Departures from Slutsky Symmetry in Household Demand Models[*]

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Abstract

Maximisation of utility by a single consumer subject to a linear budget constraint is well known to imply strong restrictions on the properties of demand functions. Empirical applications to data on households however frequently reject these restrictions. In particular such data frequently show a failure of Slutsky symmetry - the restriction of symmetry on the matrix of compensated price responses. Browning and Chiappori (1998) show that under assumptions of efficient within-household decision making, the counterpart to the Slutsky matrix for demands from a k member household will be the sum of a symmetric matrix and a matrix of rank $k-1$. We establish the rank of the departure from Slutsky symmetry for couples under the assumption of Nash equilibrium in individual demands. We show that the Slutsky matrix is the sum of a symmetric matrix and another of rank at most 2. This result implies not only that the Browning-Chiappori assumption of efficiency can be tested against other models within the class of those based on individual optimisation, but also that the hypothesis of Nash equilibrium in demands has testable content against a general alternative.

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1 Introduction

Maximisation of utility by a single consumer subject to a linear budget constraint is well known to imply strong restrictions on the properties of demand functions. Empirical applications to data on households however frequently reject these restrictions (see for example, Browning and Meghir (1991), Banks, Blundell and Lewbel (1997) or Deaton (1990)). In particular such data frequently show a failure of Slutsky symmetry - the restriction of symmetry on the matrix of compensated price responses.

At the same time, the inadequacy of the single consumer model as a description of decision making for households with more than one member is increasingly being recognised. The problem in assuming families to behave as if they had the preferences of an individual was well recognised by Samuelson (1956), who established strong conditions under which such a simplified representation would hold. A large body of recent research has investigated alternative models accommodating more realistic descriptions of within-household decision-making processes.

An important advance is made by Browning and Chiappori (1998), who show that under assumptions of efficient within-household decision making, the counterpart to the Slutsky matrix for demands from a k member household will be the sum of a symmetric matrix and a matrix of rank $k-1$. Tests on Canadian data are found to reject symmetry for couples but not for single individual households. However it is impossible to reject the hypothesis that the departure from symmetry for the sample of couples has rank 1 as required. This work is important not only in filling a gap in our theoretical understanding of demand behaviour but also in the prospect which it presents of reconciling demand theory and data on consumer behaviour.

However the assumption of efficiency is not satisfied in all models of household behaviour which have been suggested. It clearly holds, for instance, in the Nash bargaining models of Manser and Brown (1980), McElroy and Horney (1981) or McElroy (1990), and has been the central assumption of papers such as Browning, Bourguignon, Chiappori and Lechene (1994) or Bourguignon and Chippori (1994). However it is not a property of noncooperative models such as those of Ulph (1988) or Woolley (1988). While the inability of Browning and Chiappori to reject the symmetry and rank condition for couples is intriguing, it is not clear what if any power it has as a test of efficiency of intrahousehold decisions unless one understands the nature of the departure from symmetry under the principal alternative models of household decision making. In particular if noncooperative models were to give rise to a departure of similar rank then this would obviously not be a feature of demand behaviour which would be of use in discriminating between these alternative assumptions about within household behaviour. On the other hand, if the departure from symmetry under noncooperative behaviour were to be of greater rank then the Browning-Chiappori result would not only promise to reconcile assumptions of optimising behaviour with demand data but also provide evidence in favour of the collectively rational model against other descriptions of within-household decision making.

In this paper we establish the rank of the departure from Slutsky symmetry for couples under the assumption of Nash equilibrium in individual demands. We show that the Slutsky matrix is the sum of a symmetric matrix and another of rank at most 2. This result implies not only that the Browning-Chiappori assumption of efficiency can be tested against other models within the class of those based on individual optimisation, but also that the hypothesis of Nash equilibrium in demands has testable content against a general alternative.

2 A non-cooperative Nash model of household demands

2.1 General framework

Suppose a household consists of two individuals labelled A and B. The household spends on a set of m private goods q and n public goods Q. The quantities purchased by the individuals are q^A, q^B, Q^A and Q^B with total household quantities being $q = q^A + q^B$ and $Q = Q^A + Q^B$. Individual utility functions are $u^A(q^A, Q)$ and $u^B(q^B, Q)$. The two partners bring in separate incomes of y^A and y^B. Prices of the two sorts of goods are vectors p and P.

