Bayesian Target Zones

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Abstract: Several authors have postulated econometric models for exchange rates restricted to lie within known target zones. However, it is not uncommon to observe exchange rate data with known limits that are not fully 'credible'; that is, where some of the observations fall outside the stated range. An empirical model for exchange rates in a soft target zone where there is a controlled probability of the observed rates exceeding the stated limits is developed in this paper. A Bayesian approach is used to analyse the model, which is then demonstrated on Deutschemark–French franc and ECU–French franc exchange rate data.

Keywords: Bayesian estimation, griddy-Gibbs sampler, credible target zones, soft margins, European Monetary System

JEL Classification Numbers: C11, C13, F31, F33

1 Introduction

Recent turmoil on international currency markets and the subsequent disruptive economic impact on some economies, highlight the need for a re-evaluation of exchange rate management practices. The recurring risk of speculative attack in particular calls for an assessment of fixing or targeting a currency to a benchmark currency. Since the collapse of the Bretton Woods agreement in the early seventies, many countries responded to the pressure on fixed exchange rates by officially floating their currencies. Unofficially, however, many of these countries were and are still maintaining their currency within certain limits vis-à-vis a benchmark currency.

To smooth revaluations and protect the value of its currency, a central bank will often restrict the free float to a managed float. The latter (dirty) float is distinguished from a clean float by the central banks' commitment to a desirable exchange rate level. This commitment can take several forms, the extremes of which are given by an effectively fixed rate with only one to two percentage points margin of variation on the one hand, and an effectively floating rate with up to ten percentage points margin on the other. Under the narrow scenario the limits and central parity are publicly announced and their limits are defended by committed central banks. Under the generous scenario the limits (or even the central parity) are sometimes not publicly announced and the central banks are not always fully committed to defend their currency. Frenkel and Goldstein (1986) label the former as 'loud zones' and the latter as 'quiet zones.' The main difference between the two practices is the degree of credibility of exchange rate intervention by the central bank. A fairly relaxed managed float would allow the monetary authority to still smooth exchange rate movements without precommitting to defend the currency at any price (i.e., at any amount of foreign currency needed to intervene). While allowing monetary policy coordination between 'targeted' countries, such a system would still allow some domestic discretion. Without this absolute commitment to defend, speculative attacks will become highly risky for the perpetrators, see Williamson (1985). Kofman et al. (1993) provides a preliminary analysis of how such a system could function. Werner (1996) and Wren-Lewis (1997) illustrate how these soft zones moderate the economic impact of exchange rate movements.

Over the past decade, the theoretical (and empirical) exchange rate literature has focused much of its attention on publicly announced target zones with fully credible limits, i.e., the central bank intervenes whenever the currency threatens to exceed its limits. Most of this literature derives from Krugman's (1991) seminal paper in which he shows that regulated exchange rates no longer exhibit a linear relationship with the underlying fundamentals of supply and demand. If, for example, there is a fixed (and credible) upper limit, and the exchange rate is close to that limit, the probability of a further increase is limited while the probability of a decrease is relatively large. This distortion in probability symmetry (the truncation of the error distribution) will be discounted in the agents' expectations, and consequently in their demand/supply decision. This implies a non-linear S-shaped relationship between theoretical and observed exchange rates.

The empirical evidence for this so-called S-shape is, however, elusive. A major identification problem seems to be the predominance of observations that are well within the stated limits. Close to the parity, the observed versus fundamental exchange rate is almost perfectly linear. Estimation methodologies will then not be able to confirm or refute the presence of non-linearity. Most of this literature uses continuous time models, which often preclude the incorporation of typical time series characteristics as time-varying volatility and unconditional fat-tailedness of the error distribution. Koedijk et al. (1998) propose a discrete time estimation procedure for a credible target zone, which allows accommodation of these time series anomalies. Nevertheless, they still fail to find convincing evidence for the imposed non-linear relationship. An alternative discrete time target zone model proposed by Bekaert and Gray (1998) is somewhat more successful in detecting evidence of a non-linear S-shape. Their model allows for a much wider range of possible behaviours in both the (conditional) mean and variance of the observed exchange rate process than is typically considered in the standard target zone literature. The only restriction imposed on the functional form for the conditional distribution of exchange rate changes is the specification of the error distribution as truncated normal. The 'indicator' variable, which appears in both conditional mean and conditional variance specifications, relates to the position of the exchange rate in the target zone.

The non-linearity identification problem is further exacerbated if the target zone is less-than-perfectly credible. The Bekaert and Gray model is restricted to credible target zones with occasional realignments, modelled using a Poisson-like jump process. Identification of the jump parameters (probability and size of a jump) proves difficult, due to the small number of observed jumps. Miller and Weller (1991) and Mizrach (1995) developed theoretical target zone models that allow for less than credible limits. That is, they are characterized by occasional realignments (jumps) or by the constant threat of realignments. The credibility definition adopted in all of the above papers is restricted to realignment risk. Central banks are still assumed to be fully committed to a target zone (in between realignments).

Realignment risk is, however, not the only type of non-credibility of a target zone. Soft target zones – those that occasionally allow the exchange rate outside their limits – are by their very nature less-than-perfectly credible. In this paper we propose an econometric model for these soft target zones, and investigate whether they still retain the non-linearity implied by a fully credible target zone. On the one hand we expect the soft currencies to spend more time near the limits, which may alleviate the identification bias. On the other hand, as we will show, a lowering of the credibility of the target zone implies a linearisation of the S-shape.

Our approach in this paper is to extend the Bekaert and Gray model to allow for exchange rates to exceed the stated target zone (i.e., the limits are soft), while still retaining the qualitative effect of truncation. The degree to which this partial truncation affects the conditional exchange rate distribution will depend on the softness of the target zone. We provide a Bayesian estimation methodology to estimate the soft target zone model. The aim is to identify and measure the existence of a non-linear relationship based on the distortion of the underlying stochastic process caused by (soft) target zones. By explicitly modelling the credibility of the target zone, we attempt to 'avoid' the linearisation problem.

To the best of our knowledge, the only other paper advocating a Bayesian methodology to estimate target zone models is Li (1998). Despite the fact that her fully credible target zone model is slightly more complicated than Bekaert and Gray's, it is very similar and does include realignment risk. Li proposes a Metropoliswithin-Gibbs sampling methodology to avoid direct evaluation of the complicated likelihood functions.

