Interaction of Drastic and Incremental Innovations: Economic Development through Schumpeterian Waves*

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ABSTRACT – Technological progress as a major source of economic development stems from the interaction of two types of innovations, drastic and incremental. While the former sets the fundamental pace of economic progress by redefining production possibilities as Schumpeter strongly emphasized, the latter takes the basic framework as given but pushes the production possibilities frontier outwards marginally in production practice. This paper studies the dynamic interaction and effects of these endogenously-determined innovations. Upstream firms in the model "produce" drastic innovation, which turns out brand new technology and obsolesces the existing technology used by downstream firms that specialize in final goods production. After the downstream firms adopt the new technology, they can improve it further by their incremental innovations. Economic development is shaped as successive Schumpeterian waves, where each wave begins with a great leap forward in technology which is followed by a sequence of adjustments. It is found that a rise in the success probability of future drastic innovations will discourage current efforts at drastic innovation but stimulate incremental innovations. More effort by downstream firms in incremental R&D reduces upstream firms' incentives for drastic R&D. The model ensures at least one dynamic equilibrium. In the case of multiple equilibria, an equilibrium with larger investment in drastic innovation has less expenditure on incremental innovation. The comparative static analysis shows that a reduction in the expansiveness of drastic innovation, and an increase in the total sales of downstream firms and in the significance of drastic innovation will raise (reduce) drastic (incremental) R&D efforts in stationary equilibrium.

KEYWORDS: Drastic and Incremental Innovations, Development, Schumpeterian Dynamics.

JEL CLASSIFICATION: O31

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… to Professor Schumpeter, business cycles are pulsations of the rate of economic evolution.

Simon Kuznets (1954)

1. Background

In his *Theory of Economic Development*, Schumpeter (1934) presented the working of a capitalistic economy as an evolutionary process, where the business cycle results from the introduction and integration of innovations. The cycle is initiated, or triggered, by a major innovation. The initial innovation (or cluster of such innovations) is then followed by a massive diffusion of smaller innovations that exploit directly or indirectly the profit potential "announced" by the initial radical change. These further adjustments typically take place during the expansion of activity when uncertainty is reduced. When the novelty is exhausted the economic system tends toward a new equilibrium position with a higher level of welfare than the previous one. The industrial transformation that takes place during the cycle encourages firms to use new forms of production technologies, organizational structures, etc., and ill-adapted firms are eliminated.

Two important features of innovation-driven business cycles should be underlined here. The first is that economic fluctuations, or the business cycle, as presented above, was understood by Schumpeter to be *an analytical unit of reference*, a dynamic unit, (just as the equilibrium concept is a static reference point), rather than a mechanical outcome of a parameter set as in some macroeconomic models. It is the basic process by which the economy regenerates itself and reallocates resources in the integration of new information. The second feature, closely related to the first, is that the business cycle, expressing the transformation of production and markets, is in fact shaped by the *interaction* of two asymmetric and contradictory forces: creation or development on the one hand and adjustment or allocation on the other. Both "creative destruction", where new firms, families and forms of life arise through the introduction of innovations while the older forms decline in the social-economic strata, *and* the more gradual adaptation of economic structure (in Schumpeter’s terms, the “tendency towards equilibrium”) should be regarded as an outcome of the basic interplay between development and allocation processes.

While the original novelty is attributed to the action of the entrepreneur who is obviously less risk adverse than other agents and is capable of anticipating the direction of change, other firms that follow require less entrepreneurial capability. The diffusion process that finally exhausts the profit opportunities created by the initial innovation largely relies on the adjustment behavior of rational price-taking agents. Thus the “representative agents” of these two facets of economic activities - the entrepreneur and the manager, or alternatively the innovator and the imitator, are characterized by different states of information and related uncertainty. They obey a different economic rationale and operate with different global outcomes (positive and zero-sum games, respectively) and the basic sense of norms and deviations (for example, the role of market failures) is also very different.

Rosenberg (1976, 1982) expressed the composition of the two types of innovation in several of his path-breaking works. Although he did not deny the contribution of major innovation to economic development and growth, he claimed that revolutionary innovation is in fact materialized through a sequence of gradual and cumulative development steps. He presented a famous example of the car industry to argue that, although the private automobile was certainly one of the great driving forces of the industrialization in the early 20th century, it was not really (economically) present when few mechanical toys (the first automobiles) terrified horses in the countryside. The revolution was only effective once Ford’s chain production reduced costs and prices of car manufacture and made the automobile available to millions, gas stations sprung up along the highways and the whole suburban lifestyle developed.

Nelson and Winter’s “technological trajectories” (1977) contributed to a better understanding of the relation between successive innovations and the overall pattern of the constitution and development of a new technology, which is in fact a continuous representation of a major innovation. Subsequent works by Sahal (1981), Dosi (1982), Freeman et al. (1982), and Zuscovitch (1986) presented conceptual frameworks in which drastic innovations redefine the boundary of overall possibilities for economic agents through a paradigm shift. Agents then perform "normal" innovations which improve productivity and develop and exhaust the potential of the novelty until the next jump. Although these works undoubtedly offer an intellectually appealing framework for the endogenizing of major innovations in the form of a process of technological development, they remain within the sphere of what Nelson calls "appreciative theory".

New growth theories have focused for more than a decade on the relationship between technical change and growth. As is common in the neoclassical tradition, the representation of innovation is basically restricted to a process of continuous specialization via the division of labor in the Smithian tradition (Romer, 1986, 1990). Grossman and Helpman (1991) added some qualitative features to the analysis of innovation but essentially are guided by the framework of smooth growth without any jumps and discontinuities that are an essential feature of the growth process.

The new-Schumpeterian approach formalizing several Schumpeter's views of growth and innovation recognizes the contribution of both drastic and incremental innovations to economic growth and fluctuation. The work on creative destruction by Aghion and Howitt (1992) introduced a strong depreciation effect of drastic innovation on existing technology without, however, an analysis of the impact on subsequent innovations. The models of general purpose technologies (Helpman and Trajtenberg, 1998; Aghion and Howitt, 1998, ch. 8) and swarming mechanism (Justman, 1996, 1997) studied the growth and fluctuation caused by a sequence of incremental innovations resulting from an exogenously-determined drastic innovation. Jovanovic and Rob (1990), and Cheng and Dinopoulos (1996) endogenized both breakthroughs and improvements. But in their models, the economy can have only one type of innovation at any instant in time and improvements have no any effects on breakthroughs. Moreover, the Cheng-Dinopoulos model relies on a parameter indexing the exogenously accumulated basic research and scientific knowledge.

