Debt-limits and Endogenous Growth

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Abstract

This paper studies the consequences in two different endogenous growth models with labor-leisure choice of imposing debt-limits on the government budget constraint. The analysis is motivated by the constraints the Stability Pact imposed on EMU countries, emphasizing the state of financial needs. We show that relaxing debt and time schedule criteria reduces the economy's growth rate in both models, but with a stronger impact on the government in the production function model, even though consumption-to-output ratio and leisure increase. We conclude that both capital tax increases and rising debt-limits reduce financial needs, but have different effects on growth.

Keywords: balanced growth path, debt-limits, endogenous growth, government spending.

JEL Classification: E62, H3, H5, H63, 041.

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1 Introduction

Sustainability of fiscal policies is one of the major issues in Macroeconomics. Recently, in both the United States and the European Monetary Union (EMU), restrictive budgets have been approved in order to meet certain fiscal constraints in the next 5 to 10 years. In the case of the EMU countries, these fiscal constraints refer to the deficit and debt ceilings imposed by the Maastricht Treaty (the Treaty, from now on) in 1991 and lately reinforced by the Stability Pact in 1998. If we look at the figures, we can see that in average terms, the growth rate of government consumption in the EMU is higher after Maastricht (1991-1999) than before the Treaty. This result seems a little bit striking given that the expected trend for government consumption should be decreasing in order to fulfil both the deficit and debt criteria. This suggests a key question: will European countries be able to commit to the debt Maastricht criterion?

Feasibility of a policy of perpetual debt financing has been questioned over years. The importance of debt appears in the literature ever since it is realized that Ricardian equivalence hardly occurs in reality. Nowadays, countries have realized the benefits of reducing debt and deficit levels that models start looking for the optimal policy to cut down debt without entering the economy into a recession or lowering individuals' welfare.

This work introduces a new approach to the role of imposing debt-limits on growth. In this context, Uctum and Wickens [14] examine, from the econometric viewpoint, the effects of debt-ceilings on a generalization of the government intertemporal budget constraint. They find that fiscal policy is not sustainable for most industrialized countries over an infinite horizon, but is sustainable in the medium term in the absence of ceilings. That is, *imposing debt-limits generates unsustainability.* More recent papers, like Chari and Kehoe [4] and Canzoneri, Cumby and Diba [3], analyze the need of fiscal constraints in the implementation of monetary unions, specially in the case of the European monetary union. However, none of theses papers use an endogenous growth framework.

The present paper takes Barro's [1] and Romer's [12] models to be extended to debt-ceilings. The main reason for comparing these two models relies on the fact of how government spending affects the economy. In the first case, productive government spending is introduced in the production function so as to enhance both capital and labor productivity, and we analyze an externality coming from the public capital side, while in the second case, public spending enters the household's utility function and the externality arises from the private capital side.[1] As usual when working with externalities, the competitive equilibrium is inefficient. Moreover, we imposed from the beginning the existence of distortionary taxes.

We show that relaxing debt and time schedule criteria reduces the economy's growth rate in both models, but with a stronger impact when government spending enters the production function. Thus, a strict and rigid interpretation of debt-limit criteria is recommended.

Interesting results arise when comparing the effects of capital tax increases and rises in the debt-limit. Given the fact that both instruments appear to be useful to reduce financial needs, the implementation of any of them will rely on the final government's objectives either on enhancing growth or welfare.

The rest of the paper is organized as follows: section 2 describes the models and agents of the economy. In section 3, the competitive equilibrium is characterized. Sections 4 and 5 concentrate on a balanced growth path analysis and the results after the simulations. Conclusions and further extensions close the paper.

2 The Models

In this section, we present two different endogenous growth models in a general equilibrium framework. There are three types of agents in our economy: households, firms and Government. In both setups, labor is elastically supplied by agents who are endowed with one unit of time to be devoted either to leisure or to labor. The models display different externalities: the first model considered is a case where externalities arise because of public productive capital in the production function *à-la-Barro* [1]; in the second model, externalities appear due to the existence of learning-by-doing and knowledge spillovers in the productive process, *à-la-Romer* [12], and where government spending only supplies public services and enters additively into the households' utility function.

[1] We will refer to the case where government purchases a proportion of the private production in order to provide free public services to private agents (either through the production function or the utility function). In this case, we are dealing with public goods (nonrival and nonexcludable in Samuelson's terminology).

2.1 The government spending in the production function model

First, we will develop the model where government expenditure enters the production function (GPF model) enhancing both capital and labor productivity. The remainder of this section is as follows: first, households are introduced and then firms and technology follow.

2.1.1 Households

This economy consists of a large number of identical individuals. It is assumed that the representative infinite-lived household decides how much time to allocate between consumption and leisure. Agents are endowed with one unit of time to be divided between leisure, $l(t)$, and labor, $1 - l(t)$. There is an homogeneous good whose price is taken as numeraire and normalized to one. The population size is also normalized to 1, so that variables are either in gross or per capita terms. Then, the household's problem reduces to maximize his utility subject to the following budget constraint

$$\dot{a}(t) = (1-\tau)r(t)a(t) + (1-\tau_w)\omega(t)[1-l(t)] - c(t), \tag{1}$$

where $a(t)$ denotes the household's wealth, composed by the stock of capital and government bonds; τ and τ_w are flat-tax rates on wealth and labor income, respectively; and finally, $r(t)$ and $\omega(t)$ refer to the rental prices of capital and labor in terms of time t consumption. Thus, the agent's budget constraint consists of after-tax labor income, returns of capital and bonds to be spent on consumption and accumulation of more capital and government bonds.

