Bargaining, Revenue Sharing and Control Rights Allocation[*]

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Abstract

In a two-period, double moral hazard model with incomplete contracting, this paper explores the relationship between revenue sharing and control rights. Specifically, we endogenize the allocation of both the income rights and the control rights and show why the two are often bundled together in the context of a two-party joint venture. Moreover, we study how the use of different bargaining solutions for the ex post contract renegotiation game may affect the optimal allocation of income and control rights. Our results can be used to explain the commonly observed ownership structures of equity joint ventures.

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1. Introduction

The property rights theory of the firm of Grossman-Hart-Moore (henceforth GHM) identifies asset ownership with the residual control rights over the use of physical assets (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995). The theory shows in a two-agent model that when complete contracting is not possible, an agent should own an asset (i.e., have the control rights over the ex post decisions that are uncontractible ex ante) if her incentive to make relationship-specific investment is more important than that by the other agent. GHM therefore do not distinguish between ownership and control. But ownership of an asset is often regarded to entail the claim on the returns (or, more precisely, residual income) from the asset. It is in this sense the term "ownership" is used in the literature on the separation of ownership and control. In Alchian and Demsetz's (1972) classical paper, the rights to be the residual claimant are the first among a bundle of rights that define the ownership of the classical firm.

Hart (1995, pp.63-66) offers a few informal but insightful remarks on the relationship between residual income and control. First, he notes that residual income may be difficult to measure and sometimes may not even be well-defined; and if it is measurable, it is not always bundled together with residual control. He then proceeds to give a few justifications for why residual income and residual control are often bundled together. In particular, Hart comments that residual income and residual control may not be separable and the party having residual control may be able to divert some of the income away for her own private benefit.

The paper modifies the GHM model based on the above insights from Hart to achieve several related objectives. The first and the most important objective is to endogenize the allocation of both the income rights and the control rights and hence to show why the two are often bundled together in the context of a two-party joint venture. Specifically, we consider a two-period, double-sided moral hazard model in which two parties each make an unobservable investment (effort) to a joint project in the first period (ex ante); and an ex ante uncontractible action (an operating decision) involving the project is then taken in the second period (ex post). The income from the project is (stochastically) determined by the ex ante investments as well as the ex post action, which meanwhile may generate a private benefit to the party that has the control rights over the action. Departing from GHM, we assume that the income from the joint project is publicly observable, and hence an ex ante revenue-sharing contract becomes possible. We make this assumption because only when income is more or less verifiable can we study the relationship between income rights and control rights.

Our second objective is to demonstrate when GHM's key results would still hold if a contractual allocation of income is possible. We will show that GHM's results that control rights matter in the organization of economic transactions and that control rights should, at least normally, be allocated to the party whose investment incentive is relatively more important are basically unaffected by introducing the possibility of contractual revenue sharing. But we show that it is not the control rights over assets per se but rather the revenue-sharing contract that provides investment incentives; however, the allocation of control rights makes the revenue-sharing contract renegotiation-proof ex post.

Because ex post action is not contractible ex ante, it must be decided ex post. This gives rise not only to the need for allocating control rights but also to the possibility of renegotiating the ex ante revenue-sharing contract. This is because a party who has either the exclusive control rights or the joint control rights over the ex post action may use the rights as a bargaining chip to renegotiate the ex ante revenue-sharing contract. An important departure of the paper from GHM is that we consider both cooperative Nash bargaining and noncooperative alternating-offer bargaining and show how they may affect the optimal contractual arrangements. A number of recent papers (Chiu, 1998; De Meza and Lockwood, 1998; Rajan and Zingales, 1998) have shown that the specific results of GHM depend on the solution concept they use for the ex post bargaining game. If instead of using the Nash bargaining solution with the no-negotiation outcome as the disagreement point, the use of the "outside options principle", which can be derived from a Rubinstein-style alternating-offer noncooperative bargaining game (Binmore et al., 1989), may reverse the GHM results; i.e., in some situations, an agent who makes important investment decisions should not have the control rights. In addition to the above two bargaining solutions, we also consider another variant of the Nash bargaining solution in which the disagreement point is determined by what Nash (1953) calls rational threats (also see Myerson, 1991).

The third objective of the paper is to study how the use of different bargaining solutions for the ex post contract renegotiation game may affect the optimal allocation of income and control rights. We show that only under Nash bargaining with the non-negotiation outcome as the disagreement point can it be optimal for the minority shareholder, who is also the party whose investment incentive is less important, to have the control rights (i.e., revenue sharing and control rights are unbundled). Under both Nash bargaining with rational threats as the disagreement point and noncooperative alternating-offer bargaining, revenue sharing and control rights are always bundled together; in other words, the minority shareholder should never have the control rights. These results are in contrast with those of the aforementioned articles.

Finally, while ex ante investments are in relationship-specific human capital in GHM, we assume that they are embodied in the physical assets of the joint project.

In so doing, we are able to show that joint ownership can be optimal.[1] Therefore, the fourth objective of the paper is to use our results to explain the commonly observed contractual arrangements of equity joint ventures. We are able to explain, first of all, when and why the majority shareholding partner of a joint venture is normally the controlling party. Second, our results provide an explanation for a somewhat puzzling feature of joint ventures, i.e., many of them are owned 50:50 by the two companies setting them up. The 50:50 joint ownership is puzzling because it appears to provide sub-optimal incentives unless incentive provisions for both partners to make efficient investments or work efforts are equally important. An even more puzzling ownership arrangement of joint ventures is the 51:49 ownership structure in which one partner owns 51% of the equity shares and is the controlling partner of the joint venture (Dasgupta and Tao, 1998). We provide a scenario in which the 51:49 joint venture is optimal.

The question of why income and control are often bundled together has been addressed in the "one share-one vote" literature (Grossman and Hart, 1988; Harris and Raviv, 1988; Hart, 1995). Theories along that line are based on the agency costs associated with the private benefits of control and the market for corporate control. This paper adopts the idea of private benefits of control but addresses the bundling of income and control rights in the context of a joint venture between two business partners. In such a setup, corporate takeovers become less relevant.

To the extent that our results can be used to explain the ownership structure of equity joint ventures, this paper is related to the theoretical papers on joint ownership, which are surprisingly very few. Dasgupta and Tao (1998) propose a theory of equity joint ventures by distinguishing between marketable equities and unmarketable revenue-sharing contracts. In a related paper, Dasgupta and Tao (1999) explain partial ownership in an upstream firm by a downstream firm as a device to increase the former's incentive to choose relationship-specific investments over general investments. But both papers treat ownership as shares of revenue and either ignore control rights or simply assume that the majority shareholder has the control rights, which they interpret as the power to make a take-it-or-leave-it offer in bilateral bargaining. Cai (1999) develops a theory of joint asset ownership by extending the GHM model to situations where the level of investment specificity is endogenously determined. He shows that when specific investments and general investments are substitutes, joint ownership is optimal in most cases. Similar to GHM, he ignores the possibility of revenue-sharing contracts. A more closely related paper is Bai et al. (1998). They also consider the

[1] Hart (1995, pp.68-9) illustrates with an example why investments in physical assets may lead to joint ownership. But, in his example, revenue is not verifiable and the party that has the the control rights can take away all the revenue.

relationship between revenue sharing and control rights and show that under the Outside Option Principle, joint ownership makes the second-best revenue-sharing contract self-enforcing. However, they assume that if a manager controls an asset, he can quit the relationship and receive all the revenue while a manager who does not control any asset receives nothing if he quits. In our model, each of the two managers is entitled to the contractually specified share of the public revenue, which cannot be taken away by the controlling party. Moreover, in our model, joint ownership is optimal only under certain conditions and does not implement the second-best outcome.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 provides a preliminary analysis of the model. Section 4 characterizes the optimal sharing of revenue and allocation of control rights under Nash bargaining with two types of disagreement points while Section 5 considers noncooperative bargaining. Section 6 summarizes the main results and concludes the paper. All the proofs are in Appendix A.[2]

2. The Model

2.1. Model Primitives

We consider a two-period model involving two risk-neutral parties: M_1 and M_2. In period 1 (ex ante), each party can make an unobservable investment (or effort) to develop a joint project: e_1 for M_1 and e_2 for M_2. Following Kim and Wang (1998), we define a composite investment (effort), represented by $h = h(e_1; e_2)$. Let $c_1(e_1)$ and $c_2(e_2)$, which are also unobservable to other parties, denote the disutilities of M_1's and M_2's investment (effort), respectively.

Let $\mathbb{R}$ be the space of real numbers, $\mathbb{R}_+$ be the space of non-negative real numbers, and E be the space of investment (effort), which, for simplicity, is taken to be a subset of $\mathbb{R}$. We make the following technical assumptions.

Assumption 1. $h(e_1; e_2) \in \mathbb{R}$ is strictly increasing in e_1 and in e_2.

Assumption 2. $c_1(e_1) \in \mathbb{R}_+$ and $c_2(e_2) \in \mathbb{R}_+$ are convex and strictly increasing in e_1 and in e_2 respectively.

In period 2 (ex post), some further action involving the project, such as deciding on a specific type of product to produce or hiring a suitable manager to run the project, a

[2] For the convenience of the referees, the proof of the noncooperative bargaing solution, which is drawn from Binmore et al (1990), is attached in Appendix B.

$\in A$ needs to be taken, where A denotes the space of feasible actions. This action itself is assumed to be costless but affects the revenue of the project and may potentially generate private benefits to one of the two parties as well. The project yields at the end of period 2 a random revenue X; the range of which belongs to R_+. The realization of X is determined by the composite investment effort $h(e_1; e_2)$; the ex post action $a \in A$ and the state of nature $\omega \in \Omega$; where Ω denotes the space of the states of nature, which is realized at the end of period 1 or at the beginning of period 2, i.e., after the investments are made but before the action is taken. Formally, $X \equiv X(a; \omega; h)$: Departing from GHM, we assume that the revenue of the project is publicly observable and hence is contractible ex ante.

Following GHM, we suppose that ω represents a highly complex state of the world, including, for example, the state of consumer preferences. It is thus prohibitively costly to write a contract contingent on ω in sufficiently precise terms to make it enforceable by a third party. However, the realization of ω is assumed to be observable to both parties.