In the present work we focus on the interaction and duality between drastic and incremental innovations. The model represents the two features of innovation-driven business cycles highlighted earlier. The first, due to its revolutionary nature, changes the course of dynamics by defining a new framework for the productive system and redefines the basis upon which economic agents compute their action plans. The second is of an incremental and cumulative nature and each step is governed by computable investment decisions according to the prevailing basic structure. The first type of behavior is indeed historically more associated with Schumpeter where innovation "pushes" economic activities. The behavior pattern of incremental innovation and gradual development is more associated with Smithian specialization through the division of labor, and also with Schmookler's (1966) "demand pull" hypothesis where the innovative activities are attracted and oriented by the relative prices and more general economic activities. However, neither the revolutionary nor the incremental behavioral pattern is a consistent way to view the dynamics of innovation.

In order to grasp the forces that govern the interaction we treat drastic innovations as the endogenous result of the investment of upstream firms. After a new drastic innovation succeeds, each downstream firm devotes their efforts in a series of incremental innovations which can further increase their production efficiency and make them more suitable to the new environment. Within this framework, a drastic innovation opens new opportunities for incremental innovations on the one hand and incremental innovations affect the profitability of next new drastic innovation and upstream firms' R&D decisions on the other hand. In a technical sense, our model synthesizes and generalizes the case of creative destruction imposing obsolescence on the productive structure à la Aghion and Howitt (1992) with subsequent adjustments as suggested by the general purpose technologies approach (Helpman and Trajtenberg, 1998; Aghion and Howitt, 1998, ch. 8).

In the rest of the paper, section 2 presents a basic model exploring the duality between drastic and incremental innovations and illustrates how drastic innovations surge Schumpeterian business cycles and how the economy grows wavily. The model shows that upstream firms reduce their efforts at drastic R&D but downstream firms strengthen their efforts at incremental R&D if each expects the next drastic innovation to arrive sooner. On the other hand, upstream firms have less incentive to do drastic innovations if downstream firms invest more in incremental R&D. Section 3 proves that the model ensures at least one stationary equilibrium. In the case of multiple equilibria, an equilibrium with larger investment in drastic innovation has less expenditure on incremental innovation. Comparative dynamics in section 3 shows that a reduction in the expansiveness of drastic innovation, and an increase in the total sales of downstream firms and in the significance of drastic innovation will raise (reduce) drastic (incremental) R&D efforts in stationary equilibrium. The basic model is then extended to account for learning by doing and the physical capital costs of drastic innovations.

2. A Basic Model

2.1 Technologies and Innovations

In an industry, technology and physical capital are provided by an upstream incumbent. A drastic new innovation invented by an upstream firm completely obsolesces the current production process and capital stock of n downstream firms that engage in monopolistic competition in the final goods market. Therefore, the drastic innovation introduces a creative destruction, which redefines the production possibilities frontier as argued by, amongst others, Dosi (1982) and Zuscovitch (1986). Time is continuous and t_t, indexing the moment the tth drastic technology is realized, is a random variable since drastic innovations are uncertain. After the adoption of the drastically new technology, each downstream firm can improve their technology efficiency by successive incremental innovations of their own until the next drastic innovation arrives. In the time interval $t \in [t_t, t_{t+1})$, downstream firm i has a stepwise marginal cost function

$$c_i(t) = A_t \Gamma_i(t) . \tag{1}$$

In the function, A_t, resulting from the drastic innovation, is universal to all downstream firms, and technical progress achieved by drastic innovation entails its decline as t

increases. $\Gamma_i(t)$ is determined by firm *i*'s incremental R&D effort. Incremental innovations are assumed to be deterministic and take a certain length of time to fully materialize. This assumption depicts the notion discussed earlier that the adjustment process requires less entrepreneurial capabilities and is more certain in contrast to drastic innovation. To simplify notation, we assume that all incremental innovations have the same time delay, equal to a single time period. Therefore $\Gamma_i(t)$ evolves as

$$\Gamma_i(t) = \Gamma_{its} = g_{ts}\Gamma_{it(s-1)} \qquad \text{for } t \in [t_t + s, t_t + s + 1),\ s=1,2,\ldots \tag{2}$$

$$\Gamma_{it0} = 1,$$

where $g_{ts} < 1$ is the contribution of the *s*th incremental innovation to cost reduction in the *t*th Schumpeterian wave. Since incremental R&D is performed over a one-period time lag, the R&D production is

$$g_{ts} = g_{ts}(y_{its}), \tag{3}$$

where y_{its} is firm *i*'s R&D investment flow in $[t_t + s - 1, t_t + s)$. The formula implies that the productivity of the investment in incremental R&D might vary over different periods but it is identical across all downstream firms. A reasonable assumption about R&D production is that it has positive marginal returns but the gain is limited, i.e., $g_{ts}(\infty) = \bar{g}_{ts}$. Since the innovation is incremental, $\bar{g}_{ts}$ is much smaller than A_t / A_{t-1}.

This specification of technology illustrates that technological progress is in the form of a big leap forward, initiating a Schumpeterian wave, followed by a series of small steps of efficiency improvements. Another drastic innovation yields a new Schumpeterian wave. It captures part of Rosenberg's discussion (1976) of the relationship between the "announce effect" of the major innovation and the subsequent adjustments. Although the model shares some characteristics with the works by Aghion and Howitt (1992) and the models of general purpose technologies (Helpman and Trajtenberg, 1998; Aghion and Howitt, 1998, ch. 8), it goes beyond them. In the Aghion-Howitt model, downstream firms do not do any research and technological progress is generated solely by the breakthroughs of upstream firms. The general purpose technologies models presume drastic innovation are exogeneously given and incremental innovations have no impacts on them. In this sense, the present model extends their discussions to a more general setting. Furthermore, if g_{ts} is achieved by learning-by-doing (LBD) instead of by incremental innovation, the model resembles the works of Young (1993) and Stern (1997) where the LBD process, after each innovation, can further improve production efficiency. LBD is discussed in the final section.

Drastic innovations have two implications in the model. The first is that it is not firm-specific; that is, if a new drastic innovation succeeds and is adopted by downstream firms, all these firms have the same technological improvement. The second is creative destruction, in the sense that if a new drastic innovation is materialized, not only does it make previous technology and physical capital obsolete, but it also renders previous incremental innovation meaningless. This latter characteristic is reflected in the non-accumulation of $\Gamma_i(t)$ across Schumpeterian waves, i.e., after each drastic innovation $\Gamma_i(t)$ starts from $\Gamma_{it0} = 1$. In contrast, the effects of incremental innovation on efficiency are cumulative so that $\Gamma_i(t)$ continuously declines in a stepwise manner within one Schumpeterian wave. The cumulative effects exist until a new generation of technology replaces the old. In fact, much literature in the economics of innovation distinguishes between drastic and incremental innovations on the basis of the accumulation or otherwise of innovation effects. By comparison, Katz and Shapiro (1987) argued that a minor innovation marginally reduces each firm’s cost by the same amount and is irrelevant to their existing technology difference but a major innovation sweeps off existing difference and leads to the same post-innovation production cost. Moreover, incremental innovation could be made firm-specific in the model and therefore imitating other firms’ outcomes of incremental R&D could cost time and money. In order to keep the model simple, however, we assume that each downstream firm can only keep their incremental innovation secret for one period and after that period rival firms can costlessly imitate the innovation without any further time delay.[1]