The plan of this paper is as follows. The proposed soft target zone model is present in Section 2, along with a discussion of the implications the model has on the theoretical S-shape. An S-shape is still implied by the soft target zone model, however, the greater the degree of softness, the further the non-linearity is pushed outside the target zone. The Bayesian estimation methodology is presented in Section 3. We propose a griddy-Gibbs algorithm (Ritter and Tanner, 1992), and use Rao-Blackwellised estimators (Gelfand and Smith, 1990) for calculating marginal posterior summaries. A discussion of the estimation of the S-shape implied by the analysis is also included. In Section 4, the model is applied to ECU-French franc and Deutschemark-French franc exchange rate data where the limits are known, but are occasionally exceeded. Within the European Monetary System, exchange rate targeting of member currencies officially took place against the ECU. Our results indicate that de facto targeting took place against the Deutschemark. Section 5 concludes with some directions for future work.

2 An econometric model for a soft target zone

We begin by reviewing a target zone model, where the zone is perfectly credible by following the basic model of Bekaert and Gray (1998), and assuming known realignments. Consider the continuously compounded (logarithmic) exchange rate returns, $\Delta S_t = S_t - S_{t-1}$, as being determined by the regression relationship

$$\Delta S_t = x_t'\beta + \varepsilon_t , \qquad (1)$$

for $t = 2,\cdots,N$. Assuming perfectly credible upper and lower limits on the observed rates so that $L \le S_t \le U$, the error ε_t is restricted to have a truncated normal distribution with mode at zero, scale parameter equal to σ_t^2, corresponding upper limit $U_t = U - S_{t-1} - x_t'\beta$ and lower limit $L_t = L - S_{t-1} - x_t'\beta$. The probability density function of this truncated normal distribution is given by

$$p(\varepsilon_t) = \frac{\frac{1}{\sigma}\phi\left(\frac{\varepsilon_t}{\sigma_t}\right)}{\Phi\left(\frac{U_t}{\sigma_t}\right) - \Phi\left(\frac{L_t}{\sigma_t}\right)} 1_{[L_t,U_t]}(\varepsilon_t) , \qquad (2)$$

where ϕ and Φ are the probability density and cumulative distribution functions, respectively, for the standard normal distribution, and

$$1_A(\varepsilon) = \begin{cases} 1 \text{ if } \varepsilon \in A \\ 0 \text{ if } \varepsilon \notin A \end{cases} \tag{3}$$

is an indicator function of the set *A*. As in Bekaert and Gray (1998), in our discussion and example we restrict the regression coefficient vector β to correspond to a drift (a constant) and a 'position-in-the-zone' explanatory variable. Hence we use $x_{1t} = 1$ and $x_{2t} = 2(S_{t-1} - C)/(U - L)$, with C the known central parity. With perfectly credible limits, $-1 \le x_{2t} \le 1$, a value of $x_{2t} = -1$ corresponds to S_{t-1} at the lower limit *L*, and $x_{2t} = 1$ corresponds to S_{t-1} at the upper limit *U*. From this point, to simplify the discussion, we set $\sigma_t = \sigma$ for all $t=2, \ldots, N$. The specification of $x'_t \beta$ as the function determining the mode of the distribution of ΔS_t is not critical, as other specifications could be incorporated. Bekaert and Gray, for example, use a more elaborate model including inflation and interest differentials. We are instead focusing on the form of the distribution of the error term ε_t.

This truncated normal error distribution underlies the fully credible target zone models used by Bekaert and Gray (1998) and Li (1998). As discussed earlier, the advantage of the above truncated normal error distribution is that it retains the S-shape in the expected returns as a function of S_{t-1}, or equivalently, as a function of the 'position in the zone' variable, x_{2t}. This is due to the fact that under the assumption of truncated normal errors having a density function as in (2), the (conditional) expected exchange rate is

$$E(S_t | S_{t-1}, \beta, \sigma) = S_{t-1} + x_t{}'\beta + \sigma \frac{\phi\left(\frac{L_t}{\sigma}\right) - \phi\left(\frac{U_t}{\sigma}\right)}{\Phi\left(\frac{U_t}{\sigma}\right) - \Phi\left(\frac{L_t}{\sigma}\right)}. \tag{4}$$

To demonstrate the 'S-shape' corresponding to the above function, we set β' =(0,0), U = 3.0, L = 0.5, σ = 0.25 and plot $E(S_t|S_{t-1}, \beta, \sigma)$ as a function of S_{t-1} in Figure 1.

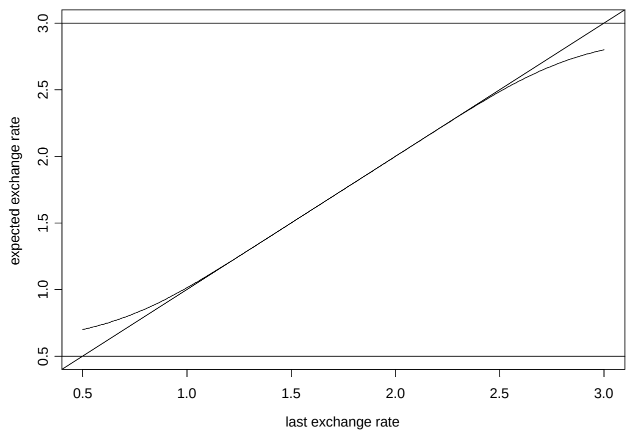


Figure 1: $E(S_t|S_{t-1}, \beta' = (0,0), \sigma = 0.25)$ for fully truncated normally distributed errors with exchange rate constrained between lower limit L=0.5 and and upper limit U=3.0. The 45 degree line represents the corresponding expected exchange rate when no limits are present.

The S-shape in Figure 1 is a result of the skewness (due to the truncation) of the distribution of ε_t as S_{t-1} moves closer to the edge of the target zone. This effect is more dramatically demonstrated by considering the effect of the truncation on the expected error term as a function of the 'position in the zone' variable as in Figure 2.

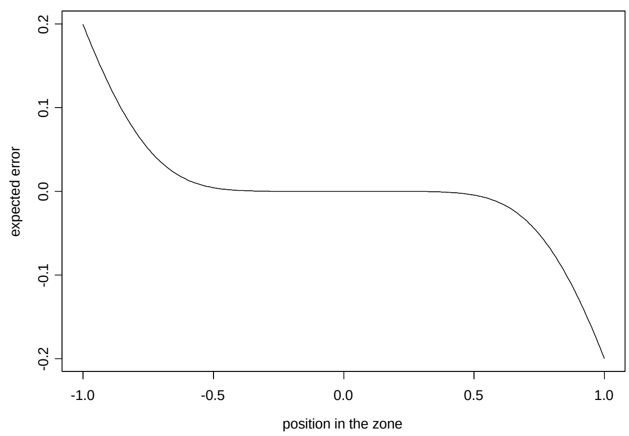


Figure 2: $E(\varepsilon_t|S_{t-1}, \beta' = (0,0), \sigma = 0.25)$ for fully truncated normally distributed errors with L=0.5 and U=3.0.

