A Theory of Firm Formation and Skills Acquisition

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December 28, 1999
Preliminary Draft

Abstract: We present a theory of production that begins with an exogenously specified set of technologies, accessible to each potential firm. The technologies used in equilibrium are endogenous. Labor skills are differentiated, and the labor skills are acquired endogenously by workers, possibly by bearing private costs, and possibly by attending school. A technology can be used by a group of agents having the appropriate skills. We allow that workers care about the production plans in their firms, and will accept lower compensation to satisfy their preferences on production plans. In a continuum model, we show what price systems are required so that competitive equilibrium exists and core outcomes are equivalent to competitive outcomes.

Keywords: clubs, production, skills, schools, groups

1. Introduction

General equilibrium theory generally begins with an exogenous set of firms, each of which has an exogenously given production set (see, for example, Mas-Colell, Whinston and Green (1995)). There is no opportunity to replicate the production set, and hence there is no distinction between the technology and the firm.

In this paper we take the view that it should be *technologies* and not *firms* that are given exogenously in the economy. The formation of a firm should be an equilibrium outcome rather than a primitive datum of the economy.

We think of a technology as a publicly accessible means of converting inputs to outputs, and we think of a firm as a group of agents employing a technology. However, not every group of agents can use every technology, since a technology will typically require different kinds of skilled labor. The technology must specify the skills required to use it. Because skills give access to valuable technologies, agents have incentives to invest in skills. The skills they acquire in equilibrium will affect which technologies are used, and the available technologies will affect which skills the agents acquire. It is this simultaneous problem of skill acquistion and firm formation that we wish to address.

We define a technology as a triple consisting of (i) the skill mix of the agents required to use it, (ii) the infrastructure of the technology and (iii) the production plan that the skill mix and infrastructure enable. Even without the additional complexity that agents must invest in skills, such a structure introduces a matching problem in the formation of firms. The firms that form in equilibrium must employ agents with different skills in a way that is consistent with the technologies, the preferences of the agents, and their endowments of private goods.

Labor skills can be acquired in schools. A school is defined by its teachers, the skills offered to the students, and resource costs. Student fees, as determined in equilibrium, depend on the skills acquired and possibly on other aspects of the school, but not on the innate abilities of the students. However we assume that unless the student makes suitable collateral investments of a private nature, he might fail to acquire the skills offered by the school. We thus assume that the acquisition of skills entails two types of costs: those borne privately by the student, and the fees he pays to the schools he attends. Conditional on deciding to acquire a skill, the student will choose his type of schooling by considering the relative costs, but also taking account of which he enjoys more. Thus he might or might not attend school in addition to making personalized investments.

Firms and schools are both instances of group formation. Both types of groups are partially defined by the "external characteristics" of their members, which for firms are work skills. In two previous papers (1999a,b) we have developed a theory of group formation that overcomes the problem of matching agents consistently in groups. In this paper we adapt that theory for the joint problems of skills acquisition and firm formation. One of the innovations of our papers (1999a,b) is to permit that each agent can belong to many groups simultaneously. The present paper depends heavily on this feature as it allows us simultaneously to solve the matching problem in firms, schools, and social clubs.

The groups discussed in the (1999a,b) papers are called "clubs". Those papers begin with an exogenously given set of "clubtypes", which are defined by the external characteristics of their members and the activities they undertake. Memberships to clubtypes are bought and sold in the market, and the number of social groups of each clubtype is endogenous. The theory below can be interpreted to mean that the external characteristics or skills required by agents for club membership can be acquired endogenously rather than given exogenously.

In this paper we extend the notion of clubtype so that it extends to production technologies and schools, and we call it a "grouptype". For grouptypes that are interpreted as technologies, the members are workers, and the production plan, which typically has private goods output as well as inputs, takes the place of private goods inputs. The external characteristics specified for the technology are interpreted as labor skills, and the activity is interpreted as working conditions.

In our model workers have preferences over memberships in grouptypes and thus especially on being workers in different technologies. Hence, our model of production accomodates the complexity that workers can have preferences over production plans, working conditions, and characteristics of their co-workers. The technology specifies, for example, whether the production plan uses environmentally sound practices or small furry animals in testing cosmetics. Because workers are allowed to care about the technology of production, agents with the same labor skills but different tastes can earn different incomes in equilibrium.

Our theory of production follows Keiding (1973) in that it has exogenous technologies and differentiated labor in a continuum model. However, in Keiding's model agents have access to a single technology, rather than to several technologies, and there is no problem of matching agents consistently in groups. Private goods are sold in a competitive market, but firms are formed cooperatively. He shows that in equilibrium, it is "as if" there are competitive prices for labor characteristics, and the core is equivalent to competitive equilibrium. Labor skills are endowed rather than acquired. Drèze (1989) also has a concept of labor management in equilibrium, where any group of agents has access to an exogenously given technology. Labor is divisible and differentiated, but again there is no matching problem. Agents do not have preferences for working in specific firms, apart from its affect on income.

Our theory of production is also in the spirit of coalition production models (see Ichiishi (1993)) in that each group of agents has production possibilities. In coalition production models, the production possibilities are given exogenously for each coalition, whereas in our model, the coalition must choose from an exogenously given set of technologies which are appropriate for the skills they have acquired. In addition, the production plan of the firm and the characteristics of co-workers can affect each agent's utility.

The plan of this paper is first to embellish our (1999b) clubs model so that it extends to technologies and schools. In Section 2 we define feasible states, optimality, the core, and competitive equilibrium, and present the existence and core-equivalence theorems. There are two aspects of the group formation model in Section 2 that differ from our (1999a,b) papers. The first difference from our previous model is that agents' external characteristics (labor skills) can be chosen endogenously rather than given exogenously as an endowment. Second, we include a private goods vector in the formal definition of a grouptype. In our previous model, this vector was an input vector, and of no direct consequence to club members.

In Section 3 we elaborate on how the general model of group formation in Section 2 applies to technologies and schools.

A potential criticism of the model in Section 2 is that the price space is very large. In addition to the prices for private goods, there is a membership price for agents with every characteristic in every type of group. In general, such a large price space is required, but in Sections 4 and 5 we give conditions under which a smaller price space will suffice. In particular, we give conditions under which the equilibrium allocations described in Section 2 are also equilibria under a price system in which workers' wages depend only on their skills, and not on the technologies where they work. Under a similar condition for schools, tuition fees depend only on the skill being learned, and not on the type of school. The latter does not preclude that students pay different total costs, as they might make collateral investments in their educations which are known only to themselves.