2.2 Decisions of Downstream Firms

Within each Schumpeterian wave, market equilibrium is determined by a Stackelberg-like process, where all upstream firms are leaders announcing their investment in drastic innovation first while all downstream firms are followers who take the upstream firms' R&D expenditures as given in their decisions.[2] Downstream firms make two decisions: price/output and investment in incremental innovations. The demand for each output is assumed to have a constant elasticity of substitution, $1/(1-q)$, i.e.,

$$q_i(t) = Ip_i(t)^{1/(q-1)} P(t)^{q/(q-1)}, \tag{4}$$

where I is a constant determined by aggregate demand and the number of downstream firms, $p_i(t)$ is the price of firm i's product and the price index is defined as

$$P(t) = \left[\frac{1}{n}\sum_{i=1}^{n} p_i(t)^{q/(q-1)}\right]^{(q-1)/q}. \tag{5}$$

Assuming the number of downstream firms is large enough so that price competition leads to a constant markup $1/q$ and the equilibrium price is equal to

$$p_i(t) = c_i(t)/q, \tag{6}$$

given the profile of marginal costs $c_i(t)$ (i=1,2,…n). Since marginal costs decline stepwise as technology is improved, so do prices.

The investment in incremental innovation is determined by the gains it generates. Since an innovative firm can maintain its efficiency superiority for one

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[1] This simplification is in contrast to Yin and Zuscovitch (1998) who argue that technology is not completely transferable. Therefore, a firm cannot apply another firm's innovations without its own R&D efforts. In that case we should introduce imitation cost and time delay.

[2] However, the competition among upstream and downstream firms differs from the standard Stackelberg model.

period, downstream firm i needs only to consider the profit reward from incremental innovation in this period. It receives the profit flow (gross of R&D investment)

$$p_i(t) = (1-q)IP(t)^{q/(1-q)}\left[c_i(t)/q\right]^{q/(q-1)} \tag{7}$$

in period $[t_t + s, t_t + s + 1)$ and incurs a cost flow y_{its} in period $[t_t + s - 1, t_t + s)$. As the probability that the next (the $(t$+1)th) drastic innovation will not succeed until t is $\exp\{-lH_{t+1}(t - t_t)\}$,[3] the firm chooses y_{its} to maximize

$$\begin{aligned} Ep_{its} = & \int_{t_t+s}^{t_t+s+1} (1-q)IP(t)^{q/(1-q)}\left[\Gamma_{it(s-1)}g(y_{its})/q\right]^{q/(q-1)} \exp\{-(r + lH_{t+1})(t - t_t)\}dt \\ & - \int_{t_t+s-1}^{t_t+s} y_{its} \exp\{-(r + lH_{t+1})(t - t_t)\}dt, \end{aligned} \tag{8}$$

where r is the interest rate. By ignoring the effect of a firm's marginal cost on the price index, symmetric equilibrium is characterized by the first-order condition[4]

$$-(1-q)I\, g'_{ts}(y_{ts}) \exp\{-(r + lH_{t+1})\}/g_{ts}(y_{ts}) \le 1,$$

$$y_{ts} \ge 0 \qquad \text{with at least one equality.} \qquad s=1, 2, \ldots \tag{9}$$

The condition implies that the expected marginal return of incremental innovation must be equal to its expected cost in the case of positive investment. A standard assumption on the investment in incremental innovation is diminishing returns so that $g'_{ts}(\cdot)$ is a monotonically increasing function and $g'_{ts}(+\infty) = 0$. This assumption ensures an interior equilibrium if it exists.

As is obvious from the first-order condition, downstream firms' investment in incremental innovation declines if upstream firms invest more in drastic innovation. The reason is simple. When upstream firms spend more money on the new generation of technology, the hazard rate is larger, i.e., it will succeed sooner. Since it will obsolesce current technology and efficiency efforts completely, the expected return from incremental innovation based on the current generation of technology is lower. This discourages downstream firms' involvement in incremental innovation. If, for example, biotechnological breakthroughs appear at a faster rate (Grabowski, 1998), this obviously

[3] Hazard rate H_{t+1} is constant in these periods and positively relates to upstream firms' investment in the $(t$+1)th drastic innovation. We will show how to obtain the probability in detail below.

[4] A variable without subscript i indicates its symmetric counterpart.

reduces the investments of pharmaceutical firms in gradual improvements in chemical-based medicines.

The model also shows that investment expenditures on incremental innovation for different periods are independent. This is a special property stemming jointly from the symmetry and the coincidence of the length of time-delay for an incremental R&D with the duration of its appropriation. This property ensures that expenditure on incremental innovation in each period is equal if the R&D production functions, $g_{ts}(\cdot)$, are identical across periods. However, a more realistic assumption is that $g_{ts}(\cdot)$ declines as s increases which implies that efficiency improvement potential is gradually exhausted.

2.3 Decisions of Upstream Firms

The success of drastic innovation is assumed to be uncertain so that when a firm wins the race of drastic innovation competition, i.e., becomes the first inventor, it monopolizes the new technology until the technology is phased out by another newer drastic innovation.[5] The monopolist charges the same lump-sum license fee to all downstream firms for access to the new technology. Then the question is what license fee will each downstream firm be willing to pay for the tth radical innovation? It is clear that they are willing to pay up to the difference of two expected profit streams: the profit of adopting the tth drastic innovation from t_t onwards without any further incremental innovation, and the profit of using the existing technology at time t_t without further incremental innovation. This profit difference reflects the net gain of a downstream firm achieved by giving up the current production method and adopting a radically new one. In the profit difference, gains from incremental innovation should be excluded because they are the contribution of a downstream firm's own R&D efforts.

From (1) and (2), the marginal cost achieved by the tth drastic innovation is A_t. But if a firm still uses the old generation of technology updated until time t_t, it has a

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[5] This can be thought of as a permanent patent race for a drastic innovation.

marginal cost $\bar{c}_{t-1} \equiv A_{t-1}\Gamma_{(t-1)T(t-1)}$, where $T(t-1)$ is the largest integer of $(t_t - t_{t-1})$. Therefore, if k downstream firms adopt the tth generation of technology while the remaining firms continue to use the old technology, the profit flow from adopting the new technology is given by:[6]

$$l_t(k,t) = (1-q)IP(k,t)^{q/(1-q)}\left\{(A_t/q)^{q/(1-q)} - (\bar{c}_{t-1}/q)^{q/(1-q)}\right\}, \tag{10}$$

where

$$P(k,t) = \left[(k/n)(A_t/q)^{q/(q-1)} - ((n-k)/n)(\bar{c}_{t-1}/q)^{q/(q-1)}\right]^{(q-1)/q}. \tag{5a}$$

With k licenses allowing access to the tth generation of technology, an upstream monopolist can receive total license fees equal to the flow $kl_t(k,t)$. To maximize the total license fees, it chooses k by solving

$$\max kl_t(k,t), \tag{11}$$

which gives the optimum k=n. This means that the monopolist always grants all downstream firms a license. The total license fees it can reap is $nl_t(n,t) = n(1-q)I\left[1-(\bar{c}_{t-1}/A_t)^{q/(q-1)}\right]$.