The optimal action a that maximizes the total ex post revenue depends on the realization of ω as well as the ex ante investments. We assume that the nature of the optimal action is prohibitively difficult to foresee or describe ex ante at date 1. For example, suppose the joint project is for the production of a technologically sophisticated widget. Before the state of nature is realized about the preferences of potential buyers for specific types of widgets, it is often impossible to know in advance the details of the widget that should be produced. Again, we assume that the action can be specified ex post in a contract without difficulty (this requires the action to be publicly observable and hence contractible in period 2).[3]

Let $a^*(\omega; h)$ denote the unique ex post efficient action. Thus, a^* maximizes the ex post revenue of the project. Let $X^*(\omega; h) \equiv X(a^*; \omega; h)$ denote the ex post maximum revenue conditional on h and ω. Let the random variable X^*; which is assumed to be strictly positive, be described by a distribution function $F(x; h)$; and $f(x; h)$ be the corresponding density function. Thus the total expected maximum income is

$$R(e_1; e_2) \equiv E(X^*) = \int x f[x; h(e_1; e_2)]dx:$$

We assume

[3] For standard justifications of incomplete contracts, see Williamson (1985), Grossman and Hart (1986) and Hart and Moore (1988). While we are aware of Maskin and Tirole (1999), who argue that unverifiability does not necessarily preclude the existence of an ex ante efficient information elicitation mechanism (contract) that makes the unverifiable information de facto verifiable, we follow GHM both for the simplicity of the assumption of incomplete contracting and for the fact that we allow ex post renegotiation of the ex ante contract, which weakens Maskin and Tirole's result. For a formal foundation of incomplete contracts, see Hart and Moore (1999) and Segal (1999).

Assumption 3. $R(e_1; e_2)$ is concave and strictly increasing separately in e_1 and in e_2.[4]

This assumption seems natural; Kim-Wang (1998) mention conditions under which it holds. We define the following notations to be used throughout the paper:

$$R_1'(e_1; e_2) = \frac{\partial R(e_1;e_2)}{\partial e_1};\ R_2'(e_1; e_2) = \frac{\partial R(e_1;e_2)}{\partial e_2};\ R_1''(e_1; e_2) = \frac{\partial^2 R(e_1;e_2)}{\partial^2 e_1};\ R_2''(e_1; e_2) = \frac{\partial^2 R(e_1;e_2)}{\partial^2 e_2};$$

$$h_1'(e_1; e_2) = \frac{\partial h(e_1;e_2)}{\partial e_1};\ h_2'(e_1; e_2) = \frac{\partial h(e_1;e_2)}{\partial e_2};\ h_1''(e_1; e_2) = \frac{\partial^2 h(e_1;e_2)}{\partial^2 e_1};\ h_2''(e_1; e_2) = \frac{\partial^2 h(e_1;e_2)}{\partial^2 e_2}.$$

As a is not contractible ex ante in period 1, it must be decided ex post in period 2.[5] We suppose that ex post control rights over a can be contracted on in period 1. We consider three possible ways of allocating the control rights: M_1 control; M_2 control; and joint control (i.e., each party has the veto rights over a). In the case of joint control, if the two parties cannot reach an agreement on a, then we suppose that no revenue from the project will be realized.[6] If a single party has the control rights over a, then in the absence of ex post bargaining, the controlling party can choose whatever a she wants to.

An important component of the model is that we assume that the controlling party can acquire a private benefit of control from choosing a sub-optimal ex post action. The non-controlling party is, however, assumed to enjoy no private benefit from the controlling party's choice. Let $B_i(a; \omega; h)$ denote the (nonnegative) private benefit to M_i when the ex ante contract allocates M_i with the control rights and a is chosen at date 2. We assume, for simplicity, $X(a; \omega; h) + B_i(a; \omega; h) < X^*$ for $a \neq a^*$ and $B_1(a^*; \omega; h) = B_2(a^*; \omega; h) = 0$. In other words, we assume that it is socially inefficient for the controlling party to enjoy any private benefit of control and that ex post efficient action a^* entails no private benefit to the controlling party. Furthermore, we assume

[4] To the extent that the investments by both parties increase the total revenue, they can be thought of as cooperative investments as in Che and Hausch (1999). With slight modifications, our model can be interpreted as a procurement model with relationship-specific cooperative investments. A difference between our model and that of Che and Hausch is that we assume revenue is contractible but the ex post action is not, while they assume the contrary.

[5] In the context of a simple procurement model studied by Hart and Moore (1988), several authors (Aghion, Dewatripont and Rey, 1994; Chung, 1991; Edlin and Reichelstein, 1996; Hermalin and Katz, 1993; Nöldeke and Schmidt, 1995) show that efficiency can be achieved by appropriately-designed simple contracts, although incomplete, without the aid of an ownership arrangement. These articles rely on the assumption that ex post trade is ex ante contractible. But in a more complex setup that goes beyond a simple procurement relationship, future actions may be difficult to contract on. This paper shows that when the future action is indeed uncontractible, control rights allocation has a role to play.

[6] While in equilibrium, $a = a^*$; the right to veto the efficient action gives each firm a bargaining instrument.

that the private benefit is only observable to the insiders (i.e., the two parties involved) but not to the outsiders, implying that it is not contractible.

Examples of unverifiable private benefits of control abound. For instance, the controlling party can use her decision power to sell the widget at an unnecessarily low price to a firm in which she or her relatives have a personal stake. This private benefit, however, can be difficult to verify if the widget is not a standard product and hence has no market price to which it can be compared. This action is inefficient from the perspective of maximizing the total revenue of the project because the private benefit the controlling party enjoys is only a fraction of the price concession she makes to the third party. Alternatively, the controlling party can choose to produce a specific type of widget that suits a related firm's need rather than a widget for an unrelated firm at a higher price.

The above setup is however too general for us to derive specific results. Therefore, for the most part of this paper (except for Section 3), we make further simplifying assumptions. We assume that the private benefits of control take the following linear form:

$$B_i(a;\omega;h) = b_i[X^*(\omega;h) - X(a;\omega;h)]; \quad \text{for } i = 1 \text{ or } 2;$$

where $b_i \in [0;1)$ is a constant. In other words, for every suboptimal action, the controlling party can acquire a private benefit that is a constant proportion of the lost revenue.

Let $q(a;\omega;h) \equiv X(a;\omega;h)/X^*(\omega;h)$; which is 1 for $a = a^*$ and less than 1 for $a \neq a^*$: The above definition can be expressed as

$$B_i(a;\omega;h) = b_i[1 - q(a;\omega;h)]X^*; \quad \text{for } i = 1 \text{ or } 2:$$

Furthermore, we assume that for any $\phi \in [0;1]$; there exists an action $a \in A$ such that $q(a;\omega;h) = \phi \in [0;1]$ for all $h \in \Re$ and $\omega \in \Omega$: In other words, the controlling party can always find a choice of action that will enable her to acquire any amount of private benefits of control between 0 and $b_i X^*$: This assumption requires the feasible set of actions A to be sufficiently rich.

With the above simplifying assumption, the controlling party M_i can be thought of as being able to act as if she can choose a q between 0 and 1 and acquire an unverifiable private benefit $b_i(1 - q)X^*$. This in turn can be interpreted in the sense that the controlling party can "cream off" a fraction of the potential total revenue, i.e. $(1 - q)X^*$; at a cost of $(1 - b_i)(1 - q)X^*$ to the joint project.[7] We will refer to

[7] This interpretation of the private benefit of control follows the way in which asset ownership is interpreted in Hart (1988), which, in showing the role of asset ownership, considers a number of scenarios in which the owner of an asset is assumed to be able either to siphon off a fraction of the

$X(a;\omega;h) = q(a;\omega;h)X^*(\omega;h)$ as the public revenue. For most part of the paper, we will speak of the choice of (or control rights over) q instead of the action a itself.

2.2. Ex Ante Contracting and Ex Post Contract Renegotiation

At the beginning of period 1, the two parties can sign an ex ante contract that allocates the public revenue from the project as well as the control rights over an ex post action involving the project in period 2. Let S be the space of feasible revenue-sharing rules, defined by $S \equiv \{s : R_+ \to R_+ \mid s \text{ is Lebesgue measurable}\}$. The revenue-sharing part of the contract gives M_1 an amount of $s_1(X) \in S$ based on the total public revenue and M_2 the rest of the revenue, i.e., $s_2(X) = X - s_1(X)$.[8] Because we are concerned with the question of the bundling of revenue sharing and control rights, we will restrict our attention to linear, budget-balancing revenue-sharing contracts. The linearity assumption seems rather natural in a double moral hazard setup. In fact, several authors have shown under fairly general conditions that a linear contract is optimal in one-period double moral hazard models (Bhattacharyya and Lafontaine, 1995; Kim and Wang, 1998; and Romano, 1994). We will also show that a linear revenue-sharing contract can achieve the second-best outcome if it is possible to write a complete state-contingent contract (see Proposition 1).

We suppose that the initial contract agreed upon by the two parties is efficient ex ante. This is plausible so long as we assume that both parties have unlimited wealth and that information is symmetric. In order to reach an agreement on such a contract, a lump-sum transfer payment between the two parties may be needed. The amount of such a payment may depend on each party's bargaining power. Since it has no incentive effect, we will ignore it throughout the paper.

The ex ante contract may be renegotiated ex post in period 2, after the investments are sunk and the state of nature is realized but before action a is taken. Since a is ex post verifiable, the two parties may want to take advantage of the opportunity to ensure that the ex post action is efficient if the initial contract may fail to achieve that. Even if it is possible to sign a renegotiation-proof ex ante contract, the possibility of ex post renegotiation may have effects on the design of the initial contract. We assume

asset's return at no extra cost or to manipulate accounting costs and receive a fraction of the extra costs as a private benefit. In demonstrating the possibility for joint ownership to be optimal, Hart (1995, pp.68-69) provides an example in which there are two parties making investments on a physical asset and the controlling party can grab all the revenue ex post without incurring any cost. He points out that when both parties' investments are important, joint ownership can be optimal. These examples are special cases of our model.

[8] We restrict ourselves to budget-balancing contracts for the well-known reason that commiting to throwing money away is not credible, because it is ex post inefficient.

that the renegotiation game always achieves an ex post efficient outcome, i.e., $a = a^*$. The distribution of payoffs among the two parties depends on which bargaining solution concepts one uses.

There are two broad types of bargaining solutions one can use for the ex post renegotiation game. One is the cooperative Nash bargaining solution. The other is derived from noncooperative bargaining games, particularly the Rubinstein-style alternating-offer games. Within the Nash solution, the choice of disagreement points is also important. One choice of the disagreement point is the no-negotiation outcome as used in most of the incomplete contract literature; the other is the so-called rational threats. We will show that different bargaining solution concepts may lead to different results.

Furthermore, we assume that there is a small cost, ε, to each party if they engage in an ex post renegotiation. It is so small that no renegotiation will be deterred by such a cost. It is nevertheless a cost so that if a non-renegotiation-proof contract achieves an outcome (apart from the small renegotiation cost) that is also achievable by a renegotiation-proof contact, the former is strictly dominated by the latter. The role of such a small renegotiation cost will become clear later in the paper. It is mostly a technical assumption that ensures the uniqueness of equilibrium.

The timing of events is illustrated in Figure 1.