The representative agent's decision is given by the following individual's optimization problem (P1.)

$$\underset{\{c(t),l(t),a(t)\}}{Max} \int_0^{+\infty} U[c(t), l(t)]e^{-\rho t}dt$$

$$subject \ \ to \ \ \dot{a}(t) = (1-\tau)r(t)a(t) + (1-\tau_w)\omega(t)[1-l(t)] - c(t)$$

$$c(t) \geq 0 \ \ for \ \ all \ \ t$$

$$a(0) = a_0 \ \ taken \ \ as \ \ given$$

$$l(t) \in [0, 1],$$

and the no-Ponzi game condition on assets

$$\lim_{t \to \infty} a(t) e^{-\int_0^t (1-\tau) r(v) dv} \geq 0, \tag{2}$$

where, for this model, $U[c(t), l(t)]$ takes the appropriate functional form according to the following CES utility function

$$U[c(t), l(t)] = \begin{cases} \int_0^{+\infty} \frac{(c(t)^{\theta} l(t)^{1-\theta})^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt & if \quad \sigma \neq 1 \\ \theta \ln c(t) + (1-\theta) \ln l(t) & if \quad \sigma = 1, \end{cases} \tag{3}$$

for $\theta \in [0, 1]$, $\rho > 0$ and $\sigma > 0$, where $c(t)$ is consumption per capita at time t; $l(t)$ is the proportion of time devoted to leisure at time t; ρ is the discount parameter, σ refers to the elasticity of substitution, which is constant, and θ reflects the household's preference between consumption and leisure.[2,3]

The Hamiltonian for problem (P.1.) is the following:

$$\begin{aligned} H[c(t), l(t), a(t), \lambda(t)] = \; & e^{-\rho t} \{ U[c(t), l(t)] + \lambda(t)[(1-\tau) r(t) a(t) + \\ & + (1-\tau_w) \omega(t)[1 - l(t)] - c(t)] \}, \end{aligned} \tag{4}$$

where $\lambda(t) = \mu(t) e^{\rho t}$ is the shadow price associated to the household's budget constraint.

The first order conditions (FOC) for an interior solution to this problem are given by

$$\theta c(t)^{\theta(1-\sigma)-1} l(t)^{(1-\theta)(1-\sigma)} = \lambda(t), \tag{5}$$

$$(1-\theta) c(t)^{\theta(1-\sigma)} l(t)^{(1-\theta)(1-\sigma)-1} = \lambda(t)(1-\tau_w)\omega(t), \tag{6}$$

$$\dot{\lambda}(t) = \lambda(t)[\rho - (1-\tau) r(t)], \tag{7}$$

[2] In our case $1 - \theta(1-\sigma) = -\frac{U''[c]c}{U'[c]}$, that is the elasticity of marginal private consumption, which is assumed to be constant. Remark that setting $\theta = 1$ (no leisure in the utility function) leads us to the general case analyzed where σ is the intertemporal elasticity of substitution.

[3] This type of preferences allows us to capture the endogenous character of leisure as pointed out in Rebelo [11], since this specification is consistent with the existence of a balanced growth path. As we will see later, this Cobb-Douglas specification of the utility function together with the constant returns to scale of the production function will allow for the existence of endogenous growth. For a more detailed discussion, see King, Plosser and Rebelo[7].

together with the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)a(t) = 0. \tag{8}$$

Equations (5) and (6) refer to the marginal utility of consumption and leisure, involving in our case, two basic margins: first, the marginal rate of substitution between $c(0)$ and $c(t)$, given by equation (5), evaluated at times 0 and t; and second, the marginal rate of substitution between $c(t)$ and $l(t)$ that equals the real wage, that is, the ratio equation (5) over equation (6).

From here, we can obtain the growth rate of consumption, $\gamma_c(t)$ as follows

$$\gamma_c(t) = \frac{1}{\sigma}[(1-\tau)r(t) - \rho]. \tag{9}$$

2.1.2 Firms and Technology

There is a large number of identical firms, but the analysis will be focused on a representative one. There is only a final good produced through a Cobb-Douglas constant returns to scale production function.[4] Markets are competitive and there is no depreciation. The inputs are capital stock, labor and labor-augmenting productivity government expenditure. The production function is given by

$$y(t) = Ak(t)^{\alpha}[(1-l(t))g(t)]^{1-\alpha}, \tag{10}$$

where low letters denote variables in per capita terms, $\alpha \in [0,1]$, $y(t)$ is output, $A > 0$ is the scale parameter, $k(t)$ is private capital, $[1-l(t)]$ is labor, and $g(t)$ is government expenditure.

We concentrate on the government expenditure to output ratio, which we denote by $\tilde{g}$, that is

$$\frac{g(t)}{y(t)} = \tilde{g}(t) \quad \text{and} \quad \tilde{g}(t) \in [0,1]. \tag{11}$$

Substituting (11) into (10) we obtain a new expression for the production function

$$y(t) = A^{\frac{1}{\alpha}}k(t)[(1-l(t))\tilde{g}(t)]^{\frac{1-\alpha}{\alpha}}. \tag{12}$$

Under the assumptions of competitive input markets and constant returns to scale in production technology, factors are paid their marginal products

$$r(t) = \alpha A^{\frac{1}{\alpha}}[(1-l(t))\tilde{g}(t)]^{\frac{1-\alpha}{\alpha}}, \tag{13}$$

[4] This type of production function displays a constant elasticity of productive factors to output.

for capital[5] and for labor

$$w(t) = (1-\alpha)A^{\frac{1}{\alpha}}k(t)\widetilde{g}(t)^{\frac{1-\alpha}{\alpha}}[1-l(t)]^{\frac{1-2\alpha}{\alpha}}. \tag{14}$$

2.2 The government spending in the utility function model

Next, we present the model where a learning-by-doing technology is used and government spending enters the utility function (GUF model).

2.2.1 Households

This new economy has the same characteristics as the one before. However, preferences change: now, government expenditure enters the utility function additively. This will affect growth, since this expenditure is financed through distorting taxes. Again, there is a single household, representing many, who maximizes (P1.), but now, the objective function changes to the following expression

$$\int_0^{+\infty} U[c(t), l(t), g(t)]e^{-\rho t}dt,$$

where preferences are described by

$$U[c(t), l(t), g(t)] = \begin{cases} \frac{[(c(t)^{\theta}l(t)^{1-\theta})^{1-\sigma}+(g(t)^{\psi})^{1-\sigma}-1]}{1-\sigma}e^{-\rho t}dt & if \quad \sigma \neq 1 \\ \theta \ln c(t) + (1-\theta)\ln l(t) + \psi \ln g(t) & if \quad \sigma = 1, \end{cases} \tag{15}$$

for $\theta \in [0, 1]$, ρ and $\sigma > 0$. In the expression above, $\psi > 0$ is a parameter measuring the impact of $g(t)$ on the welfare of the household.[6,7,8]

The rest of the household's problem remains the same, including the FOC since $g(t)$ is taken as given and is not a choice variable.

[5] Note that $r(t)$ is a function of $\widetilde{g}(t)$, the model brings in a dependence of the growth rate on the quantity of government productive services.