Since drastic innovation is uncertain, the time of upstream firm j successfully realizing the tth drastic innovation, $t_{jt} \in (t_{t-1}, \infty)$, is a continuously-distributed random variable. Following Lee and Wilde (1980) and Reinganum (1983), if firm j invests a capital flow z_{jt} in the R&D project, starting from time t_{t-1}, the probability that the firm will succeed in implementing the innovation at or before t_t is assumed to be

$$\Pr\{t_{jt} - t_{t-1} \le t_t - t_{t-1}\} = 1 - \exp\{-lh_t(z_{jt})(t_t - t_{t-1})\}. \quad t_t > t_{t-1} \tag{12}$$

Drastic innovation also has positive but diminishing returns, i.e., $h'(\cdot) > 0$ and $h''(\cdot) < 0$. Furthermore, it is assumed that $h(0) = \lim_{z\to\infty} h'(z) = 0$. For such an exponential distribution, the probability that no upstream firm will succeed until t_t but firm j succeeds in the next instant dt_t is $h_t(z_{jt})\exp\{-lH_t(t_t - t_{t-1})\}$, where

$$H_t = \sum_{j=1}^{m} h_t(z_{jt}) \tag{13}$$

[6] This simple expression depends on monopolistic competition so that $P(k,t) \cong P(k+1,t)$. For an oligopoly, the profit difference is more complex due to an integer problem (see Kamien and Tauman, 1986).

is the sum of hazard rates. Because each downstream firm is willing to pay a license fee flow $l_t(n,t)$ from t_t to t_{t+1}, the upstream monopolist's expected license revenue from the *t*th drastic innovation is

$$L_t = \int_{t_t}^{\infty} n(1-q)I\left[1-\left(\bar{c}_{t-1}/A_t\right)^{q/(q-1)}\right]\exp\{-(r+lH_{t+1})(t_{t+1}-t_t)\}dt_{t+1}$$
$$= n(1-q)I\left[1-\left(\bar{c}_{t-1}/A_t\right)^{q/(q-1)}\right]/(r+lH_{t+1}). \quad (14)$$

It should be noted that L_t is a decreasing function of t_t because $\bar{c}_{t-1}$ declines stepwise as t_t increases. The intuition is clear. If the winning upstream firm succeeds in its search for the *t*th drastic innovation later, it should expect smaller license fees since the delay leads to a smaller gap of marginal costs between old and new technologies. To ensure the model is tractable, we make an approximation on L_t. Let $t*_t$ be the expected time of the *t*th drastic innovation succeeding. When upstream firms make their investment decisions at time t_{t-1}, they expect downstream firms to have a marginal cost $\bar{c}*_{t-1} = A_{t-1}\Gamma_{(t-1)T*(t-1)}$, where $T*(t-1)$ is the largest integer of $(t*_t - t_{t-1})$. In other words, they use $\bar{c}*_{t-1}$ to approximate $\bar{c}_{t-1}$ in (14) to calculate license revenue. This approximation implies that upstream firms are indifferent to marginal variations in downstream firms' marginal costs due to the timing of drastic innovation success and the consequent variations in license revenue. What they are concerned with is how to win the drastic R&D race. Therefore, upstream firm *j* chooses investment strategy z_{jt} to maximize[7]

$$\int_{t_{t-1}}^{\infty}\left[L_t\, lh_t\left(z_{jt}\right)-z_{jt}\right]\exp\{-(r+lH_t)(t_t-t_{t-1})\}dt_t = \left[L_t\, lh_t\left(z_{jt}\right)-z_{jt}\right]/(r+lH_t). \quad (15)$$

When all upstream firms have a common prediction of H_{t+1}, the first-order condition, characterizing symmetric equilibrium z_t, is:

$$[\,lh'_t(z_t)V_t(z_t)/(r+lH_{t+1})-1][(m-1)\,lh_t(z_t)+r]-l[h_t(z_t)-z_t h'_t(z_t)]\le 0$$
$$z_t \ge 0 \qquad \text{with at least one equality,} \qquad t=1, 2, \ldots \quad (16)$$

where

$$V_t(z_t) \equiv n(1-q)I\left[1-\left(\bar{c}*_{t-1}/A_t\right)^{q/(q-1)}\right] \quad (17)$$

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[7] In this setting, physical capital costs are implicitly included in the costs of drastic innovation. We discuss the case where physical costs occur after innovation success in the final section.

is the profit stream of downstream firms, obtainable by adopting a new drastic technology.

2.4 Equilibrium

Given H_{t+1}, which is determined by the investment of upstream firms in the tth Schumpeterian wave, condition (16) provides their equilibrium investment in the tth drastic innovation, z_t,[8] taking the reaction of downstream firms (9) into account. Since $H_t = mh(z_t)$, downstream firms know the probability distribution of the tth drastic innovation succeeding since z_t is known. They therefore choose their incremental R&D investment in the (t-1)th Schumpeterian wave according to (9).[9] Equilibrium price and output in the goods market are determined by (1), (4) and (6).

Economic dynamics is thus characterized by a sequence of waves initiated by technology jumps of drastic innovations with substantial reallocation of resources and reorganization of markets. These waves represent real stages in economic development as Schumpeter (1939) and Kuznets (1954) emphasized. Within each wave a series of progressive efficiency improvements mildly change resource allocation at each adjusting step, marginally push the production frontier outward and moderately adjust production and markets in a cumulative manner. In this sense, the model consolidates Schumpeter's analytical construction of innovation-triggered business cycles.

Proposition 1. *Given H_{t+1}, there exist equilibrium investment decisions $\{y_{ts}\}$ (s=1, 2,…) and z_t that satisfy (9) and (16).*

For the proof, see the appendix.

An obvious characteristic of the model is that decisions of up- and downstream firms in each wave are interactive in conjunction with the expectation of the next drastic

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[8] z_t is decided at the beginning of the (t-1)th Schumpeterian wave and maintained during that wave.