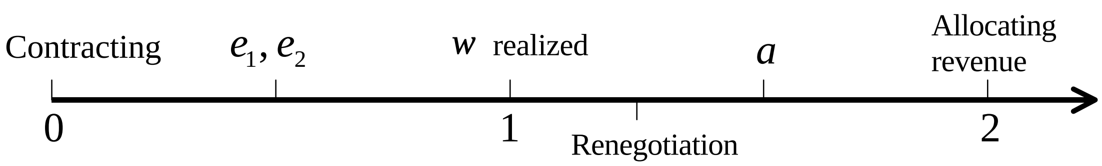

Figure 1. The Timing of Events

3. Preliminary Analysis

In this section, we first consider, as a benchmark, a state-contingent second-best revenue-sharing contract. In 3.2, we show that in the case of joint control, the only renegotiation-proof revenue-sharing contract is the 50:50 sharing rule. In 3.3, we prove a technical result regarding the optimal revenue-sharing contract among a class of renegotiation-proof contracts that are not second-best. This result will be used to prove all the main results of the paper. Finally, in 3.4, we describe an example that will be used throughout the paper to provide intuition for the analysis and to derive more specific results.

3.1. The Second Best

If a complete, state-contingent contract can be signed at the beginning of period 1, then the allocation of control rights becomes irrelevant. We assume that the initial contract is always chosen to maximize the total surplus. (Note again that a lump sum transfer payment may be needed to reflect the ex ante bargaining power of the two parties.) The second-best outcome can be achieved through a contract that specifies an optimal revenue-sharing rule and the second-period action contingent on the state of nature such that $a = a^*[\omega;\ h(e_1^*; e_2^*)]$; where e_1^* and e_2^* are the second-best investment levels to be defined below, which are assumed to be unique. The problem of maximizing the total surplus can be written as

$$V = \max_{s_i \in S;\, e_1, e_2 \in E} \ R(e_1; e_2) - c_1(e_1) - c_2(e_2) \tag{1}$$

$$\text{s.t.} \quad h_1' \int s_1(x) f_h[x; h(e_1; e_2)]dx = c_1'(e_1); \tag{1a}$$

$$h_2' \int s_2(x) f_h[x; h(e_1; e_2)]dx = c_2'(e_2); \tag{1b}$$

$$h_1'' \int s_1(x) f_h[x; h(e_1; e_2)]dx + (h_1')^2 \int s_1(x) f_{hh}[x; h(e_1; e_2)]dx < c_1''(e_1); \tag{1c}$$

$$h_2'' \int s_2(x) f_h[x; h(e_1; e_2)]dx + (h_2')^2 \int s_2(x) f_{hh}[x; h(e_1; e_2)]dx < c_2''(e_2); \tag{1d}$$

$$s_1(x) + s_2(x) = x; \ \text{for all } x \in \mathbb{R}_+; \tag{1e}$$

where (1a) and (1b) are the first-order conditions of the individuals' incentive problems, and (1c) and (2d) are the corresponding second-order conditions.

Proposition 1. If an ex ante state-contingent contract is possible, then with Assumptions 1–3, there exists a linear revenue-sharing rule

$$s_i^*(x) = \alpha_i^* x;$$

where $\alpha_i^* = \frac{c_i'(e_i^*)}{R_i'(e_1^*; e_2^*)}$; which induces the second-best investment efforts $e_i^* > 0$; determined by

$$\max_{e_1, e_2 \in E} R(e_1; e_2) - c_1(e_1) - c_2(e_2)$$
$$\text{s.t.} \quad R_1'(e_1; e_2) = c_1'(e_1) + \frac{h_1'(e_1; e_2)}{h_2'(e_1; e_2)} c_2'(e_2). \tag{2}$$

In addition, we have $0 < \alpha_i^* < 1$; and the first-best outcome cannot be achieved.

Bhattacharyya and Lafontaine (1995) are the first to provide a similar result to Proposition 1 and its proof for a special output process with the distribution function of the form $F(x; h) = F(x - h)$: Kim and Wang (1998) provide a general result without a proof. We here provide a rigorous proof under weaker conditions for the general result in Kim and Wang (1998). Romano (1994) also provides an optimal linear contract for a special double moral hazard model.

Without loss of generality, we assume throughout the paper that $\alpha_1^* < 1/2$; which can be interpreted as M_2's investment incentive being relatively more important than M_1's.

If a must be decided ex post in period 2, then there is no guarantee that the second-best outcome can be achieved. Particularly, the second-best ex ante revenue-sharing contract may not be renegotiation-proof ex post after the investments are sunk.

3.2. Joint Control

In the case of joint control, a is decided ex post jointly by both parties, and both cooperative (Nash) bargaining solution and a Rubinstein-style alternating offer bargaining game yield the same outcome. For the Nash bargaining solution, the disagreement point is clearly $(0; 0)$ for each party. Therefore, the bargaining outcome is $a = a^*$ and two parties share equally the total revenue regardless of the initial revenue sharing agreement. Such a split-the-gains solution can also be derived from an alternating-bargaining game in which there is a common discount factor and it is sufficiently close to one.

The renegotiation-proof revenue-sharing contract is thus the 50:50 sharing rule. If joint control is chosen in period 1, then, because of the small renegotiation cost, a 50:50 revenue-sharing contract will be signed in period 1.

3.3. A Lemma

In this subsection, we prove a lemma which will be used in all the proofs of the propositions in the rest of the paper.

Lemma 1. For $i \in \{1; 2\}$ and any constant $k \in (\alpha_i^*; 1]$; among all renegotiation-proof contracts with a revenue sharing rule $\alpha_i \geq k$; the contract in which $\alpha_i = k$ is the most efficient.

The lemma is intuitive. It simply says that the further the (renegotiation-proof) revenue-sharing rule deviates from the second-best one, the less efficient it becomes.

3.4. An Example

Throughout the paper, we will use an example with specific functional forms both to provide intuition for the analysis and to derive more specific results.

Let the composite investment be

$$h(e_1; e_2) = \mu_1 e_1 + \mu_2 e_2;$$

where μ_1 and μ_2 are two arbitrary positive constants. $h(e_1; e_2)$ satisfies Assumption 1. In this example, let the investment space be $E = R_+$: Let the output process be

$$X^*(\omega; h) = \tilde{A}h;$$

where $\tilde{A} \equiv A(\omega)$ is a random variable, $A(\omega) > 0$ and $E(\tilde{A}) = 1$: Thus, we have

$$R(e_1; e_2) = \mu_1 e_1 + \mu_2 e_2;$$

which satisfies Assumption 3. In other words, the expected total revenue is equal to a weighted average of investments.

We assume a linear functional form for the composite investment (effort) and the (expected) revenue function for easy calculation. There is one caveat; i.e., the precondition for the above setup is that the two parties both participate in the project in period 1. Thus when $e_1 = 0$; it does not mean that M_1 does not participate.

Let the cost of investment be

$$c_i(e_i) = \frac{1}{2}e_i^2; \qquad \text{for } i = 1; 2;$$

which satisfies Assumption 2. Then, we can easily compute the second-best investments, defined by problem (2),

$$e_1^* = \frac{\mu_1^3}{\mu_1^2 + \mu_2^2}; \qquad e_2^* = \frac{\mu_2^3}{\mu_1^2 + \mu_2^2};$$

and the second-best revenue-sharing contract, defined by Proposition 1,

$$\alpha_1^* = \frac{e_1^*}{\mu_1} = \frac{\mu_1^2}{\mu_1^2 + \mu_2^2}; \quad \alpha_2^* = \frac{e_2^*}{\mu_2} = \frac{\mu_2^2}{\mu_1^2 + \mu_2^2}:$$

Since we have assumed $\alpha_2^* > 1/2 > \alpha_1^*$; we also assume $\mu_2 > \mu_1$; which means that M_2 is more productive than M_1 or her investment is more important.

4. Optimal Contracts I: Nash Bargaining

We employ subgame perfect equilibrium as the equilibrium concept in analyzing the two-period game. Using the backward induction principle, we first analyze the ex post renegotiation game and then derive the ex ante optimal contract.

In period 2 when e_1 and e_2 are sunk and a contract on revenue sharing and control rights allocation is in place, the two parties can renegotiate on the share of revenue and bargain over a. Suppose that one of the parties, say M_i, is allocated with the control rights in the ex ante contract and that her contractual share of public revenue is α_i i.e., $\alpha_1 X$ for M_1 and $(1 - \alpha_1)X$ for M_2. If renegotiation in period 2 is not allowed, then M_i chooses a to maximize her own private payoff. In this section, we consider Nash bargaining for the renegotiation game and, because of transferable utilities, assume that the two parties split the gains from the renegotiation (see Myerson, 1991, p. 385). But there is a need to identify the disagreement point, i.e., the status quo. In Section 5, we use a noncooperative bargaining game to derive the bargaining solution. We will see that when the control rights over a are given to a single party, bargaining solution concepts matter.

4.1. No-Negotiation Outcome as the Disagreement Point

It is quite common in the literature to identify the no-renegotiation outcome as the disagreement point in Nash bargaining (see, e.g., Grossman and Hart, 1986). In this case, renegotiation will take place only if either a single party is assigned the control rights over a, and she would choose $a \neq a^*$ in the absence of renegotiation, or if joint control is stipulated in the initial contract and hence two parties must jointly decide on a. For brevity, we will also speak of the choice of (or control rights over) q instead of the action a.

In the case of single party control, the disagreement point is determined by the initial sharing contract as well as the q that the controlling party would choose unilaterally in the absence of renegotiation. Given ex ante investments e_1 and e_2 and given a revenue sharing rule α_i, the controlling party can either choose $q = 1$ to obtain the maximum public surplus $X^*(\omega, h)$ and get $\alpha_i X^*(\omega, h)$, or choose a q to maximize her own private surplus:

$$\max_{0 \le q \le 1} \alpha_i q X^*(\omega, h) + b_i(1 - q)X^*(\omega, h) \equiv [b_i + (\alpha_i - b_i)q]\, X^*(\omega, h).$$

If renegotiation is not allowed, M_i would choose $q = 1$ if $\alpha_i \ge b_i$ or $q = 0$ if $\alpha_i < b_i$.[9]

[9] Throughout the paper, we implicitly assume that the controlling party chooses $q = 1$ instead of

The following proposition characterizes the equilibrium (and optimal) ex ante contract for the case where the disagreement point for the Nash bargaining solution is determined by the payoff structure in the absence of renegotiation.

<u>Proposition 2.</u> Let (α_1^*, α_2^*) with $\alpha_1^* < \frac{1}{2}$ be the second-best sharing rule. Under Nash bargaining with the no-negotiation outcome as the disagreement point:

(1) If $\alpha_i^* \geq b_i$ for either $i = 1$ or 2 but not both, then an ex ante contract that specifies $\alpha_i = \alpha_i^*$ and gives M_i the control rights over a is the unique optimal contract and implements the second-best outcome.

