

The Growth-Inequality Relationship in A Model with Discrete Occupational Choice and Redistributive Tax¹

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Abstract

Growth-inequality relationship is reexamined in a neo-classical growth model with discrete occupational choice and incomplete markets for human capital. In our model a fiscal redistributive tax program directly impacts the steady state distribution of human capital by influencing the occupational choice of the agents. Growth and income-inequality are endogenously driven by the evolution of the proportion of innovators in the economy and the redistributive tax rate. The correlation between growth and factor shares depends crucially on the interaction between the redistributive tax policy and the initial distribution of human capital. The model predicts that the growth rate and income-inequality are positively related across countries with different redistributive tax regimes. On the other hand, countries with different redistributive tax regimes as well as different initial distribution of human capital do not show any robust correlation between growth and inequality. The correlation depends on the skill intensity of the production technology and the degree of institutional barriers to knowledge diffusion. The lesson for the cross-country growth-inequality regression is that it is necessary to adequately control for the differences in initial distribution of human capital, and technology, as well as differences in redistributive tax regimes.

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1. Introduction

Whether inequality retards or promotes growth is a long-standing theoretical and empirical issue. Recent pioneering work of Torsten Persson and Guido Tabellini (1994) report a strong negative relationship between growth and inequality. They rationalize this relationship as a politico-economic equilibrium outcome of a model economy. Although Persson and Tabellini's theoretical model provides important insight about the link between distribution and growth, their empirical cross-country growth-inequality regression has sparked further debate. Kristin Forbes (2000) argues that there is an omitted variable bias in Persson and Tabellini's cross country regressions. After correcting for this bias, she finds a positive association between growth and inequality. In fact, finding an empirical relation between growth and inequality over a cross section of countries is problematic when countries differ widely in terms of structural characteristics. Until now there is no clear consensus about the empirical relationship between growth and inequality.

Even at a theoretical level, economists differ about the relationship between growth and inequality. Woojn Lee and John. E. Roemer (1998) report in terms of their model that the relationship between private investment and inequality does not necessarily show a monotonic negative relationship. They argue, in terms of a theoretical model, that the relationship between inequality and private investment is not necessarily monotonic. Chien Chou and Gabriel Talmain (1996) consider the effect of distribution of wealth on endogenous rate of growth. They argue that the relationship between inequality and growth depends on the curvature of the labor Engel curve. Peter Orazem and Leigh Testfation (1997) develop a model where income redistribution can result in sub-optimal choices that offset the beneficial effects of income transfer. However, because of the assumption of diminishing returns, their model has steady state in level and thus does not provide insight about long-run growth.

Most of these growth models dealing with fiscal redistributive policy, however, focus mainly on the effect of redistributive policy on either growth or inequality. Little efforts have been directed towards understanding how a redistributive tax policy could interact with the growth-inequality relationship. Understanding the endogenous dynamics of

growth, distribution and fiscal policy is a major challenge facing the growth theorists nowadays, as emphasized by the following quote from Persson and Tabellini (1992):

"...Future theoretical research should try to study the joint dynamics of growth, income distribution and policy. "

A few recent papers take this issue of endogeneity of growth-inequality relationship seriously. For example, Mervyn King (1992) attempts a synthesis of the models developed by Persson and Tabellini (1994) and Phillippe Aghion and Patrick Bolton (1992). While Aghion and Bolton (1992) focus on the link from growth to distribution, Persson and Tabellini (1994) stress more on the link from distribution to growth. However, none of these models explicitly focus on the two-way link between growth and income-inequality in terms of a redistributive fiscal policy. The issue is important because the nature of the relationship between growth and inequality might depend in an important way on the type of redistributive policy chosen by the government.

In this paper we explore the link between income distribution and growth when the fiscal authority is involved in redistributive taxation of capital income. We propose a theoretical framework here to deal with this issue. As in Glenn Loury (1981), we invoke the extreme form of capital market incompleteness in the sense that the credit market is absent. Human capital is the only device for consumption smoothing and the sole engine of growth. This incompleteness of the credit market is crucial in preserving the dynastic heterogeneity in our model. Although individuals are ex-ante identical in terms of preference over dated consumption, due to past investment in human capital, they differ in terms of the endowment of human capital which evolves endogenously in the model. As a result, income inequality arises as an equilibrium outcome and it persists across generations. This happens without bringing any element of uncertainty in the production technology as in Abhijit Banerjee and Andrew Newman (1991).²

Our model also involves the externalities associated with human capital similar to Robert E. Lucas, Jr (1988) and abstract knowledge as in Paul M. Romer (1990). However, unlike Romer (1990), the total factor productivity in our model depends on the intensity of innovative activity represented by the proportion of innovators (who we call

managers) in the economy. The fraction of people choosing a managerial occupation is the driving force behind innovation in our model.³

Why is the proportion of innovators an important component of the total factor productivity? It is well known that growth is significantly determined by total factor productivity. Recent literature on endogenous growth models following Lucas (1988) and Romer (1990) stress the importance of non-rival knowledge in determining total factor productivity. In a recent paper, Edward Prescott (1998) argues that it is the extent of non-rival knowledge that a country exploits rather than the available stock of knowledge itself that accounts for the cross country disparity in income.⁴ How much available knowledge would be exploited in the economy depends on the intensity of innovations, which is traced back in the model to the proportion of innovators or managers in the economy.

In our model, a redistributive capital income taxation impacts the long-run growth through two distinct channels. First, it directly influences the steady state rate of investment by impacting the post tax return to capital. Second, it indirectly influences growth by impacting a fundamental state variable in our model, which is the occupational distribution. A change in the redistributive tax rate, by altering the steady state proportion of managers, may influence growth as well as the post-tax shares of human capital in output. The relationship between growth and income inequality is thus endogenous in our model. It is driven in this model by the interaction between the initial distribution of human capital and the redistributive tax rate chosen by the government.

The model generates various steady state relationships between growth and income inequality depending on the interaction between initial distribution of human capital and the redistributive tax rate across countries. If countries differ widely in terms of their fiscal redistributive taxation, the model predicts a robust positive association between

² In a similar spirit, Scott Freeman (1996) develops a discrete occupational choice model and demonstrates that income inequality persists across generations. Freeman (1996), however, does not address the issue of growth-inequality relationship in the context of redistributive taxation.

³ Our model is an extension of Debasis Bandyopadhyay (1993). In our model the educated elite choose a managerial occupation and undertake all the investment in human capital. The motivation for including the proportion of managers in the total factor productivity function stems from the study of Bandyopadhyay (1997) who finds that the proportion of educated people significantly explains cross country disparity in growth rates.

⁴ In a similar vein Oded Galor and Danile Tsiddon (1997) highlight the importance of high-ability individuals in determining economic growth.

growth and income inequality as in Forbes (2000). A higher redistributive capital income tax rate lowers the steady state growth rate through the usual distortionary effect of driving a wedge between the marginal product of capital and return to capital. As the redistributive tax rate increases, the steady state post tax factor share goes in favor of the worker thus making growth and inequality correlate positively.

On the other hand, if we compare countries that are heterogeneous in initial distribution of skills, correlation between growth and income inequality does not show any robust pattern. A calibration exercise with model's steady state property indicates that the correlation between growth and inequality depends critically on technology parameters involving the skill intensity and the degree of institutional barriers to knowledge.

The model thus suggests that the cross-country correlation between growth and income inequality depends on the complex interaction between the tax policy environment, initial distribution of human capital and the economy-wide technology. Different countries may be in different steady states because of different redistributive tax regimes, different initial history of human capital distribution and different technology. The choice of the sample of countries may, therefore, make a crucial difference in determining the correlation between growth and income inequality. An econometrician, without controlling for these structural differences across countries, may bias a cross-country regression of growth on income inequality.

The rest of the paper is organized as follows. In the following section, we lay out the model and its comparative statics. Section 3 derives the steady state properties of the model. Section 4 examines the growth-inequality relationship under alternative environments regarding tax policy and initial distribution of human capital and reports some calibration results regarding the growth-inequality correlation. Section 5 ends with concluding comments.

2. The Model

Consider an environment with variable human capital, labor, and a single perishable consumption good. An agent lives two periods, one as a child being attached to an adult and one as an adult when she receives a child of her own. There is a continuum of

dynasties with measure one and at each date t , a typical dynasty consists of an adult and a child. The adult has one unit of labor and h units of human capital. She earns her income by choosing between the occupations of manager and worker and then divides her income between current consumption and investment in her child's education. Investment in human capital is the only means of transferring consumption in our model. We assume incomplete markets for human capital because human capital cannot be used as collateral for loans and there is no separate tangible capital in the economy. This rules out a viable credit market in the model, which is crucial in terms of preserving dynastic heterogeneity.