[9] (9) is deduced for the tth wave. When it is applied to the (t-1) wave, the subscripts in (9) should be adjusted accordingly.

innovation. If upstream firms expect that the (t+1)th drastic innovation will succeed sooner due to higher investment, i.e., H_{t+1} is larger, they will reduce their R&D expenditures on the current drastic innovation (the tth innovation). The reason is that if the (t+1)th drastic innovation comes sooner, downstream firms will be willing to pay a smaller license fee for the tth drastic innovation since it may last a shorter period (see Figure 1 for the time structure). This is clear from (17) since for the same stream of profit difference V_t, a larger H_{t+1} implies a smaller license fee L_t. With a smaller license fee, upstream firms have less incentive to win the race of the tth drastic innovation as indicated by (15) so that they will invest less in the tth drastic innovation as required by (16).[10]

FIGURE 1 IS ABOUT HERE

So far, only the first round effects have been accounted for. When upstream firms reduce their investment in the tth drastic innovation, the downstream firms will expect that the t-generation technology will arrive later so their incentive to engage in incremental innovation based on the (t-1)-generation technology increases. According to (9), they will increase their investment series $y_{(t-1)s}$. Thus $\bar{c}^*_{t-1}$ will be reduced as downstream firms realize a greater efficiency improvement and V_t will decline.[11] A smaller V_t will naturally prevent upstream firms from investing further in the tth drastic innovation. From this analysis, we can draw the following proposition:

Proposition 2. *If all firms expect an increase in the probability of success of the (t+1)th drastic innovation, upstream firms will reduce their R&D investment in the tth drastic innovation but downstream firms will increase their incremental innovation activities to improve the efficiency in the (t-1)th Schumpeterian wave. Moreover, an increase in incremental innovation expenditure by downstream firms in the (t-1)th wave will reduce upstream firms' incentive for the tth drastic innovation further.*

[10] Holding V_t constant, the equality of (16) implicitly defines z_t as a decreasing function of H_{t+1}.

[11] Another effect that reduces V_t is the longer expected time of the tth drastic innovation to succeed.

The proposition vividly illustrates the duality between drastic and incremental innovations. On the one hand, a drastic innovation provides a new platform to perform incremental innovations. If managers of downstream firms think that this new platform will come soon they prefer to save resources now and to work on the new platform once it arrives. On the other hand, a more efficient exiting technology is more difficult to abandon. Today's efficiency could be an obstacle to tomorrow's breakthrough in development. Rosenberg (1976), for instance, describes how the steam ships have stimulated a last wave of perfection design of the clippers. The play of overlapping technological generation is a fascinating issue per se, and substitution and complementary relations need to be better explored.

3. Stationary Equilibrium

In this section, we focus on stationary equilibrium, where the investment series $\{y_{ts}\}$ and $\{z_t\}$ are wave-invariant such that $y_{ts} = y_{(t+1)s}$ and $z_t = z_{t+1}$.[12] Thus the R&D investment profits across Schumpeterian waves are identical. To ensure the existence of stationary equilibrium, we assume that each drastic innovation results in the same radical jump in technology, i.e.,

$$A_t = aA_{t-1} = a^t A_1, \tag{18}$$

and that the R&D productivity of incremental innovation based upon each drastic innovation is identical, i.e., $g_{ts}(\cdot) = g_{(t-1)s}(\cdot) = g_s(\cdot)$. To simplify the notation, it is further assumed that there exists an integer S such that $g_s(\cdot) = g(\cdot)$ (with $g'(0) = -\infty$) for all $s \le S$ and $g_s(\cdot) \equiv 0$ when $s > S$; that is, in the first S periods, downstream firms have the same production function for incremental R&D but after that they cannot improve efficiency at all. In reality, we might observe that right after a drastically new technology has been introduced efficiency improvements are relatively easy to make. Then the efficiency potential of the new generation technology is gradually exhausted and efficiency improvement is more and more difficult until it becomes economically

[12] A variable without subscript t and/or s is wave- and/or period-invariant.

infeasible.[13] Our simplification is an approximation of this process with a switch off in period *S*.

Noting the assumptions on the production function of incremental innovation, $g(\cdot)$,[14] the stationary equilibrium *y*>0 for any given $H = mh(z)$ is uniquely characterized by

$$-(1-q)I\, g'(y)\exp\{-(r+\ lH)\}/g(y) = 1 \tag{9a}$$

for each period $s \leq S$. In these periods, each downstream firm has an efficiency improvement $g = g(y)$. Since $-g'(\cdot)/g(\cdot)$ is a decreasing function, *y* declines as *H* increases. But when $s > S$, *y*=0 so that goods production has no efficiency update.

Turning to drastic innovation, the first-order condition (16) can be rewritten as

$$[r+\ lmh(z)][m\ lh(z) - z\ lh'(z) + r]/\ lh'(z)[(m-1)\ lh(z) + r] = V(z), \tag{16a}$$

where

$$V(z) = \begin{cases} n(1-q)I\left[1-\left(g^{\left[\frac{1}{lH}\right]}/a\right)^{\frac{q}{q-1}}\right] & \text{if } \left[\frac{1}{lH}\right] \leq S \\ n(1-q)I\left[1-\left(g^{S}/a\right)^{\frac{q}{q-1}}\right] & \text{if } \left[\frac{1}{lH}\right] > S \end{cases}. \tag{17a}$$

In (17a), $\left[\frac{1}{lH}\right]$ indexes the largest integer of 1/ *lH*, which is the expected time for a drastic innovation to succeed or the expected life of a generation of technology. Variable *z* affects *V*(*z*) in two ways. First, an increase in *z* reduces the expected life 1/ *lH* since $H = mh(z)$. When $\left[\frac{1}{lH}\right] \leq S$, the increase results in a larger *V* if g remains constant. The intuition is that for a shorter expected life of drastic innovations, downstream firms have less frequent opportunities to improve efficiency. Consequently, at the time that a drastically new technology is introduced, the gap between old and new technology is larger. Second, because $g = g(y)$ and *y* is a decreasing function of *z*

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[13] In his recent contribution, Weitzman (1997) provides an interesting explanation for this process of potential exhaustion by pointing out that the new principle innovation is crossed with existing technical objects.

[14] They are: $g(0)=0$, $g(+\infty)=\bar{g}$, $g'(0)=-\infty$, $g'(+\infty)=0$, $g'(\cdot)<0$ and $g''(\cdot)>0$. An example of such a function is $g(y)=(1-\bar{g})\exp\{y^{\frac{-1}{2}}\}+\bar{g}$.

determined by (9a), an increase in z discourages the downstream firms' efforts at incremental innovation, and in turn increases g and V.

Denoting the left-hand side of (16a) as $W(z)$, it is easy to show that $W(z)$ is a continuously increasing function with $W(0) \le 0$ and $\lim_{z \to +\infty} W(z) = +\infty$. On the other hand, (17a) shows that $V(z)$ is a piece-continuous increasing function with

$$V_{\min} \equiv \min V(z) = n(1-q)I\left[1-\left(g_0^S/a\right)^{q/(q-1)}\right] > 0,$$

$$V_{\max} \equiv \max V(z) = n(1-q)I\left(1-a^{q/(1-q)}\right) > 0,$$

where $g_0 = g(y_0)$ and y_0 is the solution of (9a) for H=0. Let $\bar{z}_s$ be implicitly defined as $lmh(\bar{z}_s) = s$ for s=1, 2, …, S. Figure 2 graphically illustrates $V(z)$ and $W(z)$.