As in our (1999b) paper, we assume there is a continuum of agents. In a finite economy it might be impossible to ensure that all the slots are filled in every group that some agent wants to join. Since we assume that groups are finite, the continuum allows this problem to be solved, leading to existence of equilibrium and core/competitive equivalence as well as giving a foundation for price-taking.

2. General Equilibrium with Group Formation

We first extend our (1999b) club model so that it applies to schools and firms. Instead of using the language of "clubtypes" and "clubs" as in the previous paper, we use the language of "grouptypes" and "groups".

2.1. Grouptypes and memberships

Groups are described by an exogenous set of *grouptypes*.

To define grouptypes, let Ω be a finite set of *external characteristics* of potential members, and let Γ be an abstract, finite set of *activities*. We assume there are $N \geq 1$ divisible private goods.

A *grouptype* is a triple (π, γ, y) consisting of a *profile* $\pi : \Omega \to \mathbf{Z}_+ = \{0, 1, \ldots\}$, an activity $\gamma \in \Gamma$, and a vector of private goods $y \in \mathbf{R}^N$. The profile describes the external characteristics of the group's members. For $\omega \in \Omega$, $\pi(\omega)$ represents the number of members of the group having external characteristic ω. The negative elements of y represent net inputs, and the positive elements represent net outputs. We take as given a finite set of possible grouptypes $\mathcal{G} = \{(\pi, \gamma, y)\}$.

A *membership* is an opening in a particular grouptype for an agent of a particular external characteristic; i.e., $m = (\omega, (\pi, \gamma, y))$ such that $(\pi, \gamma, y) \in \mathcal{G}$ and $\pi(\omega) \geq 1$. We write $\mathcal{M}$ for the (finite) set of memberships.

Each agent may choose many memberships in groups or none. A membership *list* is a function $\ell : \mathcal{M} \to \{0, 1, \ldots\}$, where $\ell(\omega, (\pi, \gamma, y))$ specifies the number of memberships of type $(\omega, (\pi, \gamma, y))$.

We assume that for each $\omega \in \Omega$, the set $\mathcal{G}$ contains a grouptype with a single member with the characteristic ω, and no inputs or production plan. We correspondingly define "singleton lists" $\{\ell_\omega\}_{\omega \in \Omega}$. The interpretation is that if an agent chooses a singleton list ℓ_ω, then he is choosing the characteristic ω, which might affect his utility for private goods even if he does not use that characteristic in other group memberships.

2.2. Agents

The set of agents is a nonatomic finite measure space $(A, \mathcal{F}, \lambda)$. That is, A is a set, $\mathcal{F}$ is a σ-algebra of subsets of A and λ is a non-atomic measure on $\mathcal{F}$ with $\lambda(A) < \infty$.

A complete description of an agent $a \in A$ consists of a consumption set, a personal cost function for producing his external characteristics, an endowment of private goods and a utility function.

The consumption set X_a specifies the feasible bundles of private goods and feasible lists of memberships. We assume that the number of memberships that each agent can consume is bounded by $M > 0$. Since the agent must restrict himself to a single external characteristic (which is possibly a bundle of several attributes), feasible lists include the restriction that all memberships in the list are for the same characteristic. We say that a list ℓ is *associated* with an external characteristic $\omega \in \Omega$ if $\ell(\omega', (\pi, \gamma, y)) = 0$ for all $(\pi, \gamma, y) \in \mathcal{G}$ when $\omega \neq \omega'$. We assume that $X_a = \mathbf{R}^N_+ \times \mathbf{Lists}(a)$ where

$$\mathbf{Lists}(a) \subset \{\ell : \ell \text{ is a list that is associated with some } \omega \in \Omega;$$

$$\sum_{(\omega,(\pi,\gamma,y))\in\mathcal{M}} \ell(\omega,(\pi,\gamma,y)) \leq M \ \}$$

For $\ell \in \cup_{a\in A}\mathbf{Lists}(a)$, $\ell \neq 0$, we use the notation $\omega^L(\ell)$ to denote the characteristic that is associated with the list ℓ.

Let $c_a : \mathbf{Lists}(a) \to \mathbf{R}^N_+$ represent agent a's personal cost, where $c_a(\ell)$ is the personal cost required to achieve the characteristic $\omega^L(\ell)$ associated with the list $\ell \in \mathbf{Lists}(a)$, $\ell \neq 0$. We assume that $c_a(0) = 0$. In our (1999a,b) papers, we assumed that agents were endowed directly with external characteristics. Here we enrich the model so that agents can acquire their characteristics, but at personalized costs. The cost to a particular agent of acquiring a particular characteristic might be zero. As we shall see when we interpret grouptypes as schools, agent a might produce his characteristic either by bearing a personal cost and not going to school, or by going to school. Going to school might lower his personal cost of acquiring a characteristic, but the school itself would typically require inputs.

The endowments of private goods are $e_a \in \mathbf{R}^N_+$, $a \in A$.

The utility function is defined over private goods consumptions and lists of group memberships, and is thus a mapping $u_a : X_a \to \mathbf{R}$. We assume throughout that utility functions $u_a(\cdot, \ell)$ are continuous and strictly monotone in private goods. The utility an agent obtains by consuming private goods can depend on the agent a's choice of characteristic even if he does not consume any memberships in grouptypes $(\pi,\gamma,y) \in \mathcal{G}$ with $|\pi| \geq 2$ by the choice of a singleton list ℓ_ω for some $\omega \in \Omega$.

2.3. Economies

An *economy* $\mathcal{E}$ is a mapping $a \mapsto (c_a, X_a, e_a, u_a)$ for which:

- the cost function $(a, \ell) \mapsto c_a(\ell)$ is a jointly measureable function of its arguments.
- the consumption set correspondence $a \mapsto X_a$ is a measurable correspondence
- the endowment mapping $a \mapsto e_a$ is an integrable function
- the utility mapping $(a, x, \ell) \mapsto u_a(x, \ell)$ is a jointly measurable function of its arguments.

We assume that the *aggregate endowment* $\bar{e} = \int_A e_a \, d\lambda(a)$ is strictly positive, so all private goods are represented in the aggregate.