Preferences display intergenerational altruism, and so the adult maximizes the present discounted value of consumption of her dynasty. Dynasties differ only in terms of the adult's endowment of human capital at date 0. At date t , \mathbf{Y}_t denotes the cumulative distribution of human capital among the date t adults. The history specifies the initial distribution \mathbf{Y}_0 . For simplicity, we consider a two-point distribution for \mathbf{Y}_0 , where h can take two possible values, 0 and h_0 , such that

$$(1) \quad \mathbf{Y}_0(0) = 1-m = \mathbf{Y}_0(h_0).$$

In other words, initially a fraction m of adults are skilled adults possessing nonzero units of human capital and a fraction $1-m$ of adults have no skill, or equivalently, have zero units of human capital.

Groups of adults carry out production. Each group consists of a manager and one or more workers. The output q of a group depends on the manager's human capital h , the number n^d of workers she employs and the total factor productivity level $A > 0$ such that $q = Ah^{1-a}(n^d)^a$, where $0 < a < 1$ measures the output elasticity of a worker. We assume that the total factor productivity A depends, following Lucas (1988), on a knowledge spillover process that increases with the quality of labor, measured by the economy's average human capital stock H . Following Romer (1990), we assume that the total factor productivity depends as well on the stock A_0 of non-rival knowledge.

In contrast with Lucas (1988) and Romer (1990), total factor productivity A also depends on the intensity of the innovative activity in our model. It is proxied by the proportion of managers, m . Two countries may have the same average human capital but experience different growth patterns because of different m . The knowledge spillover increases with the intensity of innovative activities in the economy, which is measured by the proportion m of adults who are managers. In particular, we assume that $A = A_0 m^\theta H^b$, where $b > 0$ is a parameter measuring the degree of externality and $\theta \geq 0$ is a parameter.

A few additional clarifications about the specification of total factor productivity involving the parameter θ are in order. Notice that for a given θ , a higher intensity of innovations (in the form of a higher proportion of managers, m) enhances the total factor productivity A . On the other hand, for a given m , a higher value of θ lowers the total factor productivity. The parameter θ thus measures the degree of institutional barriers to spillover of knowledge. If θ equals 0, there is no such barrier which means 100% of the stock of knowledge, H is exploited⁵. In such a case, the production function reduces to the Lucas-Romer technology. To summarize, at each date $t \geq 0$ the output q_t of a manager is given by

$$(2) \quad q(h, n_t^d; H_m, m_t) = A_0 m_t^\theta H_t^a h^{1-a} (n_t^d)^a, \quad t=0, 1, 2, \dots$$

Since our central interest in this paper is to understand the interaction between tax policy, growth and distribution, we assume that $b=a$. This assumption makes the aggregate production function linear in the reproducible input H , thus implying endogenous growth.⁶ Observe that there are m firms per capita in this economy, each being run by one manager. On average, there are $(1-m)/m$ workers per firm. Note that $\theta > 0$ implies that the total factor productivity in the economy decreases as number of workers per

⁵ These institutional barriers may be of the form of patent laws that prevent instant diffusion of new technology or knowledge from one firm to another.

manager increases. At any given point in time, manager's human capital is given. Also, an increase in the number of workers per manager gives rise to overcrowding and results in diseconomies of scale.

At each date $t \geq 0$ given the wage rate w_t , and the two external factors H_{mt} and m_t , a manager with h units of human capital employs n_t^d number of workers so as to

$$(3) \quad \underset{n^d > 0}{\text{Maximize}} \left| q(H_t, m_t, n_t^d, h) - w_t n_t^d \right| \quad t=0, 1, 2, \dots$$

The first order condition of (3) yields $w_t = aA_0 H_t^a m_t^\theta h^{1-a} (n_t^d)^{a-1}$, or, equivalently, the optimal number $n_t^d(h)$ of workers employed by a manager with h units of human capital is

$$(4) \quad n_t^d(h) = \left(\frac{a m_t^\theta H_t^a}{w_t} \right)^{\frac{1}{1-a}} h, \quad t=0, 1, 2, \dots$$

By (3) and (4), at each date t , the indirect profit of a manager is proportional to her human capital stock h and is given by $r_t h$, where,

$$(5) \quad r_t = (1-a) A_0 m_t^\theta H_t^a (a A_0 m_t^\theta H_t^a / w_t)^{a/(1-a)}, \quad t=0, 1, 2, \dots$$

The Government

The government in this economy undertakes a redistributive tax-subsidy program. At each date, the government levies a constant proportional tax (τ_t) on the income of the managers and lump sum transfer of z_t units to each worker.⁷ In addition, the managers

⁶ To see this, note that in a steady state the distribution of human capital coincides with the initial distribution laid out in (1). This means the workers stay as workers and managers stay as managers. In a steady state equilibrium described in section 4, workers do not invest in human capital meaning $H_t = H_{mt}$, which means the production function becomes linear in H making the production function "Ak" type à la Rebelo (1991). The details of the steady state property of the model are discussed in Section 3.

⁷ In the steady state, the level of tax on workers grows over time at a balanced rate and hence, in the steady state, a lump-sum tax is equivalent to taxing worker's income proportionally.

receive as an educational subsidy, or equivalently, as a reimbursement for their educational expenses incurred by their parents. The amount of subsidy for acquired education is, however, in constant proportion (t_2) to the manager's income level.⁸ The budget constraint of the government can thus be written as:

$$(5) \quad t_2 r_t H_{mt} + (1 - m_t) z_t = t_1 r_t H_{mt}$$

Defining $t = t_1 - t_2$ as the effective tax rate for the managers such that the budget constraint (6) can be rewritten as:

$$(6) \quad z_t = \frac{t r_t H_{mt}}{(1 - m_t)}$$

Note that in principle we do not impose any restriction on the sign of the effective redistributive tax rate t , meaning t may as well be negative, in which case we have an effective investment tax credit or an educational subsidy financed by taxing worker's income. The sign of t depends on the equilibrium property of the model.

The Breakeven Skill Level

At each date t , x_t denotes the level of *breakeven skill* such that an adult with x_t units of human capital earns an equal amount either as a manager or as a worker. By (5), x_t satisfies

$$(8) \quad \bar{w}_t = \bar{r}_t x_t, \quad t=0, 1, 2, \dots$$

where $\bar{w}_t = w_t + z_t$ is the post subsidy wage and $\bar{r}_t = (1 - t)r_t$ is the after-tax rate of return on human capital. The adult's occupation $n_t(\cdot)$ is an indicator function such that if

⁸ Notice that in the present setting, the educational subsidy is proportional to the level of rental income. The implication is that adults with higher rental income from human capital receive greater subsidy. The net subsidy determined by the fraction t is financed by wage income taxation. Rogerson and Fernandez (1995) find that a similar pattern of financing educational subsidy is an equilibrium outcome in a median voter setting.

she is a worker, $n_t(\cdot) = 1$, otherwise, if she is a manager, $n_t(\cdot) = 0$. At each date $t \geq 0$, her occupational choice $n_t(\cdot)$ and the resulting income $y_t(\cdot)$ as functions $h \geq 0$ are

$$(9) \quad \begin{aligned} n_t(h, \mathbf{t}) &= 1, \text{ if } h < x_t; & n_t(h, \mathbf{t}) &= 0 \text{ if } h > x_t; \\ n_t(h, \mathbf{t}) &= 1 \text{ or } 0, & \text{ if } h = x_t, \end{aligned}$$

$$(10) \quad y_t(h, \mathbf{t}) = n_t(h, \mathbf{t}) \cdot \bar{w}_t + (1 - n_t(h, \mathbf{t})) \cdot \bar{r}_t h.$$

Figure 1 illustrates how the breakeven skill level divides the adults into two occupational groups, workers and managers, according to their individual stock of human capital.