FIGURE 2 IS ABOUT HERE

Proposition 3. *The industry has at least one stationary equilibrium. In the case of multiple equilibria, an equilibrium with larger investment in drastic innovation has less expenditure on incremental innovation, and vice versa.*

Proof: Noting that $W(z)$ is a monotonic increasing function, the proof is similar to proposition 1 and we omit it here.

The dynamic time path of production costs, prices and outputs of downstream firms consists of a series of big random jumps and a number of small non-random steps between any two adjacent jumps. More specifically, assume that the industry is established by the first drastic innovation and define $t_1 = 0$. Then $c(t)$, $p(t)$ and $q(t)$ start from $c(0) = A_1$, $p(0) = A_1/q$ and $q(0) = IA_1/q$ according to (1)-(2) and (4)-(6). Before the next drastic innovation arrives, $c(t)$, $p(t)$ and $q(t)$ are constant except for steps at t=1,2,…,S. The size of each step in percentage terms (i.e., $\Delta c/c$, etc.) is $g = g(y)$, where y is the solution of (9a) and (16a). The arrival time t_2 of the next drastic innovation is stochastic, with probability distribution function $\Pr\{\bar{t}_2 \le t_2\} = 1 - \exp\{-\ lmh(z)(t_2 - t_1)\}$. At time t_2 a new drastic innovation succeeds and initiates a new Schumpeterian wave. Previous technology becomes obsolete and each downstream firm with marginal cost $c(t_2) = aA_1$ sets a new price $p(t_2) = aA_1/q$ to produce an output of $q(t_2) = a\,IA_1/q$ at the beginning of the second wave. After that, their cost, price and output evolve in stepwise-increase or -decrease as in the first wave. The economy grows through a cyclical process infinitely.

An interesting outcome of the model is that although agents in stationary equilibrium take the same decisions, all upstream firms spend a constant amount *z* on drastic innovations and all downstream firms invest *y* in incremental R&D, the economy still fluctuates. The Schumpeterian business cycle is rooted in the economy and without fluctuations it cannot grow. This growth pattern obviously differs from the marginal evolution of growth models by Romer (1990), Grossman and Helpman (1991), and Young (1993). In its spirit our Schumpeterian model is closer to Aghion and Howitt (1992), Justman (1997), and Stein (1997) in emphasizing the discontinuity of economic growth. However, it goes further by integrating into the business cycle through the interaction of the up-steam and downstream agents, part of the dynamics of development and allocation processes.

We turn now to study the effects of parameters of the model on stationary equilibrium. To simplify the discussion, we presume the equilibrium of (9a) and (16a) is unique or that the equilibrium does not jump from one dynamic locus to another when a parameter of the model has a marginal variation. Since *y* is inversely related to *z*, the following analysis focuses on *z*.

Consider l first. A larger l means that a drastic innovation is less expensive in the sense that it has a greater probability to be invented given the same amount of investment. It is easy to see that $\partial W(z)/\partial l < 0$ so that an increase in l shifts *W*(*z*) in Figure 2 downwards. On the other hand, (9a) shows that an increase in l reduces *y* when *H* is invariant and in turn raises $g$by (3). This effect, together with the shorter expected life of a drastic innovation, $1/lH$, shifts *V*(*z*) upwards. Thus, the intersection of *W*(*z*) and *V*(*z*) yields a larger equilibrium *z*. Intuitively, an increase in l has three effects. First, it shifts the marginal revenue curve of drastic innovation upward and the marginal cost curve downward given that the license fee is invariant,[15] resulting in higher investment by upstream firms. Second, a large l implies that the next drastic innovation will arrive sooner and that downstream firms have less incentive to improve efficiency so that V is larger, which also stimulates upstream firms' investment. Third, when the next drastic innovation arrives sooner, the expected life of a drastic innovation is shorter, a winning upstream firm can therefore reap a smaller license fee L from the same V, which discourages investment in drastic innovation. The third effect is offset by the second leading the net effect to be second-order small in comparison with the first effect. Thus, a larger l unambiguously yields more investment in drastic innovation.

From (18), a smaller a represents more significant of a drastic innovation. Accordingly, it is more attractive to upstream firms. But it has no direct impact on the behavior of downstream firms although it indirectly reduces their interests in incremental innovations through upstream firms' expenditure increase on drastic innovations. This can be shown by the upward shift of $V(t)$ and the unchanged $W(t)$ as a declines in Figure 2.

In a one-shot game of an uncertain R&D race to win a fixed technology reward, Lee and Wilde (1980) and Yin and Zuscovitch (1995) claim that the number of firms in the industry can stimulate each competitor's R&D investment. These models are quite similar to our model if it is simplified to have only upstream firms and these firms only meet once in the game. But these two conditions prevent us from reaching unambiguous conclusions on the R&D stimulation of upstream firms. The reason is that an increase in m shifts both the marginal revenue and the marginal cost curves of drastic innovations upward when the license fee, L, is fixed.[16] Moreover, a larger m implies a shorter expected life of drastic innovations with given z and induces two contradictory effects on L. On the one hand, it makes each downstream firm invest less, leading to a larger V

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[15] To see this, take the partial derivatives of (15) with respect to z_{jt} keeping L_t constant, and then set all z_{jt} equal to z.

[16] For a proof, see footnote 15.

and L. On the other hand, the monopoly power of an upstream incumbent can be maintained for a shorter period of time and it therefore receives a lower L. In Figure 2, the ambiguity is depicted by the upward shifts of both $V(z)$ and $W(z)$ curves.[17]

In the model the effects of n, the number of downstream firms, is also unambiguous, depending on the sales of each downstream firm. Clearly, $W(z)$ is independent of n. From (4) we can see that I is equal to the sales of each downstream firm. If consumer expenditure on the output of this industry is constant, which would be the case if, for instance, the utility function for composite goods of this industry and the remaining economy is Cobb-Douglas, then I is proportional to the inverse of n. Generally, I can decline more or less than proportionately with the inverse of n as n increases. Thus $V(z)$ may shift upward, remain unchanged or shift downward as more firms enter the downstream sector, depending on whether nI rises, remains constant or falls. Consequently, the expenditures on drastic innovation is higher, unchanged, or lower, respectively.

Because $1/(1-q)$ is the elasticity of substitution between any pair of goods, a larger q corresponds to a situation where the goods in the economy are more homogeneous. Although q has no direct effect on $W(z)$, its effect on $V(z)$ is ambiguous. The reason is that a larger q implies a more competitive market. It reduces the profit stream from using old as well as new generation technology.

The parameter S in the model is determined by the property of drastic and incremental innovations. On the one hand, a larger S may be the result of a more revolutionary drastic innovation since it promises a greater potential for subsequent incremental innovation. On the other hand, a large S can also result from the relative insignificance of each round of incremental innovation. Consider now an increase in S from an initial level S^* to S^*+1. This increase breaks the first segment of the initial $V(z)$ curve into two segments and leaves others unchanged. Moreover, the first segment of the new $V(z)$ is on $(0, \bar{z}_{S^*+1})$ and is below the old curve and the second segment is on

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[17] Positive partial derivatives of $V(z)$ and $W(z)$ with respect to m can prove this.