[6] For the isoelastic utility function, ψ can also be interpreted as the marginal rate of substitution between public and private goods and leisure.

[7] For the learning-by-doing model if preferences for government spending are separable (or if agent obtains no utility from government spending) then the wealth and substitution effects cancel and leisure remains unchanged, a condition we need for the balanced growth in this model.

[8] The parameter ψ is assumed to be positive (so that public consumption yields a positive marginal utility) and the following expressions must hold $-\infty < 1-\sigma < \frac{1}{1+\psi}$ and $\psi(1-\sigma) < 1$, to have a bounded utility.

2.2.2 Firms and technology

As in the model developed above, there is a large number of firms. We take again the representative firm, indexed by i. Markets are competitive and there is no depreciation. Firms face a profit maximization problem, but now the technology they use is different. There is a Cobb-Douglas production function, exhibiting learning-by-doing and knowledge spillovers externalities, that is

$$y_i(t) = Ak_i(t)^{\alpha}[1 - l(t)]^{1-\alpha}\bar{k}(t)^{1-\alpha}, \tag{16}$$

where $\bar{k}(t)$ denotes the aggregate level of capital. Notice that we already imposed the fact that aggregate and per-capita quantities coincide. We still keep the assumptions of competitive input markets so that factors are paid their marginal products that now are given by

$$r(t) = \alpha Ak_i(t)^{\alpha-1}[1 - l(t)]^{1-\alpha}k(t)^{1-\alpha}, \tag{17}$$

$$\omega(t) = (1 - \alpha)Ak_i(t)^{\alpha}[1 - l(t)]^{-\alpha}\bar{k}(t)^{1-\alpha}, \tag{18}$$

for the stock of capital and labor respectively.

Up to now, we have dealt with two different models, next we present the common factor they have, the government behavior.

2.3 Government

In both models, government chooses a path for consumption expenditure, $g(t)$, that is financed through taxes and debt, equivalently, the government needs not run a balanced budget at every moment of time.[9] Tax revenues come from flat-tax rates on capital and labor income, and debt is issued as public bonds held by the households, also taxed at a flat-tax rate τ.[10] As we assumed before, the flow of government consumption is a function of total production as denoted by equation (11).

With these assumptions the government budget constraint is the following:

$$\dot{b}(t) = r(t)b(t) + g(t) - \tau r(t)d(t) - \tau_w\omega(t)[1 - l(t)], \tag{19}$$

[9] There are other forms of government financing like inflation taxes and money. These extensions are left for future work.

[10] Notice that we use the same flat-rate tax for the whole amount of investment: stock of capital and government bonds. This could be generalized but it would imply no qualitative differences due to arbitrage equilibrium conditions.

where $r(t)b(t)$ denotes public debt expenses, $g(t)$ is the flow of public services and the rest of the terms in the equation refer to the revenues from flat-tax rates on assets and labor income. However, we will focus on the debt-to-output ratio, denoted by $\beta(t) = \frac{b(t)}{y(t)}$, in order to obtain a variable we can deal with in the balanced growth path. Then, the government budget constraint can be expressed as follows

$$\dot{\beta}(t) = [r(t) - \gamma(t)]\beta(t) + \tilde{g} - \tau r(t)\frac{d(t)}{y(t)} - \frac{\tau_w \omega(t)[1 - l(t)]}{y(t)}. \tag{20}$$

3 Competitive Equilibrium

As usual, we combine now the conditions from utility maximization with those of profit maximization, together with the budget balance for the government and market clearing conditions to characterize the competitive equilibrium of this economy.

First of all, regarding firms technology, in the CUF model we have to take into account that in equilibrium, assuming symmetry among firms, aggregate and individual stocks of capital are the same, $y_i(t) = y(t)$ and $k_i(t) = k(t) = \bar{k}(t)$, then output becomes

$$y(t) = Ak(t)[1 - l(t)]^{1-\alpha},$$

and the marginal products for capital and labor are, respectively,

$$r(t) = \alpha A[1 - l(t)]^{1-\alpha}, \tag{21}$$

and

$$\omega(t) = (1 - \alpha)Ak(t)[1 - l(t)]^{1-\alpha}. \tag{22}$$

Secondly, in a competitive equilibrium markets clear, in particular financial markets. Then, the following holds

$$d(t) = k(t) + b(t), \tag{23}$$

that is, assets demanded by the household, $d(t)$, must equal the supply of the economy: private, $k(t)$, and public assets, $b(t)$.

It remains to state the market clearing condition

$$\dot{k}(t) = y(t) - c(t) - g(t). \tag{24}$$

Definition 1 *A competitive equilibrium path for the economy described above consists of sequences for quantities* $\{c(t), l(t), k(t), b(t)\}_{t=0}^{\infty}$*, and prices* $\{r(t), w(t)\}_{t=0}^{\infty}$ *such that: (i)* $c(t), l(t), k(t)$ *and* $b(t)$ *maximize their respective individual problem; (ii) factors are paid their marginal product according to the expressions given for each model; and (iii) that satisfy both (20) and (24).*

The first order conditions characterizing the competitive equilibrium are in Appendix A.

4 A Balanced Growth Analysis

Although the original versions of both models assert the nonexistence of transitional dynamics, the models presented above do display them. However, for the sake of concreteness, the analysis concentrates on the balanced growth path.

Definition 2 *A BGP*[11] *can be defined in our case as a competitive equilibrium path in which consumption, government spending, debt and capital grow at the same rate,* γ*, where the time allocation (leisure, labor),*[12] *interest and wage rates and the fiscal variables* τ, τ_w, $\widetilde{g}$*, and* $\widetilde{\beta}$ *are constant over time.*

Time between parenthesis is removed to denote steady-state variables. Stationarity is required to study the BGP, hence for flow variables, we will study them in ratios to $k(t)$ and $y(t)$.

The rate of growth of the whole economy, γ, is

$$\gamma = \frac{1}{\sigma}[(1-\tau)r - \rho], \tag{25}$$

where the following needs to hold

$$(1-\tau)r > \rho > (1-\tau)r - \sigma(1-\widetilde{g})\frac{y}{k} \tag{26}$$

and

$$\rho > \theta(1-\sigma)\gamma. \tag{27}$$

[11] To ensure that the BGP exists for this model, it is necessary to assume that the utility function has the CES form, as it is the case here, where $\sigma > 0$. See Lucas [9] and Rebelo [11].