<Figure 1 comes here>

An adult's human capital h_{t+1} at date $t+1$ is positively related to her parent's human capital h_t and the investment s_t in her schooling made by her parent at date t . In particular,

$$(11) \quad h_{t+1} = (1 - \mathbf{d})h_t + s_t, \quad 0 < \mathbf{d} < 1 \quad t=0, 1, 2, ..$$

The above formulation presumes a positive externality $\mathbf{d} < 1$ associated with family upbringing in the tradition of Benabou (1996). It also assumes $\mathbf{d} > 0$ such that without a positive investment in schooling the current generation can transfer only a fraction $(1 - \mathbf{d})$ of existing knowledge to the future generation. Consequently, knowledge is maintained or accumulated only if a generation acquires them through investment in schooling. This feature is similar to Mankiw et al. (1992).

Following Barro (1974) we assume intergenerational altruism. At each date t , the utility v_t of the adult is a function of her family's consumption c_t and her child's utility v_{t+1} as a grown-up adult. In other words,

$$(12) \quad v_t = V(c_t, v_{t+1}) = u(c_t) + \mathbf{b}v_{t+1}, \quad t=0, 1, 2, \dots$$

We assume that $u(\cdot)$ is strictly concave, bounded above, $u(0)=-\mathbf{Y}$, $u'(0)=\mathbf{Y}$, $0 < \mathbf{b} < 1$, such

$$\text{that } v_0 = \sum_{t=0}^{\infty} \mathbf{b}^t u(c_t).$$

The adult with h units of human capital chooses a suitable occupation $n_t(h)$ following (9) and divides her income $y_t(h)$, given by (10), between consumption c_t and investment s_t such that

$$(13) \quad c_t + s_t \leq y_t(h, \mathbf{t}) \quad t=0, 1, 2, \dots$$

At $t = 0$ the optimization problem of the adult with $h \geq 0$ units of human capital is to choose a sequence $\{c_t(h, \mathbf{t}) \geq 0, s_t(h, \mathbf{t}) \geq 0, n_t(h, \mathbf{t}) \in \{0, 1\}\}_{t=0,1,2,\dots}$, so as to

$$(14) \quad \text{Maximize } \sum_{t=0}^{\infty} \mathbf{b}^t u(c_t) \text{ subject to (9)-(11) and (13). } \quad t=0, 1, 2, \dots$$

Characteristics of Equilibrium

The set of sequences $\{(c_t(h, \mathbf{t}), s_t(h, \mathbf{t}), n_t(h, \mathbf{t}), n_t^d(h, \mathbf{t}) : h \geq 0; x_t, r_t, m_t, H_{mt}, w_t, \mathbf{t}\}_{t=0,1,2,\dots}$ and the initial distribution \mathbf{Y}_0 describe the model's *equilibrium* such that at each $t \geq 0$, the labor demand $n_t^d(\cdot)$ satisfies (3), the implicit rental price r_t of human capital satisfies (5), the breakeven skill x_t satisfies (8), the sequence $\{(c_t(h, \mathbf{t}), s_t(h, \mathbf{t}), n_t(h, \mathbf{t})\}_{t=0,1,2,\dots}$ satisfies (14), and $\{H_{mt}, m_t\}_{t \geq 0}$ coincides with the same generated by the optimal sequence $\{s_t(h, \mathbf{t}), n_t(h, \mathbf{t})\}_{t=0,1,2,\dots}$, such that

$$(15) \quad m_t = \int_{\{h:n_t(h,\mathbf{t})=0\}} d\mathbf{Y}_t(h, \mathbf{t}),$$

$$(16) \quad H_{m,t+1} = (1-d) \int h d\Psi_t(h,t) + \int s_t(h,t) d\Psi_t(h,t), \quad H_0 = \int h d\Psi_0(h,t),$$

and the labor market clears such that at each date $t=0, 1, 2, \dots$,

$$(17) \quad \int_{\{h:n_t(h,t)=0\}} n_t^d(h, w_t; H_{mt}, m_t) dY_t(h,t) = 1 - m_t$$

Notice that the labor demand function does not depend on the redistributive tax rate because the tax is based on indirect profit and not on the output of the firms. On the other hand, the labor supply or equivalently occupational choice as characterized in (8) and (9) depends on the after tax wage rate. Nevertheless, the market clearing wage does not depend on t because the profit maximizing firm equates the before tax real wage to the marginal product of labor. Figure 2 illustrates the labor market equilibrium in a situation where subsidy z_t is negative. Note that because of the discrete occupational choice, the labor supply curve (called L^s schedule) is a step function. At $\bar{w}_t=0$, the breakeven skill, x_t equals zero which means L^s equals zero, because everybody chooses to be a manager. At \bar{w}_t^* which is the post tax wage when $x_t=h_0$, an adult is indifferent between the two occupations. This explains horizontal segment BC of the labor supply function over the range $1-m \leq L^s \leq 1$. The labor market equilibrium condition (17) holds at the point where the MPL schedule intersects the labor supply schedule corresponding to $L^s=1-m_t$ as shown in (17).

<Figure 2 comes here>

The goods market clears such that at each date $t=0, 1, 2, \dots$,

$$(18) \quad \int_{h \geq 0} (c_t(h,t) + s_t(h,t)) d\Psi_t(h,t) = \int_{\{h:n_t(h,t)=0\}} q_t(h, n_t^d(h); H_{mt}, m_t) dY_t(h,t).$$

The above definition yields a sequence $\{m_t, H_{mt}\}_{t \geq 0}$ of state variables that characterize the equilibrium, where H_{mt} denotes the total human capital of managers such that

$$(19) \quad H_{mt} = \int_{\{h:n_t(h,t)=0\}} h d\Psi_t(h, \mathbf{t}) \quad \text{for } t=0, 1, 2, \dots$$

By (4), (15) and (17), the equilibrium wage rate w_t is given by

$$(20) \quad w_t = aA_0 m_t^\theta H_t (1 - m_t)^{a-1} \quad t=0, 1, 2, \dots$$

By (20) the wage rate of workers increases with the economy's average human capital, H_t and the relative proportion of managers, m_t . The former positively influences the productivity of workers through an external effect, while the latter augments the relative scarcity of workers. By (5) and (20) the implicit rental r_t price of human capital is given by

$$(21) \quad r_t = (1 - a)A_0 m_t^q (1 - m_t)^a \quad t=0, 1, 2, \dots$$

By (21) the price r_t of human capital and hence the gross rate of return $r_t + 1 - \mathbf{d}$ from the investment in schooling is an inverted- U shaped function of m_t . A new manager generates an external benefit to other managers with her innovative activities. She, however, adds to the relative scarcity of workers and hence boosts the wage rate or, equivalently, the cost of production for all managers. For a low value of m_t , additional benefits are higher than additional costs and, therefore, returns to schooling increases with additional managers in the economy. A high value of m_t , however, turns the balance in the opposite direction. At $m_t = \mathbf{q}/(\mathbf{q} + a)$, the return to schooling reaches its maximum. By (8), (20) and (21) we obtain the following expression for the breakeven skill.

$$(22) \quad x_t = \frac{aH_t + t(1-a)(1-m_t)H_{mt}^{1-a}H_t^a}{(1-a)(1-m_t)(1-t)} \quad t=0, 1, 2, \dots$$

3. Steady State

Our primary concern in this paper is to understand the steady state relationship between growth and income inequality. For this reason, we shall abstract from any issue relating to the transitional dynamics of the model and only characterize the steady state equilibrium of the model's environment. We shall restrict our attention to a class of steady state equilibria in which the distribution of human capital is time invariant meaning m_t in (15) remains constant over time. In such a steady state, workers and managers do not switch their respective occupations. At any date t , workers with zero human capital thus stay as workers and managers with h_t units of human capital continue as managers by investing in their children's education. Because of manager's investment in human capital growth occurs endogenously in this environment preserving two distinct dynasties: workers and managers. By (19) in such a steady state, the average human capital grows at a constant balanced growth rate (call it γ) starting from its initial state mh_0 .

In order to characterize such a dynastic steady state, it is important to understand what incentive compatibility conditions will guarantee such a steady state equilibrium. To answer this question we need to analyze the property of the optimal investment function of the managers in steady state. We have the following proposition:

Proposition 1: *If the utility function is of the constant relative risk aversion class meaning $U(C_t) = C_t^{1-\tau}/(1-\tau)$, in the steady state, a manager invests a constant fraction $i(m, \tau)$ of her human capital in her child's education such that $\delta \leq i(m, \tau) < (1-\tau)r(m)$. The steady state growth rate is given by:*

$$(23) \quad g(m, \tau) = i(m, \tau) - d$$

where

$$(24) \quad i(m, \mathbf{t}) = (\mathbf{b}[(1 - \mathbf{t})r(m) + 1 - \mathbf{d}])^{1/\mathbf{a}} - 1 + \mathbf{d}$$

Proof: Appendix.