We wish to develop an alternative to the above truncated normal distribution that qualitatively retains this S-shape in the expected return, but that still allows for observations to lie outside the stated target zone. One way of specifying an alternative distribution for ε_t that satisfies these requirements is to retain the basic underlying normality of the errors, and then allow, say $(1-\alpha_t)\times 100\%$ of the probability to remain in the target zone, and the excess $\alpha_t \times 100\%$ probability falling outside the range. A resulting probability density function is given by

$$p(\varepsilon_t)=\frac{\frac{1}{\sigma}\phi\left(\frac{\varepsilon_t}{\sigma}\right)(1-\alpha_t)}{\Phi\left(\frac{U_t}{\sigma}\right)-\Phi\left(\frac{L_t}{\sigma}\right)}1_{(L_t,U_t]}(\varepsilon_t)+\frac{\frac{1}{\sigma}\phi\left(\frac{\varepsilon_t}{\sigma}\right)\alpha_t}{\Phi\left(\frac{L_t}{\sigma}\right)+1-\Phi\left(\frac{U_t}{\sigma}\right)}\left\{1_{(-\infty,L_t]}(\varepsilon_t)+1_{(U_t,\infty)}(\varepsilon_t)\right\} \quad (5)$$

We call the distribution of ε_t corresponding to the above density the 'α_t-truncated normal distribution'. Note that this probability α_t is allowed to vary with t because we want a higher chance of falling outside the target zone as the exchange rate S_{t-1} moves nearer to the edges of the stated zone.

As the exchange rate S_{t-1} moves closer to, say, the upper limit U, the error term ε_t moves closer to its corresponding upper limit U_t. We expect the chance of a move outside the target zone to increase as we move closer to the edge of the target zone. However, we also expect the chance of intervention from the central bank to increase, and hence we want to restrict the probability of leaving the target zone to a maximum value of, say, α^*. That is, we set

$$\alpha_t=\min\left\{\alpha^*,\Phi\left(\frac{L_t}{\sigma}\right)+1-\Phi\left(\frac{U_t}{\sigma}\right)\right\},\ \text{for all } t. \quad (6)$$

This value, α^*, might be termed a maximum 'exceedence probability'. It is the largest chance the exchange rate can have of moving outside the stated target zone in one step. As the value of α^* is a maximum probability, a large value does not necessarily imply a low degree of credibility of the target zone. The credibility at any particular point in time is measured by α_t, which would typically be quite small with large values occurring only when the exchange rate moves close to or beyond the limits. This is a convenient model for the error distribution because it nests both the non-truncated normal (α^* = 1) and fully truncated (α^* = 0) distributions. Our estimation procedure in Section 4 allows the data to choose the value of α^* and therefore we can obtain information regarding the individual α_t values.

To demonstrate the effect of the α_t-truncated normal distribution, Figure 3 displays the probability of moving (or remaining) outside the stated target zone for a given position in the zone.

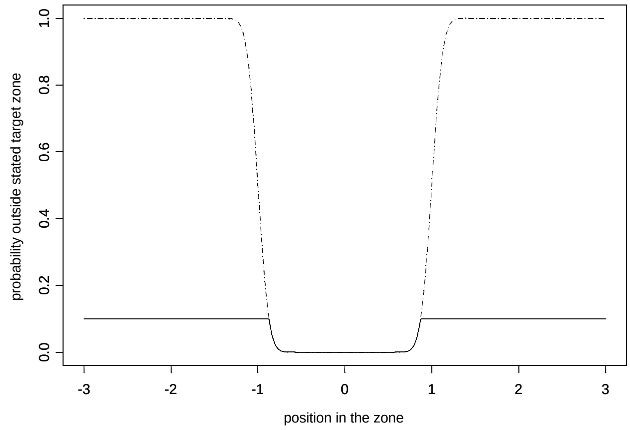


Figure 3: Controlled probability of moving (or remaining) outside the stated target zone under α_t-truncated normal distribution for a given position in the zone and with $\alpha^* = 0.1$, $\beta = (0,0)'$, $\sigma = 0.1$. The dotted line is the probability of moving outside the stated target zone under the non-truncated case of $\alpha^* = 1.0$.

Using individually constructed $\varepsilon_t = \Delta S_t - x_t'\beta$ and α_t values the likelihood function is obtained

$$L(\beta,\sigma,\alpha^*) \propto \sigma^{-(N-1)} \prod_{t=2}^{N} \phi\left(\tfrac{\varepsilon_t}{\sigma}\right) \left\{ \frac{(1-\alpha_t) 1_{(L_t,U_t]}(\varepsilon_t)}{\Phi\left(\frac{U_t}{\sigma}\right)-\Phi\left(\frac{L_t}{\sigma}\right)} + \frac{\alpha_t 1_{(-\infty,L_t]\cup(U_t,\infty)}(\varepsilon_t)}{\Phi\left(\frac{L_t}{\sigma}\right)+1-\Phi\left(\frac{U_t}{\sigma}\right)} \right\}. \qquad (7)$$

For a small value of α^* we retain the essential S-shape in the expected errors. This is because it can be shown that

$$E(\varepsilon_t | S_{t-1},\beta,\sigma) = (1-\alpha_t)\sigma \frac{\phi\left(\frac{L_t}{\sigma}\right)-\phi\left(\frac{U_t}{\sigma}\right)}{\Phi\left(\frac{U_t}{\sigma}\right)-\Phi\left(\frac{L_t}{\sigma}\right)} + \alpha_t \sigma \frac{\phi\left(\frac{U_t}{\sigma}\right)-\phi\left(\frac{L_t}{\sigma}\right)}{\Phi\left(\frac{L_t}{\sigma}\right)+1-\Phi\left(\frac{U_t}{\sigma}\right)} \qquad (8)$$

Notice that if $\alpha^* = 0$, and hence all $\alpha_t = 0$, we return to the usual expected value of a fully truncated normal random variable used in (4). Also, if $\alpha_t = \Phi(L_t/\sigma) + 1 - \Phi(U_t/\sigma)$, then the expected value of ε_t is zero and $E(S_t | S_{t-1},\beta,\sigma,\alpha_t) = S_{t-1} + x_t'\beta$. This occurs when S_{t-1} is well within the target zone, relative to σ. In this case, the expected value of the error term, ε_t, will be zero and the expected return will be given by $x_t'\beta$. This corresponds to the 'flat' middle of the S-shape. The smaller the value of α^*, the more pronounced the S-shape will be.