Each person decides on the purchases made from their own income so as to maximise their own utility subject to the spending decisions of the partner. Hence, A chooses q^A and Q^A to solve

$$\max_{q^A, Q^A} u^A(q^A, Q^A + Q^B) \text{ s. t. } pq^A + PQ^A \leq y^A, \; Q^A \geq 0$$

and B chooses q^B and Q^B to solve

$$\max_{q^B, Q^B} u^B(q^B, Q^A + Q^B) \text{ s. t. } pq^B + PQ^B \leq y^B, \; Q^B \geq 0.$$

Note that we can write these two problems equivalently as

$$\max_{q^A, Q} u^A(q^A, Q) \text{ s. t. } pq^A + PQ \leq y^A + PQ^B, \; Q \geq Q^B$$

and

$$\max_{q^B, Q} u^B(q^B, Q) \text{ s. t. } pq^B + PQ \leq y^B + PQ^A, \; Q \geq Q^A.$$

We concentrate on the case of interior solutions where both partners contribute to all public goods. In Nash equilibrium the quantities purchased will satisfy

$$\begin{aligned} q^A &= f^A(y^A + PQ^B, p, P) \\ q^B &= f^B(y^B + PQ^A, p, P) \\ Q &= F^A(y^A + PQ^B, p, P) \\ &= F^B(y^B + PQ^A, p, P). \end{aligned}$$

where $f^A(\cdot)$ and $F^A(\cdot)$ are demand functions corresponding to A's preferences and together satisfying the usual demand properties including Slutsky symmetry and $f^B(\cdot)$ and $F^B(\cdot)$ are demand functions corresponding to B's preferences, of which the same is true.

2.2 Demand responses

Demand responses follow from

$$M\begin{pmatrix} dq^A \\ dq^B \\ dQ^A \\ dQ^B \end{pmatrix} = N_1\begin{pmatrix} dy^A \\ dy^B \end{pmatrix} + N_2\begin{pmatrix} dp \\ dP \end{pmatrix}$$

where

$$M = \begin{pmatrix} I_m & 0 & 0 & -f_y^A P' \\ 0 & I_m & -f_y^A P' & 0 \\ 0 & 0 & I_n & I_n - F_y^A P' \\ 0 & 0 & I_n - F_y^B P' & I_n \end{pmatrix} = \begin{pmatrix} I_m & 0 & 0 & -a \\ 0 & I_m & -b & 0 \\ 0 & 0 & I_n & \alpha \\ 0 & 0 & \beta & I_n \end{pmatrix}$$

$$N_1 = \begin{pmatrix} f_y^A & 0 \\ 0 & f_y^B \\ F_y^A & 0 \\ 0 & F_y^B \end{pmatrix} = \begin{pmatrix} aR & 0 \\ 0 & bR \\ (I_n - \alpha)R & 0 \\ 0 & (I_n - \beta)R \end{pmatrix}$$

$$N_2 = \begin{pmatrix} f_p^A & f_P^A + f_y^A Q^{B\prime} \\ f_p^B & f_P^B + f_y^B Q^{A\prime} \\ F_p^A & F_P^A + F_y^A Q^{B\prime} \\ F_p^B & F_P^B + F_y^B Q^{A\prime} \end{pmatrix} = \begin{pmatrix} f_p^A & f_P^A + aRQ^{B\prime} \\ f_p^B & f_P^B + bRQ^{A\prime} \\ F_p^A & F_P^A + (I_n - \alpha)RQ^{B\prime} \\ F_p^B & F_P^B + (I_n - B)RQ^{A\prime} \end{pmatrix}$$

and $R = P(P'P)^{-1}$.

Hence

$$\begin{pmatrix} dq^A \\ dq^B \\ dQ^A \\ dQ^B \end{pmatrix} = M^{-1}N_1\begin{pmatrix} dy^A \\ dy^B \end{pmatrix} + M^{-1}N_2\begin{pmatrix} dp \\ dP \end{pmatrix}.$$

In typical budget surveys, we observe total household purchases rather than spending by individual members. In terms of household purchases q and Q we have

$$\begin{pmatrix} dq \\ dQ \end{pmatrix} = EM^{-1}N_1\begin{pmatrix} dy^A \\ dy^B \end{pmatrix} + EM^{-1}N_2\begin{pmatrix} dp \\ dP \end{pmatrix}$$

where

$$E = \begin{pmatrix} I_m & I_m & 0 & 0 \\ 0 & 0 & I_n & I_n \end{pmatrix}$$

is an appropriate aggregating matrix.

