Simulation-Based Exact Tests for Jump-Diffusions with Unidentified Nuisance Parameters: An Application to Commodities Spot Prices*

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Abstract

We propose to use the Monte-Carlo (MC) test technique to obtain valid p-values when testing for the presence of discontinuities in jump-diffusion models. Indeed, the LR statistic used to test for discontinuities has typically a complex non-standard distribution, for at least two reasons: the jump frequency parameter lies on the boundary of its domain, and unidentified nuisance parameters intervene under the null hypothesis. We show that, if no other (identified) nuisance parameters are present (e.g. the geometric Brownian motion case), the proposed p-value is finite sample exact. Otherwise, we derive nuisance-parameter free bounds on the null distribution of the LR and obtain exact bounds p-values. We illustrate our approach with four classes of jump diffusion models (geometric Brownian motion and logarithmic Ornstein-Uhlenbeck, with and without a GARCH(1,1) error structure), which we apply to weekly and monthly spot prices of copper, nickel, gold, and crude oil. We find significant jumps in all weekly time series, but only in a few monthly time series.

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1 Introduction

The problem of unidentified nuisance parameters is quite pervasive in econometrics. Prominent examples include tests for structural change (Andrews and Ploberger (1994)) and ARCH-in-mean tests (Bera and Ra (1995)). As is well known, when nuisance parameters are present only under the alternative hypothesis, the tests limiting null distributions are not generally chi-square. Indeed, they can take a much more complex form, *e.g.* Andrews' (1993) sup-χ^2 distribution and Hansen's (1996) χ^2 processes. More importantly, as emphasized in Hansen (1996), in several situations, the relevant limiting distributions are nuisance parameter dependent which precludes the construction of specialized critical points tables.

One early approach to dealing with the problem is the asymptotic bounds procedures proposed in Davies (1977, 1987). Hansen (1996) and Andrews (1999) have recently proposed simulation-based procedures to approximate asymptotic p-values, which is valid in settings more general than Davies'. However, it is important to remember that all the latter procedures are only asymptotically[1] valid. No finite sample exact procedures seem available for such non-regular test problems.

In this paper, we provide exact simulation-based solutions to the problem of nuisance parameters which intervene only under the alternative. Although the testing strategy we propose here is sufficiently general to suggest extensions to a wide class of parametric models[2], we focus on testing the significance of jumps[3] in jump diffusion/ARCH models.

There has been widespread interest in finance and economics for these models since Merton (1976) proposed to model stock prices with Poisson jumps superimposed on a geometric Brownian motion. This approach has since been extended to include mean-reversion and conditional heteroscedasticity, which is common in high frequency data (e.g. see Bollerslev, Chou, and Kroner 1992 or Amin and Ng 1993). Examples of well know papers using jump/diffusion models include Ball and Torous (1985), Jarrow and Rosenfeld (1985), Ahn and Thompson (1988), Akgiray and Booth (1988), Jorion's (1988), Brorsen and Yang (1994) and Bates (1991, 1996a, b). Although recognized as a potential difficulty (see, for example, Brorsen and Yang (1994)) un-identification problems have not received the attention they deserve in empirical applications on jump-tests. In fact, most of the references just

[1] In fact, Hansen (1996) operates in a local-to-zero asymptotic framework.

[2] For instance, see Bernard, Dufour, Khalaf and Genest (1999) for applications to ARCH-in-mean testing.

[3] For an operational definition of jump processes, see Merton (1990).

cited inappropriately use χ^2 critical points.

Jump-tests have recently been the subject of renewed attention; see, for example, Hilliard and Reis (1999) and Drost, Nijman and Werker (1998). Hilliard and Reis (1999) approach the testing problem as follows. Sum-of-squared errors based on the difference between observed and predicted option prices are obtained using two alternative pricing formulas, imposing and ignoring jumps. Then an F-type statistic is constructed. However, the authors provide no formal proof of the test's asymptotic validity, even though they recognize the intervening identification difficulties. Drost *et. al.* derive a kurtosis-based test for jumps. They formulate a Quasi-Maximum-likelihood (QMLE) estimate for the kurtosis after imposing normal-GARCH errors, and then implement the delta-method - based on the usual QMLE-based asymptotic standard-errors and covariance estimates - to assess the estimate's proximity-to-zero. The authors report favorable simulation evidence but warn that their experiments may not be sufficient to establish the test's overall finite sample validity.

One solution to identification problems, which we adopt in this paper (following Dufour 1997), is to use pivotal or *boundedly* pivotal test statistics, *i.e.* statistics whose null distributions are either nuisance-parameters-free or can be bounded by nuisance-parameter-free distributions. Here we consider likelihood-ratio (LR) test statistics and show first that they satisfy the boundedly pivotal property. To do this, we derive exact bounds on their null distributions explicitly. The method of proof is analytical and is similar to that used by Dufour (1989, 1997) in different yet related contexts. In addition, to obtain exact tests in the presence of unidentified nuisance parameters, we apply the Monte Carlo (MC) test procedure (Dufour (1995)) which yields exact simulation-based p-values whenever the null distributions of the underlying test statistics do not depend on unknown parameters. The main fact exploited here is that the MC p-values simulated under the null will not depend on the unidentified nuisance parameters; this follows immediately from the implications of under-identification. In other words, in many cases where unidentified nuisance parameters cause important complications, the MC test procedure, which exploits the under-identification situation from a finite sample perspective, easily yields valid p-values. As far as MC tests are concerned, tractable null distributions are not a relevant issue.[4]

An important related difficulty is the case of *restricted* alternative hypotheses. As is well known (see for example Andrews (1996, 1999)), prob-

[4] For applications of MC tests in econometrics, see Dufour and Kiviet (1996, 1997), Dufour and Khalaf (1997, 1999), Dufour, Farhat, Gardiol and Khalaf (1998).

lems similar to unidentification occur when the null hypothesis sets values on the parameter space *boundary*. This question is also relevant in the no-jump tests case considered here. It turns out however that the method we adopt to deal with unidentified nuisance parameters does not have the problems of standard asymptotic tests, in the presence of restricted alternatives.