(2) If $\alpha_i^* \geq b_i$ for both $i = 1$ and 2, then an ex ante contract that specifies $\alpha_i = \alpha_i^*$ and gives any one of the parties the control rights implements the second-best outcome.

(3) If $\alpha_i^* < b_i$ for both $i = 1$ and 2, then the second-best outcome cannot be achieved. Depending on the parameters, either single party control or joint control can be optimal. In the case of control by M_i, the optimal share of revenue for M_i is $\alpha_i = b_i$. Furthermore, if $b_1 < 1/2$, then M_1 control is more efficient than joint control; and if $b_1 > 1/2$, joint control is more efficient than M_1 control.

In either case, ex ante optimal contracts are renegotiation-proof.

$q = 0$ when she is indifferent between the two choices (for example, when $\alpha_i = b_i$ in this case); and we do not explicitly distinguish among outcomes that differ only when $\alpha_i = b_i$ (in Subsection 4.1), or $\alpha_i = (1 + b_i)/2$ (in Subsection 4.2), or $\alpha_i = \max\{b_i, 1/2\}$ (in Section 5).

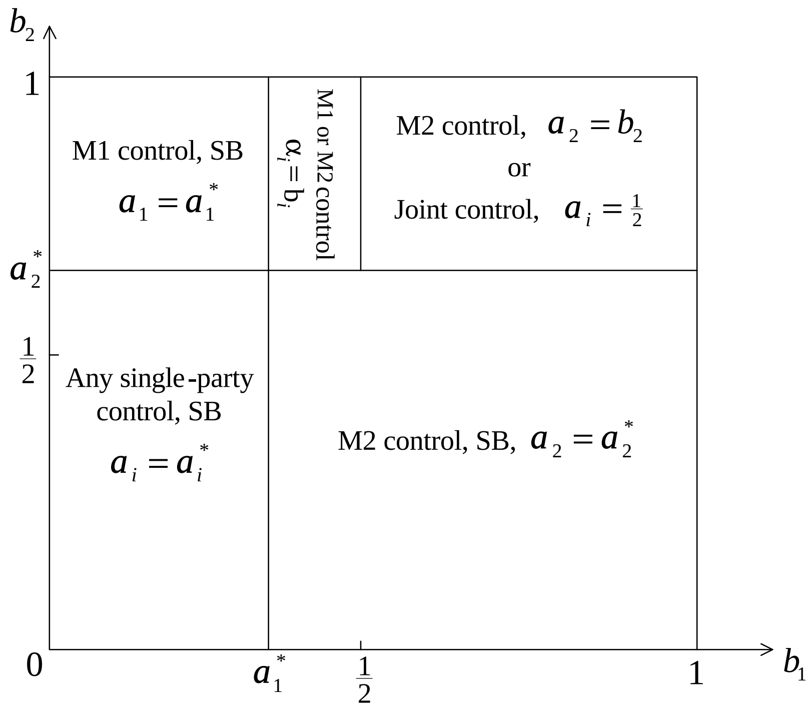

Figure 2. Illustration of Proposition 2

Part (1) of Proposition 2, which is illustrated in Figure 2, where SB stands for "second best", says that control rights should be given to the party for whom the private benefit acquired for each dollar of the total revenue lost as a result of a sub-optimal action is no greater than her second-best contractual share of the revenue. The condition $\alpha_i^* \geq b_i$ ensures that the second-best ex ante revenue-sharing agreement $\alpha_i = \alpha_i^*$ will not be renegotiated because the controlling party M_i would voluntarily choose the ex post efficient action if renegotiation is not allowed. Part (2) simply says that if this condition holds for both parties such as in the case where $b_i = 0$ for $i = 1$ and 2; then any party can have the control rights and the second-best revenue sharing rule $\alpha_i = \alpha_i^*$ is self-enforcing.

Part (3) of the proposition says that if the above condition fails to hold for either of the two parties (i.e., it is relatively easy for any of the two parties to acquire private benefits from having the control rights), then the second-best revenue-sharing rule cannot be implemented in the second period. This is because, in the absence of renegotiation, the controlling party would have incentives to choose an inefficient action since the private benefit acquired from such an action more than offsets her loss in contractual income as stipulated by a second-best sharing rule. A special case is when $b_i = 1$ for $i = 1$ and 2 (i.e., the controlling party can grab all the revenue at no cost). In this case, M_1 control is never optimal. It is efficient either for M_2 to have the control rights if her investment incentive is very important or for the control right to be held jointly if, for example, α_i^* is close to $1/2$ (i.e., when both parties' investment incentives are almost equally important).[10]

Shares of income and control rights are bundled when either M_2 has the control rights and shares the majority of revenue or when control rights are jointly held and revenue is shared equally between the two parties. On the other hand, unbundling of income and control occurs when M_1 has the control rights and shares a minority of the revenue. Given that $\alpha_2^* > 1/2 > \alpha_1^*$, bundling of income and control is more likely to occur than unbundling of the two. This can be seen more clearly from Figure 2, in which the area for M_1 control (i.e., unbundling) to be uniquely optimal is much smaller than the area in which bundling of income and control is uniquely optimal. We will see later that unbundling can be optimal only under Nash bargaining with the no-negotiation outcome as the disagreement point.

In case (3) of Proposition 2, we cannot determine precisely when a certain way of allocating control rights is optimal. In the following, we use the same example from 3.4 to derive more specific results for this case.

Given that $b_1 > \alpha_1^*$, under M_1 control, by Lemma 1, the optimal sharing rule is $\alpha_1 = b_1$ (and $\alpha_2 = 1 - b_1$), which is renegotiation-proof. Then, problem (3) becomes

$$V_1 \equiv \max_{e_1, e_2 \in E} R(e_1, e_2) - c_1(e_1) - c_2(e_2)$$

$$\text{s.t.} \quad R_1'(e_1, e_2) = c_1'(e_1) + \frac{h_1'(e_1, e_2)}{h_2'(e_1, e_2)} c_2'(e_2),$$

$$(1 - b_1) R_2'(e_1, e_2) = c_2'(e_2),$$

which gives $e_1 = b_1 \theta_1$, $e_2 = (1 - b_1)\theta_2$, and the total expected surplus

$$V_1 \equiv \left[\left(b_1 - \frac{1}{2}\right)\alpha_1^* + \frac{1 - b_1^2}{2}\right](\theta_1^2 + \theta_2^2).$$

Given that $b_2 > \alpha_2^*$, then under M_2 control, the optimal sharing rule is $\alpha_2 = b_2$. Similarly, such a contract yields $e_1 = (1 - b_2)\theta_1$, $e_2 = b_2 \theta_2$, and the total expected surplus

$$V_2 \equiv \left[\left(b_2 - \frac{1}{2}\right)\alpha_2^* + \frac{1 - b_2^2}{2}\right]\left(\theta_1^2 + \theta_2^2\right).$$

[10] Hart (1995, p.79) observes that the hold-up problem remains even when revenue-sharing is possible by noting that in the case that is equivalent to joint control in our model, either party can trigger the no-revenue outcome with the intention to renegotiate the initial revenue-sharing contract. With his implicit assumption that $b_i = 0$, our result shows that the initial contract will not be renegotiated so long as control rights are allocated to a single party regardless of which party it is. This result, however, depends on the particular bargaining concept we have used. Control rights should be given to M_2 in this case if rational threats are used as the disagreement point (see Subsection 4.2 below).

Under joint control, the renegotiation-proof sharing rule is $\alpha_i = \frac{1}{2}$. Again, we can similarly find $e_1 = \frac{\theta_1}{2}$, $e_2 = \frac{\theta_2}{2}$, and a total expected surplus

$$V_J \equiv \frac{3}{8}\left(\theta_1^2 + \theta_2^2\right).$$

In the region defined by $b_2 > \alpha_2^* \geq \frac{1}{2}$ and $b_1 < \frac{1}{2}$, we compare joint control with M_2 control. If joint control is more efficient, we have $V_J > V_2$, which is equivalent to

$$\frac{3}{8} > \left(b_2 - \frac{1}{2}\right)\alpha_2^* + \frac{1 - b_2^2}{2},$$

which, in turn, is equivalent to

$$\left(\frac{1}{2} - \alpha_2^*\right)^2 < (\alpha_2^* - b_2)^2 \qquad \text{or} \qquad \alpha_2^* - \frac{1}{2} < b_2 - \alpha_2^*.$$

That is, if $b_2 > 2\alpha_2^* - \frac{1}{2}$, we have $V_J > V_2$; otherwise $V_J \leq V_2$.

In the region defined by $\alpha_1^* < b_1 < \frac{1}{2}$ and $\alpha_2^* < b_2$, we compare M_1 control with M_2 control. The inequality $V_1 > V_2$ is equivalent to

$$\left(b_1 - \frac{1}{2}\right)\alpha_1^* + \frac{1 - b_1^2}{2} > \left(b_2 - \frac{1}{2}\right)\alpha_2^* + \frac{1 - b_2^2}{2},$$

which, in turn, is equivalent to

$$\left(\alpha_1^* - \frac{1}{2}\right)^2 - (b_1 - \alpha_1^*)^2 > \left(\alpha_2^* - \frac{1}{2}\right)^2 - (b_2 - \alpha_2^*)^2.$$

Since $\left(\alpha_1^* - \frac{1}{2}\right)^2 = \left(\alpha_2^* - \frac{1}{2}\right)^2$, we have

$$V_1 > V_2 \quad \Leftrightarrow \quad b_1 - \alpha_1^* < b_2 - \alpha_2^*.$$

Thus, if and only if $b_2 > b_1 + \alpha_2^* - \alpha_1^*$, M_1 control is better than M_2 control. The linear line $b_2 = b_1 + \alpha_2^* - \alpha_1^*$ of (b_1, b_2) passes through points (α_1^*, α_2^*) and $\left(\frac{1}{2}, 2\alpha_2^* - \frac{1}{2}\right)$.

Therefore, the efficient control rights allocation for all combinations (b_1, b_2) in the region $[0, 1] \times [0, 1]$ is well defined. The optimal revenue-sharing and control rights allocation for the example is illustrated in Figure 3.