Notice that the balanced growth rate \mathbf{g} is directly related to the steady state rate of investment, which in turn depends positively on the after tax return on schooling. Since return on schooling reaches its maximum at $m^* = \mathbf{q}/(\mathbf{q} + \mathbf{a})$, for a given \mathbf{t} , the balanced growth rate \mathbf{g} also attains its maximum at the same m^* . In Figure 3, we have illustrated this by drawing $i(m, \mathbf{t})$ and \mathbf{d} schedules. The steady state growth rate is the vertical difference between $i(m, \mathbf{t})$ and \mathbf{d} , which reaches its maximum at m^* .

<Figure 3 comes here>

Note that by there are two roots $m_L^1(\mathbf{t}) > 0$ and $m_L^2(\mathbf{t}) > 0$ that solve the equation $i(m, \mathbf{t}) = \mathbf{d}$ for any given $0 < \mathbf{t} < 1$. It follows, therefore, from Figure 3 that we can ensure a non-negative balanced growth state in this model if and only if $m_L^1(\mathbf{t}) \leq m \leq m_L^2(\mathbf{t})$.

We are now ready to characterize the incentive compatibility condition for the managers to invest in human capital. Managers find it incentive compatible to invest in human capital if the indirect lifetime utility for being a manger exceeds the indirect lifetime utility for being a worker. In other words, the incentive compatibility condition for being a manger is:

$$(25) \quad \sum_{t=0}^{\infty} \mathbf{b}^t U((1 - \mathbf{t})r(m) - i(m, \mathbf{t}))h_t \geq \sum_{t=0}^{\infty} \mathbf{b}^t U(w_t + z_t)$$

Since in the steady state there is balanced growth, meaning h_t , w_t and z_t all grow at the same rate, it is straightforward to verify that (25) holds if

$$(26) \quad h_0 \geq h^*(m, \mathbf{t}, h_0)$$

where

$$(27) \quad h^*(m, \mathbf{t}, h_0) = \frac{w(m, h_0) + \mathbf{tr}(m) \left(\frac{m}{1-m} \right) h_0}{(1-\mathbf{t})r(m) - i(m, \mathbf{t})}.$$

and $w(m, h_0)$ is the market clearing wage rate at date 0. Given an initial distribution of human capital, parameterized by (h_0, m) , and a tax rate \mathbf{t} , h^* defines the minimum level of human capital that a manager must possess to attain the same consumption as the worker at the steady state. In other words, if the initial human capital h_0 is just equal to h^* , all the managers are marginal managers in the sense that they are indifferent as to whether they will invest in human capital or not.

Our next task is to characterize the range of m over which the equilibrium describes a steady state, or equivalently, a balanced growth path. In others words, we need to know the admissible range of m over which neither managers nor workers switch their occupations. To obtain such an admissible range, it is necessary to characterize the properties of $h^*(m, \mathbf{t}, h_0)$. Using (27) it is straightforward to verify that $h^*(m, \mathbf{t}, h_0)$ is linear and homogenous in h_0 . Therefore,

$$(28) \quad h^*(m, \mathbf{t}, h_0) = \mathbf{x}(m, \mathbf{t})h_0$$

where

$$(29) \quad \mathbf{x}(m, \mathbf{t}) = \frac{\frac{am^{1+q} A_0}{(1-m)^{1-a}} + \frac{\mathbf{tr}(m)m}{1-m}}{(1-\mathbf{t})r(m) - i(m, \mathbf{t})}$$

Using (26) through (29), it is straightforward to check the following lemma:

Lemma 1: *An adult manager will invest in human capital if*

$$(30) \quad \mathbf{x}(m, \mathbf{t}) \leq 1$$

Next we consider the incentive compatibility condition for workers not to invest in human capital. Along a balanced growth path, at date t an adult worker may become a manager if and only if she invests h_t units in human capital. A worker will not undertake such an investment decision if the opportunity cost exceeds the future return. This means that

$$(31) \quad U'(w_t + z_t h_t) > \mathbf{b} U'(C_{t+1}) ((1-\mathbf{t})r_m + 1-\mathbf{d}).$$

Next we have the following lemma:

Lemma 2: *If the utility function is of the constant relative risk aversion class meaning $U(C_t) = C_t^{1-\mathbf{I}} / (1-\mathbf{I})$, inequality (30) is sufficient to guarantee (31).*

Proof: Appendix

Proposition 1 together with Lemmas 1 and 2 have the following implication. If we consider economies experiencing a balanced growth with $\mathbf{g} > 0$, the rate of investment, $i(m, \mathbf{t})$, must be positive. This requires that the managers must find it incentive compatible to invest a positive amount in the steady state. In such a case, from Lemma 2, it follows that workers do not find it incentive compatible to invest in education. The immediate implication is, therefore, that along a steady state path only managers invest in education and workers do not. Therefore, in the steady state $H_t = H_m$.⁹

We are now ready to characterize the admissible range of m over which the equilibrium is a steady state. Notice that if the inequality (30) holds, it is incentive compatible for the managers to invest in human capital and stay as managers and workers not to invest in human capital and thus stay as workers. Inequality (30) thus defines the range of m over which a steady state equilibrium exists.

⁹ Plugging this steady state condition, $H_t = H_m$ into (22) comparing with (27), it is straightforward to verify that $h^* > x$ if the investment rate $i(m, \mathbf{t}) > 0$, which will necessarily be the case in a steady state with non-negative growth as per proposition 2.

For analytical tractability, henceforth we consider a logarithmic utility function for which $I=1$. In case of a logarithmic utility function, $\mathbf{x}(m, \mathbf{t})$ in (30) can be written as:

$$(32) \quad \mathbf{x}(m, \mathbf{t}) = \frac{\left[\frac{am}{(1-a)(1-m)} + \frac{tm}{1-m} \right]}{\left[(1-b)(1-t) + (1-b)(1-d)r(m)^{-1} \right]}$$

We have now the following lemma characterizing the property of $\mathbf{x}(m, \mathbf{t})$:

Lemma 3: *For a given \mathbf{t} , $\mathbf{x}(m, \mathbf{t})$ is monotonically increasing in m .*

Proof: Appendix.

Our next task is to characterize the restrictions on the space of the initial distributions of human capital, $\mathbf{Y}_0(0)$ defined in (1) that also makes it a steady state distribution. We have the following lemma:

Lemma 4: *For a sufficiently high value of the discount factor \mathbf{b} and for any given \mathbf{t} , there exists a unique $m_c(\mathbf{t})$ as a function of \mathbf{t} such that for all $m_L^1 < m \leq m_c(\mathbf{t})$, the equilibrium with the initial distribution, \mathbf{Y}_0 with $\mathbf{Y}_0(0) = 1 - m$, describes a balanced growth state with a non-negative rate of growth.*

Proof: Appendix.

Figure 4 illustrates the restrictions set forth in Lemma 4 on the space of the initial distribution of human capital for which it is also a steady state distribution. Notice that $m_c(\mathbf{t})$ is the level of m at which $\mathbf{x}(m, \mathbf{t})$ schedule intersects the unit line once because of the monotonicity of the $\mathbf{x}(m, \mathbf{t})$ with respect to m as shown in Lemma 3. The set of initial distributions of human capital, \mathbf{Y}_0 which satisfies the steady state restrictions is, therefore, given by the set, $m_L^1 < m \leq m_c(\mathbf{t})$. In other words, under these restrictions on the space of m , the steady state equilibrium preserves the initial inequality in the distribution of human capital. In this sense, the steady state equilibrium is path dependent.

<Figure 4 comes here>

Next, we have the following proposition regarding the properties of the steady state proportion of managers:

Proposition 2:: For a given \mathbf{t} , in an economy with a non-negative steady state growth, either $m_L^1(\tau) \leq m < \min\{m_c(\mathbf{t}), m_L^2(\mathbf{t})\}$ or $m_L^1(\tau) \leq m = \min\{m_c(\mathbf{t}), m_L^2(\mathbf{t})\}$.

Proof: Appendix.

Notice that the set of steady state proportion of managers has an upper bound $m_c(\mathbf{t})$ which is a function of the redistributive tax rate \mathbf{t} . By changing \mathbf{t} the fiscal authority can alter the set of admissible proportions of managers in the steady state and thus alter the growth and steady state factor shares. In order to analyze the growth-inequality relationship, it is therefore, necessary to analyze the comparative static property of $m_c(\tau)$. We have the following lemma:

Lemma 5: $\frac{\partial m_c(\tau)}{\partial \tau} < 0$.