Below, we will be more precise about the setup and test strategy adopted but it is of interest to give a brief overview of the proposed test method. Take, for example, a size-α right-tailed test. By drawing (conditionally on the relevant nuisance parameters), N *simulated* samples conformable with the null, we can come up with N replications of the test statistic. Then a MC p-value (conditional on the nuisance parameters) can be computed from the percentage of the simulated statistics which exceed the *observed* test statistic. The test is significant at level α if the largest MC p-value (over the *relevant* nuisance parameter space) does not exceed α. Naturally, for the problem at hand, the only *relevant* nuisance parameters are those which are, in the notation of Davies (1977, 1987), *present* under the null. If no other nuisance parameters are involved, the method just described can be easily applied to deal with the under-identification problem. In situations where both identified and unidentified parameters intervene as nuisance parameters, we propose exact MC bounds tests based on the bound we derived to justify the use of the LR criteria; as will be demonstrated, the bounding statistics' cut-off points are non-standard yet may be easily obtained by simulation. For further discussion of MC bounds tests, see Dufour and Khalaf (1997, 1999).[5]

The MC test procedure is highly related to the parametric bootstrap (see the discussion in Dufour (1995)). In connection, Diebold and Chen (1996) have shown that improved p-values for structural change tests can be obtained with the bootstrap. Yet Diebold and Chen's work is also motivated by asymptotic arguments. Finally, note that whereas the results in Hansen (1996) are not directly applicable to the test problem considered here, our procedure may be applied in Hansen's framework, provided distributional assumptions are imposed so that simulated samples conformable with the null can be drawn. The procedures proposed in Andrews (1999) are - in principle - applicable here. These involve simulating the supremum of quadratic forms based on: (i) a random term whose construction requires solving a *restricted* minimization problem (over a convex cone), and (ii) first and second order derivatives of the likelihood function. The main methods we propose simply require simulated values of LR-based statistics under the (no-jump)

[5]See also Wang and Zivot (1998) for further examples on bounds tests in near-unidentified test problems.

null hypothesis. At any rate, Hansen and Andrews solutions basically serve to approximate the limiting distributions' tail probabilities. In contrast, we obtain finite sample exact p-values.

These results are then applied to investigate the existence of discontinuities in commodities spot prices using four basic models: a geometric Brownian motion (GBM) and a logarithmic Ornstein-Uhlenbeck mean reverting motion (MRM), with and without a GARCH(1,1) error structure. Ignoring jumps by assuming only a continuous time model when jumps are indeed present can have a number of well known, unpleasant consequences, such as mispricing derivative instruments, adopting misleading hedging strategies, or miscalculating the value of a portfolio.

This paper is organized as follows. The models and test strategies are presented in Section 2. Section 3 presents our empirical applications and discusses our results. Our conclusions are presented in Section 4.

2 Jump processes and Jump tests

This section presents the general framework we consider as well as the statistical inference procedures. We focus on mixtures distributions which can be written as the sum of a continuous component and a jump component. Merton (1976) proposed to formally model price discontinuities which represent, in the notation of Ball and Torous (1983), the arrival of "abnormal" information.

We impose the following basic assumptions on both DGPs.

Assumption A. Both continuous and jump processes are completely specified to allow the formulation of likelihood functions.

Assumption B. The continuous model is nested, imposing possibly boundary constraints, within the mixed model.

Assumption C. The no-jump process is *simulatable*, i.e. it is possible to obtain simulated samples drawing from the continuous process.

Assumption D. The model admits a restricted version which corresponds to a location-scale model.

Assumption E. The mixture model is *additively separable*, in the sense that the likelihood imposing no-jump constraints does not depend on the parameters associated with the jump component.

Assumption A is fundamental because we adopt Maximum-likelihood-based tests. The smooth and jump components may be derived from continuous-time models, as long as likelihood functions are available. In fact, continuous-time modeling, although desirable, is not necessary for most of the results we obtain. Furthermore, the methods we propose will be also valid in a simulated maximum likelihood framework, as long as the intervening parameters are finite dimensional and identifiable (at least under the no-jump model). Assumption B justifies the use of the likelihood ratio criterion.[6] Restrictions-to-boundary are typical in the context of jump tests and will be formally dealt with here. Assumption C is necessary because the method we propose requires to draw samples from the relevant no-jump DGP. Assumption D relates to our proposed bound and will become clear from our demonstrations below. Finally, assumption E does not seem to be a limitation for mixture models - which are typically written as the sum of a smooth and a jump component- but its usefulness will be apparent below, so we require it here. No further constraints will be imposed in this section. We take up a specific mixture model in section 3, allowing for conditional heteroskedasticity in mean-reverting and random-walk contexts.

It is useful at this stage to contrast our framework with that of Drost et. al. (1998). Whereas in our case both likelihood functions should be specified, Drost et. al. (1998)'s test requires only a pseudo no-jump model; the jump element needs not be formally modelled. In a way, this is an advantage since the estimation of jump models may present added challenges. In the LR framework we adopt here, promise of good power often rewards for the difficulties associated with estimating both constrained and unconstrained model. A simulation experiment is of course needed to formally compare the merits of both tests. In this paper, we aim to illustrate the feasibility of the LR procedure and show that all statistical complications which arise in this context can be solved relatively easily. A power study to assess the performance of the LR and LM-type test will be the subject of further work. Nevertheless, note that serious problems associated with the delta method underlying Drost et. al. (1998)'s test have recently been pointed out; see for example Dufour (1997). These problems are caused by identification difficulties and are not restricted to small samples. Our LR test does not suffer from such potential disadvantages. Indeed, we establish the test's validity, in finite samples, following the criteria proposed by Dufour (1997).

We proceed now to present our test procedure. To derive the bound on

[6] This assumption is not crucial in the sense that extensions to non-tested tests are possible. However, such procedures will not be discussed in this paper.

the null distribution of the no-jump LR criterion in general contexts, we use a key result by Dufour and Khalaf (1999) who show that it is straightforward to obtain finite-sample exact p-values for the LR no-jump test statistic in the jump-GBM model. We build on this result to construct a finite sample bound for the general case, as in Dufour (1989). Let us start with the jump-GBM model proposed by Merton (1976) which will serve as our benchmark model.

If a random variable P_t follows a GBM with Poisson jumps, it can be written:

$$dP_t = \alpha P_t dt + \sigma P_t dz_t + P_t dq_t. \tag{2.1}$$

The jump component, dq_t, equals a lognormally distributed variable Y_t such that:

$$Y_t \overset{iid}{\sim} LN(\theta, \delta^2)$$

with probability λdt, and 0 with probability $(1 - \lambda dt)$. If a jump occurs, Y_t is the ratio of P_t just after the jump ($P_t^+ = \lim_{\tau \to t,\ t>t} P_\tau$) by P_t just before the jump ($P_t^- = \lim_{\tau \to t,\ \tau<t} P_\tau$). λ is the arrival rate of jumps.