[12] The preferences specifications made assure that leisure is constant on the BGP. See Rebelo [11] for a detailed explanation of this.

to ensure endogenous growth, as well as the particular conditions regarding each model.

The government budget constraint also simplifies considerably. The BCP equation (20) reduces to

$$\dot{\beta} = [(1-\tau)r - \gamma]\beta + (1-\tau)r\frac{k}{y} + (1-\tau_w)\omega\frac{(1-l)}{y} + \tilde{g} - 1. \tag{28}$$

Integrating equation (28) backwards and using the following short-run transversality condition

$$\lim_{t \to T} e^{-\rho t}\lambda(t)\beta(t) \leq \bar{\beta}, \qquad \text{for} \quad \bar{\beta} \geq 0, \tag{29}$$

we obtain the following budget constraint

$$\bar{\beta}e^{\gamma T} \geq \beta(t)e^{\gamma T} = \beta(0) + \frac{[1-e^{-[(1-\tau)r-\gamma]T}]}{(1-\tau)r-\gamma}[\tilde{g} - 1 + \frac{(1-\tau)rk + (1-\tau_w)\omega(1-l)}{y}], \tag{30}$$

where equation (29) refers to the debt-ceiling we want to impose. It means that for a given point in time, $T > t$, the debt-to-output ratio does not surpass a given level, in this case $\bar{\beta}$.[13] This equation captures the requirement imposed on debt: the right-hand side of (30) is the maximum level of debt-to-output ratio that the economy can reach at a certain moment in time T. The left-hand side is how this level of debt is composed: on the one hand, we have the initial level of debt-to-output ratio in the economy, $\beta(0)$; on the other hand, the present value of the net public budget (expenses minus revenues).

Definition 3 *Let $\Phi(0)$ denote the government financial needs at time zero, as the difference between the present value of revenues and the present value of spending.*

The total effect on the government budget constraint of changes in fiscal variables and time to accomplish with the debt-ceiling (reflecting the financial needs, Φ) can be measured by writing equation (30) as follows

$$\Phi(0) = \beta(0) + [\frac{1-e^{-((1-\tau)r-\gamma)T}}{(1-\tau)r-\gamma}][\tilde{g} - 1 + \frac{(1-\tau)rk + (1-\tau_w)\omega(1-l)}{y}] - \bar{\beta}e^{\gamma T}. \tag{31}$$

[13] This can be particularized for the European Monetary Union as being of 60%, and for the United States to zero for a larger time horizon.

Values for Φ different from zero indicate the existence of either deficit or superavit. When Φ is around or equal to zero, the government budget constraint is balanced and there are no financial needs.[14]

Therefore, the BGP can be described by the non-negative values of $\{\gamma, l, \frac{c}{k}, \frac{y}{k}, r, \frac{w}{k}, \widetilde{g}, 1-l\}$ that satisfy the following system of equations, for the CPF model:

$$\theta(1-\tau_w)l\frac{w}{k} = (1-\theta)\frac{c}{k}, \tag{32}$$

$$\gamma = \frac{1}{\sigma}[(1-\tau)r-\rho], \tag{33}$$

$$l \in [0,1], \tag{34}$$

$$r = \alpha A^{\frac{1}{\alpha}}[\widetilde{g}(1-l)]^{\frac{1-\alpha}{\alpha}}, \tag{35}$$

$$\frac{w}{k} = (1-\alpha)A^{\frac{1}{\alpha}}(1-l)^{\frac{1-2\alpha}{\alpha}}\widetilde{g}^{\frac{1-\alpha}{\alpha}}, \tag{36}$$

$$\widetilde{g}\frac{y}{k} = 0.07, \tag{37}$$

$$(1-\widetilde{g})\frac{y}{k} = \gamma + \frac{c}{k}, \tag{38}$$

and

$$\beta e^{\gamma T} \geq \beta(0) + \frac{[1-e^{-[(1-\tau)r-\gamma]T}]}{(1-\tau)r-\gamma}[\widetilde{g}-1+\frac{(1-\tau)rk+(1-\tau_w)w(1-l)}{y}]. \tag{39}$$

For the CUF model, the system remains the same but for equations (35) and (36), that are replaced by equations (21) and (22).

In general, it is not possible to solve this model analytically. Actually, a closed form analytic solution can only be obtained for certain types of models. Models where *leisure is exogenously determined* are one of them. In this type of models the system to solve becomes block-recursive (see Stokey and Rebelo [13] for an example).[15]

[14] Another important point here is about Ricardian equivalence. This public expenditure is financed through distortionary taxes, hence Ricardian equivalence does not hold, in this context. This has two important implications: first, having a limit on debt may (and actually, will) affect the consumer's decisions; and second, debt and taxes are no longer indifferent to households. We may find that one fiscal instrument is better than the other.

[15] Moreover, the *dynamic nonsubstitution theorem of Mirrlees* [10] holds: the production possibilities surface is a linear plane and there are no transitional dynamics across steady-states. Some

5 Simulations

According to the analysis of the last section, numerical solutions need to be computed in order to obtain results that can be interpreted referring the steady-state behavior of our two economies.[16]

First, we calibrate both models and solve for a benchmark balanced growth path. As in Stokey and Rebelo [13], we compare economies that are observationally equivalent: they part from the same balanced growth path,[17] but respond differently to any parameter change. Table 1 reflects the initial values and benchmark parameters for both models.

[Insert Table 1 about here]

The benchmark balanced growth path from where both models start is given by Table 2.

[Insert Table 2 about here]

Among all the variable changes, we will focus on the rate of growth, γ, and what we called before the financial needs, Φ. Subscripts will refer to each of the models considered.

5.1 An Increase in capital tax rates

In this section we compare the effects produced on both models when taxes on capital increase from 0.3 up to $\tau = 1$, leaving all the other parameters unchanged.[18]

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examples of this are Barro and Sala-i-Martin [2] and Stokey and Rebelo [13].

However, for models like ours, where *leisure is endogenously determined*, the dynamic nonsubstitution theorem fails to hold: the production possibilities surface is strictly concave and parameter changes can lead to transitional dynamics. Now, the next step will be to solve numerically the two models we presented.

[16] All numerical computations have been carried out with GAUSS-386. Where possible, parameterization of the model follows Lucas [9] and Barro [1]. Additional data for Spain are introduced referring debt-ceilings and initial debt-to-GDP ratios.