Proof: Appendix.

4. Growth-Inequality Relationship

Growth and income inequality are endogenous in the present setting and are determined by the interaction between two fundamental state variables, the redistributive tax rate (\mathbf{t}), and the distribution of human capital parameterized by the proportion of managers (m). A change in the redistributive tax rate (\mathbf{t}) has a direct effect on the steady state growth rate and factor share via its effect on the post tax return to capital. On the other hand, it also has an indirect effect on growth and income inequality via its effect on the steady state occupational distribution, as demonstrated in Proposition 2 and Lemma 5.

To see it clearly, denote the post tax factor share as $w(m)$, which is the measure of income inequality in the present context. Note that $w(m)$ is the steady state ratio of the income of managers to the income of workers. In other words,

$$(33) \quad w(m, \mathbf{t}) = \frac{(1-\mathbf{t})r(m)h_0}{w_0 + z_0}$$

Notice that $\omega(m, \mathbf{t})$ is nothing but the relative income of the managers, which can as well be interpreted as the post tax proportional skill premium for the managers. One can easily verify that the incentive compatibility condition (25) for positive investment is sufficient to ensure that $\omega(m, \mathbf{t})$ in (33) exceeds unity.¹⁰

Next plugging (7), (20), (21) one obtains the following expression for the steady state factor share:

$$(34) \quad w(m, \mathbf{t}) = \frac{(1-\mathbf{t})(1-a)(1-m)}{(a+\mathbf{t}(1-a))m}$$

Based on this factor share the Gini coefficient of the income distribution (call it Gini) is given by:

$$(35) \quad Gini = (1-a)(1-\mathbf{t}) - m$$

The appendix provides a derivation of the Gini coefficient. Notice that the model's income Gini coefficient is state dependent. It depends on the country's initial distribution of human capital parameterized by m and the redistributive tax regime measured by \mathbf{t} .¹¹ Everything else remaining the same, a country with a higher initial inequality in the distribution of human capital (meaning lower m) will end up with greater income

¹⁰ To check this note that (25) and (26) require that $[(1-\mathbf{t})r(m)-i(m, \mathbf{t})]h_0/[w_0+z_0] \geq 1$. If $i(m, \mathbf{t}) > 0$, it means $\omega(m, \mathbf{t}) > 1$

¹¹ It is straightforward to verify that as long as $w(m, \mathbf{t}) > 1$, the Gini coefficient is positive.

inequality. Steady state growth (equation 23) and inequality (equation 35) experiences differ across countries because of difference in m and t .

Two Special Cases

(i) *Case of Positive Growth-Gini correlation:* We next investigate how, everything else equal, only a difference in t across countries may give rise to a positive cross-country correlation between growth and inequality. As per proposition 2 the economy has multiple steady state equilibria depending on m and t . Although various steady state cases are possible, take a simple case when $m_L^1(\tau) \leq m < m_c(t) < m_L^2(t)$. In this case, a change in the redistributive tax rate only alters the upper bound of the set of admissible steady state distribution of managers. It does not have any effect on the steady state proportion of managers, m . Although t is neutral in its effect on m , it has a direct effect on the steady state growth rate and income inequality ratios. One may easily verify this by observing that the growth rate, $g(m, t)$ in (23) declines and the Gini coefficient in (35) declines as well when t increases. Thus, growth and inequality correlate positively across countries differing in terms of t .¹²

One thus gets a positive correlation between growth and income inequality as found by Forbes (2000). Given m , the correlation between growth and inequality is positive as one expects in any neoclassical model such as Judd (1985), Chamely (1986) or Atkinson, Chari and Kehoe (1999) for the following reason: The workers consume their income and, therefore, do not accumulate any capital input while the managers do. By taxing the workers, to finance an investment tax credit for the managers, this would increase inequality and the rate of growth. Thus suggesting a positive correlation between growth and inequality.

¹² Two other cases are possible: (i) $m_L^1(\tau) \leq m = m_c(t) < m^*$, (ii) $m_L^1(\tau) \leq m^* < m = m_c(t)$. From Lemma 5, we know that when t is higher $m_c(t)$ is lower. The growth rate unambiguously falls in case (i) when t is higher although the effect on the Gini coefficient in (35) is ambiguous because of opposing effects of m and t . In case (ii), the effect on growth is also ambiguous because there are two opposing effects on growth when t increases. We have extensively simulated the growth and inequality for wide range of parameter values and find that the growth and Gini are always positively correlated in both these cases. The details of these results are available from the authors upon request.

(ii) *Case of Negative Growth-Gini Correlation*: Next consider another special case where the countries have the same redistributive tax rate, \mathbf{t} but differ among themselves in terms of the initial distribution of human capital parameterized by m . Using proposition 2, consider the case where $m_L^1(\tau) \leq m < m_c(\mathbf{t}) < m^*$. Here it is straightforward to verify from (23) that given \mathbf{t} , as m increases the steady state growth rate increases. On the other hand, from (35) it follows that income Gini coefficient decreases because a higher m raises the relative scarcity of worker's and thus lowers the proportional skill premium $w(m, \mathbf{t})$ in (34). The growth rate and the inequality ratio thus negatively correlate as in Persson and Tabellini (1994) for countries with the same tax regime, \mathbf{t} but with different distribution of human capital, m .

These theoretical special cases highlight the role of initial inequality in the distribution of human capital and the tax policy in determining the nature of the cross-country correlation between growth and income-inequality. In reality, countries indeed differ widely in terms of fiscal regimes, the initial distribution of human capital as well as the production technology. Enrique Mendoza, Assaf Razin and Linda L. Tesar (1994) find that even major OECD countries with similar infrastructure differ widely in terms of the effective tax rates on capital income. Robert J. Barro and J. Lee (1985) document that the proportion of educated people differs widely across countries. A natural question then arises: what does the model predict about the growth-inequality relationship for the countries that may structurally differ in terms of these three characteristics. To answer this question effectively, we resort to a calibration experiment with the model.

4.1 Calibration

In order to calibrate the model's growth-inequality correlation with the actual growth-inequality correlation, one encounters an immediate difficulty. To the best of our knowledge there is no available series for the redistributive effective tax rates across countries that comes close to our model's \mathbf{t} . Regarding the initial distribution of human capital, the closest available series is the proportion of people with secondary and higher than secondary education constructed by Barro and Lee (1993). We used this series as a proxy for the proportion m of managers in the economy.

In the absence of any available series for redistributive tax rate, we used the model's steady state property to generate a series for t using the following strategy. We picked countries only with positive growth rates to be consistent with the steady state property of the model as per proposition 2. Using their observed m values, we next generated a series for the effective redistributive tax rate, t by imposing the steady state restriction $m=m_c(t)$. The underlying assumption here is that the countries may differ in terms of the initial distribution of human capital, m because of the difference in the effective tax rates.¹³ Next we calculate the effective redistributive tax rate t for each country with a given m by solving the equation $x(m,t)=l$ using (32). The t thus obtained is the effective redistributive tax rate such that a given country's initial distribution of human capital can be identified as a steady state of the model.

Evidently, given any arbitrary m , the resulting t that solves $m=m_c(t)$ may not necessarily sustain the observed m as a steady state. Sustainability requires that t must satisfy the following three restrictions:

- (i) t must not exceed unity because in that case, the manager's after tax rental income becomes negative and she stops investing;
- (ii) $t > -a/(1-a)$ because otherwise the after tax wage income becomes negative making worker's consumption negative as evident from (34);
- (iii) t must be such that the proportional skill premium of the managers, $w(m,t)$ in (34) does not fall short of unity.¹⁴

Only a subset of countries in the data set created by Barro and Lee (1993) satisfies all three steady state restrictions. Moreover, the subset changes when we alter the parameter values. For plausible parameter values, at most 28 countries in the sample satisfy the aforementioned steady state restrictions. Figure 5 plots the relationship between the

¹³ Evidently this is a bit restrictive specification of the steady state because proposition 2 indicates that the set of admissible steady state equilibria includes $m \leq m_c(t)$. Nevertheless, we restrict our attention to this set of steady state equilibria for two reasons. First, we are interested in understanding the interaction between the tax policy and the distribution of human capital. Second, we do not have any available series for the redistributive tax rate, t .