2.4. States

A *state* of an economy is a measurable mapping

$$(x, \mu) : A \to \mathbf{R}^N \times \mathbf{R}^{\mathcal{M}}$$

A state describes choices of private goods and lists (hence, when $\mu_a \neq 0$, the external characteristics) for each agent, ignoring budget feasibility at the level of the individual and at the level of society. *Individual feasibility* means $(x_a, \mu_a) \in X_a$. *Social feasibility* entails market clearing for private goods and consistent matching of agents. To define consistency, we consider the *aggregate membership vector*, namely a vector $\bar{\mu} \in \mathbf{R}^{\mathcal{M}}$ that represents the total number of memberships of each type. We say that an aggregate membership vector $\bar{\mu} \in \mathbf{R}^{\mathcal{M}}$ is *consistent* if for every grouptype $(\pi, \gamma, y) \in \mathcal{G}$, there is a real number $\alpha(\pi, \gamma, y)$ such that

$$\bar{\mu}(\omega, (\pi, \gamma, y)) = \alpha(\pi, \gamma, y)\ \pi(\omega)$$

for each $\omega \in \Omega$. For $B \subset A$, a measurable function $\mu : B \to \cup_{a \in B} \mathbf{Lists}(a)$ is *consistent for* B if the corresponding aggregate membership vector $\bar{\mu} = \int_B \mu_a d\lambda(a) \in \mathbf{R}^{\mathcal{M}}$ is consistent. Write

$$\mathbf{Cons} = \{\bar{\mu} \in \mathbf{R}^{\mathcal{M}} : \bar{\mu} \text{ is consistent } \}$$

Cons is a subspace of $\mathbf{R}^{\mathcal{M}}$. If individual membership lists are in the agents' consumption sets, the aggregate membership vector is in the positive part $\mathbf{Cons}_+ \subset \mathbf{Cons}$.

The state (x, μ) is *feasible for* the measurable subset $B \subset A$ if it satisfies the following requirements:

(i) **Individual Feasibility** $(x_a, \mu_a) \in X_a$ for each $a \in B$

(ii) **Material Balance**

$$\int_B (x_a + c_a(\mu_a))\, d\lambda(a) - \int_B \sum_{(\omega,(\pi,\gamma,y)) \in \mathcal{M}} \mu_a(\omega, (\pi, \gamma, y)) \frac{y}{|\pi|}\ d\lambda(a) \leq \int_B e_a\, d\lambda(a)$$

(iii) **Consistency** $\int_B\ \mu_a\ d\lambda(a)$ is consistent.

The state (x, μ) is *feasible* if it is feasible for the set A itself.

A state of the economy will generally have "many" groups of some grouptypes. Because members of a group care only about the external characteristics of other members, and not about their identities, it is not necessary to distinguish different groups of the same grouptype.

2.5. Profit-Share Equilibrium

Competitive prices will be $(p, q) \in \mathbf{R}_+^N \times \mathbf{R}^{\mathcal{M}}$, where p is a vector of prices for private goods and q is a vector of prices for memberships in groups. Because utility functions are assumed monotone in private goods, the prices of private goods will be non-negative, but prices of memberships may be positive, negative or zero. In this section we prove existence of equilibrium prices such that profit in each grouptype is exactly zero. For this reason we call it a profit-share equilibrium. The existence of such an equilibrium depends on the rich set of prices defined here: Membership prices differ according to both the member's external characteristic and the grouptype.

A *profit-share equilibrium* is a feasible state (x, μ) and prices $(p, q) \in \mathbf{R}_+^N \times \mathbf{R}^{\mathcal{M}}, p \neq 0$ such that

(1) **Budget Feasibility for Agents** For almost all $a \in A$,

$$(p, q) \cdot ((x_a + c_a(\mu_a)), \mu_a) \leq p \cdot e_a$$

(2) **Optimization of Agents** For almost all $a \in A$:

$$(x'_a, \mu'_a) \in X_a \text{ and } u_a(x'_a, \mu'_a) > u_a(x_a, \mu_a) \Rightarrow (p, q) \cdot ((x'_a + c_a(\mu'_a)), \mu'_a) > p \cdot e_a$$

(3) **Budget Balance for Grouptypes** For each $(\pi, \gamma, y) \in \mathcal{G}$:

$$\sum_{\omega \in \Omega} \pi(\omega)\ q(\omega, (\pi, \gamma, y)) + p \cdot y = 0$$

Thus, at an equilibrium individuals optimize subject to their budget constraints and the sum of membership prices in a given grouptype is just enough to pay for the production plan or inputs.

A *proit-share quasi-equilibrium* satisfies (1), (3) and (2′) instead of (2):

(2′) **Quasi-Optimization** For almost all $a \in A$:

$$(x'_a, \mu'_a) \in X_a \text{ and } u_a(x'_a, \mu'_a) > u_a(x_a, \mu_a) \Rightarrow (p,q)\cdot((x'_a + c_a(\mu'_a)), \mu'_a) \geq p\cdot e_a$$

That is, no consumption bundle that is feasible and strictly preferred can cost strictly less than agent a's wealth. An equilibrium is necessarily a quasi-equilibrium.

We say that the grouptype $(\pi, \gamma, y) \in \mathcal{G}$ is *used in a profit-share quasi-equilibrium* $(p,q),(x,\mu)$ if $\int_A \mu_a(\omega, (\pi, \gamma, y))d\lambda(a) > 0$ for some $\omega \in \Omega$.

2.6. Theorems

We say a feasible state (x, μ) is *Pareto optimal* if there is no feasible state (x', μ') such that $u_a(x'_a, \mu_a) > u_a(x_a, \mu_a)$ for almost every $a \in A$. We say (x, μ) is in the *core* if there is no subset $B \subset A$ of positive measure and state (x', μ') that is feasible for B such that $u_b(x'_b, \mu'_b) > u_b(x_b, \mu_b)$ for almost every $b \in B$.

The following theorem follows from Theorem 4.1 in our (1999b) paper.

Theorem 2.1. *Let (x, μ),(p,q) be a profit-share equilibrium for an economy $\mathcal{E}$. Then the state of the economy (x, μ) is Pareto optimal and in the core.*

We say that *endowments are desirable* if for every agent a and every list $\ell \in$ **Lists**(a), $u_a(e_a, 0) > u_a(0, \ell)$. This condition is discussed in our (1999a) paper.