In discrete time, this model can be written:

$$p_t - p_{t-1} = \mu + \sigma z_t + \sum_{i=1}^{n_t} lnY_{ti} \tag{2.2}$$

where $\mu = \alpha - \frac{\sigma^2}{2}$, $p_t = \ln(P_t)$, $z_t \overset{iid}{\sim} N(0,1)$, n_t is the number of jumps between t and $t-1$, and Y_{ti} is the size of the i^{th} jump which occurs between t and $t-1$.

The parameters of the above model may be estimated by numerical maximization of the likelihood functions. In this framework, to test the null hypothesis

$$H_0 : \lambda = 0 \qquad \text{(no jump)}\ , \tag{2.3}$$

the likelihood ratio (LR) statistics is:

$$LR_{GBM} = 2[L_{Jump/GBM} - L_{GBM}], \tag{2.4}$$

where L_{GBM} and $L_{Jump/GBM}$ are respectively the maximum of the log-likelihood function (MLF) under the null and the alternative hypothesis. As emphasized above, the standard regularity conditions ensuring that the LR statistic is asymptotically χ^2 distributed under the null hypothesis are not verified. One reason is that there are two nuisance parameters θ and δ, which are not identified under H_0 (i.e., when we set $\lambda = 0$, the likelihood function no longer depends on these two parameters). Another reason is that the value of λ tested under H_0 is on the boundary of the parameter space. As a consequence, the asymptotic distribution of the LR statistic under H_0 is non-standard and quite complex. Its χ^2 approximation is no longer valid.

Dufour and Khalaf (1999) argue that a parametric bootstrap test (a Monte Carlo test, in the notation of Dufour (1995)), applied to LR_{GBM} will yield an exact p-value. Before we discuss the rationale underlying this result, it is useful to describe the MC test method as it applies to the problem at hand. Our exposition will be very brief and details are relegated to the Appendix. The procedure may be summarized as follows. Imposing H_0, *i.e.* drawing from the GBM DGP, N simulated samples are generated which yield N simulated test statistics. Then a MC p-value is obtained from the *rank* of the observed value of the test statistic within the set

$$\left[\ \textit{observed}\ \text{statistic},\quad \textit{simulated}\ \text{statistics}\ \right].$$

In other words, a *rank* exceeding $(1-\alpha)(N+1)$ is interpreted as evidence, at level α, against H_0. Dufour (1995) shows that the MC procedure yields finite sample exact p-values if the null distribution of the test statistic considered is *pivotal, i.e.* nuisance-parameter-free. In nuisance-parameter-dependent context, an exact MC test may be obtained based on the largest MC p-value over the nuisance parameter space compatible with the null hypothesis.

Our point here is that the MC p-value (calculated as just described) will not depend on θ and δ^2. This follows immediately from the implications of unidentification. Furthermore, the invariance to location and scale (μ and σ in the GBM case) is straightforward to see. Consequently, the MC test in this case will be finite sample exact. Clearly, the boundary restriction does not intervene here, since the only elements of proof concern the pivotal characteristic of the LR statistic. We summarize this result in the following Theorem.

Theorem 2.1 *Consider the jump-GBM model defined by (*2.1*). Then a Monte Carlo p-value based on the LR statistic (*2.4*) and obtained as in (*5.10*) is finite sample exact.*

We have just seen that the MC test procedure conveniently solves the un-identification problem in the jump-GBM benchmark model. Theorem 1 has further implications on the properties of the LR test in the general case. Indeed, we will next use Theorem 1 to show that the LR no-jump test in general mixed models which satisfy assumption A-E is boundedly pivotal.

Now, let L_{Mixed} and L_{Smooth} denote respectively the maximum of the likelihood function associated with the general mixed and the no-jump model. The associated likelihood ratio (LR) statistics is:

$$LR = 2[L_{Mixed} - L_{Smooth}]. \tag{2.5}$$

By construction, and using assumption D, it is easy to see that $L_{Mixed} \geq L_{Smooth} \geq L_{GBM.}$ In turn, this implies that

$$[L_{Mixed} - L_{Smooth}] \leq [L_{Mixed} - L_{GBM}]\,. \tag{2.6}$$

Now let

$$LR_B = 2\,[L_{Mixed} - L_{GBM}] \tag{2.7}$$

denote the LR statistic for testing the GBM null model against the mixed model at hand. Then inequality (2.6) implies that $LR \leq LR_B$. Furthermore, as argued in the context of Theorem 1, the null distribution of the LR_B may be simulated to obtain exact p-values. In fact, from assumptions D-E, it is easy to see that no-unknown parameters intervene in the null distribution of LR_B.[7] This provides the condition to apply the bounds-MC as described in Dufour and Khalaf (1999). The procedure may be summarized as follows (see also the Appendix). From the observed data, compute the test statistic LR. Generate N simulated samples drawing from the GBM process and compute the statistic LR_B. Then a bounds MC p-value is obtained from the *rank* of the observed value of the test statistic within the set

$$\Big[\ \textit{observed } LR \text{ statistic}, \quad \textit{simulated bounding } LR_B \text{ statistics}\ \Big].$$

Our results may be summarized as follows.

Theorem 2.2 *Consider a mixed smooth-jump model which satisfies assumptions A-E. Then the Monte Carlo bounds p-value based on the LR statistic (*2.5*) and the bounding statistic (*2.7*), obtained as in (*5.12*), is finite sample exact.*

In the next section, we focus on specific cases, namely jump-GARCH models.

3 Empirical examples

3.1 Model

To illustrate the feasibility of our proposed tests, we focus on two special cases. Consider first the random Walk model with GARCH (1,1) errors

$$\begin{aligned} p_t &= p_{t-1} + \sqrt{h_t} z_t \\ h_{t+1} &= \alpha_0 + h_t(\alpha_1 z_t^2 + \alpha_2) \end{aligned} \tag{3.8}$$

[7] Location-scale invariance is also straightforward to see here.

where $z_t \overset{iid}{\sim} N(0,1)$. Nelson (1990) shows that the diffusion limit of (3.8) is the stochastic volatility model

$$\begin{aligned} dp_t &= \sigma_t dW_{1,t} \\ d\sigma_t^2 &= \beta(\alpha - \sigma_t^2)dt + \gamma^2 \sigma_t^2 dW_{2,t} \end{aligned}$$

where $\{W_{1,t},\ t \geq 0\}$ and $\{W_{2,t},\ t \geq 0\}$ are two independent standardized Brownian motions, and $\alpha_0 > 0$, $\alpha_1 \geq 0$, and $\alpha_2 \geq 0$ are linked to $\alpha \geq 0$, $\beta \geq 0, \gamma > 0$ and the frequency of observation of p_t. His work was generalized by Drost and Werker (1996) and Duan (1997). Clearly, setting $\alpha_1 = \alpha_2 = 0$ yields a (driftless) GBM.