¹⁴ Notice that t can very well be negative indicating proportional educational subsidy or investment tax credit. In fact, in our sample, in a majority of the cases, the observed value of m warrants a negative redistributive tax rate, t .

actual proportion of educated people and the implied effective tax rate. Notice that as expected from the model, the implied effective tax rate is lower for countries that have higher historically observed m and the effective tax rate becomes negative for higher value of m .¹⁵

<Figure 5 comes here>

In the final step, using the observed m and the corresponding simulated redistributive tax rates \mathbf{t} , we calculated the growth rate using (23) and the model's Gini using (35). Since there is no clear benchmark estimates for any of the parameters in the model (particularly a and \mathbf{q}), we searched for values in the parameter space, which reproduced a world per capita growth rate around 2.5%, based on World Bank Development Indicators. From the Gini data set generated by Forbes (2000) and using the corresponding per capita growth rate series from the World Bank development indicators we ended up with 33 countries with positive growth rates. For these 33 countries, we find that the observed correlation coefficient between growth and the Gini coefficient is 0.15. The model comes close to this observed value of the growth-inequality for $a=0.45$.

Table 1: Growth-Gini Correlation for Various a values

	$a=.3$	$a=.35$	$a=.4$	$a=.45$	$a=.5$	$a=.55$
Growth-Gini Correlation	-.26	-.25	-.06	.12	.008	.37
Average Growth Rate	3.17%	2.87%	2.25%	1.78%	1.57%	1.14%

Note: The other parameter values are $\mathbf{q}=.7$, $A_0=.3$, $\mathbf{d}=.04$, $\mathbf{b}=.96$

¹⁵ The issue arises whether a negative \mathbf{t} is socially optimal in our model. Bandyopadhyay and Basu (1999) finds that for the restricted subset of steady states considered here, the optimal \mathbf{t} is negative for all admissible range of m values.

There are two noteworthy features of the simulated numbers in Table 1. First, countries with a more unskilled labor-intensive technology (a higher a) experience lower growth. Second, the correlation coefficient is highly sensitive to a change in the value of the output elasticity parameter a . Notice that there is a non-linear association between a and the growth-gini correlation coefficient. The correlation coefficient reverses sign once a crosses a threshold value of 0.4. Countries with a more skilled labor-intensive technology (low a) display negative correlation between growth and inequality while the pattern is reversed for countries with more unskilled labor intensive technology. Everything else equal, a low value of the parameter a lowers the steady state proportion of managers in the economy.¹⁶ On the other hand, a low value of a also raises the growth maximizing proportion of managers, m^* (see Figure 3). The economy will be, therefore, operating in the segment where $m < m^*$ where growth and inequality are inversely related, discussed as a special case in the previous section.

Table 2 reports the growth-inequality correlation from the model for different q values. Notice that countries with higher q tend to have a negative growth inequality correlation and a lower average growth rate. Recall from (2) that a larger value of q means a greater degree of institutional barrier to knowledge-diffusion. Everything else equal, a larger q raises the growth maximizing proportion of managers m^* thus expanding the range $m < m^*$ where countries tend to have negative correlation between growth and inequality.

Table 2: Growth-Gini Correlation for Various q Values

	$q=.5$	$q=.6$	$q=.7$	$q=.8$
Growth-Gini Correlation	.15	.006	-.06	-.21
Average Growth Rate	3.22%	2.64%	2.24%	2.13%

Note: The other parameter values are $a=.4$, $A_0=.3$, $d=.04$, $b=.96$

One may thus summarize the findings of the calibration experiment as follows. First, countries sharing the same technology may have different growth and income inequality experiences because of the difference in the redistributive tax regimes and the initial distribution of human capital. These differences in tax regimes and initial distribution of human capital may give rise to a cross-country correlation between growth and income inequality. Second, countries may differ in terms of the production technology. Depending on the technology parameters one may get widely different correlation coefficients between the growth and income Lorenz ratio.

Lessons for Cross-Country Growth-Inequality Relationship

These steady state growth-inequality calculations help us understand what could possibly drive the association between growth and income inequality across countries. It also provides the following caveat while interpreting a cross-country regression of growth on inequality. Because of the state dependent nature of the relationship between growth and inequality, no robust correlation between growth and income inequality can be obtained when countries widely differ in terms of structural characteristics in the sense described in the model.

It is difficult to undertake a precise test of the diversity of the growth-Gini correlation coefficients across countries because a long time series for Gini data for individual countries is not available. Using the Forbes' income-Gini data over the sample period 1965-90, we calculated the growth-inequality correlation for a selected sample of countries over 6 sub- periods. Figure 6 reports the growth-Gini correlation for a list of 10 countries for which growth and Gini data for all 6 sub-periods are available. Notice that the correlation coefficient differs widely across these countries although the majority of the countries have a positive growth-inequality correlation. These differences in correlation do not necessarily validate the predictions of our model. However, it at least makes us cautious that structural differences among countries in terms of technology, tax policy and initial distribution of human capital may make a difference in the pattern of growth-inequality correlation across countries.

¹⁶ This can be immediately verified from (32). $\mathbf{x}(m, \mathbf{t})$ is monotonically decreasing in a which means the $\mathbf{x}(m, \mathbf{t})$ schedule shifts upward when a is lower making $m_c(t)$ decrease.

<Figure 6 comes here>

6. Conclusion

In this paper, we address the long-standing debate about the growth-inequality relationship. We attempt to answer the following question. What drives the cross-country correlation between growth and income inequality? The theoretical and calibration results highlight the role of redistributive tax policy and its interaction with the steady state distribution of human capital in determining the correlation between growth and income inequality. Countries with widely different initial distribution of human capital and fiscal redistributive tax structure may show either a positive or negative correlation between growth and income inequality depending on the production technology.

The punchline of our model is that one is unlikely to find a systematic monotonic relationship between growth and inequality if countries differ widely in the intensity of innovations, fiscal structure as well as production technology in the sense described in the model. One, therefore, needs to use caution while interpreting a regression of the cross-country growth-inequality relationship. If the regression method does not suitably control for the intensity of innovations, the differences in tax regimes and the differences in production technology across countries, the regression estimates are likely to be biased.

Appendix

Proof of Proposition 1: Define $i_t = s_t/h_t$. Using equation (11), the law of motion for human capital, along a balanced growth path,

$$(A.0) \quad h_{t+1}/h_t = (1+\mathbf{g}) \Leftrightarrow 1+i_t-\mathbf{d}=1+\mathbf{g}$$

It immediately follows that i_t is a constant i such that

$$(A.1) \quad \mathbf{g} = i - \mathbf{d}.$$

Next note that, by (21), if $m_t = m$ then $r_t = r(m)$. Next, use the first order condition for (14) to obtain

$$(A.2) \quad C_{t+1}/C_t = \mathbf{b}^{1/\lambda} [(1-\mathbf{t})r(m) + 1 - \mathbf{d}]^{1/\lambda}.$$

On the balanced growth path, $C_{t+1}/C_t = 1 + \mathbf{g}$. It follows, therefore, that the balanced growth rate, \mathbf{g} is given by:

$$(A.3) \quad 1 + \mathbf{g} = (\mathbf{b}[(1-\mathbf{t})r(m) + 1 - \mathbf{d}])^{1/\lambda},$$

which proves (23) and (24). From (A.1) note that for $\mathbf{g} \geq 0$, it must be that $i \geq \mathbf{d}$ which establishes the lower bound for i . Using (13), next verify that the budget constraint for the manager is:

$$(A.4) \quad c_t + s_t = (1-\mathbf{t})r(m) h_t.$$

Dividing through by h_t , one can immediately check that $i < (1-\mathbf{t})r(m)$.

Proof of Lemma 2: If $U(C_t) = \frac{C_t^{1-\lambda}}{1-\lambda}$, inequality (31) can be rewritten as:

$$(A.5) \quad \frac{1}{(w_t + z_t - h_t)^I} > \frac{\mathbf{b}[(1-t)r(m) + 1 - \mathbf{d}]}{[(1-t)r(m) - i(m, t)]^I h_{t+1}^I}.$$

Note that along a balanced growth path, $h_t = h_0(1 + \mathbf{g})^t$.