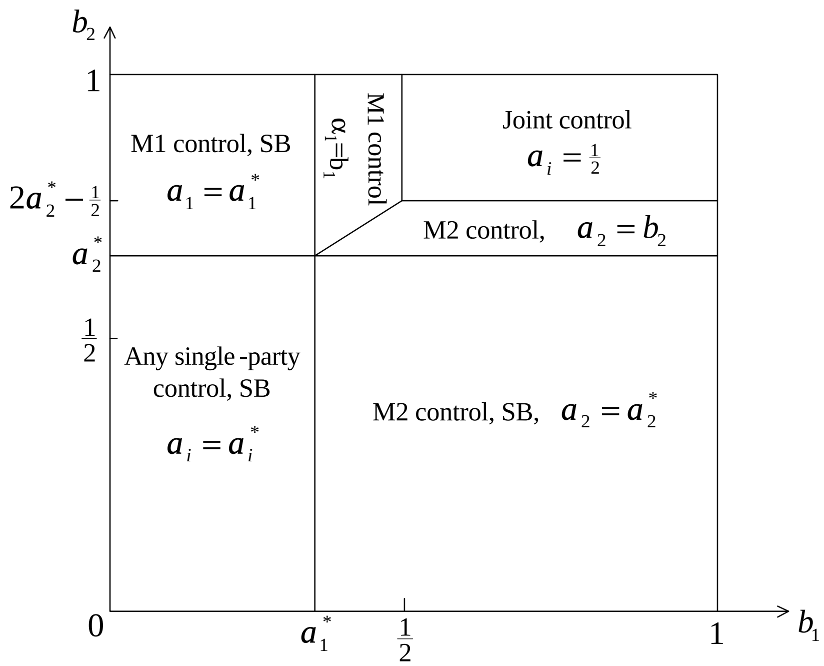

Figure 3. Optimal Contract in the Example for Proposition 2

The indeterminacy of the optimal allocation of control rights in case (3) of Proposition 2 appears to be a weakness with the use of the no-negotiation outcome as the disagreement point. Consider a rather extreme example in which $b_1 = b_2 = 0$; and $\alpha_2^* = 0.99$ and $\alpha_1^* = 0.01$ (for example, when M_1's investment is almost negligible). It does not seem to be natural for M_1 to be given the control rights in this example. Having almost nothing to lose, M_1 might ex post demand a larger share of the revenue by threatening to choose a very inefficient action. Given the fact that M_1 does not have much to lose while M_2 would lose everything if the threat is carried out, M_2 may indeed yield to M_1's demand. This discussion motivates us to consider using a different disagreement point in the Nash bargaining solution.

4.2. Rational Threats as the Disagreement Point

In the Nash bargaining solution, the payoff from renegotiation for one party increases as the disagreement payoff for the other party decreases. Hence, given the prospect of reaching a cooperative agreement, each party may try to create a more favorable disagreement point by acting more antagonistically in the bargaining process. For instance, the controlling party, in the renegotiation process, can threaten to take $q = 0$ if doing so can hurt the other party more than it hurts herself. By making such a threat, the controlling party may be able to extract a larger share of the revenue even if in the end she will not carry out the threat. This is the notion of rational threats (see Nash, 1953, and Myerson, 1991). Rational threats as the disagreement point differ from the no-negotiation outcome used above. In the latter case, when the controlling party has incentives to voluntarily choose $q = 1$ in the absence of renegotiation, then no renegotiation will take place. When rational threats are used to determine the disagreement point, the controlling party may threaten to take $q < 1$ as long as it hurts the other party more than it hurts herself if the threat is actually carried out. This may be so even if the controlling party would choose $q = 1$ if renegotiation were not allowed.

From the noncooperative game-theoretical point of view, the use of rational threats as the status quo in a bargaining game requires that the player who makes a threat be able to commit to carry out the threat if her offer (or demand) is rejected (Nash, 1953). If such a commitment cannot be made credible, then the use of "rational" threats becomes questionable. However, rational threats as the status quo is no more questionable than the no-negotiation outcome, because when taken as the status quo in a bargaining game, no-negotiation as a threat itself needs to be credible as well.

Rational threats are derived as follows. Suppose M_i has the control rights. Given a threat q_i and an initial contract (α_1, α_2), then the disagreement payoff for M_i is $[b_i + (\alpha_i - b_i)q_i]X^*(\omega; h)$. The party, M_j, who has no control rights has no instrument for threats and her disagreement payoff is $q_i(1 - \alpha_i)X^*(\omega; h)$. Therefore, the payoff that the controlling party M_i receives from the renegotiation game with such a disagreement point is

$$
\begin{aligned}
&[b_i + (\alpha_i - b_i)q_i]X^*(\omega; h) + \frac{1}{2}[1 - b_i - (\alpha_i - b_i)q_i - q_i(1 - \alpha_i)]X^*(\omega; h) \\
= &\left[\frac{1 + b_i}{2} + \left(\alpha_i - \frac{1 + b_i}{2}\right) q_i\right] X^*(\omega; h).
\end{aligned}
$$

M_i chooses a threat q_i to maximize this payoff. Thus, if $\alpha_i \geq \frac{1+b_i}{2}$, then the threat is $q_i = 1$ and M_i's payoff from a renegotiation is $\alpha_i X^*(\omega; h)$. In other words, if the controlling party M_i is given by the initial contract a share of revenue $\alpha_i \geq \frac{1+b_i}{2}$, then she will choose $q = 1$ voluntarily without further renegotiation. Otherwise, she will initiate a renegotiation and choose a threat $q_i = 0$ and correspondingly receive a payoff $\frac{1+b_i}{2}X^*(\omega; h)$ from the renegotiation. Now we are ready to establish the following proposition.

Proposition 3. Let (α_1^*, α_2^*) with $\alpha_1^* < \frac{1}{2}$ be the second-best sharing rule. Under Nash bargaining with rational threats as the disagreement point:

(1) If $\alpha_2^* \geq \frac{1+b_2}{2}$, then an ex ante contract that specifies $\alpha_2 = \alpha_2^*$ and gives M_2 control rights over a is the unique optimal contract and implements the second-best outcome.

(2) If $\alpha_2^* < \frac{1+b_2}{2}$, then the second-best outcome cannot be achieved. Depending on the parameters, either M_2 control or joint control can be optimal. M_1 control is never optimal unless $b_1 = 0$, in which case, $\alpha_1 = 1/2$. Furthermore, if M_2 has the control rights, then $\alpha_2 = \frac{1+b_2}{2}$.

In either case, ex ante optimal contracts are renegotiation-proof.

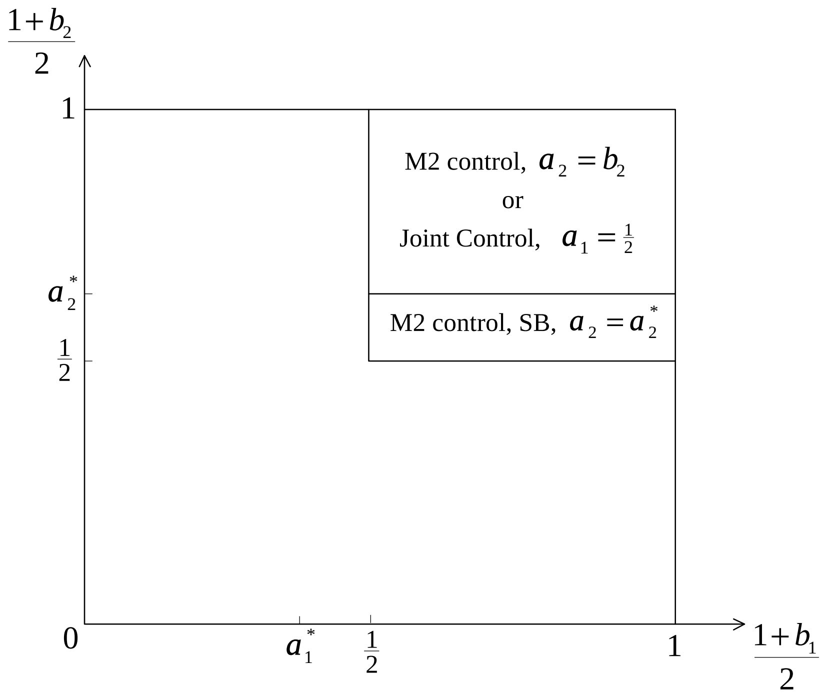

Figure 4. Illustration of Proposition 3

Proposition 3, which is illustrated in Figure 4, shows that under Nash bargaining with rational threats as the disagreement point, revenue sharing and control rights are always bundled together; in other words, either the majority shareholder, who is also the party whose investment incentive is more important, should have the control rights or the control rights should be held jointly by the two parties, each of whom has 50% of the income shares of the joint venture. M_2 control would make the second-best ex ante revenue-sharing agreement renegotiation-proof ex post (i.e., self-enforcing) when her ability to acquire private benefits is not too strong relative to her second-best contractual share of the revenue. For instance, when $b_1 = b_2 = 0$, M_2 should have the control rights while M_1 should not. In other words, in the absence of private bene…ts of control, the party whose investment incentive is more important should have the control rights. This is in contrast with the result in Proposition 2 when the no-negotiation outcome is taken as the disagreement point. The intuition is that, under Nash bargaining with rational threats as the disagreement point, when M_1 has control and has a share of revenue less than 1/2, she would demand ex post a larger share (say, 1/2) by threatening to choose the most ine¢cient action. In another special case where $b_1 = b_2 = 1$ (i.e., the controlling party can grab all the revenue at no cost), the optimal contract is the same as in the case when the no-negotiation outcome is used as the disagreement point.

We now use the same example to derive more speci…c results for Part (2) of Proposition 3. Let us de…ne $d_1 \equiv \frac{1+b_1}{2}$ and $d_2 \equiv \frac{1+b_2}{2}$: When we replace b_i by d_i; Proposition 3 becomes Proposition 2 with the exception that α_1^* cannot be greater than d_1 by assumption. Thus, the results for the parametric example of Proposition 2 can be carried over to Proposition 3 directly. Formally, we have: if $d_2 > 2\alpha_2^* - \frac{1}{2}$; then $V_J > V_2$; otherwise, $V_J \leq V_2$: Figure 5 illustrates the optimal contract for the example.