We also consider the autoregressive model with GARCH errors

$$\begin{aligned} p_t &= a_0 + a_1 p_{t-1} + \sqrt{h_t} z_t \\ h_{t+1} &= \alpha_0 + h_t(\alpha_1 z_t^2 + \alpha_2). \end{aligned} \tag{3.9}$$

Drost and Werker (Section 5, 1996) show that the diffusion limit of (3.9) is the stochastic volatility model:

$$\begin{aligned} dp_t &= \kappa(\mu - p_t)dt + \sigma_t dW_{1,t} \\ d\sigma_t^2 &= \beta(\alpha - \sigma_t^2)dt + \gamma^2 \sigma_t^2 dW_{2,t} \end{aligned}$$

where again $\{W_{1,t}, t \geq 0\}$ and $\{W_{2,t}, t \geq 0\}$ are two independent standardized Brownian motions. The discrete time coefficients a_0, $|a_1| < 1$, $c_0 > 0$, $\alpha_1 \geq 0$, and $\alpha_2 \geq 0$ are linked to $\kappa \geq 0$, $\mu \geq 0$, $\alpha \geq 0$, $\beta \geq 0$, $\gamma > 0$ and the frequency of observation of p_t through fairly complex relationships. In both models, the null hypothesis is (2.3). Observe that setting $\alpha_1 = \alpha_2 = 0$ in (3.8) and (3.9) yields a homoskedastic random-walk and autoregressive process respectively. For convenience, we have adopted below the standard notation in the no-GARCH case: when $\alpha_1 = \alpha_2 = 0$, α_0 is denoted σ^2.

To obtain the mixed process, we add the Poisson jump process introduced above to obtain the following mixed models:

$$\begin{aligned} p_t &= p_{t-1} + \sqrt{h_t} z_t + \sum_{i=1}^{n_t} lnY_{ti} \\ h_{t+1} &= \alpha_0 + h_t(\alpha_1 z_t^2 + \alpha_2) \end{aligned}$$

and

$$\begin{aligned} p_t &= a_0 + a_1 p_{t-1} + \sqrt{h_t} z_t + \sum_{i=1}^{n_t} lnY_{ti} \\ h_{t+1} &= \alpha_0 + h_t(\alpha_1 z_t^2 + \alpha_2) \end{aligned}$$

Y_{ti} is the number of jumps which occur between t and $t+1$, which follows the same assumptions as in (2.1): the arrival of jumps follows a Poisson process with arrival rate λ and that the jump-size distribution is lognormal with mean θ and variance δ^2. The hypothesis of interest is $\lambda = 0$.

The assumptions of Theorem 2 are clearly satisfied here. The possible nuisance parameters are: i) the parameters of the diffusion/GARCH process, and ii) the parameters of the jump process θ and δ (the value of λ is set to zero under the null). As argued in Section 2, unidentification under the null implies that θ and δ are not relevant. Moreover, the test problem is location-scale invariant, which leaves the GARCH (and the mean reversion when relevant) parameters as effective nuisance parameters. For the jump/GBM test we have seen that a MC p-value based on N random draws from a normal distribution (with parameters the data-based GBM-MLE) achieves size control. However, for the jump/GARCH (GBM or mean reverting) case, if we generate N simulated samples drawing form the GARCH process (with parameters the data-based GARCH-MLE), the procedure may not be reliable in finite samples. In fact, the same conditions which cause the failure of standard asymptotics given the *restricted* alternative problem may also affect the performance of such bootstrap-type corrections. To obtain the bounds p-value from Theorem 2, derive the bounding statistic which corresponds to the LR no-jump/no-GARCH test statistic and apply the bounds MC procedure.

To conclude, note that an LR bounds tests for GARCH or mean-reversion in the presence of jumps may also be obtained using similar arguments (e.g. see Saphores *et al.* (1999)).

3.2 Applications

For our empirical application, we consider weekly and monthly observations of spot prices for three commercial commodities, crude oil (West Texas Intermediate or WTI), copper and nickel, and one precious metal, gold. Daily WTI spot prices, which were provided by Natural Resources Canada, cover the period extending from 01/02/86 to 05/13/99. Daily prices of copper and nickel, obtained from the London Metal Exchange, extend from 01/03/89 to 06/30/99. Daily closing prices of gold, from the New York Metals Exchange, go from 01/03/89 to 10/15/99. Weekly series were constructed from daily data by taking the Wednesday price to avoid beginning or end of the week effects. In the rare instances were the Wednesday price was missing, we used the Tuesday price instead. Approximate monthly data were constructed by taking every four weekly observation. The four weekly time series analyzed are shown on Figures 1 to 4.

Each of the four models presented above were fitted by maximum likelihood using the procedure OPTMUM in GAUSS. Since the Poisson distribution allows for an infinite number of jumps in a finite time interval (albeit with vanishingly small probability), we had to truncate the infinite sum to estimate the parameters numerically. Like Ball and Torous (1985), who derived an upper bound for the truncation error, and Jorion (1988), we found that 10 terms gave satisfactory accuracy for the parameter values encountered.

Moreover, as remarked by Ball and Torous (1985), the likelihood functions of jump-diffusion models usually have a local maximum at $\lambda = 0$ (the no jump case). To find the global maximum of each likelihood function in the presence of several local maxima, we considered several starting points of likely values for each iteration of the bootstrap as well as for the evaluation of the maximum likelihood parameters of the observed data. Since convergence problems are mainly due to the near-unidentified region in the neighborhood of $\lambda = 0$, numerical difficulties arise most with the simulated samples which are drawn, as required, under the no-jump null. Our MC test algorithms are available upon request.

Results are presented in Tables 1-2. We report estimates and standard errors[8], the LR test and the exact MC p-value for the GBM case and the bootstrap and bounds MC p-values otherwise.

Tables 1 presents the weekly series based tests. In this case, we first observe that there is ample evidence of statistically significant jumps in each of the time series investigated, for both the Geometric Brownian Motion and for the Mean Reverting process, with and without GARCH effects. Indeed, both the Monte-Carlo and Monte-Carlo bound p-values are 0.01 for 100 replications for each time series and each of the models considered. We also observe that the jump frequency, given by λ, is fairly stable between models. It is highest for WTI, with a frequency of approximately 0.5 (which represents one jump every other week, on average). For copper, we find $\lambda \approx 0.4$ (one jump every 2.5 weeks), while for nickel λ is between 0.2 and 0.3, with a high value of 0.6 for the MRM model. It is for gold that λ is most stable with a value of ≈ 0.11 for all four models. We can notice, however, that the jump-less GBM-GARCH and MRM-GARCH (a result consistent with the findings of Schwartz (1997)) models estimated for this time series are non-stationary since $\alpha_1 + \alpha_2 > 1$.

