1979, *57*, pp. 74-91

Jensen, M.C. and W.H. Meckling, "Theory of the Firm: Managerial Behavior, Agency Costs, and Capital Structure", *Journal of Financial Economics*, 1976, *3*, pp. 305-360.

Jewitt, I., "Justifying the First-Order Approach to Principal-Agent Problems", *Econometrica*, 1988, *56*, pp. 1177-1190.

Knight, F.H., *Risk, Uncertainty and Profit*, Boston: Houghton Mifflin, 1921.

Kocherlakota, N.R., "The Effect of Moral Hazard on Asset Prices When Financial Markets are Complete", *Journal of Monetary Economics*, 1998, *41*, pp. 39-56.

Kreps, D.M., *A Course in Microeconomic Theory*, Princeton: Princeton University Press, 1990.

Leland, H. and D. Pyle, "Information Asymmetries, Financial Structure and Financial Intermediation", *Journal of Finance*, 1977, *32*, pp. 371-388.

Lisboa, M.B., "Moral Hazard and General Equilibrium in Large Economies", Working Paper, Stanford University, 1998.

Magill, M. and M. Quinzii, *Theory of Incomplete Markets*, Volume 1, Cambridge: MIT Press, 1996a.

Magill, M. and M. Quinzii, "Incentives and Risk Sharing in a Stock Market Equilibrium", in *Current Trends in Economics*, A. Alkan, C. D. Aliprantis, N.C. Yannelis eds, Berlin-Heidelberg: Springer-Verlag, 1999.

Murphy, K.J., "Executive Compensation", Marshall School of Business Working Paper, University of Southern California, 1998; forthcoming in *Handbook of Labor Economics*, *3*, O. Ashenfelter and D. Card eds, North-Holland.

Radner, R., "A Note on Unanimity of Stockholders' Preferences among Alternative Production Plans: a Reformulation of the Ekern-Wilson Model", *Bell Journal of Economics and Mangement Science*, 1974, *5*, pp. 181-184.

Rogerson, W.P., "The First Order Approach to Principal-Agent Problem", *Econometrica*, 1985, *53*, pp. 1357-1367.

Ross, S.A., " The Economic Theory of Agency: the Principal's Problem", *American Economic Review*, 1973, *63*, pp. 134-139.

Ross, S.A., "Options and Efficiency", *Quarterly Journal of Economics*, 1976, *90*, pp. 75-89.

Ross, S.A., "The Determination of Financial Structure: The Incentive-Signalling Approach", *Bell Journal of Economics*, Spring 1977, *8*, pp. 23-40.