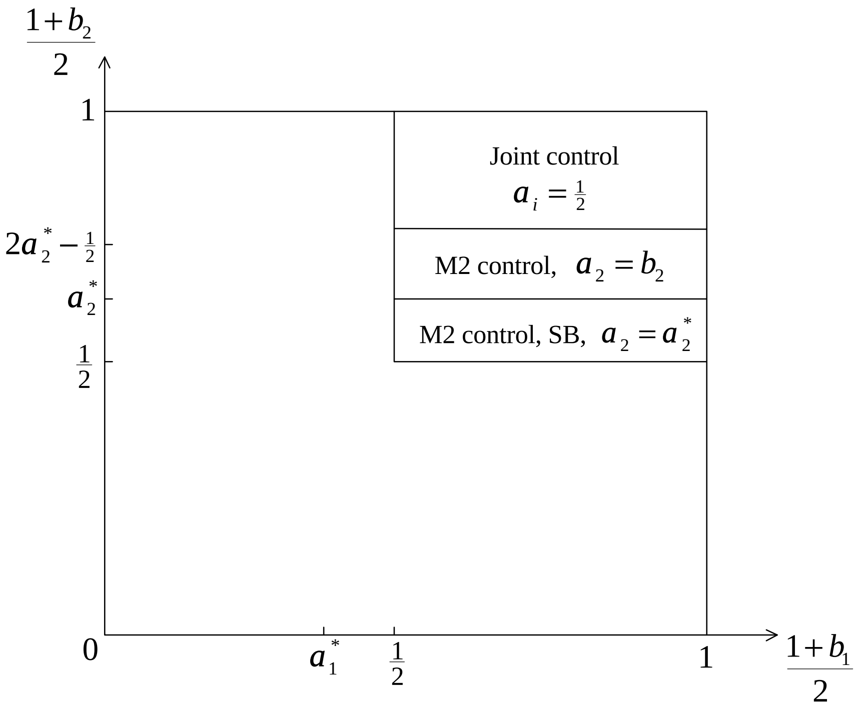

Figure 5. Optimal Contract in the Example for Proposition 3

A conclusion we can draw from this section regarding the Nash bargaining solution is that the use of di¤erent disagreement points can lead to di¤erent results. For instance, when rational threats are used, M_1 control is never optimal while it can be optimal if the no-negotiation outcome is taken to be the disagr em nt oi t. he as ba gai in solution itself does not provide any guidance on what kind of disagreement point one should use. This appears to be a weakness of the Nash bargaining solution.

From the noncooperative perspective, as is remarked above, for the no-negotiation outcome or rational threats to affect the bargaining outcome, the threats must be credible. We now turn to the analysis of optimal contracting using a noncooperative bargaining game for ex post renegotiation.

5. Optimal Contracts II: Alternating-Offer Bargaining

In this section, we use Rubinstein's alternating-offer bargaining game to model the ex post contract renegotiation.

Suppose M_i has the control rights over a. In the second period, M_i can choose not to start a renegotiation with M_j, where $j \neq i$, but to choose a unilaterally and then divide the public revenue with M_j according to the ex ante revenue-sharing rule. If M_i decides to initiate a renegotiation, then we suppose it is over a new contract that specifies the optimal action a^* (i.e., $q = 1$) and a sharing rule α_i (and hence $\alpha_j = 1 - \alpha_i$). The bargaining is thus equivalent to a division of a fixed-sized pie given the sunk investments and the assumption of risk neutrality. The rule of the renegotiation game is assumed as follows. M_i begins by proposing to M_j a new contract. M_j then accepts or refuses this proposal. If M_j accepts the offer, the bargaining game ends with an agreement as proposed by M_i. If she rejects the offer, then in the second round, M_j makes a counter-offer and M_i responds by either (1) accepting, (2) rejecting and going to the next round or (3) rejecting and unilaterally terminating the bargaining and opting out to exercise the control rights. In case (3), M_i receives $O \equiv Max_{\{q\}}[b_i + (\alpha_i - b_i)q]X^*(\omega; h)$ while M_j receives $(1 - \alpha_i)qX^*(\omega; h)$. The above procedure continues with the two parties alternating in being the proposer until either an agreement is reached or M_i opts out. There is a common discount factor $\delta < 1$. We further assume that the party who has no control rights cannot terminate the bargaining process.

The above structure of the bargaining game is similar to the structure of the alternating-offer noncooperative bargaining game with one party having the outside option (Binmore, Shaked and Sutton, 1989; Osborne and Rubinstein, 1990, pp.55-58). The difference between our bargaining game and that of Binmore et al. is that in their game when the party having an outside option exercises that option, the other party gets 0 payoff while in our game, if the controlling party chooses to exercise the control rights (i.e., unilaterally determines q), the other party receives her share of the public revenue according to the initial contract (i.e., $(1-\alpha_i)qX^*(\omega;h)$). However, it can be shown that with an infinite horizon and a discount factor approaching to 1; the "outside options principle" proposed by Binmore et al. equally applies to our bargaining game as defined above. (The proof is similar to that in Binmore et al. and is thus omitted.) Specifically, we have the following bargaining solution:

Noncooperative Bargaining Solution: Given an initial contract that assigns M_i the control rights and a share of revenue α_i, then from the renegotiation: (1) M_i receives $O \equiv \operatorname{Max}_{\{q\}}[b_i + (\alpha_i - b_i)q]X^*(\omega;h)$ if $O \geq \frac{1}{2}X^*(\omega;h)$; (2) M_i receives $\frac{1}{2}X^*(\omega;h)$ if $O < \frac{1}{2}X^*(\omega;h)$.

Obviously, $O = \alpha_i X^*(\omega;h)$ if $\alpha_i \geq b_i$; and $O = b_i X^*(\omega;h)$ if $\alpha_i < b_i$. This is because if M_i terminates the renegotiation, she will choose $q = 1$ when $\alpha_i \geq b_i$ or $q = 0$ when $\alpha_i < b_i$. The following lemma is self-evident.

Lemma 2. Suppose M_i has the control rights. If $\alpha_i \geq \max\left\{\frac{1}{2}; b_i\right\}$, then M_i's payoff from renegotiation is $\alpha_i X^*(\omega;h)$. Otherwise, if $\alpha_i < \max\left\{\frac{1}{2}; b_i\right\}$, then M_i's payoff from renegotiation is $\max\{\frac{1}{2}; b_i\}X^*(\omega;h)$.

We are now ready to prove the following proposition.

Proposition 4. Let $(\alpha_1^*;\alpha_2^*)$ with $\alpha_1^* < \frac{1}{2}$ be the second-best sharing rule. Under alternating-offer bargaining as defined above:

(1) If $\alpha_2^* \geq b_2$, then an ex ante contract that specifies $\alpha_2 = \alpha_2^*$ and gives M_2 control rights over a is the unique optimal contract and implements the second-best outcome.

(2) If $\alpha_2^* < b_2$, then the second-best outcome cannot be achieved. Depending on the parameters, either single party control or joint control can be optimal. In the case of control by M_i, the optimal share of revenue for M_i is $\alpha_i = \max\left\{\frac{1}{2}; b_i\right\}$. Furthermore, if $b_1 > 1/2$, then M_1 control is not optimal; if $b_1 \leq 1/2$, then M_1 control yields the same outcome as joint control.

In either case, ex ante optimal contracts are renegotiation-proof.

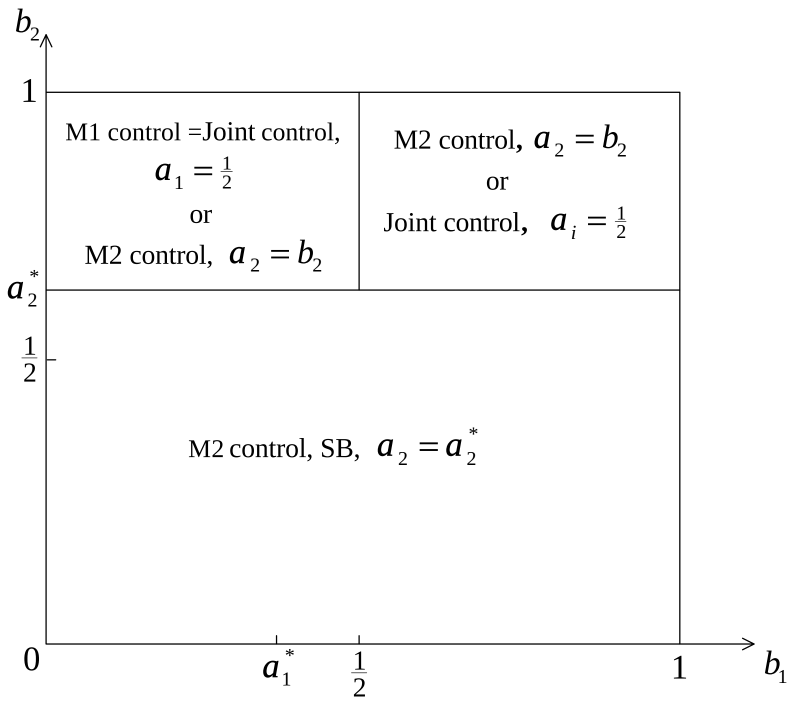

Figure 6. Illustartion of Proposition 4

Similar to the result derived under Nash bargaining with rational threats, Proposition 4, which is illustrated in Figure 6, shows that, under alternating-o¤er bargaining, income and control rights are also always bundled together. This is because, under noncooperative bargaining, the minority shareholder can threaten to choose an ine¢cient action, and by doing so can secure in the renegotiation a share of revenue no less than 50%. The di¤erence between the rational threats solution and the noncooperative bargaining solution is that the minority party (say M_1) receives in the renegotiation a share of $(1 + b_1)=2$ in the former case but a share of $1=2$ in the latter case. In either case, minority control cannot achieve the second best and, given the small renegotiation cost, is dominated by an ex ante contract that bundles the control rights with equity shares by assigning M_1 the control rights with a share of revenue $(1 + b_1)=2$ and $1=2$ (or more practically, 51% voting shares) respectively.

Di¤ering from Proposition 3, M_1 control (with a share of revenue at 50%) may be optimal under noncooperative bargaining if her ability to acquire private bene…ts from control is weak while M_2's ability to do so is not. In practice, the ownership arrangement of M_1 control with $®_1 = 1=2$ can be interpreted as a 51:49 equity joint venture with M_1 having 51% of voting shares. Such an ownership arrangement is equivalent to joint control in terms of e¢ciency. But if joint control entails, say, a decision cost, the 51:49 equity joint venture would be uniquely optimal. This may explain the prevalence of the existence of such an ownership arrangement among joint ventures.

Again, we use the example to get a specific result for part (2) of Proposition 4. Define $t_1 \equiv \max\{\frac{1}{2}; b_1\}$ and $t_2 \equiv \max\{\frac{1}{2}; b_2\}$: When we replace b_i by t_i; Proposition 4 becomes Proposition 2 with the exception that α_1^* cannot be greater than t_1 by assumption. Thus, the results for the parametric example of Proposition 2 can be carried over to Proposition 4 directly. Formally, if $t_2 > 2\alpha_2^* - \frac{1}{2}$; we have $V_J > V_2$; otherwise $V_J < V_2$: Since $\alpha_2^* \geq \frac{1}{2}$; for $b_2 \geq \alpha_2^*$; we have $t_2 = b_2$: Thus, for the region with $b_2 \geq \alpha_2^*$ in Figure 7, if $b_2 > 2\alpha_2^* - \frac{1}{2}$; we have $V_J > V_2$; otherwise $V_J < V_2$: Figure 7, which is divided into four control zones, shows the optimal allocation of revenue and control for the example.