[8] Although we report asymptotic standard errors as is usual in this literature, we warn against their use in a t-tests framework. As argued in section 2, asymptotic SE based t-tests may be seriously flawed in the presence of identification problems such as the those we are dealing with here. Following Dufour (1997), we rather use LR tests for hypotheses of interest.

Results for monthly series are presented in Table 2. This time, the statistical significance of jumps depends on the time series and on the model considered. Hence, for copper jumps are not significant at the 5% level; for nickel, there are statistically significant jumps only for the GBM; and for WTI, there are no jumps in presence of GARCH effects. The exception is gold, which exhibits jumps on average every 5.5 months ($\lambda \approx 0.18$). There are, however, no GARCH effects with the jumpless GBM or MRM (both α_1 and α_2 are null), and only ARCH(1) effects in the presence of jumps (α_2 is null). Similarly, there are only ARCH(1) effects for copper for both the GBM and MRM models.

Before discussing some more the results, we note that the Monte-Carlo bound p-value is, as expected, closer to the Monte-Carlo p-value the closest the reduced model (either a GBM or a driftless GBM) is from the model under H_0 (MRM, GBM-GARCH, or MRM-GARCH). These two p-values are thus closer for the GBM-GARCH model than for either the MRM or MRM-GARCH models.

The observed discrepancy on jump frequency (value and statistical significance of λ) between the weekly and monthly data can easily be explained: since there is more time between successive observations which anyway tend to partly cancel each other's effects (weekly observations do not all go up or down between consecutive months), a diffusion is more likely to have produced the observed sample paths. In addition, since weekly and monthly data cover the same period, we have four times less data information to estimate the jump parameters.

If we contrast these results for the MRM processes with those of Schwartz (1997) for futures prices, we observe that we obtain very small values for the coefficient of mean reversion, κ. The value of this coefficient is smallest for weekly data: between 0.006 and 0.012 for copper; ≈ 0.016 for nickel; ≈ 0.008 for gold; and 0.04 for WTI (its highest value for weekly data). These values are still far lower than those found by Schwartz for weekly futures: 0.37 for copper and 0.30 for oil for the MRM. The value of κ is higher and less influenced by the presence of GARCH effects for monthly data (≈ 0.05 for copper, ≈ 0.06 for nickel, ≈ 0.04 for gold, and ≈ 0.14 for WTI), but still very low. This can be explained by the much higher volatility of futures prices compared to spot prices.

4 Conclusions

When nuisance parameters are unidentified under the null, conventional asymptotics fail even if the sample is large. Such problems frequently occur in jump/diffusion models. In this paper we propose an approach based on (exact) boundedly pivotal statistics, which combines exact bounds and MC test procedures based on the LR no-jump test statistic. On theoretical grounds, we have established *explicitly* the criterion's boundedly pivotal characteristic. From the practical point of view, we have applied simulation-based methods to the LR and bounding statistics to obtain size-correct p-values. Although the problem we considered is highly non-regular, the solution we propose is computationally attractive and it is finite sample exact. It can be generalized to other non identification problems in the presence of nuisance parameters.

We have illustrated our proposed tests on spot price series for crude oil, copper, nickel, and gold. We have found statistically significant jumps for all time series and all models considered for the weekly data, but only in some cases for the monthly time series. This has implications for pricing derivative instruments, adopting hedging strategies, or calculating the value of a portfolio based on non renewable resources.

Table 1: Jump Price Tests For Weekly Data

	Copper		Nickel	
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0015 (0.0014)	-0.0008 (0.0017)	-0.0023 (0.0017)	-0.0022 (0.0018)
σ	0.0319 (0.0010)	0.0210 (0.0037)	0.0390 (0.0012)	0.0288 (0.0027)
λ		0.4095 (0.3210)		0.2306 (0.1452)
θ		-0.0018 (0.0040)		-0.0006 (0.0074)
δ		0.0373 (0.0107)		0.0547 (0.0129)
MLE	1108.13	1133.48	998.96	1022.67
LR	50.69 (0.01)		47.42 (0.01)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	7.5996 (0.1252)	7.5807 (0.1540)	8.7343 (0.1073)	8.6478 (0.1741)
κ	0.0126 (0.0065)	0.0104 (0.0060)	0.0170 (0.0055)	0.0117 (0.0053)
σ	0.0320 (0.0010)	0.0229 (0.0027)	0.0389 (0.0012)	0.0295 (0.0025)
λ		0.2663 (0.1785)		0.2088 (0.1279)
θ		-0.0003 (0.0055)		0.0014 (0.0079)
δ		0.0429 (0.0105)		0.0553 (0.0127)
MLE	1110.03	1134.64	1003.82	1024.91
LR	49.23 (0.01, 0.01)		42.19 (0.01, 0.01)	
	GARCH errors			
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0016 (0.0012)	-0.0016 (0.0016)	-0.0027 (0.0015)	-0.0030 (0.0020)
α_0	0.0000 (0.0000)	0.0000 (0.0000)	0.0001 (0.0000)	0.0000 (0.0000)
α_1	0.0851 (0.0245)	0.1372 (0.0522)	0.0691 (0.0258)	0.0693 (0.0308)
α_2	0.8803 (0.0390)	0.7276 (0.0995)	0.8844 (0.0509)	0.8676 (0.0745)
λ		0.4079 (0.3323)		0.6155 (0.3719)
θ		0.0017 (0.0045)		0.0004 (0.0036)
δ		0.0290 (0.0079)		0.0329 (0.0073)
MLE	1137.38	1151.05	1028.10	1040.81
LR	27.33 (0.01, 0.01)		25.40 (0.01, 0.01)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	7.4991 (0.2983)	7.4284 (0.4308)	8.6854 (0.1235)	8.5441 (0.2261)
κ	0.0058 (0.0062)	0.0054 (0.0059)	0.0150 (0.0061)	0.0114 (0.0058)
α_0	0.0002 (0.0001)	0.0001 (0.0000)	0.0003 (0.0001)	0.0000 (0.0000)
α_1	0.1187 (0.0386)	0.1097 (0.0425)	0.1085 (0.0369)	0.0007 (0.0005)
α_2	0.6788 (0.0891)	0.7020 (0.0822)	0.6544 (0.1079)	0.9714 (0.0092)
λ		0.4294 (0.3480)		0.3076 (0.2417)
θ		0.0012 (0.0043)		0.0051 (0.0061)
δ		0.0293 (0.0080)		0.0453 (0.0133)
MLE	1136.61	1151.69	1019.75	1033.73
LR	30.17 (0.01, 0.01)		27.96 (0.01, 0.01)	

Table 1 (Continued.)