The matrix M has a block upper triangular structure which makes it readily invertible. In a convenient representation

$$M^{-1} = \begin{pmatrix} I_m & 0 & -a\beta S & aT \\ 0 & I_m & bS & -b\alpha T \\ 0 & 0 & S & -\alpha T \\ 0 & 0 & -\beta S & T \end{pmatrix}$$

$$\Rightarrow \quad EM^{-1} = \begin{pmatrix} I_m & I_m & (b - a\beta)S & (a - b\alpha)T \\ 0 & 0 & (I_n - \beta)S & (I_n - \alpha)T \end{pmatrix}$$

where $T = (I_n - \beta\alpha)^{-1}$ and $S = (I_n - \alpha\beta)^{-1}$. These two matrices are intimately linked and we make free use in the exposition of results linking the two[1].

2.3 Income effects

We can derive expressions for the effects of each partner's income on household purchases of all goods

$$\Phi = \begin{pmatrix} dq/dy^A & dq/dy^B \\ dQ/dy^A & dQ/dy^B \end{pmatrix} = EM^{-1}N_1$$

$$= \begin{pmatrix} a + (b - a\beta)S(I_n - \alpha)R & b + (a - b\alpha)T(I_n - \beta)R \\ (I_n - \beta)S(I_n - \alpha)R & (I_n - \alpha)T(I_n - \beta)R \end{pmatrix}.$$

It can be shown that the two columns of this matrix are in fact identical since

$$\begin{aligned} a + (b - a\beta)S(I_n - \alpha) &= b + (a - b\alpha)T(I_n - \beta) = bS(I_n - \alpha) + aT(I_n - \beta) \\ (I_n - \beta)S(I_n - \alpha) &= (I_n - \alpha)T(I_n - \beta). \end{aligned}$$

Let ϕ denote this common column so that $\Phi = \phi \begin{pmatrix} 1 \\ 1 \end{pmatrix}'$.

The distribution of income within the household is therefore locally irrelevant to household demands provided that we are at an interior solution. This result was first established in the literature on private provision of public goods by Warr (1983), Kemp (1984) and Bergstrom, Blume and Varian (1986), then later rediscovered in the context of household demand models by Ulph (1988).

It means in the current context that income effects of price changes can be defined straightforwardly since income from any source has the same effect on household demand.

[1] In particular, $T = I_n + \beta S\alpha$, $S = I_n + \alpha T\beta$, $\alpha T = S\alpha$, $T\beta = \beta S$, $T\beta a = T - I_n$ and $\alpha\beta S = S - I_n$.

2.4 Price effects and Slutsky symmetry

Uncompensated price responses follow from

$$
\begin{aligned}
\Gamma &= \begin{pmatrix} dq/dp & dq/dP \\ dQ/dp & dQ/dP \end{pmatrix} = EM^{-1}N_2 \\
&= \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma_{11} &= f_p^A + f_p^B + (b - a\beta)SF_p^A + (a - b\alpha)TF_p^B \\
\Gamma_{12} &= f_P^A + f_P^B + (b - a\beta)SF_P^A + (a - b\alpha)TF_P^B + aRQ^{B\prime} + bRQ^{A\prime} \\
&\quad + (b - a\beta)S(I_n - \alpha)RQ^{B\prime} + (a - b\alpha)T(I_n - \beta)RQ^{A\prime} \\
\Gamma_{21} &= (I_n - \beta)SF_p^A + (I_n - \alpha)TF_p^B \\
\Gamma_{22} &= (I_n - \beta)SF_P^A + (I_n - \alpha)TF_P^B \\
&\quad + (I_n - \beta)S(I_n - \alpha)RQ^{B\prime} + (I_n - \alpha)T(I_n - \beta)RQ^{A\prime}.
\end{aligned}
$$

Browning and Chiappori (1998) define the pseudo-Slutsky matrix. In the present context we can see this as what would be calculated in place of the Slutsky matrix if the household were treated as behaving according to the unitary model. Thus the pseudo-Slutsky matrix has the form $\Psi = \Gamma + \phi \begin{pmatrix} q \\ Q \end{pmatrix}' = \begin{pmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{pmatrix}$.