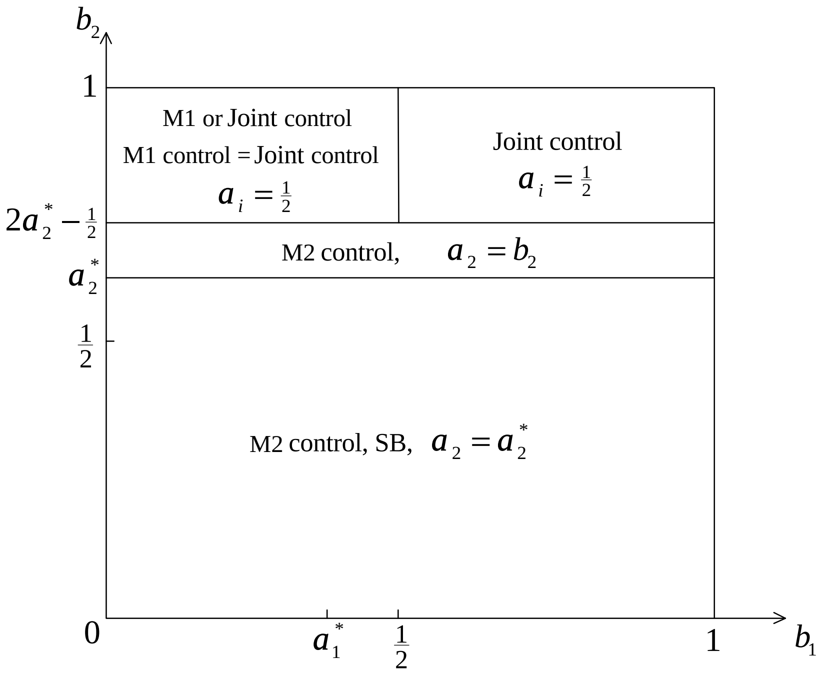

Figure 7. Control Zones in the Example for Proposition 4

6. Conclusion

In this paper, we adopt the incomplete contracts approach of Grossman-Hart-Moore to study how income rights and control rights are optimally linked to each other in the context of joint ventures. For ease of comparison, we summarize in the following table the main results of the paper as derived from the example. Each cell of the table indicates the condition(s) for one of the three ways of allocating control rights to be optimal under one of the three bargaining scenarios as well as the corresponding optimal revenue sharing (in brackets).

Table 1. Summary of Results (assuming $\alpha_1^* < \frac{1}{2}$)

	Nash Bargaining with No-Negotiation Outcome As Disagreement Point	Nash Bargaining with Rational Threats As Disagreement Point	Alternating-Offer Bargaining
M_1 Control	$\alpha_1^* \geq b_1$ [$\alpha_1 = \alpha_1^*$]. $\alpha_1^* - b_1 > \alpha_2^* - b_2$ [$\alpha_1 = b_1$] (for $\alpha_1^* < b_1 \leq \frac{1}{2}$):	Never optimal (unless $b_1 = 0$)	$b_1 \leq \frac{1}{2}$ and $b_2 > 2\alpha_2^* - \frac{1}{2}$; [$\alpha_1 = 1/2$].
M_2 Control	$\alpha_2^* \geq b_2$ [$\alpha_2 = \alpha_2^*$]. $\alpha_2^* - b_2 > \alpha_1^* - b_1$ [$\alpha_2 = b_2$] (for $\alpha_2^* < b_2 \leq 2\alpha_2^* - \frac{1}{2}$):	$\alpha_2^* \geq \frac{1+b_2}{2}$ [$\alpha_2 = \alpha_2^*$]: $2\alpha_2^* - \frac{1}{2} \geq \frac{1+b_2}{2}$ [$\alpha_2 = \frac{1+b_2}{2}$].	$\alpha_2^* \geq b_2$ [$\alpha_2 = \alpha_2^*$]: $\alpha_2^* < b_2 \leq 2\alpha_2^* - \frac{1}{2}$ [$\alpha_2 = b_2$].
Joint Control	$b_1 > \frac{1}{2}$ and $b_2 > 2\alpha_2^* - \frac{1}{2}$ [$\alpha_1 = \alpha_2 = 1/2$].	$2\alpha_2^* - \frac{1}{2} < \frac{1+b_2}{2}$ [$\alpha_1 = \alpha_2 = 1/2$].	$b_2 > 2\alpha_2^* - \frac{1}{2}$ [$\alpha_1 = \alpha_2 = 1/2$].

Several conclusions can be drawn from our analysis. First, GHM's main insight that control rights matter in the organization of economic transactions holds regardless of whether or not income is contractible and regardless of which bargaining solution concept is used. Particularly, control rights should, more often than not, be allocated to the party whose investment incentive is relatively more important (i.e., M_2); noting from the table that the condition for M_2 control is easier to be satisfied.

Second, income and control are usually bundled together; and it is always so under both Nash bargaining with rational threats as the disagreement point and noncooperative alternating-offer bargaining. Unbundling can be optimal only when Nash bargaining solution with the no-negotiation outcome is used (the upper left cell of the table corresponds to this case). Particularly, when the opportunity for acquiring private benefits is limited, a proper allocation of control rights (usually to the investment-wise more important party) makes the second-best revenue-sharing contract self-enforcing and hence provides optimal investment incentives.

Third, joint control may be optimal when there is ample opportunity for the controlling party to acquire private benefits and incentives to invest by both parties are important (i.e., α_2^* is relatively close to 1/2).

Finally, our results can be used to explain different types of ownership structure of equity joint ventures. Specifically, equity shares in a joint venture are often characterized by both the rights to share revenue and the rights of control. The majority shareholding partner of a joint venture is normally the controlling party. A 50:50 ownership structure is usually associated with joint control. A 51:49 joint venture can practically be interpreted as a contractual arrangement in which the two partners share the revenue equally but only one partner has the control rights. The upper right cell of the table corresponds to this case.

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Appendix A

A.1. Proof of Proposition 1

We will first ignore the inequality constraints (1c) and (1d); we will later prove that they are satisfied by the solution of the problem without them. Substituting (1b) into (1a) implies

$$R_1'(e_1, e_2) = c_1'(e_1) + \frac{h_1'(e_1, e_2)}{h_2'(e_1, e_2)} c_2'(e_2).$$

So, the problem is equivalent to

$$\begin{aligned} \max_{s_2 \in S;\, e_1, e_2 \in E} \quad & R(e_1, e_2) - c_1(e_1) - c_2(e_2) \\ \text{s.t.} \quad & R_1'(e_1, e_2) = c_1'(e_1) + \tfrac{h_1'(e_1, e_2)}{h_2'(e_1, e_2)} c_2'(e_2), \\ & h_2'(e_1, e_2) \int s_2(x) f_h[x, h(e_1, e_2)]dx = c_2'(e_2). \end{aligned} \tag{A1}$$

Let (e_1^*, e_2^*) be the solution to the above problem without the second constraint, i.e., the solution to the following problem:

$$\begin{aligned} \max_{e_1, e_2 \in E} \quad & R(e_1, e_2) - c_1(e_1) - c_2(e_2) \\ \text{s.t.} \quad & R_1'(e_1, e_2) = c_1'(e_1) + \tfrac{h_1'(e_1, e_2)}{h_2'(e_1, e_2)} c_2'(e_2). \end{aligned} \tag{A2}$$

Problem (A2) is not related to a contract. Given (e_1^*, e_2^*), we look for a contract $s_2(x)$ that satisfies the second condition of problem (A1). There are many such contracts. In particular, a sharing contract of the form $s_2(x) = \alpha_2 x$ will do. The second constraint of (A1) then becomes

$$\alpha_2 R_2'(e_1^*, e_2^*) = c_2'(e_2^*),$$

implying

$$\alpha_2^* = \frac{c_2'(e_2^*)}{R_2'(e_1^*, e_2^*)}.$$

This means that $(e_1^*, e_2^*, \alpha_2^*)$ is a solution of problem (A1); it is thus the solution of problem (1) without constraints (1c) and (1d). Will the second-order conditions (1c) and (1d) be satisfied by the solution? For $s_2^*(x) = \alpha_2^* x$, by Assumptions 2 and 3, we have

$$h_2'' \int s_2^*(x) f_h[x, h(e_1, e_2)]dx + (h_2')^2 \int s_2^*(x) f_{hh}[x, h(e_1, e_2)]dx = \alpha_2^* R_2''(e_1^*, e_2^*) \leq 0.$$

Condition (1d) is thus satisfied. Symmetrically, condition (1c) is also satisfied. Therefore, $(e_1^*, e_2^*, \alpha_2^*)$ is a solution of problem (1).

Furthermore, substituting (1a) into (1b) yields

$$R_2'(e_1;e_2) = c_2'(e_2) + \frac{h_2'(e_1;e_2)}{h_1'(e_1;e_2)}c_1'(e_1):$$

By Assumptions 1 and 2, we thus have $0 < \alpha_2^* < 1$: Also, since the first-best solution $(e_1^{**};e_2^{**})$ satisfies

$$R_1'(e_1^{**};e_2^{**}) = c_1'(e_1^{**}) = c_2'(e_2^{**});$$

by the constraint of (A2), the solution $(e_1^*;e_2^*)$ of problem (A2) cannot be the first best. Proposition 1 is thus proven.

A.2. Proof of Lemma 1

Without loss of generality, consider a contract that allocates M_2 with a share of revenue $\alpha_2 \geq k > \alpha_2^*$: We need to show that for all renegotiation-proof contracts with $\alpha_2 \geq k > \alpha_2^*$; the sharing rule $\alpha_2 = k$ maximizes the total expected surplus. Because the contract is renegotiation-proof, the sharing rule will be implemented (i.e., self-enforcing) in the second period and $q = 1$ will be chosen, following the proof of Proposition 1, by problem (A1), we only need to show $\alpha_2 = k$ is the solution to the following maximization problem :

$$\begin{aligned} \max_{\alpha_2 \geq k;\, e_1;e_2 \in E} \quad & R(e_1;e_2) - c_1(e_1) - c_2(e_2) \\ \text{s.t.} \quad & R_1'(e_1;e_2) = c_1'(e_1) + \tfrac{h_1'(e_1;e_2)}{h_2'(e_1;e_2)}c_2'(e_2); \\ & \alpha_2 R_2'(e_1;e_2) = c_2'(e_2): \end{aligned} \tag{A3}$$

Now consider the following problem:

$$\begin{aligned} \max_{\alpha_2 \geq k;\, e_1;e_2 \in E} \quad & R(e_1;e_2) - c_1(e_1) - c_2(e_2) \\ \text{s.t.} \quad & R_1'(e_1;e_2) = c_1'(e_1) + \tfrac{h_1'(e_1;e_2)}{h_2'(e_1;e_2)}c_2'(e_2); \\ & \alpha_2 R_2'(e_1;e_2) \leq c_2'(e_2): \end{aligned} \tag{A4}$$