	Gold		WTI	
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0004 (0.0008)	-0.0011 (0.0006)	-0.0006 (0.0021)	0.0014 (0.0020)
σ	0.0190 (0.0006)	0.0120 (0.0007)	0.0546 (0.0015)	0.0311 (0.0030)
λ		0.1115 (0.0429)		0.3982 (0.1295)
θ		0.0059 (0.0060)		-0.0048 (0.0057)
δ		0.0401 (0.0079)		0.0709 (0.0099)
MLE	1429.69	1546.29	1036.14	1095.91
LR	233.20 (0.01)		119.54 (0.01)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	5.8538 (0.0900)	5.6857 (0.2226)	2.9086 (0.0488)	2.9834 (0.0453)
κ	0.0094 (0.0059)	0.0049 (0.0041)	0.0430 (0.0111)	0.0475 (0.0109)
σ	0.0191 (0.0006)	0.0120 (0.0007)	0.0552 (0.0015)	0.0277 (0.0042)
λ		0.1120 (0.0426)		0.5391 (0.2002)
θ		0.0055 (0.0060)		-0.0063 (0.0045)
δ		0.0399 (0.0077)		0.0632 (0.0098)
MLE	431.00 1	1546.89	1043.93	1104.23
LR	231.79 (0.01, 0.01)		120.60 (0.01, 0.01)	
GARCH errors				
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0028 (0.0006)	-0.0011 (0.0006)	-0.0004 (0.0016)	0.0027 (0.0018)
α_0	0.0002 (0.0000)	0.0000 (0.0000)	0.0001 (0.0000)	0.0000 (0.0001)
α_1	0.9911 (0.1759)	0.0625 (0.0307)	0.1562 (0.0282)	0.1666 (0.0325)
α_2	0.0349 (0.0538)	0.6475 (0.1553)	0.8033 (0.0312)	0.7895 (0.0574)
λ		0.1069 (0.0522)		0.4845 (0.7822)
θ		0.0045 (0.0060)		-0.0074 (0.0119)
δ		0.0392 (0.0092)		0.0342 (0.0155)
MLE	1457.43	1545.98	1140.60	1155.46
LR	177.09 (0.01, 0.01)		29.73 (0.01, 0.01)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	5.5481 (0.2201)	5.7205 (0.1738)	2.9413 (0.0381)	3.0442 (0.0571)
κ	0.0074 (0.0041)	0.0057 (0.0042)	0.0420 (0.0116)	0.0460 (0.0117)
α_0	0.0002 (0.0000)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0000)
α_1	1.0008 (0.1770)	0.0617 (0.0310)	0.1371 (0.0271)	0.1402 (0.0302)
α_2	0.0000 (- -)	0.0000 (- -)	0.8074 (0.0318)	0.8037 (0.0384)
λ		0.1133 (0.0482)		0.5877 (0.3389)
θ		0.0044 (0.0058))		-0.0084 (0.0046
δ		0.0385 (0.0082)		0.0316 (0.0066)
MLE	1460.16	1548.85	1155.39	1170.10
LR	177.38 (0.01, 0.01)		29.42 (0.01, 0.01)	

Standard errors are in parenthesis. We report ($\widehat{p}_{MC}$) for LR-GBM and ($\widehat{p}_{bootstrap}$, $\widehat{p}_{bound}$) otherwise.

Table 2. Jump tests, Monthly Data

	Copper		Nickel	
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0069 (0.0059)	-0.0022 (0.0104)	-0.0099 (0.0073)	-0.0143 (0.0093)
σ	0.0684 (0.0042)	0.0422 (0.0267)	0.0852 (0.0052)	0.0537 (0.0086)
λ		0.9006 (1.8212)		0.5750 (0.3650)
θ		-0.0052 (0.0137)		0.0078 (0.0168)
δ		0.0568 (0.0379)		0.0883 (0.0256)
MLE	171.74	174.16	141.96	145.27
LR	4.84 (0.08)		6.62 (0.04)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	7.5894 (0.1408)	7.5286 (0.1730)	8.7354 (0.1123)	8.6082 (0.1658)
κ	0.0506 (0.0289)	0.0457 (0.0278)	0.0717 (0.0243)	0.0681 (0.0260)
σ	0.0694 (0.0043)	0.0657 (0.0043)	0.0854 (0.0053)	0.0686 (0.0109)
λ		0.0093 (0.0114)		0.2800 (0.2972)
θ		0.2194 (0.0893)		0.0280 (0.0371)
δ		0.0000 (0.1270)		0.0887 (0.0347)
MLE	173.34	174.72	146.48	147.77
LR	2.76 (0.09, .31)		2.59 (0.09, 0.33)	
	GARCH errors			
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0065 (0.0058)	-0.0057 (0.0072)	-0.0129 (0.0059)	-0.0180 (0.0130)
α_0	0.0042 (0.0007)	0.0021 (0.0009)	0.0006 (0.0003)	0.0005 (0.0004)
α_1	0.1168 (0.1214)	0.1814 (0.1432)	0.2494 (0.1035)	0.2372 (0.1012)
α_2	0.0000 (- -)	0.0000 (- -)	0.6890 (0.0923)	0.6867 (0.0972)
λ		0.3140 (0.4037)		0.0859 (0.2182)
θ		-0.0017 (0.0221)		0.0731 (0.0668)
δ		0.0768 (0.0364)		0.0000 (0.0636)
MLE	171.21	174.14	152.37	152.49
LR	5.85 (0.08, 0.15)		0.24 (0.81, 0.88)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	7.5835 (0.1547)	7.5545 (0.2854)	8.6602 (0.1488)	7.3236 (1.3505)
κ	0.0467 (0.0298)	0.0334 (0.0276)	0.0539 (0.0276)	0.0328 (0.0264)
α_0	0.0044 (0.0007)	0.0021 (0.0013)	0.0006 (0.0003)	0.0001 (0.0002)
α_1	0.0361 (0.1090)	0.1094 (0.1627)	0.0563 (0.0838)	0.0267 (0.0236)
α_2	0.0000 (- -)	0.0000 (- -)	0.8233 (0.1234)	0.8673 (0.0614)
λ		0.4313 (0.8639)		0.5864 (0.5320)
θ		-0.0013 (0.0190)		0.0703 (0.0274)
δ		0.0678 (0.0452)		0.0001 (0.0325)
MLE	173.39	175.62	153.35	155.68
LR	4.47 (0.11, 0.31)		4.64 (0.11, 0.26)	

Table 2 (Continued.)