By substitution, we can derive expressions for each of the terms

$$
\begin{aligned}
\Psi_{11} &= (bS(I_n - \alpha) + aT(I_n - \beta))R(q^A + q^B)' \\
&\quad + f_p^A + f_p^B + (b - a\beta)SF_p^A + (a - b\alpha)TF_p^B \\
&= aRq^{A\prime} + bRq^{B\prime} + f_p^A + f_p^B \\
&\quad + (b - a\beta)S(F_p^A + (I_n - \alpha)Rq^{A\prime}) + (a - b\alpha)T(F_p^B + (I_n - \beta)Rq^{B\prime}) \\
\Psi_{12} &= 2(bS(I_n - \alpha) + aT(I_n - \beta))RQ' \\
&\quad + f_P^A + f_P^B + (b - a\beta)SF_P^A + (a - b\alpha)TF_P^B \\
&= (a + b)RQ' + f_P^A + f_P^B \\
&\quad + (b - a\beta)S(F_P^A + (I_n - \alpha)RQ') + (a - b\alpha)T(F_P^B + (I_n - \beta)RQ') \\
\Psi_{21} &= (I_n - \alpha)T(I_n - \beta)R(q^A + q^B)' + (I_n - \beta)SF_p^A + (I_n - \alpha)TF_p^B \\
&= (I_n - \alpha)Rq^{A\prime} + (I_n - \beta)Rq^{B\prime} + F_p^A + F_p^B \\
&\quad - (I_n - \alpha)T\beta((I_n - \alpha)Rq^{A\prime} + F_p^A) - (I_n - \beta)S\alpha((I_n - \beta)Rq^{B\prime} + F_p^B) \\
\Psi_{22} &= 2(I_n - \alpha)T(I_n - \beta)RQ' + (I_n - \beta)SF_P^A + (I_n - \alpha)TF_P^B \\
&= ((I_n - \alpha) + (I_n - \beta))RQ' + F_P^A + F_P^B \\
&\quad - (I_n - \alpha)T\beta((I_n - \alpha)RQ' + F_P^A) - (I_n - \beta)S\alpha((I_n - \beta)RQ' + F_P^B).
\end{aligned}
$$

But then this can be shown to admit a decomposition

$$a = a^A + a^B + \Delta$$

where

$$a^A = \begin{pmatrix} f_p^A & f_P^A \\ F_p^A & F_P^A \end{pmatrix} + \begin{pmatrix} aR \\ (I_n - \alpha)R \end{pmatrix} \begin{pmatrix} q^A \\ Q \end{pmatrix}'$$

$$a^B = \begin{pmatrix} f_p^B & f_P^B \\ F_p^B & F_P^B \end{pmatrix} + \begin{pmatrix} bR \\ (I_n - \beta)R \end{pmatrix} \begin{pmatrix} q^B \\ Q \end{pmatrix}'$$

$$\Delta = \begin{pmatrix} (b - a\beta)S \\ -(I_n - \alpha)\beta S \end{pmatrix} \begin{pmatrix} (I_n - \alpha)Rq^{A'} + F_p^A \\ (I_n - \alpha)RQ' + F_P^A \end{pmatrix}' + \begin{pmatrix} (a - b\alpha)T \\ -(I_n - \beta)\alpha T \end{pmatrix} \begin{pmatrix} (I_n - \beta)Rq^{B'} + F_p^B \\ (I_n - \beta)RQ' + F_P^B \end{pmatrix}'.$$

Both a^A and a^B are individual Slutsky matrices and therefore symmetric. The departure from symmetry therefore depends on the properties of the matrix Δ. This, being the sum of two outer products of vectors is plainly of rank 2 in general.

2.5 Discussion

This is a result of considerable interest for at least two reasons. Firstly it establishes that the departure from symmetry has a greater rank in noncooperative than in cooperative models. The Browning and Chiappori test involving the rank of this matrix therefore has power against at least one alternative within the class of models involving individual optimisation. The inability to reject a rank 1 deviation from Slutsky symmetry in their paper can therefore be seen as a failure to reject cooperative behaviour against noncooperative models of the sort treated here.

Secondly it shows that the noncooperative model is itself testable against a general alternative given sufficient number of goods.

2.6 Extensions

A number of directions for extending the theory presented here provide the material for work in progress. In particular, three issues deserve specific attention. Firstly, generalisation to the case of more than two household members is a promising direction of research, although the two-person case is obviously empirically the most relevant and important for many societies. Secondly, the possibility of non-interior solutions with one or more member of the household choosing not to spend on certain public goods needs to be addressed. Thirdly, the possibility of "caring" preferences (see Becker (1991), Bourguignon and Chiappori (1994)) in which partners' preferences depend on "egoistic" utilities of both household members offer a feasible way to relax the somewhat mercenary behaviour evident in cruder versions of noncooperative models.

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