[17] Since we wanted both cases to start from the same steady-state, the adjustment is made through the technological parameter, A, whose value differs from one model to another. Actually, A has been chosen to give a maximized value of the growth rate of 2.8%, and the numerical value for α corresponds to a wide concept of $k(t)$, including physical and human capital.

[18] The reason for studying a tax increase, when the usual analysis refers to a tax cut, is because of the need for reducing debt and deficit. Most countries start from debt-to-output ratios above the required debt-ceiling, this could force them to undertake possible restrictive fiscal policies as this one.

Table 3 summarizes the results for an increase in capital tax rates.[19]

[Insert Table 3 about here]

Result 5.1.a: *An increase in the capital tax implies a fall in the growth rate and in the interest rate.*

A rise of the tax on capital income implies a substitution effect between capital and labor holding in both models, since the opportunity cost of investing increases, capital accumulation decreases in benefit of labor. Then the ratios $\tilde{g}$ and consumption-to-output grow in both cases. The reason why the fall in γ_{GPF} equals the fall in γ_{GPF} can be found in the larger adjustment in labor produced in the GPF model. Increasing taxes decreases the after-tax real return on capital inducing the agent to anticipate consumption by reducing the saving rate on capital (the household's intertemporal substitution effect in consumption).

In the GUF setup, investment falls, savings go down and consumption rises, but this rise is not so big to offset the negative effect on the capital stock, such that output falls even more than consumption. This explains the rise in the consumption-to-output ratio, due to the fact that consumption is substituted by public consumption appearing in the utility function.

Result 5.1.b: *An increase in the capital tax reduces financial needs faster in the GUF model than in the GPF one.*

Regarding financial needs, as illustrated by Figures 1 and 2, decrease and converge to zero as τ goes to one. This is important to note because increasing taxes would be a good policy to reach the debt-to-output ceiling in time and with an expansionary fiscal policy ($\tilde{g}$ goes up). However, the GUF model reduces financial

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[19] All the results we present here are percentage changes with respect to the BGP benchmark values.

reads faster than the CPF one.[20]

[Insert Figures 1 and 2 about here]

5.2 An increase in the time to accomplish with the debt ceiling

The objective of this section is to analyze how both models react when the timing to comply with the requirement on debt is "relaxed". This implies that the economies have more time to meet the required debt-to-output ratio. The increase in time horizon goes from T=100 to T=200,[21] keeping all the other variables constant. The transmission mechanism of such a change starts in the government spending-to-output ratio appearing in equation (30). Then this change affects the growth rate and indirectly the rest of variables, leading the economy to a new balanced growth path. Table 4 reports the results.

[Insert Table 4 about here]

Result 5.2.a: *An increase in the time planning reduces the growth rate in both models, but the fall is larger in the CPF model than in the CUF one. Interest rates diminish in both models.*

Taking equation (30) we can see that if T increases, the left-hand side of the equation rises, then, an adjustment is needed to keep the equation holding. In the CPF model, this adjustment comes through two ways: first, through the decrease in the public spending-to-output ratio, and this directly reduces the growth rate of the economy; and, second, in a more indirect way, through the rest of variables. Regarding the CUF model, the adjustment arises again from the fall in the public

[20] Following Ireland [5] this effect can be measured by the following expression denoting the *budget effect*:

$$\tilde{\beta} - e^{-\gamma T}[\tilde{\beta}(0) + \tilde{g} - \frac{[\tau r k + \tau_w \omega (1-l)]}{y}]. \tag{40}$$

Ireland [5] suggests a tax cut and shows that incurring in short-run deficits can be solved by the debt offset effect. We directly reduce these deficits in the short-run, since in our case, this difference tends to zero as τ goes to 1, the conclusion is that an increase in the tax on capital allows the economy to reduce the gap between debt and current deficit, and to balance the government budget constraint.

[21] Time can be seen as to be measured in weekly periods. Then the BGP would consider hardly 2 years to fulfil the requirement, and we amplify it to about 4 years.

spending-to-output ratio in equation (30), which decreases more in order to fulfil that equation. Given that $\tilde{g}$ now does not affect directly the rate of growth, we still need the rest of variables to adjust as a reaction to the increase in T. Note that in the CUF model, the fall in government spending is compensated by a corresponding increase in both leisure and consumption, which translates into lower capital accumulation, and, thus, lower growth rate.

Result 5.2.b: *An increase in the time planning worsens the financial needs more in the CUF model than in the CPF one.*

An interpretation common to both models is that the longer the time horizon to satisfy the debt-requirement, the lower the economy's growth rate. The economic rationale is that keeping taxes constant we need to make an extra effort to reach $\bar{B}$. This additional effort is made by the reduction of $\tilde{g}$. Note that in the CPF case this reduction implies a fall in the labor-productivity, and a substitution effect of labor by capital.

A striking result is that financial needs become worse. This is shown in Figures 3 and 4. Thus, an important conclusion, extensive for both models, is that, relaxing the time-horizon is not such a good policy for governments, even though they will accomplish the debt criterion, their finances will end up in a worse situation.

[Insert Figures 3 and 4 about here]

5.3 An increase in the government spending to capital ratio.

The aim here is to analyze the effects on the BGP when government spending as a ratio to capital increases a 100%, keeping the other variables unchanged.[22] Table 5 summarizes the results coming out of the numerical simulations.

[Insert Table 5 about here]

Result 5.3.a: *A rise in the government spending-to-capital ratio enhances growth in the CPF model while this variable is reduced in the CUF one.*

[22] The reader may wonder why do we change $\frac{g}{k}$ instead of $\frac{g}{y}$. There are two main reasons for this: firstly, $\frac{g}{y}$ already changes since it is considered as an endogenous variable, $\tilde{g}$; and secondly, we part from a given benchmark where $\frac{g}{k} = 0.07$, meaning that the government size in the economy is between the usual values (25-40% of output). Later on, when solving for the BGP, variables take equilibrium values, depending on the parameters and equations.

Regarding the CPF model, the rise in the interest rate is due to the positive effect that this increasing spending has on it, reflecting again the positive correlation between this variable and the growth rate.

With respect to the CUF model, the rise in the government spending implies a reduction in the interest rate, but hardly in a percentage point. Here we have a clear crowding out effect where private investment is replaced by public one. The reason is that, keeping taxes constant, a rising $\frac{g}{k}$ affects negatively growth since it only reduces capital accumulation. This happens, basically, in the CUF model, where $\tilde{g}$ only enters additively the utility function, with no other effect on the productive process.