	Gold		WTI	
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0022 (0.0033)	-0.0059 (0.0025)	-0.0022 (0.0077)	-0.0017 (0.0066)
σ	0.0393 (0.0023)	0.0231 (0.0025)	0.1013 (0.0054)	0.0663 (0.0060)
λ		0.1719 (0.0928)		0.1978 (0.0948)
θ		0.0215 (0.0193)		-0.0028 (0.0368)
δ		0.0736 (0.0201)		0.1733 (0.0414)
MLE	254.58	279.50	151.56	168.10
LR	49.85 (0.01)		33.08 (0.01)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	5.8394 (0.0962)	5.4366 (0.6979)	2.9055 (0.0543)	2.9040 (0.0462)
κ	0.0380 (0.0250)	0.0124 (0.0174)	0.1469 (0.0429)	0.1456 (0.0426)
σ	0.0397 (0.0024)	0.0235 (0.0025)	0.1048 (0.0060)	0.0703 (0.0067)
λ		0.1596 (0.0854)		0.1675 (0.0833)
θ		0.0216 (0.0202)		0.0011 (0.0397)
δ		0.0752 (0.0203)		0.1776 (0.0434)
MLE	255.77	279.60	158.10	174.85
LR	47.65 (0.01, 0.01)		33.50 (0.01, 0.01)	
	GARCH errors			
GBM	No Jumps	Jumps	No Jumps	Jumps
μ	-0.0022 (0.0033)	-0.0055 (0.0024)	0.0008 (0.0057)	-0.0101 (0.0070)
α_0	0.0393 (0.0023)	0.0005 (0.0002)	0.0008 (0.0004)	0.0005 (0.0003)
α_1	0.0000 (- -)	0.0158 (0.0523)	0.2039 (0.0636)	0.1969 (0.0680)
α_2	0.0000 (- -)	0.0000 (- -)	0.6916 (0.0745)	0.6727 (0.0824)
λ		0.1915 (0.1154)		0.0902 (0.0605)
θ		0.0194 (0.0182)		0.1313 (0.0365)
δ		0.0710 (0.0208)		0.0000 (0.0453)
MLE	254.58	278.06	181.47	183.57
LR	46.95 (0.01, 0.01)		4.19 (0.14, 0.23)	
MRM	No Jumps	Jumps	No Jumps	Jumps
μ	5.8394 (0.0962)	5.2196 (1.4759)	2.9391 (0.0435)	2.9213 (0.0458)
κ	0.0380 (0.0250)	0.0082 (0.0172)	0.1428 (0.0441)	0.1433 (0.0441)
α_0	0.0397 (0.0024)	0.0005 (0.0002)	0.0009 (0.0004)	0.0006 (0.0003)
α_1	0.0000 (- -)	0.0243 (0.0555)	0.2392 (0.0752)	0.2334 (0.0821)
α_2	0.0000 (- -)	0.0000 (- -)	0.6516 (0.0814)	0.6584 (0.0790)
λ		0.1891 (0.1178)		0.0311 (0.0317)
θ		0.0191 (0.0181)		0.1487 (0.0598)
δ		0.0709 (0.0212)		0.0000 (0.0667)
MLE	255.77	279.90	185.86	186.95
LR	48.26 (0.01, 0.01)		2.17 (0.32, 0.67)	

Standard errors are in parenthesis. We report ($\widehat{p}_{MC}$) for LR-GBM and ($\widehat{p}_{bootstrap}$, $\widehat{p}_{bound}$) otherwise.

5 Appendix: Monte Carlo tests

The Monte Carlo (MC) test procedure [Dwass (1957), Barnard (1963)] is presented in Dufour (1995) where the nuisance-parameter-dependent case is formerly treated. Here we summarize the underlying methodology.

Consider a right tailed test problem based on a given test statistic which we denote $STAT$ and suppose the null distribution of $STAT$ depends on the unknown parameter ξ. Let $STAT_0$ refer to the value of the test statistic obtained from the data. The following steps define a procedure to obtain a MC p-value conditional on ξ, which we will denote $\widehat{p}_N(STAT_0|\xi)$ where N refers to the number of MC replications.[9]

- Conditionally on ξ, draw N samples from the null DGP.
- From each simulated sample, compute the $STAT$ criterion; this yields N realizations of the statistic, namely $STAT_j,\ j = 1\ ,\ \ldots\ ,\ N$.
- Given $STAT_0$ and $STAT_j,\ j = 1\ ,\ \ldots\ ,\ N$, obtain

$$\widehat{G}_N(STAT_0\ ) = \frac{1}{N}\sum_{i=1}^{N} I_{[0,\infty]}\ (STAT_i - STAT_0),$$

$$I_A(x) = \begin{cases} 1, & if\ x \in A, \\ 0, & if\ x \notin A. \end{cases}$$

  In other words, $N\widehat{G}_N(STAT_0)$ is the number of simulated criteria $\geq STAT_0$ and

$$\widehat{R}_N(STAT_0) = N - N\widehat{G}_N(STAT_0) + 1$$

  gives the rank of $STAT_0$ in the series $STAT_0,\ \ STAT_1,\ ...,\ STAT_N$.

- The MC p-value[10] conditional on ξ corresponds to

$$\widehat{p}_N(STAT_0|\xi) = \frac{N\widehat{G}_N(STAT_0\ ) + 1}{N+1}\ .$$

Dufour (1995) proves that, if $STAT$ is pivotal and $\alpha(N + 1)$ is an integer, the test's critical region would correspond to

$$\widehat{p}_N(STAT_0) \leq \alpha, \quad 0 < \alpha < 1, \tag{5.10}$$

[9] Note that the latter p-value takes N explicitly into consideration, so that no central limit arguments on N are needed to establish the method's validity.