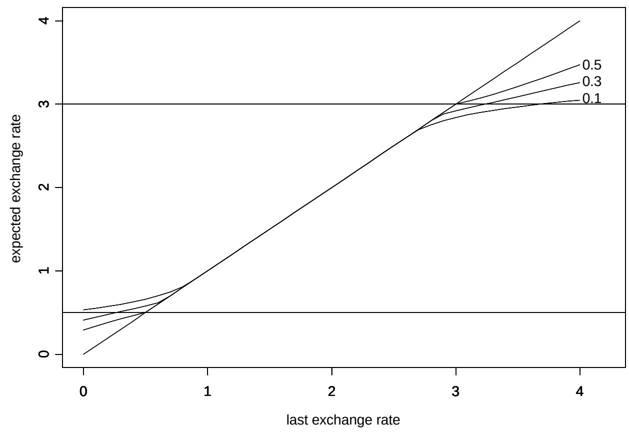


Figure 4: $E(S_t|S_{t-1}, \beta' = (0,0), \sigma = 0.25, \alpha*)$ for α_t-truncated normally distributed errors for each of $\alpha* = \{0.1, 0.3, 0.5\}$ and with L=0.5 and U=3.0.

To demonstrate the S-shape corresponding to these non-credible target zones, we consider the function in (8). We again set β' = (0,0), U = 3.0, L = 0.5, σ = 0.25 and plot $E(S_t|S_{t-1}, \beta, \sigma, \alpha*)$ as a function of S_{t-1} in Figure 4. As $\alpha*$ increases, the S-shape becomes less pronounced, with the limiting case $\alpha*$ = 1 corresponding to the stated limits having no effect, and hence a linear relationship results with $E(S_t|S_{t-1}, \beta, \sigma, \alpha*) = S_{t-1} + x_t'\beta$. In particular, if $\alpha* > 0.5$, the expected exchange rate inside the target zone is a linear function of the regressors. Hence, there is no non-linear the impact on agents expectations inside the stated target zone. However, if $\alpha* < 0.5$, there will be at least some degree of non-linearity in this relationship inside the target zone. Figure 5 demonstrates this more dramatically by presenting the expected error term as a function of the 'position in the zone' variable, x_{2t}.

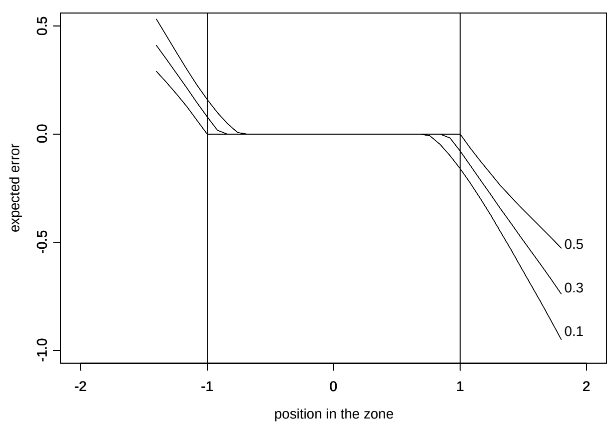


Figure 5: $E(\varepsilon_t|S_{t-1}, \beta' = (0,0), \sigma = 0.25, \alpha*)$ for α_t-truncated normally distributed errors for each of $\alpha* = \{0.1, 0.3, 0.5\}$ and with L=0.5 and U=3.0.

3 Bayesian analysis of the soft target zone model

In this section, we begin by reviewing the general Bayesian approach, and detail the means by which empirical results for our model can be obtained. A standard Bayesian analysis of an econometric model begins with the computation of the posterior distribution of the parameters in the model, given the observed data. In general, suppose the parameters of the model under investigation are denoted by the vector θ, and let the data be denoted by the vector y. The Bayesian approach requires the availability of prior distribution for the unknown parameter vector θ, and is denoted by its density function $p(\theta)$. Conditional on α^*, we choose a standard regression non-informative prior distribution (see Zellner 1971) for

$$p(\beta,\sigma|\alpha^*) \propto \frac{1}{\sigma}, \text{ for } \sigma > 0, \ -\infty < \beta_1 < \infty, \ -\infty < \beta_2 < \infty \ . \qquad (9)$$

As α^* determines the maximal exceedence probability of observations outside the stated target zone, it must lie within the unit interval. We employ a selected prior distribution from the Beta family with hyper-parameters a and b, and having density function

$$p(\alpha^*) = \frac{\Gamma(\gamma+\delta)}{\Gamma(\gamma)\Gamma(\delta)} \alpha^{*a-1} (1-\alpha^*)^{b-1}, \text{ with } \gamma > 0, \delta > 0 . \qquad (10)$$

We selected *a priori* probabilities $p(\alpha^* < 0.05) = 0.50$ and $p(\alpha^* < 0.20) = 0.95$, resulting in $\gamma = 0.95$ and $\delta = 12.6$. These values correspond to strong prior belief in fairly tightly managed exchange rates. In our application, this is consistent with the French franc participating in the exchange rate mechanism of the EMS. In other applications one might like to impose a more diffuse prior on α^*, for example, to incorporate the information that a particular target zone might not be considered so heavily managed.

The likelihood function, which is determined by the econometric model, is given by the conditional distribution of the data assuming the parameters are fixed. That is, $L(\theta) \propto p(y|\theta)$. Taking these two probability densities, we can construct the conditional density for the unknown parameter θ, given the observed data y according to Bayes' theorem

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}. \qquad (11)$$

As the data are observed, and therefore considered known, the denominator in Bayes' theorem can be viewed as a constant, and hence we have the often stated result that the posterior density is proportional to the likelihood function times the prior density, or

$$p(\theta|y) \propto L(\theta)p(\theta). \qquad (12)$$

The problem is then, not what is the analytical form of the posterior density, as we 'know' it up to a constant of proportionality, but rather what are its features? For instance, what are the mean and mode of the distribution? Is it unimodal? How can statements such as

$$P(\theta_1 \leq c|y) \qquad (13)$$

be evaluated? If the denominator in (9) can be easily obtained, using

$$p(y) = \int p(\theta|y)p(\theta)d\theta \qquad (14)$$

then any desired feature of the posterior distribution can be readily obtained.

Applying the griddy-Gibbs sampler

The likelihood function is given by the product of the conditional densities of the errors, as displayed in (7). In this problem, $\theta' = (\beta, \sigma, \alpha^*)$, where $\beta = (\beta_1, \beta_2)'$, as these are the parameters we wish to estimate. As the likelihood function is a complicated function of θ, it is difficult to obtain even a simple numerical estimate of *p(y)*. In this case, we resort to an alternative approach for obtaining features of the posterior distribution, namely the use of Markov chain Monte Carlo methods and in particular the 'griddy Gibbs sampler'. We briefly review the Gibbs sampler, Gelfand and Smith (1990), and summarise the extension to the 'griddy' Gibbs sampler, Ritter and Tanner (1992), in the context of our particular model.