($\bar{z}_{S^*+1}, \bar{z}_{S^*}$) and overlaps the old curve. Hence, only when the initial intersection of $V(z)$ and $W(z)$ falls in (0, $\bar{z}_{S^*+1}$), will an increase in S by one reduce z. Intuitively, if the initial z is more than $\bar{z}_{S^*+1}$, the expected life of a drastic innovation is shorter than S^*+1. In this case, downstream firms cannot, on average, exploit all the opportunities for incremental innovations within S periods.[18] Providing more opportunities for incremental innovations does not alter the expected license revenue obtainable by an upstream monopoly. But, if the initial z is less than $\bar{z}_{S^*+1}$, an extra round of incremental innovation can reduce the technology gap between current technology and the drastically new technology at the time the new drastic innovation is introduced. Hence, V is reduced and so is the expected license fee, and upstream firms will tend to cut their expenditures on drastic innovation.

Finally, $W(t)$ shifts upward when r rises since $\partial W(z)/\partial r > 0$. On the other hand, (9a) illustrates that the greater is r, the smaller is y, and consequently the larger is g Thus an increase in r also shifts $V(t)$ upward. This leads to ambiguous effects of r on the equilibrium.

The findings of this comparative dynamics can be summarized as follows.

Proposition 4. *Stationary investment in drastic innovation z increases but investment in incremental innovation y decreases if l or nI increases or a decreases. An increase in S reduces z and raises y if* $z < \bar{z}_{S+1}$ *and yields no effect otherwise. But the effects of m, q and r on y and z are ambiguous.*

4. Extensions of the Basic Model

4.1 Physical Capital Costs

Although the innovation may come in the form of intangible assets and blueprints, its transformation into a new production technology usually requires new physical capital such as new machine tools. A drastic innovation should phase out original physical capital and replace it with new machines embodying the new technology. This matches

[18] Note, incremental innovation has a one-period time delay.

very well with Schumpeter's perception of innovation as a "New Combination". The invention may be immaterial but the economics of innovation is driven by the need to adapt production and to reallocate resources from the previous use to the new and more productive one. In the basic model, we did not clearly distinguish between the costs of inventing a drastic new technology and the costs of producing the related physical capital. To deal with this, we can think of the R&D expenditure flow z in the basic model as the one required to invent the new technology. In order to develop the physical capital once the innovation succeeds, the monopolist has to pay a fixed amount F_t and then sell the physical capital to downstream firms in conjunction with charging a license fee. In this case, the license fee plus the capital cost equals the sum of the expected profit stream each downstream firm can obtain by adopting a new generation of technology. Hence, the monopolist's gain from a drastic innovation is $\tilde{L}_t = V(z_t)/(r+ lH_{t+1}) - nF_t$ instead of L_t. The equilibrium conditions are (9) and

$$\{ lh'_t(z_t)[V_t(z_t)/(r+ lH_{t+1}) - nF_t] - 1\}[(m-1)\, lh_t(z_t) + r] - l[h_t(z_t) - z_t h'_t(z_t)] \leq 0$$

$z_t \geq 0$ with at least one equality. t=1, 2, … (16b)

Thus the introduction of capital costs only reduces the rewards of the winner of drastic innovation by a fixed amount of capital costs and the mechanisms of the interaction between two types of innovations remain the same. Propositions 1 and 2 are still valid.

Referring to stationary equilibrium, we must assume that the capital costs for each drastic innovation are the same, i.e., $F_t = F$. Equilibrium is characterized by (9a) and

$$WF(z)=V(z), \tag{16c}$$

where $V(z)$ is still defined by (17a) but

$$WF(z) = [r + lmh(z)]\{[m\, lh(z) - z\, lh'(z) + r]/ lh'(z)[(m-1)\, lh(z) + r] + nF\}. \tag{18}$$

Since $WF(z)$ has the property that $WF(0) \leq 0$ and $\lim_{z\to+\infty} WF(z) = +\infty$, proposition 3 still holds after the introduction of fixed costs. Comparing $WF(z)$ with $W(z)$, we have[19]

[19] To simplify the notation, the variable z has been dropped. Subscripts here index partial derivatives.

$$WF_l = mnhF + W_l\,,\quad WF_n = (r + lmh)F > 0 > W_m = 0\,,\quad WF_m = lnhF + W_m > 0$$

$$WF_r = nF + W_r > 0\,,\quad WF_a = WF_S = WF_q = W_a = W_S = W_q = 0\,.$$

With the same proof of proposition 4, we have

Proposition 4a. *In stationary equilibrium z increases but y decreases if a decreases. An increase in S reduces z and raises y if* $z < \bar{z}_{S+1}$ *and yields no effect otherwise. An increase in n reduces z but raises y if nI remains constant. The effects of* l*, m, q and r on y and z are ambiguous.*

A significant change due to the introduction of capital costs is that the effect of l becomes ambiguous. This is because $W_l < 0$ and $mnhF > 0$, and hence the sign of WF_l is uncertain. If $WF_l < 0$, then an increase in l still raises *z* and reduces *y*. But when $WF_l > 0$, the effect is ambiguous and depends on the shape and relative shift of the *WF*(*z*) and *V*(*z*) curves.

4.2 Learning-By-Doing

To focus on LBD, assume downstream firms do not engage in any incremental innovation activities. When they adopt the t-generation new technology invented by an upstream firm at time t_t, their marginal cost is A_t. From practicing with the new technology they become more skillful and their costs decline *automatically* with production according to

$$c(t) = A_t \Gamma_t (t - t_t)\,. \qquad t \in [t_t, t_{t+1}) \tag{1a}$$

The learning function $\Gamma_t(t)$ decreases as t increases with $\Gamma_t(0) = 1$ and $\lim_{t\to\infty} \Gamma_t(t - t_t) = \bar{\Gamma}_t$.[20] Although the learning effects are bounded, similar to Young (1993), the model differs from Young's in that the learning effects are the result of a drastic process innovation instead of marginal progress in new product invention. It is

[20] The specification implies that the longer the technology is applied the greater are cumulative learning effects. An alternative is to assume that learning effects depend on the cumulative output and downstream firms' production decisions will take these effects into account.

plausible to assume learning effects only improve production efficiency marginally so that $\overline{\Gamma}_t$ is assumed to be much greater than A_t / A_{t-1}.

Given H_{t+1}, the equilibrium investment in drastic innovation z_t is still determined by (16b) except that $\bar{c}*_{t-1}$ in (17) becomes $\bar{c}*_{t-1} = A_{t-1}\Gamma(t*_t - t_{t-1})$. Since the first order condition is the same, proposition 1 still holds and proposition 2 is simplified such that an increase in expected future investment (a larger H_{t+1}) reduces current investment (a smaller z_t).