[10] The formula for $\widehat{p}_N(STAT_0|\xi)$ gives the *empirical probability,* conditional on ξ, to observe a value as extreme or more extreme than $STAT_0$ under the null. Consequently, $\widehat{p}_N(STAT_0|\xi)$ may be viewed as a randomized MC p-value.

where the notation is explicit about the non-dependence on nuisance parameters.[11] Formally, it is shown that the latter critical region is *exact*, in the following sense:

$$P_{(H_0)}\left[\widehat{p}_N(STAT_0)\le \alpha\right]=\alpha.$$

In the presence of nuisance parameters, Dufour (1995) shows that the test based on the region

$$\sup_{\xi\,\in\,M_0}\left[\widehat{p}_N(STAT_0|\xi)\right]\le \alpha \tag{5.11}$$

where M_0 is the nuisance parameter space under H_0, is exact at level α.

If a consistent (constrained[12]) estimate $\widehat{\xi}_n$of ξ is available, the corresponding MC p-value $\widehat{p}_N(STAT_0|\widehat{\xi}_n)$ may provide an asymptotic test. Indeed, Dufour (1995) shows that given general regularity conditions, the test based on the latter p-value has the correct size asymptotically (as $T\to\infty$), *i.e.*, under H_0,

$$\lim_{n\to\infty}\left\{P[\widehat{p}_N(STAT_0|\widehat{\xi}_n)\le\alpha]-P[\widehat{p}_N(STAT_0|\xi)\le\alpha]\right\}=0\ .$$

Note that no asymptotics on the number N of MC replications is required to obtain the latter result; this is the fundamental difference between the latter procedure and the (closely related) parametric bootstrap method. Dufour and Khalaf (1998, 1999) call the test based on simulations using a consistent nuisance parameter estimate a *local* MC (LMC) test.[13] Furthermore, they show they observe that LMC non-rejections are *exactly* conclusive in the following sense. If $\widehat{p}_N(STAT_0|\widehat{\xi}_n)>\alpha$, then the exact test defined in (5.11) is clearly not significant at level α.

Finally, in the context of boundedly pivotal statistics a *conservative*[14] p-value may be derived as follows. Suppose the bounding statistic is $STAT^*$ so that

$$STAT\le STAT^*,\quad \forall\xi\in M_0$$

and $STAT^*$ is pivotal under the null. Then, to obtain a bounds MC p-value, proceed as in the pivotal case (drawing from the relevant null DGP), computing $STAT^*$ rather than $STAT$ from the simulated samples.

- Given $STAT_0$ and $STAT_j^*$, $j=1\ ,\ \dots\ ,\ N$, obtain

$$\widehat{G}_N^B(STAT_0)=\frac{1}{N}\sum_{i=1}^{N}I_{[0,\infty]}\ (STAT_i^*-STAT_0).$$

[11] For instance, given 99 replications of a pivotal statistic, a MC test is significant at 5% if the rank of $STAT_0$ in the series $STAT_0,\ STAT_1,\ ...,\ STAT_N$ is at least 96.

[12] *i.e.* derived imposing the null.

[13] The term *local* reflects the fact that the underlying MC p-value is based on a specific choice for the nuisance parameter

[14] A test is *conservative* if rejections are conclusive. For a formal definition, see Dufour (1989).

- The bounds MC p-value

$$\hat{p}_N^B(STAT_0) = \frac{N\hat{G}_N^B(STAT_{\mathbf{0}}) + 1}{N+1} \ . \tag{5.12}$$

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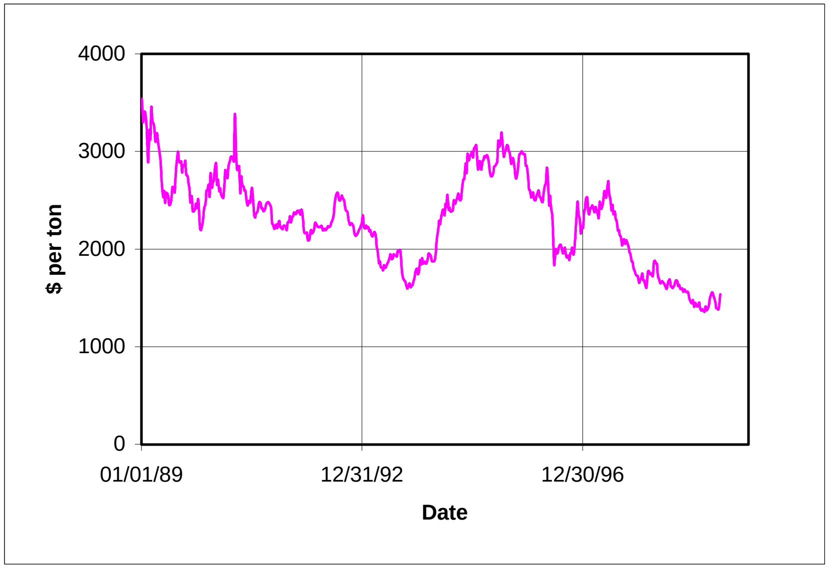

Fig. 1: Weekly Spot Price for Copper

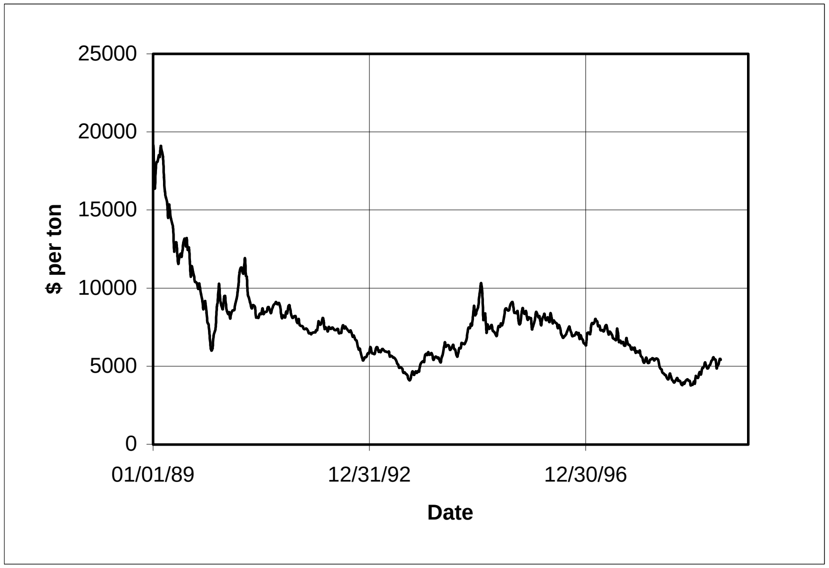

Fig. 2: Weekly Spot Price for Nickel

Fig. 3: Weekly Spot Price for Gold

Fig. 4: Weekly Spot Price for WTI