Under quite general conditions, see Tierney (1994), a Gibbs sampler can be constructed by sampling from the so-called full conditional distributions. That is, we begin by selecting starting values $\left(\beta_1^{(0)}, \beta_2^{(0)}, \sigma^{(0)}, \alpha^{*(0)}\right)$ and sampling iteratively

$$
\begin{aligned}
&\sigma^{(i)} \sim p\left(\sigma|\beta_1^{(i-1)}, \beta_2^{(i-1)}, \alpha*^{(i-1)}, y\right) \\
&\beta_1^{(i)} \sim p\left(\beta_1|\beta_2^{(i-1)}, \alpha*^{(i-1)}, \sigma^{(i)}, y\right) \\
&\beta_2^{(i)} \sim p\left(\beta_2|\alpha*^{(i-1)}, \sigma^{(i)}, \beta_1^{(i)}, y\right) \\
&\alpha*^{(i)} \sim p\left(\alpha*|\sigma^{(i)}, \beta_1^{(i)}, \beta_2{}^{(i)}, y\right)
\end{aligned}
\qquad (15)
$$

for $i = 1,2,\cdots, M$. Repeated sampling in this manner whilst updating the conditioning variables produces a sample of M parameter sets $\left(\beta^{(i)},\sigma^{(i)},\alpha*^{(i)}\right)$ which converge in distribution to the posterior distribution $p(\beta,\sigma,\alpha*|y)$. From this sample we can extract features of the posterior distribution, its marginal densities and probability intervals.

The 'griddy' Gibbs sampler simply uses the fact that we can approximately sample from the full conditional distributions in (13) by computing, for example

$$
\hat{p}\left(\beta_1|\beta_2^{(i-1)}, \alpha*^{(i-1)}, \sigma^{(i)}, y\right) \qquad (16)
$$

by evaluating

$$
L\left(\beta_1, \beta_2^{(i-1)}, \sigma^{(i)}, \alpha*^{(i-1)}\right) p\left(\beta_1, \beta_2^{(i-1)}, \sigma^{(i)}|\alpha*^{(i-1)}\right) p\left(\alpha*^{(i-1)}\right) \qquad (17)
$$

on a (univariate) grid of β_1 values and numerically normalising so that $\int \hat{p}\left(\beta_1|\beta_2^{(i-1)},\sigma^{(i)},\alpha*^{(i-1)},y\right)d\beta_1 = 1$. Then, using the standard inverse cumulative distribution function (cdf) sampling method, a sampled $\beta_1^{(i)}$ can be obtained. We can also use these numerically normalised conditional densities to construct Rao-Blackwellised marginal posterior density estimates (see Gelfand and Smith, 1990). Marginal posterior probability intervals can be obtained from these marginal posterior density estimates, or from histograms of the sampled parameters directly. The Rao-Blackwellised marginal density estimators have been shown to have smaller (Monte Carlo) mean squared error than the simple histogram approach (Gelfand and Smith, 1990).

Inference regarding the S-shape

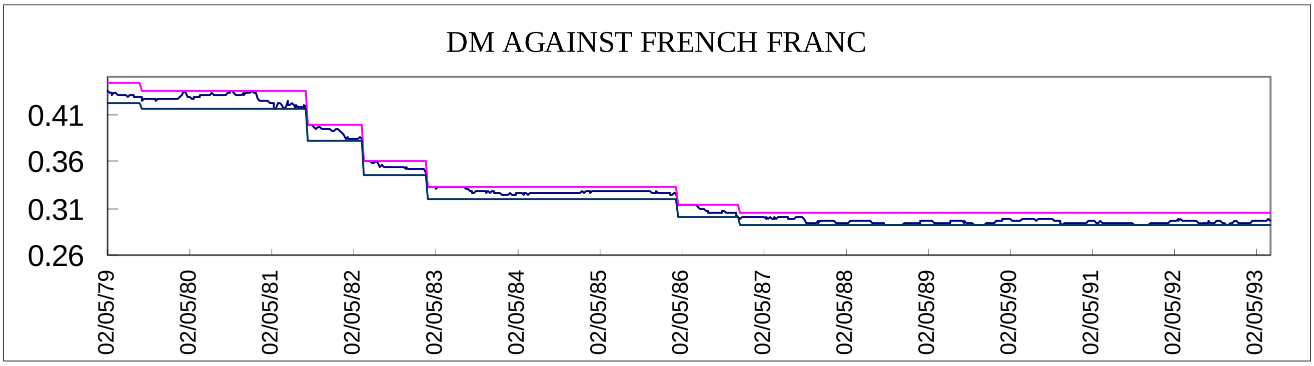
In order to produce an estimate of the S-shape curve resulting from our analysis, we recognise that for fixed S_{t-1}, the expected value $E(S_t|S_{t-1}, \beta, \sigma, \alpha*)$ is a function of the unknown parameters. Hence, we can think of this expectation as a parameter in and of itself, and construct a posterior distribution for it. For each S_{t-1} denote

$$\psi_{S_{t-1}}(\beta,\sigma,\alpha^*) = E(S_t|S_{t-1},\beta,\sigma,\alpha^*). \tag{18}$$

To construct an estimate of this distribution from the output of our Gibbs sampler, we construct a sample of M $\psi_{S_{t-1}}^{(i)} = E\left(S_t|S_{t-1},\beta^{(i)},\sigma^{(i)},\alpha^{*(i)}\right)$ values, which comprise a sample from the marginal posterior distribution of $p\left(\psi_{S_{t-1}}|y\right)$. From this sample we can obtain the upper and lower 5% probability quantiles, which yield 90% Bayesian confidence intervals, as well as point estimates for $\psi_{S_{t-1}}$, for any given value of S_{t-1}.

4 An example for the French franc

Our exchange rate data are obtained from *Datastream*. The parity and realignment data are obtained from Ungerer (1997). The sample consists of weekly observations on the Deutschemark–French franc (DM/FF) and ECU–French franc (ECU/FF) exchange rate. According to Honohan (1998), countries that participated in the Exchange Rate Mechanism (ERM) managed their currencies against the ECU (European Currency Unit, a basket of participating currencies), not against a bilateral currency. The applied literature has, however, focused on bilateral target zones (in particular the DM/FF example). To evaluate this distinction, we include both exchange rates in the analysis. Exchange rates are taken at the close of Wednesday trading. Figure 6 plots the French franc exchange rate with its bilateral and ERM limits over the period 2nd of May 1979 through 30th of June 1993 against the Deutschemark and ECU, respectively. The width of the target zone is 4.5 percent of the parity rate – symmetrically distributed. The six (DM/FF), respectively fourteen (ECU/FF) jumps illustrate the realignments in the parity rate during this period. Excluding these realignments there are 734 (726 for the ECU/FF) weekly observations for the DM/FF, 2 (47 for the ECU/FF) of which fall outside the stated target zones. The 'position in the zone' variable, x_{2t}, is constructed using the relative position of S_{t-1} in the DM/FF target zone at time t using $x_{2t} = 2(S_{t-1} - C)/(U - L)$. Note that C, U and L are not constant over the observation period, but are constant between realignments.