Result 5.3.b: *A rise in the government spending-to-capital ratio reduces financial needs faster in the CUF model than in the CPF one.*

The general result here is the fact that an increase in the $\frac{g}{y}$ ratio, as finally we obtain, is usually accompanied by a rise on taxes, reducing the net interest rate and hence cutting down growth. However, in our case, the rise in $\frac{g}{y}$ is done *without changing taxes*, then financial needs worsen as appears to be evident from Figures 5 and 6. Moreover, we have a positive effect since we end up with a higher rate of growth. It is worth noticing that this does not occur in the CUF model, where the usual results hold.

[Insert Figures 5 and 6 about here]

Summing up, the key point of this analysis is the fact that in an economy with CPF an increase in the government spending to capital ratio (an expansive fiscal policy) can be compatible with the need of holding a debt-ceiling since it increases the growth rate. However, this is not true for an economy with CUF.

5.4 An increase in the debt-to-GDP ceiling

The last experiment we are going to perform refers to the effect on the BCP of an increase in the debt-to-output ceiling, from 60% to 100% of GDP, keeping all the variables constant. This case has been discussed by administrations as "relaxing" the Maastricht criteria.[23] Table 6 reports the percentage changes on the BCP of an

[23] Actually, the European Monetary Union does not require the strictly fulfillment of the debt ceiling, but having a decreasing trend in both debt-to-output and deficit amounts. However, we did all the analysis under a strict interpretation of the Maastricht criterion: we imposed our economies to reach at least $\bar{\beta}$.

increase in the debt-to-output ratio.

[Insert Table 6 about here]

Result 5.4.a: *Increasing the debt-ceiling reduces growth more in the CPF model than in the CUF one. The same is true for interest rates.*

The rise in the debt-ceiling implies a reduction in government-to-output ratio, so that equation (30) holds. This fall in $\tilde{g}$ induces the fall in γ, as usual higher in the CPF model than in the CUF.

Result 5.4.b: *Increasing the debt-ceiling implies less financial needs in the CUF model than in the CPF one, even though they are very close to zero (no financial needs).*

An increase in β smooths the government budget constraint. The ability to issue more debt affects positively public finance, since $\tilde{g}$ hardly changes. The decrease in labor and in the interest rate could reduce the tax-base, so that a cut in government spending needs to be made to finally meet β. But the striking result here is that the final changes for CPF are very insignificant. Then, as Figure 7 shows, financial needs improve.

For the CUF case, the effects on the rate of growth, leisure and $\frac{c}{y}$ are smaller than in the CPF model. The after-tax interest rate remains almost at the same level and labor falls, $\tilde{g}$ falls too, more than in the CPF case. Again, the difference here stems from where $\tilde{g}$ appears. For the CUF model, the implications of increasing debt-ceilings only affect the government budget constraint. Looking at the state of finances, we find again good effects of rising β, as can be seen from Figure 8.

[Insert Figures 7 and 8 about here]

6 Conclusions and Extensions

The aim of this paper has been to investigate the consequences of imposing debt-limits on government's decisions. In two different endogenous growth models with labor-leisure choice, we have analyzed increases in taxes, debt-limits, time-planning and government spending-to-capital ratio.

An interesting result arises when we compare capital tax increases and rises in the debt-limit. Both policies seem to be appropriate instruments to reduce financial needs. Although the former affects more negatively growth than the latter, both increase the consumption-to-output ratio. Moreover, the improvement in financial needs is sooner achieved by the government spending in the utility function model than in the other case analyzed. The choice between fiscal instruments will finally depend on government's objectives on either enhancing growth or individuals' welfare. When the aim is to maximize growth, the best policy is to increase the debt-ceiling. On the other hand, if welfare maximization is the target, tax increases is the appropriate one.

Relaxing debt and time schedule criteria is shown to be harmful for growth in both models, but with stronger impact on the enhancing labor productivity model, even though consumption-to-output ratio and leisure increase. Thus, a strict and rigid interpretation of debt-limit criteria is recommended.

The experiment of increasing the government spending-to-capital ratio is based on the upward trend this variable shows in certain countries. In our simulations, we show that this expansionary policy affects positively the economy's growth where this additional expenditure is labor productivity-enhancing; whereas for the case where it only enhances household's utility, it appears to be growth diminishing.

However, there is an important issue not analyzed in this paper: despite of reducing the analysis just to the balanced growth path, both models present transitional dynamics. Since we stay on the BGP, we miss all the intermediate changes from one steady-state to another. This is left for future research.

Although we stated at the beginning of the paper the no consideration of imposing deficit-ceilings, this measure is also regarded in the elaboration of some countries' budgets. Uctum and Wickens [14] do consider this aspect in their analysis, and conclude that imposing deficit-ceilings make the fiscal stance of countries intertemporally inconsistent unless tax rates are raised significantly. However, unlike debt-ceilings, deficit-ceilings create a positive and persistent fiscal pressure on all countries, which lasts beyond the date these limits are imposed. Furthermore, questions regarding time-consistency of government policies provide more issues for further study.

In short, we recognize that there are many other aspects influencing growth and debt-limits. The techniques employed in this paper also allow to study many of them. Changing parameter values also serves to analyze supply-side economies as Ireland [5]. However, it should be stressed that our results will change if we let debt be endogenous and finding whether it reaches or not the required ceiling. In future work, these aspects will be investigated, focusing on fiscal discipline and its feasibility.

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A First Order Conditions for the Competitive Equilibrium

The problem the representative household faces is to maximize his constant elasticity utility function, subject to a wealth accumulation equation. The Hamiltonian for program (P.1) as follows

$$H[c(t), l(t), d(t), \lambda(t)] = e^{-\rho t}\{U[c(t), l(t), g(t)] +$$

$$+ \lambda(t)[(1-\tau)r(t)d(t) + \quad + (1-\tau_w)\omega(t)(1-l(t)) - c(t)]\}, \tag{41}$$

where $U[c(t), l(t), g(t)]$ takes the functional form from equations (3) or (15), depending on the model we are dealing with. Notice that, in the GPF model, $U_g = 0$.

The F.O.C. are given by equations (5), (6) and (7),together with the transversality condition on assets, equation (8).