If the solution to this problem $(\hat\alpha_2;\hat e_1;\hat e_2)$ satisfies $\hat\alpha_2 R_2'(\hat e_1;\hat e_2) < c_2'(\hat e_2)$; then it would also be the solution to the problem without the condition $\alpha_2 R_2'(e_1;e_2) \leq c_2'(e_2)$: Then, by (A2), $(\hat e_1;\hat e_2)$ would be the second-best $(e_1^*;e_2^*)$; which implies $k \leq \hat\alpha_2 \leq \frac{c_2'(\hat e_2)}{R_2'(\hat e_1;\hat e_2)} = \frac{c_2'(e_2^*)}{R_2(e_1^*;e_2^*)} = \alpha_2^*$: But this contradicts the assumption that $\alpha_2^* < k$: Thus, the inequality condition in (A4) must be binding, which means that problems (A3) and (A4) are equivalent. The Lagrangian function for (A4) is

$$L = R(e_1;e_2) - c_1(e_1) - c_2(e_2) + \lambda\left[\alpha_2 R_2'(e_1;e_2) - c_2'(e_2)\right]$$

$$+\mu\left[R_1'(e_1,e_2) - c_1'(e_1) - \frac{h_1'(e_1,e_2)}{h_2'(e_1,e_2)}c_2'(e_2)\right];$$

where $\lambda \le 0$. Suppose $\lambda = 0$. Then the above Lagrangian function is equivalent to the one for the maximization problem (A2), implying that the solution to (A4), $(\hat{e}_1, \hat{e}_2, \hat{\alpha}_2)$, is the same as that to (A2) and hence is the second-best solution, i.e., $(\hat{e}_1, \hat{e}_2) = (e_1^*, e_2^*)$. But this leads to a contradiction as we have shown above. Thus, $\lambda < 0$. Therefore, the optimal α_2 for (A4) must be as small as possible, i.e., $\hat{\alpha}_2 = k$.

A.3. Proof of Proposition 2

Consider a contract that gives M_i the control rights and specifies a revenue sharing rule α_i. If $\alpha_i \ge b_i$, her choice is $q = 1$. This is an ex post efficient outcome, and hence there is no mutually beneficial renegotiation. If $\alpha_i < b_i$, then M_i would choose $q = 0$ if there is no renegotiation. This is inefficient, and hence the two parties will resort to renegotiation in order to reach an ex post efficient agreement. (Recall that the fixed renegotiation cost ε is so small that ex post renegotiation cannot be deterred. That is, gains from renegotiation for each party are greater than ε.) The payoff after efficient bargaining for M_i is

$$b_i X^*(\omega, h) + \frac{1}{2}[X^*(\omega, h) - b_i X^*(\omega, h)] = \frac{1+b_i}{2} X^*(\omega, h).$$

In other words, when the initial contract gives the controlling party M_i a share of revenue less than b_i, her effective share after renegotiation is $\frac{1+b_i}{2}$, which is larger than b_i.

(1) Since $\alpha_i^* \ge b_i$, a revenue-sharing rule $\alpha_i = \alpha_i^* \ge b_i$ is renegotiation-proof and implements the second-best outcome. Clearly, all other revenue-sharing rules with $\alpha_i \ge b_i$ are sub-optimal. The sharing rules with $\alpha_i < b_i \le \alpha_i^*$ would entail renegotiation. The effective sharing rule after renegotiation would at best induce the second-best investments (this occurs only when $\frac{1+b_i}{2} = \alpha_i^*$). Given the small renegotiation cost, a sharing rule that is not renegotiation-proof is sub-optimal.

(2) This part is immediately implied by part (1).

(3) Suppose the initial contract allocates M_i with the control rights. Given that $\alpha_i^* < b_i$, all the sharing rules with $\alpha_i \ge b_i$ are, by Lemma 1, Pareto-dominated by the sharing rule $\alpha_i = b_i$. On the other hand, any sharing rule with $\alpha_i < b_i$ leads to renegotiation and the post-renegotiation share for M_i is $\frac{1+b_1}{2}$, which is greater than b_1. Thus, after the small renegotiation cost ε is taken into account, a sharing rule of

$\alpha_i = \frac{1+b_i}{2}$; which is renegotiation-proof, dominates all the sharing rules with $\alpha_i < b_i$. But a contract with $\alpha_i = \frac{1+b_i}{2} > b_i$ is, by Lemma 1, dominated by the contract with $\alpha_i = b_i$. Therefore, if M_i is given the control rights, then the optimal sharing rule is $\alpha_i = b_i$; which is renegotiation-proof. Notice that since $b_i > \alpha_i^*$; the second-best outcome cannot be implemented.

Furthermore, suppose $\alpha_1^* < b_1 < 1/2$. A joint control contract with or without a revenue sharing rule is equivalent to the renegotiation-proof contract that stipulates joint control and an equal share of revenue between two parties (i.e., $\alpha_1 = 1/2$), which, by Lemma 1, is Pareto-dominated by a renegotiation-proof contract that allocates M_1 with the control rights and a share of revenue $\alpha_1 = b_1$. In other words, M_1 control dominates joint control.

Now suppose $b_1 > 1/2$. Any contract that allocates M_1 with the control rights must optimally have the renegotiation-proof sharing rule $\alpha_1 = b_1$. Because $b_1 > 1/2 > \alpha_1^*$; by Lemma 1, such a contract is Pareto-dominated by a joint control contract with $\alpha_1 = 1/2$. Therefore, Proposition 2 is proven.

A.4. Proof of Proposition 3

(1) Given a contract that gives M_2 the control rights and specifies $\alpha_2 = \alpha_2^* \geq \frac{1+b_2}{2}$; M_2 will choose $q = 1$ voluntarily without further renegotiation and hence the second-best outcome is implemented. Any sharing rule with $\alpha_2 \geq \frac{1+b_2}{2}$ but $\alpha_2 \neq \alpha_2^*$ is clearly sub-optimal. A sharing rule with $\alpha_2 < \frac{1+b_2}{2}$ would entail renegotiation and the post-renegotiation share of revenue is $\frac{1+b_2}{2}$. Because of the small renegotiation cost, such a sharing rule is thus dominated by the sharing rule $\alpha_2 = \frac{1+b_2}{2}$; which is optimal if and only if $\alpha_2^* = \frac{1+b_2}{2}$. Next, we consider the case of M_1 control with a sharing rule α_1. Suppose $\alpha_1 < \frac{1+b_1}{2}$. The expected payoff from the renegotiation with the rational threat $q = 0$ is $\frac{1+b_1}{2} X^*(\omega; h)$. This outcome, again, can be implemented by a revenue-sharing contract $\alpha_1 = \frac{1+b_1}{2}$ without incurring the renegotiation cost. Now suppose $\alpha_1 \geq \frac{1+b_1}{2}$; which is greater than α_1^*. By Lemma 1, the optimal share must be $\alpha_1 = \frac{1+b_1}{2}$. Thus, M_1 control cannot implement the second-best outcome and is sub-optimal.

(2) Similar to the case of M_1 control, we can prove that in the case of M_2 control, the optimal sharing rule is $\alpha_2 = \frac{1+b_2}{2}$ when $\alpha_2^* < \frac{1+b_2}{2}$. It is also renegotiation-proof and cannot achieve the second-best outcome. As is shown in (1), in the case of M_1

control, the optimal sharing rule is $\alpha_1 = \frac{1+b_1}{2}$, which is greater than $1/2$ unless $b_1 = 0$. By Lemma 1, noting that $\frac{1+b_1}{2} \geq \frac{1}{2} > \alpha_1^*$, this contract is dominated by a joint control contract with $\alpha_1 = 1/2$, which is renegotiation-proof, unless $b_1 = 0$. Thus M_1 control cannot be optimal unless $b_1 = 0$, in which case, it is equivalent to the joint control.

A.5. Proof of Proposition 4

(1) The contract that specifies $\alpha_2 = \alpha_2^*$ and gives M_2 the control rights over a is renegotiation-proof and clearly implements the second-best outcome. This is because $\alpha_2 = \alpha_2^* \geq \max\left\{\frac{1}{2}, b_2\right\}$, and by Lemma 2, M_2's payoff from renegotiation is $\alpha_2 X^*(\omega; h)$. In the case of M_2 control, any renegotiation-proof sharing rule with $\alpha_2 \neq \alpha_2^*$ is clearly sub-optimal. A sharing rule that is not renegotiation-proof (i.e., when $\alpha_2 < \max\left\{\frac{1}{2}, b_2\right\}$) entails a small renegotiation cost and hence cannot be optimal. In the case of M_1 control, any renegotiation-proof sharing rule must satisfy $\alpha_1 \geq \max\left\{\frac{1}{2}, b_1\right\}$, which is greater than α_1^*, and hence cannot be optimal.

(2) The condition $\alpha_2^* < b_2$ implies $b_2 > \frac{1}{2}$, and hence is equivalent to $\alpha_2^* < \max\left\{\frac{1}{2}, b_2\right\}$. Since $\alpha_1^* < \frac{1}{2}$, we have $\alpha_i^* < \max\left\{\frac{1}{2}, b_i\right\}$ for both $i = 1$ and 2. Now suppose M_i has the control rights. If $\alpha_i \geq \max\left\{\frac{1}{2}, b_i\right\}$, then by Lemma 2, M_i's renegotiation payoff is $\alpha_i X(\omega; h)$. Thus the initial contract is renegotiation-proof. By Lemma 1, the optimal contract under the constraint that $\alpha_i \geq \max\left\{\frac{1}{2}, b_i\right\} > \alpha_i^*$ is $\alpha_i = \max\left\{\frac{1}{2}, b_i\right\}$. On the other hand, if $\alpha_i < \max\left\{\frac{1}{2}, b_i\right\}$, then the contract will be renegotiated, and M_i's payoff from renegotiation is $\max\left\{\frac{1}{2}, b_i\right\} X^*(\omega; h)$. Such an outcome, however, can be implemented by a renegotiation-proof sharing rule $\alpha_i = \max\{\frac{1}{2}, b_i\}$. We have thus proven that in the case of M_i control, the optimal sharing rule is $\alpha_i = \max\{\frac{1}{2}, b_i\}$. Because $\alpha_i^* < \max\left\{\frac{1}{2}, b_i\right\}$, the second-best outcome cannot be achieved. Furthermore, if $b_1 > 1/2$, then in the case of M_1 control, the optimal sharing rule is $\alpha_1 = \max\left\{\frac{1}{2}, b_1\right\} = b_1$. By Lemma 1, this contract is dominated by a joint control contract with $\alpha_1 = 1/2$. Thus M_1 is not optimal in this case. If $b_1 \leq 1/2$, and M_1 has control, then the optimal sharing of revenue is $\alpha_1 = 1/2$. This contract hence yields the same outcome as joint control.

End