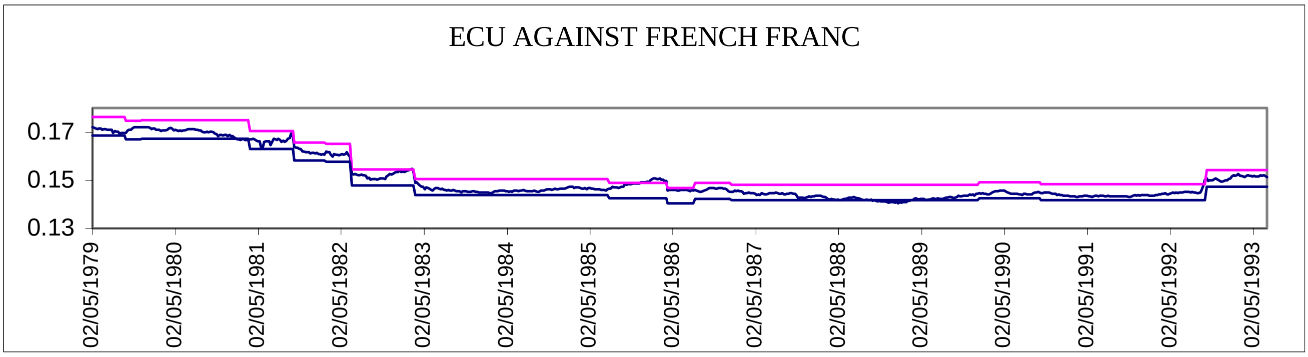


Figure 6: French franc exchange rates

Inference assuming a perfectly credible target zone ($\alpha^* = 0$)

We begin with an analysis of the fully credible target zone model. A 'griddy' Gibbs sampler was run for the model in Section 3 conditional on $\alpha^* = 0$. Rao-Blackwellised density estimates of the marginal posterior probability densities for the drift coefficient, β_1, the 'position in the zone' coefficient, β_2, and the scale parameter, σ, are given in Figure 7. As can be seen from the figures, each of the marginal posterior distributions are roughly symmetric and unimodal. Hence, the marginal distributions can be well summarised by their median (which roughly equals the mean and mode of the respective distribution) and upper and lower probability quantiles to indicate where the majority of probability lies. These values are given in column three of Table 1. Note that since $\alpha^* = 0$ corresponds to a perfectly credible zone, any observations which fall outside the stated zone are ignored and do not contribute to the analysis.

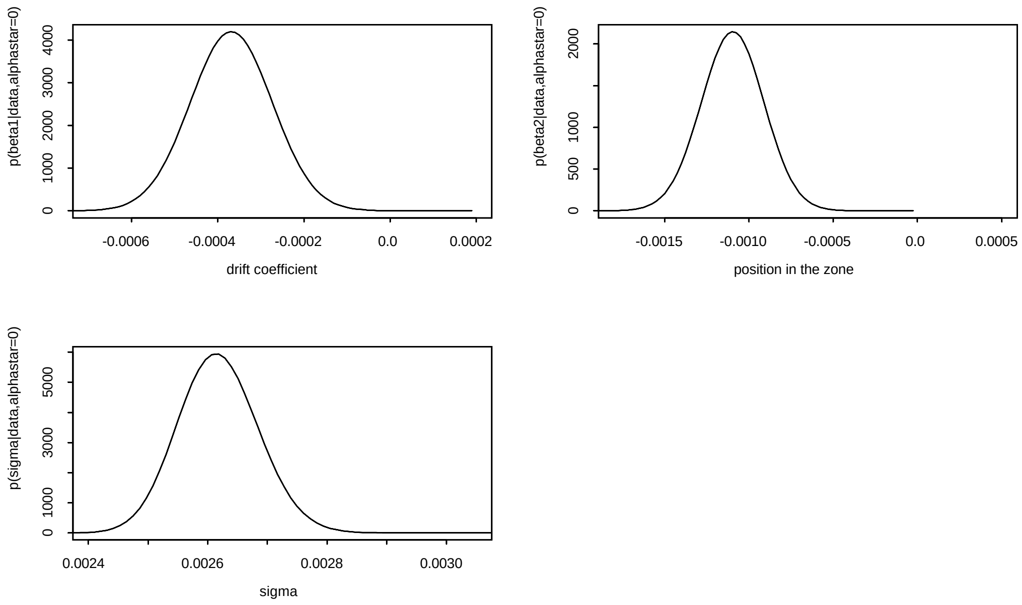


Figure 7: Posterior density functions for parameters with $\alpha^* = 0.0$, corresponding to the fully truncated errors case (DM/FF data).

A measure of target zone credibility

Next, to assess the credibility of the DM/FF target zone given our observed exchange rate series, the model in Section 3 is estimated, where in this case we include α^* as an unknown parameter. Then, 1000 iterations of the Gibbs sampler as described in Section 3 were used, resulting in the Rao-Blackwellised density estimates of the marginal posterior probability densities. The marginal posterior distributions for β_1, β_2 and σ are each roughly symmetric and unimodal. Again, the marginal distributions can be well summarised by their median and upper and lower probability 5%, and these figures are given in column four of Table 1. The marginal posterior distribution for these parameters, including α^*, are shown in Figure 8. Notice that although the 'credibility' of the target zone, measured by $1-\alpha^*$, appears to be quite high, the estimates for the other parameters are noticeably affected. We have also included in Table 1 the posterior quantile estimates of β and σ for the other extreme case of no truncation, that is when $\alpha^* = 1$. In this case, all data points (excluding realignments) are included and all of the errors are normally distributed (without truncation). This corresponds to a (Bayesian) linear regression of the exchange rate returns on the constant and 'position in the zone' variables. The means of the regression coefficient posterior distributions correspond to the ordinary least squares (OLS) estimates for those parameters.