Next using (7) and the steady state balanced growth condition, we can write

$$(A.6) \quad z_t = z_0(1 + \mathbf{g})^t,$$

where based on (7) and the steady state condition,

$$(A.7) \quad z_0 = \frac{\mathbf{t} \cdot r(m) \cdot mh_0}{(1 - m)}.$$

Using (20) and the balanced growth condition we can write

$$(A.8) \quad w_t = w_0(m, h_0)(1 + \mathbf{g})^t,$$

where

$$(A.9) \quad w_0 = \frac{aA_0 m^{1+q}}{(1 - m)^{1-a}} h_0 = w(m)h_0.$$

Using (A.6) through (A.9), and also using the balanced growth condition, (A.0), the inequality (A.5) can be rewritten as:

$$(A.10) \quad \frac{1}{\left[w(m) + \frac{\mathbf{t} \cdot r(m) \cdot m}{1 - m} - 1 \right]} > \frac{\mathbf{b}^{\frac{1}{I}} [(1-t)r(m) + 1 - \mathbf{d}]^{\frac{1}{I}}}{[(1-t)r(m) - i(m, t)](1 + \mathbf{g})}.$$

Next note from (A. 3) that along a balanced growth path

$$(A.11) \quad \mathbf{g} = \mathbf{b}^{\frac{1}{i}} [(1-t)r(m) + 1 - \mathbf{d}]^{\frac{1}{i}} - 1,$$

which means (A.10) reduces to

$$(A.12) \quad \frac{\left[w(m, h_0) + tr(m) \cdot \frac{m}{1-m} - 1 \right]}{[(1-t)r(m) - i(m, t)]} < 1.$$

Using the definition $\mathbf{x}(m, t)$, the above inequality can be rewritten as:

$$(A.13) \quad \mathbf{x}(m, t) - \frac{1}{(1-t)r(m) - i(m, t)} < 1.$$

Next since $i(m, t) < (1-t)r(m)$, the inequality (A.13) holds if $\mathbf{x}(m, t) < 1$.

Proof of Lemma 3: Taking logarithm on both sides of (32):

$$(A.14) \quad \ln \mathbf{x}(m) = \ln \left[\frac{a}{1-a} + t \right] + \ln \left(\frac{m}{1-m} \right) - \ln(1-\mathbf{b})(1-t) - \ln \left(1 + \frac{(1-\mathbf{d})(1-t)^{-1}}{r(m)} \right).$$

Differentiating ξ with respect to m , and using from (21) the fact that $\frac{r'(m)}{r(m)} = \frac{\mathbf{q}(1-m) - am}{m(1-m)}$, we get the partial derivative, ξ_m as follows:

$$(A.15) \quad \mathbf{x}_m = \frac{\mathbf{x}(m)}{m(1-m)} \left[1 - (1-\mathbf{d}) \frac{am}{(1-t)r(m) + 1 - \mathbf{d}} + (1-\mathbf{d}) \frac{\mathbf{q}(1-m)}{(1-t)r(m) + 1 - \mathbf{d}} \right].$$

Note that since $0 < a < 1$ and $0 < m < 1$, the second term in the square bracket is less than unity. Since $\mathbf{q} > 0, a > 0, 0 < m < 1, r(m) > 0$ and $\mathbf{x}(m) > 0$ it follows that $\mathbf{x}_m > 0$.

Proof of Lemma 4: Note from (32) that $\lim_{m \rightarrow 0} \mathbf{x}(m, \mathbf{t}) = 0$ and applying L'Hopital's rule, $\lim_{m \rightarrow 1} \mathbf{x}(m, \mathbf{t}) = \infty$. Since by Lemma 3, $\mathbf{x}(m, \mathbf{t})$ is monotonic in m , there exists a unique $m_c(\mathbf{t})$, at which $\mathbf{x}(m, \mathbf{t}) = 1$ holds. Note also from (32) that a higher value of \mathbf{b} implies a higher value of $\mathbf{x}(m, \mathbf{t})$ and hence a lower value of $m_c(\mathbf{t})$ but a higher value of $m_L^2(\mathbf{t})$ (see Figure 3). We only focus on the set of steady states with sufficiently high values of the discount rate \mathbf{b} such that $m_c(\mathbf{t}) < m_L^2(\mathbf{t})$. It follows that the equilibrium with the initial distribution characterized by the constant m that belongs to the interval $m_L^1(\mathbf{t}) \leq m \leq m_c(\mathbf{t})$ yields a non-negative rate of growth. This immediately proves that for all $0 < m_L(\mathbf{t}) \leq m \leq m_c(\mathbf{t})$, a steady state equilibrium with non-negative growth exists.

Proof of Proposition 2: From Figure 4 as well as Lemma 1 and 2, it follows that that when $m_L^1(\tau) \leq m < \min(m_c(\mathbf{t}), m_L^2(\mathbf{t}))$, all members of the dynasties of managers continue to invest in education at the rate $i(m, \mathbf{t})$ while the members of the dynasties of workers do not invest in education and stay as workers. Since $m_L^1(\tau) \leq m \leq m_L^2(\tau)$, we have non-negative growth in this environment.

If $m > m_c(\mathbf{t})$, from Lemma 1 and Figure 4, it follows that it is not incentive compatible for managers to undertake positive investment because $\xi(m, \tau) > 1$. The question is whether workers may find it incentive compatible to invest in education. Notice that if $\xi(m, \tau) > 1$, for some range of m values the inequality (A.13) may get reversed in which case it is possible for the workers to find it incentive compatible to invest in children's education. Thus we may have a situation where existing managers do not invest in education while all workers turn into managers by investing in their children's education in which case m approaches unity. However, this cannot be sustained in a steady state with non-negative growth because if m approaches unity the implicit rental price, $r(m)$ in (21) approaches zero making the steady state growth rate a negative number. This proves that for non-negative steady state growth rate, m cannot exceed $m_c(\mathbf{t})$.

Proof of Lemma 5: It is straightforward to verify from (32) that $\mathbf{x}(m, \mathbf{t})$ is monotonically increasing in \mathbf{t} , meaning that $\frac{\partial \mathbf{x}(m, \mathbf{t})}{\partial \mathbf{t}} > 0$. In other words, the $\mathbf{x}(m, \tau)$ function in Figure 4 shifts upward when \mathbf{t} rises. The immediate implication is that $m_c(\mathbf{t})$ is monotonically decreasing in \mathbf{t} . One can formally show this by implicitly differentiating $\mathbf{x}(m_c(\mathbf{t}), \mathbf{t})$ with respect to \mathbf{t} to obtain

$$(A.16) \quad \frac{\partial m_c(\tau)}{\partial \tau} = \frac{-\xi_\tau}{\xi_m},$$

where \mathbf{x}_t is the partial derivative of \mathbf{x} with respect to \mathbf{t} . Since $\mathbf{x}_t > 0$ and $\mathbf{x}_m > 0$, it follows that $\frac{\partial m_c(\mathbf{t})}{\partial \mathbf{t}} < 0$.

Derivation of equation (35), Gini coefficient: Define \bar{a} as the worker's steady state post tax share in income. In other words,

$$(A.17) \quad \bar{a} = \frac{(1-m)(w_0 + z_0)}{(1-m)(w_0 + z_0) + (1-t)r(m)h_0m},$$

which upon the use of (34), reduces to

$$(A.18) \quad \bar{a} = \frac{1}{1 + m(1-m)^{-1} \mathbf{w}(m, \mathbf{t})}.$$

Next, note that in the steady state, the initial distribution of human capital (1) is preserved, which means $(1-m)$ fraction of the population have \bar{a} fraction of total income. and m fraction the population have $(1-\bar{a})$ fraction of total income. The Lorenz ratio for income (*Gini*) is therefore given by:

$$(A.19) \text{ Gini} = 1 - \bar{a} - m,$$

which after simplification yields (35).