If we substitute equation (5) into (6), using equation (7), and recovering the government budget constraint and the market clearing conditions, we end up with the following system

$$\frac{\theta(1-\tau_w)\omega(t)}{(1-\theta)} = \frac{c(t)}{l(t)}, \tag{42}$$

$$\gamma(t) = \frac{1}{\sigma}[(1-\tau)r(t) - \rho], \tag{43}$$

$$\dot{k}(t) = y(t) - g(t) - c(t), \tag{44}$$

$$\dot{\beta}(t) = [(1-\tau)r(t) - \gamma(t)]\beta(t) + (1-\tau)r(t)\frac{k(t)}{y(t)} + (1-\tau_w)\omega(t)\frac{(1-l(t))}{y(t)} + \tilde{g} - 1, \tag{45}$$

also we have to take into account the transversality constraint in equation (8).

The system defined above fully describes the competitive equilibrium in our economy together with the constraints on $l(t), \tilde{g} \subset [0, 1]$.

B Tables

Table 1: Initial and benchmark parameter values			
GPF model		GUF model	
$k(0)$	1	$k(0)$	1
θ	0.5	θ	0.5
τ	0.30	τ	0.30
τ_w	0.30	τ_w	0.30
α	0.60	α	0.60
ρ	0.05	ρ	0.05
σ	1	σ	1
$\beta(0)$	69.6%*	$\beta(0)$	69.6%*
$\bar{\beta}$	60%*	$\bar{\beta}$	60%**
T	100	T	100
$\frac{g}{k}$	0.07	$\frac{g}{k}$	0.07
A	0.8615	A	0.2882

*This data is for Spain 1997, *Source: European Commission*.

**This is the debt-limit imposed by Maastricht.

Table 2: Benchmark values for the balanced growth path	
γ	2.82%
l	66.42%
$\frac{c}{y}$	51.22%
r	7.82%
$\frac{w}{k}$	22.19%
$\tilde{g}$	34.75%
$1-l$	33.58%

Table 3: Changes in endogenous variables when τ increase to one.			
GPF model		GUF model	
γ	-277.06	γ	-277.06
l	-5.06	l	-0.4
$\frac{c}{y}$	67.45	$\frac{c}{y}$	63.29
r	-259.15	r	-100
$\frac{w}{k}$	124.4	$\frac{w}{k}$	-0.47
$\tilde{g}$	43.26	$\tilde{g}$	64.71
$1-l$	15.63	$1-l$	0.78

Table 4: Changes in endogenous variables when time horizon doubles.			
GPF model		GUF model	
γ	-7.99	γ	-4.74
l	1.96	l	2.13
$\frac{c}{y}$	2.59	$\frac{c}{y}$	2.96
r	-2.89	r	-1.71
$\frac{w}{k}$	1.04	$\frac{w}{k}$	2.62
$\tilde{g}$	-0.43	$\tilde{g}$	-1.77
$1-l$	-3.89	$1-l$	-4.22

Table 5: Changes in endogenous variables when $\frac{g}{k}$ increases.			
GPF model		GUF model	
γ	33.81	γ	-0.13
l	-1.13	l	0.06
$\frac{c}{y}$	-13.02	$\frac{c}{y}$	-13.45
r	12.20	r	-0.05
$\frac{w}{k}$	9.74	$\frac{w}{k}$	0.07
$\tilde{g}$	16.25	$\tilde{g}$	25.34
$1-l$	2.24	$1-l$	-0.11

Table 6: Changes in endogenous variables when $\tilde{\beta}$ increases.			
GPF model		GUP model	
γ	-1.04	γ	-0.51
l	0.26	l	0.23
$\frac{c}{y}$	0.34	$\frac{c}{y}$	0.32
r^*	-0.38	r^*	-0.19
$\frac{\omega}{k}$	0.13	$\frac{\omega}{k}$	0.28
$\tilde{g}$	-0.05	$\tilde{g}$	-0.19
$1-l$	-0.51	$1-l$	-0.46

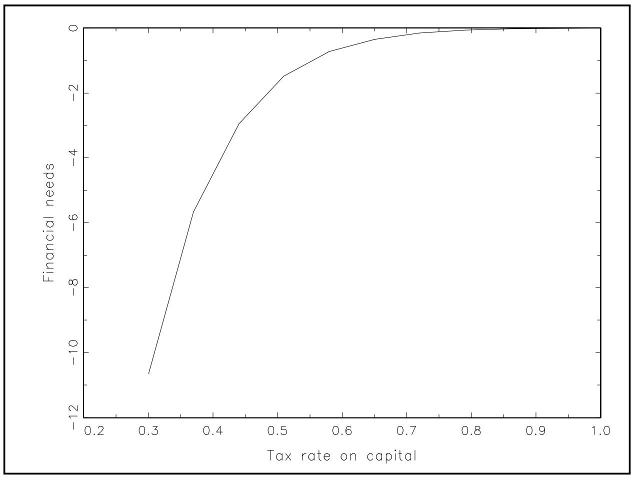

Figure 1: Changes on financial needs in the CPF model when taxes on capital rise.

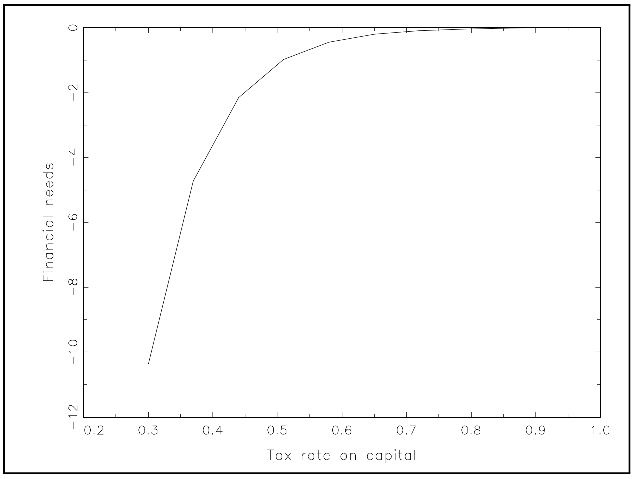

Figure 2: Changes on financial needs in the CUF model when taxes on capital rise.