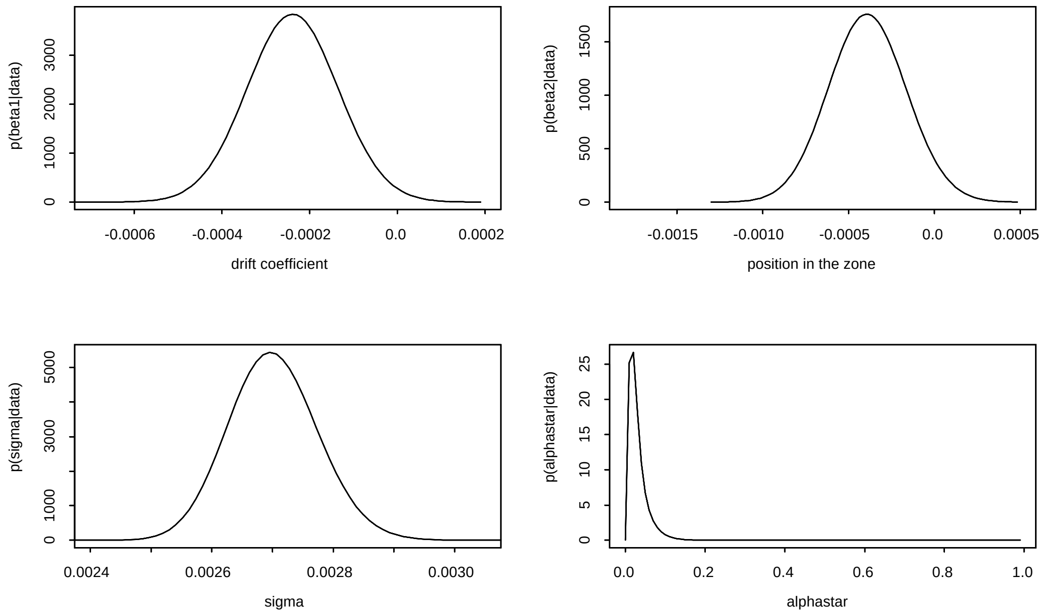


Figure 8: 'Rao-Blackwellised' posterior density estimates the soft target zone model (DM/FF data).

Estimates for the S-shape relationship between the 'position in the zone' variable and the expected error (and hence the implied expected exchange rate) are demonstrated in Figure 9a for the DM/FF data and the soft target zone model. The middle curve is the estimated pointwise expected error over a grid of x_{2t} values and the upper and lower limits are the 0.05 and 0.95 (pointwise) quantiles of this function as computed from the output of the Gibbs sampler. The estimated S-shape under the assumption of $\alpha^* = 0$ is given in 9b. Despite differing parameter estimates, the soft target zone posterior distribution of α^* having virtually all of its probability less than $\alpha^* = 0.1$, results in similar S-shape estimates. Note that under the assumption of $\alpha^* = 1$, the estimated expected error is a linear function of the 'position in the zone' variable, regardless of the parameter estimates for β_1, β_2 and σ.

Parameter	Posterior quantile	Fully credible Target zone $\alpha^*=0$	Soft target Zone $0<\alpha^*<1$	No truncation $\alpha^*=1$
α^*	5%		0.0001	
	50%		0.0201	
	95%		0.0701	
β_1	5%	-0.00053	-0.00041	-0.000467
	50%	-0.00037	-0.00024	-0.000311
	95%	-0.00022	-0.00007	-0.000149
β_2	5%	-0.001425	-0.000778	-0.001123
	50%	-0.001100	-0.000400	-0.000794
	95%	-0.000800	-0.000022	-0.000469
σ	5%	0.002508	0.002574	0.002538
	50%	0.002607	0.002695	0.002648
	95%	0.002728	0.002816	0.002767

Table 1: Posterior quantiles for parameters under three target zone models (DM/FF data).

The same analysis was completed for the ECU/FF data. The Rao-Blackwellised marginal posterior density quantile estimates are given in Table 2. In this case, the marginal posterior distribution for the credibility parameter, α^*, has 90% of its probability in the interval (0.64,0.82), which is remarkably different from the DM/FF case. The resulting estimate of the S-shape expected error function is given in figure 9c. While this figure does show evidence of an S-shape, it also shows linearity inside the stated target zone. This is not in accordance with Honohan (1998), who alleges that the empirical verification of an S-shape for ERM currencies is obscured by the fact that these currencies are regulated against the ECU instead of bilaterally (e.g. DM/FF). Clearly, there is little evidence in our results supporting the French central bank managing its currency against the ECU. The α^* values estimated from this data set are simply too large to have a significant impact on the expected return when the last exchange rate is inside the target zone. However, once outside the stated target zone there does appear to be an effect due to (partial) truncation. Perhaps some intervention did indeed occur once the stated limits were actually exceeded. By that time, the bilateral DM/FF target zone was also exceeded.

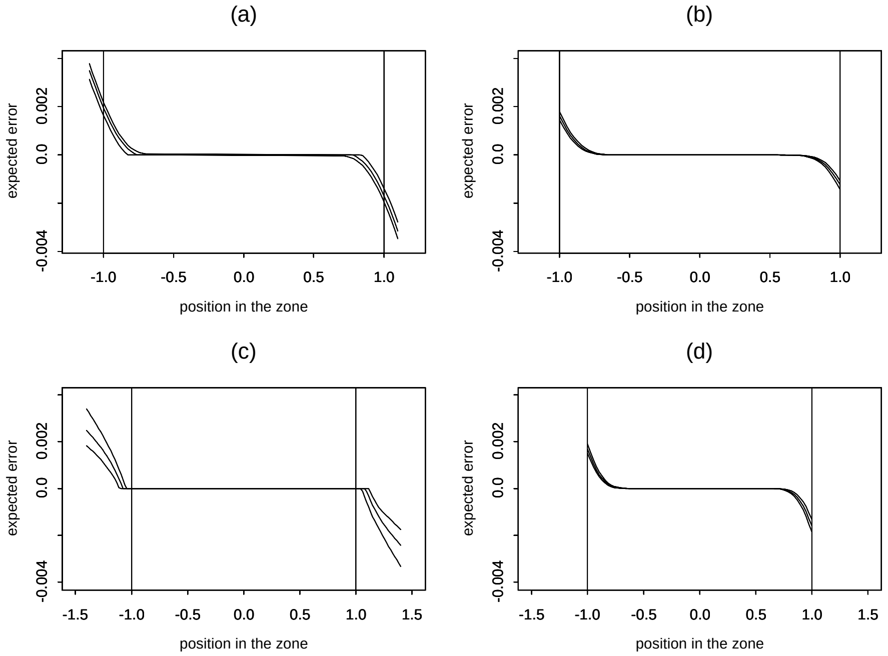


Figure 9: Posterior function estimate of the expected error curve as a function of the position in the zone, $E(\varepsilon_t | S_{t-1}, \beta, \sigma, \alpha^*)$, with lower 5%, 50% and upper 5% pointwise posterior quantiles. (a) DM/FF under soft target zone model; (b) DM/FF under fully credible target zone model (α^* = 0); (c) ECU/FF under soft target zone model; (d) ECU/FF under fully credible target zone model (α^* = 0).

Parameter	Posterior quantile	Fully credible Target zone $\alpha^*=0$	Soft target Zone $0<\alpha^*<1$	No truncation $\alpha^*=1$
α^*	5%		0.639972	
	50%		0.739952	
	95%		0.819936	
β_1	5%	-0.000280	-0.000275	-0.000223
	50%	-0.000100	-0.000100	-0.000061
	95%	+0.000075	+0.000075	+0.000097
β_2	5%	-0.000800	-0.000620	-0.000490
	50%	-0.000440	-0.000260	-0.000195
	95%	-0.000080	-0.000040	+0.000089
σ	5%	0.002300	0.002328	0.002305
	50%	0.002398	0.002426	0.002405
	95%	0.002510	0.002538	0.002513

Table 2: Posterior quantiles for parameters under three target zone models (ECU/FF data).