For stationary equilibrium, the first-order condition is (16c) but *V*(*z*) in (18) becomes

$$V(z) = n(1-q)I\left\{1-\left[\Gamma\left(1/\ lH\right)/a\right]^{q/(q-1)}\right\}. \tag{17b}$$

An important difference is that *V*(*z*) is now a continuously increasing function instead of being piece-continuous. Propositions 3 and 4a are also applicable to LBD.

APPENDIX

The Proof of Proposition 1

Arbitrarily choose a positive z_t^1 and substitute it into (9) to get a series $\{y_{ts}^1\}$ ($s = 1,2,...$). Calculate V_t^1 by applying z_t^1 and $\{y_{ts}^1\}$ in (17) and substitute it into rearranged (16) to obtain z_t^2:

$$V_t / (r + lH_{t+1}) = [\, lmh_t(z_t) - z_t \; lh'_t(z_t) + r] / \; lh'_t(z_t)[(m-1)\, lh_t(z_t) + r]. \quad \text{(A1)}$$

Use z_t^2 to obtain $\{y_{ts}^2\}$ and then V_t^2 as before. Assume $z_t^1 < z_t^2$, so that $y_{ts}^1 > y_{ts}^2$ and $V_t^1 < V_t^2$. Noting that the right-hand side of (A1) is an increasing function in z_t, substituting V_t^2 into (A1) yields $z_t^3 > z_t^2$. Repeating this process results in a monotonic increasing series $\{z_t^k\}$ and monotonic decreasing series $\{y_{t1}^k\}$, $\{y_{t2}^k\}$, … ($k = 1,2,...$). (17) illustrates that $V_t^k \le n(1-q)I$, so that z_t^k has an upper boundary and lower boundary zero. On the other hand, y_{ts}^k has an upper boundary y_{ts}^1 and lower boundary zero. Since $\{z_t^k\}, \{y_{t1}^k\}$, $\{y_{t2}^k\}, \ldots$ are bounded monotonic series, they have limits, z_t, y_{t1}, y_{t2}, …, respectively. These limits are the solution of (9) and (16) and compose the equilibrium of the model.

If $z_t^1 > z_t^2$, $\{z_t^k\}$ is monotonic decreasing and $\{y_{t1}^k\}$, $\{y_{t2}^k\}, \ldots$ are monotonic increasing. The boundaries for $\{z_t^k\}$ are zero and z_t^1. $\{y_{ts}^k\}$ has boundaries zero and $\bar{y}_{ts}$, where $\bar{y}_{ts}$ is the solution of (9) when $H_{t+1} = 0$.

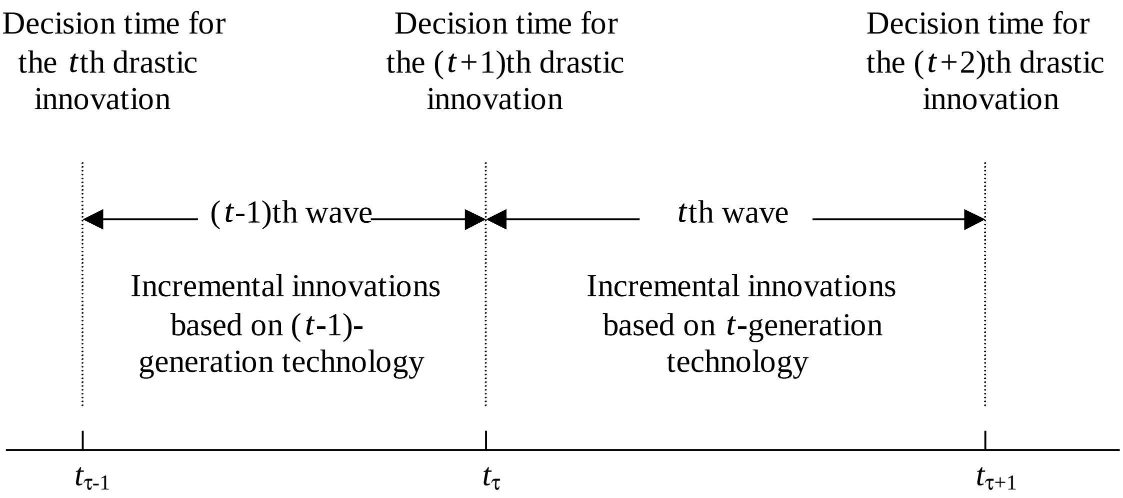

Figure 1

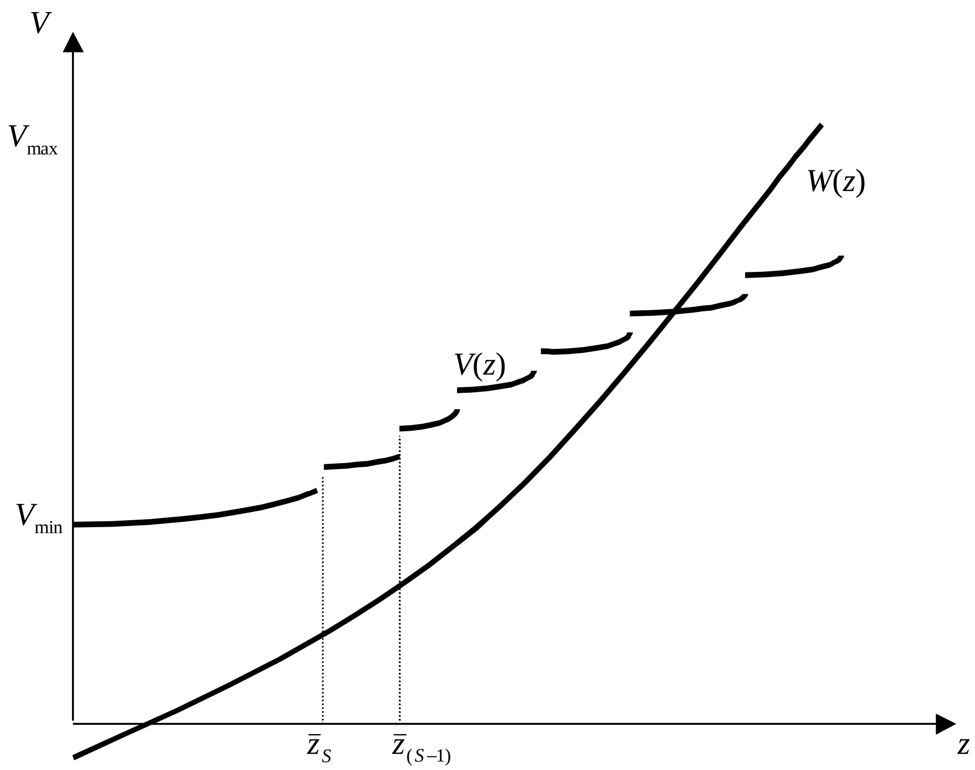

Figure 2

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