Theorem 2.2. *Let $\mathcal{E}$ be an economy in which endowments are desirable and uniformly bounded above. Then every core state can be supported as a profit-share quasi-equilibrium.*

The theorem is proved exactly as in our (1999a) paper, except that the functions $\tau_a : X_a \to \mathbf{R}^N$, $a \in A$, defined as follows, must be substituted for the function τ. The value $\tau_a(\ell)$ is equal to the resource cost to agent a for consuming any list ℓ of memberships in his consumption set. It imputes the inputs and outputs of groups to the members equally, and also includes the agent's personal cost of acquiring the characteristic $\omega^L(\ell)$.

$$\tau_a(\ell) = c_a(\ell) - \sum_{(\pi,\gamma,y)\in\mathcal{M}} \ell(\omega, (\pi, \gamma, y)) \frac{y}{|\pi|}$$

Theorem 2.3. *Let $\mathcal{E}$ be an economy. If endowments are desirable and uniformly bounded above, then a profit-share quasi-equilibrium exists.*

This theorem is proved exactly as in our (1999b) paper. There are additional considerations in going from quasi-equilibrium to equilibrium. It is not our main purpose to work out these details, so we are content to state the existence of a quasi-equilibrium.

3. Interpretations

We now refine our discussion by partitioning the set of grouptypes into three:

$$\mathcal{G} = \mathcal{T} \cup \mathcal{C} \cup \mathcal{S}.$$

The set $\mathcal{T}$ are the *technologies*. A grouptype $(\pi, \gamma, y) \in \mathcal{G}$ is in $\mathcal{T}$ if y includes positive elements (outputs). If a grouptype $(\pi, \gamma, y) \in \mathcal{G}$ satisfies $y \leq 0$, then it can be in either C or S according to whether we interpret it as a social club, comprising $\mathcal{C}$, or a school type, comprising $\mathcal{S}$.

A feature of the model that becomes particularly apparent when we discuss schools is that an external characteristic $\omega \in \Omega$ is a composite of many different "attributes", such as intelligence, cooperativeness, personal habits, the ability to teach, and work skills. The bundling in ω might be thought unsatisfying, since not all attributes are relevant for all groups. A particular ω might include the fact that the agent is a good dancer, as well as a good economist. Dancing is presumably irrelevant to a job as an economist, but relevant to social opportunities. In section 4 we investigate natural restrictions on the economy such that membership prices only reflect the attributes that are relevant.

3.1. Social Clubs

In our simpler (1999a,b) model, we discussed "clubs", but not technologies or schools. Our examples included marriages, living arrangements and athletic facilities. For some of these clubs, such as marriages or living arrangements, the notion of a profit-share equilibrium is particularly appropriate, as there is no profit-maximizing unit, but only transfers among the members, who jointly pay for the resource costs. Other clubs, such as athletic facilities, might more naturally be thought of as firms. The model here is a generalization of our previous model, with the additional feature that agents can acquire the characteristics that are relevant for their social clubs, possibly by bearing personal costs, and possibly by going to school. By introducing the input vector directly into the definition of a clubtype, we also allow that the membership prices could differ according to the input vector, although this will not happen if members are indifferent to the input vector (Proposition 4.3).

3.2. Production

We interpret $(\pi, \gamma, y) \in \mathcal{T}$ as a *technology*, and the membership $(\omega, (\pi, \gamma, y))$ as a *job*. In this conception of a production economy, it is the technologies and not the firms that are given exogenously. Each group of workers has access to an exogenous set of technologies and groups form endogenously determined in equilibrium. A firm is implicitly a group of workers using a technology.

The external characteristics of members are their work skills, and the vector y is a production plan. The activity γ establishes the working conditions of the firm, e.g., how many hours a worker of each type must contribute. Instead of being priced as an input in y, labor is compensated through the membership prices q. The salary or profit share for a job $(\omega, (\pi, \gamma, y))$ is $-q(\omega, (\pi, \gamma, y))$. The profit shares sum to the value $p \cdot y$. Typically equilibrium profit shares will be positive (the membership price is negative), since a worker would not take a job unless compensated. (On the other hand, the utility value of working in a firm could be so great that the worker would even pay to work there.) In contrast, social grouptypes would typically have positive membership prices to cover the inputs. The member is willing to pay a positive price in order to enjoy the activity and the externalities from other members.

Various production plans can be associated with the same (π, γ) in the set of technologies $\mathcal{T}$, e.g., (π, γ, y) and (π, γ, y'), where y and y' have the same outputs but different inputs. In this sense our technologies allow substitutability of inputs and outputs. However, the fact that there is a finite number of technologies restricts the substitutability of divisible inputs and outputs. An alternative production model with continuous substitutability would define a grouptype technology by (π, γ), and associate to it a production set $Y(\pi, \gamma) \subset \mathbf{R}^N$ rather than a fixed production plan. We view this as a technical advantage rather than a conceptual advantage. It has the disadvantage that workers' preferences for production plans are harder to capture. When a worker takes a job, he subscribes to the entire production set (else the firm would again be defined by a production plan instead of a production set). A limited way to introduce the workers' concerns for production plans would be that the firm could commit in advance not to use certain inputs. We have chosen not to pursue this route.

3.3. Schools

Grouptypes in $(\pi, \gamma, y) \in \mathcal{S}$ (schools) have the same formal structure as ordinary social clubs in $\mathcal{C}$. The activity γ specifies, among other things, which members are teachers and which are students, what curriculum is offered, and the physical facilities. For example, in a computer programming school, some members, say $\bar{\omega}$, would be designated by γ as "teachers", and would have both teaching skills and programming skills. Others members, say $\hat{\omega}$, would be designated by γ to be "students", and would end up with programming skills. The utility that an agent derives from attending a particular school can depend not only on the teachers and curriculum, but on "peer group effects". In addition, the personal costs of getting educated can depend on peer group effects, as in Benabou (1993).

We have allowed for two types of costs in acquiring external characteristics: school costs and personal costs. The resource cost of a school is the input vector y in the school grouptype (π, γ, y). Each member's share of these costs is designated by his equilibrium membership price. The learner will typically pay a positive price $q(\hat{\omega}, (\pi, \gamma, y))$. A teacher will typically be compensated a positive amount $-q(\bar{\omega}, (\pi, \gamma, y)) > 0$. In addition to the resource cost of the school, there may be a personal cost of acquiring the characteristic $\hat{\omega}$, specified by c_a, which depends on which schools the student attends, if any. The students' personal costs are what distinguish good learners from bad learners. Personal costs are an important determinant of how skills get distributed among agents in equilibrium. If an agent acquires the characteristic $\hat{\omega}$, his equilibrium membership list might or might not include schools, but would typically include memberships in firms, since he would not otherwise want to bear the cost of acquiring the characteristic.