These results are in contrast with the S-shape estimated under the fully credible model. If we had restricted our analysis to the fully truncated errors case, where $\alpha^*=0$, we would necessarily see an S-shape in our posterior estimates of the expected returns. The width of the pointwise confidence bands would then be a function of the posterior uncertainty in β and σ (conditional on $\alpha^* = 0$), which is necessarily smaller than when $\alpha^* > 0$ due to the exclusion of observations outside the stated target zone. More importantly, these confidence bands are strongly influenced by the imposition of the S-shape inside the stated target zone resulting from full truncation.

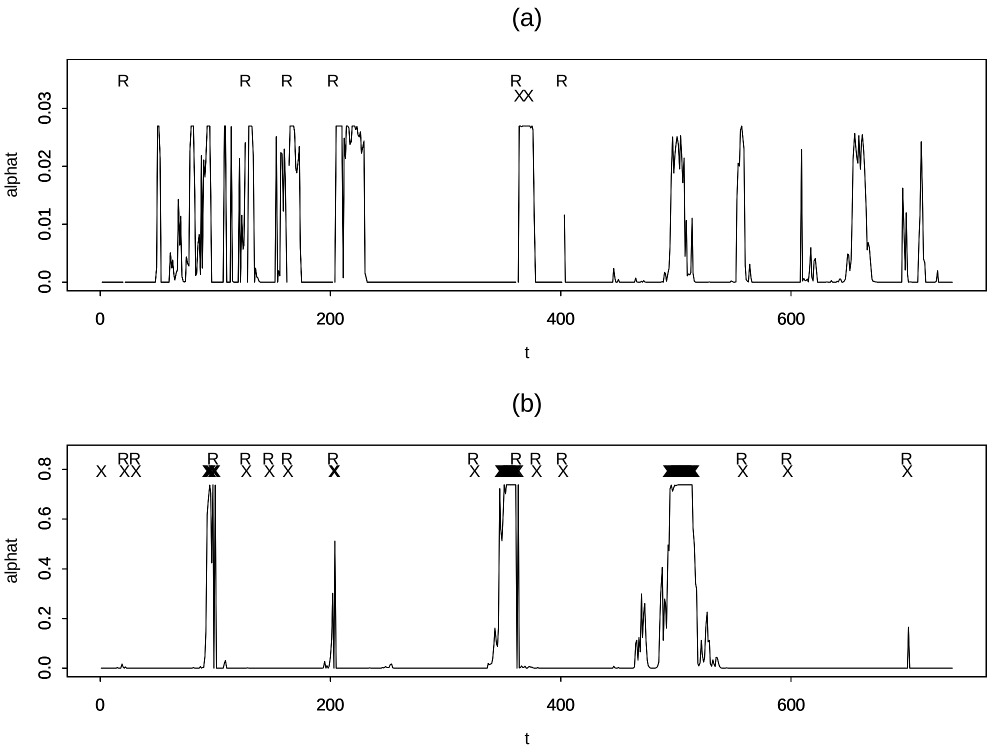


Figure 10: Posterior expected α_t values for (a) DM/FF; and (b) ECU/FF under the soft target zone model. The X-marks indicate actual exceedences of the exchange rate outside the stated target zone. The R-marks indicate actual realignments in central parity.

Finally, in Figure 10 we also report the posterior expected α_t values for the DM/FF in panel (a), and for the ECU/FF in panel (b), for the soft target zone model. These plots further corroborate our suspicion that French franc intervention did not take place against the ECU, but against the Deutschemark instead. Interestingly, the ECU-panel shows much less 'action' (even after taking into account the difference in scale) than the DM-panel. There were many more exceedences in the ECU/FF as compared to the DM/FF. Despite this, the α_t values in Figure 10 seem to indicate that the DM/FF moved closer to the edge of its stated target zone more frequently, without actually exceeding it. The ECU/FF, on the other hand, frequently surpassed the stated limits, without reverting quickly to the target zone. This is in line with the higher estimated credibility and stronger S-shape for the DM/FF than for the ECU/FF.

5 Conclusion

The empirical verification of the non-linear relationship between the observed exchange rate and its fundamental value has been notoriously difficult. So far, very little evidence has been found for a significant S-shape. In this investigation we extend a basic version of the discrete time fully credible target zone model proposed by Bekaert and Gray (1998) to incorporate soft target zones. We describe a means for computing a Bayesian analysis of the proposed model and demonstrate the approach on weekly Deutschemark-French franc and ECU-French franc exchange rate data. Our results point towards evidence of strong non-linearity in the expected exchange rate for the Deutschemark-French franc exchange rate. However, there appears to be little evidence of non-linearity in the ECU-French franc exchange rate. This seems to validate the empirical literature with its focus on bilateral target zones instead of a multilateral (ECU) target zone.

Imposing full truncation of the error terms assumes a perfectly credible target zone. If in fact some data are observed outside the stated zone, those values will necessarily be excluded from an analysis that assumes a perfectly credible zone. As shown in our ECU/FF example, imposing this assumption can result in estimates of the expected exchange rate that will appear to have a high degree of non-linearity inside the stated zone that may not be justifiable if points outside the target zone have been excluded from the analysis.

The model and analysis presented in this paper are a first step towards an empirical soft target zone model. We have necessarily kept the specification of the regression relationship simple so as to focus on incorporating ‘softness’ into the credibility of the stated target zone. The next step is to apply this model to other data sets to gain a greater appreciation for the usefulness of this specification of non-credibility, and in particular to consider the interpretation of the range of values of α^* obtained from this type of analysis.

Incorporating fat-tailed distributions, such as using a t-distribution instead of a normal distribution, would not significantly impact on the current method of computing a

Bayesian analysis. Conditional on knowing the position in the zone, the returns in our target zone model do not have a constant variance. In particular, consider for example the fully credible model with truncated normal errors. The returns having the last exchange rate near the limit of the target zone will have a smaller observed variance than those whose last exchange rate are at the central parity. Adding time-varying volatility is more difficult because the usual specifications, such as GARCH errors, rely on unconstrained error structures. Instead, we would like to give a time varying specification for the volatility for the underlying (non-truncated) error distributions corresponding to the unobscured exchange rate fundamental process. To some extent we control for the typical GARCH behaviour of exchange rate returns by choosing a weekly sampling interval. Conditional heteroskedasticity, while highly significant at a daily sampling frequency, tends to dissipate rapidly under aggregation. Also, it may be useful to relax the specification to allow for greater underlying volatility nearer to the edges of the target zone to account for the uncertainty as to whether or not the central bank will intervene.

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