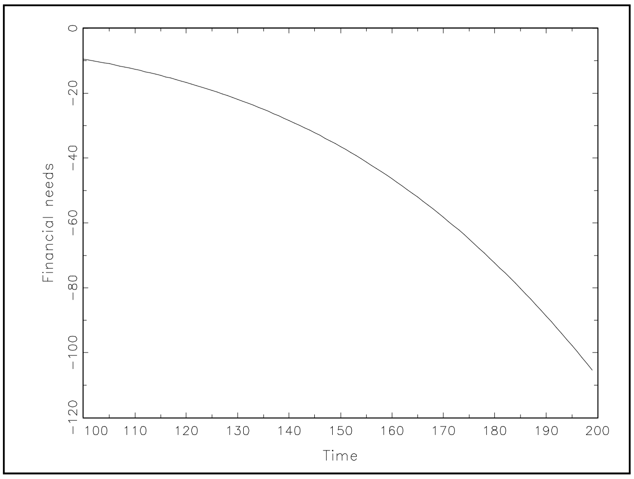

Figure 3: Changes on financial needs in the GPF model when time-schedule increases.

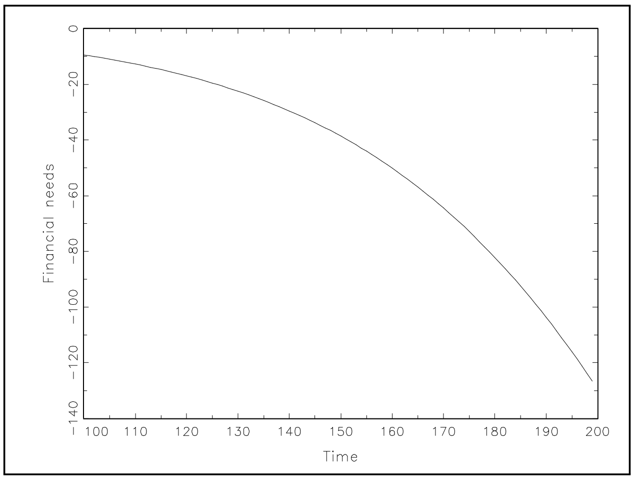

Figure 4: Changes on financial needs in the CUF model when time-schedule increases.

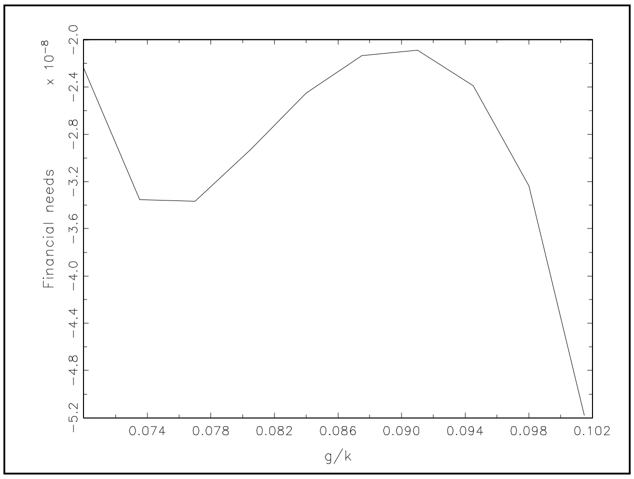

Figure 5: Changes on financial needs in the CPF model when $\frac{g}{k}$ increases.

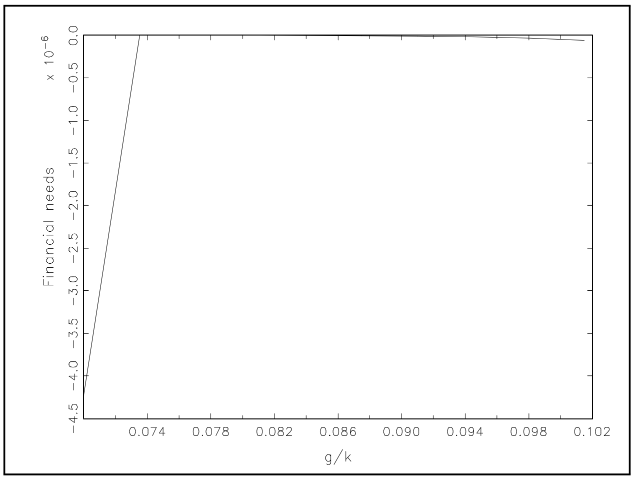

Figure 6: Changes on financial needs in the CUF model when $\frac{g}{k}$ increases.

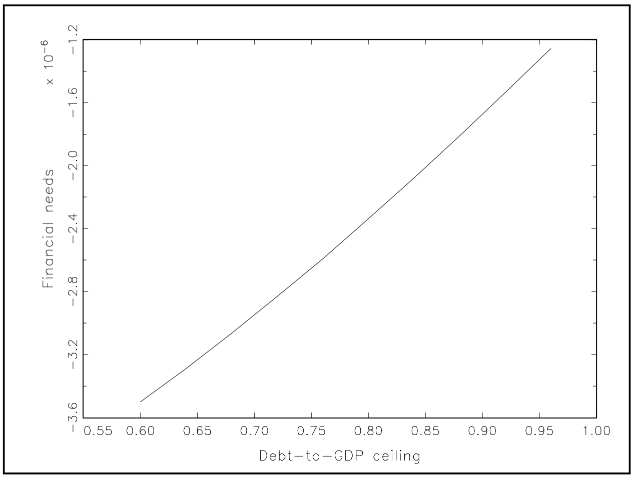

Figure 7: Changes on financial needs in the CPF model when the debt-to-output ratio rises.

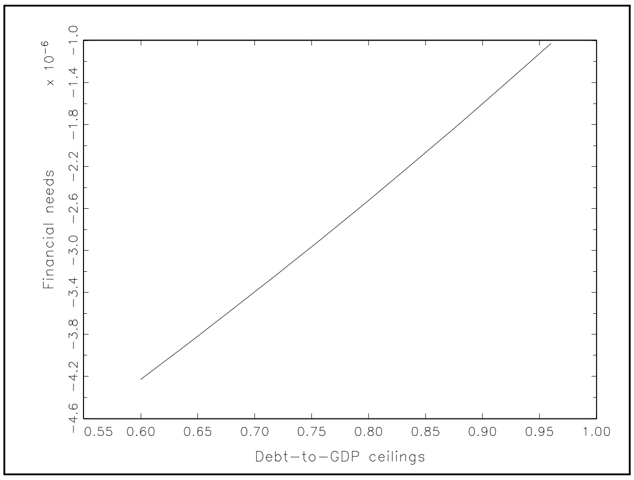

Figure 8: Changes on financial needs in the CUF model when the debt-to-output ratio rises.