Despite these interpretations, the formal model of a school has a certain symmetry between students and teachers, and it is perhaps not obvious why the payments should go one direction and not the other, namely, from students to teachers. There are two reasons this would be so. First, teachers might find teaching unpleasant, and have to be compensated for it. Second, the teacher's consumption set might be such that he cannot both be a teacher and have another job. In that case, he must choose between being a teacher and being a computer programmer, and will only be a teacher if sufficiently compensated. The teacher's students are willing to compensate him, and also cover the resource costs of the school, because their own lists of memberships include highly-paid jobs as computer programmers. Notice that we abstract from temporal issues, in particular, that education typically precedes work.

4. Pricing Relevant Characteristics

The price space in a profit-share equilibrium is very large. In particular, it permits a different membership price for each external characteristic and each grouptype. We now show that the price space can be reduced in two ways.

(1) Membership prices for two *different grouptypes* should be the same (with different prices for members with different external characteristics) if memberships in the grouptypes are substitutes in consumption, in the sense that every agent is indifferent to which he consumes. Proposition 4.3 states that this is so.

(2) Membership prices *within a given grouptype* should only depend on the aspects of each ω that are relevant to that grouptype. Thus, members with different external characteristics in Ω, who have the same *relevant* characteristics for a grouptype, should have the same membership price. Proposition 4.5 states that this is so.

To see the relevance of (1), consider the fact that membership prices can depend on the input vector y to a social club or the production plan in a technology. This is in contrast to our (1999a,b) papers, where "clubtypes" were defined only by (π, γ), and not by (π, γ, y). In our reformulation here, the grouptype is defined by (π, γ, y) where y is the input vector. This broader perspective permits different membership prices for, say (π, γ, y) and (π, γ, y'), where $y \neq y'$. It permits that agents' preferences about memberships can depend on inputs or, in the case of technologies, production plans. However if agents do not, in fact, care about y, then equilibrium membership prices should presumably be the same in two grouptypes that differ only in y. Proposition 4.3 implies that this is so, since the grouptypes (π, γ, y) and (π, γ, y') will be substitutes in consumption if agents do not care about the production plan.

Definition 4.1. For $\omega \in \Omega$, and (π, γ, y), $(\pi', \gamma', y') \in \mathcal{G}$ such that $\pi(\omega) > 0$ and $\pi'(\omega) > 0$, **the two memberships** $(\omega, (\pi, \gamma, y))$, $(\omega, (\pi', \gamma', y'))$ **are substitutes for agent** $a \in A$ if the following holds: Let the lists ℓ, ℓ' be identical except for memberships of types $(\omega, (\pi, \gamma, y))$, $(\omega, (\pi', \gamma', y'))$, where $\ell(\omega, (\pi, \gamma, y)) + \ell(\omega, (\pi', \gamma', y')) = \ell'(\omega, (\pi, \gamma, y)) + \ell'(\omega, (\pi', \gamma', y'))$. Then $\ell \in \mathbf{Lists}(a)$ if and only if $\ell' \in \mathbf{Lists}(a)$ and if $\ell, \ell' \in \mathbf{Lists}(a)$ then $u_a(x, \ \ell) = u_a(x, \ \ell')$ for all $x \in \mathbf{R}_+^N$ and $\mathrm{c}_a(\ \ell) = \mathrm{c}_a(\ \ell')$.

The definition implies that if agent a substitutes a membership $(\omega, (\pi, \gamma, y))$ for a membership $(\omega, (\pi', \gamma', y'))$, his utility remains the same and his personal costs are unchanged.

Definition 4.2. Two grouptypes (π, γ, y), $(\pi', \gamma', y') \in \mathcal{G}$ are **substitutes in consumption** if for any $\omega \in \Omega$ such that $\pi(\omega) > 0$ and $\pi'(\omega) > 0$, the memberships $(\omega, (\pi, \gamma, y))$, $(\omega, (\pi', \gamma', y'))$ are substitutes for all $a \in A$.

Proposition 4.3. *Assume that* $(p, q), (x, \mu)$ *are a profit-share quasi-equilibrium, and that* (π, γ, y), $(\pi', \gamma', y') \in \mathcal{G}$ *are substitutes in consumption. Suppose that* (π, γ, y) *is used in equilibrium. Then for every* $\omega \in \Omega$ *such that* $\pi(\omega) > 0$ *and* $\pi'(\omega) > 0$, $q(\omega, (\pi, \gamma, y)) \leq q(\omega, (\pi', \gamma', y'))$, *with equality if both grouptypes are used in equilibrium.*

Proof: The inequality follows simply from the fact that the two grouptypes are substitutes in consumption. Suppose, contrary to the inequality, that $q(\omega, (\pi, \gamma, y)) > q(\omega, (\pi', \gamma', y'))$. Then all agents for whom $\mu_a(\omega, (\pi, \gamma, y)) > 0$ would prefer to substitute memberships $(\omega, (\pi', \gamma', y'))$, which contradicts that (x, μ) is an equilibrium. Now suppose that for some ω with $\pi(\omega) > 0$ and $\pi'(\omega) > 0$ the inequality holds strictly i.e. $q(\omega, (\pi, \gamma, y)) < q(\omega, (\pi', \gamma', y'))$ and that both grouptypes are used in equilibrium. Then all agents for whom $\mu_a(\omega, (\pi', \gamma', y')) > 0$ would prefer to substitute memberships $(\omega, (\pi, \gamma, y))$. □

To see the relevance of the second restriction on prices, (2) above, suppose that Ω contains four external characteristics, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ = {economist-juggler, economist-dancer, electrician-juggler, electrician-dancer}. Only dancing ability would typically be relevant for a dancing club. As a consequence, all agents with characteristics in $\{\omega_2, \omega_4\}$, dancers, should pay the same price for membership in a dancing club, but the price would typically be different than for agents with characteristics in $\{\omega_1, \omega_3\}$, jugglers.

Symmetrically, dancing is irrelevant for an economist job. Economist-jugglers and economist-dancers should be completely substitutable in a technology (π, γ, y) where economist skills are used, but not dancing skills. Hence the wages should satisfy $-q(\omega_1, (\pi, \gamma, y)) = -q(\omega_2, (\pi, \gamma, y))$.