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Figure 1

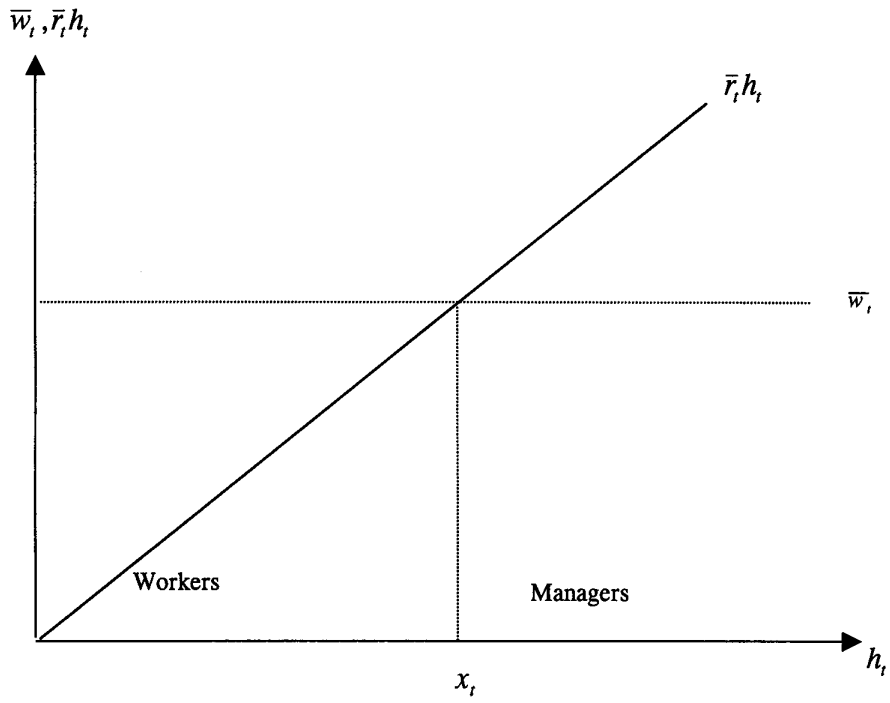


Figure 2

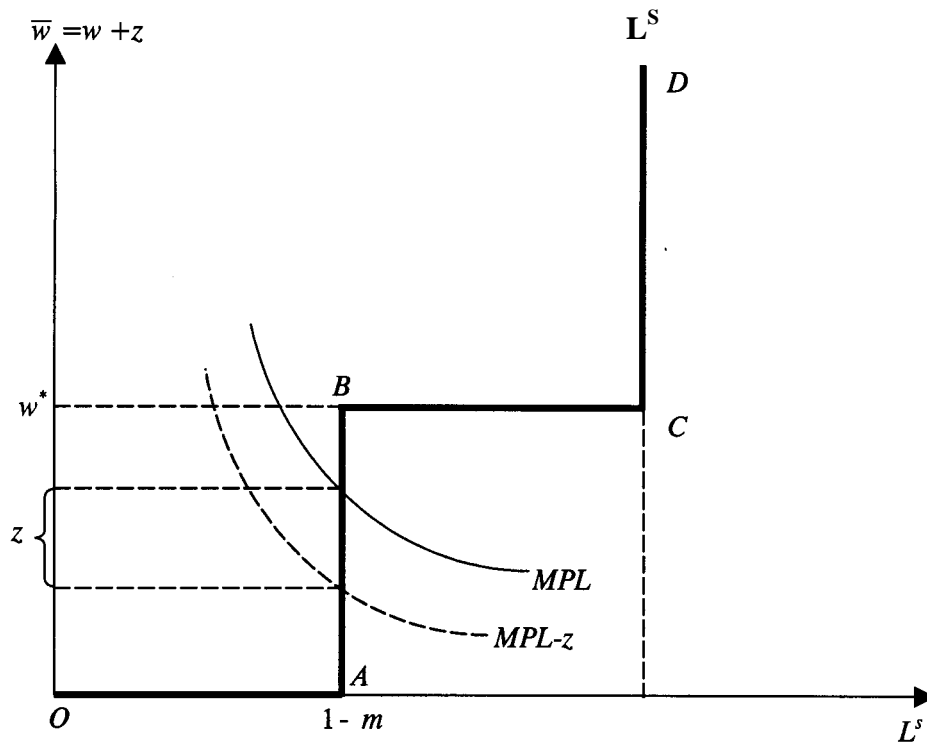


Figure 3

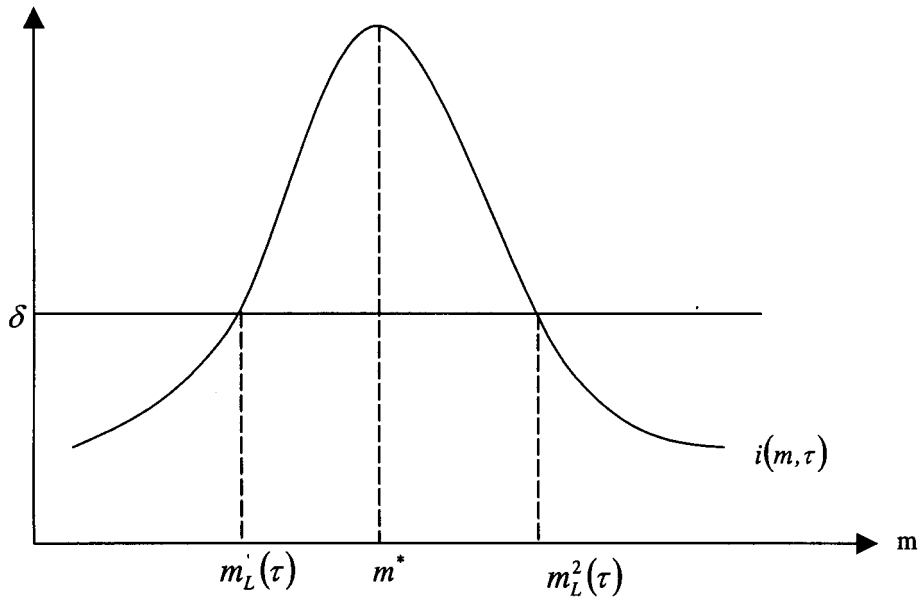


Figure 4

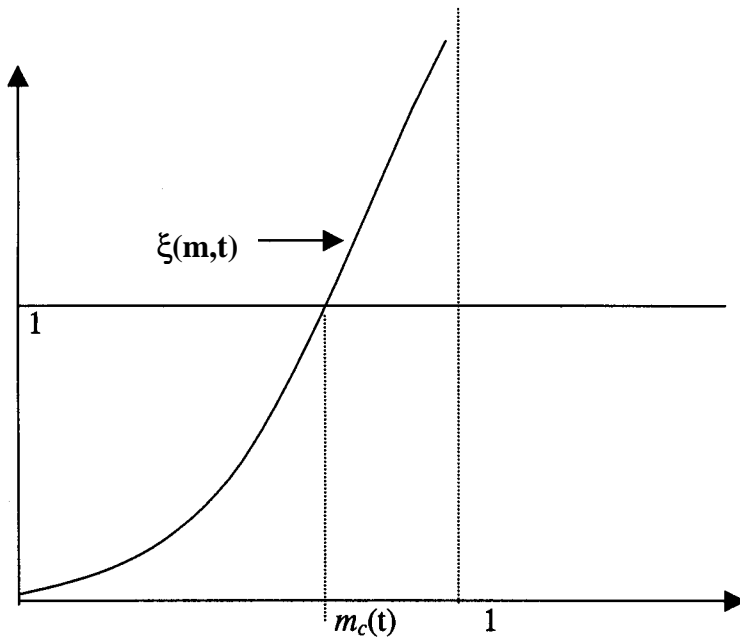


Figure 5: Model's Calibration of Implicit Tax Rates for the Observed Proportions of Skilled Labor in Countries

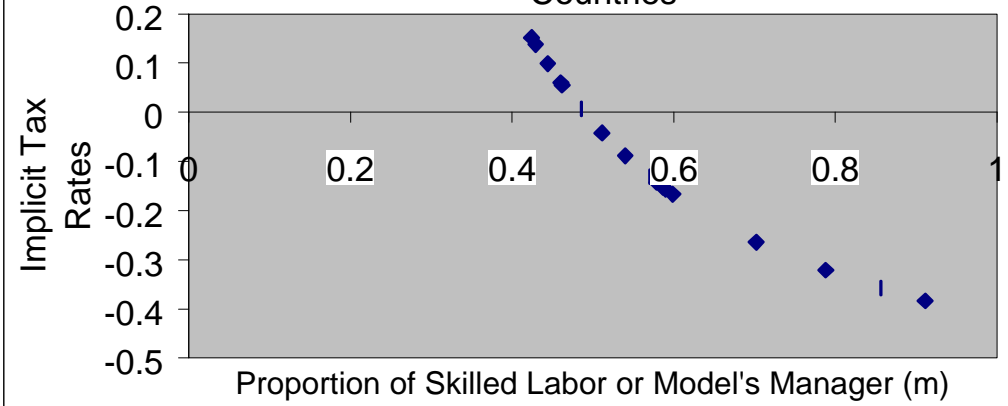


Figure 6: Growth-Gini Correlations for Selected Countries.