In fact, *all* wages, including the wages of managers and janitors, should be the same in two technologies (π, γ, y) and (π', γ, y), where π and π' are identical except that the economists have different dancing and juggling skills. That is implied by Proposition 4.3, provided that dancing and juggling do not enter the workplace.

The irrelevancy of some aspects of the external characteristic is captured in the notion that external characteristics are substitutes in a grouptype, defined below. Of course, characteristics that are substitutes in one grouptype might not be substitutes in another grouptype. In the example with dancers and economists, the characteristics $\{\omega_2, \omega_4\}$ are substitutes in the dancing grouptype, while the characteristics $\{\omega_1, \omega_2\}$ are substitutes in the technology that employs economists.

For each $\tilde{\Omega} \subset \Omega$ and $(\pi, \gamma, y) \in \mathcal{G}$ let

$$\mathcal{G}(\tilde{\Omega}; (\pi, \gamma, y)) = \{(\pi', \gamma, y) | \sum_{\omega \in \tilde{\Omega}} \pi'(\omega) = \sum_{\omega \in \tilde{\Omega}} \pi(\omega) \text{ and } \pi'(\omega) = \pi(\omega) \text{ for all } \omega \in \Omega \backslash \tilde{\Omega}\}$$

Definition 4.4. For $(\pi, \gamma, y) \in \mathcal{G}$, the characteristics in $\tilde{\Omega} \subset \Omega$ are **substitutes in the grouptype** (π, γ, y) if

1. $\mathcal{G}(\tilde{\Omega}; (\pi, \gamma, y)) \subset \mathcal{G}$.
2. For any grouptype $(\pi', \gamma, y) \in \mathcal{G}(\tilde{\Omega}; (\pi, \gamma, y))$, the two grouptypes (π', γ, y) and (π, γ, y) are substitutes in consumption.

Proposition 4.5. *(Memberships with substitute characteristics in a given grouptype have the same price.) Assume that $(p, q), (x, \mu)$ are a profit-share quasi-equilibrium for an economy $\mathcal{E}$. Let $(\pi, \gamma, y) \in \mathcal{G}$ and assume that the characteristics $\tilde{\Omega} \subset \Omega$ are substitutes in the grouptype (π, γ, y) and that (π, γ, y) is used in equilibrium. Then $q(\omega, (\pi, \gamma, y)) = q(\bar{\omega}, (\pi, \gamma, y))$, for all $\omega, \bar{\omega} \in \tilde{\Omega}$ such that $\pi(\omega)$, $\pi(\bar{\omega}) > 0$.*

Proof: Let $k = \sum_{\omega \in \tilde{\Omega}} \pi(\omega)$ and let

$$\mathcal{C}(\pi, \gamma, y) = \{(\pi^{\tilde{\omega}}, \gamma, y) \,|\, \tilde{\omega} \in \tilde{\Omega}, \pi(\tilde{\omega}) > 0\}$$

where $\pi^{\tilde{\omega}}(\tilde{\omega}) = k$, $\pi^{\tilde{\omega}}(\omega) = 0$ for $\omega \in \tilde{\Omega} \backslash \{\tilde{\omega}\}$, and $\pi^{\tilde{\omega}}(\omega) = \pi(\omega), \omega \in \Omega \backslash \tilde{\Omega}$. By assumption $\mathcal{C}(\pi, \gamma, y) \subset \mathcal{G}$. Each of the grouptypes in $\mathcal{C}(\pi, \gamma, y)$ has k members with one of the characteristics in $\tilde{\Omega}$, and no members with the other characteristics in $\tilde{\Omega}$. The number of members of types $\omega \in \Omega \backslash \tilde{\Omega}$ are the same in a group of type $(\pi^{\tilde{\omega}}, \gamma, y)$ as in (π, γ, y).

By assumption the characteristics $\tilde{\Omega}$ are substitutes in the grouptype (π, γ, y). Consider any $\tilde{\omega} \in \tilde{\Omega}$ with $\pi(\tilde{\omega}) > 0$. Since q are equilibrium prices, and (π, γ, y) is used in equilibrium, it holds that $q(\omega, (\pi^{\tilde{\omega}}, \gamma, y)) \geq q(\omega, (\pi, \gamma, y))$, for all $\omega \in \Omega$ with $\pi(\omega)$, $\pi^{\tilde{\omega}}(\omega) > 0$. Otherwise, each person who consumes a membership of type $(\omega, (\pi, \gamma, y))$ would prefer a membership of type $(\omega, (\pi^{\tilde{\omega}}, \gamma, y)))$. Since

$$p \cdot y = \sum_{\omega \in \Omega} \pi(\omega) q(\omega, (\pi, \gamma, y)) = \sum_{\omega \in \Omega} \pi^{\tilde{\omega}}(\omega) q(\omega, (\pi^{\tilde{\omega}}, \gamma, y))$$

it follows that $\sum_{\omega \in \tilde{\Omega}} \pi(\omega) q(\omega, (\pi, \gamma, y)) \geq k q(\tilde{\omega}, (\pi^{\tilde{\omega}}, \gamma, y))$. Thus $q(\tilde{\omega}, (\pi^{\tilde{\omega}}, \gamma, y)) \leq \frac{1}{k} \sum_{\omega \in \tilde{\Omega}} \pi(\omega) q(\omega, (\pi, \gamma, y))$ and hence $\pi(\tilde{\omega}) q(\tilde{\omega}, (\pi^{\tilde{\omega}}, \gamma, y)) \leq \frac{\pi(\tilde{\omega})}{k} \sum_{\omega \in \tilde{\Omega}} \pi(\omega) q(\omega, (\pi, \gamma, y))$. As $\tilde{\omega} \in \tilde{\Omega}$ was an arbitrary characterisic with $\pi(\tilde{\omega}) > 0$ we obtain by summation that

$$\sum_{\tilde{\omega} \in \tilde{\Omega}} \pi(\tilde{\omega}) q(\tilde{\omega}, (\pi^{\tilde{\omega}}, \gamma, y)) \leq \sum_{\tilde{\omega} \in \tilde{\Omega}} (\frac{\pi(\tilde{\omega})}{k} \sum_{\omega \in \tilde{\Omega}} \pi(\omega) q(\omega, (\pi, \gamma, y))).$$

This inequality yields

$$\sum_{\tilde{\omega} \in \tilde{\Omega}} \pi(\tilde{\omega}) q(\tilde{\omega}, (\pi^{\tilde{\omega}}, \gamma, y)) \leq \sum_{\omega \in \tilde{\Omega}} \pi(\omega) q(\omega, (\pi, \gamma, y)).$$

However, from above we know that $q(\tilde{\omega}, (\pi^{\tilde{\omega}}, \gamma, y)) \geq q(\tilde{\omega}, (\pi, \gamma, y))$ for all $\tilde{\omega} \in \tilde{\Omega}$, $\pi(\tilde{\omega}) > 0$. Hence, we obtain that

$$q(\tilde{\omega}, (\pi, \gamma, y)) = q(\tilde{\omega}, (\pi^{\tilde{\omega}}, \gamma, y)) = \frac{1}{k} \sum_{\omega \in \tilde{\Omega}} \pi(\omega) q(\omega, (\pi, \gamma, y))$$

for all $\tilde{\omega} \in \tilde{\Omega}$, $\pi(\tilde{\omega}) > 0$. Thus, all memberships of the type $(\tilde{\omega}, (\pi, \gamma, y))$ where $\tilde{\omega} \in \tilde{\Omega}$, $\pi(\tilde{\omega}) > 0$ have the same price, which proves the proposition. □

5. Wage Economies

Like many models of labor markets, our model allows that labor compensation depends on the workers' external characteristics $\omega \in \Omega$. In addition, it allows that wages depend on the technology of the employer. Example 5.1 shows why such a rich price system is needed, namely, because the job enters the worker's utility, either through the production plan or through direct externalities from co-workers.

Example 5.1. Consider an economy with a continuum of agents uniformly distributed on $[0, 1]$. There is a single private good. Each consumer has endowment $\frac{1}{3}$. There are two external characteristics, (c)omputer-programmers and (e)conomists, and a profile is $\pi = (\pi(c), \pi(e))$. Technologies do not have activities, so we use γ_o for a null activity. The set of technologies has three elements: $\mathcal{T} = \{((1,1), \gamma_o, 2),\ ((0,2), \gamma_o, 2\frac{1}{2}),\ ((2,0), \gamma_o, 2)\}$. The consumption sets specify that each agent can only work in one firm. For any list ℓ such that $\omega^L(\ell) = e$, $c_a(\ell) = \frac{1}{3}$ for all agents. For lists ℓ such that $\omega^L(\ell) = c$, $c_a(\ell) = \frac{1}{3}$ for $a \in [0, 0.7)$ and $c_a(\ell) = 3$ for $a \in [0.7, 1]$. All agents $a \in [0, 1]$ have the following utility function, and before becoming educated, are exactly alike except for their costs of acquiring skills.

$$u_a(x, m) = \begin{array}{lll} \frac{3}{4}x & \text{if} & m = (e, ((1,1), \gamma_o, 2)) \\ \frac{1}{3}x & \text{if} & m = (c, ((1,1), \gamma_o, 2)) \\ \frac{1}{4}x & \text{if} & m = (e, ((0,2), \gamma_o, 2\frac{1}{2})) \\ \frac{1}{2}x & \text{if} & m = (c, ((2,0), \gamma_o, 2)) \\ x & \text{if} & \text{there are no memberships} \end{array}$$

In this utility function, an agent who becomes an economist likes to work in a firm with a computer scientist, but a computer scientist does not like to work in a firm with an economist.

We assert that the following is an equilibrium. The equilibrium membership prices are $p = 1$, $q(e,\ ((1,1),\gamma_o,2)) = -\frac{1}{2}$, $q(c,((1,1),\gamma_o,2)) = -\frac{3}{2}$, $q(e,((0,2),\gamma_o,2)) = -\frac{5}{4}$, $q(c,((2,0),\gamma_o,2)) = -1$. Agents $a \in [0,0.7)$ become computer scientists. Agents $a \in [0.7,1]$ become economists. Agents $a \in [0,0.3]$ consume jobs $(c,((1,1),\gamma_o,2))$ and private goods $x = \frac{3}{2}$. Agents $[0.7,1]$ consume jobs $(e,\ ((1,1),\gamma_o,2))$ and private goods $x = \frac{1}{2}$. Agents $a \in (0.3,0.7)$ consume jobs $(c,((2,0),\gamma_o,2))$ and private goods $x = 1$. ♣

We stress the following features of the example: The wages paid to computer scientists are different in two different technologies, both of which are used in equilibrium. This is because a computer scientist must be compensated extra for working with an economist. The most productive technology per capita is comprised entirely of economists, but the technology is not used in equilibrium. Although it would provide a high wage to the economists, each economist prefers to work with a computer scientist rather than with another economist.

We now show that in a more standard labor model, where a worker does not care about the technology that employs him (for example, he cares about about his wages), the equilibrium allocations can be supported with a smaller set of prices. In fact Proposition 4.3 already reduced the price space in a way that can be interpreted for wages. If agents do not care about the production plans of their employers, but care about the characteristics of their co-workers and the working conditions in their firms, then two technologies that employ workers with the same characteristics and offer the same working conditions, but have different production plans, will be substitutes in consumption. By Proposition 4.3, the wages in such technologies will be the same, which implies that wages depend on the characteristics of the co-workers and the working conditions, but not on the production plan.

The theorem below goes further: If all jobs are substitutes in the much stronger sense that no agent cares about the characteristics of his co-workers, the working conditions of the firm, or the production plan, then wages in every technology will be the same for workers with the same skills. Theorem 5.3 is closely related to Proposition 4.3. We state it separately in order to emphasize profit-maximization rather than budget balance for technologies. The profit available in technologies that are not used in equilibrium might be negative, whereas in a profit-share equilibrium the net profit is zero for all technologies. The wages in wage equilibrium can be higher than the corresponding salaries in the profit-share equilibrium, but only in technologies that are not used in equilibrium.

We now conceive of a list ℓ as comprised of three parts $\ell = (\ell^{\mathcal{T}}, \ell^{\mathcal{S}}, \ell^{\mathcal{C}})$, where $\ell^{\mathcal{T}}$ represents the memberships in technologies (firms), $\ell^{\mathcal{S}}$ represents the memberships in schools, and $\ell^{\mathcal{C}}$ represents the memberships in other groups such as a social clubs. Correspondingly, we partition the space of membership prices into $\mathbf{R}^{\mathcal{M}} = \mathbf{R}^{\mathcal{M}^{\mathcal{T}}} \times \mathbf{R}^{\mathcal{M}^{\mathcal{S}}} \times \mathbf{R}^{\mathcal{M}^{\mathcal{C}}}$. Moreover we let $\overline{\Omega} = \{\omega \in \Omega | \text{there exists } (\pi, \gamma, y) \in \mathcal{T} \text{ with } \pi(\omega) \geq 1\}$

We define a *wage quasi-equilibrium* as a feasible state (x, μ) and prices $(p, w, (q^{\mathcal{S}}, q^{\mathcal{C}})) \in \mathbf{R}^N_+ \times \mathbf{R}^{\overline{\Omega}} \times \mathbf{R}^{\mathcal{M}^{\mathcal{S}}} \times \mathbf{R}^{\mathcal{M}^{\mathcal{C}}}$, $p \neq 0$, and such that

(W1) **Budget Feasibility for Agents** For almost all $a \in A$,

$$(p, q^{\mathcal{S}}, q^{\mathcal{C}}) \cdot ((x_a + c_a(\mu_a)), \mu_a^{\mathcal{S}}, \mu_a^{\mathcal{C}}) - \sum_{(\omega,(\pi,\gamma,y)) \in \mathcal{M}^{\mathcal{T}}} w(\omega) \mu_a^{\mathcal{T}}(\omega, (\pi, \gamma, y)) \leq p \cdot e_a$$

(W2) **Optimization of Agents** For almost all $a \in A$:

$$(x'_a, \mu'_a) \in X_a \text{ and } u_a(x'_a, \mu'_a) > u_a(x_a, \mu_a) \Rightarrow$$

$$(p, q^{\mathcal{S}}, q^{\mathcal{C}}) \cdot ((x_a + c_a(\mu_a)), \mu_a^{\mathcal{S}}, \mu_a^{\mathcal{C}}) - \sum_{(\omega,(\pi,\gamma,y)) \in \mathcal{M}^{\mathcal{T}}} w(\omega) \mu_a^{\mathcal{T}}(\omega, (\pi, \gamma, y)) > p \cdot e_a$$

(W3) **Budget Balance for Schools and Clubs**

$$\sum_{\omega\in\Omega}\pi(\omega)\ q^{\mathcal{S}}(\omega,(\pi,\gamma,y))+p\cdot y=0 \qquad \text{for } (\pi,\gamma,y)\in\mathcal{S}$$

$$\sum_{\omega\in\Omega}\pi(\omega)\ q^{\mathcal{C}}(\omega,(\pi,\gamma,y))+p\cdot y=0 \qquad \text{for } (\pi,\gamma,y)\in\mathcal{C}$$

(W4) **Profit Maximization for Technologies** For $(\pi,\gamma,y)\in\mathcal{T}$

$$-\sum_{\omega\in\overline{\Omega}}w(\omega)\pi(\omega)+p\cdot y\leq 0$$

with equality if $\int_A \mu_a(\cdot,(\pi,\gamma,y))d\lambda(a)\neq 0$.

Definition 5.2. We say that **utility and personal costs are independent of technologies** if any two grouptypes (π,γ,y), $(\pi',\gamma',y')\in\mathcal{T}$ are substitutes in consumption.

Theorem 5.3. *Suppose that the prices* $(p,(q^{\mathcal{T}},q^{\mathcal{S}},q^{\mathcal{C}}))$ *and an allocation* (x,μ) *are a profit-share quasi-equilibrium for an economy* $\mathcal{E}$. *Suppose that utility and personal costs are independent of technologies. Then there exist wages* $w\in\mathbf{R}^{\overline{\Omega}}$ *such that* (x,μ) *and* $(p,w,(q^{\mathcal{S}},q^{\mathcal{C}}))$ *are a wage quasi-equilibrium.*

Proof: By Proposition 4.3, there exist $\{w(\omega):\omega\in\overline{\Omega}\}$ such that $-q^{\mathcal{T}}(\omega,(\pi,\gamma,y)\leq w(\omega)$ for all $(\omega,(\pi,\gamma,y))\in\mathcal{M}^{\mathcal{T}}$, with equality for jobs in technologies that are used in equilibrium, where $\int_A \mu_a(\omega,(\pi,\gamma,y))d\lambda(a)>0$.

Condition (W1) (budget feasibility) follows from condition (1) of profit-share quasi-equilibrium because the prices of private goods and traded memberships are the same in both price systems.

We now show that condition (W2) (optimization) follows from (2). Suppose that (W2) is not satisfied. The bundle (x'_a, μ'_a) that improves utility must have a lower value under the price system $(p, w, (q^{\mathcal{S}}, q^{\mathcal{C}}))$ than under the price system (p, q). Otherwise (2) would not be satisfied, a contradiction. But all prices are the same in the two price systems except for wages in jobs. Thus the bundle (x'_a, μ'_a) includes a job with wage $w(\omega)$ such that $-q(\omega, (\pi, \gamma, y)) < w(\omega)$. But under the price system (p, q), there is *some* job with wage $w(\omega)$, and all jobs are substitutes. Hence, if agent a can improve utility by choosing (x'_a, μ'_a) under the price system $(p, w, (q^{\mathcal{S}}, q^{\mathcal{C}}))$, then there is a consumption bundle (x'_a, μ''_a) that will improve utility under the price system (p, q), which contradicts (2). Thus (W2) holds.

(W3) (budget balance for schools and clubs) follows immediately from (3).

We now show (W4) (profit maximization). It follows from (3) and the definition of w that

$$-w \cdot \pi + p \cdot y \leq \sum_{\omega \in \Omega} \pi(\omega) q^{\mathcal{T}}(\omega, (\pi, \gamma, y)) + p \cdot y = 0$$

for all $(\pi, \gamma, y) \in \mathcal{T}$. To show equality for a technology (π, γ, y) that is used in equilibrium, observe that $\int_A \mu_a(\omega, (\pi, \gamma, y)) d\lambda(a) > 0$ for all ω such that $\pi(\omega) > 0$. Hence $w(\omega) = -q^{\mathcal{T}}(\omega, (\pi, \gamma, y))$ for all all ω such that $\pi(\omega) > 0$. Thus (W4) follows. □

If we assumed that school memberships were substitutes, and that personal costs did not depend on the schools attended, then we could also construct a price system such that the tuition fees paid to schools depend only on the acquired skills, and not on the types of schools. However it is unreasonable to think that personal costs would not depend on the schools attended, so we have not stated such a theorem. An agent would typically be willing to attend a higher-priced school if the school reduces personal costs.

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